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Are There International R&D Spillovers Among Randomly Matched Trade Partners? A Response to Keller

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Abstract

Keller (1998) reexamines Coe and Helpman's (1995) analysis of international R&D spillovers focusing on the weights used to define the foreign R&D capital stock. Keller creates "random" weights and shows that they give rise to positive estimates of international R&D spillovers, casting doubts on the robustness of Coe and Helpman's findings. We show that Keller's "random" weights are essentially simple averages with a random error. We derive alternative random weights and present regressions showing that when they are used to define the foreign R&D capital stock, the estimated international R&D spillover estimates are nonexistent, as would be expected.

JEL Classification Numbers: C15, F12, F2, O3, O4

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Contents

I. Introduction

Keller (1998) reexamines Coe and Helpman's (1995) (CH) analysis of international research and development (R&D) spillovers focusing on the weights used to define the foreign R&D capital stock. CH define a foreign R&D capital stock for 22 industrial countries using bilateral import shares to weigh the domestic R&D capital stocks of each country's 21 trading partners. Keller presents regressions analogous to CH's using an alternative definition of the foreign R&D capital stock based on "random" weights. Keller's main conclusion is that the "randomly created trade patterns give rise to positive international R&D spillover estimates, which are often larger, and explain more of the variation in productivity across countries than if 'true' bilateral trade patterns are employed" (Keller, 1998, p.1469).

If true, Keller's results call into question the usefulness of CH's measure of foreign R&D capital, which is at the center of their empirical analysis, and cast doubts on the robustness of their finding of substantial international R&D spillovers. They also raise doubts about the results reported in Coe, Helpman, and Hoffmaister (1997) (CHH), who use an analogous measure of foreign R&D capital in their analysis of R&D spillovers from industrial countries to 77 developing countries. Keller's results are somewhat surprising, since using truly random shares to weigh domestic R & D capital stocks would tend to generate a foreign R & D capital stock that is itself a random variable, which would not, in general, be correlated with productivity.

Keller notes that his random bilateral import shares are similar to equal weights (or simple averages of trading partners' R&D capital stocks). We show that the probability distribution of Keller's "random" shares indeed concentrate tightly around the inverse of the number of trading partners, suggesting that they are not random. We derive three alternative sets of random weights that do not exhibit this property, and present their probability distributions. We then present regressions showing that when these randomly created trade patterns are used to define the foreign R&D capital stock, the estimated international R&D spillover estimates are extremely small, and the estimated equations explain less of the variation in productivity across countries than if "true" bilateral import shares are employed, as would be expected. From these results we reach the opposite conclusion from Keller, namely, that there is *no* evidence that randomly created trade patterns give rise to positive international R&D spillover estimates or do a better job of explaining productivity developments than the bilateral import shares used by CH; indeed they do worse.

In the next section we discuss the generation of random shares and present histograms or probability density functions of Keller's shares and the three random shares that we derive; analytical results underlying this section are presented in the Appendix. Section III presents the estimation results. Section IV concludes.

II. GENERATING RANDOM SHARES

Keller derives his shares in the following way (Keller, 1998, p. 1476). First, he obtains a 22×22 matrix consisting of random elements drawn from a uniform distribution in the interval 0 to 1. He then sets the main diagonal equal to zero, reflecting that a country's domestic R&D capital stock is not used in the calculation of its foreign R&D capital stock. Each column is then divided by its sum so that the shares for each country add to 1. This final step means that the shares are no longer uniformly distributed, since any single share is a nonlinear transformation of 21 uniform variables.[1]

We show in the Appendix for the simplified case of two trading partners that the division of each column by its sum concentrates the marginal distribution of the shares around the value of ½, the inverse of the number of trading partners. For more than two trading partners, we derive the marginal distribution of the shares numerically. We do this for a fixed number of trading partners, n, with n=2, 3, 4, ..., 21. For each n, we generate 100,000 replications and plot histograms to illustrate the empirical distribution of Keller's shares. These histograms are shown in Figure 1; descriptive statistics for each histogram are given in Table 1. It is clear that the empirical distribution of the shares concentrates around $1/n$, the inverse of the number of trading partners, indicating that the shares are not random. Moreover, the degree to which the distribution concentrates around $1/n$ increases as n gets large. With n=21, the number of trading partners in CH, the variance of the empirical distribution is less than 0.001, with no share exceeding the value of 0.15 in 100,000 draws!

We conclude from these results that Keller's shares are more accurately described as simple averages with a random error, rather than as random shares. Although there is no unique way of generating random shares that sum to unity, an important characteristic of any set of random weights is that their distribution does not concentrate around the inverse of the number of trading partners. We now propose three alternative ways of generating random shares, none of which have this property.

One way of thinking about the problem of generating n random shares is how to divide the interval between 0 and 1 into n random segments, with the random shares being the "length" of each segment.[2] The simplest procedure is to generate a sample of n-1 random variables that

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[1]Keller (1997) does not discuss the effect that this nonlinear transformation has on the distribution of the shares, although he does discuss the relationship between the average of the ordinary least squares (OLS) estimates and OLS estimates using the average foreign R&D capital stocks.

[2]A related problem of randomly dividing a random interval is discussed in Assche (1987) and Johnson and Kotz (1990), who consider the distribution of a variable that is uniformly distributed in a random interval.

Figure 1. Histograms of Kellers Shares

(n denotes number of bilateral trading partners)

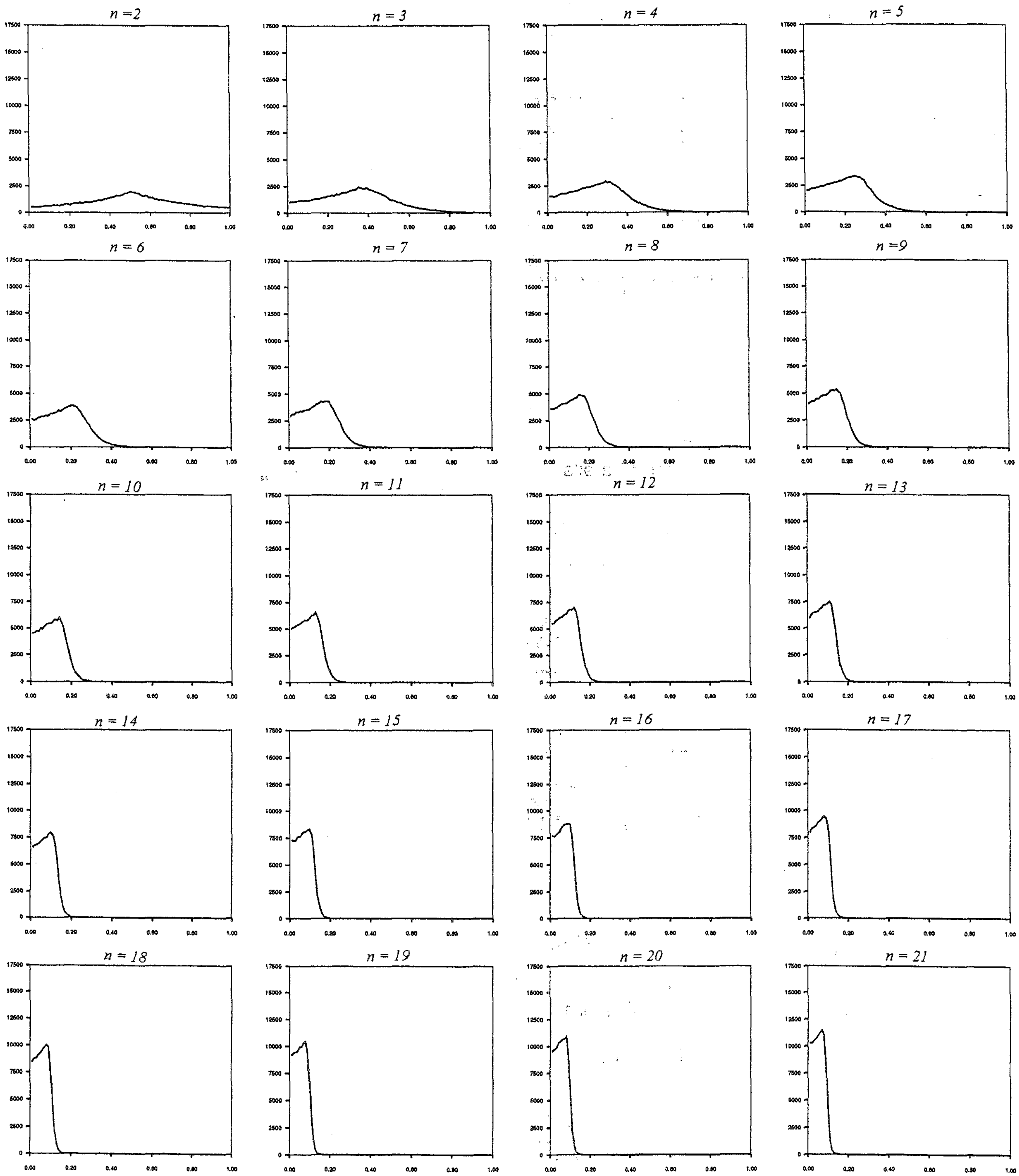


Table 1. Descriptive Statitistics for Keller's, Random Division, and Random Sequential Shares

n	Keller's shares			Random division			Random sequential		
	mean	variance	maximum	mean	variance	maximum	mean	variance	maximum
2	0.499	0.0567	1.000	0.500	0.0833	1.000	0.499	0.0834	1.000
3	0.334	0.0325	0.994	0.333	0.0556	1.000	0.334	0.0740	1.000
4	0.251	0.0195	0.971	0.250	0.0375	0.998	0.251	0.0674	1.000
5	0.200	0.0127	0.791	0.200	0.0267	0.990	0.202	0.0620	1.000
6	0.167	0.0090	0.738	0.167	0.0198	0.942	0.166	0.0557	1.000
7	0.143	0.0067	0.811	0.143	0.0153	0.924	0.142	0.0507	1.000
8	0.125	0.0051	0.568	0.125	0.0122	0.859	0.125	0.0468	1.000
9	0.111	0.0041	0.461	0.111	0.0099	0.798	0.111	0.0430	1.000
10	0.100	0.0033	0.414	0.100	0.0082	0.755	0.100	0.0403	1.000
11	0.091	0.0027	0.366	0.091	0.0069	0.752	0.091	0.0373	1.000
12	0.084	0.0023	0.369	0.083	0.0059	0.693	0.083	0.0348	1.000
13	0.077	0.0019	0.331	0.077	0.0051	0.666	0.077	0.0325	1.000
14	0.071	0.0017	0.269	0.071	0.0044	0.596	0.071	0.0302	1.000
15	0.066	0.0015	0.252	0.067	0.0039	0.547	0.066	0.0287	1.000
16	0.063	0.0013	0.209	0.063	0.0034	0.593	0.062	0.0267	1.000
17	0.059	0.0011	0.206	0.059	0.0031	0.493	0.060	0.0264	1.000
18	0.056	0.0010	0.182	0.056	0.0028	0.510	0.056	0.0249	1.000
19	0.053	0.0009	0.226	0.053	0.0025	0.521	0.052	0.0232	1.000
20	0.050	0.0008	0.163	0.050	0.0023	0.485	0.050	0.0225	1.000
21	0.048	0.0008	0.146	0.048	0.0021	0.453	0.048	0.0214	0.999

Note: Based on 100,000 replications except for the mean and variance of the random division of shares which are distributed Beta(1, n-1). The minimum for all shares is zero, for all values of n.

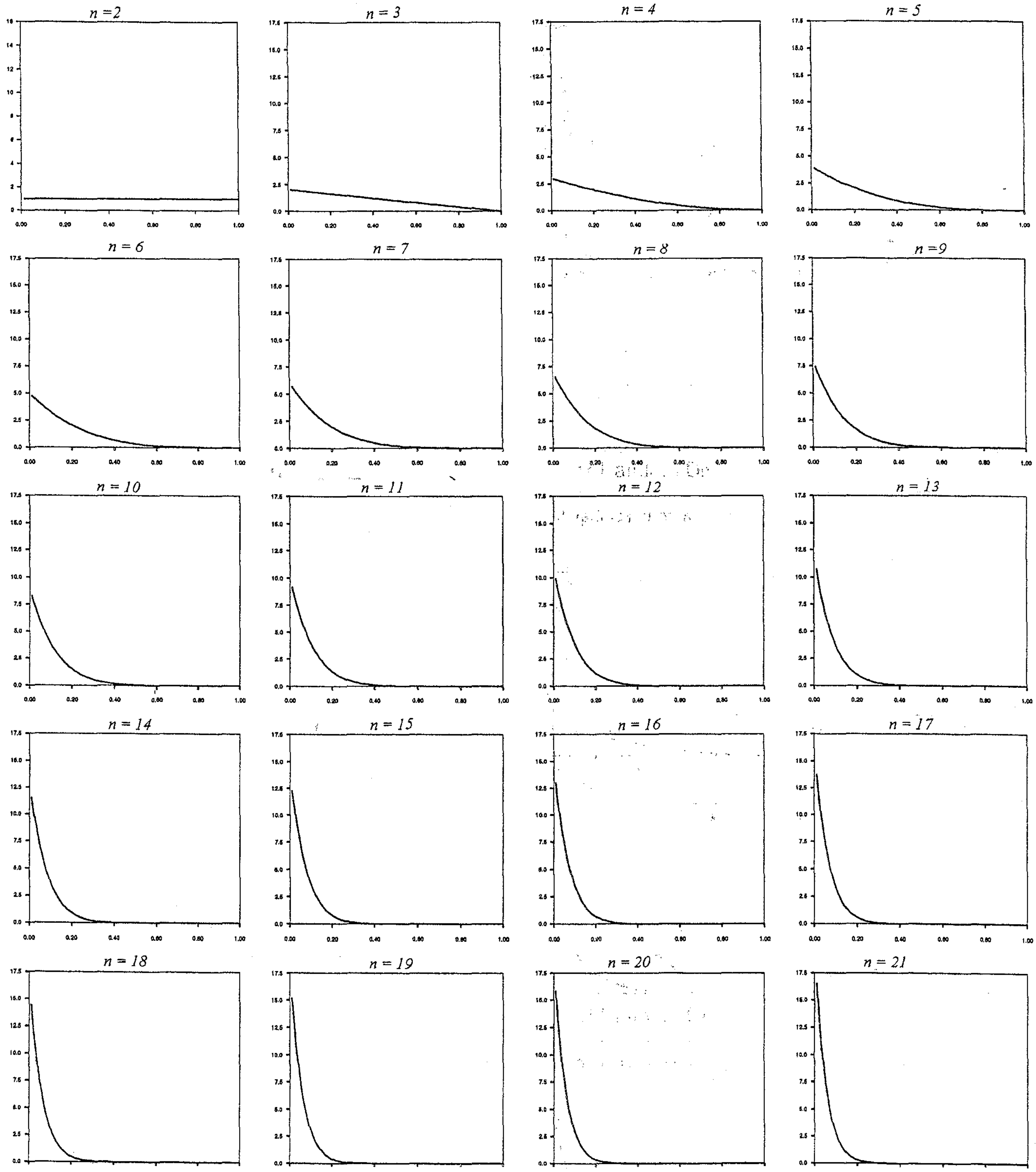
are uniformly distributed in the interval between 0 and 1. After arranging these n-1 random variables in ascending order, random shares can be obtained as the difference between the adjacent random variables; the first and last shares are defined, respectively, as the difference between zero and the first-ordered statistic and between the last-ordered statistic and one. We refer to these shares as "random division shares."

The analytical distribution of these random division shares can be obtained from known distributions of order statistics for uniform samples (see Appendix). The probability density functions, which are Beta (1, n-1), are shown in Figure 2 for n=2, 3, 4, ..., 21. The distribution of these shares is such that the mean is the inverse of the number of trading partners, as was the case with Keller's shares (see Table 1).[3] The variance of the shares also shrinks as the number of countries increases, although considerably less so than is the case with Keller's shares. The important difference, however, is that the distributions of the random division shares do not cluster around the mean, and hence the shares do not resemble a simple average. This difference is readily apparent in a comparison of Figures 1 and 2.

[3]Because these shares are identically and independently distributed and add to one, their means must equal $1/n$.

Figure 2. Probability Density Functions for Random Division Shares

(n denotes number of bilateral trading partners)



Our second method of generating random shares involves a sequential generation of n random numbers (see Appendix). The first random share, s_1, is drawn from a uniform distribution in the interval 0 to 1, the second random share, s_2, is drawn from a uniform distribution in the interval 0 to $1-s_1$, the third random share, s_3, is drawn from a uniform distribution in the interval 0 to $1-s_1-s_2$, and so on until s_{n-1} is obtained. The last share, s_n, is calculated as 1 minus the sum of the previous n-1 shares. Since on average $s_1>s_2$, $s_2>s_3$, and so on, we randomize these shares to avoid assigning (on average) the largest weight to the first country's R&D capital stock, the second-largest weight to the second country's R&D capital stock, etc. We refer to these shares as "random sequential shares."

For our purposes, an attractive feature of the random sequential shares is that they generate relatively large shares. This is apparent in Table 1 where, for n=21, the maximum value for the random sequential shares is 1.0, whereas the maximum values for Keller's and the random division shares are 0.15 and 0.45, respectively.[4] A distribution with relatively large shares more closely resembles the true bilateral import shares used by CH, where the values ranged from essentially zero to as high as 0.8 for the share of Canada's imports from the United States (CH, Table A.4; and Keller, 1998, Table 1).

As before, we obtain the statistical distribution of the random sequential shares numerically by the same Monte Carlo procedures used to generate the histograms of Keller's shares. The histograms of the random sequential shares are presented in Figure 3 and the descriptive statistics are given in Table 1. Compared with the probability density functions for the random division shares, the histograms of the random sequential shares extend further to the right, indicating the higher probability of large shares.

Our third set of random weights is derived from the actual bilateral import data used by Coe and Helpman. We obtain a 22×22 matrix of random bilateral imports by random sampling with replacement from the set of 441 (21×21) actual bilateral trade flows. After setting the main diagonal to zero, the shares are obtained by dividing each column by its sum.[5] We refer to these shares as "random sampling shares."

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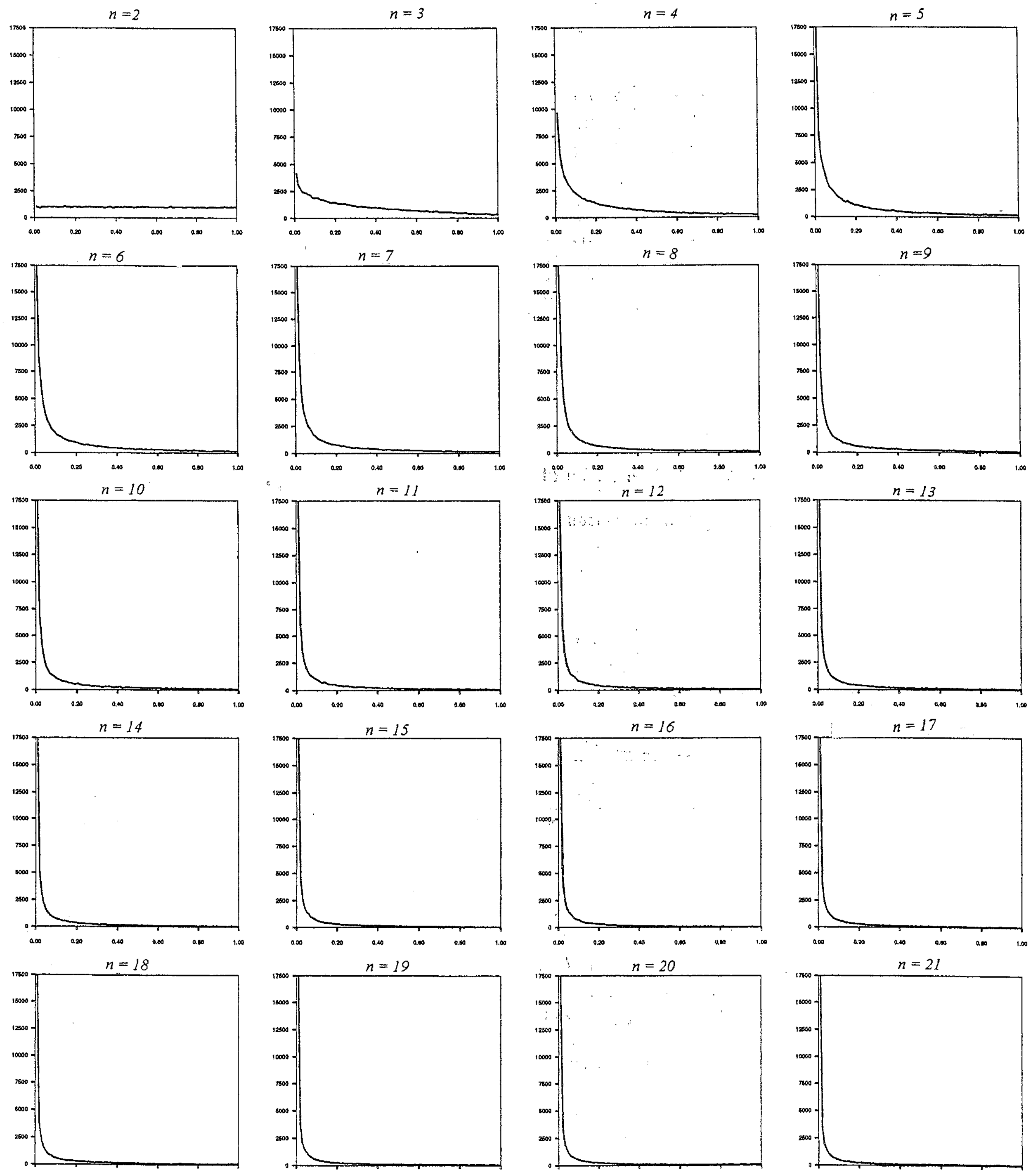
[4]For the random division shares in Table 1, the mean and variance are properties of the underlying beta distribution, whereas the maximum and minimum are obtained from a Monte Carlo generation of 100,000 replications.

[5]This procedure is similar to that used in Keller (1997) for the G7 countries and Sweden, although he resamples the bilateral trade shares directly. Creating the random shares from the original import data is preferable to randomly selecting from CH's bilateral shares because it does not limit the random shares to the same values as the true bilateral shares.

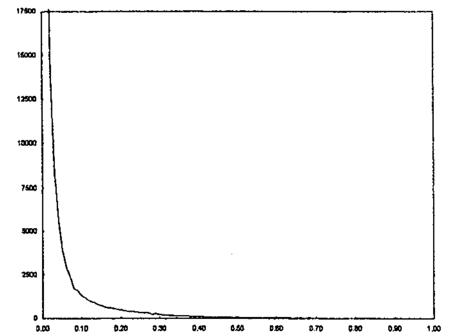
Figure 3. Histograms of Random Sequential Shares

(n denotes number of bilateral trading partners)



The histogram of the random sampling shares, which is obtained from a Monte Carlo generation of 100,000 replications using actual data for 1990 for all 22 countries, is shown in Figure 4. As with the other randomly generated shares, the distribution is nonsymmetric; it is skewed to the left with a mean of 0.048, but it is not concentrated around this mean. The maximum value of the shares is 0.85, which is similar to the maximum of the true bilateral import weights used by CH.

Figure 4. Histogram of Random Sampling Shares



III. Estimation Results

As discussed in Coe and Helpman (1995), their empirical analysis of international R&D spillovers is based on recent theoretical models of innovation-driven growth. Their simplest specification, omitting time subscripts, is

$$\log F_i = \alpha_i^0 + \alpha^d \log S_i^d + \alpha^f \log S_i^f + \epsilon_i \qquad (1)$$

where i is a country index (i=1, ..., 22), log F is the log of total factor productivity (TFP), S^d represents the domestic R&D capital stock, S^f represents the foreign R&D capital stock, and ϵ_i is an identically and independently distributed error term. This specification can be thought of as a multicountry extension of models relating TFP to only the domestic R&D capital stock, which would be a special case with α^f=0.

For each country, the foreign R&D capital stocks are defined as a weighted average of the domestic R&D capital stocks of its trading partners

$$S_i^f = \sum_{j=1}^{22} \varpi_{ij} S_j^d$$

where ϖ_{ij} are the weights for country i applied to the domestic R&D capital stocks of country j, with ϖ_{ii}=0 and $\sum_j \varpi_{ij}$= 1. These weights, of course, are the focus of this paper. In CH, the weights represent bilateral import shares. In the estimation results reported below, we also use weights defined as a simple average (i.e., ϖ_{ij}=1/21, for all $i, j, i \neq j$), as well as random division shares, random sequential shares, and random sampling shares, as discussed above.

CH report two other specifications, one of which simply allows α^d to differ between G7 and other countries

$$\log F_i = \alpha_i^0 + \alpha^d \log S_i^d + \alpha^7 G7 \log S_i^d + \alpha^f \log S_i^f + \epsilon_i \qquad (2)$$

where $G7$ is a dummy variable taking the value of 1 for G7 countries.

The third specification reported by CH allows for the interaction between foreign R&D capital stocks and the level of international trade

$$\log F_i = \alpha_i^0 + \alpha^d \log S_i^d + \alpha^7 G7 \log S_i^d + \alpha^f m_i \log S_i^f + \epsilon_i \qquad (3')$$

where m is the share of imports in GDP. Here it is important to note that CH's R&D capital stocks are indices with 1985=1. The transformation of the R&D capital stocks into indices is innocuous in equations (1) and (2), as it affects only the unreported country-specific constants. In equation 3', however, the multiplication of the import share by the log of the foreign R&D capital stock means that the estimated elasticity of TFP with respect to the import share (α^f**log** $\boldsymbol{S_i^f}$, where the bold indicates the sample average) is not invariant to the units of S_i^f, as pointed out by Lichtenberg and van Pottlesberghe de la Potterie (1998). With the R&D capital stock indices having values of about 1, **log** $\boldsymbol{S_i^f} \approx 0$, and hence the estimated elasticity of TFP with respect to the import share is about zero. If the R&D capital stocks are (correctly) not converted to indices, however, **log** $\boldsymbol{S_i^f} \approx 12$.[6] In this case, it is necessary to add the import share as an additional variable to avoid imposing cross restrictions on the estimated elasticity of TFP with respect to the import share (α^f**log** $\boldsymbol{S_i^f}$) and the foreign R&D capital stock ($\alpha^f \boldsymbol{m_i}$). For this reason, we estimate a slightly expanded version of CH's final specification

$$\log F_i = \alpha_i^0 + \alpha^d \log S_i^d + \alpha^7 G7 \log S_i^d + \alpha^f m_i \log S_i^f + \alpha^m m_i + \epsilon_i \qquad (3)$$

In this specification, the estimated elasticity of TFP with respect to the import share is $\alpha^m + \alpha^f$**log** $\boldsymbol{S_i^f}$.[7]

Table 2 reports estimation results based on equations (1), (2), and (3). The equations are estimated on pooled data for 1971-90 for 22 industrial countries. The variables are entered in level form, and CH interpret the equations as pooled cointegrating equations. We report standard errors as an indication of the precision of the coefficient estimates; however, it is not, in general, correct to interpret the ratio of the coefficient estimates to the standard errors as *t*-statistics and use them to test for the "significance" of variables because their distribution is not standard.

For each specification, we report five regressions differing only in the weights used to construct the foreign R&D capital stocks: CH's bilateral import weights, a simple average (which is similar to Keller's), random division weights, random sequential weights, and random sampling weights. Equations (1i) and (2i) essentially replicate CH's original results based on bilateral import weights. In equation (3i), which is also based on bilateral import weights, the estimated coefficients on the first four variables—the variables in equation (3') above—are very similar to those in CH. Moreover, the estimated elasticity of TFP with respect to the import share is essentially zero, that is, $\alpha^m + \alpha^f$**log** $\boldsymbol{S_i^f} \approx 0$, which is consistent with CH's original results.

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[6]Hereafter, all R&D capital stocks, S^d and S^f, refer to unindexed variables.

[7]See also the discussion in Lichtenberg and van Pottlesberghe de la Potterie (1998).

Table 2. Total Factor Productivity Estimation Results with Alternative Weights for Foreign R&D Capital
(pooled 1971-90 data, 22 countries, 440 observations)

	Equation (1)					Equation (2)					Equation (3)				
	Bilateral import	Average	Random division	Random sequential	Random resampling	Bilateral import	Average	Random division	Random sequential	Random resampling	Bilateral import	Average	Random division	Random sequential	Random resampling
	(i)	(ii)	(iii)	(iv)	(v)	(i)	(ii)	(iii)	(iv)	(v)	(i)	(ii)	(iii)	(iv)	(v)
log S^d	0.097	0.026	0.128	0.135	0.134	0.089	0.033	0.107	0.112	0.111	0.082	0.047	0.105	0.111	0.110
	(0.009)	(0.011)	(0.007)	(0.006)	(0.007)	(0.008)	(0.01)	(0.007)	(0.006)	(0.006)	(0.008)	(0.008)	(0.007)	(0.007)	(0.007)
$G7$*log S^d						0.134	0.100	0.148	0.151	0.150	0.155	0.162	0.152	0.151	0.151
						(0.016)	(0.016)	(0.016)	(0.016)	(0.016)	(0.015)	(0.014)	(0.016)	(0.016)	(0.016)
log S^f	0.093	0.222	0.013	0.002	0.005	0.060	0.177	0.009	0.001	0.003					
	(0.016)	(0.019)	(0.005)	(0.002)	(0.003)	(0.015)	(0.019)	(0.005)	(0.002)	(0.003)					
m * log S^f											0.263	0.503	0.040	0.005	0.015
											(0.04)	(0.043)	(0.014)	(0.005)	(0.008)
m											-3.187	-5.733	-0.444	-0.047	-0.154
											(0.486)	(0.487)	(0.162)	(0.071)	(0.102)
Standard Error	0.049	0.045	0.051	0.051	0.051	0.046	0.043	0.046	0.047	0.047	0.044	0.040	0.046	0.047	0.050
R^2	0.630	0.698	0.607	0.601	0.602	0.683	0.725	0.675	0.672	0.672	0.702	0.754	0.679	0.672	0.674
Adjusted R^2	0.609	0.681	0.585	0.579	0.580	0.664	0.710	0.656	0.653	0.653	0.684	0.739	0.660	0.652	0.654

Notes: The dependent variable is the log of total factor productivity. S^d is the domestic R&D capital stock; S^f is the foreign R&D capital stock; $G7$ is a dummy variable equal to 1 for G7 countries; m is the ratio of total imports to GDP. Regressions with the same lowercase roman numerals are identical except for the weights used to calculate S^f.

The regressions based on simple averages of trading partners' R&D capital stocks explain a higher proportion of the variance in TFP than those based on bilateral import shares. The estimated elasticities on foreign R&D capital, however, seem implausibly high compared with the estimated elasticities on domestic R&D capital, especially in equation (1ii). Regressions (1ii), (2ii), and, to a lesser extent, (3ii) are very similar to those reported by Keller (1998),[8] which are the basis for his conclusion that "random" import shares perform better than true bilateral import shares.

The regression results reported in Table 2 for the random division, random sequential, and random sampling weights refer to the average of the estimated coefficients and of the estimated standard deviations (in parentheses) from the 100,000 replications of the regressions corresponding to the 100,000 sets of random shares discussed in the previous section.[9] This is similar to the procedure used by Keller. Compared with the regressions based on bilateral import shares (or average shares), the three sets of regressions based on random shares uniformly have higher standard errors (except for equation (2iii)) and lower R^2s. The estimated coefficients on the domestic R&D capital stock in the random regressions tend to be much larger. The most important difference, however, is that the estimated coefficients on the foreign R&D capital stock become extremely small, both absolutely and relative to their estimated standard errors. In these random share regressions, all of the effects of R&D capital on TFP are attributed to the domestic R&D capital stock, with little or no role for foreign R&D capital, that is, there are no international R&D spillovers.

IV. Conclusions

Our conclusions are straightforward. Randomly created trade patterns *do not* give rise to positive international R&D spillovers. Moreover, there is *no* evidence that randomly created trade patterns do a better job of explaining total factor productivity developments than the bilateral import shares used by Coe and Helpman (1995). Indeed, they do worse. We show that Keller (1998) has reached the opposite conclusions because his "random" shares were not random at all, but were essentially simple averages with a random error.

As noted, regressions using a measure of foreign R&D capital based on bilateral import weights or on a simple average (or Keller's "random" shares) tend to be quite similar. This result may reflect relatively high correlations among the measures of domestic—and, hence, foreign—R&D capital stocks (see CH, Figure 2 and Table A3), or the relatively aggregate level of the data.[10] Both CH and CHH address this issue: CH (p. 884) note that they constructed a measure of foreign R&D capital based on a simple average but did not report regression results using this

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[8]Keller's third set of regressions is based on equation (3') rather than equation (3).

[9]As in CH, the random shares change from year to year, i.e., replications are done for each year.

[10]A foreign R&D capital stock computed as the sum or average of other countries' domestic R&D capital can be thought of as a measure of the world R&D capital stock, which would differ between countries only because of the exclusion of their own domestic R&D capital.

measure since the bilateral import weighted stocks were preferable on theoretical grounds; and CHH report results based on a simple average (CHH, equation (iv) of Table 3), noting that the definition of foreign R&D capital using bilateral import weights performs somewhat better than a definition using weights common to each country (p.145).

The empirical result that a measure of foreign R & D capital based on either bilateral import weights or on a simple average performs better than random weights suggests that a country's total factor productivity is related to its trading partners level of technological development. The intensity of the trade relationship, however, may not be crucial because of the public good nature of knowledge.

The issue of how R&D spillovers occur through trade is a fruitful area for further research, particularly with less aggregate data than used here. Connolly (1998), for example, presents evidence that high-technology imports positively affect both domestic imitation and innovation, based on patent data for 40 countries. Similarly, Eaton and Kortum (1996), also using patent data, find that technology diffusion between countries falls as the distance between them grows. From the perspective of a gravity model, this would suggest a positive relationship between trade and technology diffusion, although Eaton and Kortum find that the effect of distance dominates the effect of imports, a result they describe as "somewhat surprising." Finally, Coe, Helpman, and Hoffmaister (1997, Table 3) present estimation results showing that, while bilateral import weights do only a slightly better job of explaining productivity growth than a simple average (R^2 of 0.18 compared with 0.17), bilateral import weights based on imports of machinery and equipment (rather than on total imports of goods and services) perform much better (R^2 of 0.23). As CHH note, using imports of machinery and equipment rather than total imports is more consistent with the underlying theoretical model. The fact that it also performs better is evidence in favor of the hypothesis that R&D spillovers are transmitted through bilateral trading relationships.

STATISTICAL DISTRIBUTION OF ALTERNATIVE SHARES

A. Keller's Shares

We start with the statistical distribution of the "random" shares proposed by Keller (1998) for the simplified case of two trading partners (*n*=2). This case illustrates how the transformation implicit in Keller's equation (3.1) modifies the distribution of the uniform random variables that were used to generate the shares, and in doing so alters the nature of the weighted average based on these weights.

For *n*=2, Keller's equation (3.1) defines the transformation of two independent random variables that are distributed uniformly over the interval 0 to 1. The (joint) distribution of these two uniform variables is

$$f_{x1,x2} = I_{(0,1)}(x_1) \times I_{(0,1)}(x_2),$$

where $I_{(a,b)}(z)$ is the indicator function that takes the value of one when $a \leq z \leq b$ and zero otherwise. The shares of interest are defined as

$$w_1 = x_1/(x_1+x_2)$$
$$w_2 = x_2/(x_1+x_2).$$

These shares, however, are linearly dependent since by definition $w_2 = (1 - w_1)$, so the joint distribution of w_1 and w_2 is ill defined.

To overcome this problem, replace w_2 with a new variable, w_0, defined as the sum of the uniform variables, giving the following system of equations:

$$w_0 = x_1+x_2$$
$$w_1 = x_1/w_0.$$

The problem is to determine the (marginal) distribution of w_1 (Keller's shares) when x_1 and x_2 are uniform.[11] We do this by first deriving the joint distribution of w_1 and w_0, and then obtain the marginal distribution of w_1 by integrating out w_0.

Joint distribution. For continuous variables, the distribution of w_0 and w_1 is determined by the expression

$$f_{w0,w1} = |J| \times f_{x1,x2}(\, x_1=g_1^{-1}\{w_0,w_1\},\ x_2=g_2^{-1}\{w_0,w_1\}\,),$$

[11]Analyzing w_1 does not involve any loss in generality since w_2 is generated identically to w_1.

where $|J|$ is the (absolute value of the) determinant of the Jacobean of the inverse transformation that is denoted by the functions $g_1^{-1}\{w_0, w_1\}$ and $g_2^{-1}\{w_0, w_1\}$.[12] The inverse transformations are defined by the following expressions:

$$x_1 = w_0 \times w_1$$
$$x_2 = w_0 \times (1-w_1),$$

and thus $|J| = w_0$.[13] The joint distribution can be obtain by substitution

$$f_{w0,w1} = w_0 \times I_{(0,1)}(w_0 \times w_1) \times I_{(0,1)}(w_0 \times (1-w_1)).$$

Marginal distribution. As noted above, the (marginal) distribution of w_1 is obtained by integrating out w_0. This integration is simplified by determining the area over which the joint distribution is non-zero and limiting the integration to these points. For the joint distribution to be non-zero, both indicator variables must be one, which occurs when the following conditions are met: (i) $0 \leq w_0 \times w_1 \leq 1$, and (ii) $0 \leq w_0 \times (1-w_1) \leq 1$. These inequalities determine the four conditions ($w_0 \geq 0$, $w_0 \leq 1/w_1$, $w_0 \geq 0$, and $w_0 \leq 1/(1-w_1)$) that must hold for the joint distribution to be nonzero; these conditions are depicted by the shaded area of Figure 5, top left panel.[14]

Using these conditions, the joint distribution can be expressed as

$$f_{w0,w1} = w_0 \times [\ I_{]0,1/2)}(w_1) \times I_{(0,1/w1)}(w_0) + I_{(1/2,1[}(w_1) \times I_{(0,1/(1-w1))}(w_0)\ ],$$

which represents the nonzero part of the joint distribution. Therefore, the integration with respect to w_0 can be expressed as

$$f_{w1} = \int_0^{1/(1-w_1)} w_0 \times I_{]0,1/2)}(w_1)\ dw_0 + \int_0^{1/w_1} w_0 \times I_{(1/2,1[}(w_1)\ dw_0$$

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[12]See, for example, Mood, Graybill, and Boes (1963, p. 205), and DeGroot (1975, p. 134). This expression follows from the change of variables theorem in integrals; see, for example, Marsden and Hoffman (1993).

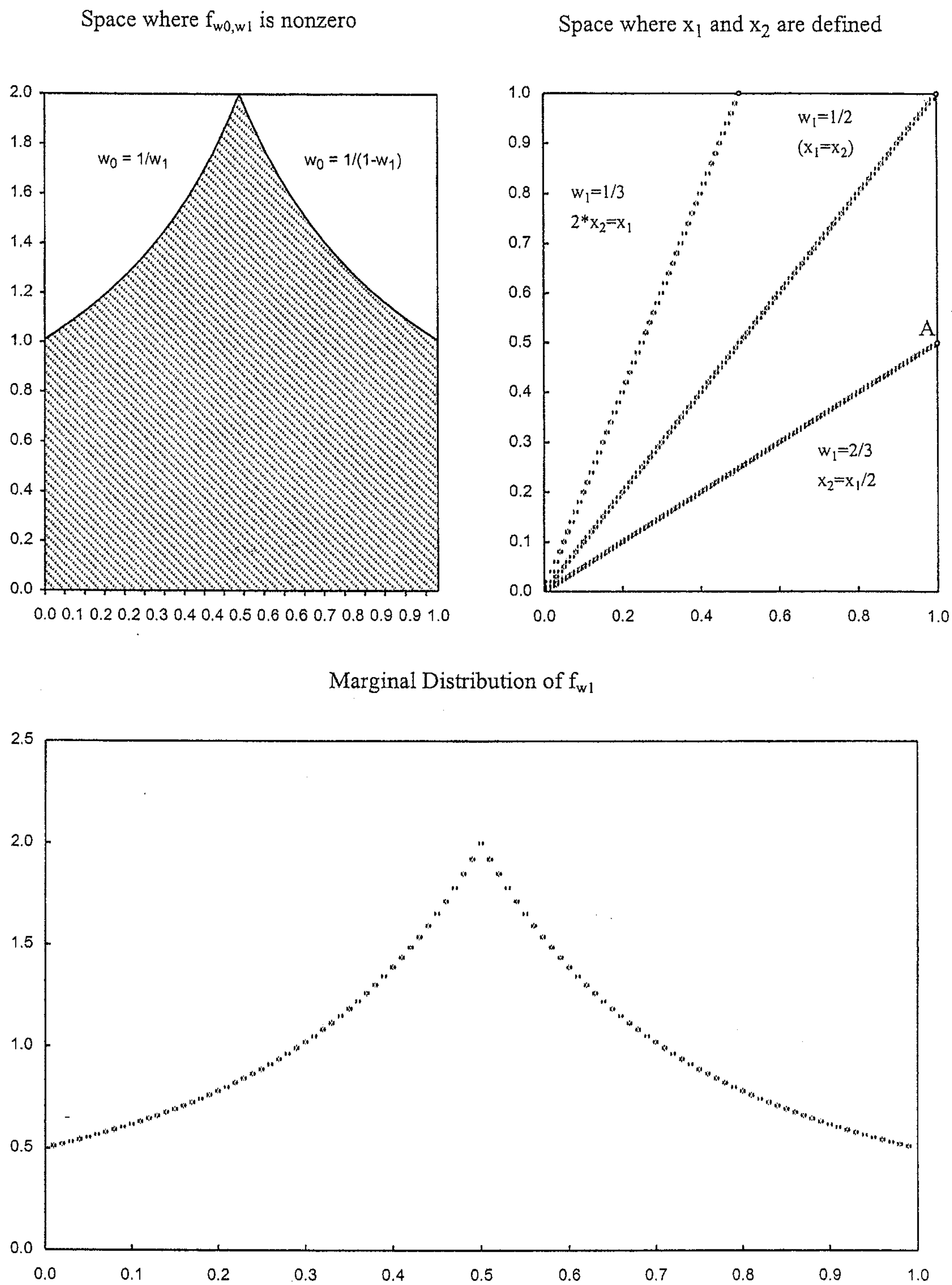
[13]The Jacobean of the inverse transformation, J, is

$$\begin{bmatrix} \delta x_1/\delta w_0 & \delta x_1/\delta w_1 \\ \delta x_2/\delta w_0 & \delta x_2/\delta w_1 \end{bmatrix} = \begin{bmatrix} w_1 & w_0 \\ (1-w_1) & -w_0 \end{bmatrix}$$

[14]These conditions are valid for $w_1 \neq 0$ and $w_1 \neq 1$.

Figure 5. Keller's Shares

(Two bilateral trading partners)



Notes: The horizontal axes in the charts show the values of w_1, x_1, and w_1, respectively, starting from the upper left corner and proceding clockwise; the vertical axes show values of w_0, x_2, and $f_{w1}(w_1)$, respectively, in the same order.

Performing the integration gives the following (marginal) distribution for the share:

$$f_{w1} = ½ \times (1 - w_1)^{-2} \times I_{]0,1/2)}(w_1) + ½ \times (w_1)^{-2} \times I_{(1/2,1[}(w_1) .$$

This expression is the distribution of Keller's shares when n=2, which is depicted in Figure 5, bottom panel. This expression illustrates how the transformation of the uniform random variables used by Keller concentrates the distribution of shares around the value of ½, that is, the inverse of the number of trading partners. Aside from the scale, the statistical distribution in Figure 5 corresponds to the empirical distribution depicted in Figure 1 for n=2.

The intuition behind the concentration can be understood by considering the two-dimensional space over which the uniform variables—x_1 and x_2—used to generate w_1 are defined (see Figure 5, upper right panel). All of the points x_1, x_2 that generate a specific value of the share, w_1, lie on a ray from the origin. The length of this ray is proportional to the frequency one would expect to observe for that specific value of w1 in a random sample. For example, a ray from the origin passing through the coordinate x_1=1, x_2=½ (point A) determines all possible combinations of x_1, x_2 that gives w_1 shares of value ⅔. As the angle of the ray increases to 45 degrees (and w_1 decreases), the length of the ray increases; at 45 degrees, the ray passes through the coordinate x_1=1, x_2=1 corresponding to w_1= ½. Since this is the longest ray from the origin (in x_1, x_2 space), w_1=½ will be the most likely observed value in a random sample. A similar argument applies when n>2. For n=3, the ray is a plane, and the plane with the greatest area is the plane corresponding to w_1=⅓; for n>4, the ray is a hyperplane, and the hyperplane with the greatest volume is that corresponding to w_1=¼; and so on.

For general n, the joint distribution of w_0, w_1, ..., w_{n-1} can be expressed as

$$f_{w0,\, w1,\, ...,\, wn-1} = w_0^{n-1} \times \prod_{i=1}^{n-1} \left(I_{(0,1)}\,(w_0 \times w_i)\right) \times I_{(0,1)}\left(w_0 \times \left\{1 - \sum_{i=1}^{n-1} w_i\right\}\right)$$

where $w_0^{n-1} = /J/$, $w_0 \times w_i = x_i$, $\forall i = 1, 2, ..., n-1$, *and* $w_0 \times \{1 - \Sigma w_i\} = x_n$. As before, the marginal distribution of w_i is obtained by (multiple) integration. Rather than attempting to do so analytically, the results of numerical integration are depicted in Figure 1.

As illustrated in Figure 1, the tendency for the distribution of Keller's shares to concentrate around $1/n$ remains as the number of trading partners increases. Since the shares are defined over the range 0 to 1, this feature of the transformation renders the distribution of these shares non-symmetric for n>2. More important, as n increases—and with it the degree of concentration around the mean—averages calculated with these weights will tend to resemble more and more *unweighted* averages with a bit of added noise.

B. Random Division Shares

Since the distribution of n-1 ordered statistics of uniform variables is Beta (1, n-1)—see Johnson, Kotz, and Balakrishnan (1994, p. 280)—the first and last shares, s_1 and s_n, are both Beta (1, n-1) because s_1 is the first-ordered statistic and s_n is one minus the last-ordered statistic. Proving that all other shares are also Beta (1, n-1) is beyond the scope of this paper. Figure 6 shows the empirical distribution of the random division shares for all trading partners when n=20 (s_1, s_2, ..., s_{20}). The empirical distributions appear to be identical, suggesting that the distribution of all the shares is the same, that is, Beta (1, n-1).

C. Random Sequential Shares

Each share is generated uniform conditional on the previously generated shares. Thus, the first share is distributed U(0, 1); the second share, conditional on the first, is distributed U(0, 1-w_1); the third share, conditional on the first two, is distributed U(0, 1-w_1-w_2); and so on. In general, the conditional distribution of the ith share is

$$f_{(w_i/w_1,w_2,\dots,w_{i-1})} = \left(1 - \sum_{h=1}^{i-1} w_h\right)^{-1} \times I_{(0,1-w_1-w_2-\dots-w_{i-1})}(w_i)$$

The joint distribution of these shares can be obtained from the product of the conditional marginal distributions because the conditional distribution is defined as $f_{x/y} = f_{x,y} \div f_y$.[15] The joint distribution is then

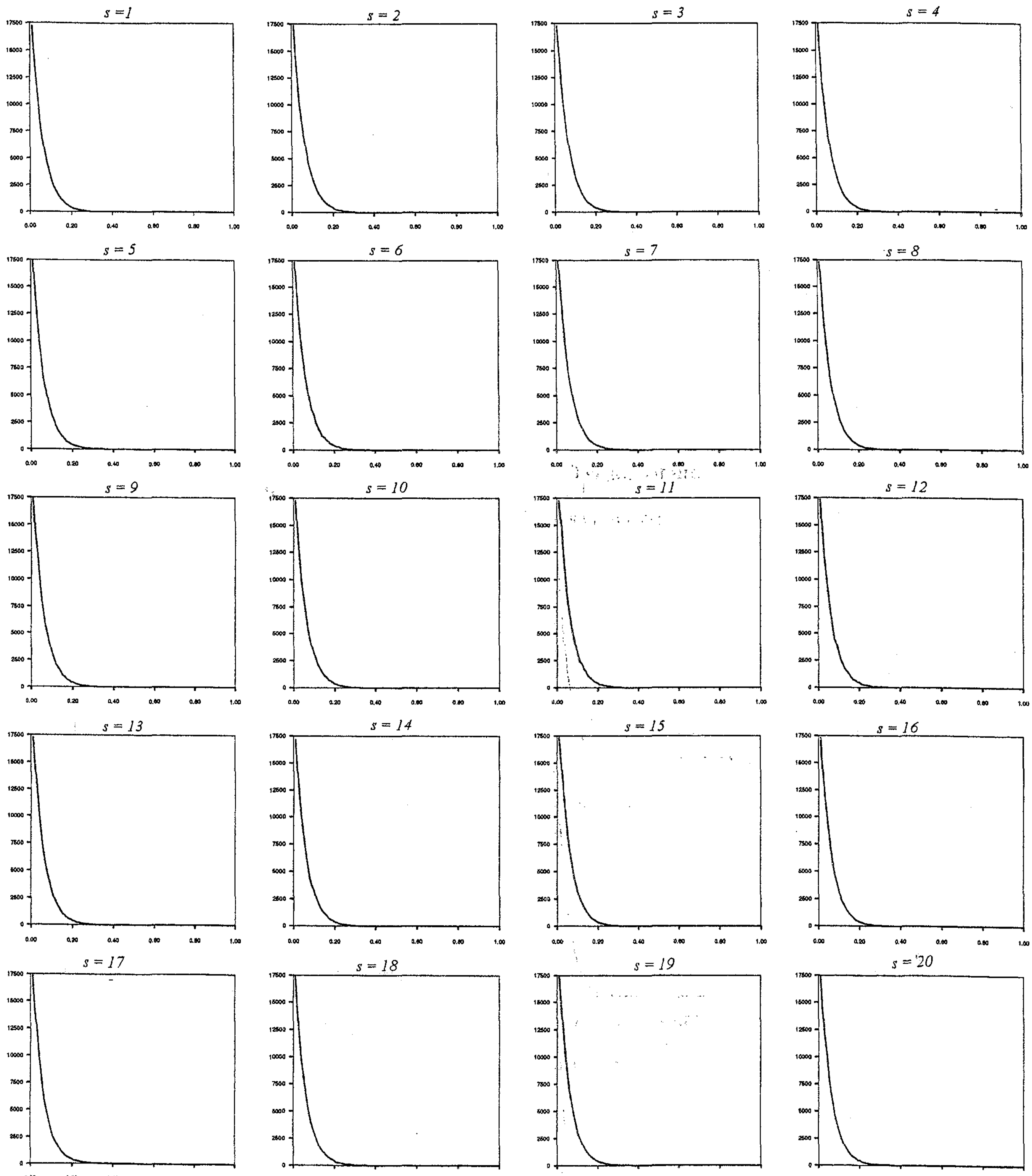
$$f_{w_1,w_2,\dots,w_{n-1}} = I_{(0,1)}(w_1) \times (1-w_1)^{-1} \times I_{(0,1-w_1)}(w_2) \times \dots \times (1-w_1-w_2-\dots-w_{n-1})^{-1} \times I_{(0,1-w_1-w_2-\dots-w_{n-1})}(w_{n-1})$$

As before, the marginal distribution of w_i is obtained by numerical integration (see Figure 3).

[15]See, for example, Mood, Graybill, and Boes (1963, p. 143), and DeGroot (1975, p. 109).

Figure 6. Histograms of Random Division Shares, n=20

(s denotes the bilateral trading partner)



References

Assche, Walter V., 1987, "A Random Variable Uniformly Distributed Between Two Independent Random Variables," *Sankhyā*, Series A, Vol. 49 (June), pp. 207-11.

Coe, David T., and Elhanan Helpman, 1995, "International R&D spillovers," *European Economic Review*, Vol. 39 (May), pp. 859-87.

———, and Alexander W. Hoffmaister, 1997, "North-South R&D Spillovers," *The Economic Journal*, Vol. 107 (January), pp. 134-49.

Connolly, Michelle, 1998, "The Dual Nature of Trade: Measuring Its Impact on Imitation and Growth," Duke University Economics Department Working Paper.

DeGroot, Morris H., 1975, *Probability and Statistics* (Reading, Massachusetts: Addison-Wesley Publishing Company).

Eaton, Jonathan, and Samuel Kortum, 1996, "Trade in Ideas: Patenting and Productivity in the OECD," *Journal of International Economics*, Vol. 40 (May), pp. 251-78.

Johnson, Norman L., and Samuel Kotz, 1990, "Randomly Weighted Averages: Some Aspects and Extensions," *American Statistician*, Vol. 44 (August), pp. 245-49.

Johnson, Norman L., Samuel Kotz, and N. Balakrishnan, 1994, *Continuous Univariate Distributions*, Vol. 2 (New York: Wiley & Sons, 2nd ed.).

Keller, Wolfgang, 1997, "How Trade Patterns and Technology Flows Affect Productivity Growth," World Bank Policy Research Working Paper No. 1831 (Washington: World Bank).

———, 1998, "Are International R&D Spillovers Trade-Related? Analyzing Spillovers Among Randomly Matched Trade Partners," *European Economic Review*, Vol. 42 (September), pp. 1469-81.

Lichtenberg, Frank, and Bruno van Pottlesberghe de la Potterie, 1998, "International R&D Spillovers: A Comment," *European Economic Review*, Vol. 42, (September), pp. 1483-91.

Marsden, Jerrold E., and Michael J. Hoffman, 1993, *Elementary Classical Analysis* (New York: W.H. Freeman & Company, 2nd ed.).

Mood, Alexander M., Franklin A. Graybill, and Duane C. Boes, 1963, *Introduction to the Theory of Statistics* (New York: McGraw-Hill, 3rd ed.).