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The Potential Role of the SDR in Diversified Currency Portfolios of Central Banks

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Abstract

This paper compares the SDR in terms of its risk-return characteristics relative to those of its five components and, on this basis, finds that the SDR has performed favorably over the period under review. In addition, several efficient portfolios including the SDR and its components are computed. These computations provide evidence that in many cases the SDR has a major weight, particularly in those portfolios which involve minimum risk and therefore would appear to be most appropriate for reserve holders. Thus, the evidence presented suggests that the SDR can play a major role in the international reserve portfolios of central banks.

JEL Classification Numbers:
4310, 4314, 4320

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I. Introduction

The depreciation of the U.S. dollar against other major currencies since 1985 has focused attention on the question of the diversification of international currency reserves. Given the large swings in exchange rates over the last several years, foreign exchange reserves held primarily in one reserve asset would have been subject to substantial changes in value. On the other hand, if reserves had been held in a number of currencies, including in particular the SDR--which is in effect a portfolio of currencies--fluctuations in the value of the stock of reserve assets held by monetary authorities would have been reduced. The question thus arises with regard to the extent to which the SDR can play a role in diversifying the currency composition of the reserves held by Fund members and thereby reducing their exposure to exchange risk.

This renewed interest is similar to that which arose in the late 1970s when the U.S. dollar depreciated substantially against most other major currencies. At that time, there was some tendency for central banks to diversify their international reserves, which were mostly composed of U.S. dollars, by shifting into other major currencies. Concurrently, there was some discussion of the role of the SDR in a diversified portfolio of reserve assets. 1/ The strengthening of the U.S. currency in the early 1980s, however, appeared to have reduced the interest of central banks in diversifying their international reserves. 2/

There have been some efforts made to canvass the views of central banks with regard to the extent to which they diversify their international reserves, the most well-known of which is that of the Group of Thirty (1982). Their report, as well as the evidence obtained by Teck and Johns (1983), indicated that there was a growing interest in the management by monetary authorities of their portfolio of reserve assets. More specifically, a number of studies have examined the actual and potential role of the SDR in reserve asset management. 3/ Kenen (1983), for example, analyzed the use of the SDR as a substitute or supplement for other reserve assets in the current international monetary system. Research related directly to the optimal composition of foreign currency reserves and the role of the SDR was carried out by Ben-Bassat (1980, 1984), Jager and de Jong (1986) and Brown, Papell and Rush (1988), while others concentrated on the characteristics of the private use of the SDR (van den Boogaerde, 1984).

1/ See Horii (1986).

2/ For a discussion of this point, see De Beaufort Winjnholds (1987) and Horii (1986).

3/ See, for example, the recent staff paper, "The SDR in the Reserve Management Practices of Monetary Authorities," (SM/87/72, 3/17/87).

This paper continues this last line of research and investigates the relative attractiveness of the SDR by measuring its performance in terms of total return and risk relative to other major reserve assets. It contains a quantitative analysis based on the mean-variance technique that involves the computation of a set of efficient portfolios that includes the five major reserve currencies and the SDR as separate investment instruments. This approach appears to be particularly suitable for a comparative analysis of the SDR relative to other reserve assets as it focuses primarily on the investment function of official currency holdings, i.e., that aspect of reserve assets that is most closely related to risk-return considerations. A transactions approach would be inappropriate as the SDR is not utilized for settling cash positions arising from international transactions except those relating to certain obligations to the Fund, nor is it used for intervening in foreign exchange markets. 1/

The basic aim of the paper is to compute the portfolio of reserve assets that involves minimum risk with emphasis on the role played by the SDR in such portfolios. This approach would appear to be especially relevant for an analysis of the composition of official reserves under the plausible assumption that central banks are risk averse. As this analysis focuses on general considerations relating to the role of the SDR in reducing the risk faced by monetary authorities in holding exchange reserves, no attempt is made in the paper to compare the actual composition of official reserves with the calculated minimum-risk portfolios of reserve assets for individual countries.

The remaining sections of this paper are as follows. Section II reviews recent trends in foreign exchange rate movements and in the composition of foreign exchange reserves. Section III describes certain institutional aspects of the SDR and its major characteristics as a reserve asset. Section IV presents the empirical results, giving particular attention to the major role played by the SDR in a minimum risk portfolio. Section V provides some concluding remarks.

II. Recent Trends in the Composition of Foreign Exchange Reserves

The volatility of the U.S. currency and the decline in the exchange rate of the U.S. dollar against most other leading currencies in the late 1970s provided the initial impetus for the diversification

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1/ For an approach to currency diversification that is explicitly transactions-oriented, see the recent paper by Dooley, Lizondo, and Mathieson (1988). The transactions motive for holding reserves is also included in the model developed by Brown, Papell, and Rush (1988).

of foreign exchange reserve holdings by central banks. This diversification involved primarily a shift from U.S. dollars into appreciating currencies, particularly into deutsche mark and to a lesser extent Japanese yen. The strengthening of the U.S. currency in the early 1980s, however, appears to have slowed the trend toward greater diversification. The subsequent weakening of the dollar against most other major currencies beginning in 1985 again appears to have been a factor in reserve diversification away from the U.S. dollar and into appreciating currencies, such as the deutsche mark and the Japanese yen. The apparent objective of central banks in taking this action was again to reduce their foreign exchange risks. 1/ The aim in this section is to review these three distinct episodes in the diversification of foreign exchange reserve holdings by the industrialized and the developing countries.

The fall of the U.S. dollar against major currencies in the late 1970s led to increased diversification of foreign exchange reserves by central banks in industrial and in developing countries, as shown in Tables 1.1 and 1.2, respectively. 2/ In the industrial countries, U.S. dollar reserves fell to 77.6 percent of total foreign exchange reserves in 1980 from 86.4 percent in 1978. Concurrently, the official holdings of deutsche mark increased to 14.4 percent in 1980 from 7.9 percent in 1978 and official holdings of Japanese yen increased to 3.5 percent in 1980 from 2.3 percent in 1978. In the developing countries, the official holdings of U.S. dollar assets fell to 60.1 percent of all foreign exchange reserve assets in 1980 from 66.6 percent in 1978, as shown in Table 1.2. At the same time, these countries increased their official holdings of pound sterling, French francs, and Japanese yen.

Over the period from 1980 to 1983, diversification of currency reserves away from dollar holdings on the part of industrial countries was interrupted, and official holdings of deutsche mark, pounds sterling, and French francs remained fairly constant. Currency reserves held in Japanese yen nonetheless continued along the rising

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1/ For a more complete discussion why countries diversified their foreign exchange reserve holdings, see Horii, A., (1986).

2/ It should be noted that changes in the composition of reserves reflect both changes in holdings of the individual reserve assets as well as exchange rate changes. Thus, a depreciation of the dollar against all other major reserve currencies would automatically increase the share of these currencies when the total stock of reserves is measured in dollar terms. Therefore the shares shown in Tables 1.1 and 1.2 reflect these valuation effects as well as actual purchases and sales of reserve assets. Nevertheless, monetary authorities presumably include these valuation effects in reaching decisions on their desired composition of exchange reserves.

Table 1.1. Share of National Currencies in Official Holdings of Foreign Exchange by Type of Currency Among Industrial Countries; 1/ End of Year 2/

(In percent)

Official Holdings	1973	1975	1976	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986	1973-1979 4/	1980 1986 5/
U.S. dollar	87.3	87.3	86.9	89.4	86.4	83.4	77.6	78.7	77.0	77.5	73.6	65.4	70.6	86.7	74.3
Pound sterling	3.9	1.1	0.7	0.9	0.7	0.8	0.8	0.7	0.8	0.9	1.6	2.0	1.2	1.3	1.1
Deutsche mark	2.6	4.0	3.8	5.5	7.9	9.7	14.4	13.1	12.5	13.1	15.2	19.8	17.6	5.5	15.1
French franc	--	0.1	0.1	0.3	0.4	0.6	0.5	0.5	0.4	0.3	0.4	0.5	0.5	0.3	0.4
Japanese yen	--	0.2	0.4	1.8	2.3	2.6	3.5	3.7	4.5	5.2	6.3	8.8	7.6	1.4	5.6
Others 3/	6.2	7.3	8.1	2.1	2.2	2.8	3.1	3.0	4.8	3.0	2.9	3.5	2.5	4.7	3.2

Source: International Monetary Fund, Annual Report, 1982, 1986 and 1987.

1/ Starting with 1979, the SDR value of European currency units (ECU) issued against the U.S. dollar is added to the SDR value of U.S. dollars, but the SDR value of ECU issued against gold is excluded from the total distribution here.

2/ For 1973, data was available only for the end of the first quarter. Data for 1974 was not available for industrial countries only.

3/ Others include: Swiss franc, Netherlands guilder holdings and a residual of unspecified currencies, which is equal to the difference between total indentified reserves and the sum of the reserves held in the currencies listed.

4/ Average shares in the period 1973-1979.

5/ Average shares in the period 1980-1986.

Table 1.2. Share of National Currencies in Official Holdings of Foreign Exchange by Type of Currency Among Developing Countries; 1/ End of Year 2/

(In percent)

Official Holdings	1973	1975	1976	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986	1973-1979 4/	1980 1986 5/
U.S. dollar	55.2	70.8	72.7	70.9	66.6	66.3	60.1	67.1	66.5	68.0	69.2	67.5	60.3	67.0	65.5
Pound sterling	13.4	6.8	3.2	2.8	3.2	3.4	5.4	3.8	4.4	4.8	4.8	4.7	4.2	5.4	4.5
Deutsche mark	13.2	8.8	10.1	13.3	15.9	16.2	16.7	13.9	13.3	11.1	10.6	10.9	10.4	12.9	12.4
French franc	3.1	2.4	1.7	2.3	2.3	2.2	3.1	2.5	2.4	2.0	1.9	2.1	2.5	2.3	2.3
Japanese yen	0.2	0.9	1.1	3.2	4.9	4.8	5.6	5.0	5.1	4.9	5.3	6.5	5.9	2.5	5.4
Others 3/	14.9	10.3	11.2	7.5	7.1	7.1	9.1	7.7	8.3	9.2	8.2	8.3	16.7	9.6	9.6

Source: International Monetary Fund, Annual Report, 1982, 1986 and 1987.

1/ Starting with 1979, the SDR value of European currency units (ECU) issued against the U.S. dollar is added to the SDR value of U.S. dollars, but the SDR value of ECU issued against Gold is excluded from the total distributed here.
2/ For 1973, data was available only for the end of the first quarter. Data for 1974 was not available for developing countries only.
3/ Others include: Swiss franc, Netherlands guilder holdings and a residual of unspecified currencies, which is equal to the difference between total identified reserves and the sum of the reserves held in the currencies listed.
4/ Average shares in the period 1973-1979.
5/ Average shares in the period 1980-1986.

trend evident throughout the 1970s. However, the appreciation of the U.S. dollar against other major currencies over this period was associated with an increase in U.S. dollar reserves held by developing countries and a decline in their holdings of pound sterling, deutsche mark, French francs, and Japanese yen. In 1984, the volatility of the U.S. dollar against most other major currencies increased significantly from the previous two years, 1/ and during 1985 and 1986 the dollar depreciated sharply against most other major currencies. While other factors may also have been important in influencing the reserve holding decisions of monetary authorities, these exchange rate changes would appear to have been a major consideration in the shift out of U.S. dollar-denominated reserve assets on the part of both industrial and developing countries from 1983-84 to 1985-86. 2/

To summarize, both industrial and developing countries have diversified their holdings of foreign exchange reserves since the 1970s. The shift toward greater diversification was more pronounced on the part of industrial countries, perhaps because developing countries in the aggregate already held a more diversified portfolio in the 1970s than the industrial countries. 3/ On balance, therefore, the holdings of the U.S. dollar as a foreign exchange reserve asset have diminished since the late 1970s, particularly in the industrial countries, and the holdings of the deutsche mark and the Japanese yen have increased.

The tendency for monetary authorities to diversify their international reserves across a number of major currencies may well indicate a desire on their part to reduce their exposure to risk arising from exchange rate fluctuations. Indeed, Table 1 provides some evidence that such diversification may have been accelerated by the volatility of exchange rates during the 1980s. As the SDR is a reserve asset that is itself a diversified portfolio of currencies, it would appear to be well designed to protect monetary authorities from large changes in the value of their currency reserves associated with

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1/ See Horii (1986).

2/ Among the other factors, it should be noted that the currency composition of the reserve assets of the major industrial countries would in particular be influenced by their exchange rate objectives rather than considerations of risk and return on alternative reserve assets. For example, the central banks of the major industrial countries intervened to stem the dollar's decline during 1986, which may in fact help account for the fact that the share of dollar assets in industrial countries' reserves rose from 1985 to 1986.

3/ For a discussion of differences in reserve diversification between industrial and developing countries, see Ben-Bassat (1984).

exchange rate fluctuations. The characteristics of the SDR as a reserve asset, particularly as they bear on this risk-reducing property, are described below.

III. The Characteristics of the SDR as a Reserve Asset

The SDR is a composite reserve asset created by the Fund in 1970 to supplement other reserve assets. 1/ To date the Fund has made two allocations; the first, of SDR 9.3 billion, in the period 1970-72 and the second, of SDR 12.1 billion, in the period 1978-81. The Fund has not made any cancellation of SDRs. At the end of 1987, SDRs held by Fund members amounted to 4 percent of their total non-gold international reserves.

The value of the SDR is expressed in terms of a basket of five major currencies: the U.S. dollar, the deutsche mark, the pound sterling, the French franc, and the Japanese yen. It is computed daily by the Fund as the sum of the value in terms of the U.S. dollar (using London noon exchange rates) of specified amounts of each of the five currencies in the basket. These amounts (currency units) are derived from both the weights assigned to each currency and the average exchange rates over a prescribed period preceding the coming into effect of a revision in the basket, which most recently was January 1, 1986. The weights of these five currencies reflect the relative importance, over a five-year period preceding a revision, of the exports of goods and services of the five countries whose currencies make up the basket, as well as the balances of those currencies held as reserves by members of the Fund.

The interest rate on the SDR is calculated as the sum of the interest rates on short-term domestic financial instruments denominated in the five component currencies, respectively, multiplied by the number of units of the currency in the basket and the value of the spot exchange rate of the currency in terms of the SDR. The interest rate on the SDR is calculated weekly (since August 1, 1983) and is paid quarterly, and thus is comparable with the returns on three-month international reserve assets. Members holding SDRs in excess of their allocations earn net interest on the excess holdings, and members holding SDRs below their allocations pay net charges at the same rate on their net use of SDRs. 2/

1/ For a description of the SDR, see Chandavarkar (1984), pp. 62-70.

2/ Prior to May 1981, the SDR interest rate was less than the combined weighted average of the interest rates on the five short-term domestic financial instruments.

While IMF members can use their SDRs to obtain an equivalent amount of usable currencies from other members, to settle financial obligations, to make donations, and to extend loans, the SDR has not developed as a major reserve asset; its use has been concentrated in transactions between members and the IMF. 1/ One of the reasons why the SDR has not developed into a reserve asset with major transactions functions reflects the fact that the SDR has liquidity characteristics which appear to make it less desirable than other reserve assets. For those countries with a balance of payments financing needs, the liquidity of the SDR is assured by the process of designation of members. Through this process, the IMF designates or selects several countries with generally strong balance of payments positions to provide convertible currencies in exchange for SDRs from countries facing balance of payments difficulties. 2/ For those countries with balance of payments financing needs, this process ensures that the SDR is a liquid and reliable source of external finance, especially when a country's net indebtedness and marginal cost of borrowing on private capital markets is high. The liquidity of the SDR, however, is somewhat circumscribed for members not facing external financing needs because other IMF members are not under an obligation to accept SDRs in exchange for usable currencies. When there is no balance of payment need, the Fund arranges transactions by agreement between members primarily by maintaining a list of those participants and prescribed holders that are ready to sell SDRs under either standing arrangements or on an ad hoc basis in transactions arranged by the Fund on their behalf. 3/ While recently the ability to engage in transactions by agreement has been facilitated by the willingness of several Fund members to stand ready to buy and/or sell SDRs in voluntary transactions, the SDR remains a less liquid reserve asset than other foreign exchange instruments, e.g., U.S. Treasury bills.

Although the SDR cannot be used directly for exchange market intervention and it is not as liquid as certain other reserve assets, it does have the property of reducing, in some cases substantially, exposure to exchange rate movements. This stability characteristic results from the fact that the SDR is a weighted average of five component currencies. This feature of the SDR implies that as long as movements in the exchange rates of the component currencies do not show a perfectly positive correlation, i.e., do not have a correlation coefficient equal to 1.0, the movements of one currency will be partially offset by smaller (in the case of positive correlation) or divergent (in the case of negative correlation) movements of the remaining currencies in the SDR. The absence of a perfect correlation

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1/ Chandavarkar (1984), p. 9 and Fawzi (1986).
2/ Chandavarkar (1984).
3/ Chandavarkar (1984).

Table 2. Pairwise Correlation Coefficients of Changes in Exchange Rates in Terms of the SDR, 1981-87

(Monthly changes)

	U.S. dollar	Deutsche mark	Pound sterling	French franc	Japanese yen
U.S. dollar	1.00				
Deutsche mark	-0.83	1.00			
Pound sterling	-0.36	0.20	1.00		
French franc	-0.80	0.87	-0.14	1.00	
Japanese yen	-0.63	0.26	-0.01	0.30	1.00

is confirmed by the empirical evidence provided in Table 2, which shows the correlation coefficients for changes in the exchange rates in terms of the SDR of the component currencies of the SDR from 1981 to 1987.

From the first column of this table, for example, it can be seen that the movements in the exchange value of the U.S. dollar (expressed in SDR terms) against the four remaining currencies (also expressed in SDR terms) are negatively correlated, i.e., when the U.S. dollar appreciated over this period, all four nondollar currencies depreciated against the SDR. Of course, as the appreciation of the dollar against the SDR reflects an appreciation against one or more of the nondollar currencies in the basket, at least one of the correlation coefficients in the first column must be negative. Because exchange rates are not perfectly correlated, the overall effect on the exchange rate of the SDR is partially or fully offset by the divergent movements in the exchange rate value of the U.S. dollar against the other currencies, resulting in greater stability of the SDR exchange rate for these currencies. This reflects the fact that the variance of the SDR exchange rate will always be lower than the weighted average of the variances of the component currencies in the basket. 1/

1/ This relationship between the variance of the SDR exchange rate and the weighted average of the variances of the component currencies is derived formally in the Appendix.

This stability characteristic is illustrated in Charts 1 to 5 which plot the quarterly average of the daily percentage movements of the bilateral exchange rates for each of the five component currencies taken in turn as numeraire, from the third quarter of 1981 to the third quarter of 1987. The thicker line in each chart represents the exchange rate of the SDR expressed in terms of the base numeraire currency. The charts show that this exchange rate, although following the general trend variation in bilateral exchange rates, fluctuates in most cases less than the bilateral exchange rates, confirming therefore the greater stability of the SDR exchange rate.

The graphical results are further confirmed by Table 3, which lists the average of absolute daily percentage movements of bilateral exchange rates from the third quarter of 1981 to the third quarter of 1987. Reading the data by column, the last row of the table confirms that the SDR exchange rate has invariably a lower average percentage movement compared with its bilateral exchange rate components, with the exception of the French franc and the deutsche mark. 1/

IV. The Role of the SDR in a Minimum Variance Portfolio

This section provides a comparison of the risk and return properties of the SDR relative to other major reserve assets. The data set for the empirical exercise, together with the working assumptions, are described first. The results of computing efficiency frontiers, which provide the means for determining the share of SDRs in a portfolio of reserve assets with minimum variability, are then presented for alternative base currencies.

1. The data set

As outlined above, the primary focus of this paper is to analyze the relative performance of the SDR in an efficient portfolio of assets denominated in those currencies that are used for official foreign currency reserves. In this context, the SDR is taken as an independent asset (currency) in which central banks can invest, and then compared with assets denominated in other major reserve currencies. The reserve assets chosen for the comparative exercise are the U.S. dollar, the Japanese yen, the pound sterling, the deutsche mark, and the French

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1/ As discussed in the next section, the lower variance in the bilateral exchange rate of the French franc with the deutsche mark reflects the participation of both of these currencies in the European Monetary System.

Chart 1. Quarterly Average of Daily Percentage Movements of Exchange Rates Against the U.S. Dollar

(Third quarter 1981 to third quarter 1987)

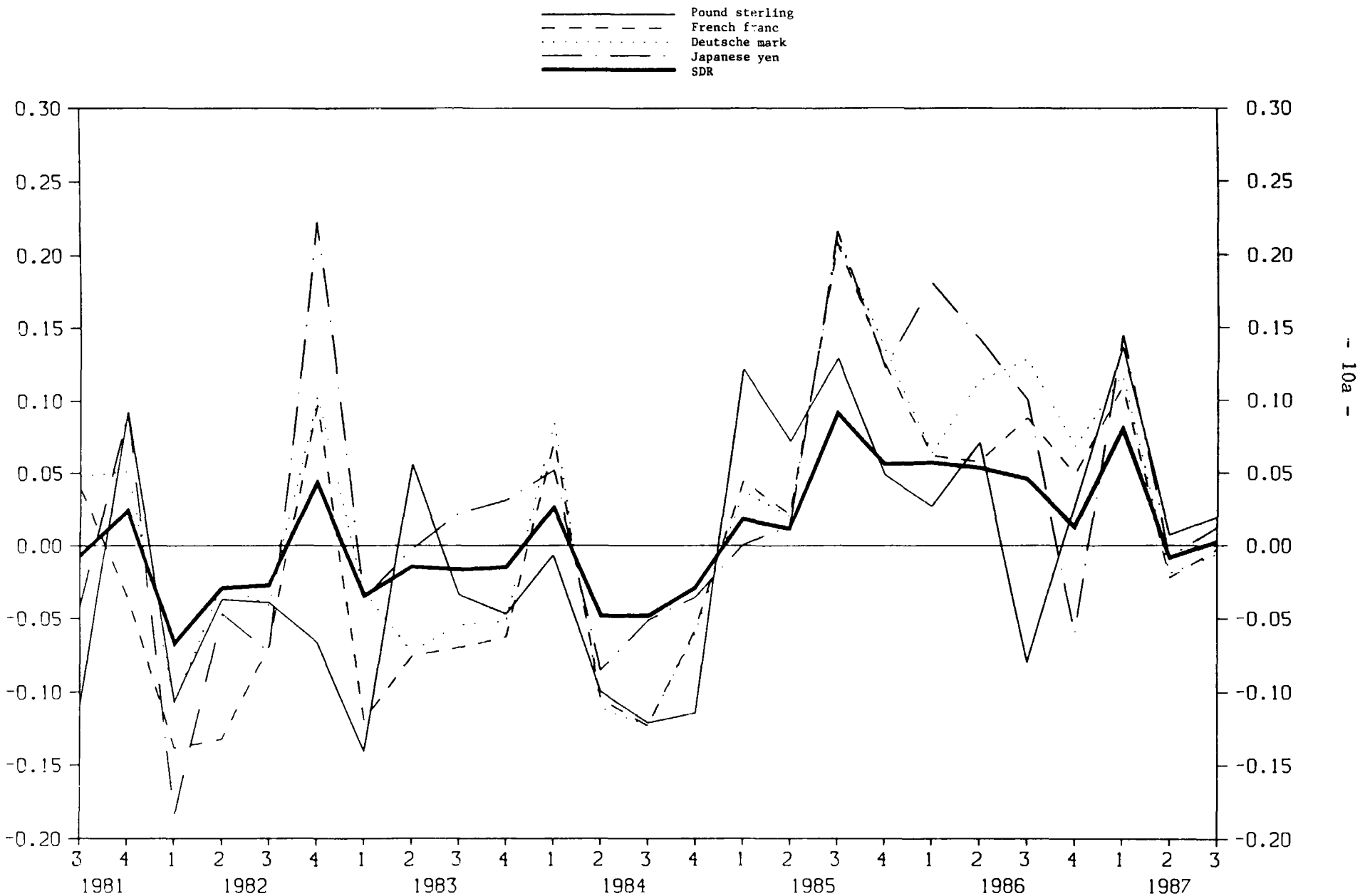

Chart 2. Quarterly Average of Daily Percentage Movements of Exchange Rates Against the Japanese Yen

(Third quarter 1981 to third quarter 1987)

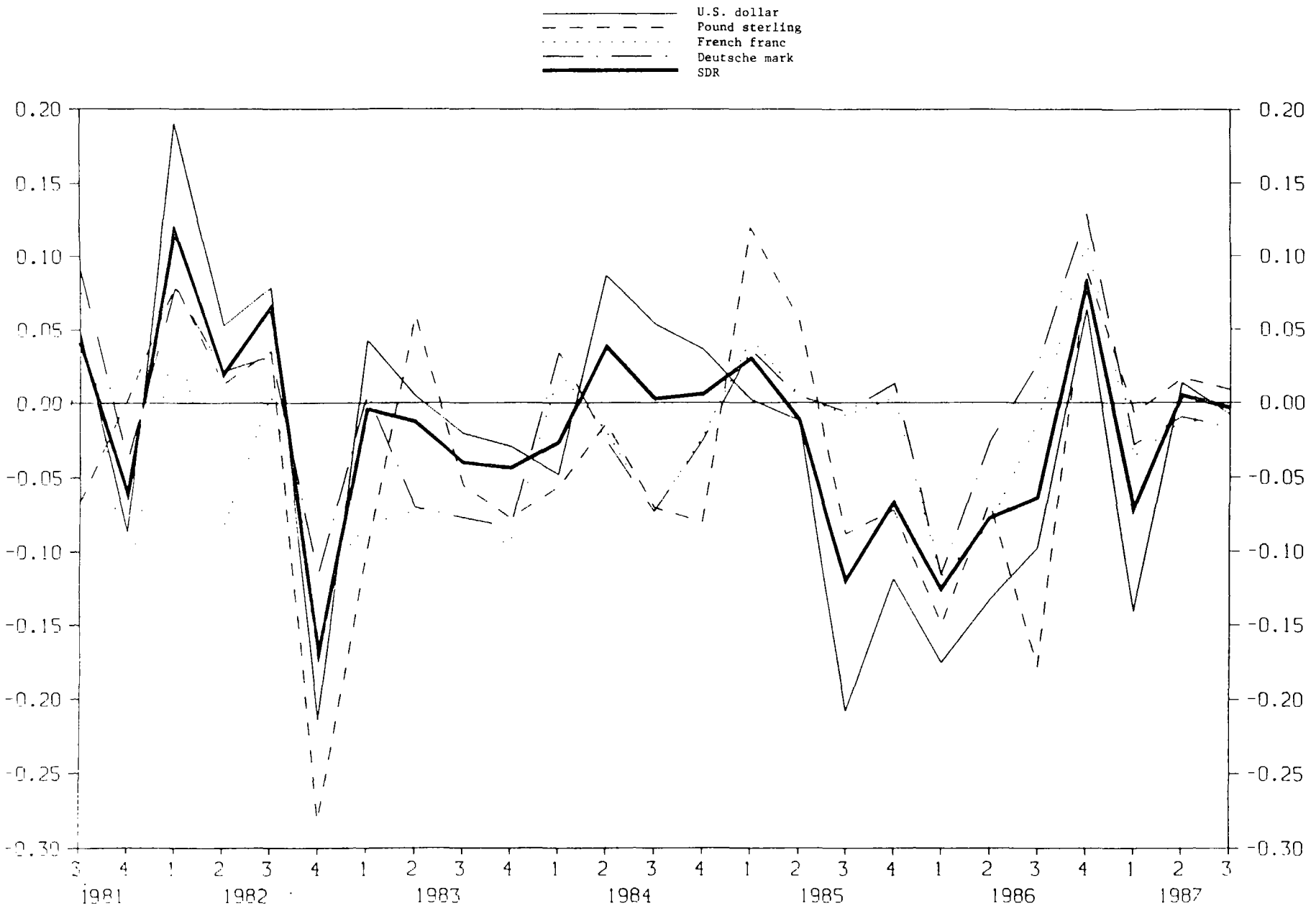

Chart 3. Quarterly Average of Daily Percentage Movements of Exchange Rates Against the Deutsche Mark

(Third quarter 1981 to third quarter 1987)

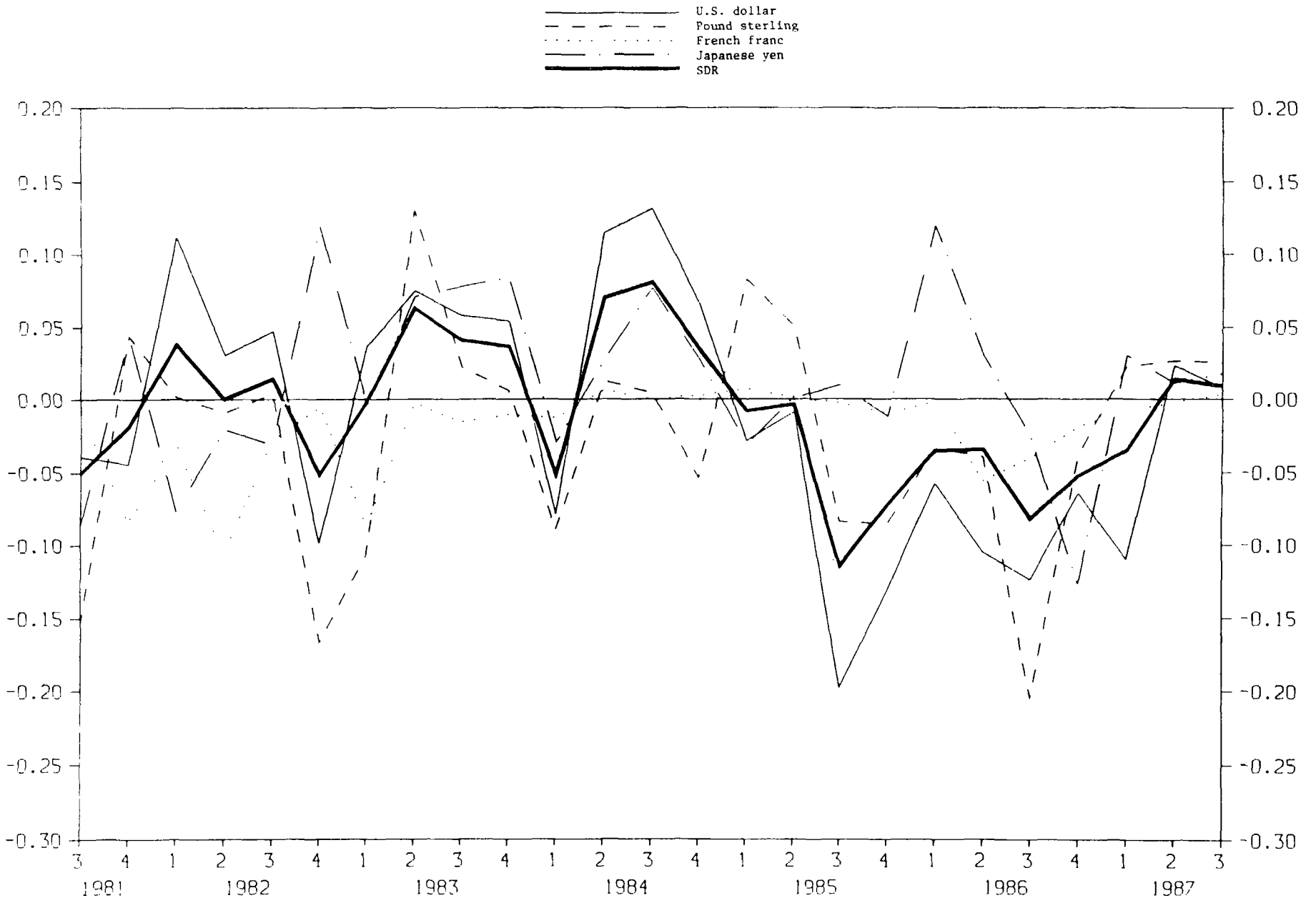

Chart 4. Quarterly Average of Daily Percentage Movements of Exchange Rates Against the Pound Sterling

(Third quarter 1981 to third quarter 1987)

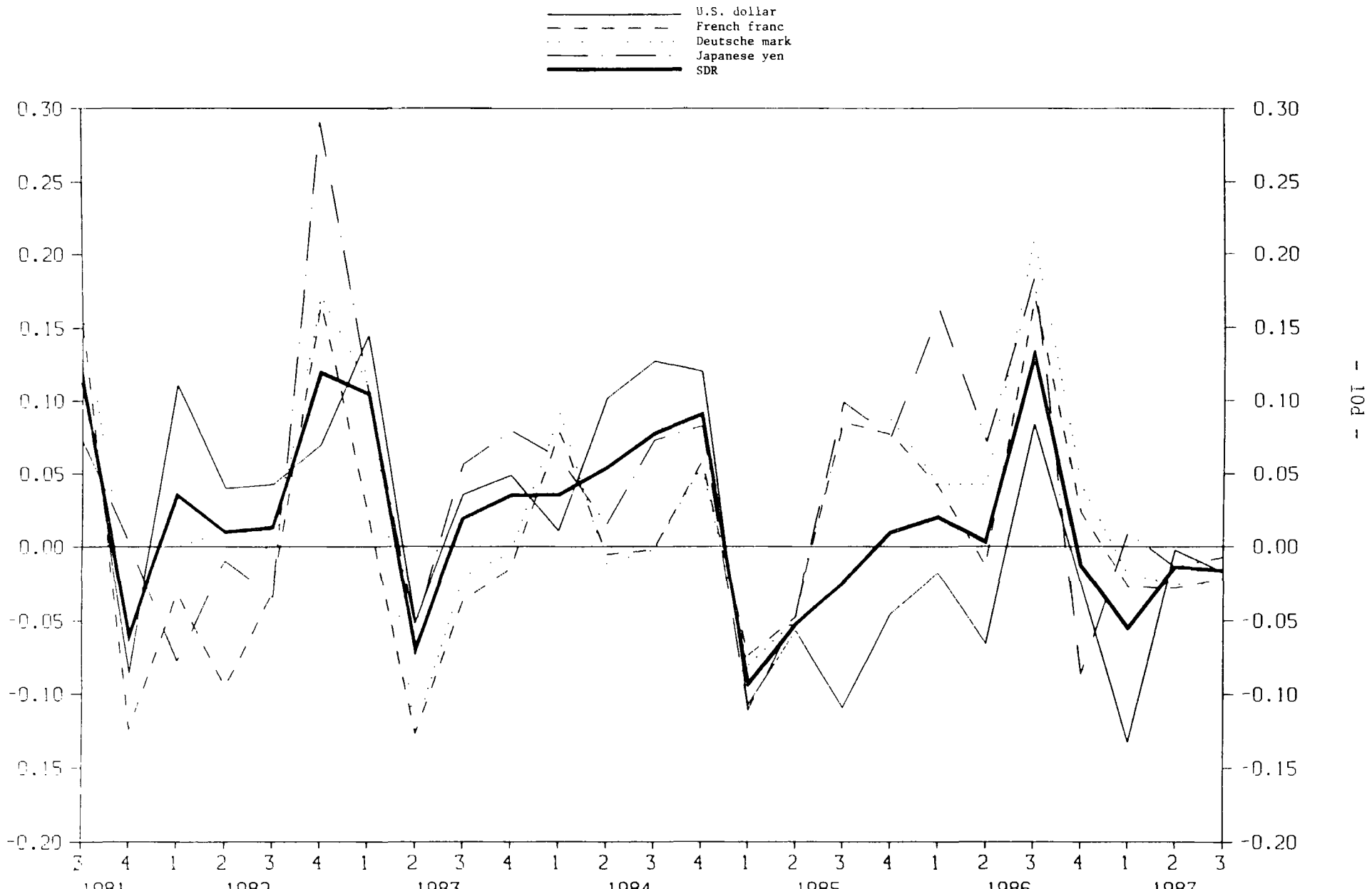

Chart 5. Quarterly Average of Daily Percentage Movements of Exchange Rates Against the French Franc

(Third quarter 1981 to third quarter 1987)

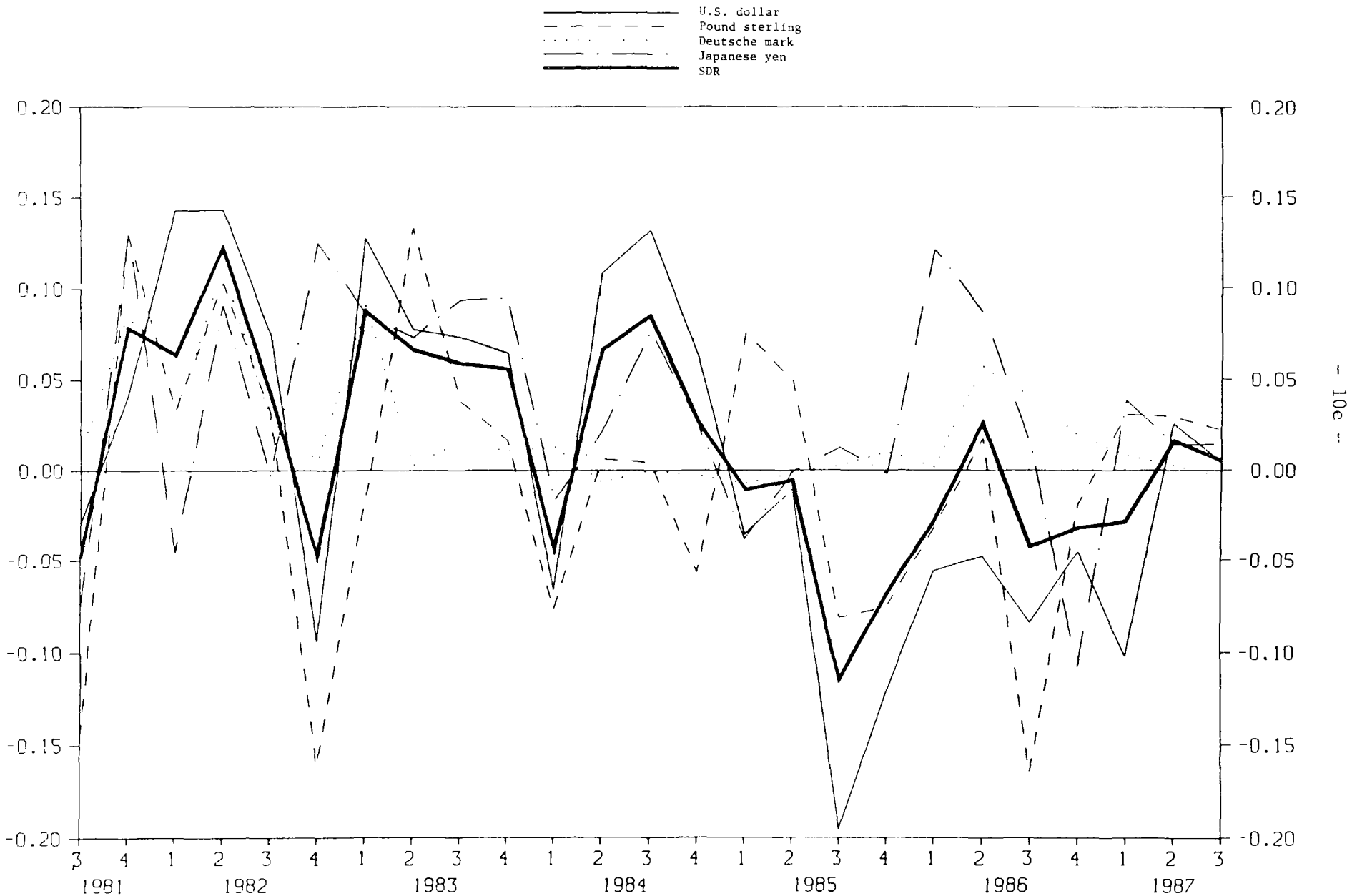

Table 3. Average Absolute Daily Percentage Changes in Bilateral Exchange Rates

(Third quarter 1981 to third quarter 1987)

	U.S. dollar	Pound sterling	French franc	Deutsche mark	Japanese yen	SDR
U.S. dollar	--	0.56	0.57	0.58	0.47	0.26
Pound sterling	0.56	--	0.39	0.39	0.49	0.37
French franc	0.57	0.39	--	0.09	0.38	0.35
Deutsche mark	0.58	0.39	0.09	--	0.37	0.33
Japanese yen	0.47	0.49	0.38	0.37	--	0.37
SDR	0.26	0.37	0.35	0.33	0.37	--

franc. These currencies are also taken as the numeraires that are relevant for expressing the value of the reserve holdings of central banks.

It is useful to distinguish two types of numeraires for measuring the returns, variances, and covariances of international reserve assets. One type of numeraire is an individual currency that serves as a unit of account. As such, the currency chosen is to some extent arbitrary, but it would seem reasonable to use the domestic currency except in those countries facing severe inflation (Jager and De Jong, 1986). In the latter situation, it is more meaningful to use as the numeraire one of the main convertible currencies, which has more stable purchasing power. A single currency numeraire, which is referred to also as the base currency, has been adopted in this paper because countries measure their gains or losses on international reserve assets in the domestic currency or one of the major reserve asset currencies. The second type of numeraire is an import currency basket (Ben Bassat, 1984, 1980, Healy, 1981) or a current account currency basket (Racine, 1988). This type of numeraire reflects the transactions of a country and is more appropriate than a single currency unit of account if the country desires to maintain the purchasing power of its international reserve assets. This paper does not attempt to define a numeraire in

this manner or compute an efficiency using this numeraire, as the analysis here is not country specific. One could, however, apply the results given below to specific countries to the extent that their international transactions are denominated in one of the major reserve currencies, or are distributed roughly in terms of the currency composition of the SDR, in which case the reserve holdings could be considered to be based on SDRs.

The primary input to the computation of efficiency frontiers is the characterization of the joint distribution of returns, i.e., the means, variances and covariances. 1/ These parameters have been derived from actual historical returns, as the primary goal of the empirical analysis is to ascertain the ex-post performance of the SDR relative to the other reserve assets. Accordingly, the distribution of the total return (the exchange rate plus interest rate) of each of the currencies was calculated on the basis of the ex-post daily movements. The historical series utilized cover the entire period from January 1, 1981 to July 31, 1987, using daily observations for interest and exchange rates. The primary reason for choosing this period was the redefinition of the SDR that became effective January 1, 1981, which reduced its component currencies from 16 to 5. The domestic instruments used are the SDR interest rate and its components--U.S. three-month Treasury bill, U.K. three-month Treasury bill, Japanese two-month private bill, three-month French interbank rate, and German three-month interbank rate. 2/

The return on currency i taken as the base currency, compared with currency j taken as the investment currency, was measured by simulating an investment in the domestic instrument of currency j and summing the interest income accrued during the maturity of the instrument plus the exchange rate gain or loss at maturity. The variation in the return was then obtained by rolling over the position on a daily basis, i.e., by taking a new equivalent position every day from January 1, 1981 to

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1/ For a description of the computation of the efficiency frontiers, see Adler and Dumas (1983), Levy and Sarnat (1982), Sharpe (1970), and Szego (1980).

2/ It should be noted that the SDR does not have a specific maturity and therefore it is not precisely comparable to those instruments that compose the SDR interest rate basket. It pays what is basically a three-month rate of return but can be converted to usable currency on two- or three-day maturity.

July 31, 1987. The mean and variance for each instrument were then computed, as well as the covariances between the rates of return. 1/

2. Comparison of individual investment alternatives

The average returns and standard deviations reported in Tables 4 to 9 provide evidence of the relatively stronger performance of the SDR compared with investments in its individual component instruments, with the obvious exception of the return on the instrument denominated in the base currency. For these investment alternatives, the standard deviation of the return is far below that on other investments because no exchange rate risk is involved. Apart from these exceptional situations, over the period covered the SDR in many cases had not only a lower variability but also a higher mean return. In these cases the SDR clearly dominates other investment alternatives, where the comparison is between a portfolio that consists exclusively of the SDR or exclusively of one alternative reserve asset. For example, for dollar-based central banks, holding SDRs would have yielded both a higher return and lower variability (aside from dollar-denominated instruments) than reserves invested in pound sterling- or French franc-denominated assets. However, in other cases the lower variability of the SDR is associated with a lower average return, so that the SDR would not clearly dominate as a single investment vehicle, except on the assumption that the lowest possible variability of return is the preferred investment strategy. Aside from investments in the base currency, in only three cases does investment in an instrument other than the SDR yield a lower variability in the return.

Rather than analyzing individually each of the tables and comparing them with each other, a more useful way to summarize the relative performance of each reserve instrument is provided by Table 10. In this table the data are presented in the form of return per unit of risk, as measured by the average total return on the investment in each currency divided by the standard deviation of the return. The higher this ratio, the higher the return on an investment for a given level of risk or, alternatively, the higher the ratio, the lower the risk for a given level of return.

1/ Real rates of return were not computed. As pointed out by Ben-Bassat (1982), nominal returns in foreign currencies do not vary significantly from real returns given the relative stability of inflation rates compared with exchange and interest rate movements. Table 4 through Table 9 list these values for each currency analyzed as well as the SDR.

Table 4. Simulation of Investments in SDR Components and in SDRs
January 1, 1981-July 31, 1987

(U.S. dollar-based investor)

	Mean	Standard deviation
U.S. dollar	9.40	3.03
Pound sterling	6.61	26.35
French franc	8.98	25.45
Deutsche mark	9.88	25.34
Japanese yen	12.90	26.60
SDR	9.67	11.58

Correlation of total return 1/

	U.S. dollar	Pound sterling	French franc	Deutsche mark	Japanese yen	SDR
U.S. dollar	1.0					
Pound sterling	-0.40	1.0				
French franc	-0.45	0.63	1.0			
Deutsche mark	-0.35	0.65	0.96	1.0		
Japanese yen	-0.47	0.52	0.79	0.76	1.0	
SDR	-0.43	0.75	0.94	0.95	0.87	1.0

Table 5. Simulation of Investments in SDR Components and in SDRs
January 1, 1981-July 31, 1987

(Pound sterling-based investor)

	Mean	Standard deviation
Pound sterling	11.08	1.80
U.S. dollar	15.84	27.46
French franc	14.32	21.68
Deutsche mark	15.23	21.15
Japanese yen	18.23	25.28
SDR	15.41	18.55

Correlation of total return 1/

	Pound sterling	U.S. dollar	French franc	Deutsche mark	Japanese yen	SDR
Pound sterling	1.0					
U.S. dollar	-0.12	1.0				
French franc	-0.27	0.40	1.0			
Deutsche mark	-0.09	0.41	0.94	1.0		
Japanese yen	-0.32	0.42	0.75	0.72	1.0	
SDR	-0.21	0.90	0.72	0.73	0.71	1.0

1/ Interest plus exchange rate return.

Table 6. Simulation of Investments in SDR Components and in SDRs
January 1, 1981-July 31, 1987

(French franc-based investor)

	Mean	Standard deviation
French franc	11.75	2.91
U.S. dollar	14.09	28.62
Pound sterling	9.92	22.41
Deutsche mark	12.79	9.05
Japanese yen	15.83	16.82
SDR	13.49	16.65

Correlation of total return 1/

	French franc	U.S. dollar	Pound sterling	Deutsche mark	Japanese yen	SDR
French franc	1.0					
U.S. dollar	0.54	1.0				
Pound sterling	0.21	0.42	1.0			
Deutsche mark	0.57	0.38	0.36	1.0		
Japanese yen	0.16	0.33	0.22	0.20	1.0	
SDR	0.55	0.96	0.56	0.49	0.47	1.0

Table 7. Simulation of Investments in SDR Components and in SDRs
January 1, 1981-July 31, 1987

(Deutsche mark-based investor)

	Mean	Standard deviation
Deutsche mark	6.98	2.73
U.S. dollar	8.27	27.30
Pound sterling	4.15	20.36
French franc	6.10	6.90
Japanese yen	10.13	16.54
SDR	7.66	15.02

Correlation of total return 1/

	Deutsche mark	U.S. dollar	Pound sterling	French franc	Japanese yen	SDR
Deutsche mark	1.0					
U.S. dollar	0.43	1.0				
Pound sterling	-0.02	0.35	1.0			
French franc	-0.01	0.20	-0.09	1.0		
Japanese yen	-0.14	0.28	0.12	0.23	1.0	
SDR	0.33	0.97	0.46	0.24	0.43	1.0

1/ Interest plus exchange rate return.

Table 8. Simulation of Investments in SDR Components and in SDRs
January 1, 1981-July 31, 1987

(Japanese yen-based investor)

	Mean	Standard deviation
Japanese yen	6.38	1.07
U.S. dollar	4.89	27.31
Pound sterling	1.02	23.94
French franc	3.08	16.04
Deutsche mark	3.97	17.04
SDR	4.34	16.48

Correlation of total return 1/

	Japanese yen	U.S. dollar	Pound sterling	French franc	Deutsche mark	SDR
Japanese yen	1.0					
U.S. dollar	0.28	1.0				
Pound sterling	-0.13	0.43	1.0			
French franc	-0.12	0.34	0.49	1.0		
Deutsche mark	-0.02	0.38	0.54	0.89	1.0	
SDR	0.15	0.94	0.63	0.58	0.64	1.0

Table 9. Simulation of Investments in SDR Components and in SDRs
January 1, 1981-July 31, 1987

(SDR-based investor)

	Mean	Standard deviation
SDR	8.96	2.3
U.S. dollar	9.12	14.40
Pound sterling	5.58	18.63
French franc	7.87	14.32
Deutsche mark	8.79	14.17
Japanese yen	11.74	16.29

Correlation of total return 1/

	SDR	U.S. dollar	Pound sterling	French franc	Deutsche mark	Japanese yen
SDR	1.0					
U.S. dollar	0.59	1.0				
Pound sterling	-0.11	-0.39	1.0			
French franc	-0.25	-0.75	0.14	1.0		
Deutsche mark	-0.07	-0.71	0.20	0.86	1.0	
Japanese yen	-0.29	-0.64	-0.37	0.40	0.35	1.0

1/ Interest plus exchange rate return.

Table 10. Return Per Unit Risk 1/
January 1, 1981-July 31, 1987

	Base currency					
	U.S. dollar	Pound sterling	French franc	Deutsche mark	Japanese yen	SDR
U.S. dollar	3.10	0.58	0.49	0.30	0.18	0.63
Pound sterling	0.25	6.16	0.44	0.20	0.04	0.30
French franc	0.35	0.66	4.04	0.88	0.19	0.55
Deutsche mark	0.39	0.72	1.41	2.56	0.23	0.62
Japanese yen	0.48	0.72	0.94	0.61	5.96	0.72
SDR	0.84	0.83	0.81	0.51	0.26	3.90

1/ $\frac{\text{Total return}}{\text{Standard deviation}}$

As one would expect from the results of the separate tables, the return per unit risk is uniformly highest for investments in the base currency, as the variance of the return does not have an exchange rate component. As a result, the return per unit risk is strongly "biased" in favor of investment in the base currency. Among the other investment possibilities, the last row of this table indicates that over the period covered--which was characterized by considerable exchange and interest rate volatility--the SDR has invariably outperformed the other assets, with the exception of the French franc and the deutsche mark. Given the construction of the SDR basket, and given that the exchange rate variations of its component currencies are not perfectly correlated, movements in the exchange value of any of the currencies included in the basket will tend to be partially offset by smaller or divergent movements in the other currencies, reducing therefore the overall variability of the exchange rate of the SDR in terms of the individual currencies. For example, with the U.S. dollar as the base currency, the return per unit of risk computed for the SDR is almost four times the lowest ratio, which is that for the pound sterling. This result reflects in part the relatively higher weight of the U.S. dollar in the SDR basket, which reduces the variability of the SDR exchange rate in terms of the dollar.

The different results obtained for the French franc and the deutsche mark are mostly attributable to the participation of both of these currencies in the European Monetary System. The narrow margin of exchange rate fluctuations allowed under this system limits the variability of total return in one of the currencies for an investor based in the other currency, reducing therefore the overall risk on the investment. However, the same is not true for the total return; in the period covered by the analysis, Tables 6 and 7 show that the total return on the SDR was higher for both French franc- and deutsche mark-based investor.

3. Comparison of alternative portfolios of reserve assets

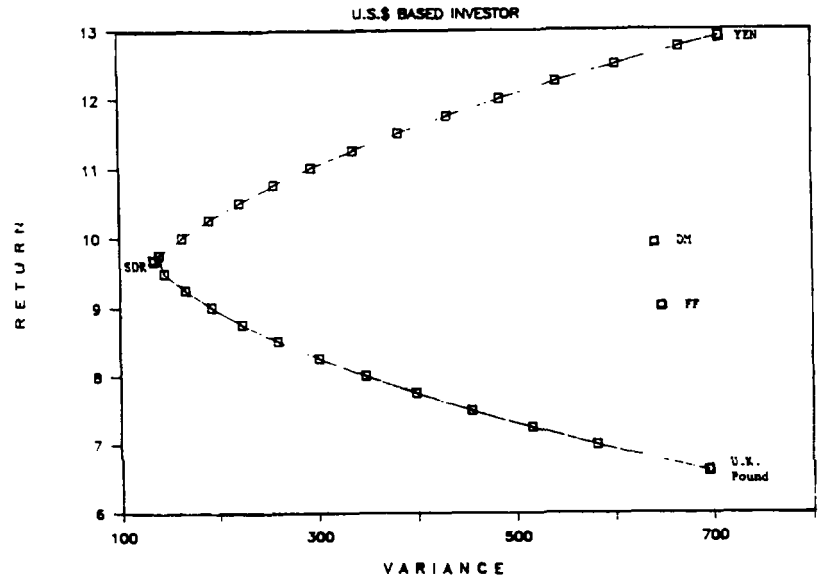
Thus far in the analysis it has been assumed that the investments in each of the currencies are mutually exclusive; and no consideration was given to the degree of correlation between the returns on the investments. In other words, it was assumed that the monetary authority allocated all of its funds to only one of the investment alternatives included in the analysis. However, one of the most important contributions of portfolio theory is that the overall risk of a stock of assets can be reduced by combining into a portfolio a number of separate instruments, or in this case, a number of currencies. 1/ In particular, the extent of risk reduction will depend on the degree of interdependence between the return on the various investments available, i.e., it will depend on the magnitude of the correlation among the returns. As long as this correlation is not perfect, i.e., the correlation coefficient is less than 1.0, the diversification of reserve assets among the various currencies will reduce the overall risk on the portfolio for a given rate of return; the lower the correlation, the greater the reduction of risk.

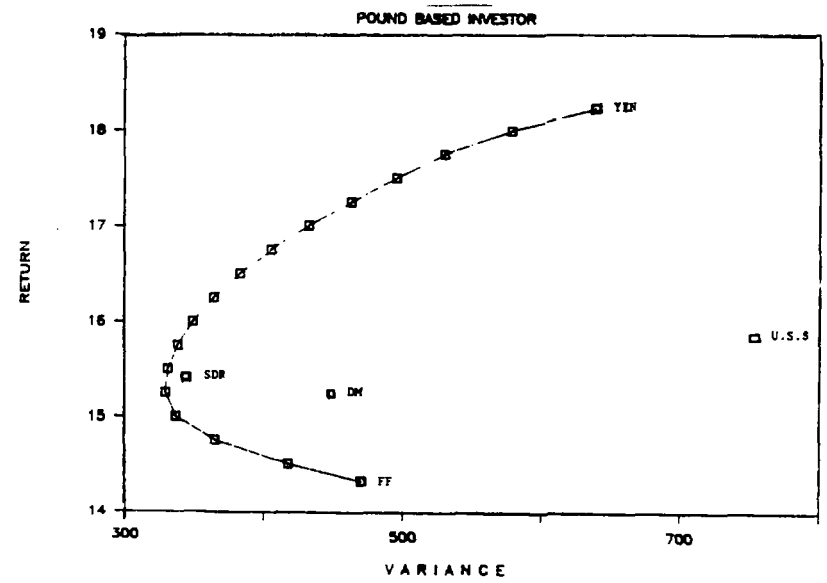
The lower panels of Tables 4 to 9 report the coefficients of correlation for the period under review among the various investment alternatives for each currency taken as a base. As can be seen from these tables, none of the investment returns are perfectly correlated, either positively or negatively. This basically means that movements in the return on the investment in one currency will tend to be partially or fully offset by smaller or divergent movements in the return of investments in the other currencies included in the portfolio, reducing therefore the overall variability for the same return on the portfolio. Taking Table 4 as a particular example of negative correlation, movements in the return on an investment in U.S. dollars will be partially or fully offset by divergent movements in the

1/ This well-known proposition of portfolio theory is described very lucidly in "Decreasing Risk by Diversification: The Portfolio Approach," Chapter 12 in Levy and Sarnat (1982).

CHART 6

EFFICIENCY FRONTIER





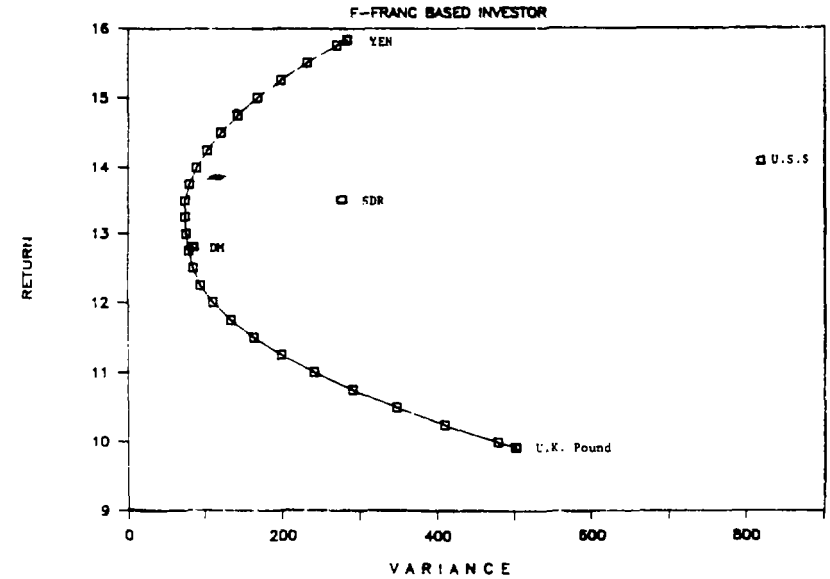
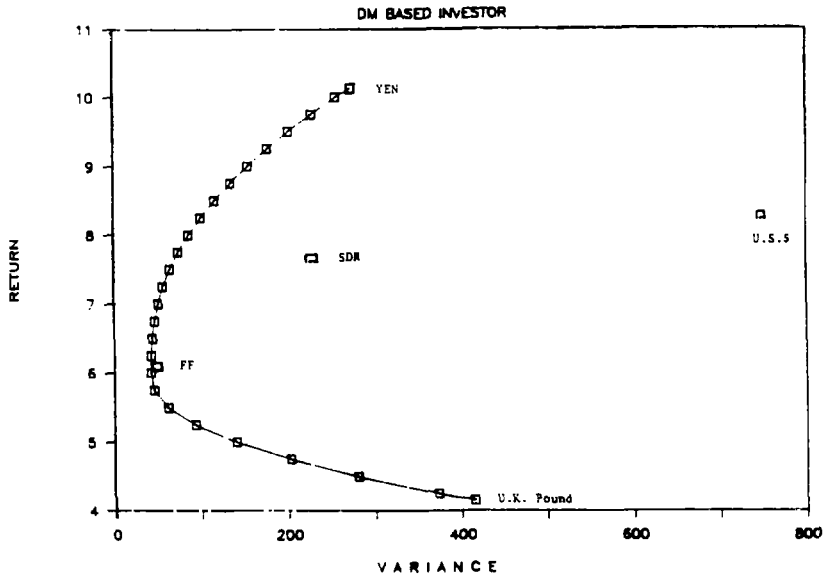
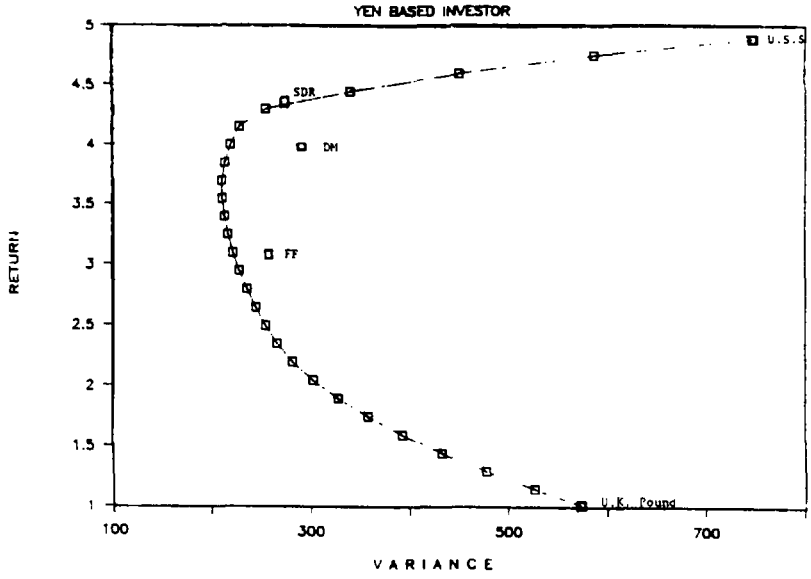
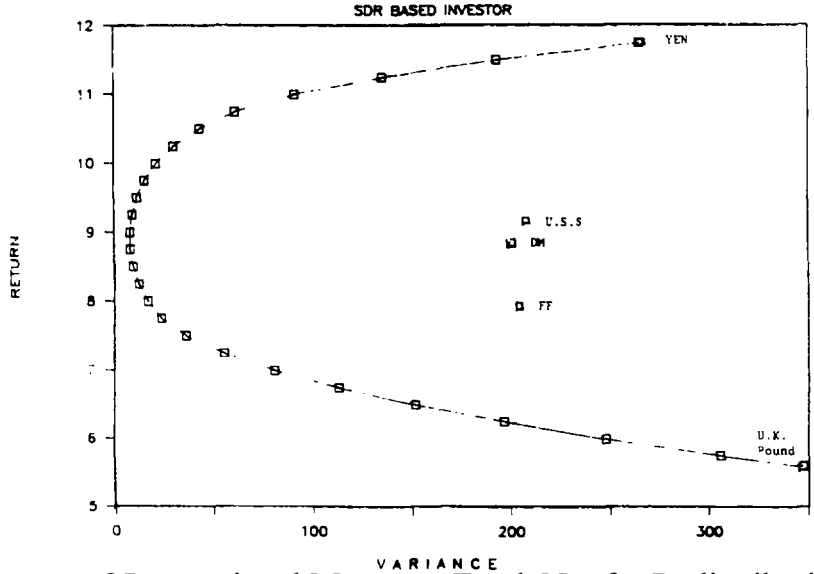


CHART 7

EFFICIENCY FRONTIER







return on investments in the other currencies, given the negative correlation existing between the return on U.S. dollar investments and the return on investments in the other currencies.

In order to quantify more precisely the extent to which a reduction in risk can be achieved by combining investments in the different currencies, a set of efficient portfolios was computed utilizing the mean-variance approach. Charts 6 and 7 show the locus of efficient portfolios for each of the currencies taken in turn as the base currency of the investor. Each point on these curves represents that combination of one or more investments in each of the currencies which minimizes the variance of the portfolio for a given level of the total return on the entire portfolio. The return (R) and variance (V) of each individual investment by itself is also plotted in the (R, V) plane. In computing the locus of efficient portfolios, the base currency was purposely excluded from the set of assets available to the reserve holder. This was done on the grounds that for the countries whose currencies are employed as reserve assets, their own currencies cannot be used by them as reserve assets, e.g., the French franc is not a feasible reserve asset for France. Results are presented below for one point on the efficiency locus--the minimum variance point--where this assumption is relaxed.

As can be seen from the charts, the results confirm the earlier findings of the generally superior performance of the SDR, in terms of risk and return, compared with its individual components. The SDR is in all cases closer to, or in some cases, on the locus of efficient portfolios, thereby providing the maximum return for a given level of risk or, alternatively, the lowest risk for a given level of return. The exceptions again are the French franc and the deutsche mark, which are extremely close to the efficiency frontier. As was noted above, this is primarily a reflection of the participation of both of these currencies in the EMS. However, in both of these cases the SDR essentially dominates two of the other remaining assets, namely, the U.S. dollar and the pound sterling, and would therefore provide the "second best" alternative for a French franc- and deutsche mark-based investor interested in diversification.

The locus of efficient portfolios, however, does not by itself specify which among the different combinations is the optimal one, i.e., the best in terms of return per unit of risk. Two different approaches could be used to identify this portfolio: the utility function approach and the capital asset pricing model (CAPM). However, neither of these approaches was utilized in this paper. The first alternative was not explored in light of the lack of consensus with

regard to the form of the utility function of central banks. 1/ The CAPM also was not applied, given the theoretical shortcomings of this model which rest on very restrictive equilibrium assumptions. 2/ In addition, a preliminary determination of the optimal portfolio in the present exercise applying the CAPM to some of the efficiency frontiers provided results which are generally similar to those of the minimum risk portfolio described below.

In light of the difficulties associated with these two approaches, it was decided to adopt the conservative assumption that the monetary authorities engaged in reserve management wish to hold the minimum risk portfolio, i.e., that combination of investments in the various currencies which yields the absolute minimum variance for the return on the portfolio of reserve assets. This minimum variance portfolio is located at the vertex of the parabola describing the efficiency frontiers in Charts 6 and 7. The composition of this portfolio appears to be particularly suitable for the analysis of the optimal composition of official foreign currency reserves, as it seems reasonable to assume that central banks are risk averse and primarily seek to preserve the value of their currency holdings.

Table 11 shows the composition of the minimum risk portfolio for each of the currencies taken in turn as the base currency. The results clearly indicate that the SDR is a substantial component in each of the minimum risk portfolios, with the exception again of the French franc and the deutsche mark. The results are particularly significant for the portfolio based in U.S. dollars, where the minimum risk is achieved by holding almost 100 percent in SDRs. As mentioned above, the U.S. dollar constituted on average almost 43 percent of the SDR basket during the period under review. Given this relatively large weight, and given that nondollar currencies in the basket tend to move in opposite direction from the dollar, an investor based in U.S. dollars would have obviously achieved greater stability by investing principally in SDR instead of investing in the other currencies.

The results are equally significant for the pound sterling-based and Japanese yen-based investor, where the SDR amount in the minimum risk portfolio is 71.8 percent and 49.2 percent, respectively. The relative weight of both of these currencies in the SDR basket is not as large as the dollar and therefore the reason for the large share of the SDR in these portfolios would not appear to be the same as for the dollar-based portfolio. It would appear that the relative stability

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1/ See Ben-Bassat (1984).
2/ See Levy and Sarnat (1982).

Table 11. Composition of the Minimum Risk Portfolio

Base currency	U.S. dollar	Pound sterling	French franc	Deutsche mark	Japanese yen	SDR	Total 1/
U.S. dollar				2.5	97.5	100	
Pound sterling			12.0	16.5		71.8	100
French franc		0.5		83.8	15.6		100
Deutsche mark		8.5	82.8		7.3	1.4	100
Japanese yen			50.8			49.2	100
SDR	42.7	11.8	16.2	9.4	19.9		100

1/ Differences from the sum of the components are due to rounding.

for a given level of return of the SDR clearly translates into a predominant role for this asset in the minimum risk portfolio based on both of these currencies.

The results obtained for the French franc and the deutsche mark are not surprising, given the participation of both of these currencies in the EMS. However, under existing EMS regulations, the participating countries are allowed to hold only working balances of other members' currencies, which means that the composition of the minimum risk portfolio suggested by this analysis will in practice not be feasible. In these two cases, given that the SDR outperforms the remaining assets in both French franc-based and deutsche mark-based portfolios, investment in the SDR basket would be a "second best" alternative and would therefore constitute the major component of a minimum risk portfolio.

The fact that the composition of the minimum-risk portfolio computed with the SDR as the numeraire has weights similar to those in the SDR basket is not surprising. Intuitively, a reserve holder that measures the return on its portfolio in SDR terms would be able to eliminate exchange rate risk by simply allocating the investment in each of the five currencies in accordance with the actual composition of the SDR basket. To the extent that the weights are precisely chosen, the variance of the total return would be equal to that

generated only by interest rate fluctuations. In this case, the SDR or a linear combination of the five currencies, would behave close to a riskless asset.

Finally, it should be emphasized that these results were obtained by imposing on the risk-minimization computation a non-negativity constraint. In other words, it was assumed that central banks did not borrow foreign currencies. The consequence is that the minimum-risk portfolio is not very diversified. However, relaxation of the non-negativity constraint would extend the analysis to that of external debt management, which would take it beyond the scope of this paper.

The analysis above has in fact focused only on the investment function of reserve assets and in particular on the determination of the most efficient portfolio composition which will yield a stock of reserves with the lowest possible risk. The asset/liability structure of the central bank or country was not taken into account in the determination of the minimum-variance portfolio. The implicit assumption is that the central bank does not hold any liability denominated in foreign currency. Clearly, if this assumption is relaxed, a more efficient strategy would be to match to the extent possible the liabilities with a similar asset. While there would be opportunity costs and liquidity constraints, the exchange rate exposure arising from the fluctuation in the value of the liability expressed in a specific currency would be entirely covered by the similar but offsetting fluctuation in the value of the asset also expressed in the same currency. For example, if a country has SDR liabilities, the best strategy would be to match them with an asset that perfectly tracks the value of the SDR in foreign exchange markets, i.e., the SDR itself.

4. The choice of the base currency

As was noted above in the discussion of the results, the choice of the base currency in which the portfolio of reserve assets is expressed has a major influence on the composition of the minimum risk portfolio. While the relative returns in the different currencies will not be affected by the choice of the numeraire, the variance and the composition of the minimum risk portfolio will change. In the above analysis the base currency was purposely dropped from the array of available assets. This was done on the basis of the assumption that the base currency was the home currency of the central bank, and therefore by definition could not be considered a reserve asset. For example, from the point of view of the United States, U.S. dollar-denominated instruments are not feasible as reserve assets. However, this approach--although consistent--is strictly speaking applicable only to the G-5 countries.

Table 12. Composition of the Minimum Risk Portfolio

(Including the base currency)

	U.S. dollar	Pound sterling	French franc	Deutsche mark	Japanese yen	SDR
U.S. dollar	87.2	--	--	--	2.0	10.8
Pound sterling	--	97.0	1.1	--	1.9	--
French franc	--	--	100	--	--	--
Deutsche mark	--	0.7	9.4	85.7	4.2	--
Japanese yen	--	00	00	00	100	--
SDR	42.7	11.8	16.2	9.4	19.9	

In order to broaden the analysis and generalize the results, a different set of efficiency frontiers was computed that included the base currency in the array of feasible reserve assets. In this case all five currencies are viable reserve assets that can be used to support the home currency in the event of excess demand for foreign exchange. As can be seen from Table 12, the resulting minimum risk portfolios are quite different from those in Table 11 and are characterized by an extremely low level of diversification.

The base currency plays a major role in the minimum risk portfolio as it behaves almost as a riskless asset. This outcome is very much in line with what could be expected, given that the total variance of the return on the base currency instrument is in fact determined only by fluctuations in the interest rate which are much smaller and less frequent than movements in the total return of the other reserve assets which also include exchange rate variations. Therefore, as noted above, the variance of the return on the base currency instrument will be considerably lower than the variances of the total returns on the remaining assets. As a consequence, in all of those cases in which the base currency also has the highest return, the minimum risk portfolio will be completely undiversified and will be composed entirely of the base currency. This is in fact the case for the Japanese yen- and French franc-based portfolios, as in both cases the instrument denominated in that currency has the lowest variance and highest return

and therefore the minimum risk portfolio is composed entirely of yen and French franc, respectively.

It should also be noted that the results given in Table 12 were derived on the basis of the simplifying assumption that the optimal portfolio was equivalent to the minimum-risk point on the efficiency locus. In this case, if the criterion constraint is the minimum variance possible and if the base currency empirically behaves as essentially a riskless asset, its weight in the optimal portfolio will be, by construction, the largest one. If, however, other criteria for the selection of the optimal portfolio would have been used, namely the utility approach or capital asset pricing model, the results would have been different, yielding in particular a more diversified portfolio.

Nonetheless, it is noteworthy that for the dollar-based portfolio, although the U.S. currency is the major component, the SDR weight is nonetheless somewhat more than 10 percent. These results underline the relatively strong performance of the basket compared with the remaining individual currencies. For a dollar-based central bank, i.e., that of a country whose international transactions are dominated by U.S. dollar-denominated transactions, this result suggests that a share of SDRs in the order of 10 percent would result in the lowest variance of the portfolio of reserve assets of the central bank. Such a figure is fairly high when compared with the share of SDRs in total non-gold reserves of Fund members equal to 4 percent at the end of 1987.

The results in Tables 11 and 12 highlight the fact that computations of shares of reserve assets using the mean-variance technique are quite sensitive to the choice of the base currency. An extension of the work of this paper would involve the computation of the share of the SDR in a minimum variance portfolio where the base currency is calculated using the currency composition of a country's international transactions. As explained in Racine (1988), such a base currency is properly measured as a weighted average of the currencies associated with a country's current account payments.

V. Concluding Remarks

The substantial fluctuations in exchange rates between the major currencies over the last ten years have exposed central banks to considerable swings in the value of their foreign exchange reserves. This paper has argued that diversification of reserves across currencies will tend to reduce the level of risk of holding foreign exchange reserves. Such a reduction in risk will occur as long as the returns on the various assets included in the portfolio are not

perfectly correlated, so that movements in the return on one asset will be partially or fully offset by smaller or divergent movements in the returns on the other assets.

Such a reduction in risk can be achieved by investing in instruments denominated in the currencies that are the components of the SDR. However, the SDR is itself a portfolio composed of five currencies each with its own weight, return and variance, and in this paper the SDR was treated as an independent asset with an associated total return (interest plus exchange rate variation) and variance. Consequently, the beneficial effects of diversification can be applied directly to the SDR. As long as the component currencies of the SDR are not perfectly correlated, the movement in the total return of one component currency will be partially or fully offset by smaller or divergent movements in the remaining currencies. In other words, the variance of the SDR will always be lower than the weighted average of the variances of the component currencies in the basket. This implies that holding reserves in the form of SDRs will tend to be subject to less exchange rate risk than reserves held in only one reserve currency.

The empirical results in this paper have shown that because of this property, the risk-adjusted rate of return on the SDR since 1981 has tended to be above that on the component instruments. In the examination of efficient portfolios of reserve assets, where the SDR was included along with its component instruments, it was found that the SDR was often a major component of portfolios that had the lowest variance in the return. Thus the evidence presented in this paper would suggest that the SDR can play an important role in the international reserve portfolios of central banks by reducing the exposure of these reserves to exchange rate risk.

One question that arises in connection with portfolio diversification relates to the comparison of the official SDR and combinations of reserve currencies available in financial markets. As SDR assets are available in private financial markets, e.g., SDR deposits, and as reserve holders can create an SDR basket by holding the component instruments separately, the ability of the official SDR to compete with these private substitutes is an important question. A number of considerations suggest, however, that the official SDR may be rather attractive as a means of diversifying reserve portfolios. First, as noted above, the SDR pays essentially a three-month interest rate, yet has the maturity associated with a two- or three-day notice deposit. Second, the transactions costs of dealing with only one ready-made composite asset already on the books of the Fund would probably be lower than creating a similar synthetic asset in the private market. Third, many Fund members have large, regular payment obligations to the Fund, and holding a fraction of their reserve assets in this form could

further reduce the costs associated with these transactions as well as reduce exposure to exchange rate risk. Finally, diversification of reserve assets through greater use of official SDRs could reduce the variability of exchange rates as some of the switches between currencies would be done "off-market."

The Variance of the SDR Relative to the Variance of a Weighted Average of its Components

The variance of the total return of the SDR expressed in a specific currency will always be lower or equal to the weighted average of the variances of returns on instruments in the basket expressed in the same currency.

$$(1) \quad \mathrm{Var(SDR)} \leq \Sigma_i \, w_i \, \mathrm{Var}(s_i)$$

where:

$\mathrm{Var(SDR)}$ = variance of the SDR interest rate
w_i = weight of currency i in the basket
$\mathrm{Var}(s_i)$ = variance of the rate of return on the instrument in currency i

The variance of the SDR is equal to:

$$(2) \quad \mathrm{Var(SDR)} = \Sigma_i \, w_i^2 \, \mathrm{Var}(s_i) + 2 \, \Sigma_i \, \Sigma_j \, w_i \, w_j \, \mathrm{Cov}_{i,j}$$

Equation (2) permits equation (1) to be rewritten as:

$$(3) \quad \Sigma_i \, w_i^2 \, \mathrm{Var}(s_i) + 2 \, \Sigma_i \, \Sigma_j \, w_i \, w_j \, \mathrm{Cov}_{i,j} \leq \Sigma_i \, w_i \, \mathrm{Var}(s_i)$$

From the definition of the correlation coefficient, we have:

$$(4) \quad \rho_{i,j} = \frac{\mathrm{Cov}_{i,j}}{\sigma i \, \sigma j}$$

and

$$(5) \quad \rho_{i,j} \leq 1$$

From equations (4) and (5) it follows directly, that:

$$(6) \quad 1 \geq \frac{\mathrm{Cov}_{i,j}}{\sigma i \, \sigma j}$$

or equivalently:

$$(7) \quad \mathrm{Cov}_{i,j} \leq \sigma i \sigma j$$

Substituting the right-hand side of (7) for the covariance in (3) gives:

$$(8) \quad \Sigma_i \, w_i^2 \, \mathrm{Var}(s_i) + \Sigma_i \, \Sigma_j \, w_i \, w_j \, \sigma_i \, \sigma_j \leq \Sigma_i \, w_i \, \mathrm{Var}(s_i)$$

This can be expressed as:

$$(8A)\quad \Sigma_i\, w_i^2\, Var(s_i) + 2\,\Sigma_i\,\Sigma_j\, w_i\, w_j \sqrt{Var(s_i)}\,\sqrt{Var(s_j)} \leq \Sigma_i\, w_i\, Var(s_i)$$

The next step involves the following property of sums, where d_i be any particular number:

$$[\Sigma_i\, d_i]^2 = \Sigma_i\, d_i^2 + 2\,\Sigma_i\,\Sigma_j\, d_i\, d_j \qquad (i \neq j)$$

Using this property and letting $d_i = w_i\,\sqrt{Var(s_i)}$, we can rewrite inequality (8A) as:

$$(9)\quad [\Sigma_i\, w_i\, \sqrt{Var(s_i)}]^2 \leq \Sigma_i\, w_i\, Var(s_i)$$

Taking the square root of both sides of (9) gives:

$$(10)\quad \Sigma_i\, w_i \sqrt{Var(s_i)} \leq \sqrt{\Sigma_i\, w_i\, Var(s_i)}$$

Now, remembering that

$$\sqrt{a + b} \leq \sqrt{a} + \sqrt{b} \quad \text{if } a,b \geq 0$$

inequality (10) can be rewritten as

$$(11)\quad \Sigma_i\, w_i \sqrt{Var(s_i)} \leq \Sigma_i \sqrt{w_i}\,\sqrt{Var(s_i)}$$

which is equal to

$$(12)\quad \Sigma_i\, w_i \leq \Sigma \sqrt{w_i}$$

and given that in this particular case:

$$w_i < 1$$

and that

$$w_i < w_i$$

inequality (10) is proven, which, by construction also proves inequality (8) and, finally, inequality (3).

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