IMF *Staff Papers*
Vol. 44, No. 4 (December 1997)

Market Information and Signaling in Central Bank Operations, or, How Often Should a Central Bank Intervene?

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A central bank must decide on the frequency with which it will conduct open market operations and the variability in short-term money market that it will allow. The paper shows how the optimal operating procedure balances the value of attaining an immediate target and broadcasting the central bank's intentions against the informational advantages to the central bank of allowing the free play of market forces to reveal more of the information available to market participants. [JEL E52, G14]

IN ALMOST ALL industrialized countries, the central bank implements policy by setting a proximate operational target for short-term money market interest rates and then using open market operations and other instruments such as standing facilities to keep rates near the target. An increasing number of developing and transition economies have developed similar operating procedures in recent years. Viewed more closely, however, practices vary widely. In particular, central banks differ significantly in the frequency with which they intervene in financial markets and the precision with which they steer market rates. The U.S. Federal Reserve and the Bank of England typically deal at least once a day with commercial banks in order to absorb or inject liquidity. In this way, these central banks largely determine certain short-term interest rates, with the Bank of England normally allowing rather greater fluctuations in very-short-term money

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market rates. In contrast, the Deutsche Bundesbank usually intervenes only through weekly repurchase tenders, resorting to ad hoc "Schnelltender" just a few times a year. Between tenders, the Bundesbank is prepared to see at least moderate day-to-day fluctuations in market rates around its central operational target. even though it is technically capable of eliminating them.

The operating procedures used by individual central banks have also evolved over time. The Bundesbank, for example, announced in 1985 that it would begin emphasizing repurchase operations in the implementation of policy; the central bank's other instruments, namely, the standing facilities and reserve requirements, were not changed concurrently. Since then, the volume transacted in the repurchase tender has increased, the tender mechanism has been refined in various ways, and the schedule of tenders made more regular (see Deutsche Bundesbank, 1995).

Given this variety and change, one may be prompted to ask what general considerations should enter into the choice of operating procedure, and in particular how precisely a central bank should attempt to achieve its immediate operational target for the interest rate. At one extreme, if the central bank knows its objective and faces no technical obstacles, why should it not intervene enough to achieve its operational target continuously? In terms of marginal decisions, what are the advantages and disadvantages of intervening, say, twice a week rather than weekly and reducing short-term interest rate fluctuations correspondingly?

This paper attempts to provide a (partial) answer to such questions by discussing a number of factors affecting both the costs and the benefits of more frequent intervention. First, some familiar arguments based on the relationship between open market operations, the other instruments available to the central bank, and the clearing and payment system will be reviewed. Then it is argued that the design of instruments can have an important effect on the information content of market prices, which both the central bank and market participants use in devising their strategies. The starting point is the observation that the central bank does not have a precise, optimal operational target set in advance (as is often assumed in the economic literature). Instead, it must actively search for and analyze information, including that contained in market prices, before deciding on its short-term strategy and estimating an appropriate operational target (and in practice central banks do indeed monitor market developments closely). It will be shown how by not intervening and thereby allowing market rates to fluctuate freely, a central bank may be able to acquire valuable information not otherwise available to it. Limiting intervention provides the central bank with more occasion to observe undistorted market trends, and therefore to acquire a clearer impression of the information available to market

participants. Some variability around the central bank's current operational target may facilitate the estimation of a better target. It will further be shown how market participants will be able to make inferences about the policy stance and other information available to the central bank from the frequency and strength of intervention. Optimally designed instruments and operating procedures will respect the need for clarity in the transmission of signals to and from the central bank and the value of information contained in market prices.

The discussion will be framed in terms of the operating procedures currently employed by central banks in most industrialized countries to implement domestic monetary policy. The central bank will be taken to have some medium-term target (say, for M3 growth, the exchange rate, or the inflation rate itself), which it attempts to achieve by appropriately steering short-term money market rates. The operational target for the short-term interest rate will depend not only on the medium-term target, but also on the information available to the central bank through different channels and its understanding of the transmission process. Since the interbank money market is constantly disturbed by inflows and outflows of liquidity and by changes in expectations, in the absence of intervention, random deviations from the central bank's operational target are inevitable. Hence, the central bank has to actively manage short-term rates, primarily through open market operations using either a formal tender or over-the-counter trading and perhaps based on repurchase agreements. The arguments of this paper are presented in the context of such an operating procedure, but they can be reformulated to address the case in which the central bank sets its operational target in terms of a quantity variable. The arguments can also be applied to economies in many stages of development, and may indeed be especially forceful in economies where financial markets are still being established and the central bank has limited capacity to collect and analyze economic data. Analogous arguments applying to the implementation of exchange rate policy can easily be formulated.

Related issues arise in the determination of longer-term policy. Edwards (1981) and Dotsey and King (1986) investigate at a macroeconomic level the connection between monetary policy and incentives for information acquisition. Recently a number of authors have applied the theory of optimal control with learning (as in Easly and Kiefer, 1988, for example) to macroeconomic policy questions (see Balvers and Cosimano, 1994; Bertocchi and Spagat, 1993; and Wieland, 1996, for example), and generally find that policymakers should allow greater fluctuations in target variables so as to improve their estimates of structural parameters. Bindseil (1997), who deals with reserve requirements, takes an approach related to that of this paper.

I. Financial Market Institutions and Central Bank Instruments

Most discussion of the optimal operating procedures to be used in the short-term implementation of monetary policy deals with effectiveness in achieving a given operational target, the strength and nature of the disturbances affecting the financial system, and the possible side effects of different instruments. Under this conventional approach, the marginal benefit of more frequent intervention can be measured in terms of reduced deviation from the operational target and possible strengthening of the transmission process, whereby monetary policy impulses radiate out into the economy at large.

Differences between central banks in the frequency of intervention can, in the first instance, be explained by differences in financial market institutions and the central bank's instruments. These factors help determine the volatility of money market interest rates in the absence of intervention, and market reaction to such intervention. Of course, how much volatility in money market rates should be permitted is itself a normative issue, so institutional differences cannot provide a complete explanation.

In countries where the payment clearing and settlement system is relatively unsophisticated and generates large, unpredictable swings in commercial bank's deposits with the central bank, the central bank may have to intervene frequently to inject or withdraw liquidity, if it wishes to forestall erratic movements in short-term interest rates. The design of the payment system (netting arrangements, time lags, the interval between settlement, the role of private clearing houses, the availability of overdrafts, etc.) may affect the sensitivity of market rates to given liquidity disturbances and may therefore influence the decision to intervene more or less often. Likewise, relatively frequent intervention may be called for when commercial banks have little expertise in, and opportunity for, liquidity management, although the central bank also needs to give banks enough incentive to develop the relevant skills. When the government keeps its funds at the central bank, as is the practice in most countries, variations in its receipts and outlays may warrant even daily offsetting injection and withdrawal of liquidity by the central bank.

Many central banks have instituted standing facilities that limit interest rate fluctuations, such as a deposit facility that sets a floor on rates and a rediscount or Lombard facility that sets a ceiling. These central banks can then afford less frequent discretionary intervention for a given level of interest rate volatility because market participants will access the facilities at their own initiative, which will have a stabilizing influence on prices. Open market operations are still undertaken more or less regularly, so the question of how frequently to intervene must still be answered, as well as the analogous question of how wide to set the interest rate "band."

When commercial banks are subject to reserve requirements that are higher than the working balances they would hold voluntarily and that have to be met on an average basis, a large buffer stock is available to absorb short-term liquidity shocks before money market rates are affected significantly. Therefore, less intervention is needed to maintain reasonably low interest rate volatility. In contrast, central banks that do not impose binding average reserve requirements on commercial banks may in contrast be motivated to intervene frequently. In the United Kingdom, for example, reserve requirements are insignificant and commercial banks hold very low working balances. The Bank of England sometimes intervenes more than once a day to offset even small exogenous fluctuations in bank liquidity that might otherwise generate large disturbances in money market rates.[1] It is, however, possible that if intervention were not so frequent, banks would learn to hold larger working balances, and private institutions would find it profitable to act in a stabilizing fashion. Furthermore, even if the marginal benefits of more frequent intervention are greater, the lower are reserve requirements, one also needs to consider the possible marginal costs. A low reserve requirement is not justification in itself for more frequent intervention.

II. Intervention and Information Inference by the Central Bank

In the previous section, it was assumed that the central bank determines an immediate operational target for the short-term interest rate in an autonomous process unconnected to the implementation of policy. Most academic literature starts from the same premise. In reality it is very difficult for central banks to determine appropriate operational targets for money market interest rates. Even when a medium-term objective such as price stability is generally accepted and an intermediate target, say for money stock growth, has been established, a great deal of information gathering, economic analysis, and indeed intuition goes into setting the operational targets from month to month, week to week, or day to day. The difficulty lies not so much in effective implementation as in deciding on the short-term aim. This short-term aim must then be communicated to market participants in a credible manner. In this paper it will be argued that the optimal intervention frequency balances the benefit of limiting deviations from an operational target once chosen and broadcasting the central bank's intentions, against the costs of obscuring information useful in estimating the best target.

[1] See King (1994), especially page 69, for a statement of this approach.

Market Information and Central Bank Information

Central banks have at their disposal large amounts of data on which to base monetary policy implementation decisions, but they do not know everything of relevance. Much information, especially on disaggregate developments, will originally be spread thinly across the economy, perhaps kept confidential by various economic agents, or compiled only with a significant lag. Such information might be aggregated in market prices but not captured in official statistics or reports. To reflect this dichotomy, suppose that in each period t the information of relevance to the implementation of short-term monetary policy can be captured by two state variables, denoted by x_t and y_t. These state variables might represent such factors as the strength of credit demand, fluctuations in the liquidity position of banks and other economic agents, disturbances from abroad, and news from the corporate sector. The state variable x_t represents relevant information that is not immediately and directly available to the central bank but is at the disposal of market participants. The state variable y_t represents the information directly available to the central bank at the start of the period. One could also interpret y_t, but not x_t, as representing shifts in the central bank's preferences and longer-term strategy.

In general, these state variables might follow any time-series process, but to illustrate the argument it will be assumed here that their levels each period evolve according to

$$x_t = \alpha x_{t-1} + e_{xt}, \quad |\alpha| < 1 \tag{1a}$$

$$y_t = \beta y_{t-1} + e_{yt}, \quad |\beta| < 1, \tag{1b}$$

where e_{xt} and e_{yt} are two identically and independently distributed (i.i.d.) random variables with mean zero and variances of σ_x^2 and σ_y^2, respectively.

The central bank implements policy by attempting to keep the price of funds in the money market, that is, the short-term money market interest rate denoted by P_t, as close as possible to an operational target that depends on these state variables. (One could reinterpret P_t as an item from the central bank's balance sheet, such as reserve money, that the central bank targets in order to implement longer-term monetary policy.) For simplicity, it will be assumed that the central bank has a quadratic loss function such that it aims to minimize expected sum of squared deviations in each period:

$$E\left\{\sum_{i=0}^{\infty} E[(P_{t+i} - x_{t+i} - y_{t+i})^2 | \Omega_{t+i}] | \Omega_t\right\}, \tag{2}$$

where E is the expectations operator, Ω_{t+i} is the information set of the central bank in period $t + i$, which, as discussed below, depends in part on its

operating procedures, and for convenience the rate of time discount has been set to zero.[2] If the central bank in fact knew both x_t and y_t every period, it would face no technical difficulty in intervening continuously to set the price and preempt any deviations from its operational target.[3] In determining its operating procedure, the central bank needs to decide whether to intervene every $N = 0, 1, 2, \ldots$ days so as to minimize the expression (2). The central bank can commit itself to following such a rule.

When in some period the central bank does not intervene, the evolution of the money market price is determined entirely by the information available to market participants, and so will be assumed to be described by

$$P_t = \alpha P_{t-1} + e_{xt}. \tag{3}$$

Thus, left undisturbed the money market equilibrium yields changes in the price of funds that perfectly reflect the current innovation in the state variable x_t but (without loss of generality) not that of y_t.[4]

Suppose that the central bank never intervenes. Then one can easily show that in each period the unconditional (average) expected loss is

$$V_0 = E\left[(P_t - x_t - y_t)^2\right] = E\left[\left(\sum_{i=0}^{\infty} \beta^i e_{yt-i}\right)^2\right] = \frac{\sigma_y^2}{1-\beta^2}. \tag{4}$$

In a period when the central bank does intervene, it can no longer observe an undisturbed market price. Instead, it receives at the start of the period a signal Δs_t that imperfectly reflects innovations in the state variable x_t:

$$\Delta s_t = e_{xt} + e_{st}, \tag{5}$$

where e_{st} is another i.i.d. random variable with mean zero and variance σ_s^2. This signal might be contained in the tenders and quotes received by the central bank when it initiates open market operations. The assertion that central bank intervention does not reveal information as well as does a freely functioning market is a somewhat ad hoc assumption, but nonetheless plausible: when the central bank conducts open market operations such as a "Schnelltender," it normally deals with only a relatively small number of money center banks, which may have incentives to behave strategically,

[2]Introducing a positive time discount rate would not affect the results significantly.

[3]This objective function with a partially unknown (operational) target can be thought of as capturing model uncertainty, as in the literature on optimal control with learning.

[4]The incentive for market participants to acquire the relevant information and how it comes to be reflected in market prices will not be discussed here, but the issue is addressed in a companion paper (Hardy, 1997), where it is shown that central bank operating procedures can affect the relative profitability of investing in the acquisition of different sorts of information, and thus the equilibrium structure and informational efficiency of financial markets.

and information available to other institutions is neglected; when the central bank organizes a tender with wide participation, the tender procedure may not induce complete revelation, and the repeated *tâtonnement* process of the market cannot function. The bidding in a quantity tender when the central bank fixes the price in advance may be especially difficult to interpret. The volume of central bank intervention can certainly contain important information on market conditions, but that information is likely to be obscured by various disturbances such as fluctuations in the quantity of cash in the hands of the public. Insofar as money fulfills a "buffer stock" function, the quantity demanded may be very interest-rate elastic and vary greatly because of transitory phenomena that are not relevant to the formulation of the central bank's operational targets.[5] In any case, the argument that follows provides sufficient, albeit not necessary, conditions for infrequent intervention to be optimal.

If intervention is conducted every period, the central bank's problem reduces the minimization of its expected loss function period by period. Its information set at time t contains only the series of signals Δs_{t-i}, $i = 0, 1, \ldots$ besides the state variable y_t. (Recall that by assumption the central bank has direct information on y_t but not on x_t.) The central bank can do no better than set the money market price as a linear function of the history of these signals and y_t broken down into its autocorrelation and innovation components:

$$P_{1t} = \sum_{i=0}^{\infty} a_{1i}\Delta s_{t-i} + c_1\beta y_{t-1} + d_1 e_{yt}. \tag{6}$$

The parameters a_{1i}, c_1, and d_1 are chosen to minimize the average loss function in each period. As shown in the Appendix, at the optimum $c_1 = d_1 = 1$, a_{1i} depends on the relative variances, and the minimized loss function V_1^* is

$$V_1^* = \frac{1}{1-\alpha^2}\;\frac{\sigma_x^2\sigma_s^2}{\sigma_x^2 + \sigma_s^2}. \tag{7}$$

The central bank will prefer to intervene every period rather than not at all if the minimized average expected loss in the former case (V_1^*) is less than that under the latter regime (V_0 given in equation (4)). From an examination of the partial derivatives of the difference ($V_1^* - V_0$) it is clear that this condition is more likely to be met the smaller is α and σ_x^2 and the larger is β and σ_y^2. Then fluctuations in the state variable known only to market participants are relatively small compared to those of the state variable

[5]With a reserve requirement, at the end of the maintenance period demand for reserve money may become very interest-rate inelastic; overnight money market rates will tend to move up or down violently in response to a small aggregate reserve shortfall or surplus. Again, such fluctuations may not contain much information of interest to the central bank.

observed by the central bank, so it is more important to keep the money market rate in line with the latter than with the former. The condition is also more likely to be met the smaller is σ_s^2, because then the central bank can better infer the information available only to market participants when it does intervene.

Suppose instead that intervention is conducted regularly every other period. Under this operating rule the central bank has only half the occasion to set the money market price as it wishes, but gains occasion to observe the uncluttered free market price and thus receive additional information on the evolution of the state variable x_t. In particular, on a nonintervention day $t-1$ it observes P_{t-1}, which allows it to infer e_{xt-1} exactly, since from equation (3) $P_{t-1} = \alpha P_{t-2} + e_{xt-1}$, and α and P_{t-2} are known. On the subsequent intervention day t, its information set thus consists of $\{y_t, \Delta s_t, e_{xt-1}, \Delta s_{t-2}, \ldots\}$. The optimal money market price can depend only on the available information variables:

$$P_{2t} = \sum_{i=0}^{\infty} (a_{2i}\Delta s_{t-2i} + b_{2i+1}e_{xt-2i-1}) + c_2\beta y_{t-1} + d_2 e_{yt}. \tag{8}$$

Moreover, the central bank is aware that its intervention in period t will affect the market price in the nonintervention period $t+1$. At $t+2$ it will intervene again and so have a chance to correct deviations from the operational target that arose in $t+1$. Therefore, its problem is to choose the parameters a_{2i}, b_{2i+1}, c_2, and d_2 so as to minimize the expected squared deviation of the actual price from its operational target averaged over periods t and $t+1$, based on current information and knowledge of the history of the state variables:

$$V_2 = (1/2)E[(P_t - x_t - y_t)^2 + (P_{t+1} - x_{t+1} - y_{t+1})^2 [y_t, \Delta s_t, e_{xt-1}, \ldots]. \tag{9}$$

The solution is derived in the Appendix. The parameters c_2 and d_2 are less than or greater than unity depending on whether α is greater than or less than β, respectively; in this way deviations of the money market "price" from the state variable y are evened out between periods t and $t+1$. The minimized average loss function is

$$V_2^* = \tfrac{1}{2}\left[\frac{1}{1-\alpha^2}\frac{\sigma_x^2\sigma_s^2}{\sigma_s^2+\sigma_s^2} + \frac{(\alpha-\beta)^2}{1+\alpha^2}\frac{\sigma_y^2}{1-\beta^2}\right]. \tag{10}$$

The central bank would do better by intervening every other period rather than every period if the difference in the minimized expected loss functions,

$$V_2^* - V_1^* = \tfrac{1}{2}\left[\frac{(\alpha-\beta)^2}{1+\alpha^2}\frac{\sigma_y^2}{1-\beta^2} - \frac{1}{1-\alpha^2}\frac{\sigma_x^2\sigma_s^2}{\sigma_x^2+\sigma_s^2}\right], \tag{11}$$

is negative. In general, the sign of this difference is ambiguous, but one can determine its partial derivatives with respect to the various parameters in order to assess when it is likely to be larger or smaller. The difference is positively related to σ_v^2, so that if variance in the component of the operational target determined by the central bank's own information is large enough or if deviations are costly enough, intervention every day is superior to intervention every other day. It is negatively related to σ_x^2 and σ_s^2 for the reasons given above related to the importance of the state variable x_t and the ability of the central bank to make inferences about it based either on free or "intervened" market prices. Frequent intervention can thus be counterproductive if the signal received in the course of intervention is sufficiently poor compared to that available in undisturbed market prices, and both are sufficiently important relative to the central bank's own information and preferences. A larger value of α (the autocorrelation coefficient for the state variable x) implies that the market price and the operational objective evolve more similarly during the nonintervention days. A larger α also entails that the central bank's errors in estimating e_{xt} in any one period are largely carried forward into subsequent periods, so it is more valuable to learn its true value by not intervening. Therefore, less frequent intervention is more likely to be optimal, the larger is α. A larger β implies that changes in the state variable y_t are more persistent; since in the absence of intervention the market price does not reflect these follow-on changes, frequent intervention is more likely to be desirable, the larger is β.

If the variances of the random variables, their autocorrelations, and the weight attached to them in the central bank's objective function differ greatly, it could be worthwhile to intervene every third period or even less frequently. If the central bank intervenes once every N days, it would set a price according to the rule

$$P_{Nt} = \sum_{i=0}^{\infty}\left(a_{Ni}\Delta s_{t-Ni} + \sum_{j=1}^{N-1} b_{Ni+j}e_{xt-2N-j}\right) + c_N\beta y_{t-1} + d_N e_{yt}. \qquad (12)$$

It is straightforward, if laborious, to determine the optimal pricing rule and the minimized average expected loss from intervening every $N = 3, 4, \ldots$ days and confirm that as N tends to infinity, the optimized V function converges back to the level obtained for $N = 0$ given in equation (4).

The incentive for limiting intervention could be reduced if the undisturbed market price did not fully and solely reflect innovations in the state variable, that is, if $(P_t - P_{t-1})$ was affected by "noise" from short-term disturbances that are irrelevant for monetary policy purposes. However, under certain circumstances the reverse could occur: in the presence of such "noise" the central bank may require a longer sample of undisturbed market prices to be able to distinguish longer-term developments that are rele-

vant for policy purposes from short-term volatility.[6] The introduction of an irrelevant "noise" term in the signal y_t received directly by the central bank would, however, increase the incentive to limit intervention; market prices and information from other sources may need to be combined to filter out what is of true importance. One could also include an error term in the central bank's operating rule to capture the possibility that it does not always extract information fully efficiently or indeed know its own preferences exactly.

Ad Hoc Intervention

So far it has been assumed that the central bank commits itself to a rule to intervene every given number of days and sets the parameters for that rule in advance. However, it could be that the central bank cannot so commit itself or chooses not to. In particular, on any particular day the central bank may determine whether or not to intervene only after it has received information on the innovation in the state variable y_t, and then must solve what amounts to an "optimal stopping rule" problem of when to intervene and when to observe unintervened prices so as to improve its information set. In general, the best choice for each period is likely to depend in a complex way on expectations of future intervention and on the history of past intervention, which affects the current information set.

To illustrate considerations that go into this choice, assume that the market price in period $t-1$ is known to be exactly at the operational target, that is, $P_{t-1} = x_{t-1} + y_{t-1}$. The central bank then learns the realization of $y_t = \beta y_{t-1} + e_{yt}$ and has to decide whether or not to intervene. If it does not intervene, the market price evolves to $P_t = \alpha P_{t-1} + e_{xt}$, and so the immediate loss is

$$\begin{aligned} V_0' &= (P_t - x_t - y_t)^2 = (\alpha P_{t-1} + e_{xt} - x_t - y_t)^2 \\ &= (\alpha(x_{t-1} + y_{t-1}) + e_{xt} - x_t - y_t)^2 = (\alpha y_{t-1} - y_t)^2, \end{aligned} \tag{13}$$

where equation (1a) has been used to simplify. By not intervening, the central bank can observe the realization of e_{xt}, which is relevant to the decision

[6]This possibility can arise if the market price evolves according to the rule

$$P_t = \alpha P_{t-1} + e_{xt} + e_{nt} - \gamma e_{nt-1},$$

where e_{nt} is an i.i.d. disturbance term that is irrelevant to the choice of the operational target, provided that γ is sufficiently large that the negative moving average component offsets the autocorrelation component.

of whether or not to intervene in $t+1$. The loss from not intervening must be at least V_0 and will be greater by an amount that depends on the realizations of the random variables next period and on whether the central bank intervenes. If the central bank does decide to intervene, it anticipates that it will set a price

$$P_t = \alpha x_{t-1} + a_0 \Delta s_t + y_t$$

once it learns Δs_t. Therefore, the immediate expected loss contingent on the information set available when the decision must be made is

$$\begin{aligned} E[(P_t - x_t - y_t)^2 | x_{t-1}, y_{t-1}, y_t] &= E[(\alpha x_{t-1} + a_0 \Delta s_t + y_t - x_t + y_t)^2 | x_{t-1}, y_{t-1}, y_t] \\ &= E[(a_0 - 1)e_{xt} + a_0 e_{st}]^2] \\ &= \frac{\sigma_x^2 \sigma_s^2}{\sigma_x^2 + \sigma_s^2}, \end{aligned}$$

when a_0 is chosen optimally. However, the central bank will not have learnt the realization of e_{xt} precisely, and any estimation error it makes will be perpetuated in subsequent periods whether or not it intervenes. The importance of this carryover will depend on the degree of autocorrelation displayed by the state variable x_t; the sum of the losses from the estimation error can easily be shown to be

$$V_1' = \frac{1}{1-\alpha^2} \frac{\sigma_x^2 \sigma_s^2}{\sigma_x^2 + \sigma_s^2}. \tag{14}$$

Thus, a sufficient condition for the central bank to intervene is that $(V_1' - V_0')$ be negative. An examination of equation (13) shows that if, for example, y_{t-1} and y_t are both positive and $(\alpha y_{t-1} - y_t) < 0$, then V_0' is a decreasing function of y_{t-1} and an increasing function of y_t, and vice versa if they are both negative; the larger the innovation in the central bank's state variable y_t away from its central value, the more likely is intervention.[7] An examination of equation (14) shows that V_1' is an increasing function of α, σ_x^2, and σ_s^2. A higher value for α increases the importance of the estimation error in future periods, a higher σ_s^2 increases the magnitude of the error, and a higher σ_x^2 signifies that variations in the state variable x_t are greater in magnitude and receive more weight in the central bank's objective function, so an increase in any of these variables makes intervention less likely.

A further extension would be based on the assumption that the central bank receives the signal Δs_t at the start of each period, whether or not it

[7]The result that the central bank may refrain from intervention to correct small derivations from its immediate monetary policy target bears some resemblance to the conclusions in Orphanides and Wilson (1996) on "opportunistic" monetary policy.

intervenes. If possible, the central bank would wish to assess the signal before deciding when to intervene, and it is easy to work out the decision criterion under simple circumstances. In general, an exceptionally large absolute value of Δs_t would indicate a large change in the state variable x_t, which the market itself will incorporate into prices better than the central bank can, so intervention is less likely to be warranted.

III. Information Inference by Market Participants

It has been shown how the design of monetary policy operating procedures can facilitate the extraction of market information by the central bank. Yet information can also flow in the opposite direction, and the central bank may be very anxious to convey its assessment of the economic situation and its future intentions to market participants.[8] This concern is likely to be strongest when the central bank wishes to broadcast "news" about longer-term developments and its policy stance; investment decisions, the pricing of long-term securities and exchange rates, wage negotiations, and general economic confidence would be affected by such news, and thus the monetary policy transmission process initiated. Here too the design of the operating procedure may improve the precision with which prices reveal information.[9] In general, however, one might expect to be presented with a trade-off, whereby practices that help the central bank to infer market information, such as limiting intervention, make it more difficult for market participants to filter out the central bank's signal from "noisy" prices. Such a trade-off will be illustrated here.

In the context of the model presented above it is plausible to suppose that the central bank would be concerned about the accuracy with which market participants can estimate y_t, the state variable representing the central bank's private information. Because changes in y_t are persistent, the market estimate of the state variable in any one period t will be highly correlated with expectations of its value in periods $t+1$, $t+2$, ... Therefore, expectations of short-term interest rates in subsequent periods and uncertainty about them will depend on the level and precision of this estimate. Current forward rates and long-term interest rates, which may be especially important in the monetary policy transmission process, will adjust correspondingly. More precisely, the central bank's loss function in any one period will be assumed to depend on both the expected deviation of the spot price from its target and

[8]However, under certain circumstances the central bank may wish to make intentionally imprecise announcements (see Garfinkel and Oh, 1995).

[9]A central bank might send signals through various other channels, such as changes in the rates applied at its standing facilities, but the information content of changes in rates on open market operations need not thereby be diminished.

the expected squared deviation of market participants' estimate of y_t from its true value:

$$E\left(\sum_{i=0}^{\infty}\{\omega E[(P_{t+i}-x_{t+i}-y_{t+i})^2|\Omega_{t+i}]+(1-\omega)E[(\hat{y}_{t+i}-y_{t+i})^2|\Gamma_{t+i}]\}|\Omega_t\right), \quad (15)$$

where $\omega\epsilon[0,1]$ is a weight, and $\hat{y}_{t+i}$ is the market participants' best estimate of y_t based on their information set Γ_{t+i}. This information set could be derived from market participants' knowledge of the economy generally and observations of the central bank's past actions, which indicate its policy reaction function, and the central bank's public pronouncements. In addition, as will be shown, the central bank's operating procedures can affect Γ_{t+i}.

Policy Signaling Through Open Market Operations

Here it will be assumed that the central bank does not announce its private information in advance of intervention operations or cannot do so credibly.[10] Indeed, the practice of many central banks is to conduct most open market operations by soliciting quotes from money center banks or by conducting a tender where neither the price nor the quantity is set in advance. Under such procedures, market participants must make inferences about the central bank's information and policy stance based on the money market price determined through intervention. By assumption, the realized value of the state variable x_t is already known to market participants. To simplify the presentation, it will be assumed that the true value of y_{t-1} is revealed at the start of the next period t. It will also be assumed that the announcement of the tender results from the previous period is sufficiently detailed that the market participants can infer Δs_{t-1}, which allows them to identify e_{st-1} because they already know the realization of x_{t-1}. Therefore, market participants need to make inferences about current disturbances only; the uncertainty in the estimation of y_t concerns just the value of e_{yt} and e_{st}.[11]

When the central bank never intervenes, no information about e_{yt} is revealed, so $\hat{y}_t = \beta y_{t-1}$ and the expected squared deviation of the estimate is σ_y^2. The average expected loss function can be shown to be

[10] The question of why the central bank cannot simply announce its information immediately will not be addressed here. During the 1970s the U.S. Federal Reserve did indeed announce whether changes in its discount rate were determined by "technical" or "policy" considerations. However, words are always open to interpretation and ambiguity, and the central bank may be tempted to make use of this margin for vagueness and obscurity, so concrete action may be necessary to make statements convincing.

[11] The information set of market participants at t is $\{x_t, P_t, x_{t-i}, y_{t-1}, \Delta s_{t-1}, \ldots\}$.

$$W_0 = \omega \frac{\sigma_y^2}{1-\beta^2} + (1-\omega)\sigma_y = \omega V_0 + (1-\omega)\sigma_y. \tag{16}$$

Suppose now that the central bank intervenes every day. The central bank will follow a pricing rule of the general form[12]

$$P_{1t} = \sum_{i=0}^{\infty}\left[a_{1i}(e_{xt-i} + e_{st-i})\right] + c_1 y_{t-1} + d_1 e_{yt}. \tag{17}$$

(The parameters a_{1i}, c_1, and d_1 need not be equal to those derived in Section II). The information set of market participants is $\Gamma_t = \{x_t, y_{t-1}, \ldots\}$ and they face a standard "signal extraction" problem in estimating y_t, the state variable known to the central bank. As is shown in the Appendix, the expected squared error of the market participant's best estimate of y_t is now

$$\min E[(\hat{y}_t - y_t)^2 | \Gamma_t] = \frac{a_{10}^2\sigma_s^2}{a_{10}^2\sigma_s^2 + d_1^2\sigma_y^2}\sigma \tag{18}$$

Again, the central bank effectively has a two-period problem. Using equations (17) and (18) in equation (15) yields the expected loss function that is to be minimized through choice of the parameters of the pricing rule:

$$W_1 = \omega V_1(a_{10}, a_{11}, \ldots, c_1, d_1) + (1-\omega)\frac{a_1^2\sigma_s^2}{a_{10}^2\sigma_s^2 + d_1^2\sigma_y^2}\sigma_y^2. \tag{19}$$

In the Appendix it is shown that the optimal parameter a_0 of the pricing rule is now smaller than when V_1 alone is minimized, and d_1 is larger; when the central bank wants to signal its private information, it reacts relatively little to innovations in the state variable known already to market participants, and relatively strongly to innovations in its private information. Strengthening the transmission from the central bank to market participants requires that the current market price deviate more from what would otherwise be its optimum level, for a given frequency of intervention.

One can also confirm the intuition that a central bank will want to intervene more often, the more importance it attaches to transmitting its own information. Consider the difference between the expected loss function when intervention occurs every period, as given in the minimized value of equation (19), and the expected loss when no intervention is undertaken (equation (16)):

$$W_1^* - W_0 = \omega V_1^*(a_{10}, a_{11}, \ldots, c_1, d_1) + (1-\omega)\frac{a_{10}^2\sigma_s^2}{a_{10}^2\sigma_s^2 + d_1^2\sigma_y^2}\sigma_y^2 - \omega V_0 - (1-\omega)\sigma_y^2. \tag{20}$$

[12] Again, the model can be reformulated in terms of the quantity of intervention.

In general, the sign of this expression is ambiguous. Suppose, however, that the variances and parameters of the model are such that the minimized V_1^* just equals V_0, so the central bank is indifferent between intervening and not when $\omega = 1$ and transmitting information to the market is not a concern. The central bank's minimized expected loss function W_1^* cannot be larger than it would be if it sets the money market price to minimize V_1. But the second term on the right-hand side of equation (20) is certainly smaller than the last term because market participants are able to make better inferences about the state variable y_t when the central bank does intervene and send a signal (the expression on the right-hand side of equation (18) is certainly smaller than σ_y^2). Hence, under these conditions, the difference $(W_1^* - W_0)$ is negative and intervening every day becomes a superior strategy. Therefore, intervention is likely to be more frequent, the more weight the central bank attaches to the broadcasting of its own information and intentions.

It can readily be confirmed that intervening every day gains in value to the central bank compared with intervening every 2, 3, . . . days as ω increases. In particular, when the variance of the state variable known to the central bank is large relative to that of the "noise" term e_{st}, the central bank has a lot of "news" that it would like to broadcast every day, while frequent intervention is not very costly in terms of loss of precision in its own estimates of market information. Therefore, more frequent intervention is preferred.

Policy Signaling and Quantity Tenders

Some central banks may have available mechanisms whereby they commit themselves to setting an intervention price in advance of their open market operations. An example of such a mechanism is a quantity tender, as used on occasion by the Deutsche Bundesbank, whereby the central bank announces an interest rate at which it will conduct repurchase transactions in a forthcoming tender and commercial banks submit bids for the amount they wish to transact. In this way, the market participants get a very clear signal of the central bank's private information, but the central bank must decide on its operational target for the interest rate without the benefit of observing the tender bids. The nature of this trade-off can be illustrated here.

Assume first that the central bank undertakes a quantity tender every period. Then its information set consists of $\{y_t, \Delta s_{t-1}, y_{t-1}, \Delta s_{t-2}, \ldots\}$; it lacks the information about the current innovation in the state variable x_t that would be contained in the signal Δs_t received through the bidding process. Its pricing rule cannot depend on e_{xt} or e_{st}, and will take the form

$$P_t' = \sum_{i=0}^{\infty} a_{1i}' \Delta s_{t-i} + c_1' \beta y_{t-1} + d_1' e_{yt}. \tag{21}$$

Recall that, by assumption, the central bank announces y_{t-1} and Δs_{t-1} at the start of period t. Market participants have an information set $\{x_t, P_t', x_{t-1}, y_{t-1}, \Delta s_{t-1}, P_{t-1}', \ldots\}$, from which they can infer the value of y_t exactly. The expected squared deviation of their estimate from the true value is zero. The Appendix contains a derivation of the central bank's minimized expected loss function, which is compared with the loss function in the case where the central bank conducts an interest rate tender. It is shown that when the central bank receives only a very noisy signal of market information, or when variations in the state variable y predominate, a quantity tender is advantageous. However, the more important is market information, the more likely it is that the operational interest rate target should not be set in advance of the tender.

So far, the central bank has been assumed to have private information on changes in one state variable. One could construct a situation in which the operational target is affected by permanent and transitory shocks, and in addition the central bank is concerned with transmitting information specifically about longer-term developments so as to influence expectations.[13] Very frequent intervention that partially accommodates daily or hourly fluctuations might in effect generate "noise" that makes it difficult for market participants to identify when the central bank is conducting stabilizing or accommodative operations and when, alternatively, it is initiating policy shifts.

In a more complex policy "game" one would have to consider a richer array of possible strategies where the revelation of information is not always desired. For example, less frequent intervention could be preferred if it facilitates the intentional not sending of a signal, as when the central bank wishes to allow interest rates to drift "silently" up or down. Less prominent involvement in determining money market interest rates may also help reduce political pressures on the central bank to keep interest rates always low.

IV. Conclusions

It has been shown how making room for the play of market forces by not intervening too frequently can offer the central bank itself occasion to extract valuable information contained only in free market prices. This is not to say that feedback from financial market prices to policies should dominate in the determination of operational targets, let alone intermediate goals. Rather, the additional information yielded by this source complements in an important way that directly available to the central bank and the central bank's own preferences and longer-term forecasts. The actual practices of

[13]To model such a situation, the central bank could be assumed to receive a signal z_t that is a serially uncorrelated random variable affecting only the current operating target, so its loss function would be $W'' = \omega E[(P_t - x_t - y_t - z_t)^2|\Omega_t] + (1-\omega)E[(\hat{y}_t - y_t)^2|\Gamma_t]$.

central banks, which take developments in interbank interest rates and other financial market prices into account when deciding how to use their instruments, reflect this complementarity. Market-based instruments are not only effective in achieving given operational targets, but may also help the central bank in choosing the right targets. A central bank has many sources of information; in an economy with well-developed financial markets, the central bank can observe not only money market rates but also, for example, the whole term structure. However, the term structure will in large measure reflect market expectations about future developments, notably in the central bank's own policy stance. Hence, information in the term structure complements rather than replaces information on current money market conditions contained in short-term rates.

A central bank needs to know how well its actions convey information to market participants. The money market rates determined through intervention are likely to be interpreted as signaling the central bank's assessment of the economic situation and its future policy stance. A clearer signal can typically be sent if intervention is more frequent and if more weight is placed on the central bank's private information, even at the expense of permitting deviations from the operational target and a reduction in the information available to the central bank. Signal extraction requires that market rates be allowed to fluctuate relatively freely around an operational target, whereas signal transmission is strengthened when intervention dominates the determination of market prices.

Similar considerations may be applied to the design of other monetary policy instruments. For example, a central bank must decide on which interest rate to focus. It may be that the very-shortest-term rates for overnight money are subject to considerable "noise" from minor disturbances to the supply of, and demand for, liquidity that are irrelevant for monetary policy purposes. The requirements of signal extraction and signal transmission may prompt the central bank to concentrate on a slightly longer-term rate. In addition, many central banks maintain one or more standing facilities at which commercial banks may borrow or deposit funds under certain conditions. The availability of these standing facilities often has the effect of imposing more or less tight bounds on money market interest rates. These bounds should not be too narrow; money market rates need to be free to fluctuate enough for the central bank to extract clear signals and for market participants to have sufficient incentive to acquire information and develop well-grounded trading strategies. Likewise, a central bank seeking to keep the exchange rate within a "band" should consider informational implications when determining the width of the band, and when deciding how frequently it should intervene through trading foreign exchange or adjusting domestic interest rates to defend the band.

APPENDIX

Derivation of Results

In this Appendix some of the results discussed in the main text are derived in detail. The first section presents the derivation of the central bank's minimized loss function as the frequency of intervention varies. The second section deals with the inference of central bank information by market participants. The third section considers the case in which the central bank conducts a quantity auction.

Intervention and Information Inference by the Central Bank

Intervention Every Period

If intervention is conducted every period, the central bank's information set at each time t consists of $\{y_t, \Delta s_t, y_{t-1}, \Delta s_{t-1}, \ldots\}$, on the basis of which it sets the money market price:

$$\begin{aligned} P_{1t} &= \sum_{i=0}^{\infty} a_{1i}\Delta s_{t-i} + c_1\beta y_{t-1} + d_1 e_{yt} \\ &= \sum_{i=0}^{\infty} a_{1i}(e_{xt-i} + e_{st-i}) + c_1\beta y_{t-1} + d_1 e_{yt}. \end{aligned} \tag{22}$$

Using equations (1a), (2), (5), and (22) and the independence of the disturbance terms, the loss function to be minimized can now be written as an infinite moving average

$$\begin{aligned} V_1(a_1, a_2, \ldots, c_1, d_1) &= E\left[(P_{1t} - x_t - y_t)^2 | \Omega_t\right] \\ &= E\left(\left\{\sum_{i=0}^{\infty}\left[(a_{1i} - \alpha^i)e_{xt-i} + a_{1i}e_{st-i}\right]\right.\right. \\ &\qquad \left.\left. + (c_1 - 1)\beta y_{t-1} + (d_1 - 1)e_{yt}\right\}^2\right) \\ &= \sum_{i=0}^{\infty}\left[(a_{1i} - \alpha^i)^2\sigma_x^2 + a_{1i}^2\sigma_s^2\right] \\ &\qquad + (c_1 - 1)^2\beta^2 y_{t-1}^2 + (d_1 - 1)^2 e_{yt}^2. \end{aligned} \tag{23}$$

It is easy to derive the first-order conditions for a minimum:

$$\partial V_1 / \partial a_{1i} = 2(a_{1i} - \alpha_i)\sigma_x^2 + 2a_{1i}\sigma_n^2 = 0 \tag{24a}$$

$$\partial V_1 / \partial c_1 = 2(c_1 - 1)\beta y_{t-1}^2 = 0 \tag{24b}$$

$$\partial V_1 / \partial d_1 = 2(d_1 - 1)e_{yt}^2, \tag{24c}$$

which can be solved to yield that $c_1 = d_1 = 1$ and

$$a_{1i} = \frac{\sigma_x^2}{\sigma_x^2 + \sigma_s^2}\alpha^i. \tag{25}$$

These parameter values are substituted into equation (22), so after simplifying the minimized loss function V_1^* is found to be

$$V_1^* = \sum_{i=0}^{\infty}\left[\alpha^{2i}\left(\frac{\sigma_x^2}{\sigma_x^2+\sigma_s^2}-1\right)^2\sigma_x^2 + \alpha^{2i}\left(\frac{\sigma_x^2}{\sigma_x^2+\sigma_s^2}\right)^2\sigma_s^2\right]$$
$$= \frac{1}{1-\alpha^2}\frac{\sigma_x^2\sigma_s^2}{\sigma_x^2+\sigma_s^2}. \tag{26}$$

Intervention Every Other Period

If intervention is conducted every other period, the central bank's information set is $\{y_t, \Delta s_t, e_{xt-1}, \Delta s_{t-2}, \ldots\}$. The optimal money market price depends linearly on the available information variables:

$$P_{2t} = \sum_{i=0}^{\infty}(a_{2i}\Delta s_{t-2i} + b_{2i+1}e_{xt-2i-1}) + c_2\beta y_{t-1} + d_2 e_{yt}. \tag{27}$$

The central bank must consider the expected squared deviation of the actual price from its operational target averaged over periods t and $t+1$, based on current information and knowledge of the history of the state variables. Hence, its expected loss function becomes

$$V_2 = (1/2)E[(P_t - x_t - y_t)^2 + (P_{t+1} - x_{t+1} - y_{t+1})^2 | y_t, \Delta s_t, e_{xt-1}, \ldots]$$
$$= (1/2)E[(P_t - x_t - y_t)^2 +$$
$$(\alpha P_t + e_{xt+1} - \alpha x_t - e_{xt+1} - y_t - e_{yt+1})^2 | y_t, \Delta s_t, e_{xt-1}, \ldots], \tag{28}$$

where use is made of equations (1a) and (1b) describing the time-series behavior of the state variables. By substituting the pricing rule (27) into expression (28), average expected loss that is to be minimized can be written as

$$V_2(a_{2i}, b_{2i+1}, c_2, d_2) =$$
$$\tfrac{1}{2}E\left(\begin{aligned}&\left\{\sum_{i=0}^{\infty}\left[(a_{2i}-\alpha^{2i})e_{xt-2i} + a_{2i}e_{st-2i} + (b_{2i+1}-\alpha^{2i+1})e_{xt-2i+1}\right]\right.\\ &\left.\quad + (c_2-1)\beta y_{t-1} + (d_2-1)^2 e_{yt}\right\}^2\\ &+\left\{\alpha\sum_{i=0}^{\infty}\left[(a_{2i}-\alpha^{2i})e_{xt-2i} + a_{2i}e_{st-2i} + (b_{2i+1}-\alpha^{2i+1})e_{xt-2i+1}\right]\right.\\ &\left.\quad + (\alpha c_2-\beta)\beta y_{t-1} + (\alpha d_2-\beta)e_{yt} - e_{yt+1}\right\}^2\end{aligned}\right),$$

which can easily be formulated in terms of unconditional variances as

$$V_2 = 1/2\left[(1+\alpha^2)\sum_{i=0}^{\infty}\left((a_{2i}-\alpha^{2i})^2\sigma_x^2 + a_{2i}^2\sigma_s^2 + (b_{2i+1}-\alpha^{2i+1})^2\sigma_x^2\right)\right.$$
$$\left. + \frac{(c_2-1)^2 + (\alpha c_2-\beta)^2}{1-\beta^2}y_t^2 + \sigma_y^2\right]. \tag{29}$$

under the assumption that all disturbances are independent. The optimal choice of parameters is determined using the first-order conditions to be

$$a_{2i} = \frac{\sigma_x^2}{\sigma_x^2 + \sigma_s^2}\alpha^{2i}, \qquad b_{2i+1} = \alpha^{2i+1}, \qquad c_2 = d_2 = \frac{1+\alpha\beta}{1+\alpha^2}.$$

The minimized average loss function can then be shown to be

$$V_2^* = \tfrac{1}{2}\left[\frac{1}{1-\alpha^2}\,\frac{\sigma_x^2\sigma_s^2}{\sigma_x^2+\sigma_s^2} + \frac{(\alpha-\beta)^2}{1+\alpha^2}\,\frac{\sigma_v^2}{1-\beta^2}\right]. \tag{30}$$

The difference between the minimized loss functions ($V_2^* - V_1^*$) when the central bank intervenes every other day versus every day is given in equation (11). The partial derivatives of this difference with respect to α and β are

$$\frac{\partial(V_2^* - V_1^*)}{\partial\alpha} = -\frac{(\alpha-\beta)(1-\alpha\beta)}{(1+\alpha^2)^2(1-\beta^2)}\sigma_v^2 - \frac{\alpha}{(1-\alpha^2)}\,\frac{\sigma_x^2\sigma_s^2}{\sigma_x^2+\sigma_s^2}. \tag{31a}$$

$$\frac{\partial(V_2^* - V_1^*)}{\partial\beta} = \frac{(\alpha-\beta)(1+\alpha\beta)}{(1-\beta^2)^2(1+\alpha^2)}\sigma_v^2, \tag{31b}$$

both of which can be either positive or negative. However, if $1 > \alpha > \beta > 0$, then equation (31a) is certainly negative and (31b) is certainly positive. Under these conditions, a larger α implies that the market price and the operational objective evolve more similarly during the nonintervention days. A larger α also entails that the central bank's errors in estimating e_{st} in any one period are largely carried forward into subsequent periods, so it is more valuable to learn its true value by not intervening. Therefore, less frequent intervention is more likely to be optimal the larger is α. A larger β implies that changes in the state variable y_t are more persistent; yet in the absence of intervention the market price does not reflect these follow-on changes, so more frequent intervention is desirable.

Information Inference by Market Participants

Intervention Every Period

The information set of market participants is $\Gamma_t = \{x_t, y_{t-1}, \ldots\}$. As is apparent from an examination of equations (16) and the time-series behavior of the state variables given in equations (1a) and (1b), with this information set the market participants can identify a signal q_t, defined by

$$q_t \equiv a_{10}e_{st} + d_1 e_{yt}.$$

The best estimator of y_t is $\hat{y}_t = \beta y_{t-1} + \xi_1 q_t$. The parameter ξ_1 is chosen to minimize the expected squared deviation, that is,

$$\begin{aligned}
\xi_1 &= \operatorname{argmin}_{\xi_1} E[(\hat{y}_t - y_t)^2 \mid \Gamma_t] \\
&= \operatorname{argmin}_{\xi_1} E[(\xi_1 q_t - e_{yt})^2] = \operatorname{argmin}_{\xi_1} E[(\xi_1(a_{10}e_{st} + d_1)e_{yt}) - e_{yt})^2] \\
&= \operatorname{argmin}_{\xi_1} [\xi_1^2(a_{10} + d_1)^2\sigma_y^2 + \sigma_s^2] \\
&= \frac{\sigma_y^2}{a_{10}^2\sigma_s^2 + d_1^2\sigma_y^2}.
\end{aligned} \tag{32}$$

One can proceed to derive that

$$\min E[(\hat{y}_t - y_t)^2 | \Gamma_t] = \frac{a_{10}^2 \sigma_s^2}{a_{10}^2 \sigma_s^2 + d_1^2 \sigma_y^2} \sigma_y^2. \tag{33}$$

Substituting equations (17) and (33) into (15) yields the expected loss function that is to be minimized through choice of the parameters of the pricing rule, specifically,

$$\begin{aligned} W_1 &= \omega E\left(\left\{ \sum_{i=0}^{\infty} \left[(a_{1i} - \alpha^i) e_{xt-i} + a_{1i} e_{st-i} \right] + (c_1 \beta - \beta) y_{t-1} + (d_1 - 1) e_{yt} \right\}^2 \right) + \\ &\quad (1 - \omega) \min E[(\hat{y}_t - y_t)^2 | \Gamma_t] \\ &= \omega V_1(a_{10}, a_{11}, \ldots, c_1, d_1) + (1 - \omega) \frac{a_{10}^2 \sigma_s^2}{a_{10}^2 \sigma_s^2 + d_1^2 \sigma_y^2} \sigma_y^2. \end{aligned} \tag{34}$$

It can easily be verified from the first-order conditions for a minimum that the optimal a_{1i}, $i = 1, 2, \ldots$, and c_1 are unchanged from the values obtained above when the central bank did not intentionally signal its information to market participants (see equation (25)). For the other parameters, a_0 and d_1, of the pricing function the first-order conditions are

$$\omega \frac{\partial V_1}{\partial a_{10}} + (1 - \omega) \frac{2 a_{10} \sigma_s^2 d_1^2 \sigma_s^2}{(a_{10}^2 \sigma_s^2 + d_1^2 \sigma_y^2)^2} \sigma_y^2 = 0 \tag{35a}$$

$$\omega \frac{\partial V_1}{\partial d_1} - (1 - \omega) \frac{2 a_{10}^2 \sigma_y^2 d_1 \sigma_s^2}{(a_{10}^2 \sigma_s^2 + d_1^2 \sigma_y^2)^2} \sigma_y^2 = 0. \tag{35b}$$

Now, $\partial V_1 / \partial a_{10}$ and $\partial V_1 / \partial d_1$ are increasing functions of a_{10} and d_1, respectively, and equal zero when V_1 is minimized (see equations (24a) and (24b)). The second terms of both (35a) and (35b) are always positive, the former entering with a positive sign and the latter with a negative. Therefore, the optimal a_0 is now smaller than when V_1 alone is minimized, and d_1 is larger: when the central bank wants to signal its private information, it reacts relatively little to innovations in the state variable known already to market participants, and relatively strongly to innovations in its private information. Strengthening the transmission from the central bank to market participants requires that the current market price deviate more from what would otherwise be its optimum level, for a given frequency of intervention. It will be useful later to have written the optimal parameter values as

$$a_0 = \frac{\sigma_x^2}{\sigma_x^2 + \sigma_s^2 + d_1^2 K}, \quad K = \frac{1 - \omega}{\omega} \frac{\sigma_s^2 \sigma_y^4}{(a_{10} \sigma_s^2 + d_1^2 \sigma_y^2)^2}, \quad d_1 = \frac{\sigma_y^2}{\sigma_y^2 - a_{10}^2 K}.$$

Then the minimized expected loss function can be written as

$$\begin{aligned} W_1^* &= \omega \left[\frac{\sigma_x^2 \sigma_s^2 (\sigma_x^2 \sigma_s^2 + 2 d_1^2 K)}{(\sigma_x^2 + \sigma_s^2 + d_1^2 K)^2} + \frac{\alpha^2}{1 - \alpha^2} \frac{\sigma_x^2 \sigma_s^2}{\sigma_x^2 + \sigma_s^2} + a_{10}^2 K^{2^2} \sigma_y^2 \right] \\ &\quad + (1 - \omega) \frac{a_{10}^2 \sigma_s^2}{a_{10}^2 \sigma_s^2 + d_1^2 \sigma_y^2} \sigma_y^2. \end{aligned} \tag{36}$$

Policy Signaling and Quantity Tenders

Assume first that the central bank undertakes a quantity tender every period. Then its information set consists of $\{y_t, \Delta s_{t-1}, y_{t-1}, \Delta s_{t-2}, \ldots\}$; it lacks the information about the current innovation in the state variable x_t that would be contained in the signal Δs_t received through the bidding process. Its pricing rule cannot depend on e_{xt} or e_{st}, and will take the form

$$P_t' = \sum_{i=1}^{\infty} a_{1i}' \Delta s_{t-i} + c_1'\beta y_{t-1} + d_1' e_{yt} = \sum_{i=1}^{\infty}\left[a_{1i}'(e_{xt-i} + e_{st-i})\right] + c_1'\beta y_{t-1} + d_1' e_{yt}. \tag{37}$$

Recall that by assumption the central bank announces y_{t-1} and Δs_{t-1} at the start of period t. Market participants have an information set $\{x_t, P_t', x_{t-1}, y_{t-1}, \Delta s_{t-1}, P_{t-1}', \ldots\}$, from which they can infer the value of y_t exactly. The expected squared deviation of their estimate from the true value is zero. Therefore, the central bank's expected loss function becomes

$$W_1' = \omega E\left(\left\{-e_{xt} + \sum_{i=1}^{\infty}\left[(a_{1i}' - \alpha^i)e_{xt-i} + a_{1i}' e_{st-i}\right] + (c_1'\beta - \beta)y_{t-1} + (d_1' - 1)e_{yt}\right\}^2\right) + (1-\omega)0,$$

or in terms of variances

$$W_1' = \omega\left\{\sigma_x^2 + \sum_{i=1}^{\infty}\left[(a_{1i}' - \alpha^i)^2\sigma_x^2 + a_{1i}'^2\sigma_s^2\right] + (c_1'\beta - \beta)^2 y_{t-1}^2 + (d_1' - 1)^2\sigma_y^2\right\}^2.$$

It is easy to establish that the optimal parameters are $a_{1i} = \sigma_x^2\alpha^i/(\sigma_x^2 + \sigma_s^2)\ \forall\, i \geq 1$, and $c_1 = d_1 = 1$, which are the same as when the central bank does not signal its information. The minimized expected loss function is thus

$$W_1^{*\prime} = \omega\left(\sigma_x^2 + \frac{\alpha^2}{1-\alpha^2}\frac{\sigma_x^2\sigma_s^2}{\sigma_x^2 + \sigma_s^2}\right). \tag{38}$$

The difference between the minimized expected loss functions (19) and (38) is

$$W_1^* - W_1^{*\prime} = \omega\{[(a_{10} - 1)^2 - 1]\sigma_x^2 + a_{10}^2\sigma_s^2 + (d_1 - 1)^2\sigma_y^2\} + (1-\omega)\frac{a_{10}^2\sigma_s^2}{a_{10}^2\sigma_s^2 + d_1^2\sigma_y^2}\sigma_y^2, \tag{39}$$

where it has been established that $0 < a_{10} < 1$ and $d_1 > 1$. Clearly, the last term on the right-hand side of equation (39) is positive, so for ω sufficiently small (when conveying information to the market participants is given overwhelming weight), $(W_1^* - W_1^{*\prime}) > 0$, in which case holding a quantity tender with a preannounced price is optimal. The expressions $a_{10}^2\sigma_s^2$ and $(d_1 - 1)^2\sigma_y^2$ are certainly positive. Hence when either σ_s^2 is very large (the signal of market information is noisy) or σ_y^2 is very large (the state variable y is very variable), a quantity tender is advantageous. The expression involving σ_x^2 is necessarily negative, so the more important is market information, the more likely it is that the operational interest rate target should depend on the results of the tender.

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