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Abstract

State-contingent debt instruments such as GDP-linked warrants have garnered attention as a potential tool to help debt-stressed economies smooth repayments over business cycles, yet very few studies of the empirical properties of these instruments exist. This paper develops a general framework to estimate the time-varying risk premium of a state-contingent sovereign debt instrument. Our estimation framework applied to GDP-linked warrants issued by Argentina, Greece, and Ukraine reveals three stylized facts: (i) the risk premium in state-contingent instruments is high and persistent; (ii) the risk premium exhibits a pro-cyclical pattern; and (iii) the liquidity premium is higher and more volatile than that for plain-vanilla government bonds issued by the same sovereign. We then present a model in which investors fear ambiguity and that can account for the cyclical properties of the risk premium.

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1 Introduction

The market for state-contingent debt instruments (SCDIs) remains sparse, despite the elaborate theoretical depictions of the idea being around for decades (e.g. Bailey 1983;Lessard and Williamson 1987; Shiller 1993, Shiller 2003; Borensztein and Mauro 2004) with interest revived in the aftermath of both the eurozone crisis (e.g. Blanchard, Mauro, and Acalin 2016; IMF 2017) and the COVID-19 crisis (Cohen et al., 2020) and examples going back at least as far back as the 19th century (Confederate States 20-year bonds issued in 1863, convertible to warrants for cotton at a below-market price). Despite the economic rationale for SCDIs, such as increased fiscal space and reduced interest payment burden in recessions, the overwhelming majority of sovereign debt in the market is in the form of traditional, plain-vanilla bonds.

Several explanations have been put forward for the limited SCDI markets and the associated concern of high levels of risk premia, ranging from a lack of liquidity in initial and, possibly, prospective trading, the need to compensate investors for the novelty and/or complexity of the instruments, concerns about manipulation of the underlying data that determines the criteria for the stipulated repayments as well as broader adverse selection and moral hazard (Bailey 1983; IMF 2017; Cohen et al. 2020). Recent theoretical work has demonstrated that information and commitment imperfections can prevent full risk sharing (Levy, 2016) and investor preferences for robustness can generate the high levels of risk premia observed in the data (Roch and Roldan, 2021). Despite the renewed and growing interest in policy, practitioner, and academic circles during the COVID-19 pandemic, still relatively little is known about the empirical properties of SCDI and SCDI risk premia.

This paper develops a model to analyze and quantify the risk premium of a state-contingent sovereign debt instrument. We extend the analysis in Costa, Chamon, and Ricci (2008), who empirically examine the Argentine experience with GDP-indexed warrants. We are able to expand the sample window both geographically and chronologically, as more countries have issued warrants of similar form thereafter. An overarching aim of our study is to understand how SCDIs differ from plain-vanilla bonds in terms of their pricing, volatility, and investor base and to analyze and interpret the long-run path of risk premia. To the best of our knowledge, this paper is the first to document the long-term variation of the risk premia in state-contingent debt instruments over multiple countries.

We first present a general framework to infer the path of risk premia on a SCDI. We focus on GDP-indexed warrants among several variants of SCDIs as they account for the largest share issued in terms of notional values. The premium is extracted from the difference between the actual trading price of a GDP-linked warrant and its model-implied price, after accounting for the default premium and the liquidity premium. The exercise involves two new cases in addition to Argentina (warrants issued in the context of the 2005 debt restructuring): Greece (warrants issued in the context of the 2011-12 debt restructuring) and Ukraine (warrants issued in the context of the 2015 debt restructuring). For each case, we disentangle risk premia into three parts: (i) the default premium, implied by the CDS spread of a plain-vanilla bond, (ii) the liquidity premium, implied by the bid and ask price spread, and (iii) the residual, which is not attributable to default or liquidity and is specific to the SCDI. Hence, the residual premium is referred to as the SCDI premium.

The empirical exercise reveals three stylized facts.

First, the SCDI premium is high and persistent. The average premia during the five-year period after issuance is 12.5%, 4.25%, and 6.65% for Argentina, Greece, and Ukraine, respectively. Notably, the premia appear to be cyclical rather than following a clear downward path. Earlier studies such as Costa, Chamon, and Ricci (2008) using a couple of years of data after issuance have suggested that the SCDI premium is high at issuance but declines over time, which has then been interpreted as the SCDI premium corresponding to a 'novelty premium' that disappears as market participants become familiar with the new instrument. We observe that this early finding for Argentina does not hold up over an extended period of analysis and the premium increases again. Further evidence hinting that the SCDI premium is more than a novelty premium comes from the cases of Greece—where we actually observe an increase over time—and Ukraine—where we observe a rather steady pattern even four years after issuance.

Second, the SCDI premium seems to exhibit a pro-cyclical pattern (i.e., it is lower when the economy is in a recession). This is in contrast to the patterns observed for the default premium and the liquidity premium, which tend to be higher when industrial production is not growing as fast. A large literature in empirical asset pricing has documented that risk premia of various assets such as stock, bond, and currency returns tend to be counter-cyclical (see, among others, Fama and French 1989, Campbell and Cochrane 1999, Lustig, Roussanov, and Verdelhan 2014). Indeed, in our data, the default premium implied by the CDS spread moves counter-cyclically. A new model is called for to reconcile this with the procyclicality of the SCDI premium, provided that our inference is accurate. We conduct several robustness checks to rule out other confounding factors.

Third, the liquidity premium for GDP-linked warrants is higher and more volatile than that for the plain-vanilla government bonds issued by the same sovereign.

We then present a simple model with robust preferences that could explain the cyclical properties of the SCDI premium. The model closely resembles that in Roch and Roldan (2021), which in turn follow Pouzo and Presno (2016). In the setup, global investors fear ambiguity in the future path of economic variables when they price financial securities issued by a small open economy government. The government issues two types of securities. One is a perpetual bond whereby investors receive a fixed coupon rate with no maturity date. The government fails to disburse a coupon only if the growth rate of the economy falls below a threshold. The default of interest payment occurs automatically whenever the growth rate crosses the line. Besides the perpetual bond, the government issues a GDP-linked warrant. The coupon size of the GDP-linked warrant is proportional to the gap between realized growth rate and a threshold. There is no maturity date for the GDP-linked warrant. The payment threshold of the GDP-linked warrant is assumed to be higher than the default threshold of the perpetual bond, which is a common feature in real-world GDP warrants.

The key mechanism we highlight is the following: global investors with robust preferences (i.e. investors that dislike ambiguity) are concerned that the probabilistic model they use to forecast the growth rate of a small open economy is potentially misspecified, thereby exaggerating the likelihood of a bad state. This distortion makes the investors require a higher premium when a small deviation from the investors' baseline forecast can change the coupon size widely in the next period. In the case of the perpetual plain-vanilla bond, the payment volatility is largest when the economy enters a recession and there is increased risk of default. In the case of the GDP-linked warrant, the volatility surges when the current GDP growth rate is in the vicinity of its payment threshold. The premium of the GDP-linked warrant therefore moves procyclically relative to the default premium on the perpetual bond. Note that, under the standard expected utility model, such distortion of subjective probability is non-existent. So the size of the SCDI premia is small and barely responds to business cycle fluctuations. Essentially, the payment structure of the GDP-linked warrant is more sensitive to the types of probability distortions that the robust preference lenders fear during an economic expansion, so a heavier discount is applied.

This paper contributes to the literature on state-contingent debt instruments primarily by documenting the empirical properties of GDP-linked bonds. The theoretical literature has long predicted the benefits of these instruments while cautioning on the practical obstacles to their issuance. Among them, Borensztein and Mauro (2004) study the benefits of state-contingent debt as an instrument to moderate the cyclicality of fiscal policy. Incorporating state-contingencies into standard sovereign default models, Hatchondo and Martinez (2012) and Bertinatto et al. (2017) both find that GDP-linked debt instruments generate large welfare gains provided that the issuing governments designed them optimally. Essentially, GDP-linked bonds allow the government to eliminate default risk while providing credits, thereby reducing the volatility of consumption relative to income in equilibrium. Roch and Roldan (2021) develop a model with robust investors to explain the size of the premia associated with a state-contingent debt instrument. We extend their model to study the time-varying properties. Notably, the robust preference story differs qualitatively from a simple investor risk aversion story (such as the one presented in Levy (2016)) because risk-aversion is unlikely to generate procyclicality as investors' consumption tends to be correlated with the country's output process.

The rest of the paper is organized as follows. In Section 2, we provide a brief background on GDP-linked bonds and describe how we select the instruments we empirically study. Section 3 presents a general framework to quantitatively measure the time-varying premium of SCDIs. We also discuss the calibration strategy. Section 4 documents stylized facts on risk premia. Section 5 extends the model to account for the cyclical properties of the risk premia. Section 6 discusses further implications. Section 7 concludes.

2 Background: GDP-linked Warrants

Building on IMF (2017) and Pina (2020), we first create a list of SCDIs from a historical point of view. We then gather information on the broad features (e.g. adjustment mechanism, issued in the context of debt restructuring or not; tradeable or not, etc.). As shown in Table 7 in Appendix, SCDIs vary widely in their type and indexation details. Most are linked to GDP or a range of commodity prices, others stipulate payments to be conditional on the occurrence of a natural disaster. In order to maintain comparability across the cases we would study, we narrow down our attention to those linked to GDP. In addition, given the differences noted between SCDIs issued as part of debt restructuring versus those issued during normal times, we constrain the sample to the former. Focusing further on the tradeable cases, we identify the individual securities using ISIN codes and collect detailed information from Bloomberg on their characteristics (currency denomination, jurisdiction, issuance volume, base rate, credit rating, market price, etc.). In addition, we collect similar information on other securities issued by the same sovereign to compare their price dynamics. Where possible, we also collect information on the holders of both SCDIs and the other securities.

This process leaves us with three cases to study: Argentina, Greece, and Ukraine. For three of the other potential candidates (Bosnia and Herzegovina, Honduras, and Ivory Coast), we are not able to locate the ISIN codes while for two others (Bulgaria and Costa Rica), we find the ISIN codes but price data are not available.[1]

Argentina issued GDP-linked warrants during the 2005 debt restructuring. These instruments are characterized by the three rules that stipulate repayment: (i) a level condition: actual real GDP must exceed baseline real GDP (the base case GDP, measured in 1993 pesos); (ii) a growth condition: growth in actual real GDP must exceed growth in baseline real GDP; (iii) a cap: the cumulative amount of past payments should not exceed 0.48 per unit of security (in its corresponding currency). Payment then equals a fraction of excess nominal GDP (equal to the excess of real GDP over the base case, multiplied by the GDP deflator of the current year), to be distributed among the units of notional GDP-linked securities (the fraction would have been 5 percent if participation in the debt exchange had been 100 percent; since participation was 76 percent, the fraction is 76 percent of 5 percent, i.e. 3.8 percent). Note that these conditions may not be satisfied in a given year but then met in the next, hence missed

[1] Note that all of these instruments were issued before 2000. Two other cases, SCDIs issued by Portugal in 2013 and Singapore in 2001, are similar in structure to the cases of Argentina, Greece, and Ukraine but are not part of a debt restructuring and are non-tradeable.

payments can be recovered.

Similarly, Greece issued GDP-linked warrants in the process of the 2011-12 debt restructuring. These warrants are again characterized by three rules: (i) a level condition: nominal GDP must exceed a base case nominal GDP specified to be a certain value from 2014 to 2020, then equal to the 2020 value; (ii) a growth condition: the real GDP growth rate must exceed the baseline growth rate; (iii) a cap: 1% of the nominal value of the original instrument. Payout then equals a notional amount that decreases each year multiplied by 1.5 times the difference between the real growth rate in that year and a baseline growth rate. Contrary to the Argentine warrants, missed payments in one year are not recovered in the next.

Ukraine issued GDP-linked warrants during the 2015 debt restructuring. These warrants feature the same three rules: (i) a level condition: nominal GDP must reach $125.4 billion (calculated using the average hryvnia/dollar rate), compared to about $82 billion at the time of issuance (albeit at a depressed exchange rate); (ii) a growth condition: the real GDP growth rate must exceed 3%; (iii) a cap: 1% of GDP, applicable from 2021 to 2025. Payments are calculated as 15% of nominal GDP times excess real GDP growth exceeding 3% if growth is between 3 and 4, plus 40% of nominal GDP times excess real GDP growth beyond 4% if growth is faster than 4.

In Section 3.2, we translate these narrative descriptions to payment formulas in order to derive the model-implied price of these instruments.

3 Estimation of SCDI Premia

In this section, we present a general framework to estimate the time-varying premium of a state-contingent debt instrument. State-contingent debt instruments in principle can take various forms. Our paper focuses on GDP-linked warrants whereby coupon disbursement is determined by the GDP growth rate of a country, primarily motivated by real-world examples and data availability, as explained in Section 2.

3.1 General Framework

We estimate the time-varying premium of a GDP-linked warrant by comparing its value implied by a country's fundamentals with the actual trading price in the data. Primarily following Costa, Chamon, and Ricci (2008), we develop a Monte Carlo pricing exercise to come up with a theoretical value of different instruments. We then compare these to market prices to gauge the magnitude and evolution of any premium above and beyond what would be attributed to liquidity and defaults risks.

The model-implied price of a GDP-linked warrant is defined as the discounted value of expected coupons that will be paid until its maturity year T. For currency j and month m, we can express the end-of-month price of a GDP warrant as:

$$\hat{p}_m(j, r_m^{scdi}(j)) = \mathbb{E}_m \left[\sum_{t>=m}^{T} \frac{c_t(j)}{1 + r_{t,m}(j) + r_m^{scdi}(j)} \right] \tag{1}$$

where t indexes year and $c_t(j)$ denotes annual coupon payment. The size of a coupon is a function of nominal GDP in constant prices, Y_t, real GDP growth, Y_t/Y_{t-1}, and exchange rate, e_t, between local currency and foreign currency j. The formula vary across countries. Repayment of principal is typically not required. The institutional details vary across the three cases we study, as summarized in Section 2. We explain further how these are translated into the formula later.

The risk-premium of a GDP-linked warrant is derived from the discount rate. The discount rate consists of three components. One is the standard discount rate, $r_{t,m}(j)$, which applies to the country's plain-vanilla sovereign bond denominated in currency j. The standard rate is simply measured as the risk-free interest rate plus default premium inferred from credit default swap. Second, we consider a residual discount rate, $r_m^{scdi}(j)$, which equates the market price of a security with the model-implied price estimate such that:

$$p_m^{bid}(j) = \hat{p}_m(j, r_m^{scdi}(j)) \tag{2}$$

Note that $p_m^{bid}(j)$ indicates the bid price of a security. Essentially, we interpret $\{r_m^{scdi}(j)\}_m$ as the additional time-varying premium besides the standard yield, for buyers who intend to hold the GDP-linked warrant until maturity. Finally, we measure the liquidity premium by looking at the yield differentials between the bid and ask prices of a GDP-linked warrant. That is, $r_m^{liq}(j)$ is defined as the spread equating

$$p_m^{ask}(j) = \hat{p}_m(j, r_m^{scdi}(j) + r_m^{liq}(j)) \tag{3}$$

where $p_m^{ask}(j)$ is the ask price of a GDP-linked warrant at each period m. Liquidity premium is intended to capture the gap between the bid and ask prices.

To quantitatively measure these premia, we next turn to stochastic processes of output, Y_t, GDP deflator, D_t, and exchange rate, e_t. Let $y_t \equiv \log(Y_t/Y_{t-1})$ denote the real GDP growth rate, and $\hat{y}_{m,t}$ denote an expected growth rate for year t projected at time m. We assume that the real GDP growth evolves according to

$$y_{t+1} - \hat{y}_{m,t+1} = \theta_y(y_t - \hat{y}_{m,t}) + \varepsilon_{y,t} \quad \text{for} \quad m \leq t \leq T \tag{4}$$

and log change of the GDP deflator follows

$$d_t = \hat{d}_{m,t} + \varepsilon_{d,t} \quad \text{for} \quad m \leq t \leq T \tag{5}$$

where $\varepsilon_{y,t}$ and $\varepsilon_{d,t}$ are error components drawn from a bi-variate normal distribution $N(0, \Sigma_{y,d})$. Finally, nominal exchange rate is assumed to follow

$$e_t = \hat{e}_{m,t} + \varepsilon_{e,t} \quad \text{for} \quad m \leq t \leq T \tag{6}$$

where $\varepsilon_{e,t}$ is drawn from $N(0, \sigma_e^2)$. This modelling framework embodies key features of GDP-linked warrants, for which interest disbursement depends on the realization of economic variables.[2]

Our estimation framework, as well as some functional forms of stochastic processes, is based on the Monte Carlo simulation method proposed by Costa, Chamon, and Ricci (2008), yet there are several notable differences. First, unlike their paper, we do not separate out real exchange rate and CPI processes from nominal exchange rates. This is due to the lack of reliable data for Argentinian CPI from 2007-2015, which is contained in our sample period and countries.[3] Also, while we simply add US treasury yields to the CDS premium of Argentinian bonds for the base discount rate, Costa, Chamon, and Ricci (2008) shifts this discount rate curve vertically to make it consistent with the price of Argentinian USD discount bonds. $\varepsilon_{d,t}$ is modelled to follow a chi-square distribution.

[2]One simplifying assumption here is that the correlations between $\varepsilon_{e,t}$ and $\varepsilon_{y,t}$, and between $\varepsilon_{e,t}$ and $\varepsilon_{d,t}$ are muted. We made this assumption because data on $\hat{e}_{m,t}$ is limited (1-year and 2-year forward rates) while $\hat{y}_{m,t}$ and $\hat{d}_{m,t}$ have longer series. A more general specification of $\{y_t, d_t, e_t\}$ can also be utilized, which we leave as future work. As we present momentarily, coupons of the Ukraine and Greek GDP-linked warrants are denominated in their domestic currencies, so the exchange rates have no bearing on their coupon payment.

[3]In 2014, the Argentinian government acknowledged that they had deliberately underreported CPI statistics during this period. See Cavallo, Cruces, and Perez-Truglia (2016) for an extensive discussion on the issue.

3.2 State-contingent Coupons

Next we specify the coupon payment formula. As explained in Section 2, we focus on GDP-linked warrants. Further to preserve comparability, we focus on cases that share a common structure where coupons are disbursed only if the GDP growth rate (or nominal level) exceeds a threshold and the coupon rate is proportional to the growth rate of the country. These criteria and data availability narrow down the set of countries to three:[4] Argentina (2005), Greece (2012) and Ukraine (2015).

Argentina issued GDP-linked warrants in the 2005 debt restructuring process. Annual coupons of the Argentinian GDP warrants are determined as follows:

$$c_t^{ARG}(j) = \frac{\gamma(j)}{20e_t(j)}(Y_t - Y_t^c)D_t \times \mathbb{I}_{\{Y_t > Y_t^c\}} \times \mathbb{I}_{\left\{Y_t/Y_{t-1} > Y_t^c/Y_{t-1}^c\right\}}$$

The formula show that GDP warrants are characterized by two features. First, the coupon size in year t is proportional to the gap between Y_t, nominal GDP in the 1993 peso price, and Y_t^c, GDP cutoff specified in the warrant contract. The payment is scaled up by GDP deflator, D_t, and adjusted by exchange rate, $e_t(j)$, and a currency-specific constant $\gamma(j)$. Second, coupons are paid only if a nominal GDP level exceeds the cutoff level and a real GDP growth rate exceeds the growth rate of cutoffs. Argentinian GDP-linked warrants were issued in four currencies: US dollar, euro, Japanese yen and Argentinian Peso, and some of these warrants were issued in different locations. In this paper, we analyze the one denominated in the US dollar and issued under the US law, given the similarity of price dynamics across these instruments as documented in Costa, Chamon, and Ricci (2008).

Likewise Greek government issued a GDP-linked warrant as a part of the debt restructuring nego-

[4] We rule out "Brady Bonds," which were issued by Latin American countries in the late 1980s and has state-contingencies such as embedded call options, due to data availability.

tiation in 2012. Annual coupons of the Greek GDP warrants are given by

$$c_t^{GRC} = \min\left\{1.5 * \left(\frac{Y_t}{Y_{t-1}} - \frac{Y_t^c}{Y_{t-1}^c}\right), 0.01\right\} \times \mathbb{I}_{\{Y_t > Y_t^c\}} \times \mathbb{I}_{\left\{Y_t/Y_{t-1} > Y_t^c/Y_{t-1}^c\right\}}$$

The Greek GDP warrant has three features. First, the size of a coupon is proportional to the gap between real GDP growth rate Y_t/Y_{t-1} and its cutoff, Y_t^c/Y_{t-1}^c. Second, the payment is denominated in euros so there is no need for conversion of currency. Third, the maximum rate of coupon is capped by 1 percent.

Ukraine is the third country that offered a GDP-linked warrant as sweetener in the course of debt restructuring process. The coupons of Ukrainian GDP warrants are given by

$$\begin{aligned} c_t^{UKR} = 0.15 * \mathcal{E}_t * \max\left\{\left(\frac{Y_t}{Y_{t-1}} - 1.03\right), 0.01\right\} Y_{t-1}(1 + D_t) \times \mathbb{I}_{\{Y_t > Y_t^c\}} \times \mathbb{I}_{\{Y_t/Y_{t-1} > 1.03\}} \\ +0.4 * \mathcal{E}_t \left(\frac{Y_t}{Y_{t-1}} - 1.04\right) Y_{t-1}(1 + D_t) \times \mathbb{I}_{\{Y_t > Y_t^c\}} \times \mathbb{I}_{\{Y_t/Y_{t-1} > 1.04\}} \end{aligned}$$

for year 2024 onwards. Prior to 2024, there is a cap whereby the maximum coupon cannot exceed one percent of the GDP. The Ukrainian GDP warrant has two features. First, the size of a coupon is proportional to the gap between real GDP growth rate Y_t/Y_{t-1} and its cutoffs. There are two cutoffs, 3 percent growth and 4 percent growth. Second, the payment is denominated in the Ukrainian hryvnia but the actual payment is converted to the dollar.

3.3 Calibration

Our next step is to calibrate the parameters of the model. Essentially, our model is characterized by three stochastic processes: GDP growth process (eq 4), GDP deflator process (eq 5), and exchange rate process (eq 6). We use various market forecasts and historical data to calibrate the parameters of the model.

First, to calibrate the growth process, we turn to the average market consensus of GDP growth rates published by Consensus Economics. The first two-year forecasts are released on a monthly basis, so we use these values directly. Growth forecasts for year T+3 through year T+10 are released on a quarterly basis (or biannual basis for older data). We convert the quarterly values into monthly values by linear interpolation. For year 11 to the maturity, we assume that the real GDP growth converges to 3 percent. Quantitatively, the long-term growth rate has little impact on prices because of future discounting. The persistence parameter, θ_y, is estimated from historical data of each country. Table 1 presents the sample horizons that we use for estimating θ_y. We turn to a milestone of each economy such as the end of a hyper-inflation period or accession to a monetary union to truncate the sample period. For older data series where GDP forecast data does not exist, we use the historical average GDP growth rate as a substitute for $\hat{y}_{m,t}$ to estimate θ. This framework is in line with the assumptions in early research such as Costa, Chamon, and Ricci (2008).

Second, the GDP deflator process is calibrated in a similar manner. As for $\{\hat{d}_{m,t}\}_{m,t}$, we extract 1-year, 2-year, ..., and 5-year GDP deflator forecasts from the World Economic Outlook database. We again convert them into monthly values by linear interpolation. We then extrapolate the 5-year forecasts to projections for the rest of the years. Given θ_y, the forecasted values for GDP growth rates and deflators,

Table 1: Parameter Values

	Paramter Values			
Country	θ_y	$\Sigma_{y,d}$	Period	Milestone
Argentina	0.7239	[0.003, -0.001; -0.001, 0.006]	1992-2005	Hyperinflation ended in 1992
Greece	0.2296	[0.001, 0.000; 0.000, 0.000]	2002-2018	Joined the eurozone in 2001
Ukraine	0.6553	[0.004, -0.003; -0.003, 0.01]	1995-2016	Hyperinflation ended in 1995

Notes: This table presents parameter values of our baseline analysis and the sample periods that we use for estimating these parameters. θ_y is the persistence parameter in the real GDP growth process. Σ is the variance-covariance matrix of the error terms in the GDP and deflator processes.

we jointly estimate Σ from residuals via Maximum Likelihood Estimation. In a similar manner, the projected exchange rates, $\{\hat{e}_{m,t}\}$, are derived from 1-year and 2-year forward exchange rates, which are good proxies for market forecasts on future exchange rates. We assume that nominal exchange rate remains constant after year 3.

Finally, we turn to the risk-free interest rate plus default premium inferred from credit default swaps to calibrate the baseline discount rate, $r_{t,m}(j)$. In cases where this rate does not exist (e.g. a short period time when trading halted due to debt restructuring), we interpolate the values between the two nearest existing records. Such cases are rare in our sample period. In the Appendix, we provide more detailed information about when this is the case.

3.4 Algorithm

Our final step is to run Monte-Carlo simulations to measure the discounted value of a GDP-linked warrant and derive its premia. The Monte-Carlo simulations are conducted in four steps:

1. Calibrate θ, Σ, and σ_e prior to running simulations.
2. For each month m in year T, extract market forecast data $\{\hat{y}_{m,T+t}\}_{t=0,1...,30}$, $\{\hat{d}_{m,T+t}\}_{t=0,1...,30}$ and $\{\hat{e}_{m,T+t}\}_{t=0,1,2}$. Draw random numbers and compute (4), (5), and (6) until the maturity year.
3. For each month m, compute discounted expected values of the GDP-linked warrant.
4. Find r^{scdi} and r^{liq} such that the discounted values are equated to the actual trading prices.

Table 2 presents one example of the Monte Carlo simulation. The table illustrates the dispersion of economic variables (20th, 40th, 60th and 80th percentiles in size) across 500 simulation seeds.[5] The SCDI premium is derived from these simulations as will be seen momentarily.

[5]The model fits next year growth rates realtively well; actual realization of GDP growth rate often falls within the range between the 40th and 60th percentiles. This is due to the reasonable forecast accurancy among market participants. GDP deflator is forecasted less accurately than GDP growth rate.

Table 2: Monte Carlo Simulations (Example)

Variable	Country	Simulated Results (T+1) 20th	40th	60th	80th	Actual Data (T+1)
GDP Growth (T=2018)	Argentina	-5.278	-2.444	0.795	4.269	-2.088
	Greece	-0.254	1.703	3.173	4.811	1.871
	Ukraine	1.169	2.797	4.289	6.015	3.233
GDP Deflator Percentage Change (T=2018)	Argentina	23.3	29.3	34.9	40.9	50.6
	Greece	-0.507	0.573	1.531	2.695	-0.379
	Ukraine	9.278	10.600	11.715	13.090	8.130
Coupon Rates (T=2018)	Argentina	0	0	0	1.861	0
	Greece	0	0	0	0	0
	Ukraine	0	0	0	0	0

Notes: This table displays dispersion of economic variables (20th, 40th, 60th and 80th percentiles in size) across 500 simulation seeds in our Monte Carlo simulation for year 2018. The actual values in the data are placed on the last column as a benchmark.

3.5 Estimated SCDI Premia

We apply this framework to estimate the time-varying risk premium of GDP-linked warrants and analyze its properties. Table 3 presents the summary statistics of our estimates. In the second column, SCDI premium refers to the residual premium that we defined in (2) while liquidity premium is r_t^{liq} in (3). For illustration, we take the average of these estimates each year and each country since the issuance of the securities. The numbers are expressed in percentage terms.

Figures 1, 2 and 3 visualize the results on a monthly frequency. Panel (a) of each figure displays a comparison between the model-implied value of a GDP-linked warrant, using only the CDS spread and risk free rate to discount coupons, and its actual trading price. The time-varying spread between these two prices is used to infer the SCDI premium associated with the GDP-linked warrant. Panel (b) displays the estimated SCDI premium. The black dotted line indicates the trend component that was extracted through HP filter. Panel (c) displays decomposition of the three premia: SCDI, default, and liquidity. Panel (d) presents the comovement between the detrended SCDI premium and CBOE Volatility Index, an indicator for global risk aversion. [6]

In the discussion that follows, we focus on documenting three components of the SCDI premia: secular trend, business cycle comovement, and liquidity during crises, and how they comove with various economic indicators.

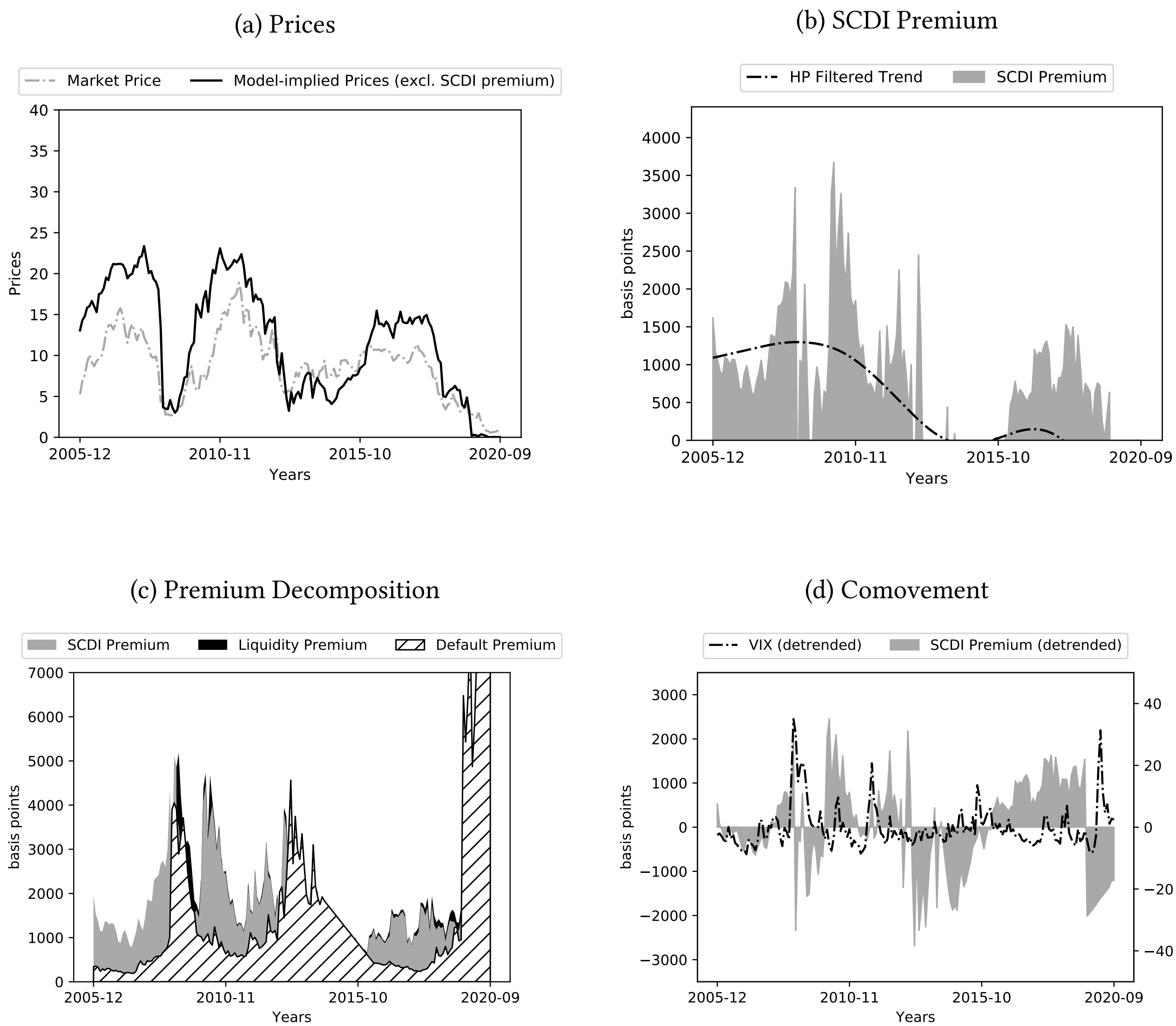
Table 3: Summary Statistics

		Year								
Year	Premium Type	2005	2006	2007	2008	2009	2010	2011	2012	
Argentina	SCDI	16.21	9.61	8.89	16.43	6.96	24.36	9.10	8.60	
	Default	3.35	2.75	2.96	11.96	23.24	8.94	7.14	11.88	
	Liquidity					8.64	1.88	0.49	0.72	
		Year								
Year	Premium Type	2013	2014	2015	2016	2017	2018	2019	2020	Avg.
Argentina	SCDI	-7.73	-10.91	-6.13	5.03	9.85	10.39	-7.03	-29.79	6.24
	Default	27.84	18.86	10.95	4.69	3.25	4.33	28.07	98.67	11.34
	Liquidity	1.46	1.07	1.07	0.53	0.57	1.39	1.19	0.00	1.73
Greece	SCDI	-1.38	2.59	8.69	12.55	15.22	17.03	18.15	11.58	10.55
	Default	17.07*	17.07*	17.07	10.16	6.79	3.36	2.52	1.28	9.41
	Liquidity	0.26	0.53	4.07	3.40	4.21	1.40	1.88	2.39	2.26
Ukraine	SCDI				4.67	6.32	7.36	6.17	5.27	5.96
	Default				6.93*	6.93	4.51	5.66	5.60	5.93
	Liquidity				0.22	0.18	0.14	0.13	0.22	0.18

Notes: This table displays summary statistics of our estimates. In the second column, SCDI premium refers to the residual premium that we defined in (2) while liquidity premium is r_t^{liq} in (3). For illustration, we take average of these estimates each year and each country since the issuance of the securities. All numbers are expressed in percentage terms. The numbers with asteriks are extrapolated values from the earliest traded prices in cases where trading halted due to debt restructuring.

[6] Note that the sharp drop in the model-implied price in 2020 seems to be clearly a function of built-in persistence in the GDP process, which is excessive for a shock like COVID-19 as GDP recovery in this case has been expected by investors to be much faster than usual. The expectation of a V-shaped recovery has supported warrant prices and reduced risk premia relative to the theoretical price which assumes high persistence of GDP growth. We confirm the robustness of the results in a couple of exercises that compute statistics excluding the pandemic period and that fit a different theta persistence parameter for the this period.

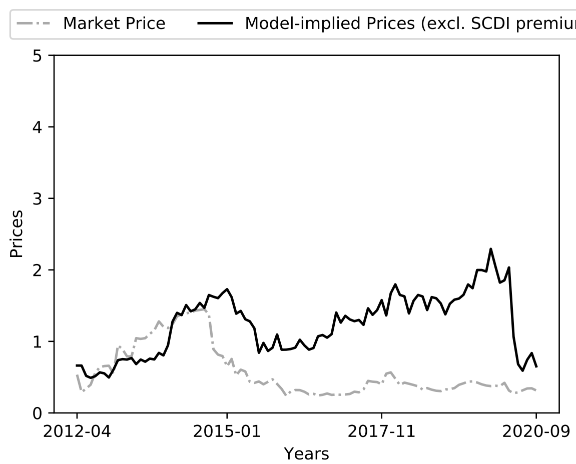
Figure 1: Argentina



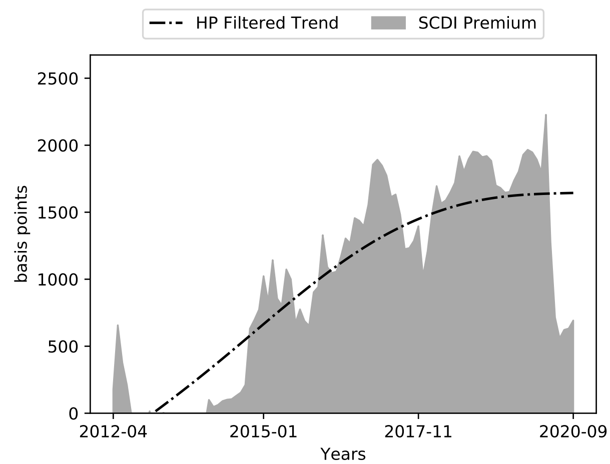
Notes: This figure plots the SCDI premium of (US dollar-denominated) Argentinian GDP-linked warrant on a monthly frequency. Panel (a) of each figure displays a comparison between the model-implied value of a GDP-linked warrant, excluding the SCDI premium, and its actual trading price. Panel (b) displays the estimated SCDI premium. The black dotted line indicates the trend component that was extracted through HP filter. Panel (c) displays decomposition of the three premia: SCDI, default and liquidity. Panel (d) presents the comovement between the detrended SCDI premium and CBOE Volatility Index.

Figure 2: Greece

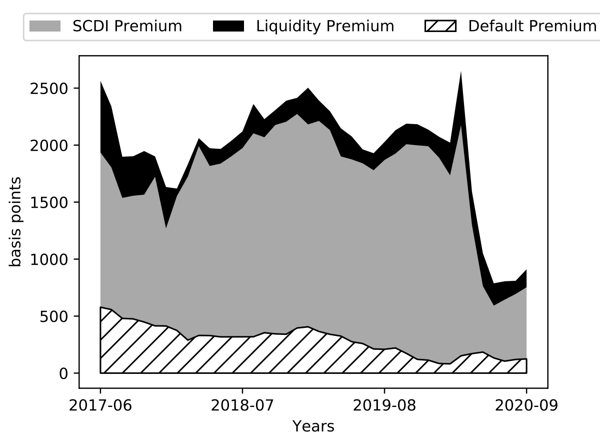
(a) Prices



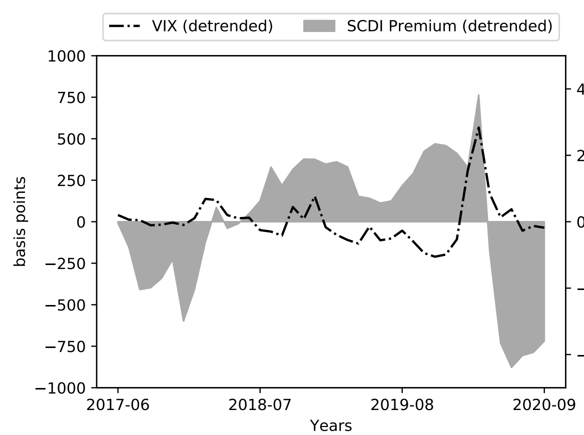
(b) SCDI Premium



(c) Premium Decomposition

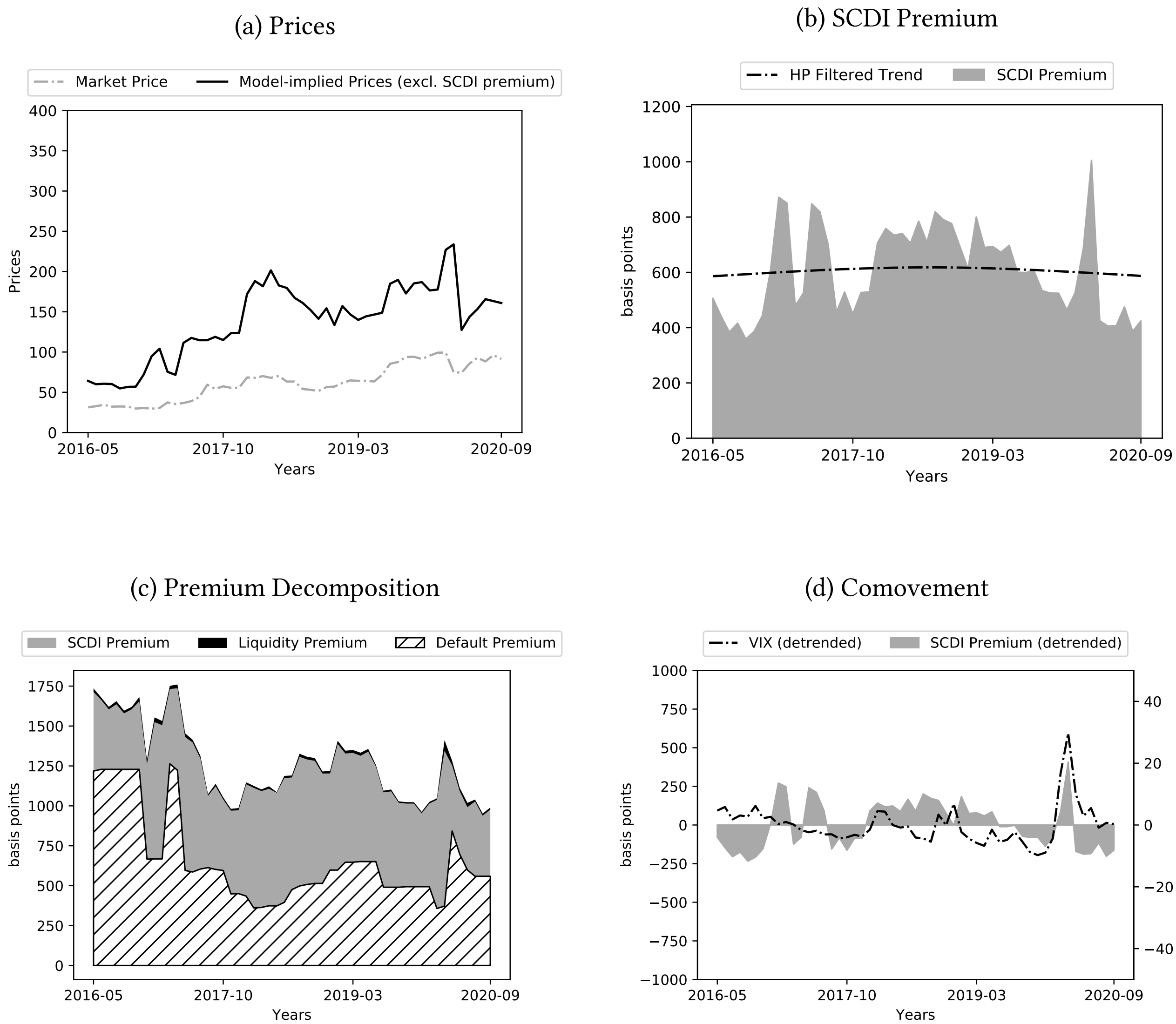


(d) Comovement



Notes: This figure plots the SCDI premium of Greek GDP-linked warrant on a monthly frequency. Panel (a) of each figure displays a comparison between the model-implied value of a GDP-linked warrant and its actual trading price. Panel (b) displays the estimated SCDI premium. The black dotted line indicates the trend component that was extracted through HP filter. Panel (c) displays decomposition of the three premia: SCDI, default and liquidity. Panel (d) presents the comovement between the detrended SCDI premium and CBOE Volatility Index.

Figure 3: Ukraine



Notes: This figure plots the SCDI premium of Ukraine GDP-linked warrant on a monthly frequency. Panel (a) of each figure displays a comparison between the model-implied value of a GDP-linked warrant, excluding the SCDI premium, and its actual trading price. Panel (b) displays the estimated SCDI premium. The black dotted line indicates the trend component that was extracted through HP filter. Panel (c) displays decomposition of the three premia: SCDI, default and liquidity. Panel (d) presents the comovement between the detrended SCDI premium and CBOE Volatility Index.

4 Discussion

4.1 Persistence of the SCDI Premium

One of our key findings is that the SCDI premium is high, persistent and shows no sign of a downward secular trend for most of our sample period. Table 3 illustrates this point; the SCDI premium fluctuates widely over business cycles, yet there is no sign of a long-term decline except the latest period in Argentina. The first two-year averages of the SCDI premium are 9.36 %p, -0.21%p,[7] and 6.12 %p, while the averages over the next three years are 15.78 %p, 8.70% and 7.18% for Argentina, Greece and Ukraine respectively. A sizable premium exists in the market even after a reasonable timeframe has passed since the issuance of each security.

Stylized Fact 1. *SCDI Premium is high, persistent and shows no sign of a downward trend over the first five years since issuance.*

This finding is in contrast with early research such as Costa, Chamon, and Ricci (2008), which documents a substantial decline in the SCDI premium (around 600 basis points) during the first two years after the issuance of Argentinian GDP-linked warrants. Their result suggests that a sizable portion of the SCDI premium could be attributed to "novelty premium," which arises from unfamiliarity with these new assets or the absence of liquidation markets. The premium is likely to decay over time, according to this view, as market participants become more used to trading the new assets.

What we highlight here is that, in hindsight, this observation reflects only a short-term aberration. In Table 3, the estimated SCDI premium trends downward for the first two years in Argentina, which

[7] A negative premium occurs when the model-implied value of a GDP-linked warrant is lower than the actual trading price. Mechanically, this can occur for various reasons. One explanation is a lag in data availability. While the prices of an SCDI and CDS are updated daily, the growth forecast is released only on a monthly basis. A quick upward movement of the economy may push down the SCDI premia to a negative territory for a short period of time until growth forecasts are fully updated.

echoes the early result of Costa, Chamon, and Ricci (2008). Extending the sample period, our result shows that the trend is reversed in year 2 and, rather, there is a steady increase in the SCDI premium during the first five years if we rule out the cyclical components of the monthly time-series. In the case of Greece and Ukraine, our estimation results do not show any sign of a downward trend. These results imply that a large portion of the SCDI premium can be potentially attributed to a permanent feature of a GDP-linked warrant.

4.2 Cyclicality

Next, we investigate the comovement between the SCDI premium and business cycles. A large literature in empirical asset pricing has documented that risk premia of stock, bond and currency returns tend to increase in economic downturns. Consequently, one may expect the SCDI premium to be counter-cyclical in the same manner.[8] In the discussion that follows, we investigate the cyclical properties of the premia and compare their magnitudes.

Table 4 presents the contemporaneous correlation between the three premia of a GDP-linked warrant and monthly variables including industrial production, unemployment rate, and the average earning yield in the local stock market. These variables are used as a proxy to capture the state of the economy at a monthly frequency. A negative correlation with growth in industrial production (or a positive correlation with cyclical component of unemployment) indicates that the premium is counter-cyclical, implying that the premium tends to rise during recessions. In the last two columns, we explore comovement with global financial indicators such as the CBOE Volatility Index (VIX) and the US federal fund rate. The correlations with these variables indicate the degree of synchronization with global financial conditions.

[8]Following the convention in the literature, we call premium to be counter-cyclical if the premium tends to rise in recession. Premium is called pro-cyclical when the premium tends to drop in recession.

Our results summarized in Table 4 show that the SCDI premium has been less counter-cyclical than the default premium of the plain-vanilla government bonds in various dimensions. While the default premium (proxied by the CDS spread of a 5-year government bond) exhibits negative correlation with growth in industrial production over 12 month, 6 month and 1 month horizons, the SCDI premium displays positive correlations with them. In the global context, the SCDI premium shows a positive correlation with the VIX but this correlation is weaker than the one the default premium has with the VIX. The procyclicality of the SCDI premium is more clearly visualized in Panel (c) of Figure 1, 2, and 3. One illustrative example, besides the general pattern, is that the SCDI premia dropped substantially

Table 4: Contemporaneous Correlations

Premium Type	Horizon	*(a) Domestic*			*(b) Global*	
		IP	Unemp.	Earning Yields	VIX	US MP Shock
SCDI Premium	12m	0.1420 (0.0541)				
	6m	0.0208 (0.0546)				
	1m	0.0443 (0.0546)	0.0274 (0.0795)	-0.1749 (0.0783)	0.0115 (0.0796)	0.0236 (0.0795)
Default Premium	12m	-0.1647 (0.0539)				
	6m	-0.1001 (0.0544)				
	1m	-0.0069 (0.0546)	0.2769 (0.0764)	-0.0500 (0.0795)	0.0806 (0.0793)	0.0267 (0.0795)
Liquidity Premium	12m	-0.1269 (0.0539)				
	6m	-0.1269 (0.0544)				
	1m	-0.1269 (0.0546)	0.0715 (0.0794)	0.0716 (0.0794)	0.1413 (0.0788)	-0.0433 (0.0795)

Notes: This table displays contemporaneous correlations between the three premia of a GDP-linked warrant and economic indicators such as industrial production (IP), unemployment rate (Unemp.), the average earning yield (Earning Yields) in local stock market, CBOE volatility index (VIX) and US monetary policy shock estimated by Bu, Rogers, and Wu (2019) (US MP Shock). We use changes in industrial production over 12 month, 6 month and 1 month horizons. For the rest of variables, we simply detrend the values through HP filter.

at the onset of the COVID-19 crisis, whereas the default premium rose.

Stylized Fact 2. *The SCDI premium is less counter-cyclical than the default premium in government bond returns.*

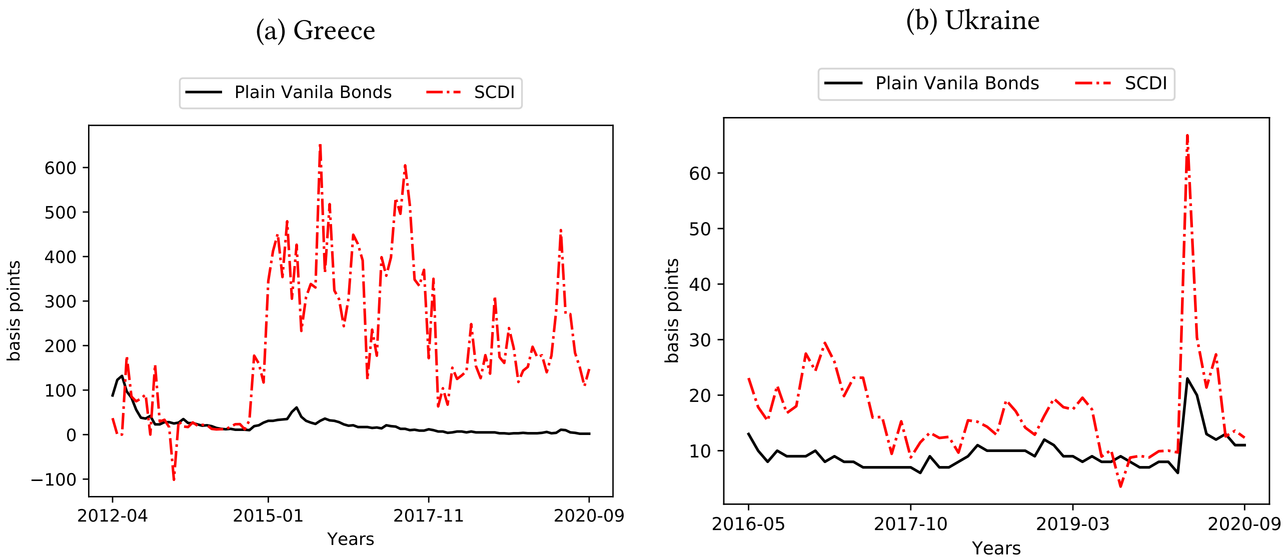
There are several possible confounding factors to account for the relative procyclicality of the SCDI premium. For one, and as observed earlier, there is a possibility that the procyclicality of the SCDI premium during the recent COVID crisis could be artificially driven by our assumption on the GDP shock persistence. In other words, the duration of the COVID shock could be shorter than the average duration of typical recession incidents, so the model-implied price of a GDP warrant could be underestimated. In Figure 8, we explore how the persistence assumption influences the calculations. One may also argue that the procyclicality of the SCDI premium may be an artifact of the chosen decomposition. In other words, default premium on plain-vanilla sovereign bonds tends to be high in a recession which forces the SCDI premium to be low if the total discount factor for SCDIs remains constant. Our main point still stands: the total discount factor for SCDIs moves less counter-cyclically than the default premium, which in turn makes — given our decomposition — the residual discount factor moves procyclically.

4.3 Liquidity

Finally, we turn to the liquidity premium component in GDP-linked warrant markets. Figure 4 displays a plot of the liquidity premia in GDP-linked warrants in tandem with the liquidity premia in plain government bonds. Due to data limitation, we only present plots for Greece and Ukraine. The mean values of the SCDI liquidity premia are 210.42 bps and 18.43 bps whereas the liquidity premia of the plain 10-year government bonds are 40.97 bps and 11.40 bps for Greece and Ukraine respectively. During our sample periods, the liquidity premium in GDP-linked warrant markets tends to be higher and fluctuates more widely than the liquidity premium in plain-vanilla government bond markets.

Stylized Fact 3. *Liquidity premium in GDP-linked warrant markets is higher and fluctuates more widely than liquidity premium in plain-vanilla government bond markets.*

Figure 4: Liquidity Premia



Notes: This figure displays a plot of liquidity premia in GDP-linked warrants and in government bonds with 10-year maturity.

5 Model

In this section, we develop a model of sovereign debt investment with robust preferences to account for the cyclical properties of the SCDI premium. Conceptually, our model closely resembles the one in Roch and Roldan (2021). Global investors price financial securities issued by a small open economy government, but they are concerned about ambiguity in the future path of economic variables. The government issues two types of securities. One is a perpetual bond whereby investors receive a fixed coupon rate with no maturity date. The government automatically defaults on coupon payment if the growth rate of the economy falls below a certain threshold. Besides the perpetual bond, the government issues a GDP-linked warrant. A coupon size of the GDP-linked warrant is proportional to the gap between realized growth rate and a threshold, while there is no maturity date for the GDP-linked warrant. The payment threshold of the GDP-linked warrant is assumed to be higher than the default threshold of the perpetual bond.

The key intuition here is the following: investors with robust preferences are concerned that the probabilistic model they use to forecast the growth rate of a small open economy is potentially misspecified, thereby exaggerating the likelihood of a bad state. This distortion makes the investors require a higher premium when a small deviation from the investors' baseline forecast can change the coupon size widely in the next period. In the case of the perpetual plain-vanilla bond, the payment volatility is maximized when the economy turns to recession due to the increased risk of default. In the case of the GDP-linked warrant, the volatility surges when the current GDP growth rate is in the vicinity its payment threshold. The premium of the GDP warrant therefore moves procyclically relative to the default premium on the perpetual bond. Essentially, the payment structure of GDP warrant is more sensitive to the types of probability distortions that the robust preference lenders fear during an economic expansion, so a heavier discount is applied.

Note that such distortion of subjective probability is non-existent under the standard expected utility model. So the size of SCDI premium is small, especially when correlation between the country's output and investors' consumption is low, and barely responds to business cycles. In this sense, robust preferences work as extra risk aversion, raising premium across all the states (e.g. current GDP growth rate) of the economy. [9] But the increase is not uniform. The premium is increased more acutely

[9]In fact, one key motivation for the ambiguity aversion assumption is to fit the size of risk premia. One may suggest alternative hypotheses under the classical expected utility framework, such as procyclicality of growth risk (i.e. volatility of growth may be high when growth rate is high) to account for some qualitative patterns. Yet, such explanations in the end require additional modelling ingredients to fit the quantitative size of risk premia and its variations.

during good times, when there is more uncertainty about payment of GDP-linked warrant in the next period while coupon payment of the plain-vanilla bond is assured. This observation is also noteworthy because investor risk-aversion in the standard model creates risk premia, but is unlikely to generate procyclicality (on the contrary, as shown in Lizarazo (2013), it generates more countercyclicality of premia).

5.1 Setup

To illustrate this mechanism, we extend the baseline model by incorporating a continuum of identical global investors with robust preferences a la Hansen and Sargent (2001) and Roch and Roldan (2021). The total mass of the investors is normalized to one. Each lender has a utility function of the form

$$(1-\beta)U_t = (1-\beta)\log(C_t) + \frac{\beta}{1-\gamma}\log(\mathbb{E}_t[exp\{(1-\beta)(1-\gamma)U_{t+1}\}]$$

where γ indicates coefficient of relative risk aversion. This form of utility is often referred as the multiplier utility, and has been used to capture a decision maker's aversion to model misspecification. From the lenders' first-order conditions, we can derive a stochastic discount factor:

$$m_{t+1,t} = \left(\beta\frac{C_t}{C_{t+1}}\right)\left(\frac{exp(-U_{t+1}/\omega)}{E_t[exp(-U_{t+1}/\omega)}\right) \tag{7}$$

where $\omega = -\frac{1}{(1-\beta)(1-\gamma)}$. As noted by Tallarini (2000), this utility function is a special case of robust (or ambiguity-averse) preferences. As $\gamma \to 1$, the mental cost of considering model deviations become prohibitively high, so the model converges to the rational expectations framework. Let c_t denote log consumption. The consumption process of international lenders is simply given by

$$c_{t+1} - \bar{c} = \theta_c(c_t - \bar{c}) + \varepsilon_{c,t} \tag{8}$$

The implicit assumption here is that the international lenders are hand-to-mouth households who face the exogenous endowment process as above. The consumption process of global investors is not affected by output of a small open economy.

Next, we model the output process of a small open economy. Let g_t denote the growth rate of output at period t. The output growth evolves as follows:

$$g_{t+1} - \bar{g} = \theta_y(g_t - \bar{g}) + \varepsilon_{y,t} \tag{9}$$

Consider two types of financial securities. One is a plain-vanilla bond, which we denote by PV, whereby investors receive a fixed interest rate perpetually with no principal repayment. The government defaults on the coupon payment only if the growth rate of the economy falls below a threshold, $\underline{g}^{PV}$. The second type is a GDP-linked warrant, which we denote by $SCDI$. The size of the SCDI coupon is proportional to the gap between realized growth rate and a threshold.[10] PV pays r^{PV} while

[10]Differently from the cases we empirically analyze, the SCDI in the model only has growth thresholds and no level conditions. This allows tractability and is more practical for stationarity, at the cost of ignoring the possibility that some of the procyclicality may stem from the level of GDP, as discussed further below.

SCDI pays r^{SCDI}.

$$r^{PV}(g_t) = \begin{cases} \bar{r} & \text{if } g_t \geq \underline{g}^{PV} \\ \\ 0 & \text{Otherwise} \end{cases}$$

$$r^{SCDI}(g_t) = \begin{cases} \alpha(g_t - \underline{g}^{SCDI}) & \text{if } g_t \geq \underline{g}^{SCDI} \\ \\ 0 & \text{Otherwise} \end{cases}$$

where $\underline{g}^{SCDI} > \bar{g} > \underline{g}^{PV}$. This assumption reflects the fact that SCDIs are issued to avert payment during downturns thereby helping avoid default. α is chosen such that $p^{SCDI}(\bar{g}) = p^{PV}(\bar{g})$. This is an simplifying assumption to normalize the values. In real-world cases, α can take an arbitrary number.

The stochastic discount factors are used to price the two government securities. Let $s_t \equiv (c_t, g_t)$ denote the pair of state variables at period t. In view of (7), the market prices of PV and SCDI are determined implicitly by the following equations

$$p^{SCDI}(s_t) = \mathbb{E}_t\left[\left(\beta\frac{C_t}{C_{t+1}}\right)\left(\frac{exp(-U_{t+1}/\omega)}{E_t[exp(-U_{t+1}/\omega}\right)(r^{SCDI}(s_{t+1}) + p^{SCDI}(s_{t+1}))\right]$$

$$p^{PV}(s_t) = \mathbb{E}_t\left[\left(\beta\frac{C_t}{C_{t+1}}\right)\left(\frac{exp(-U_{t+1}/\omega)}{E_t[exp(-U_{t+1}/\omega)]}\right)(r^{PV}(s_{t+1}) + p^{PV}(s_{t+1}))\right]$$

Note that $\frac{exp(-U_{t+1}/\omega)}{E_t[exp(-U_{t+1}/\omega]}$ is added to the standard consumption Euler equation because of the robust preferences of investors. This term reflects the distortion of global investors' belief on the probability distribution of states variables.

The model solution is derived in two steps. First, we solve the recursive utility through the value function iteration method. After obtaining these solutions, we next solve for $p^{SCDI}(s_t)$ and $p^{PV}(s_t)$ by again applying value function iterations. These price functions, along with $r^{PV}(g_t)$ and $r^{SCDI}(g_t)$, are used to compute the expected returns on the securities. That is,

$$R^i(s_t) \equiv \mathbb{E}_t[(p^i(s_{t+1}) + r^i(g_{t+1}))/p^i(s_t)]$$

for $i = SCDI$ or PV. The SCDI premium is defined as the gap between $R^{SCDI}(g_t)$ and $R^{PV}(g_t)$, where g_t indicates the growth rate of the economy in the current period, t. Essentially, the slope of $R^{SCDI}(g_t) - R^{PV}(g_t)$ measures the procyclicality of the SCDI premium. In the discussion that follows, we measure the slopes of this function when investors are endowed with robust preferences and when investors are standard rational agents.

5.2 Numerical Example

Given this setup, we turn to a numerical example to study the procyclicality of SCDI premium. The aim of this exercise is to investigate qualitative patterns of different models, and show that the model with robust preferences can account for the cyclical pattern of the premia more effectively. Table 5 presents the parameter values that we use for the numerical example.

Table 5: Parameter Values

Parameter	Description	Values
ω	Ambiguity Aversion	0.25
β	Time Discount Factor	0.95
α	SCDI Coefficient	10
$\bar{r}$	Plain-vanilla Bond Coupon Rate	0.04
$(\underline{g}^{PV}, \underline{g}^{SCDI})$	Cutoffs	(-0.02, 0.01)
(θ_c, θ_g)	AR(1) Coefficients	(0.5, 0.85)
(σ_c, σ_g)	Noise Standard Deviations	(0.5, 0.5)
ρ	Noise Correlation	0.3
$(\bar{c}, \bar{g})$	Means	(0, 0)

Notes: This table presents the set of parameter values that we used in the numerical example.

Panel (a) of Figure 5 displays a plot of SCDI premia. The black solid line indicates the SCDI premium when investors have robust preferences. The magnitude of the SCDI premium is small. The degree of risk aversion manifested by the log utility in the standard rational expectation model is not sufficient to generate a meaningful size of the premium on the SCDI security. The red dotted line indicates the SCDI premium in the case of robust preferences. The numerical example shows that, under a reasonable set of parameters, one can generate a sizable SCDI premium and that the size of the premium varies widely depending on the current growth rate of the economy. The premium is highest when the growth rate is close to the payment threshold of the SCDI, which is due to the ambiguity-aversion property of robust preferences. In Table 6, we conduct sensitivity checks to see to what extent the quantitative results depend on parameter values.

Panel (b) illustrates that the SCDI premium tends to move relatively more procyclically because of the shape of $R^{SCDI}(g_t) - R^{PV}(g_t)$ with respect to g_t. The shaded bars in panel (b) plots a randomly-generated trajectory of growth process following (eq 9). The two lines indicate how the demeaned SCDI premium comoves along the process under standard and robust preferences. A positive contemporaneous correlation with growth rate is more pronounced under the robust preference setup.

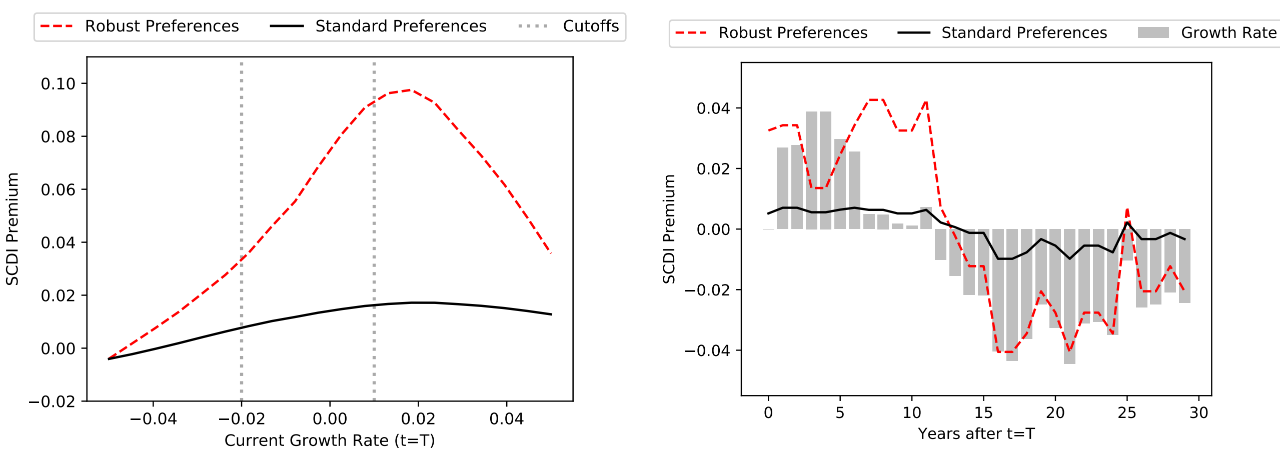
Figure 6 plots expected returns on the perpetual bond and the SCDI, respectively. The solid black lines indicate the expected returns when investors are standard utility maximizers, whereas the dotted red lines indicate the case with robust preferences. Panel (a) shows that robust preferences generate heavier price discounts when the growth rate of the economy is close to the default range. The downward slope of the black line is due to the increased default probability in the next period when the current growth rate is low. With robust preferences, investors become extra-cautious so the bond yield rises. Similarly, panel (b) shows that the gap between the two preference settings is widest when the economy is in a good state.

As mentioned in the model setup, the SCDI in the model makes payments conditional only on the growth outcomes while all three of the SCDIs studied also have level conditions on GDP in addition to growth thresholds. So, in the data, some of the procyclicality may stem from the fact that growth tends to be elevated as an economy rebounds from a low level of GDP. This is when payouts are still expected to be low and, therefore, prices are low and premia high, generating procyclicality of risk premia relative to growth (but not relative to the GDP level). Having no level conditions, the model does not capture the additional procyclicality these dynamics would generate.

Figure 5: Simulation Results

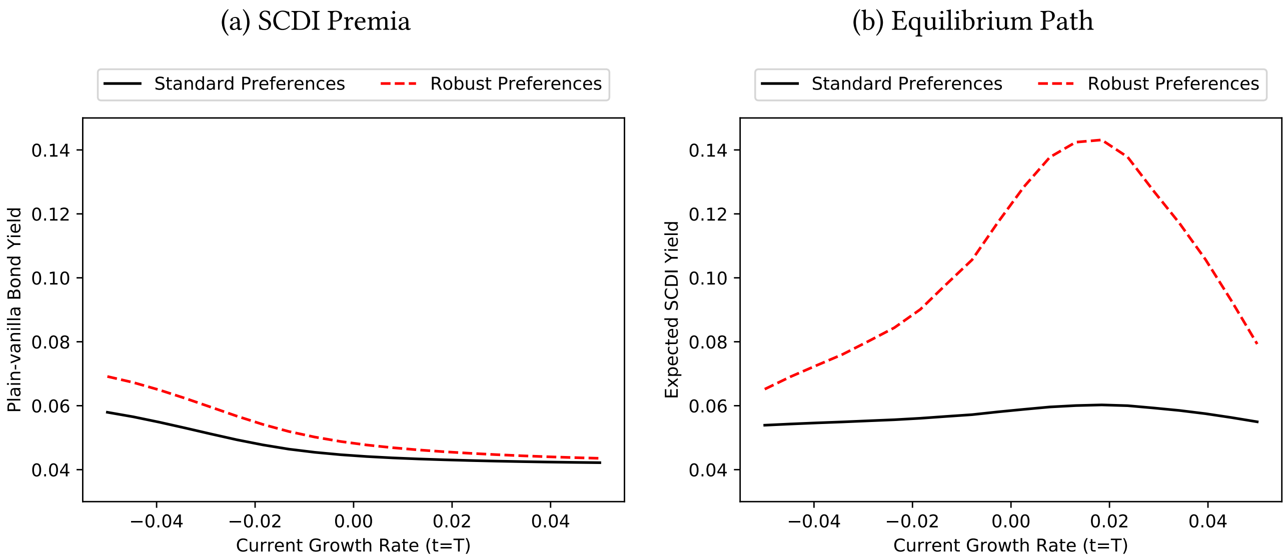
(a) SCDI Premia

(b) Growth Trajectory and Demeaned SCDI Premia



Notes: Panel (a) displays a plot of SCDI premia. The black solid line indicates the SCDI premium when investors have standard preferences. The red dashed line indicates the SCDI premium in the case of robust preferences. The gray dotted lines represent the default cutoff of perpetual bond and the payment cutoff of SCDI respectively. Panel (b) displays an illustrative trajectory of (demeaned) SCDI premia under standard and robust preferences.

Figure 6: Expected Yields



Notes: This plots expected returns on the perpetual bond and SCDI respectively. The solid black lines indicate the expected returns when investors are standard utility maximizers, whereas the dotted red line indicates the case with robust preferences. Panel (a) shows that robust preferences generate heavier price discounts when the growth rate of the economy is close to the default range. Similarly, panel (b) shows that the gap between the two preference settings is widest when the economy is in a good state.

Table 6: Sensitivity Check (SCDI Premia)

	Growth Rate	-0.03	-0.01	0	0.01	0.03
Baseline	Robust	0.016	0.047	0.071	0.019	0.020
	Standard	0.004	0.013	0.017	0.019	0.020
High theta	Robust	0.011	0.028	0.037	0.019	0.019
($\omega = 0.5$)	Standard	0.005	0.014	0.017	0.019	0.019
Low theta	Robust	0.022	0.091	0.142	0.021	0.022
($\omega = 0.1$)	Standard	0.007	0.015	0.019	0.021	0.022
High alpha	Robust	0.019	0.122	0.213	0.020	0.020
($\alpha = 11$)	Standard	0.003	0.013	0.017	0.020	0.020
Low alpha	Robust	0.025	0.089	0.135	0.019	0.019
($\alpha = 9$)	Standard	0.005	0.013	0.016	0.019	0.019

Notes: This table presents sensitivity check results with respect to ambiguity aversion parameter, ω, and payment slope of GDP-linked warrant, α. The values represent SCDI premia under standard preferences and robust preferences.

6 Further Implications and Areas for Research

6.1 Debt Restructuring

The model we develop in this paper can be used to derive testable implications on the price movement of GDP-linked warrants. One such implication is the timing: the issuance of GDP-linked warrants is most effective when the economy is at the trough of a business cycle. This is because investors require a relatively higher risk premium on plain-vanilla bonds because the ambiguity in a country's debt repayment capacity is worsened in recession. GDP-linked warrants, on the other hand, face a relatively milder discount during recessionary periods. The payment volatility of GDP-linked warrants is maximized when the economy turns to a good state. Thus, the cost of capital the government has to pay when issuing GDP-linked warrants can be lower if the current output is low and debt repayment capacity is questioned than that in expansionary periods. This time-varying ambiguity premium provides one explanation on why most GDP-linked warrants that have been issued to date came out when the issuing countries underwent a debt restructuring process (Table 7).

6.2 Cross-country Characteristics

Our paper also has implications on cross-country characteristics of GDP-linked warrants. Our model shows that GDP-linked warrants that have more kinks on their payment structure tend to have a higher SCDI premium especially when kinks are located on a more probable region of the state. As noted in Roch and Roldan (2021), the optimally designed GDP warrant takes the form of linear indexation with minimal use of payment cutoffs.[11] Besides the payment structure, moral hazard on the government's end (or the ambiguity in the presence of moral hazard) may also increase the discount rate applied to

[11] See Roch and Roldan (2021) for more extensive discussion on the optimal design of state contingent debt instruments.

SCDIs in general. Such a discount may be larger for countries with less credible institutions or a poor track record in sovereign debt markets. The cyclical properties of SCDI premium can also vary among countries with different fiscal credibility or a poor track record in international sovereign debt markets, or both. By the same token, it is quite conceivable that a country starts with a countercyclical SCDI risk premium and graduate to a procyclical one as it gains institutional credibility and reputation over time, thus ultimately reaping the theoretical benefits of a SCDI.

Another characteristic that might have a bearing on the potential SCDI market of a country is the level of financial development and existing investor base. In principle, the results from our analysis indicate that countries wishing to keep SCDI premium at a reasonable level may want to market their SCDIs to particular classes of investors that are less averse to ambiguity in economic variables determining payouts (because, for instance, they hold a well-diversified portfolio of SCDIs issued by heterogeneous countries). Available data, though sparse and limited, suggests a diverse set of investors in most cases, with the top identified debtholders accounting for less than 5 percent of outstanding bonds (Table 8). The exception is the peso-denominated Argentine SCDI, with a third of outstanding bonds held by domestic nonbanks.

Due to the limited number of issuance cases, we do not attempt to corroborate these channels through a formal analysis of the available data and instead leave it as future research. As more countries consider issuing GDP-linked warrants,[12] we believe that a more fruitful discussion on cross-country characteristics would become feasible. More issuances outside debt restructuring episodes could particularly shed additional light on the benefits and costs of SCDIs. Likewise, investor base data could potentially be used to explain cross-country variations such as the very different levels and volatility of liquidity premia in Greece and Ukraine.

[12] Suriname is a recent example, see debt restructuring proposal dated June 2021.

6.3 Inflation-indexed Bonds

Another type of state-contingent debt instruments, which has been issued mostly by advanced economies, is inflation-indexed bonds. For issuing government, inflation-indexed bonds offer similar benefits to GDP-linked warrants because a lower output growth is often accompanied with a low inflation in view of the Phillips Curve. In the hypothetical world where there is a monotonous one-to-one mapping between inflation and GDP growth, a GDP-linked warrant can be designed to substitute inflation-indexed bonds. In this sense, inflation-indexed bonds can be understood through the lens of the conceptual framework we develop in this paper. The absence of payment cutoff would help lower the cost of capital when global investors are averse to model ambiguity. We view that such comparison, substitutability, or complementarity among different types of state-contingent instruments would be an interesting venue for future research.

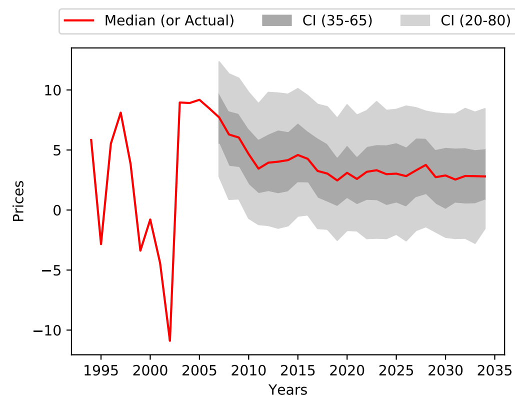
7 Conclusion

This paper has presented a quantifiable model to study the time varying premia of state-contingent debt instruments across countries. Major debt-stressed economies, such as Argentina and Greece, have issued GDP-linked warrants as a viable solution to streamline cash payment and retain debt repayment capacity over business cycles. The recent COVID-19 crisis has also renewed interest in more flexible debt instruments to avert economic crisis. Yet, there has been relatively little empirical research to uncover various properties of state-contingent debt instruments. This paper has developed a general framework to infer the path of risk premium in state-contingent debt instruments such as GDP-linked warrants. We have highlighted the three stylized facts. First, the risk premia in state-contingent instruments are high and persistent. Second, the premia exhibit a pro-cyclical pattern. Third, the liquidity premium is higher and more volatile than that for plain-vanilla government bonds issued by the same sovereign. We then present a model with robust preferences that could explain the cyclical properties of the risk premium. While our paper focuses on GDP-linked warrants among many types of state-contingent debt instruments, we view our work as a stepping stone for studying more general cases such as bonds indexed to commodity prices or equipped with escape clauses.

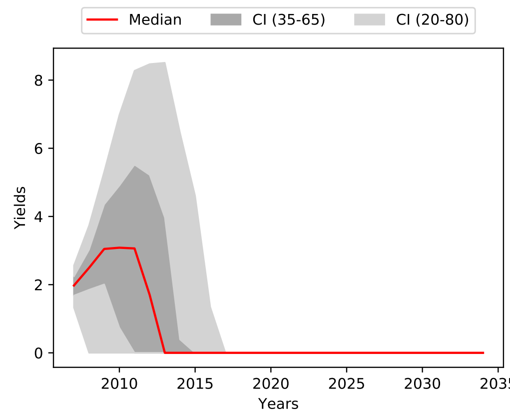
A Appendix

Figure 7: Fan Charts

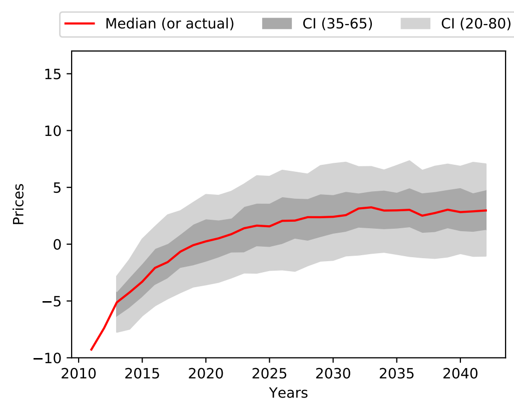
(a) GDP Growth (Argentina)



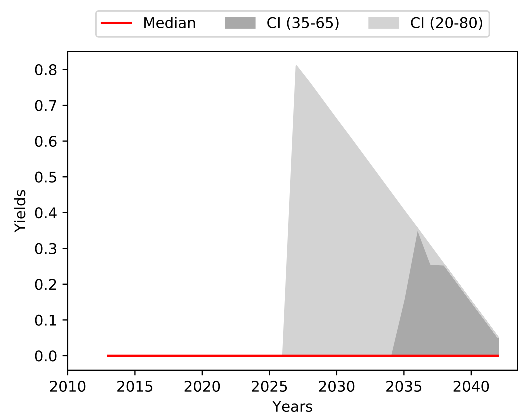
(b) Coupon Payment (Argentina)



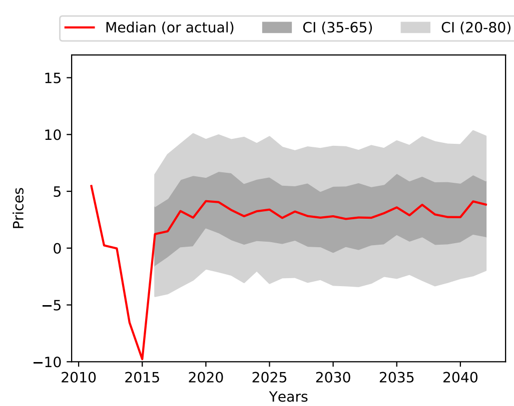
(c) GDP Growth (Greece)



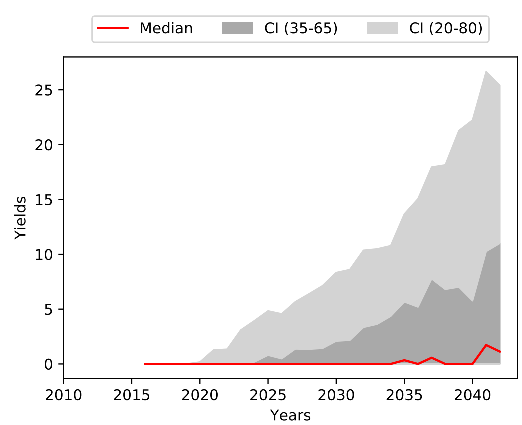
(d) Coupon Payment (Greece)



(e) GDP Growth (Ukraine)



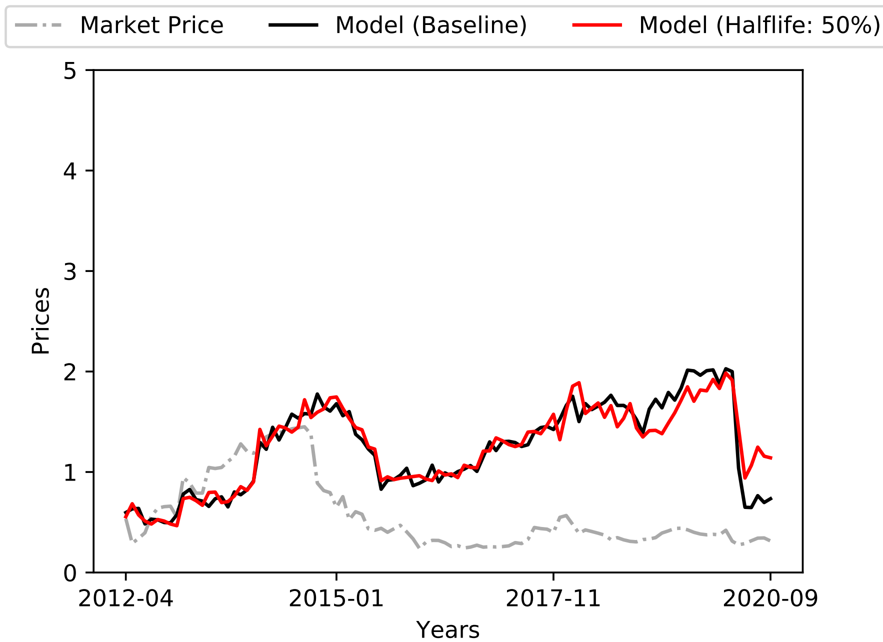
(f) Coupon Payment (Ukraine)



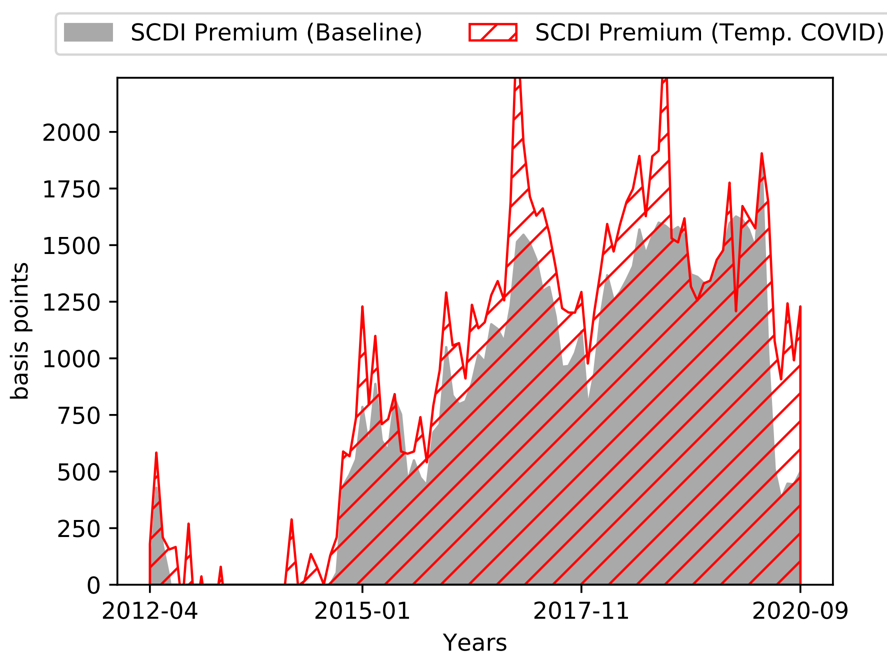
Notes: This figure displays dispersion of economic variables (20th, 40th, 60th and 80th percentiles in size) across 500 simulation seeds in our Monte Carlo simultion for year 2018.

Figure 8: Persistence of GDP Shocks (Greece)

(a) SCDI Prices



(b) SCDI Premium



Notes: This figure presents sensitivity of the model-implied prices of Greek SCDI with respect to the GDP persistence parameter. In the baseline, the persistence parameter is set to such that the half-life of a GDP shock is 2.65 years, which is the historical average. In the alternative scenario, the half-life is assumed to be 1.32 years during 2020-2025.

Table 7: Selected List of SCDI Issuances

Issuing Sovereign	Type of Instrument	Indexation	Indexation Detail	Year Issued	Years Used	Tradeable	Part of Debt Restructuring
Algeria	Loan	Commodity	Oil	1989	1989	No	No
Argentina	Warrant	GDP	Real GDP growth	2005	2005-35	Yes	Yes
Bolivia	Bond	Commodity	Tin	1992		Yes	Yes
Bosnia and Herzegovina	Warrant	GDP	GDP (German prices)	1997	1997-	Yes	Yes
Bulgaria	Warrant	GDP	Unclearly defined	1994	1994-2002	Yes	Yes
Burkina Faso and etc. (a)	Loan	Exports	Nominal Exports	2007	2007-	No (official)	No
CSA (b)	Bond	Commodity	Cotton	1863		Yes	No
Costa Rica	Warrant	GDP	GDP level	1990		Yes	Yes
France	Bond	Commodity	Gold	1952	1952-2012		No
France	Bond	Industrial Production		1956			No
France	Bond	Commodity	Gold	1973	1973-88	Yes	No
Greece	Warrant	GDP	Real GDP growth	2012	2012-42	Yes	Yes
Grenada	Bond	Natural Disaster	Hurricane and etc.	2015	2015-28	No	Yes
Grenada	Bond	Revenues	Investment program	2015	2015-30	No	Yes
Guyana and etc. (c)	Loan	Commodity	Oil	2005	2005-14	No (official)	No
Honduras	Warrant	GDP	Real GDP growth	1989		Yes	Yes
India	Bond	Commodity	Oil	2002	2002, 2005-10	No	Yes
India	Bond	Commodity	Gold	2015	2015-	No	No
Ivory Coast	Warrant	GDP	Real GDP growth	1997		Yes	Yes
Malaysia	Loan	Commodity	Palm Oil			No	No
Mexico	Bond	Commodity	Oil	1977	1977-80	Yes	No
Mexico	Warrant	Commodity	Oil	1990		Yes	Yes
Mexico	Bond	Natural Disaster	Earthquake	2006	2006-09	No	No
Mexico	Bond	Natural Disaster	Earthquake, Hurricane	2009	2009, 2012	Yes	No
Nigeria	Warrant	Commodity	Oil	1992		Yes	Yes
Papua New Guinea	Loan	Commodity	Copper			No	No
Peru and etc. (d)	Bond	Natural Disaster	Earthquake	2018	2018	No (official)	No
Portugal	Bond	GDP	Real GDP growth	2013	2013-17, 2017-	No	No
Singapore	Share	GDP	Real GDP growth	2001	2001-08	No (official)	No
Turkey	Bond	Revenues	SOE Revenues	2009	2009-12	Yes	No
Ukraine	Warrant	GDP	Real GDP growth	2017	2019-38	Yes	Yes
Uruguay	Warrant	Terms of Trade		1991		Yes	Yes
Uruguay	Bond	Wages	Nominal	2014		No	No
Venezuela	Warrant	Commodity	Oil	1990		Yes	Yes

Notes: This table presents selected examples of state contigent debt instruments. See Pina (2020) and IMF (2017) for a more comprehensive list of SCDIs and their modalities. (a) Burkina Faso, Mali, Mozambique,Senegal, and Tanzania (b) Confederate States of America (c) Guyana, Nicaragua, Haiti, Belize, Jamaica, Antigua, Dominica, Grenada, St. Kitts & Nevis, St. Vincent & the Grenadines, and the Dominican Republic (d) Peru, Colombia, Chile, and Mexico.

Table 8: Ownership Structure

Country	Currency	ISIN	Type	Institution	Percent Share
Greece	Euro	GR0138014809	SCDI	Anonymous	95.08
				Foreign banks	0.07
				Foreign Nonbanks	4.2
				Domestic Banks	0.36
				Domestic Nonbanks	0.29
Argentina	US Dollar	ARARGE03E154	SCDI	Anonymous	97.94
				Foreign Nonbanks	0.49
				Domestic Banks	1.29
				Domestic Nonbanks	0.28
Argentina	Peso	ARARGE03E147	SCDI	Anonymous	67.67
				Foreign Nonbanks	1.47
				Domestic Nonbanks	30.86
Argentina	US Dollar	US040114GM64	SCDI	Anonymous	97.22
				Foreign Nonbanks	2.15
				Domestic Nonbanks	0.63
Argentina	Euro	XS0209139244	SCDI	Anonymous	99.63
				Foreign Nonbanks	0.3
				Foreign Banks	0.01
				Domestic Nonbanks	0.04
				Domestic Banks	0.02
				Domestic Banks	0.02
Argentina	US Dollar	XS0501197262	SCDI	Anonymous	98.98
				Domestic Nonbanks	1.02
Argentina	Peso	ARARGE03E121	Plain vanilla	Anonymous	96.36
				Foreign Nonbanks	0.66
				Foreign Banks	0.16
				Domestic Nonbanks	2.35
				Domestic banks	0.47
Argentina	US Dollar	ARARGE03E113	Plain vanilla	Anonymous	52.51
				Foreign Nonbanks	17.18
				Foreign Banks	1.53
				Foreign Nonbanks	3.18
				Domestic Nonbanks	21.58
				Domestic banks	3.99
Ukraine	US Dollar	XS1303929894	SCDI	Anonymous	74.52
				Foreign Banks	5.06
				Foreign Nonbanks	20.42
Ukraine	US Dollar	US903724AW28	SCDI	Anonymous	98.46
				Foreign Nonbanks	1.49
				Foreign Banks	0.05

Notes: This table presents ownership shares of SCDIs and plain-vanilla bond counterparts reported in Bloomberg as of October 7, 2020. Nonbanks include insurance companies.

Table 9: Contemporaneous Correlations (Argentina)

Premium Type	Horizon	(a) Domestic			(b) Global	
		IP	Unemp.	Earning Yields	VIX	US MP Shock
SCDI Pre	12m	0.1364				
	6m	-0.0091				
	1m	0.0789	0.0884	-0.1919	0.0262	0.0432
Default Pre	12m	-0.2508				
	6m	-0.2161				
	1m	0.0295	0.5498	0.1765	0.2780	-0.4853
Liquidity Pre	12m	-0.2661				
	6m	-0.2661				
	1m	-0.2661	0.1674	0.1015	0.1112	-0.1326

Notes: This table displays contemporaneous correlations between the three premia of a GDP-linked warrant and economic indicators such as industrial production (IP), unemployment rate (Unemp.), the average earning yield (Earning Yields) in local stock market, CBOE volatility index (VIX) and US monetary policy shock estimated by Bu, Rogers, and Wu (2019) (US MP Shock)

Table 10: Contemporaneous Correlations (Greece)

Premium Type	Horizon	(a) Domestic			(b) Global	
		IP	Unemp.	Earning Yields	VIX	US MP Shock
SCDI Pre	12m	0.2631				
	6m	0.2315				
	1m	0.0177	-0.4968	-0.0868	-0.1382	0.5228
Default Pre	12m	-0.2558				
	6m	-0.1534				
	1m	-0.0334	0.3129	-0.2635	-0.0330	0.0843
Liquidity Pre	12m	0.1883				
	6m	0.1883				
	1m	0.1883	-0.2198	-0.0064	0.3001	-0.0683

Notes: This table displays contemporaneous correlations between the three premia of a GDP-linked warrant and economic indicators such as industrial production (IP), unemployment rate (Unemp.), the average earning yield (Earning Yields) in local stock market, CBOE volatility index (VIX) and US monetary policy shock estimated by Bu, Rogers, and Wu (2019) (US MP Shock)

Table 11: Contemporaneous Correlations (Ukraine)

Premium Type	Horizon	(a) Domestic			(b) Global	
		IP	Unemp.	Earning Yields	VIX	US MP Shock
SCDI Pre	12m	0.1505				
	6m	0.0707				
	1m	-0.1432	-0.0954	0.0057	0.1942	0.3350
Default Pre	12m	-0.1048				
	6m	0.1634				
	1m	0.0622	0.4101	0.1522	0.2628	-0.1181
Liquidity Pre	12m	0.0546				
	6m	0.0546				
	1m	0.0546	-0.0246	-0.1115	-0.1754	0.0060

Notes: This table displays contemporaneous correlations between the three premia of a GDP-linked warrant and economic indicators such as industrial production (IP), unemployment rate (Unemp.), the average earning yield (Earning Yields) in local stock market, CBOE volatility index (VIX) and US monetary policy shock estimated by Bu, Rogers, and Wu (2019) (US MP Shock)

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