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Collection Lags and the Optimal Inflation Tax

A Reconsideration

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The observation that inflation reduces real revenues when there are lags in tax collection has long been a strong argument against seigniorage. However, with the exception of Dixit, who used a general equilibrium model to reject this argument, the optimal taxation literature has not analyzed how collection lags affect desired tax structures. This paper reexamines the issue using an overlapping generations version of Dixit's model. It is shown that depending on the size of the expenditure ratio and the specification of the collection cost function, lags may increase, leave unchanged, or reduce the desired rate of inflation. [JEL E51, E62, H21]

A BASIC ISSUE in both public finance and monetary economics is the desirability of using inflationary finance to generate government revenue. In the monetary literature, the orthodox position is associated with Friedman's (1969) optimum quantity of money rule, which argues that the nominal rate of interest should be zero.[1] The public finance

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[1] Friedman argues that to allocate resources efficiently in a monetary economy, the social marginal benefit of money must be brought in line with its social marginal cost. Since fiat money is (almost) costless to produce, the nominal rate of interest should be set at zero—for example, by contracting the money supply at a rate equal to the real rate of interest. See Woodford (1990) for an extensive discussion of the optimum quantity of money rule.

literature has been dominated by Phelps (1973), who uses a model with money specified in the utility function to show that seigniorage can be part of a second-best tax system. Phelps' starting point is the observation that to implement Friedman's rule, lump-sum transfers must be feasible. If they are not, taxation of all commodities—including consumption and liquidity—may be required to raise government revenue.[2] Later contributions to the public finance literature criticize Phelps' treatment of liquidity as a separate commodity, focusing on fiscal inefficiencies to rationalize the use of seigniorage.[3] With the exception of Kimbrough (1986), who salvages Friedman's rule in a second-best setting, the inefficiencies assumed in this literature (such as positive foreign nominal rates of interest, collection costs, or a large underground economy) introduce distortions of their own for ordinary taxes. Resorting to the monetary financing of deficits helps reduce these inefficiencies and allows seigniorage to coexist with consumption (or income) taxes in optimal tax menus.

With the exception of Dixit (1991), however, the optimal inflation literature has not considered the implications of collection lags for the argument concerning seigniorage in second-best settings. As has long been emphasized by Tanzi (1978), inflationary finance may further weaken the public finances if high rates of inflation combine with long collection lags to erode the real value of ordinary fiscal revenues.[4] Tanzi's argument has been challenged recently by Dixit, who provides a welfare

[2] Kimbrough (1986), on the other hand, argues that Friedman's rule may be optimal even in second-best environments. His reasoning is based on the view (formalized in the shopping-time model) that fiat money is not a final good but rather an intermediate input in the transactions technology. A theorem on second-best taxation from Diamond and Mirrlees (1971) then applies. This theorem guarantees that if the production function exhibits constant returns to scale and all final goods are taxable, then intermediate inputs ought not to be taxed. For a discussion of the applicability of Friedman's rule in the shopping-time monetary model, see Végh and Guidotti (1993).

[3] See Frenkel (1987), Mourmouras (1991), and Végh (1989).

[4] Admittedly, the applicability of current abstract theories of inflation (especially of the steady-state variety) to macroeconomic stabilization problems is limited. For instance, the collection period (which is exogenous in our analysis as well as in Dixit's) is in reality an endogenous variable that depends, among other things, on the prevailing rate of inflation. In periods of high inflation, in particular, rational governments attempt to protect the real value of revenues by shortening the collection period. It is interesting to note that theorists allow income tax rates to be changed costlessly while exogenously holding fixed the collection interval, even though in practice the former is a "stickier" policy instrument than the latter. In particular, the frequency of tax payments can be changed simply by instructing the tax administration to issue new implementing orders, while a change in the tax rate must (in democratic systems) go through long legislative processes. See Tanzi (1992) on this point and on the other limitations of current normative tax theories.

analysis of inflation that incorporates collection costs and lags into a version of Végh's (1989) model. Dixit observes that rational governments will, in a general equilibrium environment, react to the presence of lags by adjusting all taxes, not just the rate of inflation. Since this changes prices and the real cost of collections, there is a richer menu of possibilities to consider than under a partial equilibrium framework. Dixit provides two examples that reverse the traditional argument. In the first, the length of the collection lags is irrelevant for the optimal choice of inflation, as if full interest were charged to compensate for the delay in tax payments. More interesting is the second case in which the presence of lags raises the excess burden of income taxes, thereby warranting greater reliance on seigniorage than when no lags exist.

While relying on the public finance approach to inflation, this paper reconsiders the optimal mix of inflation and costly income taxes for alternative specifications of the monetary model and the collection cost technology. In particular, an optimal-tax analysis is performed in the context of Samuelson's (1958) consumption loans model, in which, unlike the shopping-time model, the major distortion caused by money-financed deficits is that on intertemporal consumption allocations. Several interesting results emerge. First, regardless of the length of the collection lag, optimal inflation is proportional to the marginal cost of income tax collections, implying that price stability ought to be pursued whenever these collections are costless at the margin. Second, and in accordance with the Ramsey "inverse elasticity rule," the optimal rate of inflation is inversely related to both the marginal propensity to consume and the interest elasticity of real money balances. Third, the desired rate of inflation in a lag-ridden economy is lower than the desired rate in a lag-free economy if g, the share of government spending in GDP, is below a certain threshold. Thus, Dixit's unconditional rejection of the traditional presumption—that collection lags ought to reduce the optimal rate of inflation—is unwarranted on theoretical grounds. As a practical matter, the threshold value of g, which is crucial for the comparison, turns out to be a function of the marginal cost of collection (Table 1 and Figure 1).

These results owe to a combination of factors. First, in the present model the efficiency trade-off is between money-financed deficits, which lower real interest rates and distort intertemporal choice, and income taxes, which require real resources for collection. This leads naturally to Ramsey formulae that incorporate marginal collection costs as well as the interest and income elasticities of currency demand.[5] By contrast, in the

[5] This distortion, which is not always made explicit in welfare analyses of the costs of inflation, seems to be important in practice—particularly in developing

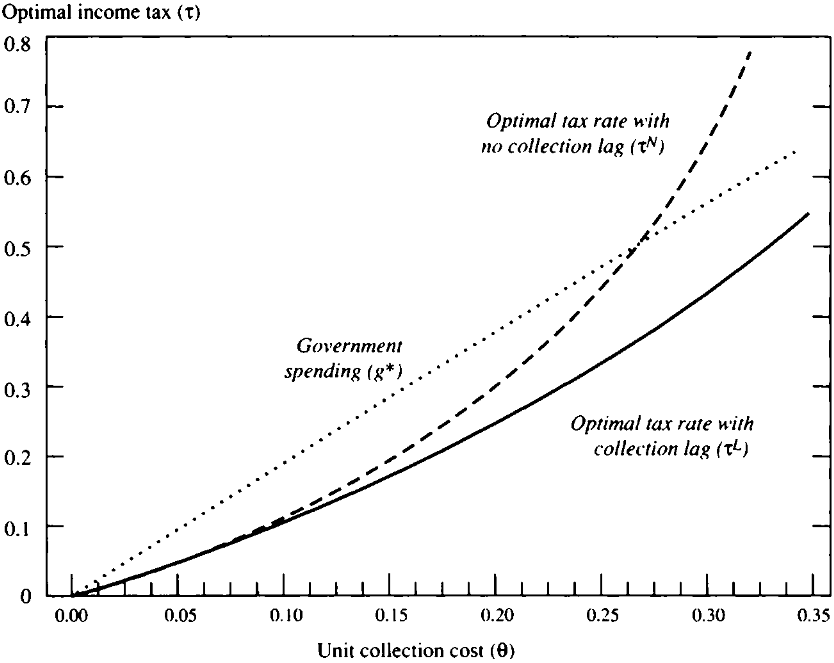
Table 1. *Critical Values of Government Spending (g) and Optimal Income Tax and Inflation (τ, π) as Functions of the Unit Cost of Collections (θ)*

Unit collection cost (θ)	Inflation		Income tax		Government spending (g^*)
	π^N	π^L	τ^L	τ^N	
0.00	0.0000	0.0000	0.0000	0.0000	0.0000
0.01	0.0204	0.0204	0.0101	0.0101	0.0199
0.02	0.0417	0.0417	0.0204	0.0204	0.0396
0.03	0.0638	0.0638	0.0309	0.0310	0.0591
0.04	0.0870	0.0870	0.0417	0.0418	0.0785
0.05	0.1111	0.1111	0.0526	0.0529	0.0976
0.06	0.1364	0.1364	0.0638	0.0644	0.1166
0.07	0.1628	0.1628	0.0753	0.0761	0.1355
0.08	0.1905	0.1905	0.0870	0.0883	0.1542
0.09	0.2195	0.2195	0.0989	0.1009	0.1727
0.10	0.2500	0.2500	0.1111	0.1139	0.1911
0.11	0.2821	0.2821	0.1236	0.1274	0.2094
0.12	0.3158	0.3158	0.1364	0.1415	0.2276
0.13	0.3514	0.3514	0.1494	0.1563	0.2456
0.14	0.3889	0.3889	0.1628	0.1717	0.2636
0.15	0.4286	0.4286	0.1765	0.1878	0.2815
0.16	0.4706	0.4706	0.1905	0.2048	0.2993
0.17	0.5152	0.5152	0.2048	0.2228	0.3170
0.18	0.5625	0.5625	0.2195	0.2417	0.3347
0.19	0.6129	0.6129	0.2346	0.2619	0.3524
0.20	0.6667	0.6667	0.2500	0.2833	0.3700
0.21	0.7241	0.7241	0.2658	0.3062	0.3876
0.22	0.7857	0.7857	0.2821	0.3308	0.4053
0.23	0.8519	0.8519	0.2987	0.3572	0.4229
0.24	0.9231	0.9231	0.3158	0.3857	0.4406
0.25	1.0000	1.0000	0.3333	0.4167	0.4583
0.26	1.0833	1.0833	0.3514	0.4503	0.4762
0.27	1.1739	1.1739	0.3699	0.4871	0.4941
0.28	1.2727	1.2727	0.3889	0.5275	0.5121
0.29	1.3810	1.3810	0.4085	0.5720	0.5303
0.30	1.5000	1.5000	0.4286	0.6214	0.5486
0.31	1.6316	1.6316	0.4493	0.6765	0.5671
0.32	1.7778	1.7778	0.4706	0.7383	0.5858
0.33	1.9412	1.9412	0.4925	0.8081	0.6047
0.34	2.1250	2.1250	0.5152	0.8873	0.6240
0.35	2.3333	2.3333	0.5385	0.9782	0.6435

Notes: (τ^N, π^N) = desired income tax and inflation pair with no collection lag; (τ^L, π^L) = desired income tax and inflation pair with collection lag; π is an absolute figure over a 20-year period; θ, τ, and g^* are absolute figures.

countries. In these countries, the ability of savers (especially small ones) to index asset returns through financial markets is hindered by prohibitive transaction costs and the primitive state of development of these markets. See Wallace (1980) on this point.

Figure 1.
Critical Value of Government Spending and Optimal Income Tax Rates



Notes: g^* = critical value of the ratio of government spending to GDP as a function of the unit cost of income tax collections; τ^N = optimal income tax rate in absence of collection lags; and τ^L = optimal income tax rate in presence of collection lags.

shopping-time model used by Végh (1989) and Dixit (1991), a version of Irving Fisher's theory of interest is maintained, according to which the inflationary process does not affect real interest rates, the whole profile of which is taken to be exogenous (Dixit, p. 645). Second, although the introduction of payment lags reduces tax collections in real terms at a given positive rate of inflation, it also raises desired real currency balances, leading to complicated changes in tax bases and the optimal tax menu. Finally, unlike Dixit who maintains (p. 648) that "the nominal collection cost technology exactly keeps pace with inflation," this paper allows real collection costs to vary with inflation. This feature proves to be important in optimal tax calculations as well.

This paper, then, describes first a model of collection lags and inflation, as well as presents the public finance analysis and derives the Ramsey formula for an economy in which there are no (significant) lags in income tax collections. Then, it calculates the optimal rate of inflation for two parametric examples corresponding, respectively, to Dixit's benchmark specification of constant marginal collection cost and the more realistic increasing marginal cost case. Next, the impact of a one-period collection lag on the optimal tax structure is analyzed, and the optimal rates of inflation are compared for the lag and lag-free environments. Finally, the paper examines the sensitivity of these results to changes in the specification of the collection cost function and proves a neutrality proposition for a plausible alternative functional form.

I. The Model

Consider the following simple version of the consumption loans model. The economy comprises an infinite sequence of generations who live two periods and overlap with one another. Each generation is the same size, which for simplicity is normalized to one. Perfect foresight is assumed. At each date $t = 1, 2, \ldots,$ the representative young agent is endowed with $\bar{n}$ units of labor and a technology $f(n)$, which allows him or her to produce a single perishable consumption good. The production function $f(\cdot)$ satisfies $f(0) = 0, f' \geq 0, f'' \leq 0$. If real gross income is $w = f(\bar{n})$ and $\tau(t)$ is the proportional rate of taxation applied to such income, after-tax real income is $A(t) \equiv [1 - \tau(t)]w(t)$. A fiat currency issued by the government is the only asset in the economy. Agents are retired in the second period of life and must rely on accumulated currency balances to purchase the consumption good. If $p(t)$ denotes the price level at t, the rate of inflation between t and $t + 1$ is $\pi(t) \equiv [p(t + 1) - p(t)]/p(t)$, while the real gross rate of return on currency balances is $R(t) \equiv 1/[1 + \pi(t)]$. The end-of-period economywide stock of currency is denoted $M(t)$. The entire pretransfer stock of currency at $t = 1$ is owned by the old agent belonging to generation 0.

Let $c_t(k)$ denote consumption in period k by an agent born at t, and let $m(t)$ denote his or her nominal currency holdings at the end of t. Given the current and expected future price levels, $p(t)$ and $p(t + 1)$ respectively, and $\tau(t)$, the competitive choice problem is to select a nonnegative vector $[c_t(t), c_t(t + 1), m(t)]$ that maximizes lifetime utility

$$u[c_t(t), c_t(t + 1)] \quad (1)$$

subject to

$$c_t(t) + m(t)/p(t) \leq [1 - \tau(t)]w \quad \text{and}$$
$$c_t(t + 1) \leq m(t)/p(t + 1). \quad (2)$$

In an interior solution (to be assumed throughout), the marginal rate of substitution between first- and second-period consumption, u_1/u_2, must be equal to the ratio of relative prices $p(t)/p(t+1) \equiv R(t)$. The condition $u_1 = Ru_2$ together with equation set (2) at equality may be used to derive consumer demand schedules $c_t(t) = c_1[A(t), R(t)]$ and $c_t(t+1) = c_2[A(t), R(t)]$, and a demand schedule for real currency balances $m(t)/p(t) = s[A(t), R(t)]$. In what follows, $\epsilon(s,R) \equiv \partial \ln(s)/\partial \ln(R)$ and $\epsilon(s,A) \equiv \partial \ln(s)/\partial \ln(A)$ denote, respectively, the interest and income elasticities of real currency demand, and $s_A \equiv \partial s/\partial A$ denotes the marginal propensity to save out of current income.

Government spending on public goods and services excluding collection costs is financed via a flat-rate income tax and seigniorage. Public goods enter individual utility functions in a separable manner that does not affect preference orderings of private goods. The sequence of government expenditures on public goods, $\{G(t), t = 1, 2, \ldots\}$ may therefore be treated as exogenous. Income taxes require real resources for collection. Specifically, if τw is gross tax revenue in real terms, then net revenue is $\tau w - \phi(\tau w)$ where $\phi(\cdot)$ is a monotonic increasing function describing the resource cost of income tax collections faced by the government. The function ϕ satisfies $\phi(0) = 0, \phi' \geq 0, \phi'' \geq 0$. The government cash-flow constraint for period $t = 1, 2, \ldots$ may be written:

$$G(t) = \tau(t)w(t) - \phi[\tau(t)w(t)] + [M(t) - M(t-1)]/p(t). \tag{3}$$

Given $\bar{n}$ and $M(0)$, a *perfect-foresight competitive equilibrium* is a set of sequences $\{c_t(t), c_{t-1}(t), s(t), M(t), p(t), \pi(t), R(t), G(t), \tau(t)\}$ that satisfy the conditions of individual optimization and are consistent with market clearing and the sequence of government budget constraints for all $t = 1, 2, \ldots$. A *stationary equilibrium* is a set of scalars $(c_1, c_2, \pi, R, G, \tau)$ together with a value $c_0(1) = M(0)/p(1)$ and geometrically increasing levels of $p(t), M(t)$ for all $t = 1, 2, \ldots$ satisfying the following:

$$u_2(A - s, Rs) = Ru_1(A - s, Rs) \tag{3a}$$

$$G = \tau w - \phi(\tau w) + (1 - R)s(A, R) \tag{3b}$$

$$G(1) = \tau(1)w(1) - \phi[\tau(1)w(1)] + [M(1) - M(0)]/p(1) \tag{3c}$$

$$M(1)/p(1) = s(A, R) \tag{3d}$$

$$M(t+1)/M(t) = p(t+1)/p(t) = 1/R. \tag{3e}$$

Note that the term $1 - R \equiv \pi/(1 + \pi)$ in equation (3b) is the effective rate of taxation on real currency balances (the "inflation tax rate").

II. Optimal Taxation

In this section, the model above is used to illustrate conditions under which inflationary finance can be part of a second-best tax menu. In particular, a Ramsey formula is derived that makes explicit the dependence of the desired rate of inflation on the specification of the collection cost function and such variables as the marginal propensity to consume and the income and interest elasticities of money demand. Basically, the assumption that income taxes require real resources for collection introduces the distortion necessary to allow seigniorage to be a part of the optimal government finance strategy. Thus, while higher rates of inflation reduce the real rate of interest and distort intertemporal choice, higher rates of income taxation entail higher collection costs. In an optimal position, marginal excess burdens of the two taxes are equalized. In some specifications, this involves trading off some distortion in the intertemporal margin to economize on the resource costs of income taxes.

Let the social welfare criterion be the steady-state utility of each generation $t = 1, 2, \ldots$, denoted $V(A, R) \equiv u(A - s, Rs)$. The authorities set the pair (R, τ) to maximize V subject to equation (3b) and the functional form of the real money demand function $s(A, R)$ dictated by private optimization. Letting μ denote the Lagrange multiplier associated with equation (3b) and assuming an interior solution, the first-order necessary conditions of this problem are equation (3b) and

$$u_1 = \mu[1 - \phi' - (1 - R)s_A], \tag{4a}$$

$$u_2 = \mu[1 - (1 - R)s_R/s]. \tag{4b}$$

Dividing equation (4a) by equation (4b) and using the identity $u_1 = Ru_2$ yields

$$R = \frac{1 - \phi' - (1 - R)s_A}{1 - (1 - R)s_R/s}. \tag{5}$$

Equation (5) may be solved for the *optimal inflation tax* $1 - R$ as a function of the marginal cost of collection ϕ' and the income and interest elasticities of real money demand. Cross-multiplying and rearranging terms lead to the following simple formula:

$$1 - R = \frac{\phi'}{1 - s_A + \epsilon(s, R)}. \tag{6}$$

The Ramsey formula (6) demonstrates, first, that *the desired inflation tax is proportional to the marginal cost of collection* ϕ'. Thus, regardless of the values of the income and interest elasticities of money demand, authorities should strive for price stability ($R = 1$ or $\pi = 0$) if tax collections are costless at the margin. The reason for this result is that in the present model agents can neither engage in untaxed home production nor value leisure directly. Since labor endowments are inelastically supplied in the taxed activity, costless flat-rate income taxes are equivalent to lump-sum taxes.[6] If costless income taxes are used to raise 100 percent of revenue it is possible for the economy to achieve its first-best allocation (Samuelson's golden rule). Given the assumption of zero population growth, this objective amounts to a zero net real interest rate target or, equivalently, the pursuit of a stable price level.

Equation (6) also suggests that the optimal inflation tax is *inversely* related to the marginal propensity to consume $1 - s_A$ and the interest elasticity of real money demand $\epsilon(s, R)$. A one-unit increase in the marginal propensity to consume lowers by one unit the real value of desired currency balances carried forward to the following period. As such, it reduces the base of the inflation tax and raises the excess burden of a given rate of inflation. Finally, and for entirely analogous reasons, the optimal inflation rate is negatively related to the interest elasticity of real currency demand.

III. Examples of Optimal Tax Menus

To gain additional insight into the nature of optimal taxes, two special cases are considered below that lead to closed-form solutions of the optimal (τ, π) pair. The first case corresponds to Végh's and Dixit's benchmark specifications of logarithmic utility and constant unit collection costs; the second corresponds to the more realistic case of increasing marginal collection costs.

Suppose first that consumer preferences are given by $u(c_1, c_2) = \log(c_1) + \beta \log(c_2)$, where $\beta > 0$ is the subjective time discount factor. Consumer optimization leads to a constant saving rate $[s = \beta/(1 + \beta)]$

[6]The presence of untaxed activities would alter this result. If workers value leisure or have employment opportunities in the underground economy, proportional income taxes would distort the margin of choice by consumers—the labor-leisure choice in the first case and the regular-underground employment in the second. In either case, efficient taxation would entail positive income and inflation taxes even in the absence of collection costs. Mourmouras (1991) provides an explicit analysis.

and interest-inelastic real currency demand $[\epsilon(s,R) = 0]$. The Ramsey formula (6) then becomes $1 - R = (1 + \beta)\phi'$. Assuming, in addition, that the marginal cost of collection is constant, say $\phi' = \theta_1 > 0$, then optimal inflation is (a) constant and independent of the level of government spending; and (b) inversely related to the subjective rate of time preference $1/\beta - 1$.

Given the constant desired value of $1 - R$, the optimal income tax rate can be computed from equation (3b). Letting $g \equiv G/w$ denote the share of government spending in national income, the optimal value of τ is

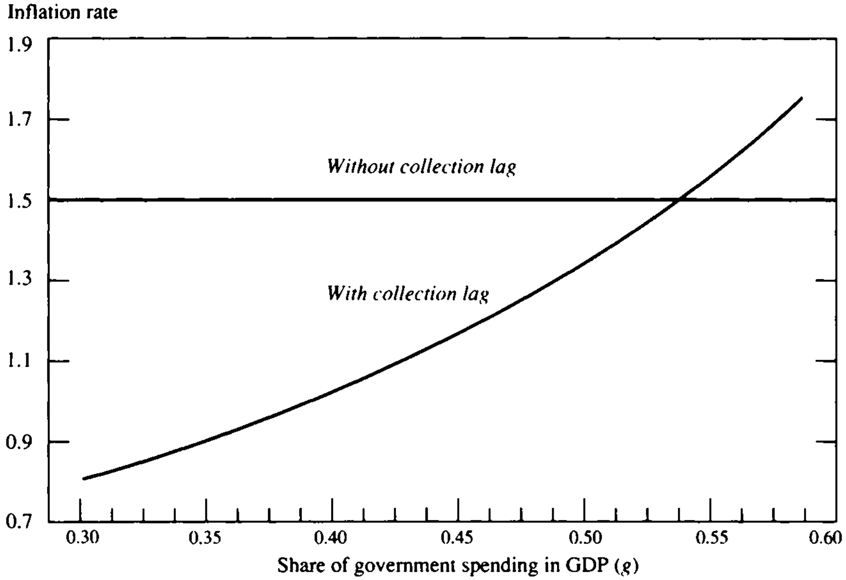
$$\tau = \frac{g - \beta\theta_1}{1 - (1 + \beta)\theta_1}. \tag{7}$$

According to equation (7), income taxes are positive provided that the unit cost of collection is not too large (in the sense of satisfying $g > \beta\theta_1$ and $1 > [1 + \beta]\theta_1$). As expected, the optimal income tax rate in this range is inversely related to the unit cost of collection. For example, if there is no discounting of the future ($\beta = 1$) and the unit cost of collection is 10 percent ($\theta_1 = 0.10$), then desired inflation is 25 percent per period ($\pi = 0.25$), regardless of the size of the government budget.[7] If, in addition, government spending net of collection costs is 40 percent of GDP ($g = 0.40$), then the income tax rate is $\tau = 0.375$. In this environment, where tax collections are 10 percent of gross budgetary revenue, or 3.75 percent of GDP, gross budget revenues are 37.5 percent of GDP, resulting in an optimal money-financed deficit of 6.25 percent of GDP. As a second example, if the unit cost of collection is only 5 percent ($\theta_1 = 0.05$) and $g = 0.40$, the optimal policy is calculated to be $(\pi, \tau) = (0.111, 0.389)$. These examples demonstrate that doubling the efficiency of the tax administration (that is, reducing unit collection costs by half) could reduce the desired rate of inflation by more than half. Figures 2 and 3 relate optimal policy to the fundamentals of the economy by drawing optimal (τ, π) pairs against g and θ_1, respectively. Table 2 shows optimal (τ, π) pairs for selected values of θ_1.

The conclusion that inflation is part of an *interior* optimal tax package is in sharp contrast with the results obtained by Frenkel (1987). This author studied a log-linear cash-in-advance model of labor-leisure choice and showed that if the marginal cost of income tax collections is constant, then the optimal policy involves financing government expenditures via inflation taxes *in their entirety*. This result stems directly from Frenkel's specification of money demand and his assumption that inflation is cost-

[7] Assuming the length of the period to be approximately 20 years, this translates to an annual rate of inflation of 1.1 percent.

Figure 2.
Optimal Inflation and Optimal Income Tax with Fixed Collection Cost ($\theta = 0.30$)



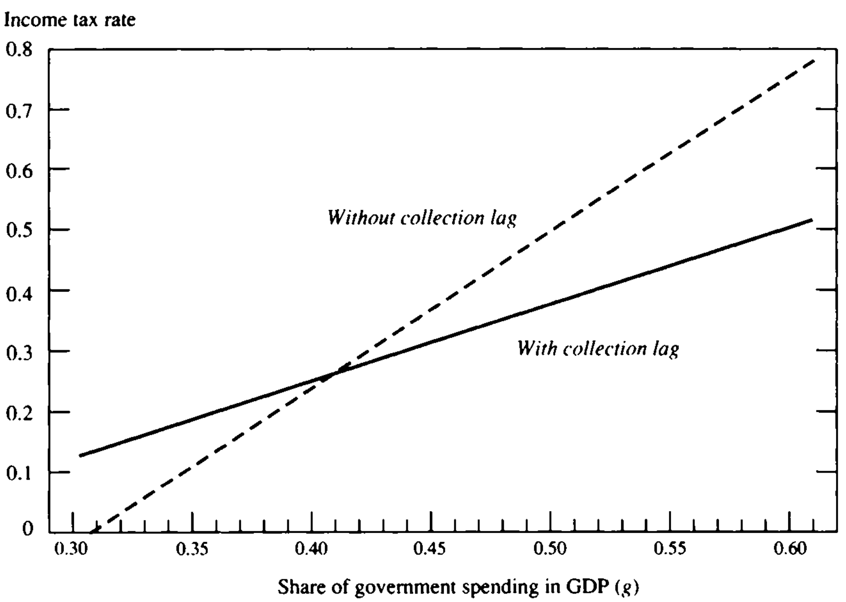
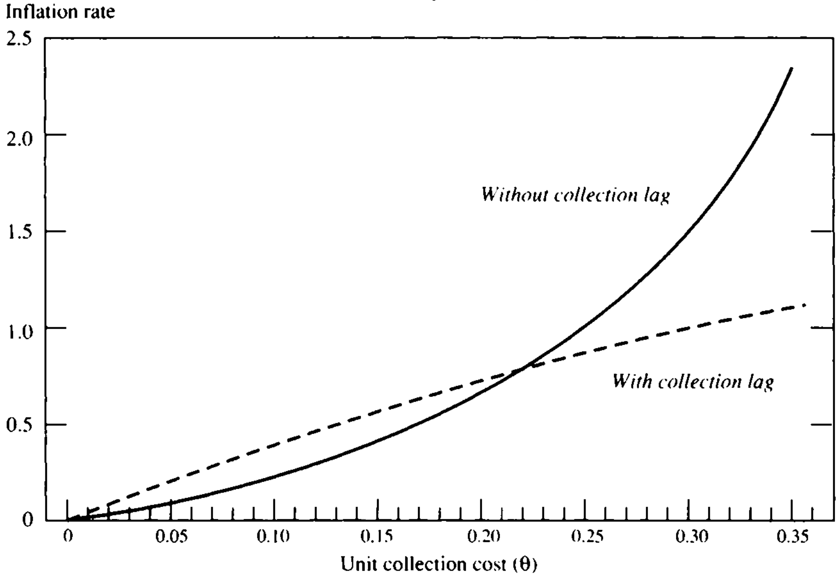


Figure 3.
Optimal Inflation and Optimal Income Tax with Fixed Government Spending ($g = 0.40$)



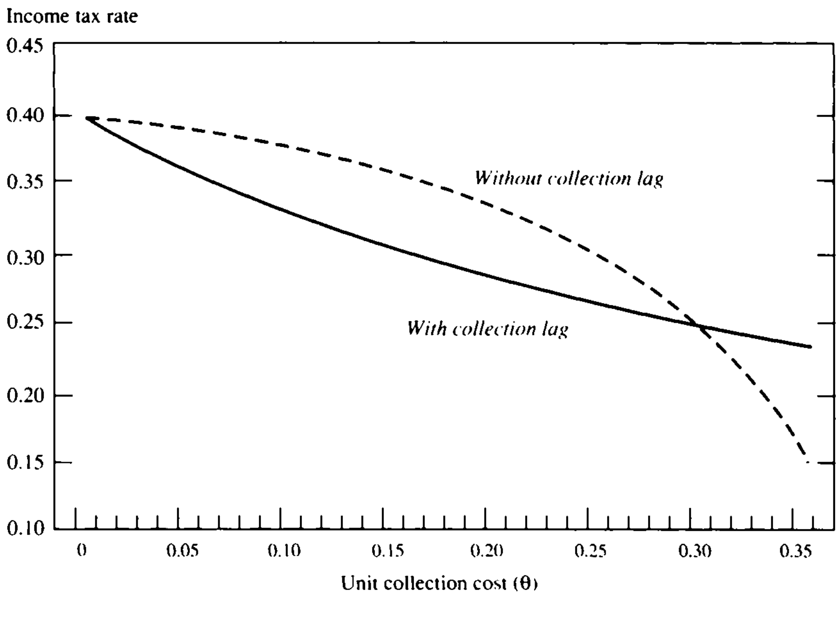


Table 2. *Size of Government Spending and Optimal Taxation: Selected Values of Desired (τ, π) Pairs as Functions of* g

Government spending (g)	Unit collection cost (θ)	Inflation		Income tax	
		π^N	π^L	τ^N	τ^L
30	30	4.69	2.93	0.00	13.16
31	30	4.69	2.99	2.50	14.34
32	30	4.69	3.04	5.00	15.52
33	30	4.69	3.10	7.50	16.70
34	30	4.69	3.16	10.00	17.88
35	30	4.69	3.21	12.50	19.07
36	30	4.69	3.27	15.00	20.25
37	30	4.69	3.34	17.50	21.43
38	30	4.69	3.40	20.00	22.62
39	30	4.69	3.46	22.50	23.81
40	30	4.69	3.53	25.00	25.00
41	30	4.69	3.59	27.50	26.19
42	30	4.69	3.66	30.00	27.38
43	30	4.69	3.73	32.50	28.58
44	30	4.69	3.80	35.00	29.77
45	30	4.69	3.87	37.50	30.97
46	30	4.69	3.95	40.00	32.17
47	30	4.69	4.02	42.50	33.37
48	30	4.69	4.10	45.00	34.57
49	30	4.69	4.18	47.50	35.78
50	30	4.69	4.26	50.00	36.98
51	30	4.69	4.35	52.50	38.19
52	30	4.69	4.43	55.00	39.40
53	30	4.69	4.52	57.50	40.61
54	30	4.69	4.61	60.00	41.82
55	30	4.69	4.70	62.50	43.03
56	30	4.69	4.80	65.00	44.25
57	30	4.69	4.89	67.50	45.46
58	30	4.69	4.99	70.00	46.68
59	30	4.69	5.10	72.50	47.90
60	30	4.69	5.20	75.00	49.13

Notes: g is expressed as a percent of GDP; π^N and π^L are expressed in annual rates in percent; τ^N, τ^L, and θ are expressed in percent.

less while income taxes require resource costs. In particular, Frenkel assumes demand for real currency balances to be proportional to income. Since the bases for the income and inflation taxes are then proportional, the costless inflation tax will always be preferred over the costly income tax. By contrast, in the overlapping generations model, the two tax bases are separate but related. On the one hand, income taxes are levied on current income at the source; on the other hand, seigniorage subjects to tax that part of current income which is *not* consumed in the present period, creating a distortion of intertemporal choice. This distinction creates a nontrivial trade-off between the two types of taxes that is optimally exploited by the policymaker as reflected in equation (6).

In this section, the more realistic case of increasing marginal collection costs is taken up. Following Végh (1989) and Dixit (1991), suppose that the function ϕ is quadratic:

$$\phi(\tau w) = \theta_1(\tau w) + (\theta_2/2)(\tau w)^2, \qquad \theta_1 > 0 \quad \text{and} \quad \theta_2 > 0. \tag{8}$$

With the collection cost function as specified in equation (8) and logarithmic utility, it turns out that the optimal income and inflation tax rates are linearly related. Substituting $\phi' = \theta_1 + \theta_2(\tau w)$, $\epsilon(s, R) = 0$, and $1 - s_A = 1/(1 + \beta)$ into equation (6) yields the exact relation between τ and π:

$$1 - R \equiv \pi/(1 + \pi) = (1 + \beta)[\theta_1 + \theta_2 \tau w]. \tag{9}$$

To compute the reduced forms of τ and π, the right-hand side of equation (9) may be substituted into the government budget constraint (3b). After some algebra, the optimal value of τ is seen to be a solution to the following equation:

$$(\tfrac{1}{2} + \beta)\theta_2 w\tau^2 - [1 - (1 + \beta)\theta_1 + \beta\theta_2 w]\tau + g - \beta\theta_1 = 0. \tag{10}$$

The roots of equation (10) are real if $\Delta = [1 - (1 + \beta)\theta_1 + \beta\theta_2 w]^2 - 4(\tfrac{1}{2} + \beta)\theta_2 w(g - \beta\theta_1)$ is positive, a condition that will always hold if g is not "too large." The reduced form of τ is then given by the smaller of the two roots, namely

$$\tau = \frac{1 - (1 + \beta)\theta_1 + \beta\theta_2 w - \Delta^{1/2}}{2(\tfrac{1}{2} + \beta)\theta_2 w}. \tag{11}$$

There are two differences between (9)–(11) and (6)–(7). First, in equation (9) the greater fiscal inefficiency raises the optimal inflation tax (note that $1 - R$ as given in equation (9) is uniformly greater than $(1 + \beta)\theta_1$ in the range $\tau \epsilon [0, 1]$). Second, equation (9) implies that higher values of τ (or, equivalently, g) will be associated with higher values of π. (That is, $\partial\tau/\partial g > 0$, as can be established by differentiating equation (10) with respect to g. From equation (9), it then follows that $\partial(1 - R)/\partial g > 0$, implying that the two tax instruments are normal.) By comparison, in the benchmark specification, π is constant and independent of g.

IV. Collection Lags and Optimal Inflation

In order to assess the impact of collection lags on the optimal tax menu, the basic model is now extended to incorporate a one-period delay in income tax payments. As before, a stream of government spending $[G(t)]$ is financed by income taxes $[\tau(t)]$ and currency issues $[M(t) - M(t-1)]$ for $t = 1, 2, \ldots$. Let the nominal income tax liability accrued at t, $\tau(t)p(t)w(t)$, become due at $t+1$. The payment lag allows workers (who also own the firms in the economy) to use these balances for an additional period and, assuming positive inflation, to reduce their tax liability in real terms. To the initial conditions $\bar{n}$ and $M(0)$ must now be added the nominal tax liability in the initial period, namely $\tau(0)p(0)w(0)$. (The real value of this nominal revenue is endogenous, as it depends on $p(1)$, the price level at time period 1.)

The delay in tax payments requires an elaboration of the time at which collection costs are incurred. Given the payment lag, nominal collections at t are $p(t-1)\tau(t-1)w(t-1)$. The real value of these collections in period t is $p(t-1)\tau(t-1)w(t-1)/p(t) = R(t-1)\tau(t-1)w(t-1)$, which is reduced by higher inflation. Two plausible formulations are considered. In this section, it is first assumed that real collection costs at t, $\phi(t)$, are a function of the real value of revenues collected at t. In other words, $\phi(t) = \phi[R(t-1)\tau(t-1)w(t-1)]$. This implies that the real cost of collecting a given nominal liability is *lowered* by inflation. While this is a plausible formulation, it must be emphasized that it is entirely ad hoc.[8] An alternative is to write the collection cost function as $\phi(t) = \phi[\tau(t-1)w(t-1)]$, implying that costs of collection are completely indexed for inflation.[9] This exercise is undertaken in a later section.

Formally, the private optimization problem is now to maximize $u[c_t(t), c_t(t+1)]$ subject to equations (12)–(13) below:

$$p(t)c_t(t) + m(t) \leq p(t)w(t), \tag{12}$$

$$p(t+1)c_t(t+1) \leq m(t) - \tau(t)p(t)w(t). \tag{13}$$

[8] Note that this formulation is consistent with the view that *nominal* collection costs at t, say $\Phi(t)$, are proportional to the nominal value of the liability to be collected, $\tau(t-1)p(t-1)w(t-1)$. Dividing through $\Phi[\tau(t-1)p(t-1)w(t-1)] = \theta_1\tau(t-1)p(t-1)w(t-1)$ by $p(t)$ shows that real collection costs, $\phi(t) = \Phi(t)/p(t)$, are equal to $\theta_1\tau(t-1)p(t-1)w(t-1)/p(t)$, or that $\phi(t) = \theta_1 R(t-1)\tau(t-1)w(t-1)$. This specification corresponds to Végh's and Dixit's benchmark cases of constant real marginal collection costs.

[9] The third alternative, namely when real collection costs are raised by higher inflation, is left as an exercise for the interested reader.

Using the notation developed earlier, equations (12)–(13) become

$$c_t(t) + s(t) \leq w(t) \tag{12'}$$

and

$$c_t(t+1) \leq R(t)s(t) - R(t)\tau(t)w(t). \tag{13'}$$

In equation (12′), $s(t)$ is an agent's *gross* real currency balance at the end of t; in equation (13′), the term $R(t)\tau(t)w(t)$ is his or her effective real tax liability. Combining (12′) and (13′), the agent's consumption set is

$$c_t(t) + c_t(t+1)/R(t) \leq A(t) \equiv [1 - \tau(t)]w(t). \tag{14}$$

As before, the solution to this problem is a pair of consumer demand schedules $c_1(A, R)$ and $c_2(A, R)$ and a real currency demand schedule $s = w - c_1$. It will be observed that given $R(t)$ and $\tau(t)$, individual consumption sets are not affected by payment lags. Anticipating the tax liability to be incurred one period ahead, agents react to the change in the timing of taxes by altering their financial decisions. The resulting increase in real currency holdings broadens the base of the inflation tax, thereby altering the government's choice set and the outcome of the optimal tax calculation.

Turning to the government budget constraint, since payment of taxes accrued at $t - 1$ is not made until t, we have that

$$\begin{aligned} p(t)G(t) = p(t-1)\tau(t-1)w(t-1) \\ - p(t)\phi[p(t-1)\tau(t-1)w(t-1)/p(t)] \\ + M(t) - M(t-1). \end{aligned} \tag{15}$$

Dividing equation (15) by $p(t)$ yields

$$\begin{aligned} G(t) = R(t-1)\tau(t-1)w(t-1) \\ - \phi[R(t-1)\tau(t-1)w(t-1)] \\ + [M(t) - M(t-1)]/p(t). \end{aligned} \tag{16}$$

In equation (16), the Tanzi effect is reflected in the term $R\tau w = \tau w/(1 + \pi)$ in the right-hand side: given the statutory rate τ, the real value of *gross* income tax collections is lowered by higher inflation. In addition, and provided that ϕ' is less than unity, the real value of *net* income tax collections, $R\tau w - \phi(R\tau w)$, is raised by lower rates of inflation. The latter statement can be established by differentiating $R\tau w - \phi(R\tau w)$ with respect to R, holding w and τ constant. This yields

$$(\partial/\partial R)[R\tau w - \phi(R\tau w)] = \tau w(1 - \phi') > 0 \qquad \text{if } \phi' < 1. \tag{17}$$

Stationary equilibria are pairs (R, τ) satisfying

$$G = R\tau w - \phi(R\tau w) + (1 - R)s(A, R). \tag{18}$$

Given a feasible choice of (R, τ), equilibrium sequences for prices and money are completely specified once the initial price level and currency issue, $p(1)$ and $M(1)$ respectively, are determined. Given values for G and the initial conditions $[p(0), \tau(0), w(0), M(0)]$, $p(1)$ and $M(1)$ may be calculated using the government budget constraint for the initial period $t = 1$,

$$G = p(0)\tau(0)w(0)/p(1) + M(1)/p(1) - M(0)/p(1), \tag{19}$$

and the stationary value of the real stock of currency $M(1)/p(1) = s(A, R)$. In principle, equations (19) and (3c) can be used to compare the initial price levels in the lag-free and lag-ridden economies. This comparison, which would establish the change in contemporaneous prices resulting from shorter payment lags, is beyond the steady-state comparisons undertaken in this paper and is left as an exercise for the interested reader.

The presence of payment lags introduces two mutually opposing forces that in principle have an ambiguous net effect on total "revenue" (seigniorage plus income taxes). On the one hand, owing to the Tanzi effect, higher inflation lowers the real value of revenue from a given rate of income tax. On the other hand, the increase in real money demand associated with the presence of collection lags makes a *given* rate of inflation more "productive." As suggested by Dixit, to ascertain how payment lags affect the desired rate of inflation in the face of these forces, the optimal tax problem must be recalculated in its entirety.

Formally, given the private decision rules $c_1(A, R)$, $c_2(A, R)$, and $s = w - c_1$, the government selects (τ, R) to maximize $u(w - s, Rs - R\tau w)$ subject to

$$G = R\tau w + (1 - R)s - \phi(R\tau w). \tag{20}$$

In the Appendix, it is shown that the optimal rate of inflation is given by

$$\pi = \frac{\phi'}{1 - s_A + \epsilon(s, R) + [\epsilon(s, A)\tau/(1 - \tau) - 1]}. \tag{21}$$

The Ramsey formula (21) retains the basic characteristics of equation (6). First, desired inflation continues to be proportional to the marginal cost of collection, and price stability should still be pursued whenever $\phi' = 0$. Second, the optimal inflation rate is inversely related to the marginal propensity to consume and the interest elasticity of money demand. To ascertain the relative magnitude of desired inflation in the lag-free and lag-ridden environments, equations (6) and (21) may be compared directly. However, since τ is endogenous and the terms in the right-hand sides of (6) and (21) are functions, a simple comparison cannot be established without resorting to additional assumptions about preferences.

Sharper results are possible for the benchmark case considered by Dixit and Végh in which the functions in (6) and (21) are constants. When this comparison is undertaken it turns out that a theoretical case can be made for the orthodox presumption, at least for levels of government spending below a certain threshold.[10] Assuming $u(c_1, c_2) = \log(c_1) + \log(c_2)$ and $\phi' = \theta > 0$, optimal inflation π is easily calculated to be

$$\pi = 2\theta(1 + \tau)/(1 - \tau). \tag{22}$$

The reduced forms of τ and π can be computed analytically as follows. The real money demand function $s = w(1 + \tau)/2$ may be substituted into the stationary form of the government budget constraint (equation (18)). After some rearranging, the following relation between τ and R is established:

$$\tau = \frac{2g - (1 - R)}{2R(1 - \theta) + (1 - R)}. \tag{23}$$

Equation (23) can also be written in a form that is more convenient for subsequent substitutions:

$$\frac{1 + \tau}{1 - \tau} = \frac{1 - \theta + (1 + \pi)g}{1 - \theta + \pi - (1 + \pi)g}. \tag{23'}$$

Substituting the right-hand side of (23′) into (22) yields the following equation:

$$\pi = 2\theta \frac{1 - \theta + (1 + \pi)g}{1 - \theta + \pi - (1 + \pi)g}. \tag{24}$$

Equation (24) is seen to be equivalent to the quadratic equation

$$(1 - g)\pi^2 - (\theta + g - 1 + 2\theta g)\pi - 2\theta(1 - \theta + g) = 0. \tag{25}$$

[10] Analysis of the more realistic case of *quadratic* collection costs yields similar results. The proof of this assertion for the quadratic case involves some tedious algebra—finding the roots of a fourth-order polynomial equation—and is available upon request.

The roots of equation (25) are real if $\Delta = (\theta + g - 1 + 2\theta g)^2 + 8(1 - g)\theta(1 - \theta + g)$ is positive, in which case the reduced form of π, denoted π^L, is given by

$$\pi^L = \frac{(\theta + g - 1 + 2\theta g) + \Delta^{1/2}}{2(1 - g)}. \tag{26}$$

Given π^L from equation (26), the reduced form for τ^L may then be computed from equation (22). The pairs (τ^L, π^L) satisfying equations (26) and (22) are drawn against g in Figure 2 and against θ in Figure 3. It may be observed that even in the benchmark specification, the introduction of a payment lag makes desired inflation rise with g. This outcome may be contrasted with the results of Section II, where the optimal rate of inflation in the benchmark case was shown to be independent of g.[11] Given the definition $R \equiv 1/(1 + \pi)$, equation (6) describing the desired rate of inflation π^N may be written

$$\pi^N = 2\theta/(1 - 2\theta). \tag{27}$$

In the top panel of Figure 2, equation (26) is the upward-sloping curve, while equation (27) is the horizontal line. For a given value of θ, the relative size of inflation in the two environments depends on the size of g. In particular, there is a threshold value of g, say $g^* > 0$, which solves $\pi^L(g^*) = \pi^N(g^*)$ given that $\pi^L(g) < \pi^N(g)$ whenever $0 < g < g^*$ and that $\pi^L(g) > \pi^N(g)$ whenever $g > g^*$. In accordance with the orthodox position, the desired rate of inflation is lower in the presence of collection lags for all values of g not exceeding the threshold. The numerical value of g^* depends on the unit cost of collection and can be calculated analytically. Setting $\pi^L = \pi^N = 2\theta/(1 - 2\theta)$ in equation (26) yields the following expression for $g^*(\theta)$:

$$g^*(\theta) = \theta(2\theta^2 - 3\theta + 2)/(1 - \theta). \tag{28}$$

Figure 1 draws $g^*(\theta)$ and the associated $\tau(\theta)$ functions for the lag-free and lag-ridden economies. Table 1 provides illustrative calculations of g^* and the associated (τ, π) pairs for selected values of θ. Notice that g^* is strictly increasing in θ, with $g^*(0) = 0$ and $g^*(0.5) = 1$. It can be shown that for values of θ greater than approximately 0.27, $g^*(\theta)$ is strictly convex, implying that as the degree of fiscal inefficiency grows, the critical value of g grows at an increasing rate. Moreover, g^* approaches unity as θ approaches 0.5 from below.

[11] If marginal collection costs are increasing, inflation continues to be an increasing function of g.

V. An Invariance Proposition

In this section, we explore the implications of altering the specification of the collection cost technology. This is important because, as emphasized by Dixit (1991), no deep theory exists to explain the nature of these costs. It will be recalled that in the preceding section real collection costs were assumed to be a stable and increasing function of real *realized* tax revenues, $\phi = \phi(R\tau w)$, implying that, for given statutory tax rates and nominal collections, real collection costs are lowered by higher inflation. In this section, the function ϕ is written as $\phi = \phi(\tau w)$, so that collection costs are assumed to be a function of *accrued* real revenues—or, that nominal costs of collection rise in proportion with prices. The main result is a neutrality proposition establishing that the optimal (τ, R) pair is independent of the collection lag in the benchmark specification of constant marginal collection costs and logarithmic utility.

Formally, $R \geq 0$, and τ are selected to maximize social welfare

$$U = \log(c_1) + \log(c_2) \tag{29}$$

subject to the resource constraint (30) and private sector demand and supply schedules (31)–(34):

$$G = (1 - R)s + R\tau w - \phi(\tau w) \tag{30}$$

$$c_1 = \tfrac{1}{2}w(1 - \tau) \tag{31}$$

$$c_2 = \tfrac{1}{2}w(1 - \tau)R \tag{32}$$

$$s = \tfrac{1}{2}w(1 + \tau). \tag{33}$$

This problem is equivalent to selecting $R \geq 0$ and τ to maximize

$$U = 2\log(1 - \tau) + \log(R) \tag{34}$$

subject to

$$G = (1 - R)\tfrac{1}{2}w(1 + \tau) + R\tau w - \phi(\tau w). \tag{35}$$

In the constant marginal cost case, the problem above can be solved directly by substitution. Assuming $\phi(\tau w) = \theta\tau w$ for some $\theta > 0$, equation (35) may be used to write τ as a function of R and θ:

$$\tau = (2g + R - 1)/(R + 1 - 2\theta). \tag{36}$$

Since equation (36) implies

$$1 - \tau = 2(1 - \theta - g)/(R + 1 - 2\theta), \tag{37}$$

the objective function, equation (34), can be written as a function of a single variable R:

$$U = 2\log[2(1-\theta-g)/(R+1-2\theta)] + \log(R). \tag{38}$$

Maximizing U with respect to $R \geq 0$ is equivalent to maximizing

$$\Omega(R) \equiv \log(R) - 2\log(R+1-2\theta). \tag{39}$$

The first and second derivatives of Ω may be written

$$\Omega' = 1/R - 2/(R+1-2\theta) \tag{40}$$

and

$$\Omega'' = -1/R^2 + 2/(R+1-2\theta)^2. \tag{41}$$

From equation (40), the optimal value of R is $1-2\theta$, implying that the optimal inflation rate does not change if lags are removed.

Returning to the general specification of the collection cost function, the problem of maximizing equation (29) subject to equations (30)–(33) may be solved by forming the Lagrangian

$$L = 2\log(1-\tau) + \log(R) + \lambda[(1-R)\tfrac{1}{2}w(1+\tau) + R\tau w - \phi(\tau w) - G]. \tag{42}$$

The first-order necessary conditions for this problem are

$$1/R = \lambda w[(1+\tau)/2 - \tau], \tag{43}$$

$$2/(1-\tau) = \lambda w[(1-R)/2 + R - \phi']. \tag{44}$$

Dividing equation (43) by (44) yields $1 - R = 2\phi'$, which is identical to the formula for the inflation tax rate derived under the assumption that there were no collection lags ($\beta = 1$, $s_A = 0.5$, and $\epsilon(s,R) = 0$ in equation (6)). To prove that collection lags are neutral, it remains to show that the reduced form for τ is unchanged as well. This is first established for the linear marginal collection cost case: substituting equation (33) and $1 - R = 2[\theta_1 + \theta_2\tau w]$ into equation (30) and solving for τ yield the same formula as equation (10). Upon substitution, the optimal value of τ must solve

$$g = \theta_1 + \theta_2\tau w)(1+\tau) + (1 - 2\theta_1 - 2\theta_2\tau w)\tau - \theta_1\tau - (\theta_2/2)w\tau^2, \tag{45}$$

which, after a few steps of algebra, is equivalent to

$$0 = (3/2)\theta_2 w\tau^2 + (1 - 2\theta_1 + \theta_2 w)\tau - (g - \theta_1). \tag{46}$$

Inspection of equation (46) reveals that it is equivalent to equation (10) for $\beta = 1$. Finally, to establish the equivalence for the constant marginal cost case, simply note that equation (46) with $\theta_2 = 0$ implies equation (7). This establishes the claim that the presence of collection lags leaves unaltered the optimal (R, τ) pair in both the linear and quadratic collection cost specifications.

A note of caution is in order when interpreting this invariance proposition. While the proposition does suggest that the optimal inflation tax *rate* $1 - R$ is invariant to the lag, it does not suggest invariance of inflation tax *revenue*, where the latter is defined as the product of the inflation tax rate and the real currency stock $(1 - R)s(A, R)$. Analogously, this proposition does not suggest invariance of income tax revenue. Clearly, with inelastic labor supply, a positive rate of inflation, and an unchanged rate of labor income taxation, the introduction of collection lags lowers the effective yield of labor taxes. According to the proposition, all revenue losses are made up by higher real seigniorage earnings. In the presence of an unchanged rate of inflation, the additional seigniorage earnings are possible by the higher stock of real balances accumulated to meet future income tax liabilities.

VI. Conclusions

This paper has analyzed how the presence of collection lags affects the optimal inflation rate in an overlapping generations version of the Frenkel-Végh model of costly income taxation. Ramsey pairs (τ, π) were derived from first principles, and the traditional argument, that inflation ought to reduce the optimal rate of inflation, was confirmed for cases where the level of government spending in GDP was below a threshold level. Dixit's conclusions were confirmed for tax rates above the threshold. This demonstration casts some doubt on the claim that, as a general principle, the Tanzi hypothesis is inconsistent with optimal tax theory.

APPENDIX

The Appendix derives the optimal tax formula applicable in Section IV. The following notation is used:

w = real pretax income;

τ = income tax rate;

$A = (1 - \tau)w$ = after-tax income;

c_1 = consumption in first period of life;
c_2 = consumption in second period of life;
G = rate of real government purchases;
$\phi(\cdot)$ = collection cost function;
$\phi'(\cdot)$ = marginal cost of income tax collections;
π = rate of inflation;
$R = 1/(1+\pi)$ = gross real yield of currency;
$1 - R$ = rate of inflation tax.

Given the private decision rules $c_1(A,R)$, $c_2(A,R)$, and $s = w - c_1$, the pair (τ, R) is selected to maximize indirect utility $u(w - s, Rs - R\tau w)$ subject to

$$G = R\tau w + (1-R)s - \phi(R\tau w). \qquad \text{(A1)}$$

The Lagrangian expression for this problem is

$$L = u(w - s, Rs - R\tau w) + \lambda[R\tau w + (1-R)s - \phi(R\tau w) - G]. \qquad \text{(A2)}$$

The first-order necessary conditions for an interior solution are

$$\tau\text{: } u_1(-s_A A_\tau) + u_2 R(s_A A_\tau - w) + \lambda[wR(1-\phi') + (1-R)s_A A_\tau] = 0, \qquad \text{(A3)}$$

and

$$R\text{: } u_1(-s_R) + u_2(s + RS_R - \tau w) + \lambda[\tau w(1-\phi') - s + (1-R)s_R] = 0. \qquad \text{(A4)}$$

Substituting $A_\tau = -w$ and $u_1 = Ru_2$ into equations (A3)–(A4) yields

$$u_2 R = \lambda[R(1-\phi') - (1-R)s_A], \qquad \text{(A5)}$$

and

$$u_2(s - \tau w) = -\lambda[\tau w(1-\phi') - s + (1-R)s_R]. \qquad \text{(A6)}$$

Equations (A5)–(A6), in turn, imply that

$$\frac{-R}{s - \tau w} = \frac{R(1-\phi') - (1-R)s_A}{\tau w(1-\phi') - s + (1-R)s_R}. \qquad \text{(A7)}$$

Next, cross-multiplication in equation (A7) yields

$$-R\tau w(1-\phi') + Rs - R(1-R)s_R = sR(1-\phi') - s(1-R)s_A - \tau wR(1-\phi') + \tau w(1-R)s_A. \qquad \text{(A8)}$$

After some simplification, equation (A8) can be written

$$-R(1-R)s_R = -sR\phi' - s(1-R)s_A + \tau w(1-R)s_A \Rightarrow \qquad \text{(A9)}$$

$$(1-R)Rs_R/s = R\phi' + (1-R)s_A - (1-R)s_A\tau w/s. \qquad \text{(A10)}$$

Note that the expression Rs_R/s on the left-hand side of equation (A10) is the interest elasticity of real currency demand $\epsilon(s,R)$, and the term $s_A\tau w/s$ equals $\epsilon(s,A)\tau/(1-\tau)$. Equation (A10) may be rewritten as follows:

$$(1-R)\epsilon(s,R) = R\phi' + (1-R)s_A - (1-R)\epsilon(s,A)\tau/(1-\tau). \qquad \text{(A11)}$$

Collecting the $(1 - R)$ terms in equation (A11) and rearranging yield

$$(1 - R)[\epsilon(s,R) - s_A + \epsilon(s,A)\tau/(1 - \tau)] = R\phi'. \quad \text{(A12)}$$

Since the factor $(1 - R)/R$ equals the rate of inflation π, equation (A12) yields the Ramsey rule for inflation in the presence of collection lags:

$$\pi = \frac{\phi'}{\epsilon(s,R) - s_A + \epsilon(s,A)\tau/(1 - \tau)}. \quad \text{(A13)}$$

Equation (A13) corresponds to Ramsey formula (21).

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