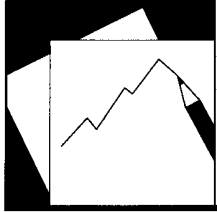


Debt Sustainability, Public Investment, and Natural Resources in Developing Countries: the DIGNAR Model



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Giovanni Melina, Shu-Chun S. Yang, Luis-Felipe Zanna

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Debt Sustainability, Public Investment, and Natural Resources in Developing Countries: the DIGNAR Model*Prepared by **Giovanni Melina, Shu-Chun S. Yang, Luis-Felipe Zanna**

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Abstract

This paper presents the DIGNAR (Debt, Investment, Growth, and Natural Resources) model, which can be used to analyze the debt sustainability and macroeconomic effects of public investment plans in resource-abundant developing countries. DIGNAR is a dynamic, stochastic model of a small open economy. It has two types of households, including poor households with no access to financial markets, and features traded and nontraded sectors as well as a natural resource sector. Public capital enters production technologies, while public investment is subject to inefficiencies and absorptive capacity constraints. The government has access to different types of debt (concessional, domestic and external commercial) and a resource fund, which can be used to finance public investment plans. The resource fund can also serve as a buffer to absorb fiscal balances for given projections of resource revenues and public investment plans. When the fund is drawn down to its minimal value, a combination of external and domestic borrowing can be used to cover the fiscal gap in the short to medium run. Fiscal adjustments through tax rates and government non-capital expenditures—which may be constrained by ceilings and floors, respectively—are then triggered to maintain debt sustainability. The paper illustrates how the model can be particularly useful to assess debt sustainability in countries that borrow against future resource revenues to scale up public investment.

JEL Classification Numbers: Q32; E22; E62; F34

Keywords: Natural resources; Public Investment; Debt Sustainability; Small Open DSGE Models; DIGNAR; Developing Countries

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I. INTRODUCTION

Natural resource revenues are a significant source of fiscal revenues in many developing countries, especially in Africa. Given tremendous infrastructure needs and borrowing constraints in these countries, resource revenues are valuable to finance public investment, which has been claimed to be important for growth and economic development. For countries that expect future production, resource revenues can also serve as a collateral for accessing international financial markets, making it possible to build up public capital before revenues arrive. However, natural resource revenues may bring the infamous natural resource curse to the recipient economy—a negative relationship between resource abundance and growth.¹

The positive growth effects of public investment have long been recognized in the theoretical literature. Agénor (2012) distinguishes several channels through which increases in public capital may affect growth, including the one associated with positive productivity and cost-saving effects—more public capital raises the productivity of labor and private capital and lowers the unit costs. The other channels correspond to (i) a complementary effect on private capital, where more public capital increases the rate of return on private capital; (ii) a crowding-out effect, when increases in public capital requires *domestic* financing and, therefore, displaces (or crowds out) private investment, and (iii) a “Dutch vigor” effect, where higher public capital can raise the total factor productivity through positive learning-by-doing externalities (see Berg et al. (2010)). These channels provide support to the “Big Push” proposal that Sachs (2005), among others, has advocated for many poor developing countries: a substantial increase in public infrastructure spending financed with more aid and debt relief to increase growth and reduce poverty. Scaling up public investment in developing countries, however, may not always enhance growth. Low public investment efficiency and absorptive capacity constraints are among them, as they can significantly discount the growth benefits of public investment (Berg et al. (2013) and van der Ploeg (2012a)). Also, in resource-rich countries, spending resource revenues domestically may lead to Dutch disease, hurting the competitiveness of traded good sectors and, hence, growth (e.g., van der Ploeg (2011a) and van der Ploeg and Venables (2013)).

History reveals that, during windfalls, resource-rich developing countries that plan to increase public investment together with external borrowing may bear substantial debt risks. Soaring oil prices in the 1970s promoted many oil-exporting countries (e.g., Algeria, Ecuador, Indonesia, Nigeria, Trinidad and Tobago, and Venezuela as documented in Gelb (1988)) to undertake ambitious investment projects, jointly financed by oil revenues and external borrowing. In the early 1980s, the increase in interest rates and the collapse of oil prices contributed to a sequence of debt crises in Latin America, resulting in the “lost

¹Sachs and Warner (1995) show that resource rich countries grow on average one percent less during the period of 1970-89 after controlling for initial income per capita, investment, openness and rule of law. In spite of many later studies supporting the existence of a natural resource curse (Sachs and Warner, 1995, 1999; van der Ploeg and Poelhekke, 2009), the empirical literature that followed pointed to mixed results, suggesting also the possibility of a natural resource “blessing” (van der Ploeg, 2011b).

decade” with little or negative economic growth—per capita income plummeted and poverty increased (Carrasco (1999)). In fact, Manzano and Rigobon (2007) argue that these countries used natural resources as collateral for debt, leading to excessive borrowing when commodity prices in the 1970’s were going up. But once prices plummeted in the 1980s, debt crises became inevitable. In theory, external commercial borrowing to scale up public investment in poor developing countries can bring some benefits but also involve significant risks, as suggested by Buffie et al. (2012): poor execution of projects, sluggish fiscal adjustments to service debt, or persistent negative economic shocks can easily threaten debt sustainability.

The previous discussion suggests that assessing the growth and debt sustainability effects of public investment plans in resource-rich developing countries is not an easy task. Nevertheless country teams at the Fund are frequently asked to provide such assessments. To do so, they can use the IMF-World Bank debt sustainability framework (IMF-WB DSF), but this framework can be subject to criticisms. Some suggest that IMF-WB DSF does not contain a fully consistent analytic framework to create projections that accounts for the public investment growth nexus.² To address this criticism, Buffie et al. (2012) construct a consistent model-based framework that has been used at the Fund to provide debt sustainability and growth assessments of public investment surges in low-income countries (LICs).³ This framework, however, does not have a natural resource sector and, thus, it may not be suitable for resource-rich countries. On the other hand, the framework developed in Berg et al. (2013) contains a resource sector but abstracts from debt accumulation; thus, the model can only be used to evaluate the effects of a public investment increase financed by resource revenues.

To fill this modeling gap, in this paper, we present a DSGE model of a small open economy, which can be used for assessing debt sustainability and growth effects in a resource-rich developing country that combines resource revenues and borrowing to scale up public investment. We name the model the “Debt, Investment, Growth, and Natural Resources (DIGNAR)” model, and it basically merges the debt model of Buffie et al. (2012) with the natural resource model of Berg et al. (2013). DIGNAR differs from the Buffie et al. model by adding a natural resource sector—so it accounts for resource GDP—and distinguishing between the resource sector and non-resource traded good sector. DIGNAR differs from the Berg et al. model by including several debt instruments such as concessional debt, external commercial debt, and domestic debt. Other key features of DIGNAR include large share of poor households that do not have access to financial markets, learning-by-doing externalities in the traded good production to capture potential Dutch disease from spending resource

²Eaton (2002) and Hjertholm’s (2003) have raised concerns that IMF-WB’s debt projections are not derived from an integrated, internally consistent macroeconomic framework. For instance they generally do not make an explicit linkage between the public investment that the proposed nonconcessional borrowing is meant to finance and the resulting growth that should make the operation self-financing. This inflates debt indicators, such as debt-to-GDP ratios, creates a bias toward conservative borrowing limits, and can amount “to sacrificing growth to imprecisely known debt sustainability risks” (Wyplosz, 2007).

³The Buffie et al. model has been used to complement the IMF-WB DSF. It has been applied to several countries including Burkina Faso, Cape Verde, Côte d’Ivoire, Ghana, Liberia, Rwanda, Senegal, and Togo.

revenues, inefficiency and absorptive capacity constraints for public investment, a time-varying depreciation rate of public capital that accelerates with the lack of maintenance of installed capital, and a relatively detailed fiscal specification, to be explained next.

The main objective of DIGNAR is to help governments facing volatile and exhaustible resource revenues to make public investment decisions. A fast investment scaling-up pace may lead to fast accumulation of public capital and higher non-resource growth. However, as more resource revenues are devoted to public investment, less can be saved, leaving the economy vulnerable to future negative resource revenue shocks. Also, when determining the public investment scaling up magnitude, it is important to consider the financing needs of sustaining capital after resource revenues are exhausted, to ensure long-lasting growth benefits from more public capital. To highlight the important role of saving in maintaining macroeconomic stability, the resource fund introduced in the model plays the role of a fiscal buffer. Given exogenous paths of public investment, concessional borrowing, aid, resource production, prices, and revenues, a resource fund is drawn down if there is a revenue shortfall or accumulates higher savings if there is excessive revenue. When the fund reaches a chosen lower bound, a government can choose one or more fiscal instruments contemporaneously or resort to borrowing to close the fiscal gap. Debt accumulation then triggers fiscal adjustments. Debt dynamics are determined by the speed of fiscal adjustments, specified in the fiscal rules of the adjustment instruments. As a result, the resource fund in DIGNAR can maintain macroeconomic stability by stabilizing public and private consumption paths.⁴

There are four fiscal instruments to close the fiscal gap: the consumption and labor income tax rates, on the revenue side, and government consumption and transfers, on the expenditure side. These instruments can be constrained by corresponding ceilings or floors. In practice, raising tax rates beyond a certain level may not be feasible, due to weak institutions in enforcing revenue collection or lack of political support. To maintain minimal functions, government consumption cannot be lowered than the level required to cover its operating costs, and transfers cannot be lower than zero. Also, we assume that the resource fund can be subject to a minimal level of assets, so the fund can serve as a saving commitment device.⁵ Moreover, to model ambitious public investment plans, DIGNAR introduces a functional form that can be used to parameterize the degree of front-loading and the level of scaling-up magnitude in the longer horizon, convenient to construct an investment scaling-up path.

In this paper, we also illustrate how DIGNAR can be used for policy analysis. We calibrate the model to an average LIC and analyze various investment scaling-up paths. We consider two hypothetical scenarios of resource revenue inflows, similar to the qualitative patterns of a

⁴Different from Berg et al. (2013), where the saving rate of a resource windfall into a resource fund is constant and transfers adjust to satisfy the government budget constraint, the resource fund in DIGNAR can maintain a stable fiscal regime, in which government consumption, tax rates, and transfers do not have to adjust to maintain fiscal sustainability while scaling up public investment.

⁵To see the role of this feature in practice see the model application to Kazakhstan in Minasyan and Yang (2013).

country that anticipates a future resource windfall. The simulated investment approaches include (i) the spend-as-you-go approach, which invests all resource windfall each period without saving, and (ii) the delinked approach, which combines investment and saving such that government spending is a-cyclical along with resource revenue flows. Simulations for different degrees of investment front-loading, investment efficiency, and return to public capital are also pursued to show their role in the debt sustainability analysis. Country applications of DIGNAR so far include Mozambique (Melina and Xiong, 2013) and Kazakhstan (Minasyan and Yang, 2013).

Like the models in Buffie et al. (2012) and Berg et al. (2013) that follow the approach of developing DSGE models for LICs, we see several advantages of using DIGNAR for policy analysis, as summarized in Berg et al. (2014). Models like DIGNAR offer a framework for organizing thinking and incorporating empirical evidence, such as returns to public capital and public investment efficiency. Also, they can be used to systematically produce alternative macroeconomic and policy scenarios, which make transparent the linkages of different assumptions and their effects on macroeconomic outcomes in an internally consistent framework.

The remainder of this paper is organized as follows. In Section II, we present the structure of the model and discuss extensively all its features. We discuss our calibration to an average LIC in Section III. In Section IV, we present and discuss the policy scenarios. Finally, Section V concludes.

II. THE DIGNAR MODEL

DIGNAR is a real three-sector model of a small open economy embellished with multiple types of public sector debt, multiple tax and spending variables, and a resource fund. The country produces a composite of traded good and a nontraded good using capital k , labor L , and government-supplied infrastructure k_G . It is also endowed with natural resources whose production and prices are assumed to be exogenous. Since the time horizon is 20+ years, the model abstracts from money and all nominal rigidities.⁶ We lay out the model in stages, starting with the specification of households.

A. Households

There are two types of households who live infinitely and are distributed over the unit interval. A fraction ω own firms and have access to capital markets. They are referred to as intertemporal optimizing or Ricardian households and denoted by the superscript OPT . The remaining fraction $1 - \omega$ are poor and financially constrained. They are referred to as

⁶The nominal side and New Keynesian features may be added if the model is used to study the short-run policy effects of fiscal management to resource revenue flows.

rule-of-thumb or hand-to-mouth and do not have access to capital and financial markets. They consume all of their disposable income each period and are denoted by the superscript *ROT*.

Both types consume a consumption basket c_t^i , which is described as a constant-elasticity-of-substitution (CES) aggregate of traded goods $c_{T,t}^i$ and nontraded goods $c_{N,t}^i$. Thus, the consumption basket is

$$c_t^i = \left[\varphi^{\frac{1}{\chi}} (c_{N,t}^i)^{\frac{\chi-1}{\chi}} + (1 - \varphi)^{\frac{1}{\chi}} (c_{T,t}^i)^{\frac{\chi-1}{\chi}} \right]^{\frac{\chi}{\chi-1}}, \quad \text{for } i = OPT, ROT, \quad (1)$$

where φ indicates the nontraded good bias and $\chi > 0$ is the intra-temporal elasticity of substitution.

The consumption basket is the numeraire of the economy, and $p_{N,t}$ represents the relative price of nontraded goods, and s_t corresponds to the relative price of traded goods to the consumption basket. Assuming that the law of one price holds for traded goods implies that s_t also corresponds to the real exchange rate, defined as the price of one unit of foreign consumption basket in units of domestic basket.

Minimizing total consumption expenditures subject to the consumption basket (1) yields the following demand functions for each good:

$$c_{N,t}^i = \varphi p_{N,t}^{-\chi} c_t^i, \quad \forall i = OPT, ROT, \quad (2)$$

and

$$c_{T,t}^i = (1 - \varphi) s_t^{-\chi} c_t^i, \quad \forall i = OPT, ROT. \quad (3)$$

The unit price of the consumption basket is

$$1 = \left[\varphi p_N^{1-\chi} + (1 - \varphi) s_t^{1-\chi} \right]^{\frac{1}{1-\chi}}. \quad (4)$$

Both types of households provide labor service ($L_{T,t}^i$ and $L_{N,t}^i$, $i = OPT, ROT$) to the traded and the nontraded good sectors, denoted by subscripts *T* and *N*, respectively. Total labor L_t^i has the following CES specification to capture imperfect substitutability between the labor amounts supplied to the two sectors:

$$L_t^i = \left[\delta^{-\frac{1}{\rho}} (L_{N,t}^i)^{\frac{1+\rho}{\rho}} + (1 - \delta)^{-\frac{1}{\rho}} (L_{T,t}^i)^{\frac{1+\rho}{\rho}} \right]^{\frac{\rho}{1+\rho}}, \quad \text{for } i = OPT, ROT, \quad (5)$$

where δ is the steady-state share of labor in the nontraded good sector, and $\rho > 0$ is the intra-temporal elasticity of substitution. Let $w_{T,t}$ and $w_{N,t}$ be the real wage rates paid in each sector, and w_t be the real wage index. Maximizing the household's total labor income

$(w_t L_t^i = w_{T,t} L_{T,t}^i + w_{N,t} L_{N,t}^i)$ subject to aggregate labor (5) yields the following labor supply schedules for each sector:

$$L_{N,t}^i = \delta \left(\frac{w_{N,t}}{w_t} \right)^\rho L_t^i, \quad \text{for } i = OPT, ROT, \quad (6)$$

and

$$L_{T,t}^i = (1 - \delta) \left(\frac{w_{T,t}}{w_t} \right)^\rho L_t^i, \quad \text{for } i = OPT, ROT. \quad (7)$$

The real wage index is

$$w_t = [\delta w_{N,t}^{1+\rho} + (1 - \delta) w_{T,t}^{1+\rho}]^{\frac{1}{1+\rho}}. \quad (8)$$

1. Intertemporal Optimizing Households

A representative intertemporal optimizing household maximizes its utility

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t^{OPT}, L_t^{OPT}) = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\sigma} (c_t^{OPT})^{1-\sigma} - \frac{\kappa^{OPT}}{1+\psi} (L_t^{OPT})^{1+\psi} \right] \right\}, \quad (9)$$

subject to the following budget constraint:

$$\begin{aligned} (1 + \tau_t^C) c_t^{OPT} + b_t^{OPT} - s_t b_t^{OPT*} &= (1 - \tau_t^L) w_t L_t^{OPT} + R_{t-1} b_{t-1}^{OPT} - R_{t-1}^* s_t b_{t-1}^{OPT*} \\ &+ \Omega_{T,t} + \Omega_{N,t} + \vartheta^K \tau^K (r_{T,t}^K k_{T,t-1} + r_{N,t}^K k_{N,t-1}) + s_t r m_t^* + z_t - \mu k_{G,t-1} - \Theta_t^{OPT*}. \end{aligned} \quad (10)$$

E_0 is the expectation operator at time 0; $\beta \equiv [(1 + \varrho)]^{-1}$ is the subjective discount factor; and ϱ is the pure rate of time preference. σ is the inverse of the inter-temporal elasticity of substitution of consumption, while ψ is the inverse of the inter-temporal elasticity of substitution of the labor supply. κ^{OPT} is the disutility weight of labor and τ_t^C and τ_t^L are the tax rates on consumption and labor income, respectively. The intertemporal optimizing households have access to government bonds b_t^{OPT} that pay a (gross) real interest rate R_t . They can also borrow from abroad—so b_t^{OPT*} is liabilities to the rest of the world—by paying an interest rate R_t^* . These households also receive profits, $\Omega_{T,t}$ and $\Omega_{N,t}$, from firms that are in the traded and nontraded good sector. The term $\vartheta^K \tau^K (r_{T,t}^K k_{T,t-1} + r_{N,t}^K k_{N,t-1})$ is a tax rebate that optimizing households receive on the tax levied on the firms' return on capital.⁷ $r m_t^*$ denotes remittances from abroad and z_t corresponds to government transfers. μ is the user fees charged for public capital $k_{G,t}$ services, and $\Theta_t^{OPT*} \equiv \frac{\eta}{2} (b_t^{OPT*} - b_{t-1}^{OPT*})^2$ are portfolio adjustment costs associated to foreign liabilities, where η controls the degree of

⁷Because of the common wedge between tax burden imposed and tax revenues accrued to the government in developing countries, we assume that a fraction ϑ^K of the tax revenue related to capital income does not enter the government budget constraint. Introducing this wedge also allows us to match the observed initial low private investment flows observed in most LICs.

capital account openness and b^{OPT*} is the initial steady-state value of private foreign debt.⁸ Note that a variable without a time subscript refers to the steady-state value of such variable.

Let λ_t be the Lagrange multiplier to the budget constraint (10). The first-order conditions with respect to c_t^{OPT} , L_t^{OPT} , b_t^{OPT} , and b_t^{OPT*} are

$$\lambda_t (1 + \tau_t^C) = (c_t^{OPT})^{-\sigma}, \quad (11)$$

$$\kappa^{OPT} (L_t^{OPT})^\psi = \lambda_t (1 - \tau_t^L) w_t, \quad (12)$$

$$\lambda_t = \beta E_t (\lambda_{t+1} R_t), \quad (13)$$

and

$$\lambda_t = \beta E_t \left[\frac{\lambda_{t+1} s_{t+1} R_t^*}{s_t - \eta (b_t^{OPT*} - b^{OPT*})} \right]. \quad (14)$$

We assume that the private sector pays a constant premium u over the interest rate that the government pays on external commercial debt $R_{dc,t}$, such that

$$R_t^* = R_{dc,t} + u. \quad (15)$$

2. Rule-of-thumb Households

Rule-of-thumb households have the same utility function as that of intertemporal optimizing households, so

$$U(c_t^{ROT}, L_t^{ROT}) = \frac{1}{1 - \sigma} (c_t^{ROT})^{1 - \sigma} - \frac{\kappa^{ROT}}{1 + \psi} (L_t^{ROT})^{1 + \psi}. \quad (16)$$

Their consumption is determined by the budget constraint

$$(1 + \tau_t^C) c_t^{ROT} = (1 - \tau_t^L) w_t L_t^{ROT} + s_t r m_t^* + z_t - \mu k_{G,t-1}, \quad (17)$$

while static maximization of the utility function gives the following labor supply function:

$$L_t^{ROT} = \left[\frac{1}{\kappa^{ROT}} \frac{1 - \tau_t^L}{1 + \tau_t^C} (c_t^{ROT})^{-\sigma} w_t \right]^{\frac{1}{\psi}}. \quad (18)$$

⁸These adjustment costs also ensure stationarity in this small open economy model, as discussed in Schmitt-Grohe and Uribe (2003).

3. Aggregation

With two types of households, aggregate consumption, labor, privately-owned government bonds, and foreign liabilities are computed as follows.

$$c_t = \omega c_t^{OPT} + (1 - \omega) c_t^{ROT}, \quad (19)$$

$$L_t = \omega L_t^{OPT} + (1 - \omega) L_t^{ROT}, \quad (20)$$

and

$$b_t = \omega b_t^{OPT}; \quad b_t^* = \omega b_t^{OPT*}. \quad (21)$$

B. Firms

The economy has three production sectors: (i) a nontraded good sector indexed by N ; (ii) a (non-resource) traded good sector indexed by T ; and (iii) a natural resource sector indexed by O . Since resource-rich developing countries tend to export most resource output, we assume that the whole resource output is exported for simplicity.

1. Nontraded Good Sector

Nontraded good firms produce output $y_{N,t}$ with the following Cobb-Douglas technology:

$$y_{N,t} = z_N (k_{N,t-1})^{1-\alpha_N} (L_{N,t})^{\alpha_N} (k_{G,t-1})^{\alpha_G}, \quad (22)$$

where z_N is total factor productivity, $k_{N,t}$ is end-of-period private capital, $k_{G,t}$ is the end-of-period public capital, α_N is the labor share of sectoral income, and α_G is the output elasticity respect to public capital.

Capital installed in the nontraded good sector evolves according to

$$k_{N,t} = (1 - \delta_N) k_{N,t-1} + \left[1 - \frac{\kappa_N}{2} \left(\frac{i_{N,t}}{i_{N,t-1}} - 1 \right)^2 \right] i_{N,t}, \quad (23)$$

where $i_{N,t}$ represents investment expenditure, δ_N is the capital depreciation rate, and κ_N is the investment adjustment cost parameter. The investment adjustment costs follow the representation suggested by Christiano et al. (2005).

The representative nontraded good firm maximizes its discounted lifetime profits weighted by the marginal utility of consumption of the intertemporal optimizing households λ_t . These profits are given by

$$\Omega_{T,0} = E_0 \sum_{t=0}^{\infty} \beta^t \lambda_t \left[p_{N,t} y_{N,t} - w_{N,t} L_{N,t} - i_{N,t} - \tau^K r_{N,t}^K k_{N,t-1} \right], \quad (24)$$

where $r_{N,t}^K = (1 - \alpha_N) p_{N,t} \frac{y_{N,t}}{k_{N,t-1}}$ is the (gross) return to capital. Let $\lambda_t q_{N,t}$ be the Lagrange multiplier associated with the law of motion of capital, where $q_{N,t}$ is the sectoral Tobin's q . Then, the first-order conditions with respect to $L_{N,t}$, $k_{N,t}$, and $i_{N,t}$ are given by

$$w_{N,t} = \alpha_N p_{N,t} \frac{y_{N,t}}{L_{N,t}}, \quad (25)$$

$$q_{N,t} = E_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \left((1 - \delta_N) q_{N,t+1} + (1 - \tau^K) (1 - \alpha_N) p_{N,t+1} \frac{y_{N,t+1}}{k_{N,t}} \right) \right], \quad (26)$$

and

$$\begin{aligned} \frac{1}{q_{N,t}} = & \left[1 - \frac{\kappa_N}{2} \left(\frac{i_{N,t}}{i_{N,t-1}} - 1 \right)^2 - \kappa_N \left(\frac{i_{N,t}}{i_{N,t-1}} - 1 \right) \frac{i_{N,t}}{i_{N,t-1}} \right] \\ & + E_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \kappa_N \frac{q_{N,t+1}}{q_{N,t}} \left(\frac{i_{N,t+1}}{i_{N,t}} - 1 \right) \left(\frac{i_{N,t+1}}{i_{N,t}} \right)^2 \right]. \end{aligned} \quad (27)$$

2. Traded Good Sector

Analogously to the nontraded good sector, firms in the traded good sector produce traded output with the following technology

$$y_{T,t} = z_{T,t} (k_{T,t-1})^{1-\alpha_N} (L_{T,t})^{\alpha_N} (k_{G,t-1})^{\alpha_G}. \quad (28)$$

To capture the common Dutch disease effects associated with spending resource revenues, we assume that the total factor productivity in this sector, $z_{T,t}$, is subject to learning-by-doing externalities:

$$\frac{z_{T,t}}{z_T} = \left(\frac{z_{T,t-1}}{z_T} \right)^{\rho_{z_T}} + \left(\frac{y_{T,t-1}}{y_T} \right)^{\rho_{y_T}}, \quad (29)$$

where $\rho_{z_T}, \rho_{y_T} \in [0, 1]$ control the severity of Dutch disease. This specification is a variation of the one in Matsuyama (1992) and Krugman (1987).⁹ It implies that there are no permanent effects of learning by doing on output or productivity. But deviations of traded sector output from the trend do imply persistent productivity effects.

⁹See also Adam and Bevan (2006) and Torvik (2001), among others.

Private capital in the traded sectors is accumulated according to

$$k_{T,t} = (1 - \delta_T) k_{T,t-1} + \left[1 - \frac{\kappa_T}{2} \left(\frac{i_{T,t}}{i_{T,t-1}} - 1 \right)^2 \right] i_{T,t}. \quad (30)$$

Like nontraded good firms, a representative traded good firm maximizes the following discounted lifetime profits:

$$\Omega_{T,0} = E_0 \sum_{t=0}^{\infty} \beta^t \lambda_t [y_{T,t} - w_{T,t} L_{T,t} - i_{T,t} - \tau^K r_{T,t}^K k_{T,t-1}]. \quad (31)$$

The first-order conditions with respect to $L_{T,t}$, $k_{T,t}$, and $i_{T,t}$ are given by

$$w_{T,t} = \alpha s_t \frac{y_{T,t}}{L_{T,t}}, \quad (32)$$

$$q_{T,t} = E_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \left((1 - \delta_T) q_{T,t+1} + (1 - \tau^K) (1 - \alpha_T) s_{t+1} \frac{y_{T,t+1}}{k_{T,t}} \right) \right], \quad (33)$$

and

$$\begin{aligned} \frac{1}{q_{T,t}} = & \left[1 - \frac{\kappa_T}{2} \left(\frac{i_{T,t}}{i_{T,t-1}} - 1 \right)^2 - \kappa_T \left(\frac{i_{T,t}}{i_{T,t-1}} - 1 \right) \frac{i_{T,t}}{i_{T,t-1}} \right] \\ & + E_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \kappa_T \frac{q_{T,t+1}}{q_{T,t}} \left(\frac{i_{T,t+1}}{i_{T,t}} - 1 \right) \left(\frac{i_{T,t+1}}{i_{T,t}} \right)^2 \right]. \end{aligned} \quad (34)$$

3. Natural Resource Sector

Since often most natural resource production in resource-rich developing countries is capital intensive, and much of the investment in the resource sector is financed by foreign direct investment in LICs, natural resource production is simplified in the model as follows.

Resource production follows an exogenous process

$$\frac{\tilde{y}_{O,t}}{\tilde{y}_O} = \left(\frac{\tilde{y}_{O,t-1}}{\tilde{y}_O} \right)^{\rho_{y_o}} \exp(\varepsilon_t^{y_o}), \quad (35)$$

where $\rho_{y_o} \in (0, 1)$ is an auto-regressive coefficient and $\varepsilon_t^{y_o} \sim iid \mathcal{N}(0, \sigma_{y_o}^2)$ is the resource production shock. We assume that resource production is small relative to world production; hence, the international commodity price (relative to the foreign consumption basket), $p_{O,t}^*$, is

taken as given and evolves as

$$\frac{p_{O,t}^*}{p_O^*} = \left(\frac{p_{O,t-1}^*}{p_O^*} \right)^{\rho_{po}} \exp(\varepsilon_t^{po}), \quad (36)$$

where $\rho_{po} \in (0, 1]$ is an auto-regressive coefficient and $\varepsilon_t^{po} \sim iid \mathcal{N}(0, \sigma_{po}^2)$ is the resource price shock. Resource GDP in units of the domestic consumption basket corresponds to

$$y_{O,t} = s_t p_{O,t}^* \tilde{y}_{O,t}. \quad (37)$$

and therefore total real GDP y_t in this economy can be defined as

$$y_t = p_{N,t} y_{N,t} + s_t y_{T,t} + y_{O,t}. \quad (38)$$

C. The Government

The government flow budget constraint is given by

$$\begin{aligned} & \tau_t^C c_t + \tau_t^L w_t L_t + (1 - \vartheta^K) \tau^K (r_{T,t}^K k_{T,t-1} + r_{N,t}^K k_{N,t-1}) + s_t g r_t^* \\ & + \mu k_{G,t-1} + t_{O,t} + b_t + s_t d_t + s_t d_{c,t} + s_t R^{RF} f_{t-1}^* \\ & = p_t^G (g_t^C + g_t^I) + z_t + R_{t-1} b_{t-1} + s_t R_d d_{t-1} + s_t R_{dc,t-1} d_{c,t-1} + s_t f_t^*, \end{aligned} \quad (39)$$

where besides the tax revenues from consumption, labor income and capital income— $\tau_t^C c_t$, $\tau_t^L w_t L_t$, and $\sum_{j=T,N} (1 - \vartheta^K) \tau^K r_{j,t}^K k_{j,t-1}$ —the government also receives international grants, $g r_t^*$, user fees, $\mu k_{G,t-1}$, and resources-related royalties, $t_{O,t}$. As in Buffie et al. (2012), the user fee charged on public capital is computed as a fraction f of recurrent costs: $\mu \equiv f p_t^G \delta_G$. The resource revenues collected each period, on the other hand, correspond to

$$t_t^O = \tau^O s_t p_{O,t}^* \tilde{y}_{O,t}, \quad (40)$$

where τ^O is a constant royalty rate that can be made time-varying, if necessary. The government has three debt instruments: external concessional debt, d_t , external commercial debt, $d_{c,t}$, and domestic debt, b_t . Concessional loans extended by official creditors are taken as exogenous in the model and charge a constant (gross) real interest rate R_d . On the other hand, the gross real interest rates paid on external commercial debt incorporates a risk premium depending on the deviations of total external public debt to GDP ratio from its initial steady state. That is

$$R_{dc,t-1} = R^f + v_{dc} \exp \left[\eta_{dc} \left(\frac{d_t + d_{c,t}}{y_t} - \frac{d + d_c}{y} \right) \right], \quad (41)$$

where R^f is a (constant) risk-free world interest rate, y_t is total GDP and v_{dc} and η_{dc} are structural parameters. We now proceed to describe the spending variables and the resource fund that also appear in the budget constraint (39).

1. Government Purchases

Government purchases comprise government consumption (g_t^C) and public investment (g_t^I). Like private consumption, government expenditure, $g_t \equiv g_t^C + g_t^I$, is also a CES aggregate of domestic traded goods, $g_{T,t}$ and domestic nontraded goods, $g_{N,t}$. Thus,

$$g_t = \left[\nu_t^{\frac{1}{\chi}} (g_{N,t})^{\frac{\chi-1}{\chi}} + (1 - \nu_t)^{\frac{1}{\chi}} (g_{T,t})^{\frac{\chi-1}{\chi}} \right]^{\frac{\chi}{\chi-1}}, \quad (42)$$

where ν_t is the weight given to nontraded goods in government purchases. We assume that government purchases have the same intra-temporal elasticity of substitution $\chi > 0$ as that of private consumption.

Minimizing total government expenditures $p_t^G g_t = p_{N,t} g_{N,t} + s_t g_{T,t}$, subject to the government consumption basket (42), yields the following public demand functions for each good:

$$g_{N,t} = \nu_t \left(\frac{p_{N,t}}{p_t^G} \right)^{-\chi} g_t, \quad \forall j = N, T, \quad (43)$$

and

$$g_{T,t} = (1 - \nu_t) \left(\frac{s_t}{p_t^G} \right)^{-\chi} g_t, \quad \forall j = N, T, \quad (44)$$

where p_t^G is the government consumption price index in terms of units of the consumption basket, defined as

$$p_t^G = [\nu_t p_N^{1-\chi} + (1 - \nu_t) s_t^{1-\chi}]^{\frac{1}{1-\chi}}. \quad (45)$$

Note that ν_t is time-varying. As we focus on the effects of additional government spending in the form of government investment, the weight given to nontraded goods for the additional government spending, ν_g , can differ from its steady state value, ν , i.e.,

$$\nu_t = \frac{(p^G g) \nu + (p_t^G g_t - p^G g) \nu_g}{p_t^G g_t}. \quad (46)$$

2. Public Investment Efficiency, Absorptive Capacity Constraints, and Public Capital Depreciation

Public investment features inefficiency and absorptive capacity constraints. Hulten (1996) and Pritchett (2000) argue that often high productivity of infrastructure can coexist with very low returns on public investment in developing countries, because of inefficiencies in investing. As a result, public investment *spending* does not necessarily increase the stock of productive capital and, therefore, growth. Similarly, absorptive capacity constraints related to technical capacity and waste and leakage of resources in the investment process—which

impact project selection, management, and implementation—can have long lasting negative effects on growth, as suggested by Esfahani and Ramirez (2003), among others.

To reflect these inefficiencies and constraints, we assume that effective investment \tilde{g}_t^I ($\bar{\gamma}_t^{GI}$) is a function of the public investment growth rate ($\bar{\gamma}_t^{GI}$) relative to its steady state value, and $\bar{\gamma}_t^{GI} \equiv \frac{g_t^I}{\bar{g}^I} - 1$. Specifically,

$$\tilde{g}_t^I = \left\{ \begin{array}{ll} \bar{\epsilon} g_t^I, & \text{if } \bar{\gamma}_t^{GI} \leq \bar{\gamma}^{GI} \\ \bar{\epsilon} (1 + \bar{\gamma}^{GI}) \bar{g}^I + \epsilon (\bar{\gamma}_t^{GI}) [1 + \bar{\gamma}_t^{GI} - \bar{\gamma}^{GI}] \bar{g}^I, & \text{if } \bar{\gamma}_t^{GI} > \bar{\gamma}^{GI} \end{array} \right\}, \quad (47)$$

where $\bar{\epsilon} \in [0, 1]$ represents steady-state efficiency and $\epsilon (\bar{\gamma}_t^{GI}) \in (0, 1]$ governs the efficiency of the portion of public investment exceeding a threshold $\bar{\gamma}^{GI}$, in percent deviation from the initial steady state. We assume that $\epsilon (\bar{\gamma}_t^{GI})$ takes the following specification:

$$\epsilon (\bar{\gamma}_t^{GI}) = \exp [-\varsigma_\epsilon (\bar{\gamma}_t^{GI} - \bar{\gamma}^{GI})] \bar{\epsilon}. \quad (48)$$

In other words, if the growth rate of government investment expenditure from the initial steady state exceeds $\bar{\gamma}^{GI}$, then the efficiency of the additional investment decreases, reflecting the presence of absorptive capacity constraints. The severity of these constraints is governed by the parameter $\varsigma_\epsilon \in [0, \infty)$.

The law of motion of public capital is described as

$$k_{G,t} = (1 - \delta_{G,t}) k_{G,t-1} + \tilde{g}_t^I, \quad (49)$$

where $\delta_{G,t}$ is a time-varying depreciation rate of public capital in the spirit of Rioja (2003). Since insufficient maintenance can shorten the life of existing capital, we assume that the depreciation rate increases proportionally to the extent to which effective investment fails to maintain existing capital.¹⁰ Therefore

$$\delta_{G,t} = \left\{ \begin{array}{ll} \phi \delta_G \frac{\delta_G k_{G,t-1}}{\tilde{g}_t^I}, & \text{if } \tilde{g}_t^I < \delta_G k_{G,t-1} \\ \rho_\delta \delta_{G,t-1} + (1 - \rho_\delta) \delta_G, & \text{if } \tilde{g}_t^I \geq \delta_G k_{G,t-1} \end{array} \right\}, \quad (50)$$

where δ_G is the steady-state depreciation rate, $\phi \geq 0$ determines the extent to which poor maintenance produces additional depreciation, and $\rho_\delta \in [0, 1)$ controls its persistence.

3. The Resource Fund

We introduce a resource fund in the model along the lines of Berg et al. (2013). A resource windfall is defined as resource revenues that are above their initial steady-state level, i.e.,

¹⁰Adam and Bevan (2013) find that accounting for the operations and maintenance expenditures of installed capital is crucial for assessing the growth effects and debt sustainability of a public investment scaling-up.

$t_t^O - t^O$. Let f_t^* be the foreign financial asset value in a resource fund. Each period, the resource fund earns interest income $s_t (R^{rf} - 1) f_{t-1}^*$, with a constant gross real interest rate R^{rf} . The resource fund evolves by the process

$$f_t^* - f^* = \max \left\{ f_{floor} - f^*, (f_{t-1}^* - f^*) + \frac{f_{in,t}}{s_t} - \frac{f_{out,t}}{s_t} \right\}, \quad (51)$$

where $f_{in,t}$ represents the total fiscal inflow, $f_{out,t}$ represents the total fiscal outflow, and $f_{floor} \geq 0$ is a lower bound for the fund that the government chooses to maintain. If no minimum savings are required in a resource fund, the lower bound can be set at zero. At each point in time, if the fiscal inflow exceeds the fiscal outflow, the value of the resource fund increases.¹¹ Instead, if the resource fund is above f_{floor} , any fiscal outflow that exceeds the fiscal inflow is absorbed by a withdrawal from the fund. Whenever the floor of a resource fund binds, the fiscal gap is covered via borrowing and/or increases in taxes (on consumption and factor incomes) or cuts in government non-capital expenditures (government consumption and transfers). Later we explicitly define $f_{in,t}$ and $f_{out,t}$ and explain in detail the mechanism to close a fiscal gap.

One of the purposes of the model is to analyze the effects of investing a resource windfall. The simulations presented in this paper focus on two investing approaches: the spend-as-you-go approach and the delinked investing approach. These approaches are formulated as follows.

- **Spend-as-you-go approach (SAYG).** With spend-as-you-go, the resource fund stays at its initial level ($f_t^* = f^*, \forall t$), and the entire windfall is spent in public investment projects:

$$p_t^G g_t^I - p^G g^I = \left(\frac{t_t^O}{s_t} - \frac{t^O}{s} \right). \quad (52)$$

- **A delinked investment approach.** With delinked investing, a scaling-up path of public investment is specified as a second-order delay function,

$$\frac{g_t^I}{g^I} = 1 + [1 + \exp(-k_1 t) - 2 \exp(-k_2 t)] g_{nss}^I, \quad (53)$$

where g_{nss}^I is the scaling-up investment target expressed as percentage deviation from the initial steady state, $k_1 > 0$ represents the speed of adjustment of public investment to the new level, and $k_2 \geq k_1$ represents the degree of investment frontloading. In particular, if $k_1 = k_2 = 0$, public investment stays at its original steady-state level, i.e., $g_t^I = g^I \forall t$. If instead $k_1 \rightarrow \infty$, public investment jumps to the new steady-state level immediately. Lastly, if $k_2 = k_1$, public investment increases gradually and is not

¹¹To guarantee that the resource fund is not an explosive process, we assume that in the very long run, a small autoregressive coefficient $\rho_f \in (0, 1)$ is attached to $(f_{t-1}^* - f^*)$. The model is typically solved at a yearly frequency for a 1000-period horizon. The coefficient ρ_f is activated after the first 100 years of simulations.

frontloaded. The mechanics of this functional form on public investment trajectories are illustrated in Figures 1 and 2.

In addition to the above two approaches, the MATLAB code associated with the model allows for analyzing an *exogenously specified* public investment path, either proposed by authorities or recommended by a country team.

4. The Fiscal Gap

We borrow the structure of the fiscal gap and the mechanisms to cover it, from Buffie et al. (2012). But here we increase the number of fiscal instruments and take into account the dynamics of the fund. Given the paths of public investment, concessional borrowing, and foreign grants, algebraic manipulation of the budget constraint of the government (39) allows us to rewrite it as follows:

$$gap_t = f_{out,t} - f_{in,t} + s_t (f_t^* - f_{t-1}^*), \quad (54)$$

where

$$gap_t = \Delta b_t + s_t \Delta d_{c,t} + (\tau_t^C - \tau^C) c_t + (\tau_t^L - \tau^L) w_t L_t - p_t^G (g_t^C - g^C) - (z_t - z), \quad (55)$$

$$f_{in,t} = \tau^C c_t + \tau^L w_t L_t + (1 - \vartheta^K) \tau^K (r_{T,t}^K k_{T,t-1} + r_{N,t}^K k_{N,t-1}) + t_{O,t} + \mu k_{G,t-1} + s_t a_t^* + s_t gr_t^* + s_t (R^{RF} - 1) f_{t-1}^* + s_t \Delta d_t, \quad (56)$$

and

$$f_{out,t} = p_t^G g_t^I + p_t^G g^C + z + (s_t R_d - 1) d_{t-1} + (R_{dc,t-1} - 1) s_t d_{c,t-1} + (R_{t-1} - 1) b_{t-1}. \quad (57)$$

Equation (55) says that covering the fiscal gap entails domestic and/or external commercial borrowing or adjustments in various fiscal instruments. By combining equations (51) and (54), we can see that if $f_t^* > f_{floor}$, then $gap_t = 0$; i.e., the resource fund absorbs any fiscal gap and no fiscal policy adjustments are needed. On the other hand, when $f_t^* = f_{floor}$, the gap satisfies $gap_t > 0$ and it needs to be covered by the fiscal adjustments to be explained next.

5. Covering the Fiscal Gap

The split of government borrowing between domestic and external commercial debt, to help cover the gap, occurs according to the following simple rule:

$$\varkappa \Delta b_t = (1 - \varkappa) s_t \Delta d_{c,t}, \quad (58)$$

where $\varkappa \in [0, 1]$. Given concessional borrowing and grants, this rule accommodates the limiting cases of (i) supplementing this concessional borrowing with borrowing exclusively in domestic markets ($\varkappa = 0$) and (ii) supplementing concessional borrowing with accumulating more external commercial debt ($\varkappa = 1$).

Debt sustainability, however, requires that eventually revenues have to increase and/or expenditures have to be cut in order to cover the entire gap. To calculate the debt stabilizing (target) values of (i) the consumption tax rate, (ii) the labor income tax rate, (iii) government consumption, and (iv) transfers, the following equations are used:

$$\tau_{\text{target},t}^C = \tau^C + \lambda_1 \frac{\text{gap}_t}{c_t}, \quad (59)$$

$$\tau_{\text{target},t}^L = \tau^L + \lambda_2 \frac{\text{gap}_t}{w_t L_t}, \quad (60)$$

$$g_{\text{target},t}^C = g + \lambda_3 \frac{\text{gap}_t}{p_t^G}, \quad (61)$$

and

$$z_{\text{target},t} = z + \lambda_4 \text{gap}_t, \quad (62)$$

where λ_i , $i = 1, \dots, 4$ split the fiscal burden across the different fiscal instruments, satisfying $\sum_{i=1}^4 \lambda_i = 1$. Tax rates and expenditure items are then determined according to the policy reaction functions

$$\tau_t^C = \min \{ \tau_{\text{rule},t}^C, \tau_{\text{ceiling}}^C \}, \quad (63)$$

$$\tau_t^L = \min \{ \tau_{\text{rule},t}^L, \tau_{\text{ceiling}}^L \}, \quad (64)$$

$$\frac{g_t^C}{g^C} = \max \left\{ \frac{g_{\text{rule},t}^C}{g^C}, g_{\text{floor}}^C \right\}, \quad (65)$$

and

$$\frac{z_t}{z} = \max \left\{ \frac{z_{\text{rule},t}}{z}, z_{\text{floor}} \right\}, \quad (66)$$

where τ_{ceiling}^C and τ_{ceiling}^L are the maximum levels of the tax rates that can be implemented, and g_{floor}^C and z_{floor} are minimum deviations of government consumption and transfer from their initial steady-state values. All these ceilings and floors are determined exogenously and reflect policy adjustment constraints that governments may face. In turn, $\tau_{\text{rule},t}^C$, $\tau_{\text{rule},t}^L$, $g_{\text{rule},t}^C$, and $z_{\text{rule},t}$ are determined by the following fiscal rules,

$$\tau_{\text{rule},t}^C = \tau_{t-1}^C + \zeta_1 (\tau_{\text{target},t}^C - \tau_{t-1}^C) + \zeta_2 (x_{t-1} - x), \quad \text{with } \zeta_1, \zeta_2 > 0, \quad (67)$$

$$\tau_{\text{rule},t}^L = \tau_{t-1}^L + \zeta_3 (\tau_{\text{target},t}^L - \tau_{t-1}^L) + \zeta_4 (x_{t-1} - x), \quad \text{with } \zeta_3, \zeta_4 > 0, \quad (68)$$

$$\frac{g_{\text{rule},t}^C}{g^C} = \frac{g_{t-1}^C}{g^C} + \zeta_5 \frac{(g_{\text{target},t}^C - g_{t-1}^C)}{g^C} - \zeta_6 (x_{t-1} - x), \quad \text{with } \zeta_5, \zeta_6 > 0, \quad (69)$$

and

$$\frac{z_{\text{rule},t}}{z} = \frac{z_{t-1}}{z} + \zeta_7 \frac{(z_{\text{target},t} - z_{t-1})}{z} - \zeta_8 (x_{t-1} - x), \quad \text{with } \zeta_7, \zeta_8 > 0, \quad (70)$$

where ζ 's control the speed of fiscal adjustments, and $x_t \equiv \frac{b_t + s_t d_{c,t}}{y_t}$ is the sum of domestic and external commercial debt as a share of GDP.

D. Identities and Market Clearing Conditions

To close the model, the goods market clearing condition and the balance of payment conditions are imposed. The market clearing condition for nontraded goods is

$$y_{N,t} = \varphi p_{N,t}^{-\chi} (c_t + i_{N,t} + i_{T,t}) + \nu_t \left(\frac{p_{N,t}}{p_t^G} \right)^{-\chi} g_t. \quad (71)$$

The balance of payment condition corresponds to

$$\frac{ca_t^d}{s_t} = gr_t^* - \Delta f_t^* + \Delta d_t + \Delta d_{c,t} + \Delta b_t^*, \quad (72)$$

where ca_t^d is the current account deficit

$$\begin{aligned} ca_t^d = & c_t + i_{N,t} + i_{T,t} + p_t^G g_t + \Theta_t^{OPT*} - y_t - s_t r m_t^* + (R_d - 1) s_t d_{t-1} \\ & + (R_{dc,t-1} - 1) s_t d_{c,t-1} + (R_{t-1}^* - 1) s_t b_{t-1}^* - (R^{RF} - 1) s_t f_{t-1}^*. \end{aligned} \quad (73)$$

III. CALIBRATION

The model is calibrated to an average LIC, which is assumed to start its exploitation on liquefied natural gas (LNG). Other types of commodities and other stages of exploitations can be accommodated by imposing an exogenous path of resource quantities and prices. The model is at the annual frequency. Table 1 summarizes the baseline calibration, which is explained as follows.

- *National accounting.* To reflect LIC averages of the last decade in the IMF World Economic Outlook database, trade balance is set at 6 percent of GDP, government consumption and public investment are set at 14 and 6 percent of GDP, respectively,

and private investment is set at 15 percent of GDP. We choose the shares of traded goods to be 50 percent in private consumption and 40 percent in government purchases, as government consumption typically have a larger component of nontraded goods than private consumption. Since the economy is at the early stages of exploitation, the share of natural resources is assumed to be only 1 percent of GDP at the initial steady state.

- *Assets, debt and grants.* We assume that government savings are small initially, only 1 percent of GDP ($RF_{share} = 0.01$). For government domestic debt, concessional debt and grants, we rely on LIC averages of the last decade as in Buffie et al. (2012). This implies $b_{share} = 0.20$, $d_{share} = 0.50$, and $gr_{share} = 0.04$. To highlight the financial constraints faced by LICs in international capital markets, we set $b_{share}^* = 0$ and $d_{c,share} = 0$.
- *Interest rates.* We set the subjective discount rate ρ such that the real annual interest rate on domestic debt ($R - 1$) is 10 percent. Consistent with stylized facts, domestic debt is assumed to be more costly than external commercial debt. We fix the real annual risk-free interest rate ($R^f - 1$) at 4 percent. The premium parameter v_{dc} is chosen such that the real interest rate on external commercial debt ($R_{dc} - 1$) is 6 percent, and the real interest rate paid on concessional loans ($R_d - 1$) is 0 percent, as in Buffie et al. (2012). We assume no additional risk premium in the baseline calibration, implying $\eta_{dc} = 0$. The parameter u is chosen to have $R = R^*$ in the steady state, required by (13) and (14). Based on the average real return of the Norwegian Government Pension Fund from 1997 to 2011 (Gros and Mayer (2012)), the annual real return on international financial assets in the resource fund ($R^{RF} - 1$) is set at 2.7 percent.
- *Private production.* Consistent with the evidence on Sub-Saharan Africa (SSA) surveyed in Buffie et al. (2012), the labor income shares in the nontraded and traded good sectors correspond to $\alpha_N = 0.45$ and $\alpha_T = 0.60$. In both sectors private capital depreciates at an annual rate of 10 percent ($\delta_N = \delta_T = 0.10$). Following Berg et al. (2013), we assume a minor degree of learning-by-doing externality in the traded good sector ($\rho_{Y_T} = \rho_{z_T} = 0.10$). Also as in Berg et al. (2010), investment adjustment costs are set to $\kappa_N = \kappa_T = 25$.
- *Households preferences.* The coefficient of risk aversion $\sigma = 2.94$ implies an inter-temporal elasticity of substitution of 0.34, which is the average LIC estimate according to Ogaki et al. (1996). We assume a low Frisch labor elasticity of 0.10 ($\psi = 10$), similar to the estimate of wage elasticity of working in rural Malawi (Goldberg, 2011). The labor mobility parameter ρ is set to 1 (Horvarth, 2000), and the elasticity of substitution between traded and nontraded goods is $\chi = 0.44$, following Stockman and Tesar (1995). To capture limited access to international capital markets, we set $\eta = 1$ as in Buffie et al. (2012).
- *Measure of intertemporal optimizing households.* Since a large proportion of households in LICs are liquidity constrained, we pick $\omega = 0.40$, implying that 60 percent of households are rule-of-thumb. Depending on the degree of financial development of a country, the measure of intertemporal optimizing households can be

lower than 40 percent in some SSA countries. Based on data collected in 2011, Demirguc-Kunt and Klapper (2012) report that on average only 24 percent of the adults in SSA countries have an account in a formal financial institution.

- *Mining.* Resource production shocks are assumed to be persistent with $\rho_{yo} = 0.90$. Based on Hamilton's (2009) estimates, we assume resource prices follow a random walk so $\rho_{po} = 1$. The royalty tax rate τ^O is set such that the ratio of natural resource revenue to total revenue at the peak of natural resource production is substantial, almost 50 percent of total revenues. In this case $\tau^O = 0.65$. When applying the model to individual countries, the resource tax rate should be calibrated to match the share of resource revenue in total revenues in the data.
- *Tax rates.* Consistently with data collected by the International Bureau of Fiscal Documentation in 2005-06, the steady-state taxes on consumption, labor and capital are chosen so that $\tau^C = 0.10$, $\tau^L = 0.15$, and $\tau^K = 0.20$, respectively. This combination of tax rates and the implied inefficiency in revenue mobilization implies a non-resource revenue of around 18 percent of GDP at the initial steady state.
- *Fiscal rules.* We impose a non-negativity constraint for the stabilization fund by setting $f_{floor} = 0$. In the baseline calibration, fiscal instruments do not have floors or ceilings. This translates in setting, for instance, $g_{floor}^C = z_{floor} = -100000$ and $\tau_{ceiling}^C = \tau_{ceiling}^L = 100000$. The baseline calibration also implies that the whole fiscal adjustment takes place through changes in external commercial borrowing and consumption taxes. This is achieved by setting $\kappa = \lambda_1 = 1$, $\lambda_2 = \lambda_3 = \lambda_4 = 0$, $\zeta_3 = \zeta_5 = \zeta_7 = 1$, and $\zeta_4 = \zeta_6 = \zeta_8 = 0$ in the fiscal rules. To smooth tax changes, we choose an intermediate adjustment of the consumption tax rate relative to its target ($\zeta_1 = 0.5$) and a low responsiveness of the consumption tax rate to the debt-to-GDP ratio ($\zeta_2 = 0.001$). The selection of values for these policy parameters should be guided by the policy scenario that the team wants to simulate as well as by what they consider feasible as a fiscal adjustment.
- *Public investment.* Public investment efficiency is set to 50 percent ($\bar{\epsilon} = 0.5$), following Pritchett's (2000) estimates for SSA countries. The annual depreciation rate for public capital is 7 percent ($\delta^G = 0.07$). The home bias for government purchases ν and for investment spending above the initial steady-state level ν^g are 0.6 and 0.4, respectively. The smaller degree of home bias in additional spending reflects that most of the investment goods are imported in LICs. The output elasticity to public capital α^G is set at 0.15, implying a marginal net return of public capital of 28 percent at the initial steady state. This is in the high end of the range of returns reported by Buffie et al. (2012). The severity of public capital depreciation corresponds to $\phi = 1$ and the change in the depreciation rate of public capital is assumed to be a persistent process by setting $\rho_\delta = 0.8$. In the baseline, absorptive capacity constraints start binding when public investment rises above 75 percent from its initial steady state ($\bar{\gamma}^{GI} = 0.75$). The calibration of absorptive capacity constraints with $\varsigma_\epsilon = 25$ implies that the average investment efficiency approximately halves to around 25 percent when public

investment spikes to around 200 percent from its initial steady state. For illustrative purposes, in the delinked investment approach, we set the planned long-term scaling up of investment such that public investment at the new steady state is 80 percent higher than at the initial steady state ($g_{nss}^I = 0.80$).

IV. SCALING UP PUBLIC INVESTMENT WITH A RESOURCE WINDFALL

The hypothetical scenarios we analyze assume that the economy discovers a sizable reserve of natural gas, and that production will reach full capacity several years later. With formidable development needs, the government plans to start investment before resource exploitation is fully in place. To do this, we assume the government uses the prospected natural resource revenues as a collateral to borrow commercially, creating challenges to ensure fiscal sustainability and macroeconomic stability.

In the baseline scenario, the production of LNG increases gradually to reach full capacity by 2021 and then starts to decline after 2035. At peak, we assume a production of about 1500 millions of cubic feet per year. For the initial years of simulations, we use the oil price forecast per barrel available in the World Economic Outlook of the IMF, multiplied by the conversion factor for full oil parity (0.1724), which yields the price in dollars per million of BTUs. The projection of the LNG price in the baseline scenario assumes a non-volatile path, fluctuating around the mean price. The adverse scenario assumes that from 2025 onwards, the resource revenue quickly declines, due to both reduced production quantity and large negative shocks to LNG prices.

A. The Spend-As-You-Go Approach versus the Delinked Investment Approach

We begin the analysis of policy scenarios by considering two investment approaches and assuming there is no commercial or domestic borrowing to finance public investment increases. With the *spend-as-you-go* (SAYG) *approach*, the government spends all of its resource windfall in public investment each period and the stabilization fund remains at its initial steady state, as analyzed in Richmond et al. (2013) for Angola. Policymakers faced with impoverished and demanding populations could easily find the SAYG approach appealing because of its related immediate increase in consumption and investment. With the *delinked investment approach*, the government combines investment spending with savings in a resource fund, consistent with the sustainable investing approach analyzed in Berg et al. (2013). We assume both approaches resort only to the consumption tax rate to close any fiscal gap by setting $\lambda_1 = 1, : \lambda_2 = \lambda_3 = \lambda_4 = 0, \zeta_1 = \zeta_3 = \zeta_5 = \zeta_7 = 1$, and $\zeta_2 = \zeta_4 = \zeta_6 = \zeta_8 = 0$ in the fiscal rules.

Figures 3 and 4 compare the two investment approaches under two resource revenue scenarios: the dotted-dashed lines refer to the SAYG approach and the solid lines correspond to the delinked investment approach. With SAYG, public investment does not increase much

because of the initial low LNG production. With the delinked approach, public investment scales up gradually with no overshooting ($k_1 = 0.20$, $k_2 = 0.20$). Since the scaling-up is deliberately chosen to be commensurate with the magnitudes of resource revenues, the investment path does not require a large increase in tax rates.

The main difference between the two investment approaches is that the SAYG approach results in a volatile path for public investment, mirroring the volatility of resource revenue flows. Fiscal volatility is translated into macroeconomic instability as shown by fluctuations in macro variables. In contrast, the delinked approach can build up a fiscal buffer and maintain a stable spending path without major fiscal adjustments. Comparing the two scenarios of resource revenues, the economy can build a bigger stabilization fund (of around 150 percent of GDP) under the baseline scenario than under the adverse scenario of rapidly declining resource revenues—it only peaks at around 25 percent of GDP.

Another concern with the SAYG approach is the reduced public investment efficiency during the years when resource revenue flows accelerate. Sudden accelerations in public investment expenditures make the economy more prone to bumping into absorptive capacity constraints, translating into lower efficiency. As shown in Figure 3, with the SAYG approach, public investment accelerates to an extent that average investment efficiency drops from a baseline value of 50 percent down to almost 25 percent. Also, when public investment significantly drops (due to a sharp decline in the natural resource revenue), failure to maintain public capital leads to a higher depreciation rate than the steady-state level.

In the baseline scenario without negative shocks, SAYG can perform reasonably well as it leads to a higher accumulation of public capital than the delinked approach. As a result, non-resource output, private consumption and investment may reach a higher level than that with a delinked approach. However, in the presence of negative shocks to the resource revenue, as captured under the adverse scenario, the delinked approach performs much better, leading to overall more public capital, real non-resource output, private consumption, and investment. Moreover, in both scenarios, a delinked approach delivers a more resilient and stable growth in non-resource GDP and a less volatile real exchange rate. The greater real exchange appreciation induced by SAYG (in periods of particularly high resource revenue) leads to greater negative learning-by-doing externalities and thus a larger decline in traded output, and thus more severe Dutch disease effects.

Lastly, the two revenue scenarios assume that the reserve of natural gas will deplete after 2040, and this has important consequences for public capital under SAYG. If the public investment level cannot be maintained (like with SAYG), public capital built with the resource windfall eventually declines back to the initial steady-state level. Consequently, the growth benefits of more public capital also diminish. Thus, when determining a scaling-up magnitude, financing needs to sustain capital should be accounted for to ensure long-lasting growth benefits from a resource windfall.

B. Front Loading Public Investment with Commercial Borrowing

Under the constraints of no additional borrowing, any front-loading of public investment is not fiscally feasible unless the government chooses to sharply increase taxes (or significantly cut non-capital expenditures). In this section, we analyze the effects of a front-loaded investment path financed jointly by resource revenues and commercial borrowing.

Figures 5 to 7 compare the public investment effects under different degrees of frontloading. All three investment paths eventually reach a long-run investment level 80 percent higher than the level in the initial steady state ($k_1 = 0.20$). The dotted lines represent a conservative path ($k_2 = 0.10$), in which public investment is scaled up slowly enough, so it does not require significant debt accumulation when LNG production is low initially. The solid lines represent a gradual path ($k_2 = 0.20$) with a small degree of frontloading. The dashed lines correspond to an aggressive path ($k_2 = 0.70$), which generates a pronounced overshooting of public investment. During the two peak years, public investment is around 100 percent from the initial steady-state level.

In terms of fiscal adjustment, we assume that the government makes use of external commercial borrowing ($\varkappa = 1$) to close the fiscal gap when the stabilization fund reaches its lower bound. Also, the consumption tax rate is used as the adjustment instrument that stabilizes debt in the long run ($\lambda_1 = 1$).¹² Since tax collection in LICs is generally weak, we assume there exists a ceiling for the consumption tax rate at 12.5 percent and it is difficult to increase the tax rate by more than 2.5 percentage points in the short run.

In both resource revenue scenarios (baseline and adverse), front loading investment results in no savings in the stabilization fund and rising public debt. As expected, the debt increase is most pronounced with the aggressive investment path under the adverse scenario. In contrast, with either the conservative or gradual path, public debt as a share of GDP does not increase significantly. Also, the increase in the consumption tax rate is smaller than that with the aggressive path. Moreover, the conservative path is able to accumulate some savings in the resource fund even under the adverse scenario.

When the economy can resort to external commercial borrowing, front-loading public investment can advance the benefits of expected resource windfalls, relative to the case with no borrowing. If the degree of front-loading is not excessive and/or the economy does not experience particularly bad shocks, public debt can be stabilized despite taxing constraints. In this respect, the model can serve as a tool to determine a proper front-loading degree under various assumptions on the rate of return to public capital, fiscal policy, and projections of resource revenues. Among the three investment paths analyzed here, the aggressive path

¹²The model allows for a flexible arrangements of using various fiscal instruments—the consumption and labor tax rates, government consumption, and transfers to households—to maintain debt sustainability. The analysis presented here uses only the consumption tax rate as an example.

signals a likely unfeasible path, since government debt appears to be on an explosive path under the adverse scenario.

C. Domestic versus External Commercial Borrowing

Instead of using external commercial borrowing, the government can use domestic borrowing to finance an investment scaling-up. Figure 8 compares the macroeconomic effects of the two borrowing strategies (domestic versus external commercial borrowing). The solid lines refer to domestic borrowing ($\varkappa = 0$), and the dotted-dashed lines reflect external borrowing ($\varkappa = 1$). The public investment path is the same as the aggressive frontloading path depicted in Figure 5.

The most important difference is that external borrowing brings in additional financial resources, while domestic borrowing shifts domestic resources away from the private sector into the public sector. Because of this and the fact that the real interest rate rises more with domestic borrowing, private investment is crowded out more with domestic borrowing. And since the amount of domestic borrowing is higher under the adverse scenario, the crowding-out effect is more pronounced than that under the baseline scenario. The higher interest rate associated with domestic borrowing also feeds into higher interest payments, more accumulation of public debt, and on average higher consumption tax rates to stabilize debt. This has important consequences for debt sustainability under the adverse scenario: with external commercial borrowing, public debt remains stable; while with domestic borrowing, public debt becomes unsustainable.¹³

D. Public Investment Efficiency, Return on Public Capital, and Debt Sustainability

Among various aspects of the model, the public investment efficiency and the return to public capital are particularly important in shaping the macroeconomic effects of public investment. Figure 9 reports the effects of improving the investment efficiency (ϵ) and the output elasticity with respect to public capital (α_G). It considers three different assumptions. The solid lines reflect the baseline calibration ($\epsilon = 0.5$ and $\alpha_G = 0.15$); the dotted-dashed lines assume efficiency increases from 0.5 to 0.7 over time; and the dotted lines correspond to improving efficiency together with $\alpha_G = 0.18$. The figure is depicted for the case of external commercial borrowing and the adverse natural resource scenario.

¹³Our simulation results appear to favor external commercial borrowing to domestic borrowing, mainly due to the reduced crowding-out effect with external borrowing. Since the model only accounts for shocks to resource prices and quantity, it does not capture the increased vulnerability from a higher stock of external debt resulting from other economic shocks. For example, an unexpected shock that depreciates the real exchange rate would expand the size of foreign liabilities, threatening debt sustainability, as the negative terms-of-trade shock analyzed in Buffie et al. (2012).

As shown in Figure 9, improving efficiency and/or raising the return on public capital deliver better macroeconomic outcomes than those from the baseline calibration. Higher efficiency generates more public capital for a given investment level, which then helps produce more non-resource output, leading to higher income and, therefore, more consumption. If public capital also becomes more productive, these positive macroeconomic effects are further amplified. In the example provided for the combined changes (dotted lines), the additional growth rate in non-resource GDP is doubled in the long run.

On the fiscal side, government debt is on an explosive path with the baseline calibration, given the adverse natural resource path. However, with the efficiency improvement, the same investment path turns out to be fiscally sustainable as the additional positive effect on non-resource GDP growth is capable of generating enough non-resource revenues that close the fiscal gap. Thus, we can see that for the same resource revenue flows and same investment paths, different investment efficiencies and returns to public capital can easily change the outlook of debt sustainability.

V. CONCLUSIONS

This paper presents the DIGNAR model that can be used to assess debt sustainability and growth effects of public investment scaling-ups in resource-abundant developing countries. The model has most of the relevant developing country features of the frameworks developed in Buffie et al. (2012) and Berg et al. (2013), including public investment inefficiencies, absorptive capacity constraints, and learning-by-doing externalities that can deliver Dutch disease effects. But it also introduces novel features especially in the fiscal policy structure. DIGNAR can accommodate flexible fiscal arrangements, with domestic and external commercial borrowing as options to close the fiscal gap in the short-to-medium run, and several fiscal instruments (taxes and expenditures rules) to maintain debt sustainability in the long run. The model also has a resource fund that can be used as a fiscal buffer as well as a saving device—a minimal level of savings greater than zero can be imposed.

To illustrate how to use DIGNAR in policy analysis, the paper calibrates the model to an average LIC and constructs some hypothetical and stylized resource revenue scenarios, including an adverse scenario of declining revenues. It then discusses the macroeconomic effects of different investment approaches (spend-as-you-go and delinked investment) under different borrowing schemes as well as different frontloading degrees of public investment. The simulation exercises show how DIGNAR can serve as an analytical tool to search for the scaling-up magnitude of public investment that can sustain public capital—financed with both resource windfalls and borrowing—while still being consistent with debt sustainability.

The analysis reveals the importance of considering country-specific information that can be mapped into parameter values of the model. When this is not possible for some parameters, sensitivity analysis can be conducted for these parameters, as the paper shows for the public investment efficiency and the return to public capital under a negative resource revenue

scenario. Also, the analysis only focuses on two resource revenue scenarios, but in reality the degree of resource revenue uncertainty can be greater than what is depicted here. One way to address this issue is to conduct simulations under a wide range of resource revenue scenarios that account for the historical resource price volatility and likely production profiles. The probability of an unfavored outcome associated with an investment path can then serve as an indicator whether a proposed investment path is overly aggressive (see the analysis for Angola in Richmond et al. (2013)).¹⁴

DIGNAR is an integrated macroeconomic framework that may be useful in constructing the scenarios necessary for debt sustainability analysis of resource-rich developing countries. Judgment is still critical to calibrate, construct and interpret these scenarios. But DIGNAR can help make explicit the assumptions underlying the projections, organize policy discussions based on different simulated scenarios, apply empirical information, and allow more systematic risk assessments. In this regard, DIGNAR can be used to complement the IMF-WB DSF, when applied to natural resource-rich developing countries.

¹⁴Work at the research department of the Fund is attempting to incorporate uncertainty about shocks and parameters more systematically in the debt sustainability model by Buffie et al. (2012), while maintaining the non-linear structure, to construct confidence bands around debt trajectories. A similar approach could in principle be taken for DIGNAR.

Parameter	Value	Definition	Parameter	Value	Definition
exp_{share}	0.51	Exports to GDP	ρ_{y_o}	0.90	Persist. of the mining production shock
imp_{share}	0.45	Imports to GDP	f	0.50	User fees of public infrastructure
g_{share}^C	0.14	Govt. consumption to GDP	τ^L	0.05	Labor income tax rate
g_{share}^I	0.06	Govt. investment to GDP	τ^C	0.10	Consumption tax rate
i_{share}	0.15	Private investment to GDP	τ^K	0.20	Tax rate on the return on capital
yO_{share}	0.01	Natural resources to GDP	$ffloor$	0	Lower bound for the stabilization fund
gT_{share}	0.40	Share of tradables in govt. purchase	\varkappa	1	Adjust. share by external commercial debt
cT_{share}	0.50	Share of tradables in private consumption	λ_1	1	Adjust. share by consumption tax
RF_{share}	0.01	Stabilization fund to GDP	λ_2	0	Fiscal adjust. share by labor tax
b_{share}	0.20	Govt. domestic debt to GDP	λ_3	0	Fiscal adjust. share by govt. consumption
b_{share}^*	0	Private foreign debt to GDP	λ_4	0	Fiscal adjust. share by transfer
d_{share}	0.50	Concessional debt to GDP	ζ_1	0.5	Adjust. speed of consumption tax to target
$d_{c,share}$	0	Govt. external commercial debt/GDP	ζ_2	0.001	Consumption tax response to debt/GDP
gr_{share}	0.04	Grants to GDP	ζ_3	1	Adjust. speed of labor tax to target
$(R - 1)$	0.10	Domestic net real int. rate	ζ_4	0	Labor tax response to debt/GDP
$(R^{RF} - 1)$	0.027	Foreign net real int. rate on savings	ζ_5	1	Adjust. speed of govt. consumption to target
$(R_d - 1)$	0	Net real int. rate on concessional debt	ζ_6	0	Govt. consumption to debt/GDP
$(R^f - 1)$	0.04	Net real risk-free rate	ζ_7	1	Adjust. speed of transfer to target
$(R_{dc,0} - 1)$	0.06	Net real int. rate on external commercial debt	ζ_8	0	Transfer response to debt/GDP
η_{dc}	0	Elast. of sovereign risk	g_{floor}^C	$-\infty$	Floor on real govt. consumption
α_N	0.45	Labor income share in nontraded sector	z_{floor}	$-\infty$	Floor on transfer
α_T	0.60	Labor income share in traded sector	$\tau_{ceiling}^C$	$+\infty$	Ceiling on consumption tax
δ_N	0.10	Depreciation rate of $k_{N,t}$	$\tau_{ceiling}^L$	$+\infty$	Ceiling on labour income tax
δ_T	0.10	Depreciation rate of $k_{T,t}$	ν	0.6	Home bias of govet. purchases
ρ_{yT}	0.10	Learning by doing in traded sector	ν_g	0.4	Home bias for additional spending
ρ_{zT}	0.10	Persist. in TFP in traded sector	α_G	0.15	Output elast. to public capital
κ_N	25	Investment adjust. cost, nontraded sector	δ_G	0.07	Depreciation rate of public capital
κ_T	25	Investment adjust. cost, traded sector	$\bar{\epsilon}$	0.50	Steady-state efficiency of public investment
ψ	10	Inverse of Frisch labor elast.	g_{nss}^I	0.80	Planned long-term scaling up
σ	2.94	Inverse of intertemporal elast. of substitution	k_1	-	Speed of scaling up plan
ρ	1	Intratemporal substitution elast. of labor	k_2	-	Degree of frontloading
ω	0.40	Measure of optimizers in the economy	ρ_δ	0.80	Persist. of deprecia. rate of public capital
χ	0.44	Substitution elast. b/w traded/nontraded goods	ϕ	1	Severity of public capital depreciation
η	1	Elast. of portfolio adjust. costs	ς_ϵ	25	Severity of absorptive capacity constraints
τ^O	0.65	Royalty tax rate on natural resources	$\bar{\gamma}^{GI}$	0.75	Threshold of absorptive capacity
ρ_{p_o}	1	Persist. of the commodity price shock			

Table 1. Baseline calibration

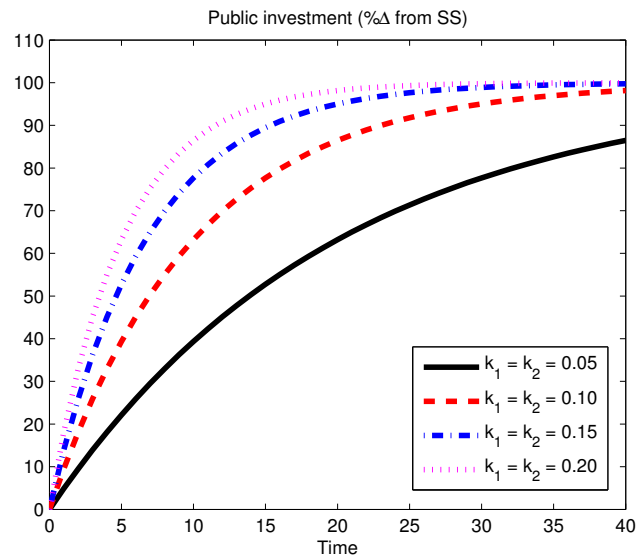


Figure 1. Different speeds of investment scaling-ups. X-axis is in years.

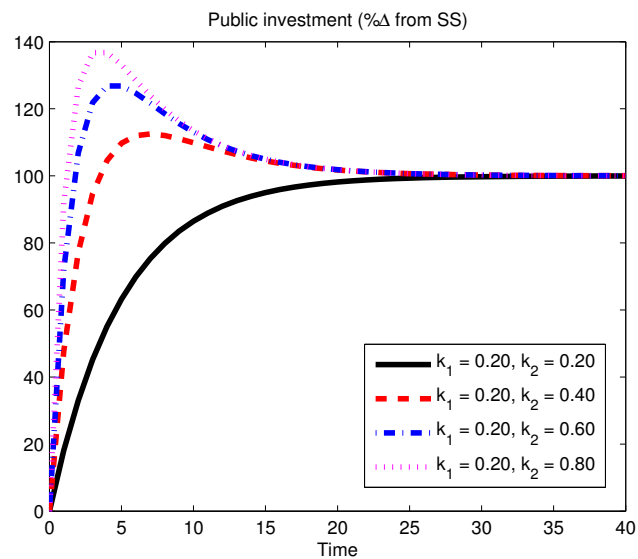


Figure 2. Different degrees of frontloading in investment scaling-ups. X-axis is in years.

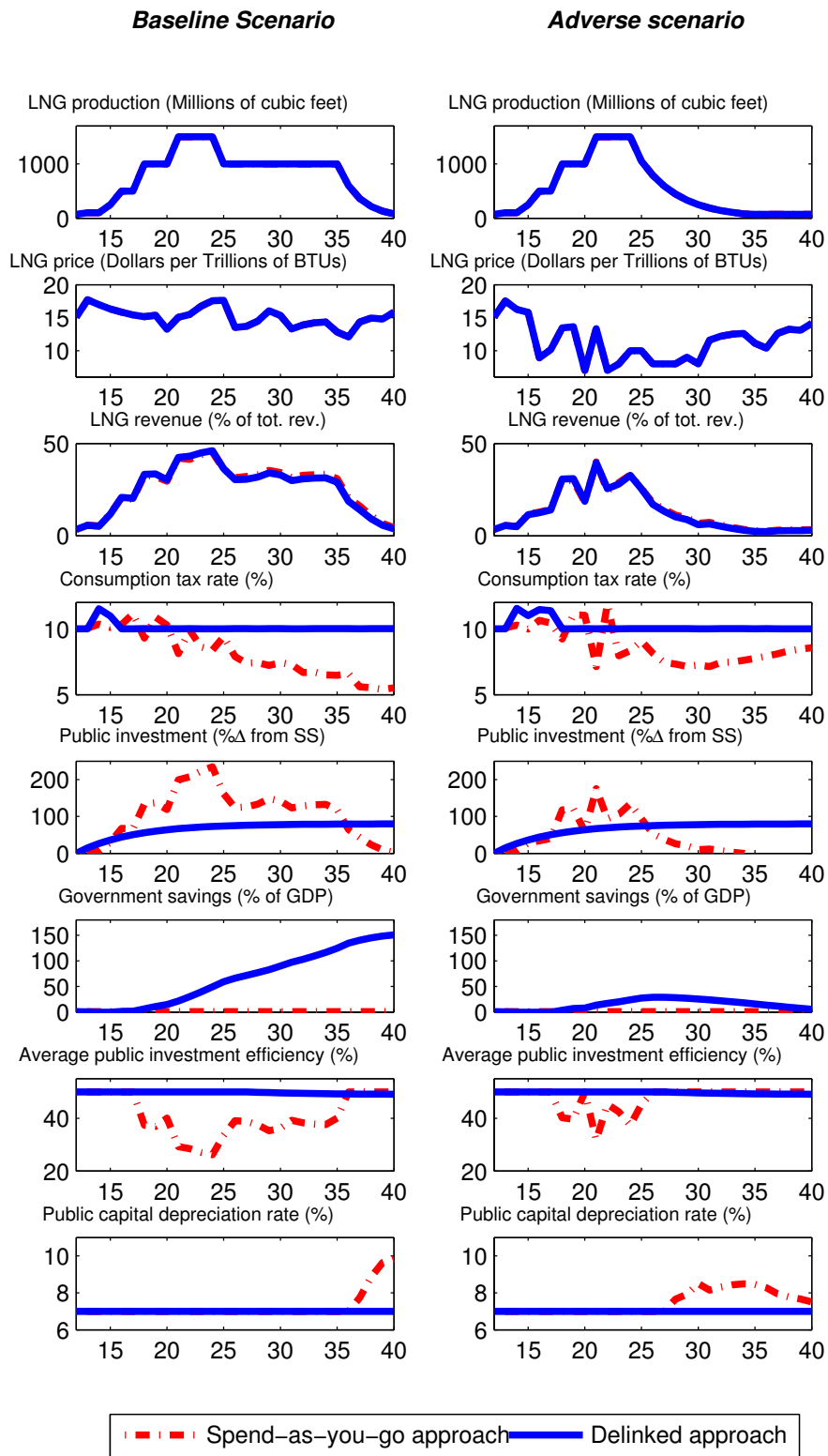


Figure 3. Spend-as-you-go vs. delinked investment approach: no additional commercial borrowing. X-axis is in years.

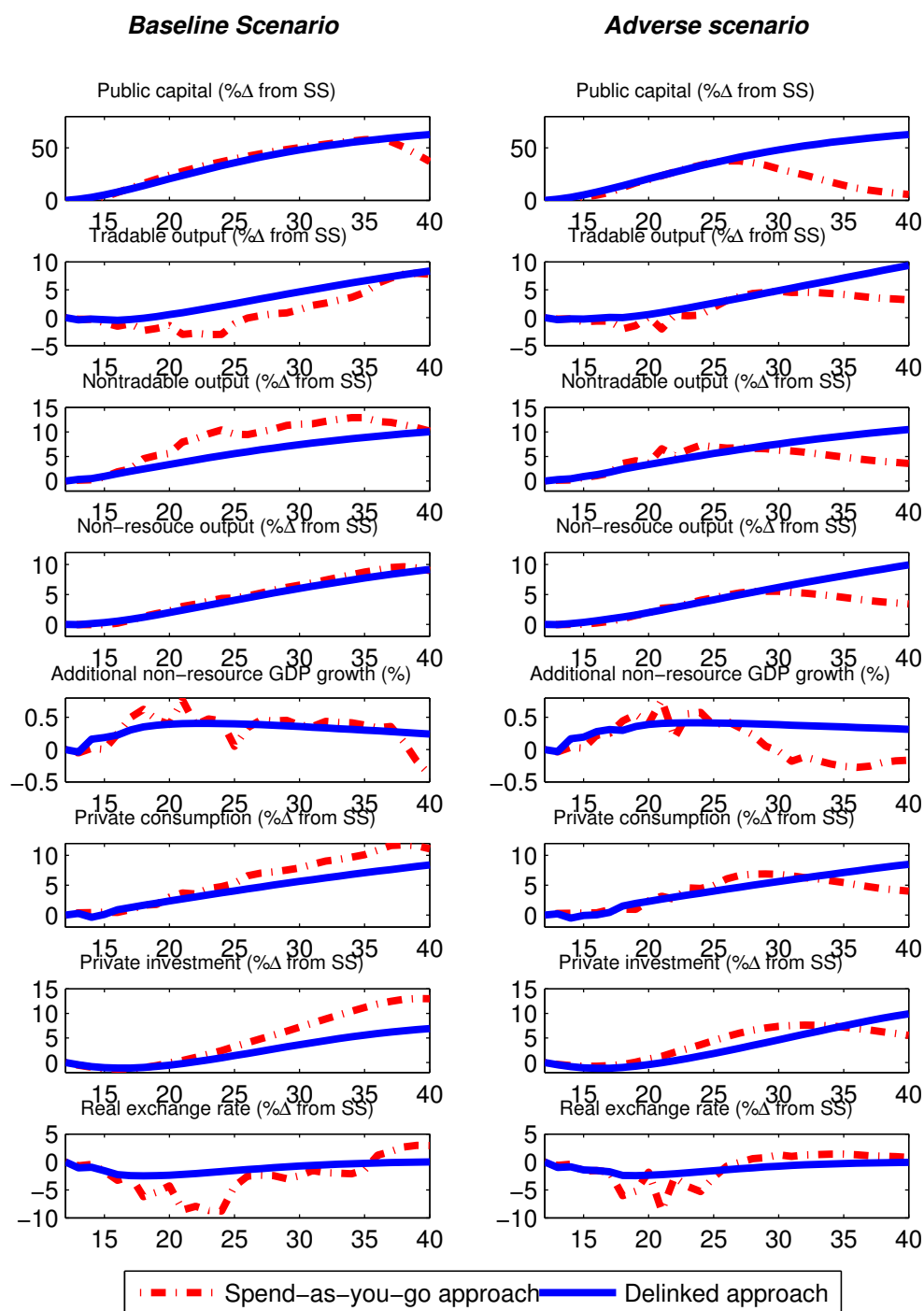


Figure 4. Spend-as-you-go vs. delinked investment approach (continued): no additional commercial borrowing. X-axis is in years.

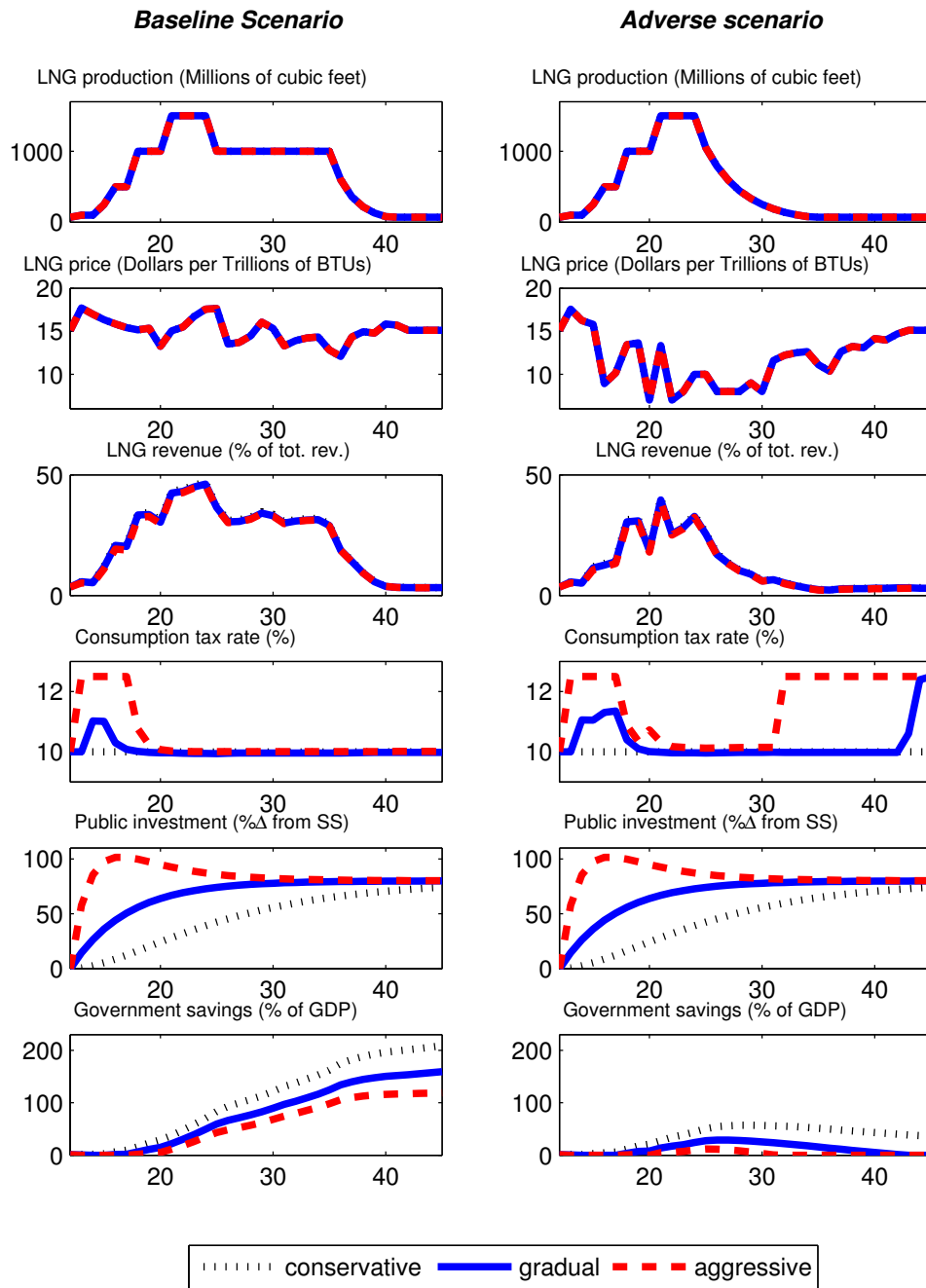


Figure 5. Various degrees of investment frontloading: external commercial borrowing. X-axis is in years.

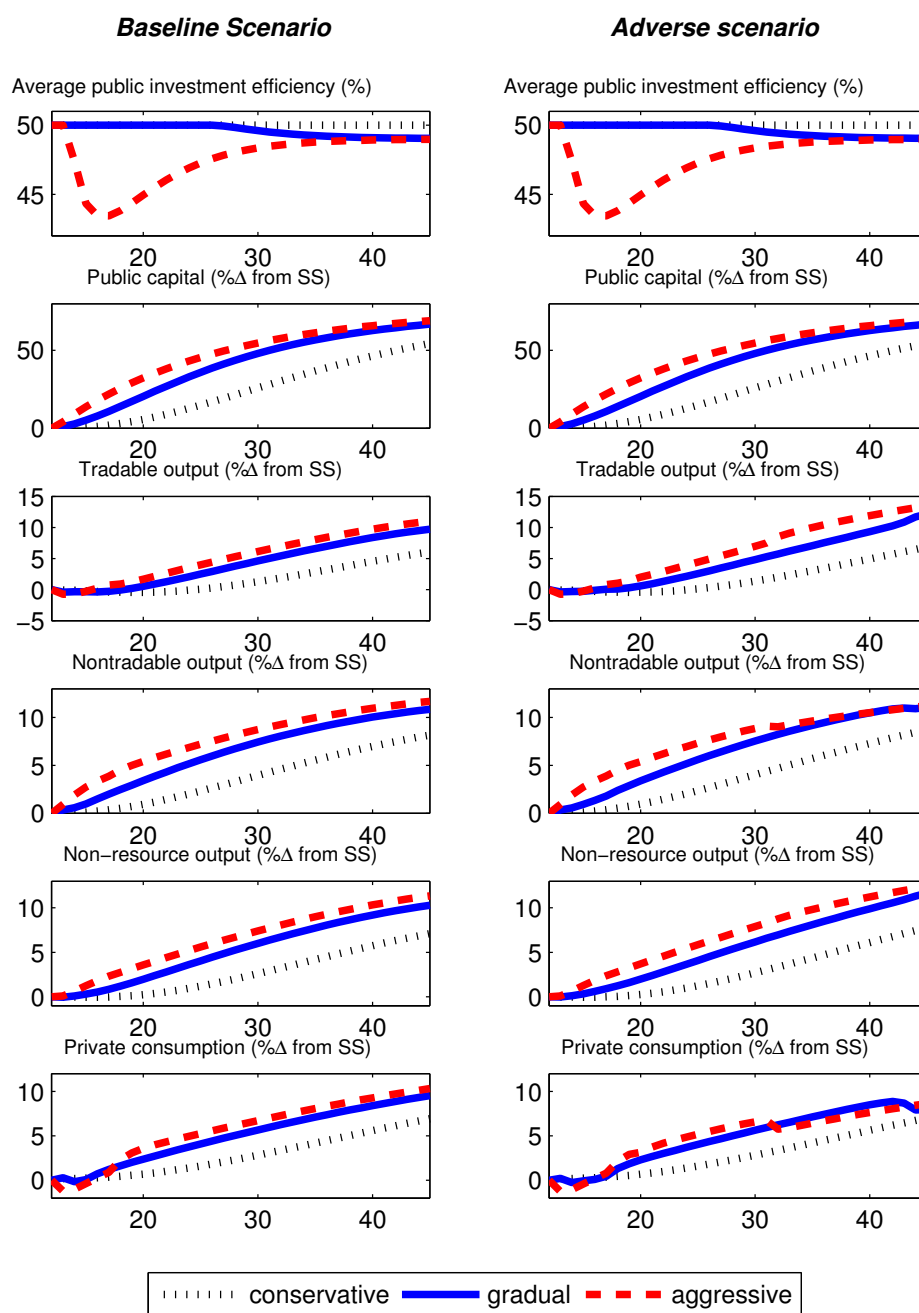


Figure 6. Various degrees of investment frontloading (continued): external commercial borrowing. X-axis is in years.

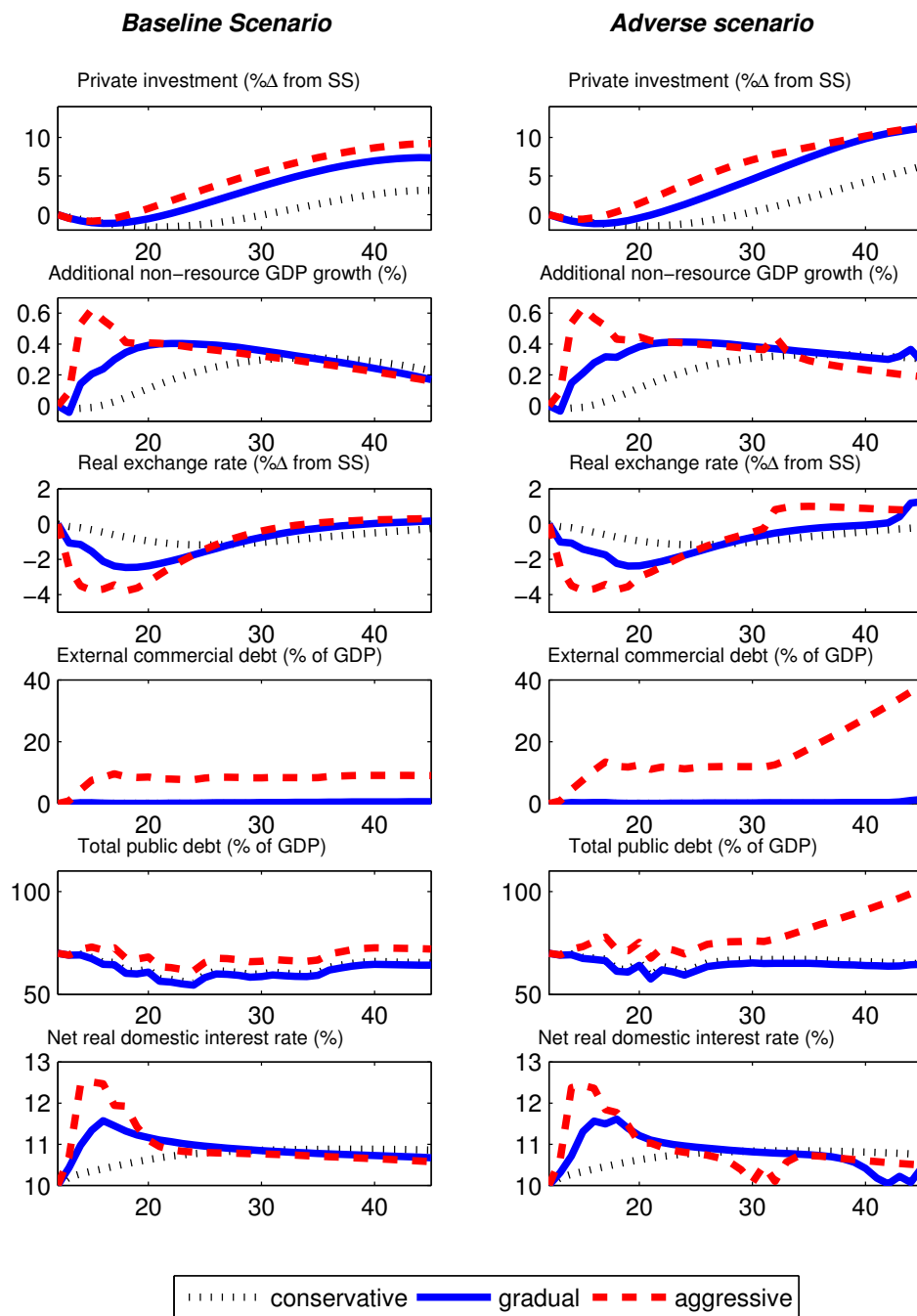


Figure 7. Various degrees of investment frontloading (concluded): external commercial borrowing. X-axis is in years.

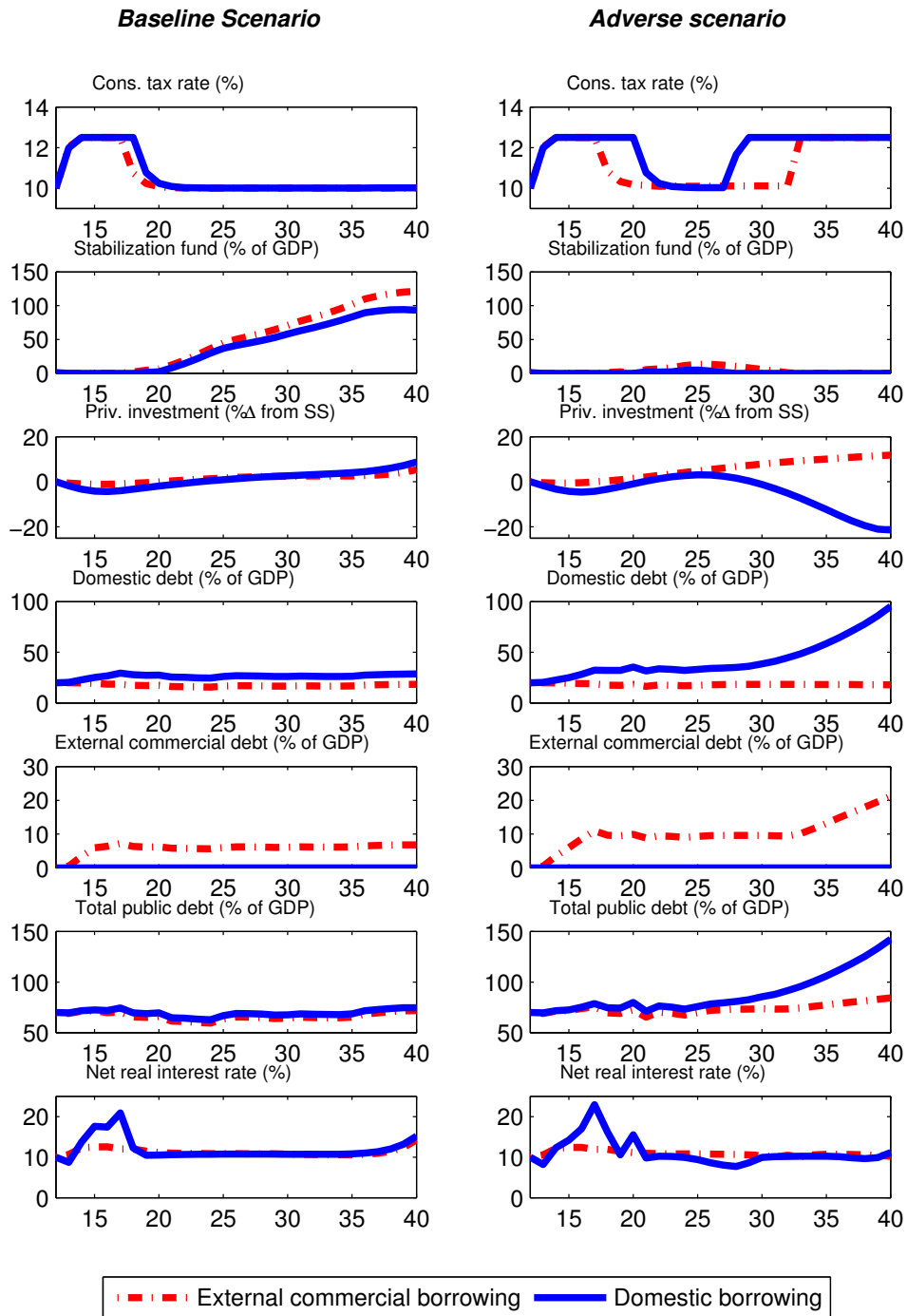


Figure 8. Domestic vs. external commercial borrowing. X-axis is in years.

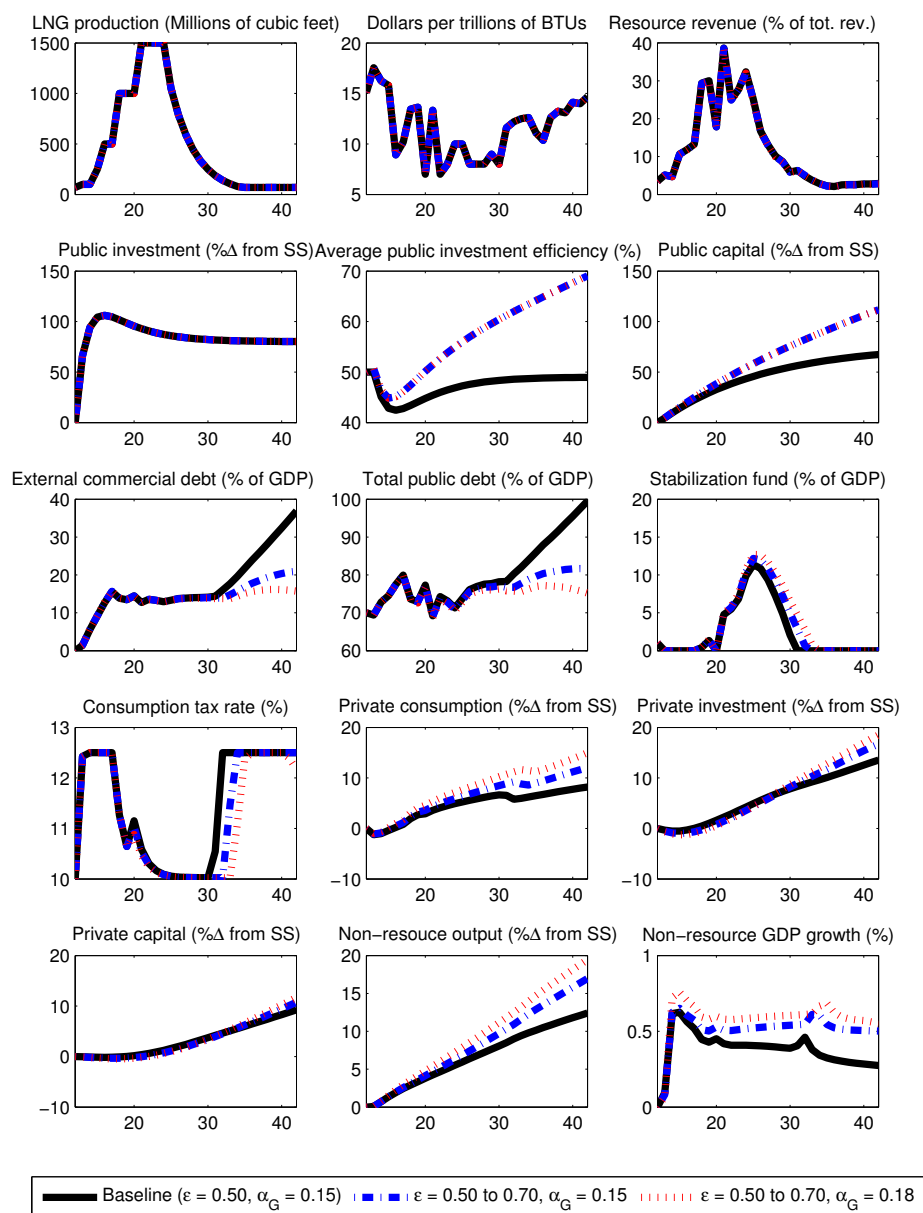


Figure 9. Increasing public investment efficiency and/or higher return to public capital: external commercial borrowing and adverse natural resource scenario. X-axis is in years.

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