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Government Spending and Inflationary Finance

A Public Finance Approach

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The dependence of the inflation tax on the level of government spending is analyzed in a public finance context. The key feature of the model is that it recognizes the possibility that conventional taxes, such as the consumption tax, carry increasing marginal collection costs. As a result, the inflation tax becomes an increasing function of government spending. Furthermore, the more inefficient the tax-collection system is, the larger is the rise in the nominal interest rate for a given increase in government spending. A numerical analysis of the model provides additional insights into these relationships. [JEL 311]

> It is common to speak as though, when a Government pays its way by inflation, the people of the country avoid taxation. We have seen that this is not so. What is raised by printing notes is just as much taken from the public as is a beer-duty or an income tax. What a Government spends the public pay for. There is no such thing as an uncovered deficit. But in some countries it seems possible to please and content the public, for a time at least, by giving them, in return for the taxes they pay, finely engraved acknowledgments on water-marked paper. The income-tax receipts, which we in England receive from the Surveyor, we throw into the wastepaper basket; in Germany they call them bank-notes and put them into their pocketbooks; in France they are termed Rentes and are locked up in the family safe.
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> — Keynes (1924, pp. 68–69)

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THE STUDY of the inflation tax in a public finance context has received increasing attention in the literature ever since Phelps (1973) pioneered this approach. Given that lump-sum taxes are not feasible in the real world and that the inflation tax is only one of several taxes that a government can resort to, it is certainly arguable that analyzing the problem of inflationary finance in a public finance context is the appropriate way of dealing with this important issue.[1] Alternative ways of approaching the optimum quantity of money rule have either relied on partial equilibrium models or ignored distortions existing elsewhere in the economy (that is, outside the money market).[2]

Phelps (1973) introduces real money balances into the utility function and reaches the conclusion that an optimal policy calls for a positive inflation tax.[3] If money is modeled as reducing transaction costs, however, Kimbrough (1986) shows that, even if distortionary taxes have to be used, it is not optimal to resort to inflationary finance.[4]

If one views the role of money as reducing the amount of time needed to transact (that is, shopping time), Kimbrough's (1986) result cannot explain the widespread use of inflationary finance that takes place, especially in developing countries (as documented by Fischer (1982)). It has long been recognized, however, that the costs of collection, enforcement, and evasion (hereafter referred to simply as collection costs) are negligible compared to other taxes. As Keynes (1924, p. 46) put it, inflationary finance "is the form of taxation which the public find hardest to evade and even the weakest Government can enforce, when it can enforce nothing else." Aizenman (1987) incorporates this characteristic of the inflation tax by assuming that a consumption tax, which is the other tax available to the government in addition to the inflation tax, carries collection costs. He concludes that a positive inflation tax is called for.[5]

[1] Frenkel (1987) stresses the importance and usefulness of attacking the problem of inflationary finance in a public f nance context.

[2] See Friedman (1969), Johnson (1970), Bewley (1980), and Jovanovic (1982), among others.

[3] The definition of the inflation tax used in this paper follows Auernheimer (1974). The consumer's expenditure on holding money accrues to the government because, by issuing non-interest-bearing debt (money), the government avoids interest costs on the public debt. Phelps (1973) discusses alternative definitions of the concept "inflation tax".

[4] On the generality of Kimbrough's (1986) result, see Guidotti and Végh (1988). They consider alternative assumptions about the transaction technology and the cases of both consumption and income taxation. For a related discussion, see also Woodford (1989).

[5] Aizenman (1983) derives a relationship between collection costs and government spending, but not in a public finance context. Instead of incorporating

The purpose of this paper is to take a comprehensive look at the relationship between government spending and inflationary finance in a public finance context. If it is assumed that alternative taxes are costly to collect and money is modeled as in Kimbrough (1986), it is shown that constant marginal collection costs imply that the optimal inflation tax is independent of the level of government spending.[6] It is generally believed, however, that higher government spending or higher budget deficits (or both) lead to higher inflation as governments seek additional sources of revenue. The main result of the paper provides an explanation for such a policy: if marginal collection costs are an increasing function of revenues from the consumption tax, the optimal inflation tax is an increasing function of government spending. Furthermore, the optimal nominal interest rate is a convex function of government spending; that is, the higher is the level of government spending, the larger is the increase in the nominal interest rate that results from a given increase in government spending. The efficiency of the tax-collection system, as indicated by higher marginal collection costs for a given level of tax revenues, is shown to play a crucial role as well. Specifically, the more inefficient is the tax-collection system, the higher is the optimal interest rate for a given level of government spending, and the larger is the increase in the interest rate for a given rise in government spending. Interestingly enough, although the optimal nominal interest rate is an increasing function of government spending, the share of revenues from the inflation tax turns out to be a decreasing function of government spending.[7]

The paper is organized as follows. Section I presents the consumer's and the government's optimization problems and addresses the general equilibrium solution for the case of constant marginal collection costs. Section II specifies some functional forms in order to tackle the theoretical analysis of the case of increasing marginal collection costs. The analysis in this section shows the effects on the optimal inflation tax of both an increase in government spending and parametric changes in the marginal collection cost schedule. A graphical illustration highlights the difference between the cases of constant and increasing marginal collec-

collection costs, Aizenman (1986) assumes that consumption taxes are not feasible and studies the optimal combination of capital controls, tariffs, and inflation in financing government spending. Végh (1989) shows that the presence of currency substitution also renders the use of inflationary finance optimal.

[6]This benchmark case draws on Végh (1987).

[7]To simplify the analysis and sharpen the economic intuition, specific functional forms are used in deriving most results. These assumptions are spelled out in the text as they are incorporated into the analysis.

tion costs. To gain further insights, Section III presents a numerical analysis of the model. Finally, Section IV offers some concluding remarks.

I. The Model

In this section the consumer's and the government's optimization problems are introduced. The presentation of the consumer's problem, although self-contained, will be brief because it follows Kimbrough (1986).

The Consumer's Problem

Consider a small open economy that operates under flexible exchange rates.[8] There is only one (tradable and nonstorable) good. Labor is the only factor of production and is taken to be the numeraire. The consumer may hold two assets: domestic money and an internationally traded bond whose real rate of return is constant and equal to r. Transacting is a costly activity in this economy in that it requires the use of shopping time (s). The consumer is endowed with one unit of time each period, so that his or her time constraint is $s_t + n_t + h_t = 1$, where n denotes labor and h leisure. The good is produced under a technology having constant returns to scale and given by $y_t = n_t$, where y denotes production of the good and where units have been so defined that producing one unit of the good requires one unit of labor.

The transaction technology is given by

$$s_t = v\left[\frac{m_t}{c_t(1+\theta_t)}\right]c_t(1+\theta_t)$$

$$v'(X)\leq 0, \qquad v''(X)>0, \qquad v'(X^s)=0, \qquad 0\leq X\leq X^s, \tag{1}$$

where m denotes real money balances, c stands for consumption, θ represents the consumption tax, and $X \equiv m/c(1+\theta)$ (hereafter referred to as relative money balances).[9] The function $v(X)$ indicates the amount of time that the consumer spends shopping per unit of consumption expenditure. Additional relative money balances bring about positive

[8] The analysis can, however, be readily reinterpretated to apply to a closed economy.

[9] Given the nonnegativity constraint on the nominal interest rate, the restriction on the domain of $v(X)$ implies no loss of generality because the consumer chooses X^s when the nominal interest rate is zero.

but diminishing reductions in shopping time. There is a level of relative money balances, $X = X^s$, such that $v'(X^s) = 0$; that is, gains from holding relative money balances are exhausted. It will be assumed that $v(X^s) = 0$.[10]

The consumer faces the following maximization problem:

$$\max_{\{c_t, h_t, m_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U(c_t, h_t),$$

subject to

$$b_0 + \sum_{t=0}^{\infty} [1/(1+r)]^t (1 - h_t)$$

$$= \sum_{t=0}^{\infty} [1/(1+r)]^t \{c_t(1+\theta_t) + s_t + [i_t/(1+i_t)]m_t\}, \tag{2}$$

where $U(c, h)$ is twice-continuously differentiable, strictly concave, and exhibits positive and decreasing marginal utilities; β is the constant subjective discount factor; c and h are normal goods; b_0 denotes initial holdings of the internationally traded bond; i is the domestic nominal interest rate; and s is given by equation (1). The optimality conditions for this problem are

$$\frac{U_c(c_t, h_t)}{U_h(c_t, h_t)} = (1+\theta_t)[1 + v(X_t) - v'(X_t)X_t], \qquad t = 0, 1, \ldots \tag{3}$$

$$-v'(X_t) = I_t, \qquad t = 0, 1, \ldots, \tag{4}$$

where $I \equiv i/(1+i)$ is the inflation tax.[11] Equation (3) shows that the consumer equates the marginal rate of substitution between consumption and leisure to the relative price of consumption, which consists of its direct price, $1 + \theta$, plus the increase in transaction costs associated with consuming an additional unit of the good, which is given by the term in brackets on the right-hand side of equation (3). Equation (4) states that the consumer holds relative money balances up to the point at which their marginal benefit, in terms of reduced transaction costs, equals their opportunity cost.

[10] This assumption—as opposed to assuming that $v(X^s)$ equals a positive contant—ensures that, if there are no collection costs associated with the consumption tax, the optimal inflation tax is zero (as pointed out by Guidotti and Végh (1988)). For the purposes of this paper, this assumption is made in order to isolate the effects of the presence of collection costs on the optimal inflation tax.

[11] The condition $\beta = 1/(1+r)$, needed to ensure the existence of a steady state, has been assumed. It follows that there are no intrinsic dynamics in the model in the sense of Obstfeld and Stockman (1985).

The Government's Problem

The government faces an exogenously given path of spending, $\{g_t\}_{t=0}^{\infty}$, which will be taken to be constant over time ($g_t = g_0$ for all t). Before formally stating the government's problem, it is necessary to define the equilibrium price functions, which follow implicitly from equations (3) and (4):

$$(1+\theta_t) \equiv y^{\theta}(c_t, h_t, m_t) = \frac{U_c(c_t, h_t)}{U_h(c_t, h_t)}[1 + v(X_t) - v'(X_t)X_t]^{-1} \tag{5}$$

$$I_t \equiv y^{I}(c_t, h_t, m_t) = -v'(X_t), \tag{6}$$

where $X \equiv m/cy^{\theta}(c, h, m)$. As shown below, the government chooses an optimal sequence $\{c_t, h_t, m_t\}_{t=0}^{\infty}$. By substituting these optimal values into equations (5) and (6), the optimal tax sequence $\{\theta_t, I_t\}_{t=0}^{\infty}$ follows. This procedure ensures that the social optimum can be the outcome of a competitive equilibrium.

The government's problem is given by:[12]

$$\max_{\{c_t, h_t, m_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U(c_t, h_t)$$

subject to

$$\sum_{t=0}^{\infty} [1/(1+r)]^t g_0 = \sum_{t=0}^{\infty} [1/(1+r)]^t \{[1-\phi(\theta_t c_t)]\theta_t c_t + I_t m_t\} \tag{7}$$

$$b_0 + \sum_{t=0}^{\infty} [1/(1+r)]^t (1-h_t) = \sum_{t=0}^{\infty} [1/(1+r)]^t$$
$$\cdot \{[1+\phi(\theta_t c_t)\theta_t]c_t + g_0 + v(X_t)\, c_t(1+\theta_t)\}, \tag{8}$$

where $\phi(\theta c) \geq 0$ satisfies $\phi'(\theta c) \geq 0$ and $[\phi''(\theta c)\theta c + 2\phi'(\theta c)] \geq 0$. To understand the role played by $\phi(\theta c)$, note that total collection costs from the consumption tax as a function of revenues from the consumption tax, $T(\theta c)$, are given by $T(\theta c) = \phi(\theta c)\theta c$, so that $\phi(\theta c)$ can be interpreted as average collection costs. If $\phi(\theta c) = 0$, collection costs are zero and the problem reduces to that of Kimbrough (1986). If $\phi(\theta c) > 0$ and $\phi'(\theta c) = 0$ (that is, $\phi(\theta c) = \phi_o$, where $0 < \phi_o < 1$ is a parameter), total collection costs are linear, and thus marginal collection costs are constant (that is, $\partial T/\partial(\theta c) = \phi_o$). In contrast, if $\phi'(\theta c) > 0$ and $[\phi''(\theta c)\theta c + 2\phi'(\theta c)] > 0$,

[12] The specification of the government revenues presupposes that the government acts honestly in the sense of Auernheimer (1974). This implies, as pointed out by Calvo (1978), that problems of time-inconsistency are assumed away.

total collection costs are a convex function and thus marginal collection costs are increasing (that is, $\partial T/\partial(\theta c) = \phi'(\theta c)\theta c + \phi(\theta c) > 0$ and $\partial^2 T/\partial(\theta c)^2 = \phi''(\theta c)\theta c + 2\phi'(\theta c) > 0$). Therefore, both cases of constant marginal collection costs and increasing marginal collection costs can be addressed within this analytical framework. The former case serves as a benchmark, and the latter is the main focus of the paper.

For notational simplicity θ_t and I_t figure in equations (7) and (8) instead of $[y^{\theta}(c_t, h_t, m_t) - 1]$ and $y^I(c_t, h_t, m_t)$, respectively; it should be kept in mind, however, that (c_t, h_t, m_t) are the only variables that appear in the government's problem. Equation (7) is the government's intertemporal budget constraint. Equation (8) is the intertemporal resource constraint for the economy as a whole.

Carrying out the government's maximization problem, substituting into these optimality conditions the partial derivatives of the equilibrium price functions, and resorting to the consumer's first-order conditions allow the general equilibrium of this economy to be represented by the following set of five equations in five unknowns, c, h, X, θ, and I:[13]

$$-v'(X) + \frac{v''(X)X}{1 + v(X) - v'(X)X + v''(X)X^2} \cdot [-\phi(\theta c) - \phi'(\theta c)\theta c - v(X) + v'(X)X] = 0 \tag{9}$$

$$g_0 = [1 - \phi(\theta c)]\theta c + IXc(1 + \theta) \tag{10}$$

$$1 - h = c(1 + \theta) + v(X)c(1 + \theta) + IXc(1 + \theta) \tag{11}$$

$$\frac{U_c(c, h)}{U_h(c, h)} = (1 + \theta)[1 + v(X) - v'(X)X] \tag{12}$$

$$-v'(X) = I. \tag{13}$$

The interpretation of equation (9) will be addressed in detail below. Equation (10) is the government's budget constraint. (Recall that this economy is always in the steady state; therefore, borrowed funds cannot be a source of revenue.) The first term on the right-hand side of equation (10) represents net revenues from the consumption tax; the second one represents revenues from the inflation tax (note that Im can be rewritten as $IXc(1 + \theta)$). Equation (11) is the consumer's steady-state budget

[13] In what follows, time subscripts wil be dropped. Because government spending is constant over time and undergoes only unexpected and permanent changes, the adjustment of the economy will be instantaneous because, as indicated earlier, there are no intrinsic dynamics in the model. For simplicity, it has been assumed that $b_0 = 0$.

constraint. The left-hand side shows the consumer's nonleisure resources, and the three terms on the right-hand side indicate the consumer's uses of these resources (expenditure on consumption, which includes the consumption tax; transaction costs, or shopping time; and the cost of holding real money balances, respectively). Equations (12) and (13) are the consumer's first-order conditions (3) and (4), which are reintroduced here for convenience. This system fully describes the general equilibrium of the economy. The parameters of the system are g and the function $\phi(\theta c)$.

Equation (9) is derived from the government's optimality condition for real money balances. It can be viewed as determining the optimal level of relative money balances, X, for a given value of θc. It proves useful to begin by showing the result, suggested by Végh (1987), that, when marginal collection costs are constant, the optimal inflation tax does not depend on government spending. For this purpose, let $\phi(\theta c) = \phi_o$, where $0 \leq \phi_o < 1$. In this case, equation (9) reduces to

$$-v'(X) + \frac{v''(X)X}{1 + v(X) - v'(X)X + v''(X)X^2} \cdot [-\phi_o - v(X) + v'(X)X] = 0. \tag{14}$$

It is immediately apparent that this equation implicitly defines the optimal level of relative money balances, X^o, solely as a function of the collection costs parameter, ϕ_o; that is, $X^o = X(\phi_o)$. Given X^o, equation (13) yields the optimal inflation tax, I^o. Note that Kimbrough's (1986) result follows as a particular case; that is, $I^o(\phi_o = 0) = 0$. From equation (14), it can be shown that $I^o(0 < \phi_o < 1) > 0$ and that $[dI^o/d(\phi_o)] > 0$ (assuming, for simplicity, that $v'''(X) = 0$).[14] The intuition behind these results is as follows. When collection costs are zero, the use of the inflation tax would entail a loss of resources from positive transaction costs. The use of the consumption tax, instead, carries no extra cost in terms of resources. Therefore it is optimal to use only the consumption tax to finance government spending. When collection costs are positive, however, the use of the consumption tax also implies a loss of resources to the economy. Hence, it is optimal to use both taxes. Two aspects of these results are worth noting. First, the optimal inflation tax does not depend on the level of government spending.[15] (The intuition behind this

[14] Because $I \equiv [i/(1+i)]$ is an increasing function of i, it makes no qualitative difference whether one refers to I or i.

[15] In the presence of currency substitution and consumption taxation (but no collection costs), the optimal inflation tax is also independent of the level of government spending. The optimal inflation tax is solely determined by the foreign nominal interest rate (Végh (1989)).

key feature is addressed below.) Second, from a mathematical point of view, the assumption of constant marginal collection costs makes the analysis fairly simple because, as just shown, equation (9) constitutes a reduced-form equation for X, given implicitly by equation (14), which is all that is needed to determine, from equation (13), the optimal inflation tax. In contrast, when there exist increasing marginal collection costs, equation (9) ceases to be a reduced form for X, and the task of determining the optimal inflation tax is no longer so simple. The next section addresses this issue.

II. Solution of the Model

For reasons just mentioned, it proves convenient to adopt specific functional forms to deal with the optimal inflation tax in the presence of increasing marginal collection costs. Two assumptions will be made: first, the utility function takes the logarithmic form $U(c, h) = \log(c) + \log(h)$; second, the transaction costs technology can be represented by a quadratic function, $v(X) = X^2 - X + d$.[16] Under these assumptions, equations (9) and (11)–(13) become

$$1 + d + X^2 - 2X[1 + 2d + \phi(\theta c) + \phi'(\theta c)\theta c] = 0 \tag{15}$$

$$\frac{h}{c(1+\theta)} = 1 + d - X^2 \tag{16}$$

$$1 - h = c(1+\theta) + (X^2 - X + d)c(1+\theta) + IXc(1+\theta) \tag{17}$$

$$X = (1/2)(1 - I). \tag{18}$$

Equation (10) remains unchanged. By combining equations (16) and (17), and making use of equation (18), it follows that $h = 1/2$. Owing to the logarithmic nature of the utility function, the amount of leisure (and hence the amount of nonleisure resources available to this economy in each period) is constant. If $h = 1/2$ is substituted into equation (16), the following obtains:

$$c(1+\theta) = (1/2)\frac{1}{1 + d - X^2}, \tag{19}$$

which expresses total expenditure on consumption solely as a function of relative money balances, X. Substituting equation (19) into the govern-

[16]Note that, in view of equation (18) below, $X^s = 1/2$, so that it is required that $d = 1/4$ for $v(X^s) = 0$ to be satisfied. Clearly, this particular function satisfies the assumption laid out in equation (1) for $0 \leq X \leq X^s$.

ment's budget constraint, given by equation (10), and using equation (18), one obtains equation (20) below. Equation (20) together with equation (15) (reintroduced, for convenience, as equation (21)) form a two-equation system in two unknowns: θc and X. For notational simplicity, denote gross revenues from the consumption tax, θc, as z (that is, $z \equiv \theta c$):

$$[1 - \phi(z)]z + (1/2)\frac{X - 2X^2}{1 + d - X^2} = g_0 \tag{20}$$

$$1 + d + X^2 - 2X[1 + 2d + \phi(z) + \phi'(z)z] = 0. \tag{21}$$

This system determines equilibrium values of z and X and thus, by way of equation (18), of z and I. Figures 1 and 2 illustrate the joint determination of I and z for the cases of constant and increasing marginal collection costs, respectively. Schedule FE (for fiscal equilibrium) in Figures 1 and 2, which depicts equation (20), is drawn for a given value of government spending (g_0). It slopes downward because an increase in z (gross revenues from the consumption tax) brings about, around an optimum, an increase in net revenues so that, for a given level of spend-

Figure 1. *Determination of* I *and* z*: Constant Marginal Collection Costs*

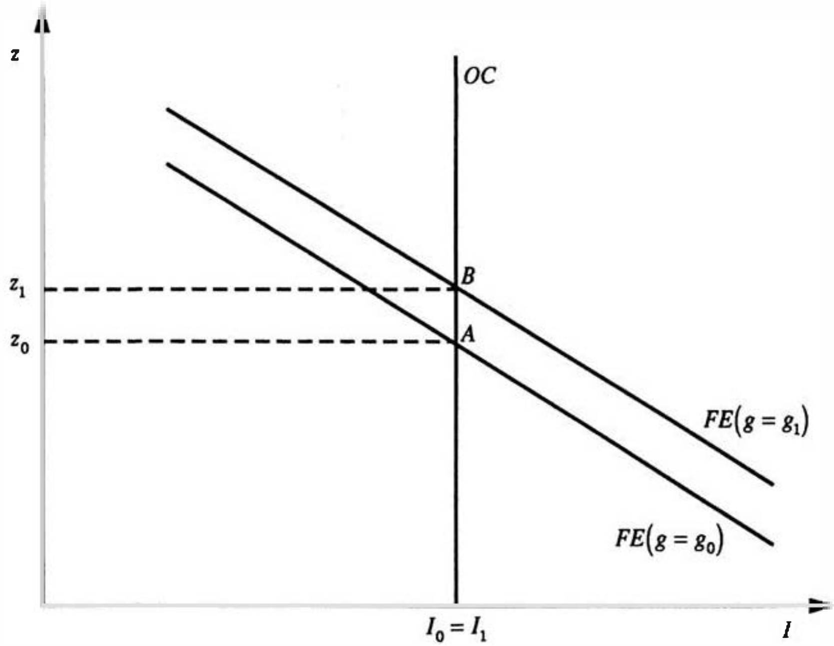
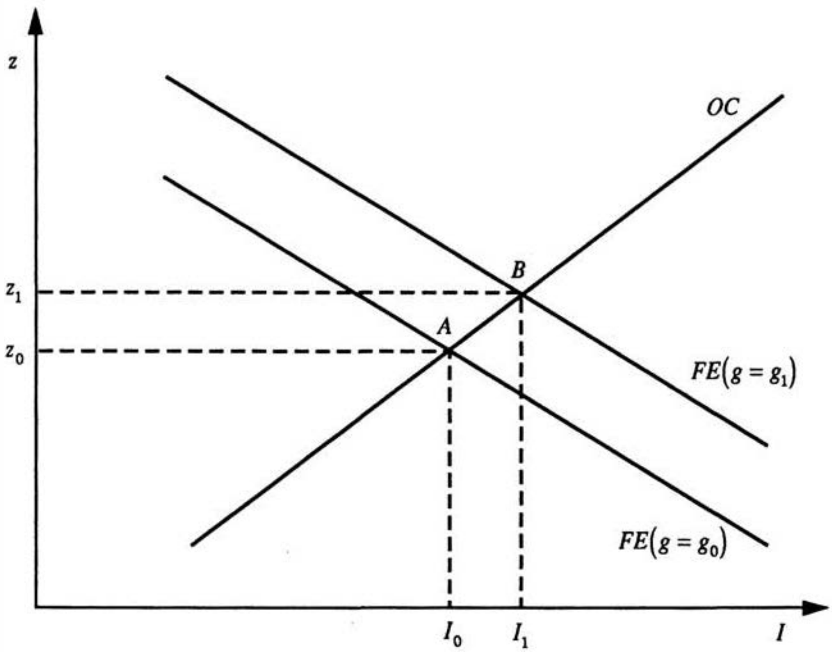


Figure 2. *Determination of* I *and z: Increasing Marginal Collection Costs*



ing, the inflation tax has to be reduced in order to decrease revenues from money creation. The analytical expression for the slope of the *FE* schedule is

$$\frac{dz}{dI} = (1/4)\left[\frac{X^2 - (1+d)4X + (1+d)}{(1+d-X^2)^2}\right] \cdot \left[\frac{1}{1 - \phi(z) - \phi'(z)z}\right] < 0. \qquad (22)$$

It can be shown that, at an optimum, $[X^2 - (1 + d)4X + (1 + d)]$ is negative while $[1 - \phi(z) - \phi'(z)z]$ is positive, which enables one to sign equation (22). To understand the intuition behind the signs of these two terms, rewrite equation (22) as

$$[1 - \phi(z) - \phi'(z)z]dz + (-1/4)\frac{X^2 - (1+d)4X + (1+d)}{(1+d-X^2)^2}dI = 0. \qquad (23)$$

The coefficient of dz indicates the marginal increase in net revenues from the consumption tax that results from an increase of one unit in

gross revenues. It seems clear that, at an optimum, this coefficient cannot be negative because, if it were, net revenues from the last unit of gross revenues would be negative. A lower z would not only increase net revenues but would also increase, other things being equal, private consumption (see equation (19)). A similar argument applies to the coefficient of dI in equation (23). This coefficient represents the marginal increase in revenues from money creation when I is increased by one unit. Clearly, at an optimum, this coefficient cannot be negative because it would mean that, at the margin, revenues from money creation would be negative. Reducing I would both increase revenues from money creation and decrease transaction costs.

The schedule given by equation (21) is depicted in Figures 1 and 2 as OC (for optimality condition). This condition can be interpreted as follows. As shown in Kimbrough (1986), the inflation tax acts as a tax on consumption because it taxes an intermediate good, money, that is used to decrease the effective cost of consumption, given $(1 + \theta)[1 + v(X) - v'(X)X]$. The consumer is indifferent, therefore, to alternative combinations of the consumption and the inflation tax that imply the same effective tax on consumption. Thus, the government can be seen as choosing, for a given effective tax on consumption, that combination of the consumption and the inflation tax that minimizes the resource loss. Hence, schedule OC can be interpreted as the locus of points (I, z) where the loss of resources that results from collecting a given amount of revenue is minimized. This implies, roughly speaking, equating at the margin the resources lost in collecting both taxes. The loss in resources associated with the inflation tax is given by the higher transaction costs that result from lower real money balances. When marginal collection costs are constant (Figure 1), OC is a vertical line because, given that the marginal cost of collecting the consumption tax is constant, there is only one value of I that equates both margins. When marginal collection costs are increasing (Figure 2), OC slopes upward because an increase in z, for given I, raises marginal collection costs so that a rise in I, which by equation (13) increases marginal transaction costs, is required to equate margins once again. The analytical expression for the slope of the OC schedule is given by

$$\frac{dz}{dI} = -\frac{X^2 - (1 + d)}{4X^2[\phi''(z)z + 2\phi'(z)]} > 0. \tag{24}$$

As the denominator tends to zero (constant marginal costs), the schedule OC becomes a vertical line, as shown in Figure 1. When the denominator is positive (increasing marginal costs), the slope is positive because the numerator is negative, as illustrated in Figure 2. (Recall that

$d = 1/4$ and $X \leq 1/2$.) As shown in Figures 1 and 2, the intersection of the $FE(g = g_0)$ schedule and the OC schedule determines the equilibrium at point A. The optimal values of I and z are denoted by I_0 and z_0.

Effects of an Increase in Government Spending

Consider an increase in government spending from g_0 to g_1. Refer first to the benchmark case in which marginal collection costs are constant (that is, when $\phi(z) = \phi_o$, where $0 < \phi_o < 1$). In that case, as already indicated, the schedule OC becomes a vertical line at I_0 in Figure 1. The upward shift in the FE schedule implies that the new equilibrium (point B) occurs at the same inflation tax (that is, $I_1 = I_0$). Analytically, the change in z is given by $(dz/dg) = [1/(1 - \phi_o)]$. Net revenues from the consumption tax, therefore, increase by the same amount that government spending does. In other words, the increase in government spending is solely financed by additional revenues from the consumption tax.[17]

Figure 2 illustrates the case in which marginal collection costs are increasing. The increase in government spending from g_0 to g_1 shifts the equilibrium to point B, thus causing the inflation tax to rise to I_1. Revenues from the consumption tax become z_1. The analytical expressions for these changes are given by

$$\frac{dI}{dg} = -\frac{4X[\phi''(z)z + 2\phi'(z)]}{\Delta} > 0 \tag{25}$$

$$\frac{dz}{dg} = \frac{X^2 - (1+d)}{X\Delta} > 0, \tag{26}$$

where

$$\Delta \equiv [1 - \phi(z) - \phi'(z)z]\{[X^2 - (1+d)]/X\}$$
$$+ X[\phi''(z)z + 2\phi'(z)]\{[X^2 - 4(1+d)X + (1+d)]/(1+d-X^2)^2\} < 0.$$

Intuitively, the increase in government spending means that, for a given I, higher revenues from the consumption tax are needed, and this increases marginal collection costs. A rise in I, which increases marginal transaction costs, is therefore needed to equate both margins again.

[17]The result that the inflation tax remains the same is a general one, as indicated earlier. The specification of the model adopted in this section, however, also implies that when marginal collection costs are constant, revenues from the inflation tax remain unchanged (that is, real money balances are independent of g). This feature may not be robust under alternative specifications.

Note that it would be conceptually incorrect to interpret the increase in the inflation tax along the following lines: if needs for revenue increase, it seems reasonable that both taxes be raised to meet them. The case of constant marginal collection costs clearly refutes this interpretaton. It is the fact that the marginal collection costs of the consumption tax rise when government spending increases that triggers the rise in the optimal nominal interest rate.[18]

Effects of Different Marginal Collection Cost Schedules

Consider now the effects of parametric changes in the marginal collection cost schedule on the increase in the optimal inflation tax that results from a given rise in government spending. Suppose that $\phi(z) = kz$ (where $k \geq 0$ is a parameter), which implies that marginal collection costs are given by $2kz$ (that is, they are increasing in z except in the case in which $k = 0$), and consider the initial equilibrium that results when there is no government spending ($g_0 = 0$).[19] It is useful to concentrate on this initial equilibrium because both this equilibrium and the translation of the *FE* schedule (around this equilibrium) do not depend on the parameter k, so that one can compare the outcomes for different values of k at an analytical level. In the next section, the numerical analysis will suggest that the results generalize to any positive initial value of g.

Figure 3 illustrates the effects of different values of k. When k is zero, the *OC* schedule coincides with the vertical axis. The higher is k, the flatter is the *OC* schedule. The reason is that a higher value of k implies that a given increase in gross revenues from the consumption tax results in a larger rise in marginal collection costs; therefore, a higher rise in marginal transaction costs is needed, and this requires a larger increase in I. The initial equilibrium is at the origin, where the three *OC* schedules intersect the $FE(g = 0)$ schedule. Because there is no government spending, both the consumption and the inflation tax are zero. If government spending rises, the *FE* schedule shifts upward. The horizontal shift (that is, holding I constant) is given by $dz = dg$. When $k = 0$, the new equilibrium is at point A ($I = 0$, z_0). When k is positive—say, $k_1 > 0$—the

[18] If the (steady-state) budget deficit is defined as government spending minus net revenues from the consumption tax, it follows from the analysis that higher government spending leads to larger budget deficits and higher nominal interest rates.

[19] This specification of $\phi(z)$ enables one to isolate the distortion introduced by the need to finance a positive level of government spending. In other words, if $\phi(z) = \phi_o + kz$, the optimal inflation tax is positive even when $g = k = 0$, as follows f om the previous discussion.

new equilibrium is at point $B(I_1, z_1)$. A higher k—for instance, $k_2 > k_1$—yields point $C(I_2, z_2)$ as the new equilibrium. Thus, for a given increase in government spending, the larger is the parameter k, the higher is the increase in the inflation tax. Analytically, the corresponding expressions are:

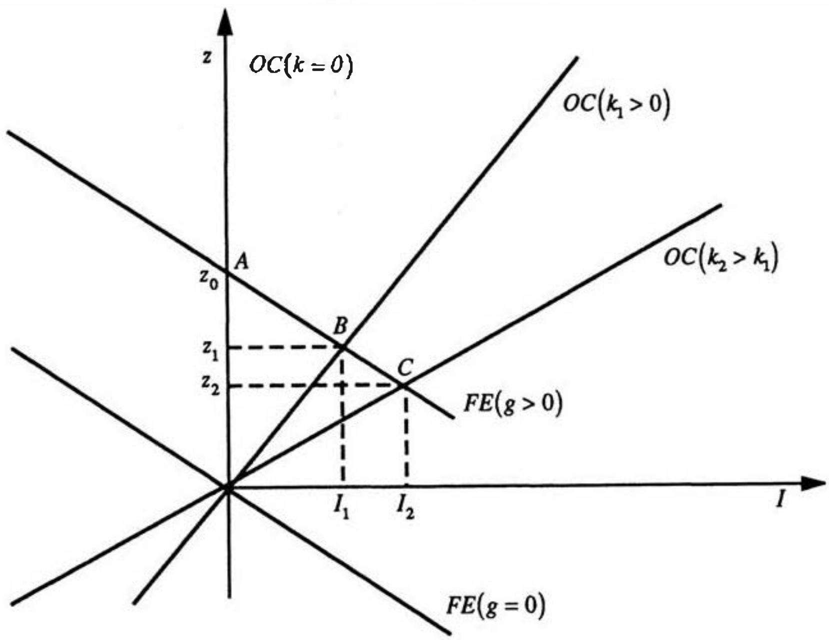
$$\frac{dI}{dg} = -\frac{8X^s k}{\Delta(g=0)} \gtrsim 0 \qquad \text{if } k \gtrsim 0 \tag{27}$$

$$\frac{dz}{dg} = \frac{(X^s)^2 - (1+d)}{X^s \Delta(g=0)} > 0, \tag{28}$$

where $\Delta(g = 0) = -(2 + k) < 0$ denotes the determinant of the system evaluated at the inital equilibrium. The signs of the different expressions follow from the fact that $X^s = 1/2$ and $d = 1/4$.

Because $\Delta(g = 0)$ is a decreasing function of k, it follows that expression (27) is an increasing function of k (note that a rise in k increases the numerator by more, in proportional terms, than it does the absolute

Figure 3. *Effects of Parametric Changes in Marginal Collection Costs*

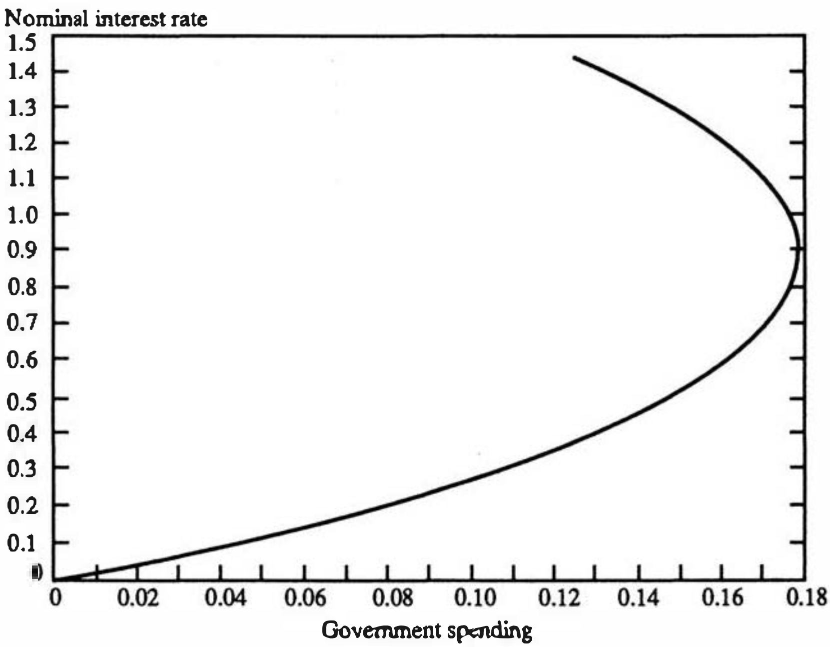


value of the denominator), whereas expression (28) is a decreasing function of k, which confirms the graphical representation. Given that $z = 0$ at the initial equilibrium, the increase in net revenues from the consumption tax is the same as the rise in gross revenues in all three cases examined. Because a higher value of k can be interpreted as representing less efficient tax-collection systems, the model predicts that a given increase in government spending will increase the inflation tax the most in countries with the least efficient tax-collection systems.

III. Numerical Analysis of the Model

In this section, a numerical analysis of the model makes it possible to gain further insights into the relationship between government spending, collection costs, and the optimal interest rate. The main results are the following. First, the nominal interest rate is a convex function of government spending; in other words, the higher is the level of government spending, the larger is the increase in the nominal interest rate that results from a given increase in government spending. Second, the more

Figure 4. *Optimal Nominal Interest Rate as a Function of Government Spending* ($k = 2$)



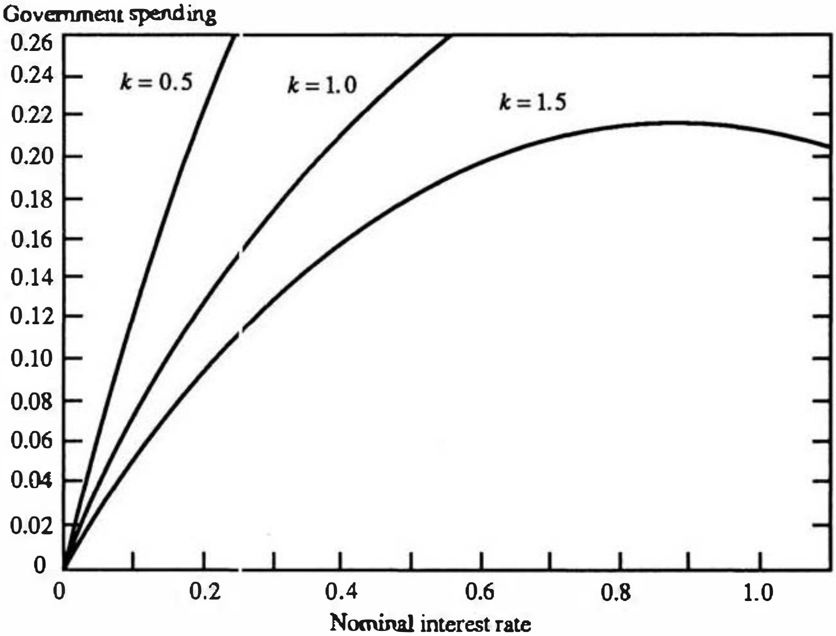
inefficient the tax-collection system is, the higher is the optimal nominal interest rate for a given level of government spending. Third, the share of revenues (relative to total revenues) of the inflation tax is a decreasing function of government spending.

For the purposes of the numerical analysis let $\phi(z) = kz$, $k > 0$, so that marginal collection costs are increasing. Figure 4 shows the optimal nominal interest rate as a function of government spending when $k = 2$. The backward-bending portion of the curve should be disregarded because those points, although they are feasible (in the sense that they satisfy equations (20) and (21) and do not violate the restriction that consumption be positive), do not represent optimum points, as follows from the theoretical analysis. The optimal interest rate is a convex function of government spending. This should come as no surprise given the convexity of total collection costs and the convexity of the transaction technology. Note that there is a limit to the amount of government spending that can be financed (in this case, just below 0.18, which is equivalent to 36 percent of nonleisure resources). Naturally, the maximum amount of government spending that can be financed depends on the magnitude of k. Lower values of k imply that higher amounts may be financed. When the upper bound on the level of government spending that can be financed is about to be reached, a small increase in government spending implies a substantial rise in the optimal interest rate.

Figure 5 shows the optimal nominal interest rate as a function of government spending for different values of the parameter k. (Note that the independent variable, government spending, is shown along the y-axis.) Two conclusions emerge from the picture. First, as regards the influence of k on the optimal *level* of the nominal interest, Figure 5 indicates that, for a given level of government spending, the larger is k, the higher is the optimal nominal interest rate. Therefore, if the value of k were to be reduced by rendering the tax-collection system more efficient, the nominal interest rate would fall, even if the level of government spending remained unchanged. Second, as regards the effects of k on the *changes* of the nominal interest rate, Figure 5 shows that, for a given increase in government spending, the higher is k, the larger is the increase in the nominal interest rate. This result was obtained analytically in the previous section when the initial level of government spending is zero; the numerical analysis thus suggests that the result remains valid for any initial positive value of government spending. Note also that the convexity of the optimal interest rate is confirmed for all values of k considered.[20]

[20]In the case in which $k = 0.5$, the convexity becomes more evident for higher levels of government spending.

Figure 5. *Optimal Nominal Interest Rate for Different Values of* k



Finally, the numerical analysis also reveals that the share of total revenues accounted for by the (net) revenues from the consumption tax is an increasing function of government spending, whereas the opposite is true of the share of revenues from the inflation tax. Different values of k were considered, and all yielded the same result: the share of revenues from the inflation tax decreases as government spending rises.[21] As one would expect, higher values of k imply a larger share of revenues from money creation. It is also the case, however, that higher values of k result in a more rapid fall of the share of revenues from the inflation tax.

IV. Conclusions

This paper has analyzed the relationship between government spending and inflationary finance in a public finance framework. It has been

[21] One would conjecture that this result, in particular, may not be robust under alternative specifications of the model because of the crucial role played by the particular form of the demand for money.

shown that, when marginal collection costs are increasing, a rise in government spending leads to an increase in the optimal nominal interest rate. A more inefficient tax-collection system implies a higher increase in the nominal interest rate for a given increase in government spending. The numerical analysis of the model suggested additional results. First, the nominal interest rate is a convex function of government spending; in other words, the higher is the level of government spending, the larger is the increase in the nominal interest rate for a given increase in government spending. Second, a more inefficient tax-collection system implies a higher optimal nominal interest rate for a given level of government spending. Third, the share of revenues from the inflation tax decreases as government spending increases.

From a policy perspective, the model lends support to the notion that, especially in developing countries, a reduction in government spending should lead to a reduction in the inflation tax because the ultimate reason for the existence of a high inflation tax lies in the fact that the government's need for revenues coexists with increasing marginal costs of collecting alternative taxes. An improvement in the efficiency of the tax-collection system should also be viewed as an important component in any attempt to reduce the reliance on the inflation tax. As the analysis has suggested, even if government spending is kept constant, a lower nominal interest rate would result from a more efficient collection of alternative taxes.

As regards extensions of this work, interesting issues may arise when a two-country world is considered. Végh and Guidotti (1989), for example, assume that both countries differ in both the level of government spending and the efficiency of the tax-collection system and examine the consequences on the optimal taxation structures of both countries of having to share a common inflation tax (due to fixed exchange rates) or a common consumption tax.

Owing to the absence of intrinsic dynamics in the model, the economy studied here is always in the steady state. Therefore, the analysis cannot consider the possibility of borrowed funds as a source of government revenue. An interesting extension of this work would be to introduce sources of dynamics in the model, thus giving the government the possibility of not balancing its budget in each period, and to study the optimal policy response to changes in government spending. One source of dynamics might be the assumption that the consumption tax cannot be adjusted instantaneously to its new level because of, say, the need for congressional authorization and other necessary administrative changes. This could explain a higher reliance on the inflation tax in the short run than in the long run after a permanent increase in government spending.

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