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ABSTRACT: We present alternative methods for calculating and interpreting the influence of exogenous shocks on historical episodes within the context of DSGE models. We show analytically why different methods for calculating shock decompositions can generate conflicting interpretations of the same historical episodes. We illustrate this point using an extended version of Drautzburg and Uhlig's (2015) model of the U.S. economy, focusing on the periods 1964–1966, 1979–1987, 2006–2009, 2016–2020 and 2020–2023. We argue that the best method for analyzing particular episodes is one which isolates the influence of the shocks during the period under consideration and where the initial conditions represent the system's distance from balanced growth path at the beginning of the episode.

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* Authors would like to thank Thorsten Drautzburg and Harald Uhlig for comments and advice on an earlier draft.

WORKING PAPERS

Evaluating Historical Episodes using Shock Decompositions in the DSGE Model

Prepared by Zamid Aligishiev, Michael Ben-Gad, and Joseph Pearlman[1]

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1 Introduction

Dynamic Stochastic General Equilibrium (DSGE) models estimated using Bayesian techniques are now a standard framework for analyzing the behavior and evolution of macroeconomies. In reduced form these are state space models where the evolution of each variable over the entire sample period can be represented as the combined effects of the different exogenous shocks over time.[1] In this paper we describe alternative methods for calculating shock decompositions for historical subsamples. We argue there is one that is particularly suited for this purpose but which surprisingly has rarely been employed by the DSGE literature. This preferred method isolates the contribution of each shock solely within the subsample.

Two common approaches are typically applied in the DSGE literature to analyze historical episodes using shock decompositions. One is to simply present or focus on a subsample of the historical decomposition for the entire sample, as in Del Negro *et al.* (2013), Brzoza-Brzezina and Kolasa (2013) or Cardani *et al.* (2022). An issue with this approach is that it does not separate the impact of the shocks of interest during a particular time span from the impact of those that preceded them. Due to the recursive nature of DSGE models, the impacts of these previously occurring shocks can be considerable. An alternative approach employed, by e.g. Drautzburg and Uhlig (2015) - henceforth DU, is to calculate the change in the entire sample decomposition from a particular start date. As we demonstrate below, although this differencing procedure nets out the levels of the variables from an initial starting point of interest, it does not completely isolate the decomposition from the impact of prior shocks, which may be persistent; the persistence of even relatively minor disaggregated shocks during a few previous periods can skew the interpretation of individual episodes quite significantly. A third shock decomposition method is to isolate the effects of shocks within that time span from those of shocks that occurred earlier. This approach has a direct correspondence with the state space form of the model and so is a natural one to employ; but it has neverthless been overlooked by the literature. The purpose of this paper is to correct this oversight.[2] We demonstrate how interpretations of a particular episode can differ depending on the shock decomposition chosen by applying all three versions to an updated and extended version of DU's (2015) model, which we describe below.

Why does this matter? We think this is important for two reasons. The first is the centrality of responses of agents and government to innovations in the DSGE framework. When looking at a model's explanation of a particular episode it is therefore the effect of innovations in that episode that should be highlighted. If one believes that monetary policy was unexpectedly tight during, e.g., the term of U.S. Federal Reserve Chairman Paul Volcker, then one should look at the effects of the contemporaneous monetary policy shocks in order to analyze the effects of monetary policy in this period. Secondly, one could also view the empirical analysis as a way of identifying when and where the model is a good approximation to the data (and where it

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[1] The literature is a very large one, but Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2003, 2007) are widely regarded as seminal contributions.

[2] A possible explanation for why this is almost never applied in the literature is that thus far there is no Dynare facility for this.

is not). Large shocks in the estimated model without historical corroboration, or the absence of large shocks when history points to their existence, may be seen as evidence that the model needs to be amended to better account for these episodes. This is the way the literature has evolved, so that, e.g. DU adapt the Smets and Wouters (2007) model by including additional wedges between the policy rate and private and government borrowing rates in order to fit the model to the data for the 2007-2008 financial crisis episode. To assess the need for and explanatory power of these adaptations in the 2007-2008 crisis period, one needs to isolate the shocks to these wedges during this period. This is what the third shock decomposition approach described in this paper does.

We describe the role of shocks in the context of the reduced-form state space representation of DSGE models in Section 2. We then present our suggested method for isolating the impact of exogenous shocks for a particular period alongside the two alternatives mentioned above. In Section 3 we briefly present our extended version of DU's (2015) DSGE model, which we use to illustrate the differing interpretations of particular recent episodes in U.S. macroeconomic history. We chose the DU model as it is built on the seminal Smets and Wouters (2007) model, but expanded to include a role for fiscal policy and financial frictions. These are important features of the 2007-2008 financial crisis, which is one of the historical subperiods we focus on below. The model also allows government investment to generate a degree of increasing returns.

In Section 4 we present the historical decomposition for detrended output across the entire sample period, starting in 1948Q2 and ending in 2023Q1. The US economy experienced an eight month recession at the end of the second world war as factories devoted to war production either closed or switched to civilian production (Meltzer (2010)). Our analysis begins at the end of the recovery in the quarter before the second postwar recession that commenced at the end of 1948 and lasted for nearly a year. From the start of the Korean war in mid 1950 to the start of the great recession in 2008 detrended output appears relatively elevated relative to steady state in nearly every quarter. From then it declines and remains permanently below its detrended steady state value from mid 2015 onward.

In Section 5 we show how the different methods for calculating shock decompositions can generate very different interpretations of the same historical episodes. We first consider the period of relatively high growth from 1964Q2 to 1966Q1. Whereas two of the decompositions suggest fiscal policy played an important role in raising output, perhaps associated with implementation of President Johnson's tax cuts at the beginning of this period and implememtation of the 1964 Economic Opportunity Act (EOA) in 1964Q4, our preferred decomposition suggests that much of the good performance of the US economy during this period can be attributed to the momentum generated by the legacy of shocks from previous quarters.

Similarly, we analyze the period surrounding the financial crisis, from 2006Q4 to 2009Q3. According to the most commonly used decomposition, the year preceding the crisis could be characterized as a time of relative stability, where the benefits of expansionary monetary policy and positive shocks to both government and private investment were partly offset by the downward pressure asserted by intermediate-term interest rates and the shocks to total factor productivity and preferences. From 2007Q4 onwards we see a subtle but growing imbalance between these forces that from one quarter to the next tips the economy further and further into recession. An alternative decomposition interprets the financial crisis as largely driven by negative preference shocks combined with an intensification of previous negative shocks to private investment. According to this decomposition, the turmoil in the financial markets that characterized this period, i.e. the observed rise in both corporate bond and term spread, had either a muted effect on output in the case of the corporate spread, or for the case of the five-year spread a mild pro-cyclical impact. Furthermore, neither the conventional nor unconventional elements of monetary policy implemented by the US Federal Reserve appear to have had much of an ameliorating effect. By contrast, our preferred decomposition assigns a far more important role to interest rate spreads in explaining the evolution of the crisis alongside the preference shock. Prior to the recession, expansionary monetary policy is not contemporaneous but is mostly the lagged effect of earlier periods. This decomposition also clearly identifies the extraordinary countercyclical monetary and in particular fiscal policies implemented during the crisis as shocks to the ordinary evolution of government expenditure and the estimated Taylor rule.

We explain how these counterintuitive and often contradictory interpretations arise from the particular shock decomposition formula used to analyze the same estimated shocks from our model. However, not always do the different decomposition methods yield such divergent interpretations. We examine the period from 1979 to 1987, which corresponds to the eight years that Paul Volcker served as Chairman of the US Federal Reserve. All three methods highlight both the profound negative impact of the contractionary monetary policy Volcker implemented during his term and the positive impact of shocks to private investment during the mid 1980's.

We also consider the behavior of the economy during the period corresponding to the last four years prior to the economic downturn associated with the onset of the Covid pandemic, from 2016Q2 to 2020Q1. According to our preferred decomposition, private investment and price markups had an increasingly negative effect on output, all of which augment the ongoing impact generated by previous negative preference shocks as well as shocks to government spending and monetary policy. At the same time, there is evidence of a very modest expansionary role for both tax and monetary policy. By contrast, the most commonly used decomposition suggests that monetary policy initially plays a very significant positive but diminishing role, while another suggests that monetary policy exerts ever-increasing downward pressure on output. Similarly, they each assign important, though opposing, roles for the intermediate-term bond spread over the policy rate.

The parameters of our model are estimated using data from 1948Q2 to 2020Q1—the last quarter before the economic disruptions caused by global covid pandemic significantly affected the US. We then use the model to consider covid's impact on output from 2020Q2 to 2023Q1. Here the different methods all assign a similarly large contractionary effect to the intertemporal preference shock followed in magnitude by the drop in private investment. Our preferred method is largely in accord with the differencing approach regarding the countervailing positive impact of fiscal policy and TFP as well. However, this method also suggests that at the start of the pandemic, the economy was already performing below what the model determines the long-run balanced growth path, and the accumulated impact of previous shocks mildly exacerbated the impact of the crisis. This is the episode that also features in Section 6 where we separate the impacts of shocks that accrue from the start of the episode under examination from the continuing impact of legacy shocks from earlier periods. The resulting figure combines information from the traditional approach with our preferred method and provides the best overall representation of how the different shocks affect the variables under analysis.

In the Appendix we describe the data extensions and resultant changes to variance and historical decompositions of the DU model used in Section 5. We think the model is of interest in itself as it provides an interesting account of the evolution of the US macroeconomy and the influence of monetary and fiscal policy over the period 1948Q2-2023Q1. The model extends the DU two-agent New Keynesian (TANK) DSGE model with private and public capital to include twelve data series and twelve exogenous shocks. The additional data series are the interest rates for intermediate-term and long-term government debt, which are added to the policy rate and short-term corporate bond rate that were included in the original DU paper. Thus the extended model has four interest rates, all determined by data. This is important addition, as these interest rates often do not move together during the periods of interest, and not including them generates counterfactual responses in the model. The inclusion of two additional observed variables necessitates the inclusion of additional exogenous shocks, one of which we choose to be a preference shock. Secondly we update and enhance the data used in the model. We improve the measure of public debt so that it comprises net, rather than gross, federal debt, and also state and local debt, and we extend our sample period to the first quarter of 2023. Since the policy rate controlled by the Federal Reserve reached the effective zero bound during the financial crisis and its aftermath, we incorporate the shadow rate calculated by Wu and Xia (2016) in our estimation. We demonstrate by means of variance decomposition and historical shock decomposition for the period 1948-2023 the effect of our modifications to the model. With regard to the decompositions, we find that the inclusion of a preference shock plays a significant role in our model, with large preference shocks having negative impacts on consumption in the third quarter of 2001, coinciding with the events of 9/11, in the last two quarters of 2008, coinciding with collapse of Lehman Brothers and in 2020Q2 at the start of the covid pandemic. The estimated model also provides an intuitive picture of the evolution of the U.S. economy during the seven and a half decades following World War II, and in particular highlights the impact of the Federal Reserve's tight monetary policy during the 1980s and early 1990s.

2 Decomposition

The model is solved as a log-linear first order approximation around the balanced growth path which takes the form, once the saddle path dynamics have been incorporated:

$$\mathbf{s}_t = \mathbf{A}\mathbf{s}_{t-1} + \mathbf{B}\boldsymbol{\varepsilon}_t \tag{1}$$

$$\mathbf{x}_t = \mathbf{C}\mathbf{s}_t \tag{2}$$

where $\mathbf{s}_t$ is a vector of m state variables, $\mathbf{x}_t$ a vector of n observables, and $\boldsymbol{\varepsilon}_t$ a vector of exogenous shocks of dimension equal to or greater than n. Matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are appropriately dimensioned. Iterating the state equation forward means that at any point t:

$$\mathbf{x}_t = \mathbf{C}\mathbf{A}^t\mathbf{s}_0 + \sum_{j=1}^{t} \mathbf{C}\mathbf{A}^{t-j}\mathbf{B}\boldsymbol{\varepsilon}_j. \tag{3}$$

One can use this equation for any period to decompose the deviation of a variable from its balanced growth path into the contribution from each exogenous shock and the initial condition. For a stable equilibrium, $\mathbf{A}^t$ should tend to zero as t becomes larger, and so at the end of the sample almost all the variation in variables is explained by the exogenous shocks.

When examining the shock decomposition over a historical period from $t+1$ to $t+h$, an obvious procedure is to subsume both the initial condition $\mathbf{s}_0$ and all shocks prior to $t+1$ into a new term that represents the state of the economy at time t, $\mathbf{s}_t$, and to treat only subsequent shocks within the shock decomposition:

$$\mathbf{x}_{t+h} = \mathbf{C}\mathbf{A}^h\mathbf{s}_t + \sum_{j=t+1}^{t+h} \mathbf{C}\mathbf{A}^{t+h-j}\mathbf{B}\boldsymbol{\varepsilon}_j,\ h \in \{1, ..., k\}\,. \tag{4}$$

This decomposition (DC1) isolates shocks occurring after time t, the second term, from those that occurred in previous periods. The first term captures the initial condition at the start of the episode and represents how distant the variable of interest is at that moment from its balanced growth path value. The value of this term in the ensuing periods captures how much the evolution of the variable is attributable to the convergence properties of the model, rather than the contribution of the subsequent shocks.[3]

The most common procedure, however, is to obtain the smoothed shocks for the entire sample, and then extract the shock decomposition for the period being analyzed.[4] This is the approach adopted by Brzoza-Brzezina and Kolasa (2013), who use quarterly US data from 1970 to 2010 to estimate three different models, and then compare the shock decompositions from those models starting in 2000. This is also the approach in Del Negro *et al.* (2013), who estimate a model from 1984Q1 to 2013Q1, and then evaluate the shock decomposition from 2007. Starting from date $t+1$ continuing through date $t+k$, we can use (3) to generate

$$\mathbf{x}_{t+h} = \mathbf{C}\mathbf{A}^{t+h}\mathbf{s}_0 + \sum_{j=1}^{t} \mathbf{C}\mathbf{A}^{t+h-j}\mathbf{B}\boldsymbol{\varepsilon}_j + \sum_{j=t+1}^{t+h} \mathbf{C}\mathbf{A}^{t+h-j}\mathbf{B}\boldsymbol{\varepsilon}_j,\ h \in \{1, ..., k\}\,. \tag{5}$$

The first two terms of this expression are incorporated into $\mathbf{C}\mathbf{A}^h\mathbf{s}_t$ in (4), representing the initial values up to the start of the episode. However the shock decomposition (DC2) of the authors above, and currently in Dynare 4.5.7, uses the last two terms, with the first term representing the effect of the initial values. Thus, shock decompositions calculated this way place significant weight on the impacts of shocks that occurred before the event, and as the effects of these shocks can be highly persistent, the interpretation of any episode may be highly influenced by events that happened years earlier.[5]

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[3] We have programmed a general code in Matlab for calculating this shock decomposition that can be implemented for a model estimated using Dynare 4.5.7, available upon request.

[4] In the software package Dynare, the shock_decomposition command generates this.

[5] Despite a (non-exhaustive, admittedly) trawl of the literature, we have found no recently published papers that use DC1 rather than DC2. Anzoategui *et al.* (2019) do use this decomposition in a nonlinear model that takes account of the zero lower bound, but given the nonlinearity, they ignore initial effects. We have it on good authority, though, that most prominent central banks use DC1 when analyzing historical shock decompositions.

One possible way to correct for the influence in DC2 of much earlier shocks is to examine the difference in shock decompositions before and after an event that occurred from period t through $t+h$:

$$\mathbf{x}_{t+h}-\mathbf{x}_t=\mathbf{C}(\mathbf{A}^{t+h}-\mathbf{A}^t)\mathbf{s}_0+\sum_{j=1}^{t}\mathbf{C}\mathbf{A}^{t-j}(\mathbf{A}^h-I)\mathbf{B}\boldsymbol{\varepsilon}_j+\sum_{j=t+1}^{t+h}\mathbf{C}\mathbf{A}^{t+h-j}\mathbf{B}\boldsymbol{\varepsilon}_j,\ h\in\{1,\ldots,k\}\,. \tag{6}$$

This is the approach (DC3) taken by DU, who estimate their model of the U.S. economy from 1948 to 2018 and present their decomposition of output for the last four quarters in the sample to evaluate the cause of the financial crisis. The first term once again does not represent the state of the economy at time t, only whatever effects remain from the initial conditions at the beginning of the sample; the second two terms then provide the shock decomposition. Thus, while differencing nets out the initial value of the variables, it does nothing to reduce the impact of the previous shocks on the net changes. This is because the shock decomposition comprises not merely the last term in (6), but the second-to-last term as well, and that still incorporates the influence of prior shocks from the very initial period onwards.

3 The Model

We demonstrate how the different methods of shock decomposition can yield starkly different interpretations of historical subperiods by applying all three methods of shock decomposition to shocks generated by a medium-sized DSGE model—our updated and extended version of DU, which is itself built on the seminal Smets and Wouters (2007), but expanded to include a role for fiscal policy and financial frictions and increasing returns to scale generated by an externality in public capital. Using twelve rather than ten time series necessitates including two additional shocks: to long term government yields and a shock to the optimising agents' time preference. We describe the model briefly below and the data extensions in the Appendix.

DU estimate a model of the U.S. economy for the period between 1948 and 2008 which includes three separate interest rates: the intermediate-term rate at which the government borrows, r^b; the corresponding rate for private firms, r^k; and the policy rate determined by the Federal Reserve, r^f. These interest rates implicitly define three corresponding wedges between each of the three pairs as depicted in Figure 1. We improve a little on their model. Specifically, DU's estimation only includes two data series for these interest rate variables: data for r^f, along with the wedge ω^b, representing the difference between r^b and r^k. This means that while the distance between r^b and r^k is determined by the data, the levels of these two variables are determined within the model, along with the other two wedges (see Figure 1).

We introduce an additional interest rate, r^l, on long-term government bonds, which creates an additional wedge that allows us to isolate financial frictions from differences stemming from shifts in the yield curve.[6] Hence our model includes four interest rates and four corresponding wedges, all determined by data: ω^f represents the wedge between the policy rate r^f, which is also an overnight interest rate and the five-year interest rate at which the U.S. government does

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[6] DU use the series provided by FRED that represents the wedge between corporate bonds with "maturities 20 years and above" and the 10 year treasury bond.

(a) Drautzburg and Uhlig (2015)

(b) This paper

Figure 1: The relationship between the different interest rates and corresponding wedges in the model. Red circles designate the observed time series used to estimate the model in (a) Drautzburg and Uhlig (2015) and (b) this paper.

most of its borrowing; ω^b represents the difference between the yields on five-year and twenty-year government bonds; and ω^l is the yield spread between twenty-year government bonds and twenty-year corporate bonds. The wedge ω^l represents the sort of financial frictions associated with recessions, isolated from the behavior of the yield curve represented by both ω^f and ω^b. These three wedges together sum to the wedge ω^k.

The inclusion of two additional data series necessitates the inclusion of additional shocks to provide sufficient degrees of freedom for the estimation. We follow Justiniano, Primiceri and Tambalotti (2011) and add a preference shock, which they show has significant explanatory power for consumption movements in post-war U.S. data. Including this preference shock has the benefit of incorporating unmodeled effects on consumer demand, such as the impact from the start of the financial crisis of the drop in house prices (Mian and Sufi, 2018). We model the preference shock in a standard way, following Lindé and Trabandt (2018), as a multiplicative shock, ς_p, to the discount factor, β, so that optimizing agent j's utility function with an external habit is represented by

$$U = E\left[\sum_{s=0}^{\infty} \varsigma_{p,t}^{s}\beta^{s}\frac{1}{1-\sigma}(C_{t+s}(j) - hC_{t+s-1})^{1-\sigma}\exp(\frac{\sigma-1}{1+\nu}n_{t+s}(j)^{1+\nu})\right]$$

where

$$\log\varsigma_{p,t} = \rho_P \log\varsigma_{p,t-1} + \varepsilon_{p,t}, \qquad \varepsilon_{p,t} \sim N(0,\sigma_{\varepsilon p}^2)$$

and $C_t(j)$ is time t consumption of optimizing agent j, C_t is the time t aggregate consumption across all optimizing agents, and $n_t(j)$ represents the optimizing agent's time t labor supply. The preference shocks only enter the Euler equation; the effect of a positive preference shock is to effectively increase the discount factor, which results in higher savings and less consumption by the fraction of agents that are not credit constrained in line with intuition.

Intermediate goods are produced by firms using a Cobb-Douglas technology that combines effective private capital, which is the stock $K_t^p(i)$ produced in the prior period multiplied by a capital utilization term $u_t(i)$, and effective labor, the input supplied by households $n_t(i)$ multiplied by trend growth μ^t. Production is also subject to a fixed cost Φ, and and an external effect generated by the contemporaneous stock of government capital K_t^g:

$$Y_t(i) = \tilde{\epsilon}_t^a \left(\frac{\left(K_t^g\right)^\Lambda}{\int_0^1 Y_t(j)\,dj + \Phi\mu^t} \right)^{\frac{\zeta}{1-\zeta}} \left(u_t(i)\,K_t^p(i)\right)^\alpha \left(\mu^t n_t(i)\right)^{1-\alpha} - \Phi\mu^t \tag{7}$$

Aggregating firm-level output using $Y_t = \int_0^1 Y_t(i)\,di + \Phi\mu^t$ yields

$$Y_t = \tilde{\epsilon}_t^a \left(K_t^g\right)^{\zeta\Lambda} \left(u_t K_t^p\right)^{\alpha(1-\zeta)} \left(\mu^t n_t\right)^{(1-\zeta)(1-\alpha)} - \Phi\mu^t. \tag{8}$$

The production functions (7) and (8) differ from DU in only one respect: by including the exponent Λ as an estimated parameter, we permit government capital to potentially generate nonconstant returns to scale for the aggregate economy. Marginal costs for firms are now

$$MC_t = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)} \frac{W_t^{1-\alpha}\left(R_t^k\right)^\alpha \mu^{-(1-\alpha)t}}{\left(\frac{\left(K_{t-1}^g\right)^\Lambda}{Y_t + \mu^t\Phi}\right)^{\frac{\zeta}{1-\zeta}} \epsilon_t^\alpha}, \tag{9}$$

which in turn enters firms' first order conditions, determining the demand for capital and labor, and also enters the equations that determine the price of final goods and the profits remitted to households. As in DU, households supply differentiated labor to unions, which sell it to labor packers. Calvo pricing obtains in both the goods and labor markets, and when unions have the opportunity to adjust wages, they set them to maximize the welfare of a representative household.[7]

4 Estimation and Historical Decomposition 1948–2023

The model is estimated using Dynare.[8] The priors and posteriors of the model's parameters are set out in Tables 2 and 3 in the Appendix. Corrado *et al.* (2021) use US data from 1985Q1 to 2020Q3 to estimate a model in which shocks may be drawn from two different distributions, one associated with disasters. Cardani *et al.* (2022) incorporate covid-specific shocks to facilitate the estimation of their DSGE models in the US and from 1998Q4 to 2021Q4 for the Euro area, respectively. Our approach is different, we estimate our model using quarterly time series starting from 1948Q2 through to 2020Q1, but use the model's parameters to analyse the behaviour of the economy till 2023Q1. We choose not to add any special pandemic-related adjustments—preferring to interpret the impact of covid using only conventional shocks.

Table 1 in the Appendix displays the variance decomposition for the whole sample period; however, the relative importance of specific shocks differs over time in interesting and significant

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[7] This matches the code for the estimated model developed by DU though the article itself asserts that unions maximise the welfare of rational optimizing households only.

[8] We estimate DU's model using the code they supply (see https://ideas.repec.org/c/red/ccodes/14-44.html). We are able to replicate their estimated model, aside from the variance decomposition, for the whole sample period. In addition, we found a small error in the original code for the linearization of the model's profit equation and provide our own estimates for a corrected version of their model in Table 1.

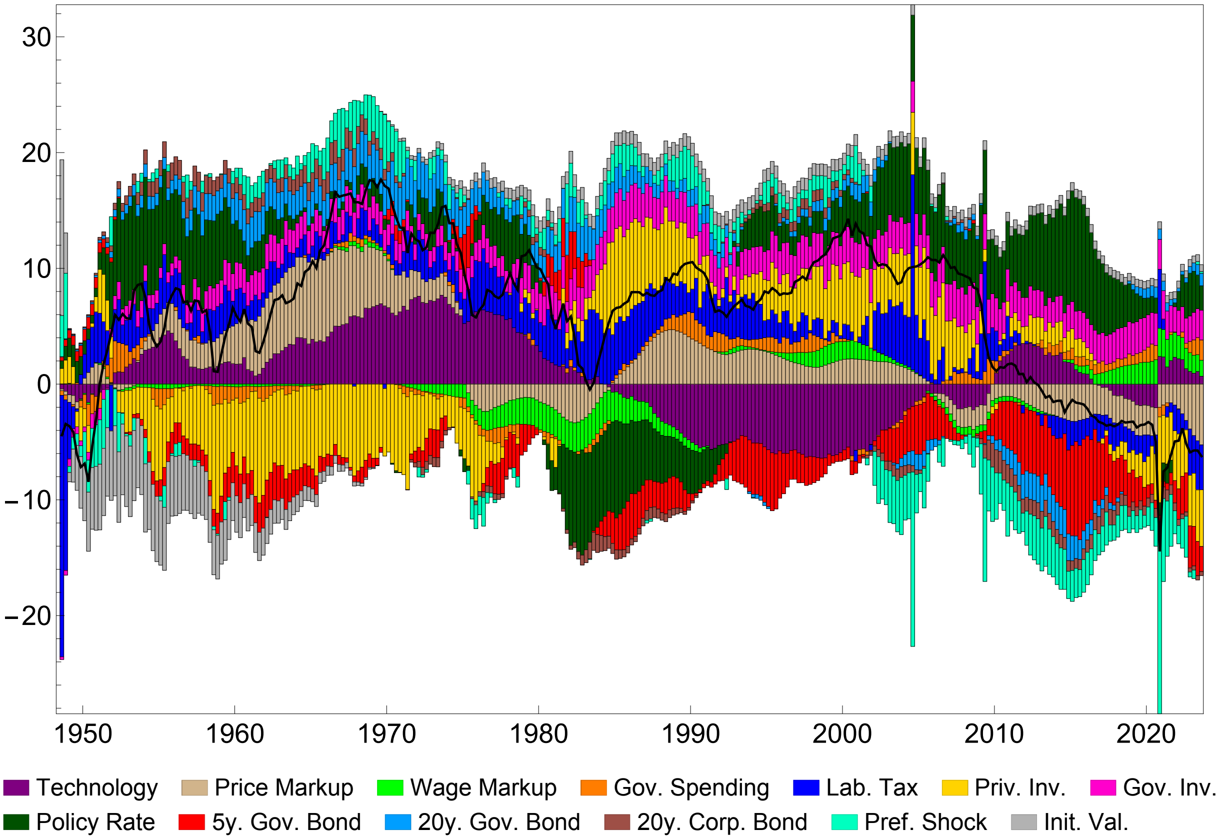

Figure 2: Shock decomposition 1948Q2–2023Q1, calculated using the decomposition formula (6). The black line represents output.

ways. Figure 2 illustrates the evolution of the shock decompositions over time using the shock decomposition formula in (6), where the shock decompositions in each time period sum to the log of output relative to trend (this corresponds to the procedure in Dynare). To better see the influence of both monetary policy, broadly defined to include the policy rate as well as the corporate spread and two term spreads in our model, and fiscal policy, defined as labour taxes as well expenditure by the government on both consumption and investment, we isolate two subsets of shocks from Figure 2 together with the underlying data on the four different interest rates and debt in Figures 3 and 5.

Figures 2 and 3 show the following patterns: very expansive monetary policy during the 1950s that continued in a more muted fashion during the 60s and 70s, followed by a sharp contraction during the 1980s associated with the anti-inflationary policies instituted by Fed Chairman Paul Volcker in 1979 upon his appointment by President Jimmy Carter, that continue under Chairman Alan Greenspan till 1992, through the George H.W. Bush administration. From then on, the effect of monetary shocks has been largely expansionary, save for a brief interlude at the start of the pandemic. In particular, during the period between 2009Q3 to 2015Q4 when the shadow policy rate descended below zero, the pattern we see in the data is closely mirrored in the manner in which monetary policy shocks influence output.

The four interest rates are highly correlated with each other, but the gaps between them fluctuate. Beyond the increases in interest rates that correspond to rises in policy rate, widening

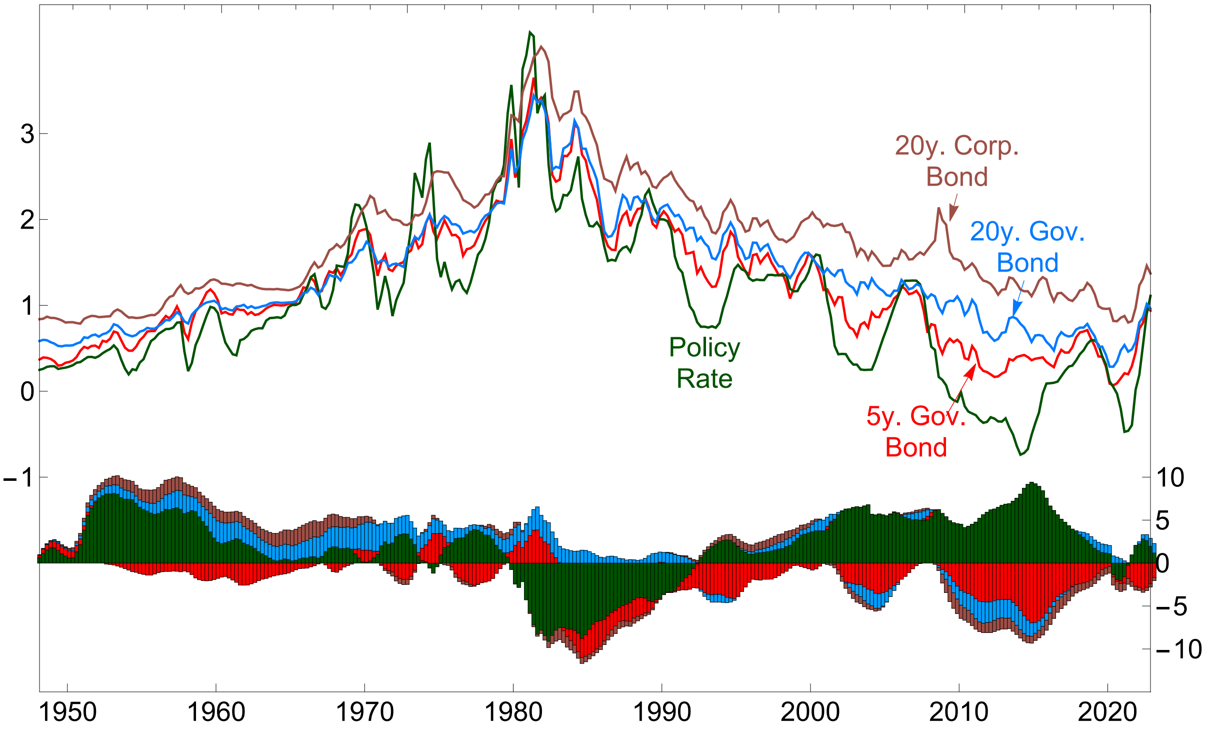

Figure 3: Left Axis: Interest Rates data used in estimating the model in this paper. Green: Policy Rate, March 1948 to June 1954 3-Month Treasury Bill: Secondary Market Rate, from the start of the subprime mortgage crisis in April 2007 through March 2023 shadow rate calculated by Wu and Xia (2016), and for all other months the Federal Funds Rate, coverted to quarterly rates. Red: 5-year U.S. Treasury coupon note yield 1953Q2 to 2023Q1, supplemented by data calculated by Ibbotson (2016) on intermediate-term U.S. Treasury bond yields for the period between 1948Q2 to 1953Q1. Blue: 20-year U.S. Treasury coupon note yields supplemented by data calculated by Ibbotson (2016) on long-term U.S. Treasury bond yields for the period from 1948Q2 to 1953Q1 and 1987Q1 to 1993Q3. Brown: Corporate bond yield, Moody's Baa index at quarterly rates. Right Axis: The historic shock decompositions for the impact of the four interest rates from Figure 2 on output.

spreads appear to depress output in Figure 3 and that is particularly true for the interest on five-year government bonds. Further evidence for the negative relationship between the interest rate and the impact of the spreads can be seen in Figure 4, where we plot the impact of the policy rate, 5 year government bond, 20 year government bond, and 20 year corporate bond policy rate against the policy rate r_t^f and three spreads ω_t^f, ω_t^b, and ω_t^l in the corresponding data.

In terms of post-war fiscal policy, note that from 1948Q2, ten quarters after the end of the second world war to the start of the Korean War in the second quarter of 1950, our measure of the overall U.S. net debt burden was still declining from 61.2% of GDP to 43.8%. In the two decades from 1960Q4 to 1979Q4 it steadily dropped again from 45.1% to 25.3%. From then the debt burden began to rise, and at an accelerated rate following the enactment of the first phase of the Economic Recovery Tax Act on October 1 1981.[9] It stabilised at about 49% in the early

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[9] Congress passed the legislation on August 4, 1981, and President Ronald Reagan signed it into law on August 13, 1981.

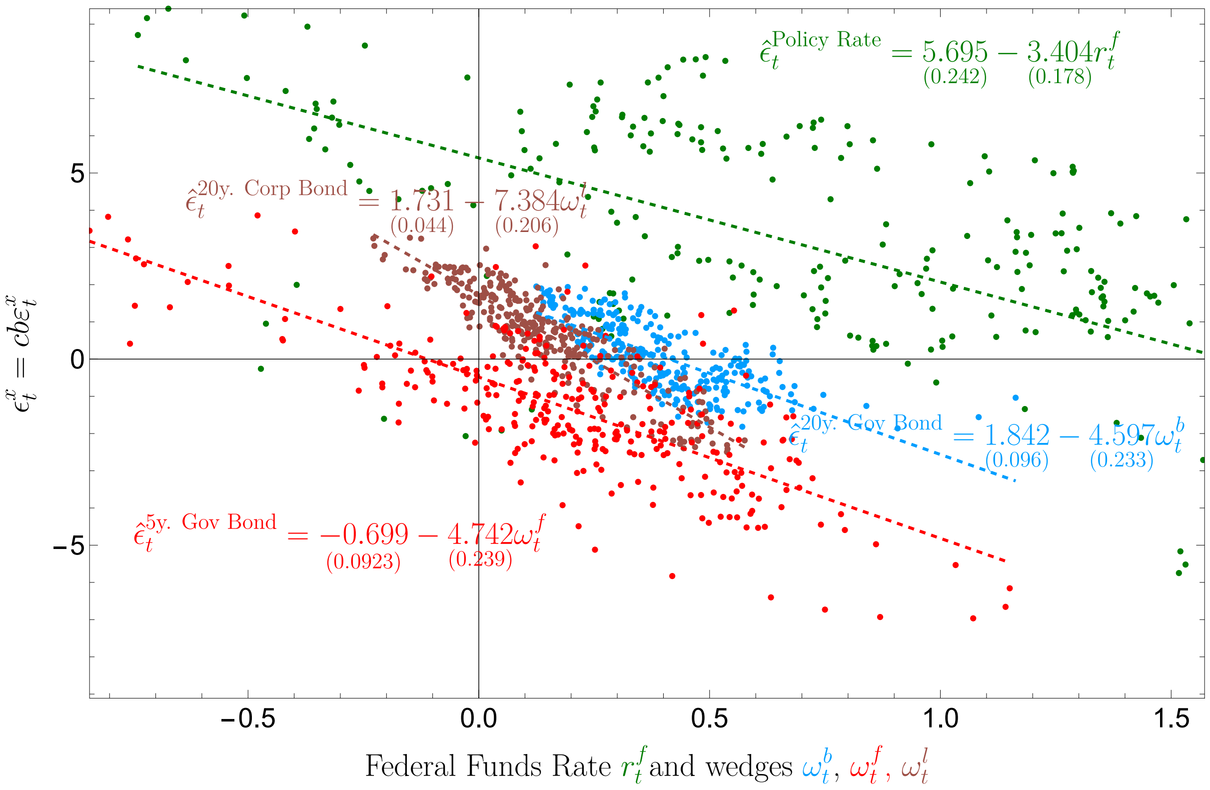

Figure 4: Horizontal Axis: Policy rate r_t^f, and interest rate spreads ω_t^f, ω_t^b, and ω_t^l. Vertical Axis: Effect on output from historical shock decomposition for policy rate, 5 year government bond, 20 year government bond, and 20 year corporate bond.

and mid 1990's and then began to drop again, reaching 30.7% in 2001Q2. From then, following the events of 9/11, the debt grew rapidly (the debt/GDP ratio fluctuates as much as it does rather than increasing nearly monotonically, only because of the unusually large fluctuations in the denominator during the great financial crisis and the covid pandemic).

The impact of this history on output is expressed most directly through the labor tax, to a lesser extent through government expenditure, as well as indirectly through the government bond. If we first divide the entire sample period into three parts, we can see in Figure 2 that the impact of the labor tax is negative in the immediate post world war II period, when our measure of the debt is falling steeply and output is below trend. In subsequent years, the economy recovers and as the debt resumes its decline during the 1960's, labor taxes no longer need adjust to stabilize it, and so their impact on output is nearly always positive. This still remains the case during the 1980s when the debt burden reverses course and begins to increase. Only from 2012Q2, when the debt burden crosses the threshold of 75% of GDP does the impact of the labour tax shocks become uniformly negative, save for 2020Q2 and 2020Q3 at the start of the covid pandemic.

Returning to Figure 3, the combined effect on output of the 5-year and 20-year government bonds fluctuates around zero throughout the late 1940s to the early 1960s (the former mostly negative, the latter mostly positive) and largely positive till the early 1980s. From then on, as the debt burden is rising, the combined effect of both types of bonds is nearly always negative.

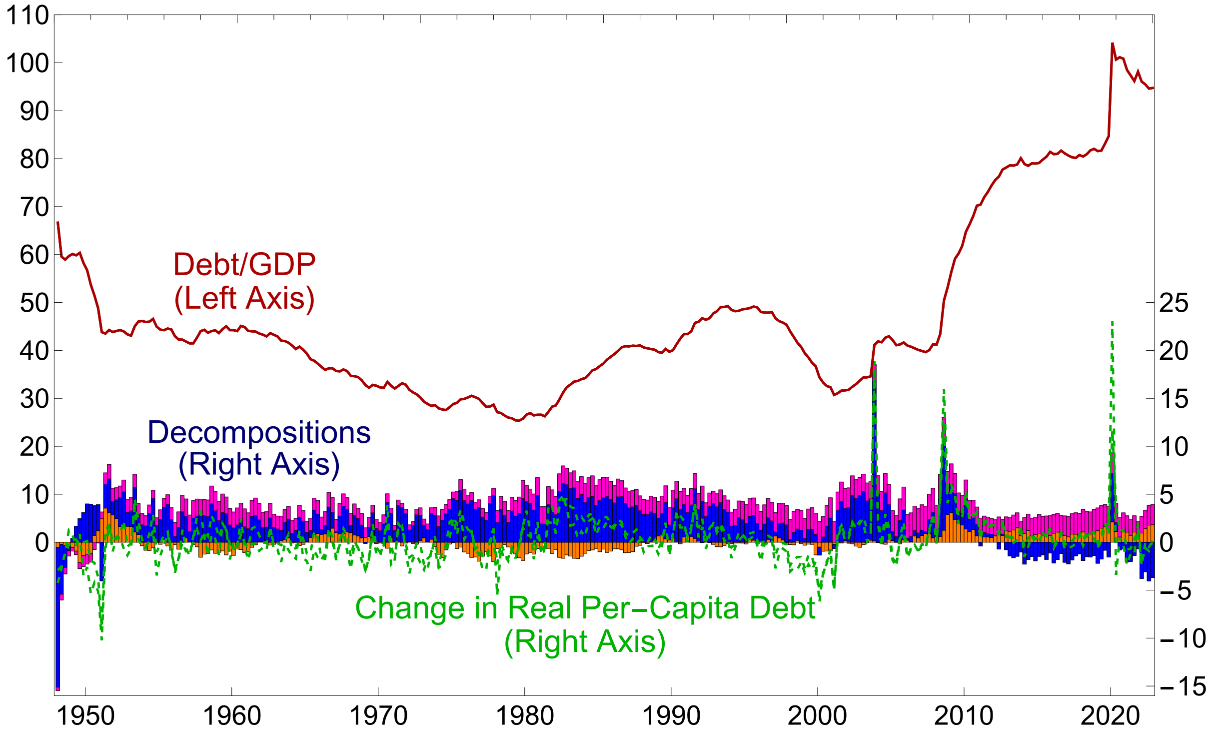

Figure 5: Left Axis: Our series of total net Debt/GDP. Right Axis: The historic shock decompositions for the impact of government consumption, labour taxation and government investment from Figure 2 and the change in real per-capita debt (demeaned) on output used in the model.

We also see in Figure 2 strong effects for shocks to the shadow price on private investment, the price markup, the technology shock and government investment. Private investment exerts a negative effect starting from mid 1951 that intensifies through the rest of the decade which only begins to dampen in the mid 1970's. From 1981Q1 to the start of the recession that lasted from 1989Q4 to 1991Q1 the effect on output is strongly positive, and positive thereafter before declining precipitously during the great financial crisis in late 2008. From 2016Q3 to 2023Q1 the impact is always negative, with a particularly large effect in 2020Q2 coinciding with the start of the covid pandemic.

The impact of the price markup is positive between 1953Q2 to 1974Q3 and again from 1984Q2 to 2005Q1. From then on its effect is negative, increasingly so towards the end of the sample. The technology shock exerts an increasingly positive effect on output from 1950Q4 till the end of the 1970's, thereafter it begins to decline. From 1987Q4 to 2009Q2 it is negative, though it is declining for much of the period that precedes the start of the pandemic. Finally, government investment exerts a positive effect on output from 1951Q2—reflecting the strong point estimate we find in our estimations for the external effect generated by public capital.

Throughout much of the last half of the twentieth century the influence of preference shocks in Figure 2 is typically positive, but it turns decidedly negative in the third quarter of 2001 and remains negative thereafter, suggesting a long-lasting decline in the value of $\varsigma_{p,t}$ beginning in the third quarter of 2001 (coinciding with the events of 9/11), intensifying during most of 2003 (the U.S. invasion of Iraq), before gradually dissipating by the end of 2005. Before then, in 2004Q1 there is a very large negative shock which offsets the positive shock from the tax in labour, perhaps a response to passage of the tax cuts in the Jobs and Growth Tax Relief Reconciliation Act signed into law in May 2003. A new sharp decline appears in the last quarter of 2008, in the aftermath of the first and second bail outs of Fannie Mae and Freddie Mac in July and September 2008 and the collapse of Lehman Brothers, also in September 2008.[10] Finally, there is one more large negative shock in 2020Q2 which coincided with the start of the covid pandemic in the US.

5 Comparing Alternative Shock Decompositions

We compare the alternative shock decompositions by discussing the model's explanation of key episodes in U.S. macroeconomic history, which we argue provides a generally intuitive picture of the evolution of the U.S. economy during the seven decades following World War II. The model highlights the impact of the Federal Reserve's tight monetary policy during the 1980s and early 1990s and, compared to DU, ascribes a far more important role for bond spreads, particularly the part of the yield curve associated with intermediate-term maturities. This effect is particularly pronounced from the late 1980s onwards, and intensifies still further during the financial crisis.

To understand how the three different methods (4)–(6) for calculating shock decompositions in Section 2 can in certain instances, generate profoundly different interpretations of particular historical episodes, we first isolate the instantaneous impact of the values of the shock process itself represented by the term CBe_j in (3), on output for every quarter during two different three decade long periods, between 1948Q2 and 1985Q4 in Figure 6a and between 1986Q1 and 2023Q1 in Figure 6b.

In Figure 6 the preference shocks, along with shocks to the labor tax and private investment, and to a lesser extent government spending and technology, appear to dominate throughout both the subsamples. Still, across the different time periods, differences in the patterns of shocks do emerge. The raw monetary policy shocks in Figure 6a were more amplified (and mostly negative) during the late 1970s and early 1980s, but much more muted during the prior and subsequent periods. The shocks to private investment in Figure 6b become notably more muted starting from the early twenty-first century. How do these patterns express themselves when fed through the three different shock decompositions formulae (4)–(6)? We focus primarily on (4) and (5) for the three episodes and then discuss the decomposition (6).

Consider the two-year period of rapid above trend U.S. growth from the second quarter of 1964—the period that followed the signing into law of the Tax Reduction Act of 1964 by President Johnson. In Figure 7a we apply our preferred shock decomposition (4) to output during this period, where the vector of shocks for 1964Q2 corresponds to the shocks for the same quarter in Figure 6a. Throughout this period the implication of this decomposition

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[10] We find the preference shocks, but not the other shocks, are negatively correlated with the cyclically adjusted price earnings ratio calculated by Robert Shiller, http://www.econ.yale.edu/ shiller/data/ie_data.xls. This suggests that investors bid up the price of equities, relative to earnings, in response to a rise in the willingness to defer consumption and save.

is that both the corresponding shocks to private investment and labor taxation we observe in Figure 6a exert downward pressure on output, partly mitigated by a variety of other shocks, but particularly the price markup and government investment. However, this decomposition method best highlights that in this instance, most of the divergence of output above its steady state value is the cumulative effect of the previous shocks to that date, which are far more important than the shocks which occurred during the relevant period. We contrast these conclusions with those that might be drawn from the shock decompositions generated using (5) and (6) in Figures 7b and 7c.

According to the standard decomposition (5) used to generate the results in Figure 7b the largest contribution to the expansion starting from 1964Q4 is the reduction in price markups, with far smaller roles for technology, labor taxes, government investment, and the interest rates on corporate and 20-year government bonds, along with the preference shock. Mitigating these effects are the negative effects generated by shocks to private investment and 5-year government bonds. Note that (5) merely isolates a sub-period of the decomposition in Figure 2, and so the legacy of the initial conditions from the beginning of the sample in 1948 have nearly all dissipated by 1964.

The reason the shock decomposition in Figure 7b seems so disconnected from the pattern of underlying shocks in Figure 6a for the corresponding period stems from the way the formula in (5) conflates the impact of the shocks occurring in the time period with those occurring beforehand. Hence, although the influence of the preference shocks and shocks to the labor tax appear to have the greatest impact on output in Figure 6a across the entire span of ten years, the manner in which they fluctuate between positive and negative values prior to 1964 means that when aggregating their cumulative effect they largely cancel each other out, which mitigates their influence on output in the periods that follow as measured by (5). By contrast, smaller shocks to government investment are mostly positive and shocks to the interest rate on 5-year government bonds largely negative in the periods prior to 1964Q2, and it is this consistency that ultimately inflates their perceived cumulative impact on the evolution of output from 1964Q2 onwards.

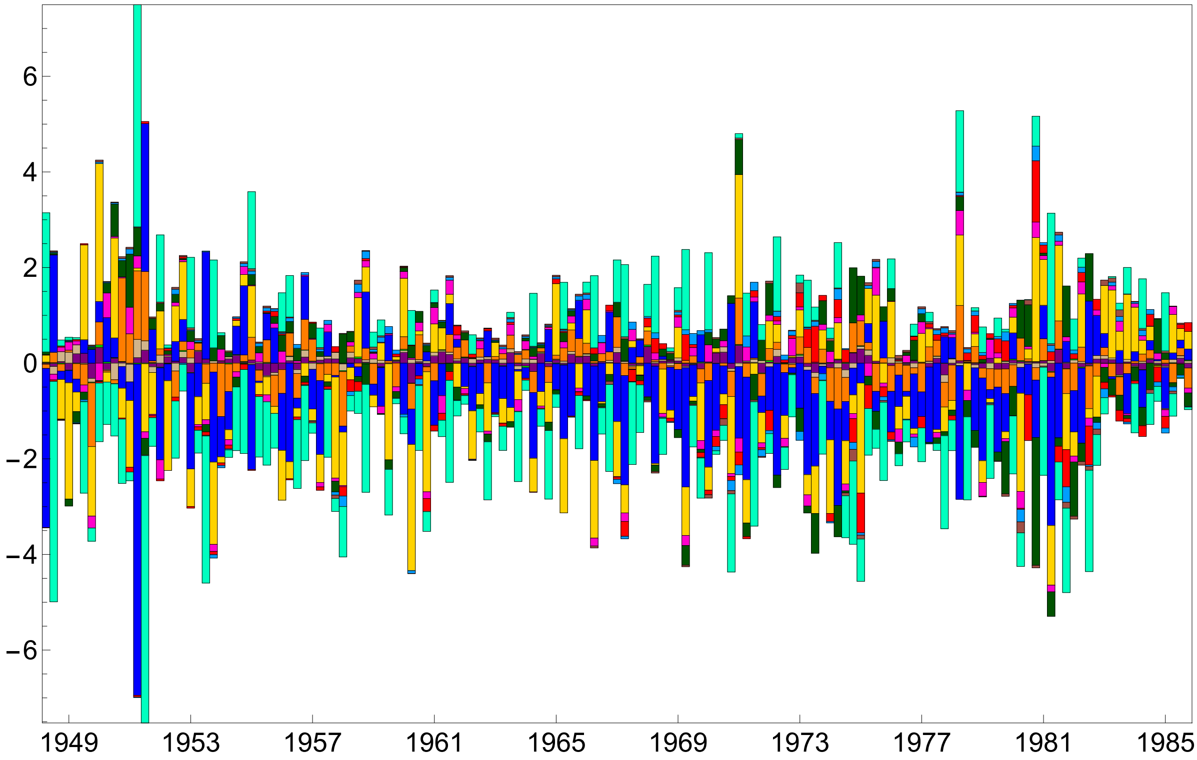
More broadly, the two methods answer different questions. What we learn from (5) is the cumulative effect across time, of a particular type of shock on the variable of interest in each period. By contrast, (4) isolates the impact of each type of shock from those that preceded it, starting at the initial period within the episode under consideration.

One alternative to the standard decomposition, designed to isolate the impact of the shocks within the time period of interest, is to once again follow DU and apply the differencing formula in (6). Yet in practice, analysis of the shocks' impact is still predicated on their previous behavior. This procedure merely draws a contrast between the cumulative impact of the different shocks up to the episode under examination and their subsequent behavior during the episode that is analyzed. For example, the effect of the shocks to 5-year government bonds in the late 1950s and early 1960s in Figure 6b are largely negative; they play little role in explaining the evolution of output from 1964Q2 to 1966Q1. Nonetheless in Figure 7c their positive influence grows over time, because their absence contrasts with their steady and mostly negative influence in the periods prior to 1964Q2. Unlike the standard decomposition in Figure 7b, here the influence of the price markup is much more modest, and the shocks to government investment nearly disappear because their behavior before and after 1964Q2 is so similar—whatever role they played is obscured by differencing. Furthermore, because it is differencing the variable, this type of shock decomposition tells us nothing about where the economy is in 1964Q2 relative to steady state.

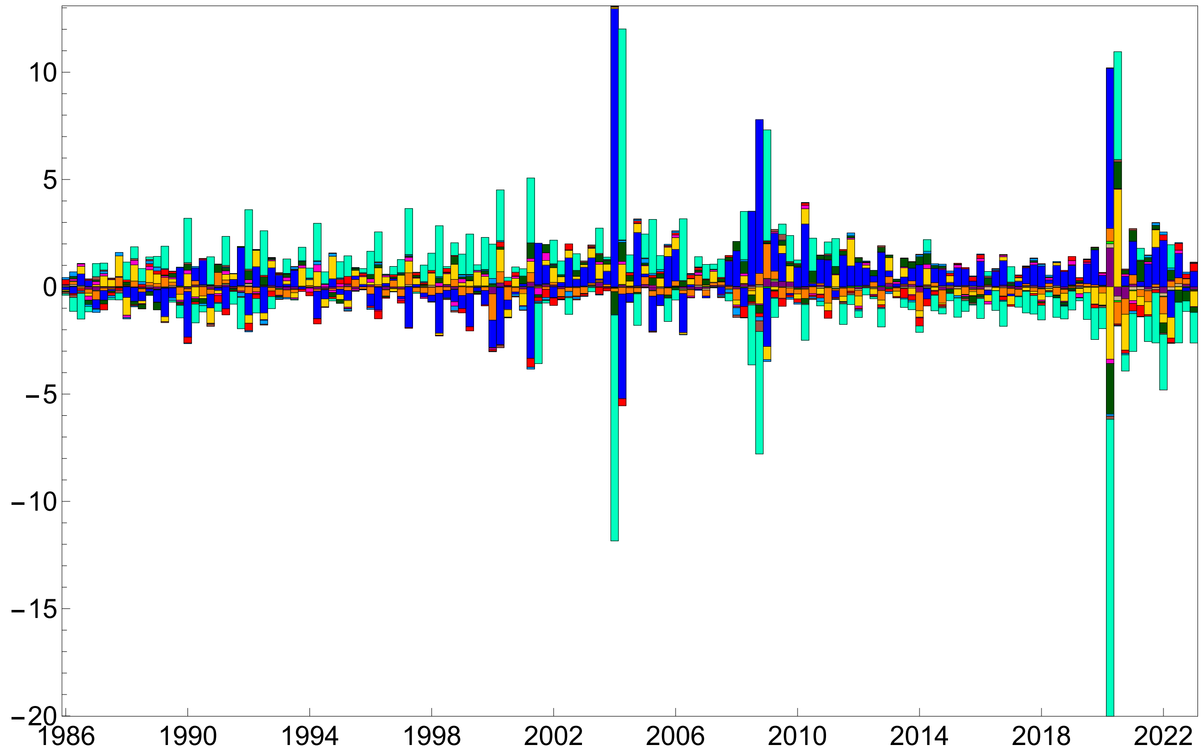
That distinction is particularly important when we consider the immediate run-up to the financial crisis. Figure 8a demonstrates that according to the model, output in the last quarter of 2006 was well above its steady-state level, and that much of its subsequent decline in the next four quarters can be attributed to convergence. By the first quarter of 2008, in addition to convergence, a set of very small shocks appears to exert negative pressure on the economy, offsetting the positive effects of expansionary monetary policy, shocks to the labor tax, and smaller contributions from government investment and the preference shock. From the third quarter of that year, as the recession intensified, some of the decline in output can be ascribed to the accumulating impacts of the shocks to preferences, and from the next quarter the shock to private investment. The largest effect emanates from the shocks to three of the model's four interest rates (one on corporate bonds, r^k, and two on government bonds, r^b and r^l), which counteract the effect of the positive shocks to the other interest rate that relates to monetary policy (the policy rate, r^f). In terms of Figure 1b, all of these changes imply a large increase in ω^k that we observe in the data, which in turn is comprised of increases in ω^b, ω^l and ω^f that we also observe.

By contrast, the standard shock decomposition in Figure 8b, obtained by using (5), suggests rather counterintuitively that not much happened in the economy throughout the period from 2006Q4 to 2009Q3, aside from a large negative preference shock in 2008Q4. The price markup plays a small negative role throughout the entire period, and monetary policy and government investment are expansionary long before the crisis begins. There is some downward pressure on output from the technology shock, but that largely dissipates as the economy veers into severe recession—replaced by small negative shocks to private investment and the interest rates on corporate bonds. Mostly what appears to generate the crisis in 2008-2009 is the intensification of the negative shocks from the cost of government borrowing and preferences. The picture that emerges in Figure 8a is one of growing turbulence as the shocks amplify from early 2008 onwards, whereas Figure 8b presents a picture of an economy that is at first finely balanced between offsetting sets of both positive and negative shocks and is then finally tipped into severe recession by very subtle shifts in their relative importance.

The shocks in Figure 6b are particularly large during 2008, yet the shock decomposition (5) once again applies large weights to events that occurred before the end of 2006. Therefore, because the shocks to government investment prior to the episode under consideration are small but consistently positive, and their impact sufficiently persistent, they exert a strong positive influence on the shock decomposition in Figure 8b. Comparing Figures 8a and 8b we can see how throughout the entire period, as the impact of previous positive monetary policy shocks gradually dissipate, they are gradually replaced with new ones to keep their total impact steady. The raw shocks to preferences are small but uniformly positive between 2006Q4 and 2007Q3, yet Figures 8b and 8c register the negative effects in Figure 6b from earlier periods. Similarly,



(a) 1948Q2 to 1985Q4



(b) 1986Q1 to 2023Q1

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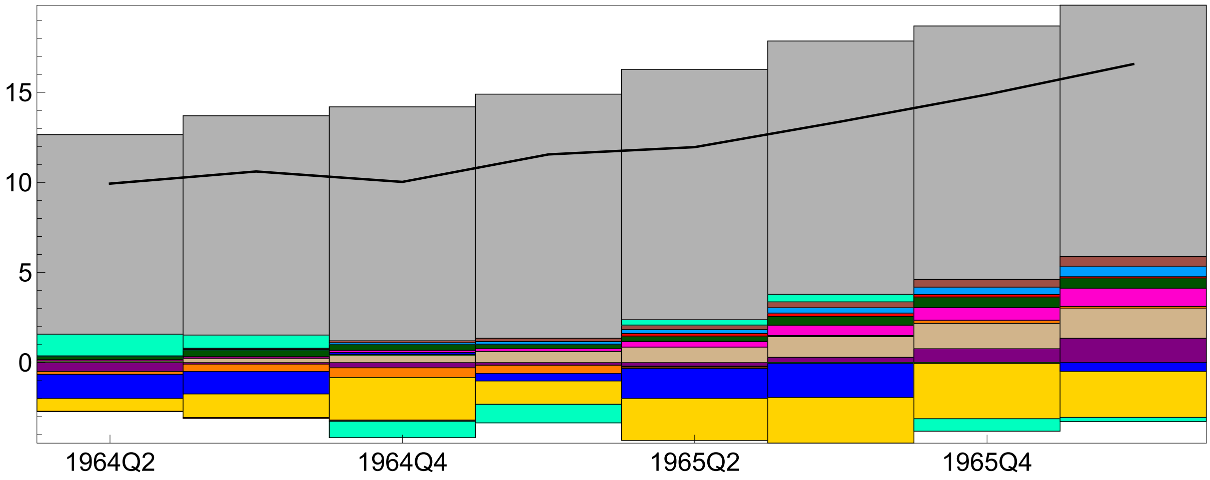
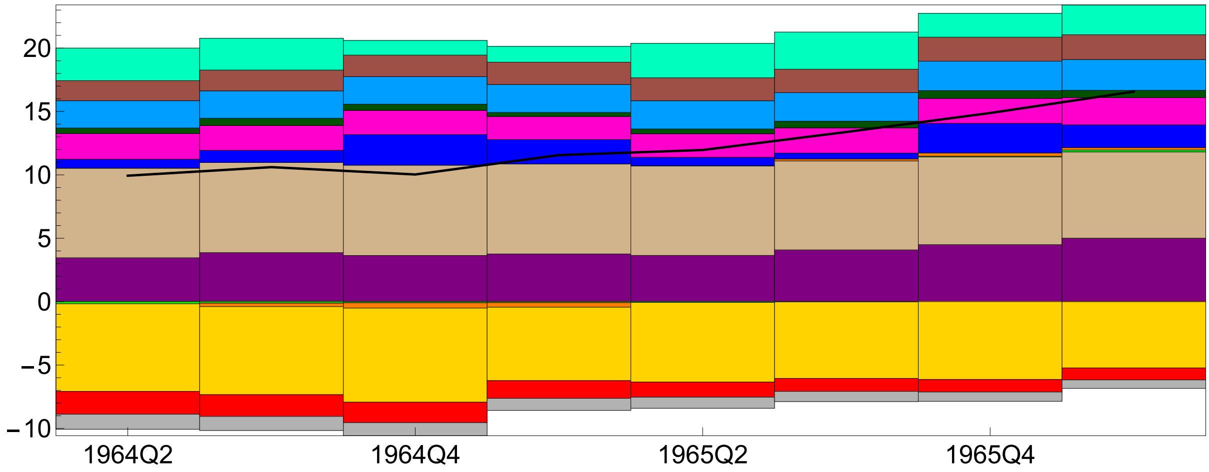
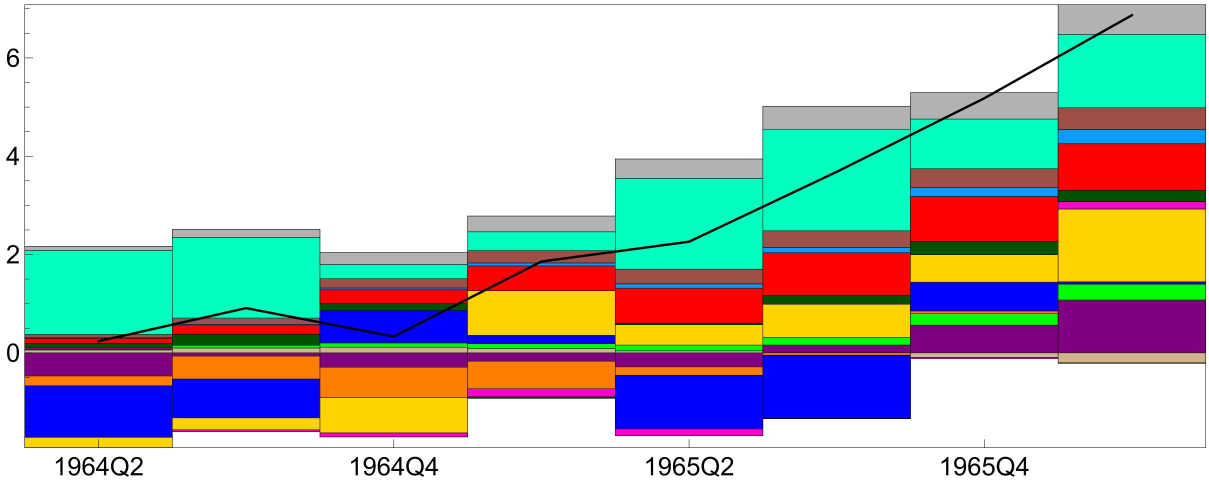
Figure 6: Instantaneous impact of the shock process $\mathbf{CB}\boldsymbol{\varepsilon}_j$ on output from (3) for 1948Q2 to 1985Q4 and 1986Q1 to 2023Q1.



(a) State space shock decomposition formula (4) for output



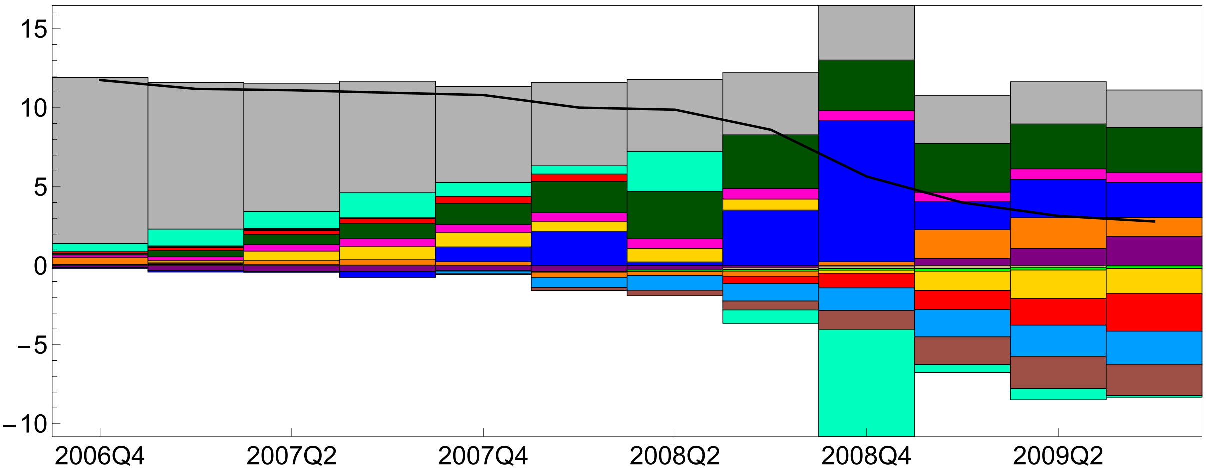
(b) Standard shock decomposition formula (5) for output



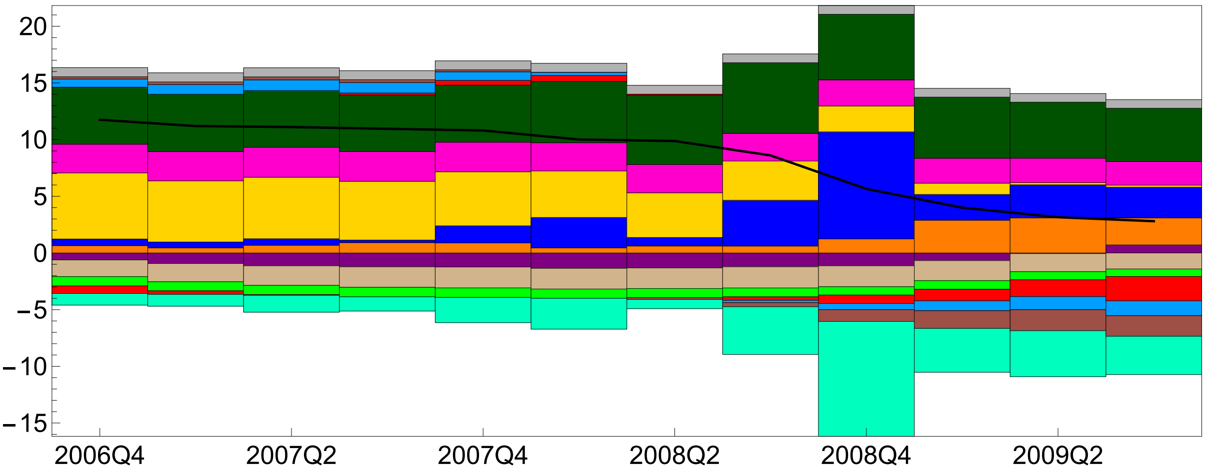
(c) Differencing shock decomposition formula (6) for output

■ Technology ■ Price Markup ■ Wage Markup ■ Gov. Spending ■ Lab. Tax ■ Priv. Inv. ■ Gov. Inv.
■ Policy Rate ■ 5y. Gov. Bond ■ 20y. Gov. Bond ■ 20y. Corp. Bond ■ Pref. Shock ■ Init. Val.

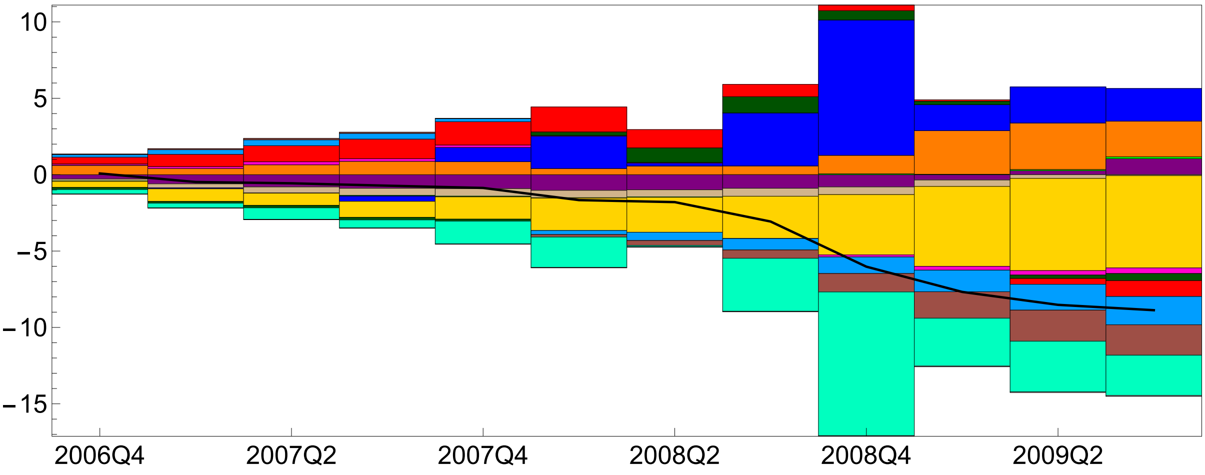
Figure 7: Shock decompositions for 1964Q2–1966Q1, calculated using the decomposition formulae (4)-(6). The black line represents output in (a) and (b) and the deviation of output from the period prior to the decomposition in (c).



(a) State space shock decomposition formula (4) for output

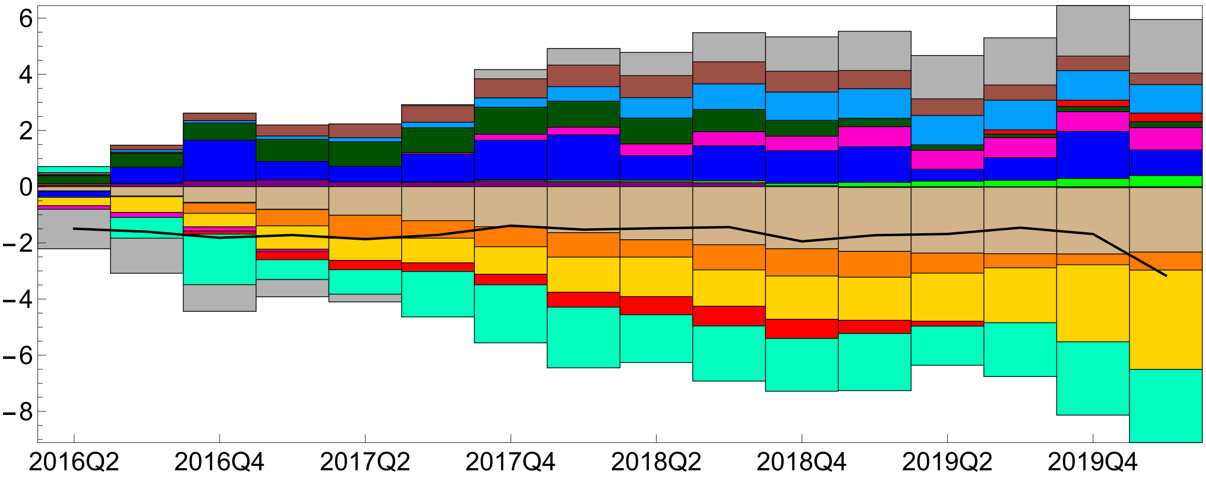


(b) Standard shock decomposition formula (5) for output

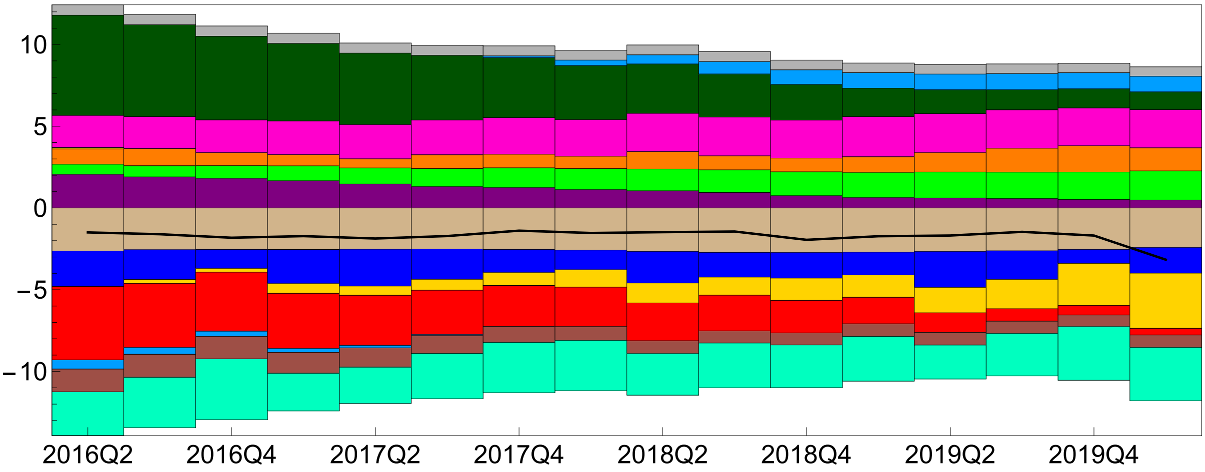


(c) Differencing shock decomposition formula (6) for output

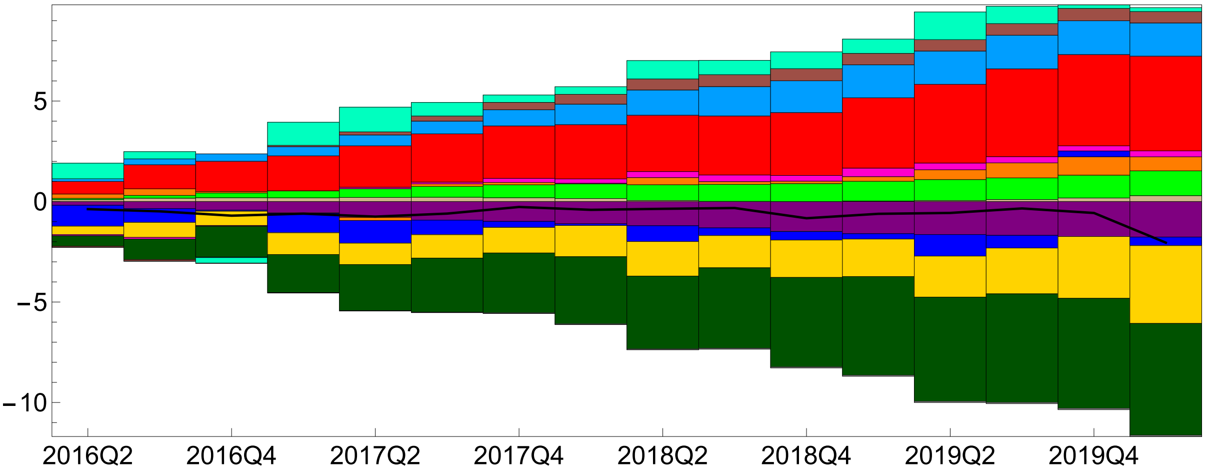
Figure 8: Shock decompositions for 2006Q4–2009Q3, calculated using the decomposition formulae (4)-(6). The black line represents output in (a) and (b) and the deviation of output from the period prior to the decomposition in (c).



(a) State space shock decomposition formula (4)



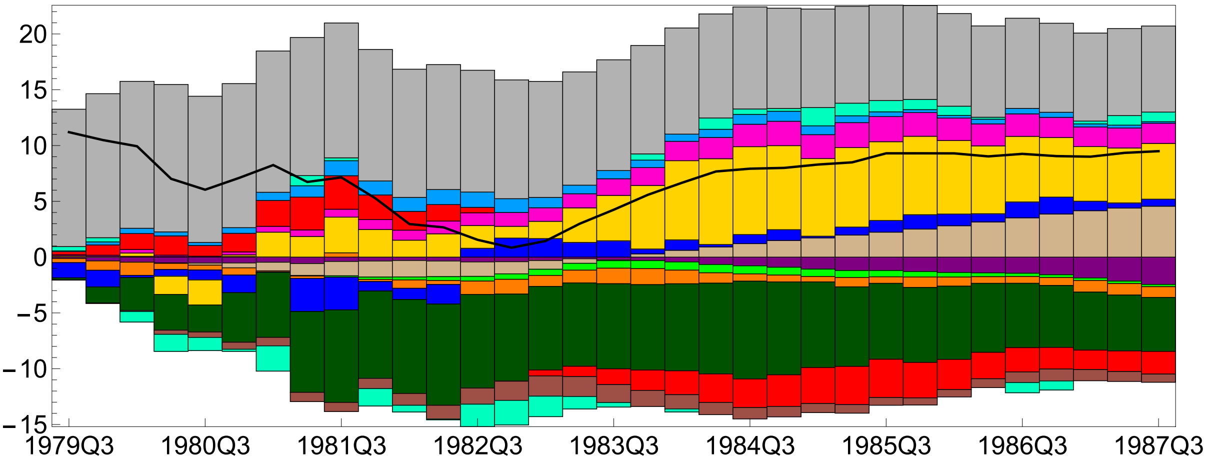
(b) Standard shock decomposition formula (5)



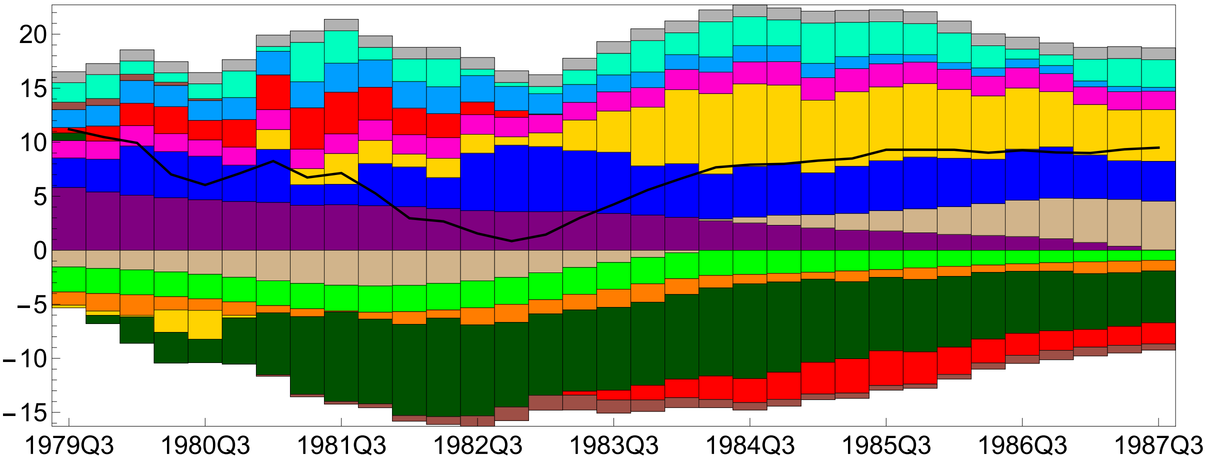
(c) Differencing shock decomposition formula (6)

■ Technology ■ Price Markup ■ Wage Markup ■ Gov. Spending ■ Lab. Tax ■ Priv. Inv. ■ Gov. Inv.
■ Policy Rate ■ 5y. Gov. Bond ■ 20y. Gov. Bond ■ 20y. Corp. Bond ■ Pref. Shock ■ Init. Val.

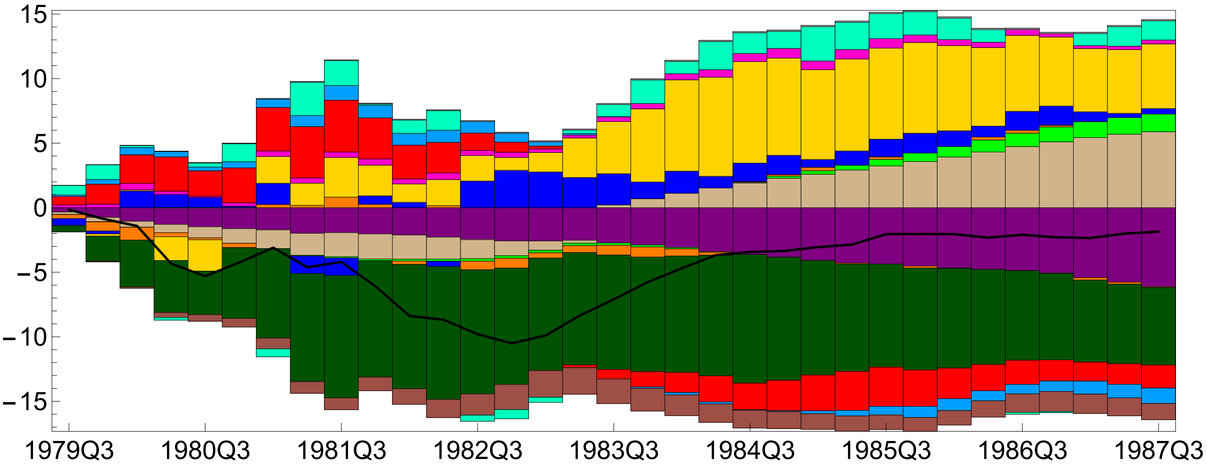
Figure 9: Shock decompositions for 2016Q2–2019Q1, calculated using the decomposition formulae (4)-(6). The black line represents output in (a) and (b) and the deviation of output from the period prior to the decomposition in (c).



(a) State space shock decomposition formula (4) for output



(b) Standard shock decomposition formula (5) for output



(c) Differencing shock decomposition formula (6) for output

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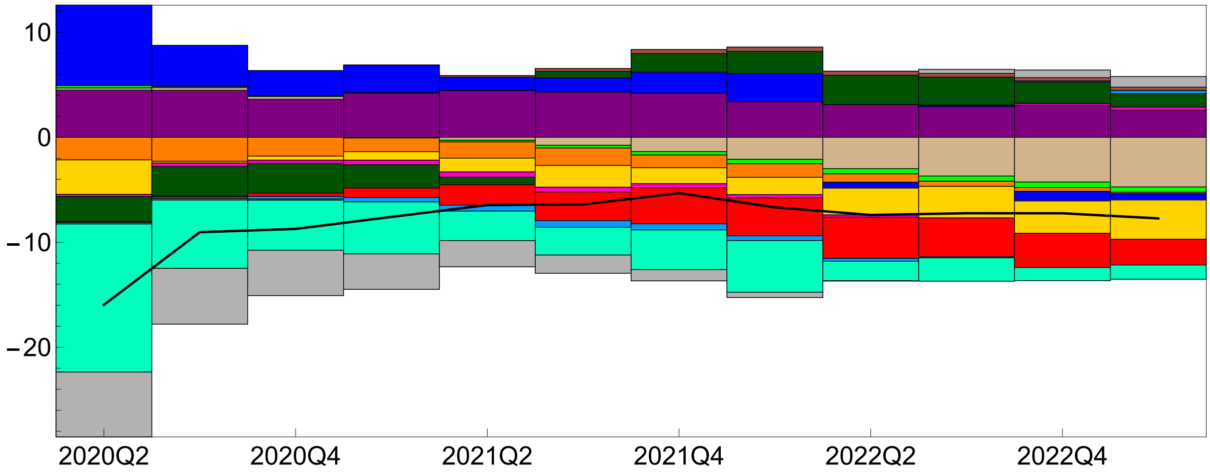
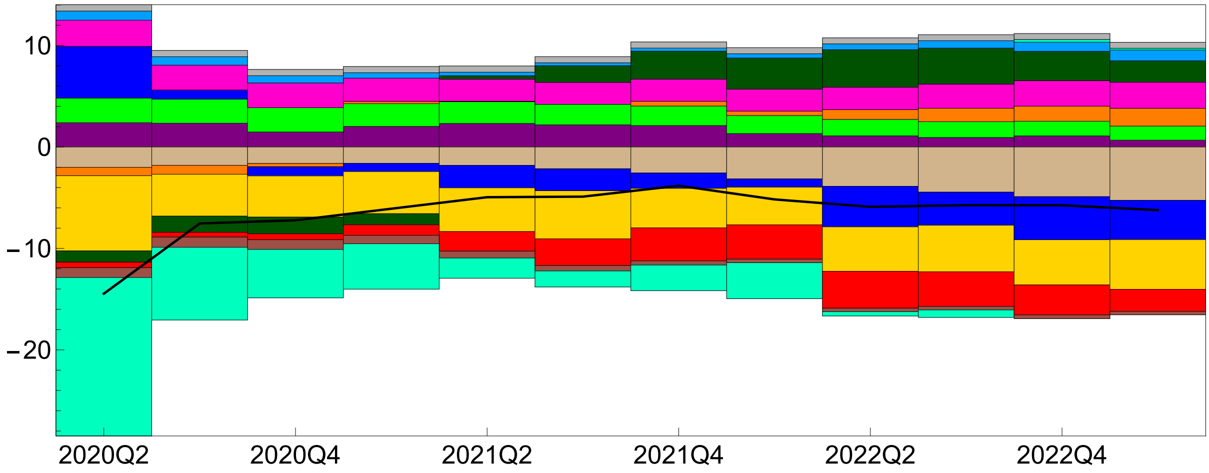
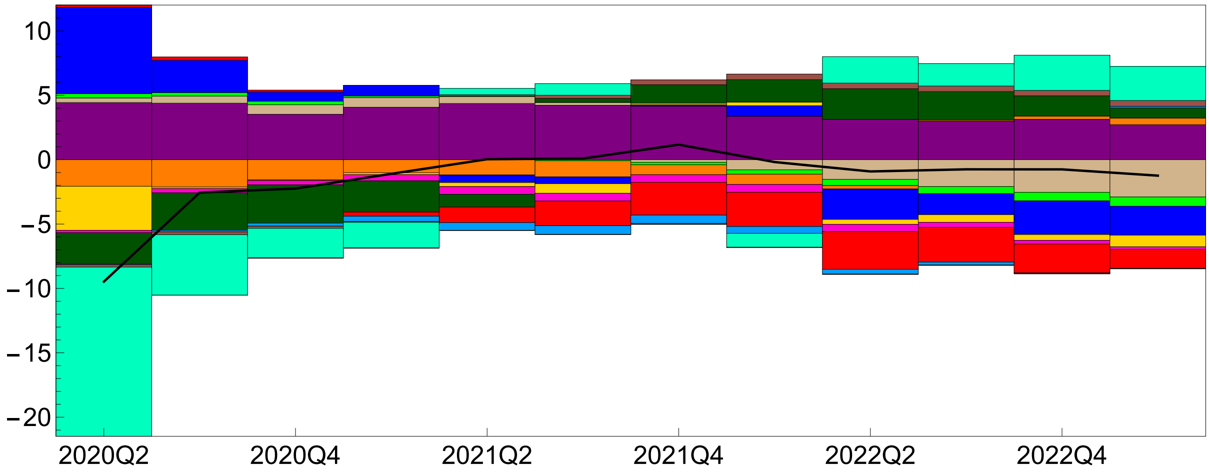
Figure 10: Shock decompositions for 1979Q3–1987Q3, calculated using the decomposition formulae (4)-(6). The black line represents output in (a) and (b) and the deviation of output from the period prior to the decomposition in (c).



(a) State space shock decomposition formula (4) for output



(b) Standard shock decomposition formula (5) for output



(c) Differencing shock decomposition formula (6) for output

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🟩 Policy Rate 🟥 5y. Gov. Bond 🟦 20y. Gov. Bond 🟫 20y. Corp. Bond 🟩 Pref. Shock ⬜ Init. Val.

Figure 11: Shock decompositions for 2020Q2–2023Q1, calculated using the decomposition formulae (4)-(6). The black line represents output in (a) and (b) and the deviation of output from the period prior to the decomposition in (c).

the shocks to the price markup in the period prior to the end of 2006 are extremely small, but they have a visible impact in Figure 8b because they are persistently negative.

Depending on the pattern of the shocks, particularly prior to the period under consideration, the differencing method (6) and our preferred decomposition method (4) can on occasion deliver somewhat similar interpretations of a particular episode. This is the case when we compare Figures 8a and 8c, though the latter suggests the negative shock to private investment played a much greater role in the crisis.

Now consider how the estimated shocks in Figure 6b are translated into the shock decomposition in Figure 9a for the period between 2016Q2 and 2020Q1, which corresponds to the last four years prior to the large drop in economic activity at the onset of the pandemic. Detrended output throughout the period is flat and unusually constant. Private investment, price markups, the preference shocks, government spending and technology all exert downward pressure on output. At the same time, output is bolstered by the shocks to labor taxes and both private and public 20-year bonds. Monetary policy too plays a modest role in supporting output from the second half of 2016 but this effect dissipates by 2019. More importantly, seven years after the end of the great recession and following years of below-trend growth in its aftermath, the prior accumulation of negative shocks has left output well below its steady-state value. In the period that follows, the receding legacy over time of the initial conditions in Figure 9a means that according to our model, despite the prevalence of negative shocks, output is growing at its trend rate because these negative shocks are counteracted by the effect of convergence. Moreover, the gradually growing amplitude of both the positive and countervailing negative shocks in Figure 9a match the pattern in Figure 6b. The shock decomposition in Figure 9b derived from (5) yields a very different interpretation of the same history.

Note that in the periods both before and after 2016, the preference shocks and shocks to the labor tax in Figure 6b are prominent but fluctuate. By contrast, the direct impact of the shocks to government investment are small but consistently positive before 2016Q2, but negative in that particular quarter. However, the decomposition in Figure 9b registers a positive and significant impact on output, because it expresses the accumulating effects from those previous periods, carried forward. Similarly the impact of the price markup is very small in Figure 6b, yet because it is consistently negative its cumulative influence is inflated.

Focusing on the impact of monetary policy, one useful feature of the decomposition in (6) does emerge. Whereas Figure 9a and 9b emphasise the expansionary impact of monetary policy that prevailed from 1992Q3, the negative shocks that feature in Figure 9c highlight the contrast with the period prior to 2016Q2.

There is far greater congruence between the results of all three decomposition methods when we analyze the period from 1979Q3 to 1987Q3, which coincided with Paul Volcker's term as Chairman of the U.S. Federal Reserve. All three decompositions in Figure 10 capture the initial negative impact on output associated with the deflationary policies implemented during Volcker's term. They all suggest that the negative impact of these policies persisted throughout much of the 1980s reinforced by the increase in government borrowing, long after the economy recovered, which each of the decompositions suggests was largely driven by positive shocks to private investment and to a lesser degree by declines in the price markup.

Still, applying the differencing methodology (6) can produce a shock decomposition for the last four years prior to the impact of the covid pandemic in Figure 9c that diverges from the shock decompositions in both Figures 9a and 9b. Differencing removes the negative influence of the price markup shock and the positive influence of government investment in Figure 9b that reflects their cumulative impact from earlier periods. At the same time, during the period from 2016Q2 to 2019Q1 the shocks to monetary policy in Figure 6b are muted. However, just as Figure 9b reflects the positive impact of monetary policy on output from earlier in the decade rather than contemporaneous shocks, the influence of the negative sign in the second term in (6) means that differencing produces a pattern that is almost its mirror image in Figure 9c. Similarly, 5-year bonds and technology have positive but receding impacts on output in Figure 9b and negative and intensifying impacts in Figure 9c. The impact of the preference shock is negative in Figure 9b and largely positive in Figure 9c. Finally, unlike our preferred decomposition, the differencing methodology does not demonstrate the influence of the initial conditions prior to 2016Q2 via the convergence properties of the model and their subsequent contribution to the evolution of output. The fact that three very different interpretations of the same episode, using shocks generated by the same model, emerge in Figure 9 strongly suggests that when using DSGE models to interrogate the past, the method chosen to calculate the shock decomposition can be decisive.

Finally, for the case of the last three years in the sample, coinciding with the start of the covid pandemic and its aftermath, the different methods in Figure 11 generate very similar patterns as was the case for the period associated with Paul Volker's efforts to control US inflation. Generally, the largest effect is the initial negative impact of the preference shock, followed by private investment. In our preferred decomposition in Figure 11a, shocks to monetary policy and government spending also contribute to the initial sharp downturn, partly ameliorated by positive shocks from the labour tax and technology and this pattern is largely the same for the differencing method in Figure 11c.

6 Combining Decompositions

We prefer the decomposition formula (4) for the way it isolates the impact of shocks from the start of each historical episode. By contrast the decompositions generated by the alternatives (5) and (6) contain the legacy of earlier shocks prior to the initial period being analysed. Nonetheless those previous shocks do influence the evolution of the variable under consideration, only their lingering influence is amalgamated into the gray columns in Figures 7a through 9a.

Figure 12 provides one option for combining the most salient information in all three decompositions, in this case for the period between 2020Q2 and 2022Q3. The solid columns are identical to those in Figure 11a, whereas the striped columns represents the difference between the decomposition in (4) and (5). The former once again isolate the impact of the raw shocks that start at the beginning of the period while the latter disaggregate the combined impact of prior shocks rather than just their net impact. The labour tax shocks exert a positive influence in Figures 11a to 11c, but their magnitudes are different. In Figure 12 we can see that the shock to the labour tax in 2020Q2 played an important role in counteracting the large negative pref-

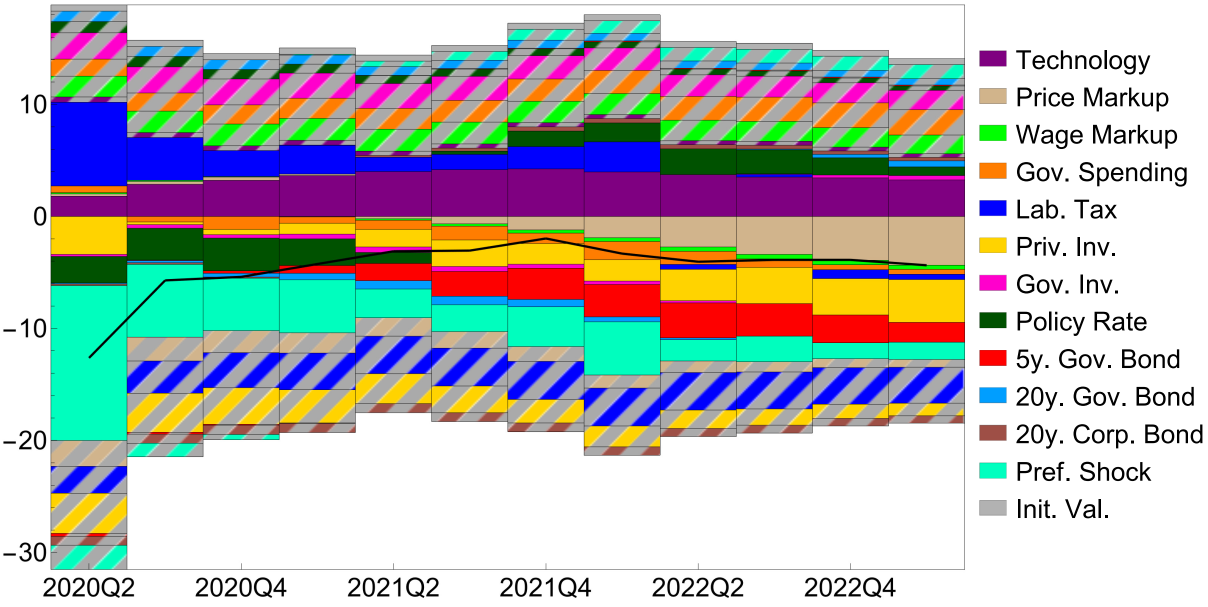

Figure 12: Shock decompositions for 2020Q2–2023Q1, combining the decomposition formulae (4) represented by solid bars and (5) represented by striped bars. The black line represents output in (a) and (b).

erence shock associated with the start of the pandemic. By contrast the influence of previous shocks to the labour tax during this period were entirely negative, highlighting the importance of the change in policy.

7 Conclusion

The analysis of shock decompositions offers useful insights both into the workings of DSGE models and for evaluating how the shocks in these models may help to explain particular historical periods. Our analysis of the impact of the deflationary policies implemented by the Federal Reserve during the 1980s and the period coinciding with the onset of the covid pandemic demonstrates that the three different methods we compare can on occasion yield roughly similar interpretations of a particular episode. However, as we also demonstrate, this is not generally the case and the different methods will often generate very different results. Our preferred method is the one that is designed to isolate the impact of shocks within the episode being analyzed from the impact of previous shocks.

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Appendix

An Extended Drautzburg-Uhlig Model of the U.S. Economy

In this section we present more fully the extended version of the DU model used in Section 5. We first describe the enhanced data set used in the model and then demonstrate, by means of variance decomposition, the effect of our modifications to the model.

A1: Modifications to the Data Compared with DU

Of the twelve data series used in our estimation, eight are the same as those found in DU, starting in 1948Q2, but are updated and extended to cover the additional forty one quarters from 2009Q1 to 2020Q1. These are (1) per-capita output: chained 2005 real GDP, growth rates; (2) per-capita consumption: private consumption expenditure, growth rates; (3) per-capita investment: private fixed investment, growth rates; (4) per-capita government investment, growth rates; (5) per-capita hours worked: civilian employment index × average nonfarm business weekly hours worked index, demeaned log; (6) inflation: GDP deflator, quarterly growth rates; (7) wages: nonfarm business, hourly compensation index, growth rates; and (8) policy interest rate: from March 1948 to June 1954 we use the 3-Month Treasury Bill: Secondary Market Rate, from April 2007 through March 2023 the shadow rate calculated by Wu and Xia (2016), and for all other months the Federal Funds Rate, all coverted to quarterly rates. The remaining four are as described in this section: (9) corporate bond yield: Moody's Baa index at quarterly rates; (10) per-capita net public debt (described below), demeaned log; (11) 5-year U.S. Treasury coupon note yield supplemented by data calculated by Ibbotson (2016) on intermediate-term U.S. Treasury bond yields for the period between 1948Q2 and 1953Q1; and (12) 20-year U.S. Treasury coupon note yields supplemented by data calculated by Ibbotson (2016) on long-term U.S. Treasury bond yields for the period from 1948Q2 to 1953Q1. We use the non-institutionalized population in the US (BLS series LNU00000000Q) to derive per-capita variables.

To highlight the importance of including data on government bond yields directly in the model, we plot in Figure 3 this data series alongside the implied interest rate on government debt that is generated by DU's original model and using their data. This shows that in DU, the interest rate on government bonds plays a significant role in equilibrating the model, causing it to descend below the zero lower bound on several occasions across the sample period and inducing a counterfactual degree of volatility not observed in the data. Note that the volatility is particularly elevated during the last two quarters of 2008, a phenomenon not observed in the data.

Aside from including the additional time series, the specific choice of 5-year Treasury bonds over the 10-year bonds better matches the maturity structure of U.S. government debt (see Greenwood *et al.* (2015) and Blanchard (2019)). Another benefit of this approach is that while the yield on corporate bonds is available at monthly frequencies for the entire sample period, the yields on both 10-year Treasury bonds and the corresponding spread used by DU are only available from April 1954.[11] The interest rates intermediate-term (5-year) U.S. Treasury coupon notes are also unavailable till April 1954, so we augment the series for the missing periods with the intermediate-term (5-year) interest rates calculated by Ibbotson (2016).

In addition to the interest rate on government borrowing, we also introduce an alternative measure of the public debt itself, derived as the sum of federal government debt securities and the total liabilities of state and local governments, excluding employee retirement funds (the series FL314122005.Q and FL214190005.Q in the *Financial Accounts of the United States*, published by the Board of Governors of the Federal Reserve System). Unlike the gross federal debt series at par value calculated by the Dallas Federal Reserve and used by DU, our measure of public debt excludes intergovernmental federal debt (particularly nonmarketable debt held by the Social Security Trust Fund), but includes the total public liabilities of states and localities, excluding employee retirement funds. This better matches the way debt is characterized in the model's government budget constraint.[12] The difference between our data series and the original data series used by DU is depicted in Figure 5 in the Appendix. Though the two series mostly move in tandem, their series begins the sample period approximately 30% of GDP above the series we constructed. During the intervening years the two series begin to converge until 1983 after which they increasing diverge as our series grows more slowly. Note the big changes in the debt series in 2020 and after reflect changes in both the debt and sharp fluctations in GDP.

A2: Resulting Differences in Variance and Shock Decompostion

Table 1 compares the historical variance decomposition for both DU's model and our extended model. Not surprisingly the addition of the eleventh and twelfth shocks detracts from the importance of most of the original ten. Of the three large shocks in DU – monetary policy, technology and private investment, which together account for nearly 60% of the variance—only the contribution of monetary policy remains nearly undiminished. The price markup, labor tax, government spending, and government investment all recede in importance. The wage markup nearly disappears entirely. More importantly, once we add the two yields on government bonds as additional observable variables, the shock associated with the spread between policy rate and the 5-year bond associated with government borrowing—ω^f in Figure 1(b)—assumes a much greater role, accounting for 22.27% of the fluctuations in output for the entire sample.

Table 1 shows that the variance decomposition of the two models assigns nearly identical values to monetary policy, between 18% and 19% of the total variation in output.

While the patterns of output decomposition across the two models are broadly similar. In the decompositions for our model presented in Figure 2, the initial conditions in 1948 imply output is below the values associated with balanced growth, but then output fluctuates closer to and more often below the horizontal axis across the enlarged sample, implying that the model

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[11] When estimating their model, DU assume the spread is zero from 1948 through 1953.

[12] The model does not distinguish between the different levels of government or the different tax rates they impose. At the same time, a dollar of tax collected on earnings either through income tax or the Federal Insurance Contributions Act (FICA) has the same impact on net federal debt (our measure), but if collected through FICA actually increases the stock of gross debt.

Table 1: Variance decomposition for this paper versus Drautzburg and Uhlig's model. [†]Drautzburg and Uhlig's (2015) model 1948Q2 to 2008Q4, with small correction to code.

Shocks	Drautzburg and Uhlig[†]	This paper
Technology	21.35	16.00
Price Markup	8.58	6.91
Wage Markup	9.17	1.47
Lab. Tax	8.26	6.94
Gov. Spending	3.39	3.29
Priv. Inv.	18.77	16.80
Gov. Inv.	4.75	6.16
Policy Rate	18.77	18.40
5 y. Gov. Bonds	–	7.22
10 y. Gov. Bonds	6.36	–
20 y. Gov. Bonds	–	2.62
20 y. Priv. Bonds	1.20	1.78
Pref. Shock	–	12.41

We are using the posterior modes in Table 2 for these calculations.

is perhaps producing a somewhat more intuitive measure of trend growth.

B: Estimation

Table 2: Results from Metropolis-Hastings (parameters)

	Prior			Posterior					
				Mode		Mean			
	Dist.	Mean	Stdev.	Estimate	Stdev.	Estimate	Stdev.	HPD inf	HPD sup
Returns to scale Λ	norm	1.000	0.5000	1.2826	0.4158	1.276	0.3249	0.7995	1.8565
Capital share α	norm	0.300	0.0500	0.2213	0.0101	0.224	0.0104	0.2072	0.2412
Risk aversion σ	norm	1.500	0.1000	1.0965	0.0861	1.115	0.0856	0.9729	1.2547
Habit h	beta	0.700	0.0500	0.7958	0.0259	0.798	0.0256	0.7580	0.8400
Inv. lab. elast. ν	norm	2.000	0.1000	2.0657	0.0987	2.064	0.0982	1.9056	2.2289
Disc. factor $100 \times \frac{1-\beta}{\beta}$	gamm	0.250	0.1000	0.0692	0.0291	0.084	0.0326	0.0319	0.1336
Calvo wage ζ_w	beta	0.400	0.1000	0.8762	0.0189	0.876	0.0185	0.8456	0.9066
Calvo price ζ_p	beta	0.400	0.1000	0.8429	0.0265	0.828	0.0379	0.7715	0.8842
Wage index ι^w	beta	0.400	0.1500	0.3289	0.1041	0.347	0.1014	0.1790	0.5118
Price index ι^p	beta	0.400	0.1500	0.1072	0.0555	0.123	0.0553	0.0357	0.2074
Cap. util. ψ_u	beta	0.400	0.1500	0.5825	0.0732	0.597	0.0714	0.4777	0.7129
$1 + \frac{\text{Fixed cost}}{\text{GDP}}$	norm	1.250	0.1250	1.7715	0.0700	1.777	0.0695	1.6619	1.8908
Taylor smoothing ρ_R	beta	0.750	0.1000	0.8762	0.0159	0.880	0.0158	0.8542	0.9060
Taylor infl. ψ_1	norm	1.500	0.1500	1.2102	0.1225	1.270	0.1206	1.0660	1.4642
Taylor out. gap. ψ_2	norm	0.125	0.0500	0.0526	0.0122	0.057	0.0132	0.0357	0.0781
Taylor Δout. gap. ψ_3	norm	0.125	0.0500	0.0593	0.0080	0.060	0.0080	0.0468	0.0733
$\frac{\text{Gov. spending}}{\text{GDP}}$	norm	0.500	0.2500	0.5000	0.2500	0.511	0.2351	0.1132	0.8870
Adjust. cost	norm	4.000	1.5000	6.5865	1.0153	6.722	0.9940	5.1067	8.3627
Gov. adjust. cost	norm	4.000	1.5000	7.3381	1.1036	7.535	1.0808	5.7424	9.2983
Trend $\mu - 1$	beta	0.450	0.1000	0.4432	0.0089	0.440	0.0094	0.4247	0.4555
Budg. bal. speed $\frac{\psi_\tau - 0.025}{0.175}$	beta	0.300	0.2000	0.0018	0.0035	0.008	0.0066	0.0001	0.0168
Mean spread ω^f	gamm	0.500	0.1000	0.4004	0.0385	0.400	0.0398	0.3362	0.4666
Mean spread ω^b	gamm	0.500	0.1000	0.9564	0.1011	0.964	0.1020	0.7968	1.1315
Mean spread ω^l	gamm	0.500	0.1000	0.2301	0.0371	0.234	0.0378	0.1717	0.2942
Mean infl.	gamm	0.625	0.1000	0.3715	0.0562	0.385	0.0579	0.2912	0.4806
Mean hours	norm	0.000	2.0000	1.5210	0.7682	1.643	0.8099	0.2899	2.9367
Mean bonds	norm	0.000	0.5000	-0.0239	0.4999	-0.021	0.5028	-0.8423	0.8103
AR(1), technology ρ_a	beta	0.500	0.2000	0.9645	0.0109	0.965	0.0104	0.9487	0.9824

(Continued on next page)

Table 2: (continued)

	Prior			Posterior					
				Mode		Mean			
	Dist.	Mean	Stdev.	Estimate	Stdev.	Estimate	Stdev.	HPD inf	HPD sup
AR(1), gov. bond spread ω_f	beta	0.500	0.1000	0.8887	0.0272	0.892	0.0252	0.8510	0.9332
AR(1), pref. shock ρ_P	beta	0.500	0.1000	0.1255	0.0357	0.131	0.0357	0.0728	0.1877
AR(1), gov. spending ρ_g	beta	0.500	0.2000	0.9920	0.0027	0.991	0.0030	0.9868	0.9961
AR(1), inv. price ρ_x	beta	0.500	0.1000	0.5684	0.0514	0.572	0.0514	0.4871	0.6564
AR(1), monetary ρ_r	beta	0.500	0.1000	0.2911	0.0470	0.296	0.0465	0.2189	0.3719
AR(1), price markup ρ_π	beta	0.500	0.1000	0.8172	0.0501	0.820	0.0539	0.7334	0.9128
AR(1), wage markup ρ_w	beta	0.500	0.1000	0.9228	0.0220	0.912	0.0220	0.8794	0.9487
AR(1), tax ρ_τ	beta	0.500	0.2000	0.5243	0.0514	0.534	0.0518	0.4478	0.6183
AR(1), gov. inv. price$\rho_{x,g}$	beta	0.500	0.2000	0.9811	0.0044	0.980	0.0048	0.9723	0.9880
AR(1), corp. spread	beta	0.500	0.2000	0.9127	0.0216	0.914	0.0211	0.8803	0.9490
AR(1), term spread ρ_{term}	beta	0.500	0.2000	0.9386	0.0139	0.937	0.0137	0.9155	0.9599
MA(1), Price markup $\theta^{\lambda,p}$	beta	0.500	0.2000	0.6376	0.1030	0.626	0.1099	0.4525	0.8058
MA(1), Wage markup $\theta^{\lambda,w}$	gamm	0.980	0.0500	0.9004	0.0261	0.886	0.0261	0.8464	0.9292

Table 3: Results from Metropolis-Hastings (standard deviation of structural shocks)

	Prior			Posterior					
				Mode		Mean			
	Dist.	Mean	Stdev.	Estimate	Stdev.	Estimate	Stdev.	HPD inf	HPD sup
s.d. tech.	invg	0.100	2.0000	0.4521	0.0205	0.454	0.0208	0.4188	0.4871
s.d. bond	invg	0.100	2.0000	0.1495	0.0064	0.151	0.0065	0.1398	0.1610
s.d. pref.	invg	4.000	4.0000	12.8913	1.9536	13.483	2.0027	10.1839	16.7559
s.d. gov.	invg	0.100	2.0000	0.3516	0.0152	0.355	0.0157	0.3285	0.3799
s.d. tax	invg	3.000	2.0000	5.8597	0.2590	5.924	0.2643	5.4934	6.3614
s.d. mon. pol.	invg	0.100	2.0000	0.1862	0.0084	0.189	0.0085	0.1744	0.2025
s.d. price mark.	invg	0.100	2.0000	0.1675	0.0178	0.166	0.0180	0.1372	0.1967
s.d. wage mark.	invg	0.100	2.0000	0.3171	0.0171	0.320	0.0174	0.2912	0.3482
s.d. corp. spread	invg	0.100	2.0000	0.0651	0.0027	0.066	0.0028	0.0611	0.0702
s.d. term. spread	invg	0.100	2.0000	0.0539	0.0023	0.054	0.0023	0.0504	0.0580
s.d. inv. price	invg	0.100	2.0000	0.8968	0.0803	0.904	0.0810	0.7693	1.0358
s.d. gov. inv. price	invg	0.100	2.0000	0.5565	0.0576	0.558	0.0579	0.4640	0.6490

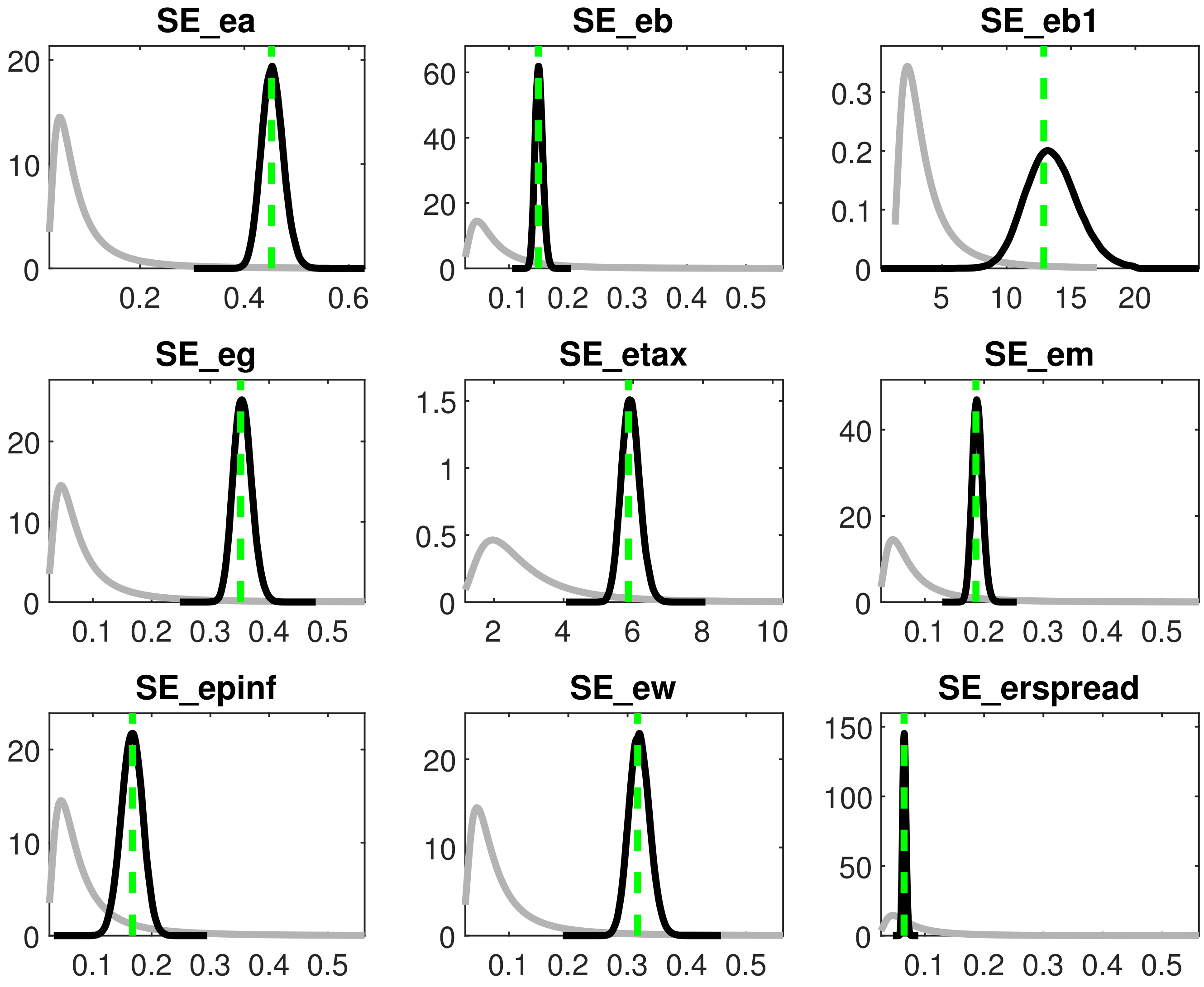

Figure 13: Priors and posteriors.

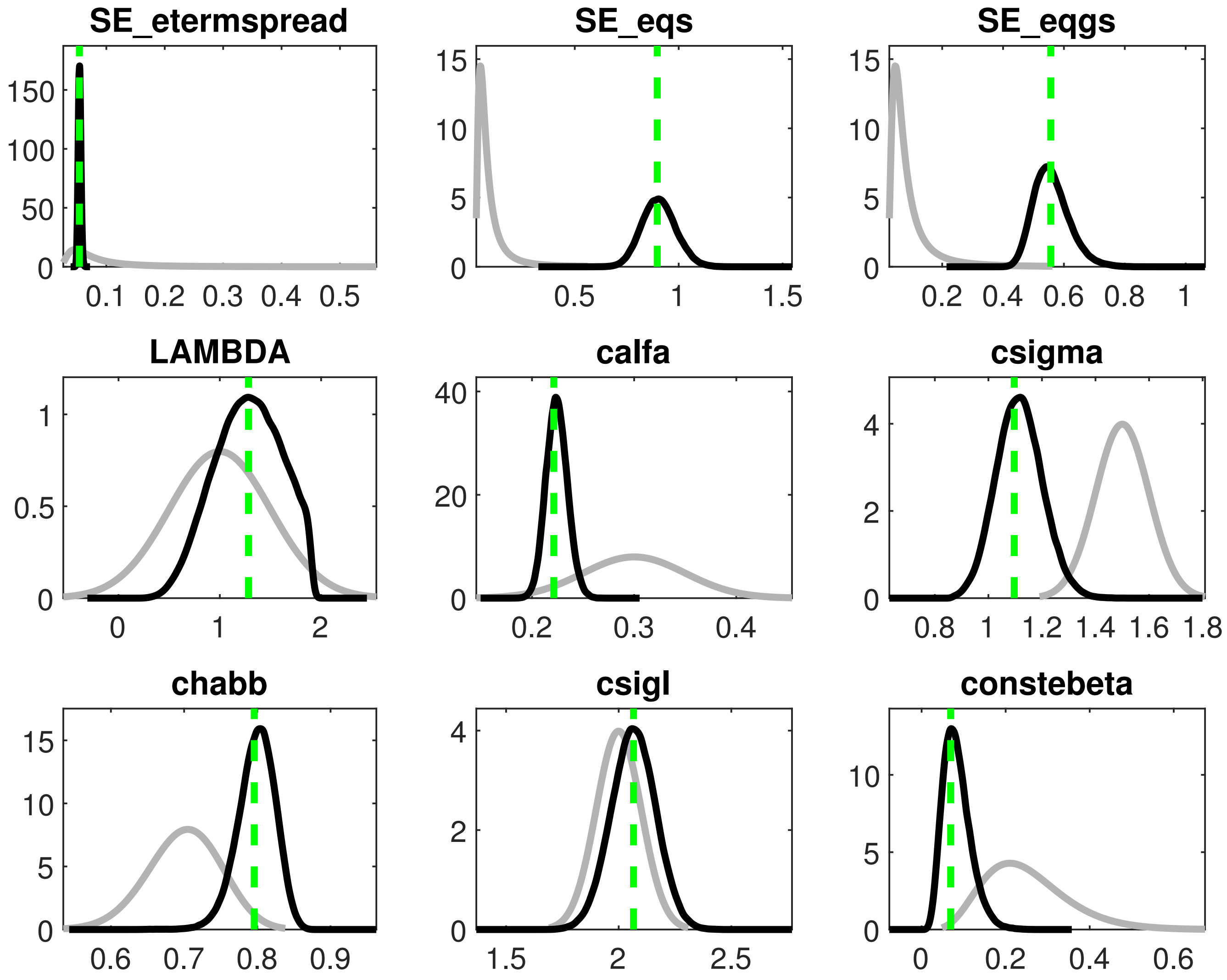

Figure 14: Priors and posteriors.

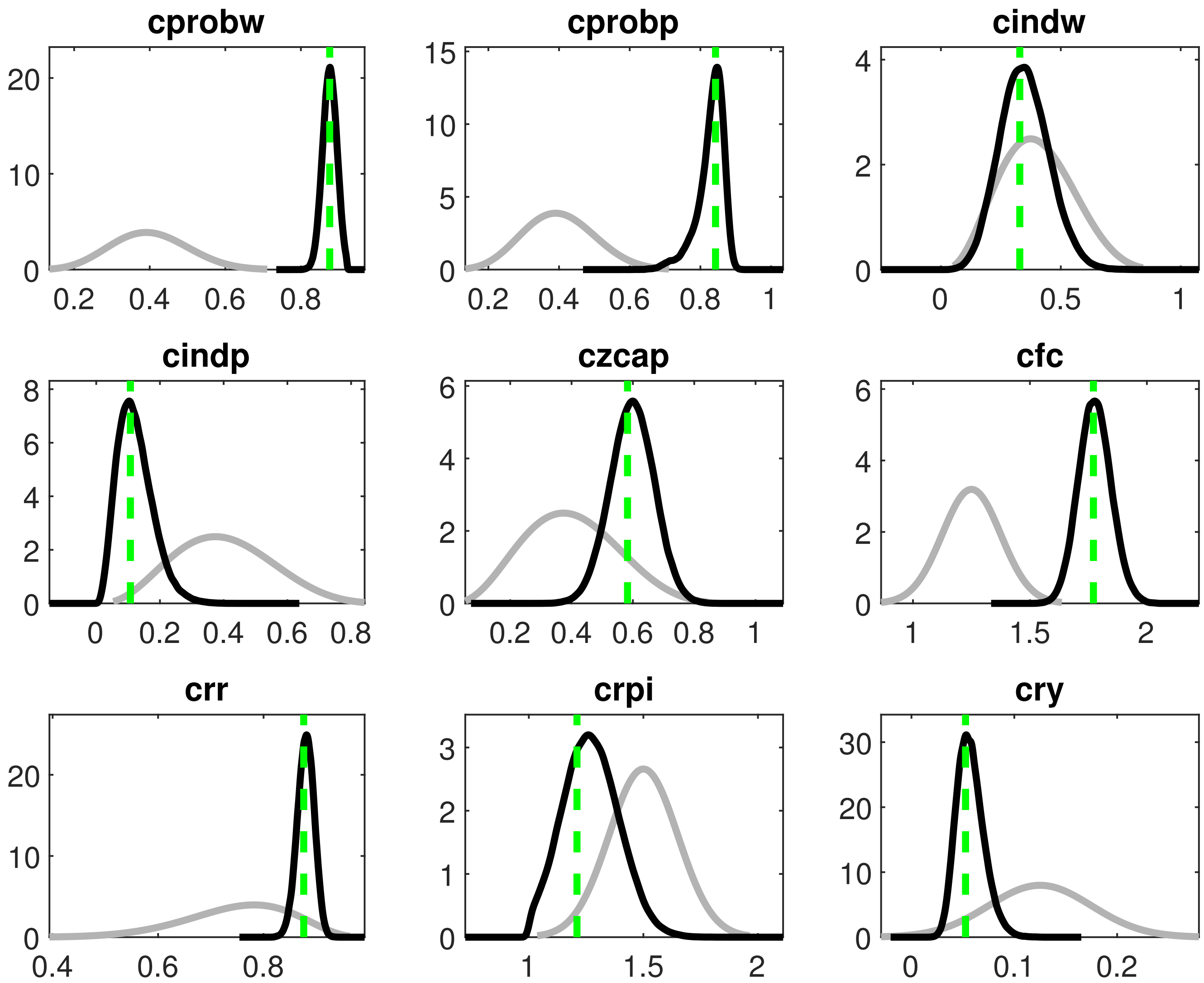

Figure 15: Priors and posteriors.

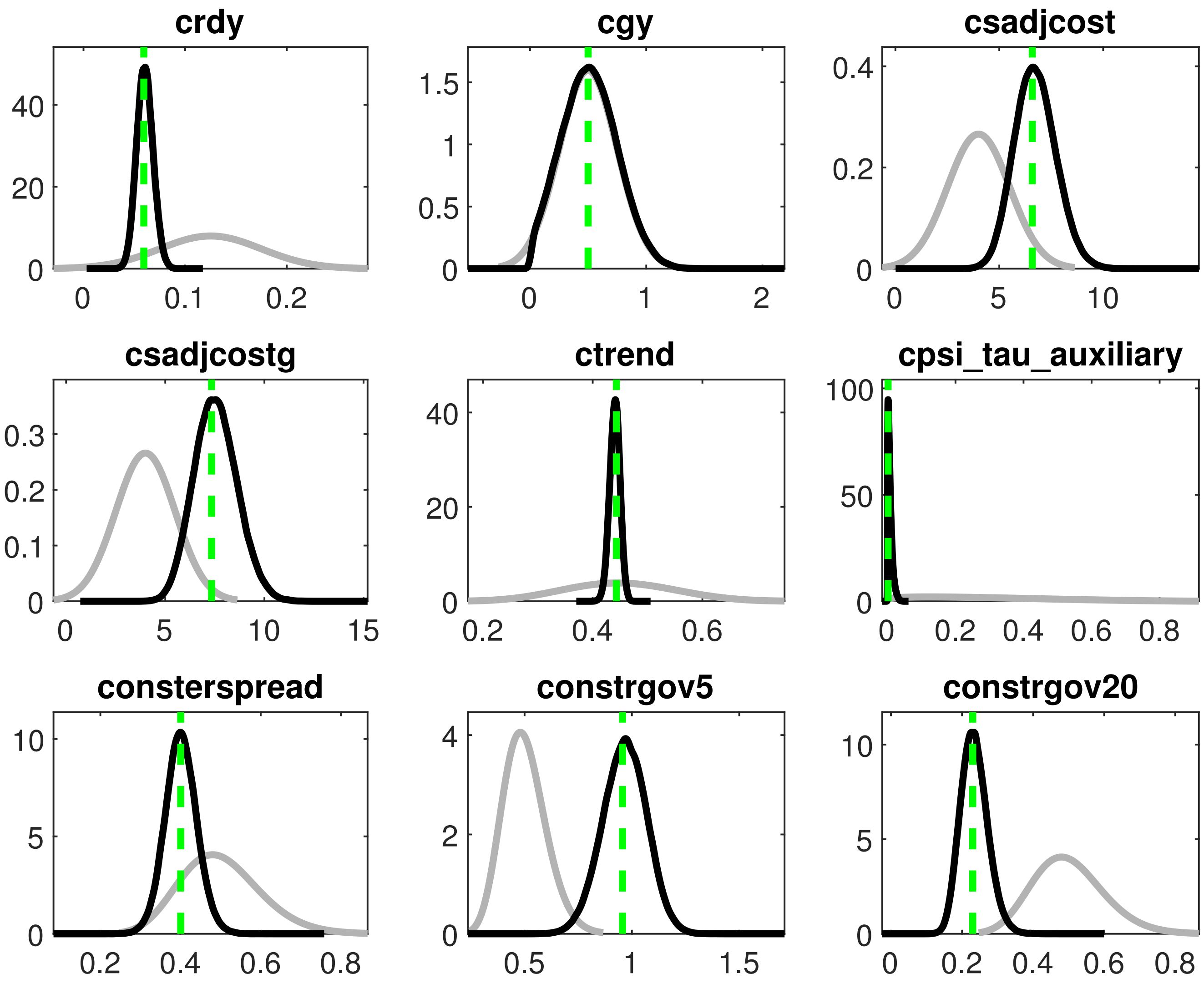

Figure 16: Priors and posteriors.

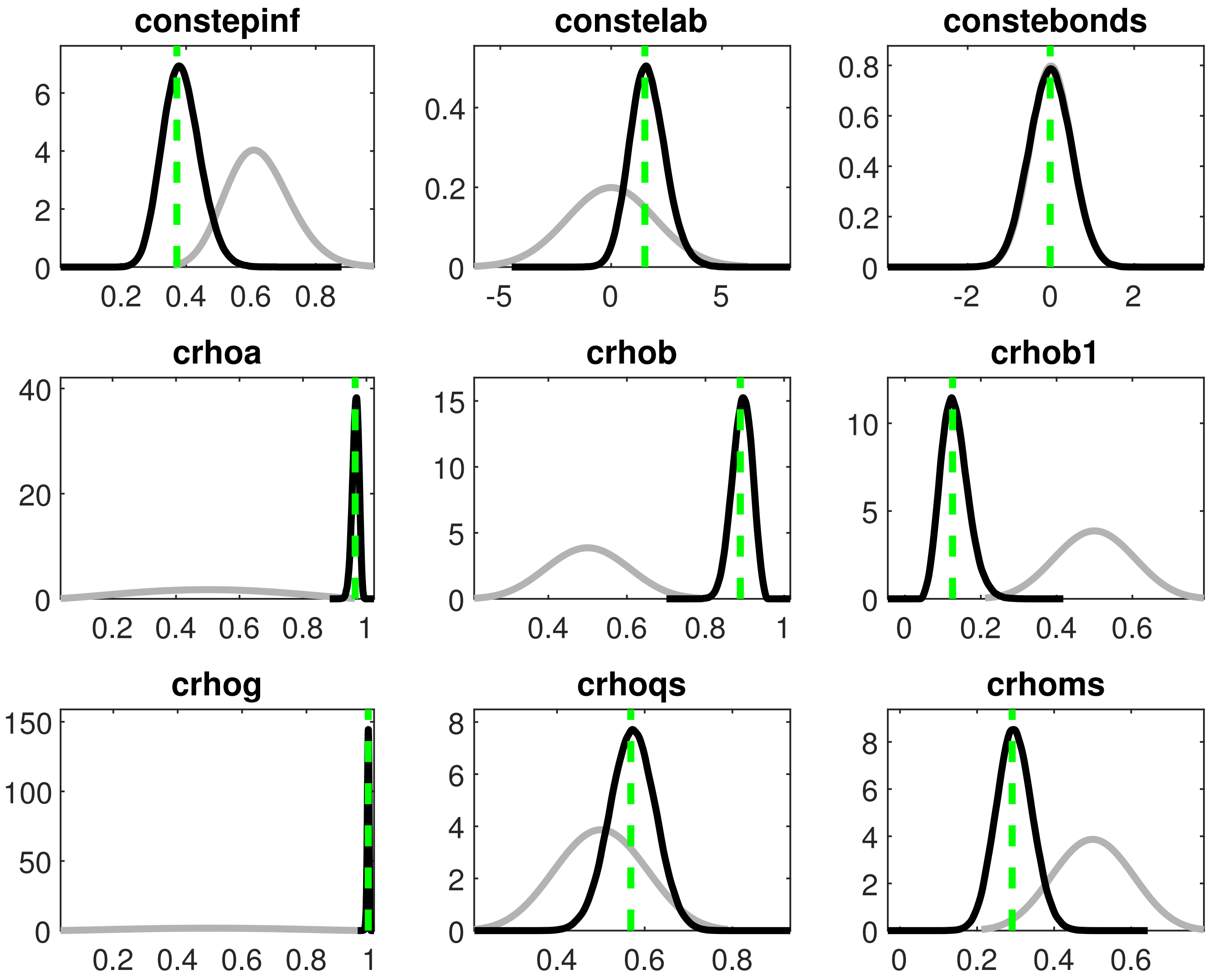

Figure 17: Priors and posteriors.

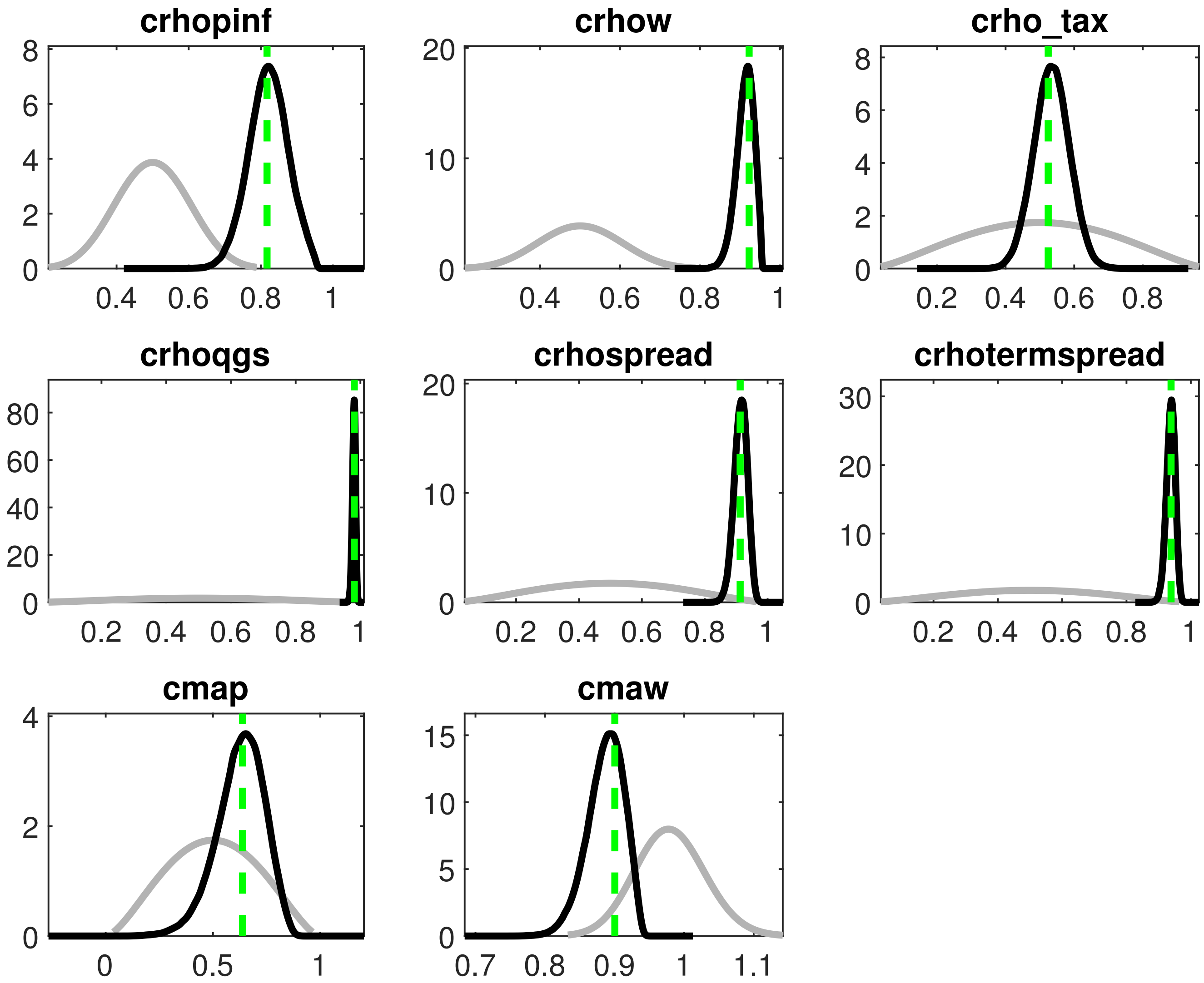

Figure 18: Priors and posteriors.

PUBLICATIONS

Evaluating Historical Episodes using Shock Decompositions in the DSGE Model

Working Paper No. WP/2025/051