The Monetary Approach to the Exchange Rate: Some Empirical Evidence

JOHN F.O. BILSON *

This paper examines the empirical validity of a simple monetary model of exchange rate determination. The model is characterized as "monetary" because it assumes the existence of a stable money demand function and integrated world markets. The monetary model provides a useful tool for exchange rate analysis because it (a) clearly defines the role of speculation among the determinants of the exchange rate, (b) provides a simple definition of the equilibrium exchange rate, and (c) directly relates the equilibrium rate to the underlying instruments of monetary policy.

Professor Friedman, in his classic defense of flexible exchange rate regimes, stressed that flexible exchange rates need not be unstable. He wrote,

The ultimate objective is a world in which exchange rates, while free to vary, are in fact highly stable. Instability of exchange rates is a symptom of instability in the underlying economic structure. ²

These words offer little consolation to those who have experienced the volatile exchange rates of the current floating rate period. Many market participants appear to have regressed from Friedman's logic toward the belief that the exchange rate is determined by speculation and "market psychology" rather than by underlying economic conditions. This belief has been encouraged by the lack of a generally accepted

*Mr. Bilson, economist in the External Adjustment Division of the Research Department, is a graduate of Monash University, Melbourne, and the University of Chicago. During 1977-78 he is on leave from Northwestern University, where he is a member of the faculty.

¹The author gratefully acknowledges the advice and suggestions of Professors Robert Z. Aliber, Rudiger Dornbusch, Stephen P. Magee, and the late Harry G. Johnson. A special debt is due to Jacob A. Frenkel, whose detailed comments on an earlier draft substantially improved the quality of the paper.

²Friedman (1953), p. 158.
economic theory of the determination of the exchange rate, since, without such a theory, it is difficult to define the elements of the "underlying economic structure" that have been responsible for the erratic movements in the rates. This theoretical vacuum, induced primarily by the inability of economic models based upon trade flows to explain exchange rate movements in the inflationary environment of the 1970s, has been rapidly filled by a number of papers stressing the role of asset markets in the determination of the exchange rate. The asset market models concentrate on the mechanisms through which the exchange rate eliminates incipient capital flows, including adjustment in real money balances through exchange rate-induced price level variation and adjustments in nominal interest rates through changes in the expected rate of exchange rate depreciation.

The primary object of this paper is to examine the empirical validity of a simple asset market model of exchange rate determination. The model is characterized as "monetary" because it is based upon two assumptions associated with the "monetary approach to the balance of payments" and is carried over, in the manner suggested by Johnson, into the study of flexible rates. These assumptions are that the demand for money is a stable function of a limited number of aggregate economic variables and that, in the absence of transportation costs and restrictions upon trade, the law of one price will hold in international markets. In the monetary model, the law of one price appears in the form of the purchasing-power-parity and interest rate parity conditions that link international price and interest rate movements to developments in the foreign exchange market. In the simple model, these arbitrage conditions are assumed to hold at each point in time, although some dynamic aspects will be introduced into the money market analysis.

The model is tested using monthly data for the Federal Republic of Germany and the United Kingdom over the period from April 1970 to May 1977. The sample period includes the last years of the fixed rate system in order to increase the number of observations and because the fundamental assumptions of the monetary approach, as stated above, are not dependent upon the particular money supply rule followed by the central bank. This does not mean, of course, that the endogeneity of the money supply under fixed exchange rates can be ignored econometrically, but it does imply that the parameters of the


money demand function should be invariant with respect to the exchange rate regime. The deutsche mark/pound (DM/pound) rate provides an interesting and difficult test of the monetary approach. The interest lies in the fact that there has been a great deal of variation in the economic factors stressed by the theory, so that the influence of these factors on the exchange rate should be clearly discernible. The difficulty associated with the DM/pound rate is obvious—not only has this rate appreciated more rapidly than most other exchange rates but it has also exhibited sharp short-term fluctuations around the trend. The empirical analysis examines whether this history is consistent with the predictions of the monetary model.

The primary object of the paper is, therefore, to test a particular hypothesis concerning the determinants of the exchange rate, rather than to provide a detailed description of the behavior of the DM/pound rate. Consequently, the specific institutional factors that influence exchange rates and interest rates in the two countries are ignored, and no attempt is made to account for the endogeneity of the money supply, as is done, for example, by Artus (1976). Despite these limitations, the results may be useful as a foundation for individual country studies and as an indication of the relationship between monetary policy and the exchange rate.

I. The Monetary Theory of the Exchange Rate

The monetary theory is not new. Jacob Frenkel has traced its origins back to Ricardo and has uncovered the following insightful statement made by Lord Keynes in 1924:

What, then, has determined and will determine the value of the franc? First, the quantity, present and prospective, of the francs in circulation. Second, the amount of purchasing power which it suits the public to hold in that shape.  

Frenkel himself states the basis of the theory in the following words:

Being a relative price of two assets (moneys), the equilibrium exchange rate is attained when the existing stocks of the two moneys are willingly held. It is reasonable, therefore, that a theory of the determination of the relative price of two moneys should be stated conveniently in terms of the supply of and the demand for these moneys.

In the following, a simple monetary model of exchange rate determination will be developed that will reflect these statements. In order to lead directly into the empirical analysis, the model will be stated

\[6\] Frenkel (1976), p. 201.
using the explicit functional forms that will be estimated in the following sections. The demand for money, which is the central behavioral equation of the monetary approach, is assumed to be of the Cagan functional form:  

\[ \frac{M}{P} = ke^{-\epsilon y} \eta \]  

(1)

where \( M \) = the stock of money demanded; \( P \) = the price level; \( i \) = the rate of interest; \( y \) = the level of real income; and \( k, \epsilon, \) and \( \eta \) are parameters. In the simple model considered, real income and the money supply are assumed to be exogenous, and movements in the nominal rate of interest are assumed to primarily reflect exogenous movements in the expected rate of inflation. 

Money market equilibrium is consequently brought about through movements in the price level.

Although the monetary approach is based upon the theoretical concept of purchasing power parity, monetary models do not typically trace the influence of the exchange rate through conventional price indices like the consumer price index, the wholesale price index, or even traded goods price indices. As Magee has demonstrated, even the traded goods indices reflect contracts that may have been made up to 18 months previously and that will consequently appear to adjust slowly to the exchange rate. 

The belief that conventional price indices are not accurate reflectors of market prices has led to the practice of treating the "true" price index as an unobservable variable whose ratio, for any two countries, is defined by the exchange rate. If, for example, the true U. K. price index measures the number of pounds required to purchase a hypothetical composite good, and the true price index of the Federal Republic of Germany measures the number of deutsche mark required to purchase the same good, then the ratio of the German price index to the U. K. price index yields, by definition, the number of deutsche mark required to purchase one pound, which is, also by definition, the exchange rate between the two currencies. The purchasing-power-parity condition, as represented in equation (2), should be interpreted in this manner:

\[ S = \frac{P}{P^*} \]  

(2)

See Cagan (1956).

These assumptions are relaxed in structural asset market models, for example, Artus (1976) and Knight (1976). This paper should be considered as an investigation of one component of a wider structural model.

where \( S \) = the current spot rate (DM per pound); \( P \) denotes the German price index; and \( P^* \) denotes the U. K. price index.

Substituting equation (1) into equation (2), and assuming an identical money demand function for the foreign country, the relative money demand function described in equation (3) is derived.

\[
\frac{M}{SM^*} = k \left( \frac{y}{y^*} \right)^{\eta} e^{-\epsilon(i^*-i^*)}
\]

(3)

First consider the left-hand side of this equation; \( M \) represents the outstanding stock of deutsche mark, \( M^* \) the outstanding stock of pounds, and \( SM^* \) the DM value of the U. K. money stock. Hence, the ratio on the left-hand side of the equation is the relative real quantity of the two currencies: expressed in terms of the purchasing-power-parity condition, it is the number of units of the composite good that could be purchased by the Federal Republic of Germany's money supply relative to the number that could be purchased by the U. K. money supply. The right-hand side of the equation describes the factors that determine the relative demand for the two currencies—the interest rate differential and the real income ratio.

The interest rate differential represents the relative holding cost of the two currencies compared to other real and financial assets. If covered interest arbitrage equates the real return on assets denominated in the two currencies, the Fisher condition may be used to express the nominal interest rate differential as the difference between the expected rates of inflation of the two price indices. Thus an increase in the expected rate of inflation in the U. K., caused, for example, by a breakdown in wage negotiations, will be reflected in higher nominal interest rates on assets denominated in sterling. The increase in British interest rates will decrease the demand for British currency and depreciate the pound relative to the DM. From another angle, the expected inflation rate differential may be linked, through the purchasing-power-parity condition, to the expected rate of depreciation of the exchange rate. Speculation consequently enters the relative money demand function through the interest rate differential.

Again, as in the case of the purchasing-power-parity condition, the recent exchange rate literature presents a different interpretation of the role of speculation in the demand for money than is found in traditional empirical work on the demand for money. The traditional view, most often represented by the adaptive expectations hypothesis, considered the expected rate of inflation as a long distributed lag of past actual rates of inflation, so that current changes in the exogenous variables
were assumed to have little or no influence on it. Recent evidence, based upon the assumption that the forward premium is a market measure of the expected rate of depreciation, suggests that the adaptive model of expectations formation is not appropriate in the foreign exchange market. The premium has not moved in the stately fashion predicted by the adaptive expectations hypothesis but has appeared to be as erratic as the spot rate itself.

An explanation for this behavior is provided by the rational expectations models of the money market developed by Sargent and Wallace. Under the rational expectations hypothesis, the expected rate of inflation does not reflect past rates of inflation but is a rational forecast of future inflation, based upon all currently available information. Under these conditions, a monetary expansion not only triggers a depreciation of the exchange rate directly but it may also induce a further depreciation by creating expectations of future monetary growth. Bilson (1978) and Mussa (1976) have extended the Sargent and Wallace model to the problem of exchange rate determination; these models support the view, expressed by Friedman, that exchange rate instability is a symptom of instability in the underlying economic structure.

The second factor in the relative money demand function is the ratio of the real incomes of the two countries. Real incomes are included in the function as both a proxy for the wealth of domestic residents, who presumably have a preference for domestic currency, and as an index of the volume of transactions undertaken in the currency. It is possible that some currencies will develop into international means of payment, so that the level of domestic income will cease to be a good index of the volume of transactions. Under these conditions, the interest rate differential will be the predominant factor in the relative money demand function, as stressed by papers on currency substitution. This does not appear to be an important empirical problem in the DM/pound case. It is also noticeable that the monetary theory predicts that an increase in real income will appreciate the exchange rate under all circumstances. This prediction distinguishes the theory from the elasticities approach, which predicts an appreciation only in the case of export-led growth, since, with domestic growth, the increase in the demand for imports would tend to depreciate the rate.

The relative money demand function can be solved for the exchange rate to yield

\[ \text{exchange rate} = \frac{\text{real incomes ratio}}{\text{interest rate differential}} \]

10 See Sargent and Wallace (1973). This issue is examined in greater depth in the last section of the present paper.

11 See, for example, Calvo and Rodriguez (1977) and Girton and Roper (1976).
This equation forms the basis for the tests that will be undertaken in the next section of the paper. These tests are based upon the following proposition: that if the monetary approach is "correct," the parameter estimates obtained from the sample data should be consistent with other estimates of the money demand function. The sample evidence would not support the approach if, for example, higher real income depreciated the exchange rate, even if the predictive power of the model were high. However, given consistency in the estimates of the parameters, the model should be capable of predicting the exchange rate with as much accuracy as simple alternative models. Finally, if dynamic patterns of adjustment were considered, the asset market models would not be consistent with extremely long-lived adjustment patterns, since many of the papers on the monetary approach have stressed the rapid adjustment of the exchange rate to changes in the relative demand for, and supply of, money. The first task is to state these conditions in a formal, testable form. After the tests have been discussed, an informal evaluation of the usefulness of the monetary approach in explaining the evolution of the DM/pound rate will be presented.

II. The Empirical Evidence

In order to estimate equation (4), data are required for the money supplies, the levels of real income, and the interest rate differential. Current data on the money supplies are readily available on a monthly basis. The M2 definition of the money supply is used for the Federal Republic of Germany, and the M3 definition for the United Kingdom. A preliminary investigation suggested that these definitions were more suitable in the exchange rate equation than the narrower M1 definition was. Industrial production indices are used as proxies for the levels of real national income in both countries. Since these indices are widely used as leading indicators of gross national product, and are closely observed by exchange market participants, this substitution appears to be justified, at least for the short period of time considered in the analysis. Finally, the forward exchange premium, which is considered

\[
S = \frac{M}{M^*} \left[ \frac{y}{y^*} \right]^{-\eta} k^* \frac{k}{e^{(i-i^*)}}
\]
as a market estimate of the expected rate of depreciation, is used as a proxy for the interest rate differential. 13

In specifying the shift factor, \( k/k^* \), an attempt is made to allow for some exogenous movement in the relative demand for the two currencies. Of the factors responsible for such a change, the degree of uncertainty about monetary and fiscal policy would appear to be important. In addition, the declining role of the pound as an international means of payment and as a reserve currency may also be responsible for a secular decline in the demand for pounds relative to deutsche mark. These factors are accounted for by allowing for a trend in the shift factor, as specified in equation (5),

\[
\ln(k/k^*) = k_0 + \lambda t
\] (5)

where \( k_0 \) is a constant and \( \lambda \) is the rate of growth in the relative money demand.

Almost all of the postwar studies of the demand for money employ some form of distributed-lag mechanism in order to take account of the slow adjustment of the actual price level to the equilibrium price level. 14 The simplest and most common is the partial adjustment mechanism, whereby the change in the price level is proportional to the difference between the actual and equilibrium values of prices. This assumption is adopted in the following analysis, where it is assumed that the actual exchange rate adjusts toward the equilibrium rate according to equation (6),

\[
\ln(S) - \ln(S_{-1}) = \gamma[\ln(S) - \ln(S_{-1})]
\] (6)

where \( \gamma \) denotes the partial adjustment coefficient and \( S \) denotes the equilibrium exchange rate that is defined in equation (4). Substituting equations (4) and (5) into (6) yields, with the addition of an error term, the final estimating equation,

\[
\ln(S) = \beta_0 + \beta_1 \ln(M) + \beta_2 \ln(M^*) + \beta_3 (i-i^*) + \beta_4 \ln(y)
+ \beta_5 \ln(y^*) + \beta_6 t + \beta_7 \ln(S_{-1}) + u
\] (7)

where \( \beta_0 = \gamma k_0; \beta_1 = \gamma; \beta_2 = -\gamma; \beta_3 = \gamma \varepsilon; \beta_4 = -\gamma \eta; \beta_5 = \gamma \eta; \beta_6 = \gamma \lambda; \) and \( \beta_7 = 1 - \gamma. \)

13Treasury bill rate differentials were also considered, but were found to be less successful in explaining variation in the exchange rate. The forward premium is a better empirical proxy for the type of interest rate stressed by monetary theorists, since it is predominantly influenced by speculative factors. The exchange rate and forward exchange premium were taken from the Harris Bank data tape, which was provided by Professor Robert Z. Aliber of the Graduate School of Business, University of Chicago.
14See Goldfeld (1973) for a survey of the empirical literature on the demand for money.
The monetary model suggests the following hypotheses concerning the coefficients in equation (7):

(1) $\beta_1 / (1 - \beta_7) = -\beta_2 / (1 - \beta_7) = 1$. This hypothesis reflects the presumption that the system is homogeneous of degree one in nominal variables.

(2) $-\beta_4 / (1 - \beta_7) = \beta_5 / (1 - \beta_7) = \eta$, the income elasticity of the demand for money.

(3) $-\beta_3 / (1 - \beta_7) = \epsilon$, the semi-elasticity relating the interest rate to the demand for money.

If the actual values of the income elasticity and the interest rate semi-elasticity were known, the hypothesis on the coefficients could be tested by means of a simple $F$-test. The tests are complicated by the fact that the exact values of these coefficients are not known, so that a range of values may be considered consistent with the monetary approach. For this reason, it is necessary to formulate the tests in a stochastic manner by stating the probability limits within which the coefficients are expected to lie. The following statements are assumed to be consistent with generally accepted views on the money demand function:

(a) The 95 per cent confidence limits for the long-run elasticity relating the exchange rate to the money supply are 0.9 and 1.1.

(b) The 95 per cent confidence limits for the long-run income elasticity of the demand for money are 0.5 and 1.5.

(c) The 95 per cent confidence limits for the interest rate semi-elasticity are 0 and 3.

For the income elasticity, a fairly wide range of values are assumed to be consistent with the monetary approach, because the industrial production index is used as a proxy for the level of real national income. This elasticity must consequently also take account of the possibility that the elasticity of real income with respect to industrial production is not exactly unity. The interest rate semi-elasticity implies that confidence limits for the interest elasticity are 0 and $-0.3$ if the nominal rate of interest is 10 per cent. Assuming normality, these statements imply the following set of linear restrictions on the coefficients in equation (7):

$$
\begin{bmatrix}
\beta_0 \\
\beta_1 \\
\beta_2 \\
\beta_3 \\
\beta_4 \\
\beta_5 \\
\beta_6 \\
\beta_7
\end{bmatrix} = \begin{bmatrix}
1 \\
-1 \\
1.5 \\
-1 \\
1.5 \\
1 \\
1 \\
1
\end{bmatrix} + (1 - \beta_7) \begin{bmatrix}
\nu_1 \\
\nu_2 \\
\nu_3 \\
\nu_4 \\
\nu_5 \\
\nu_6 \\
\nu_7 \\
\nu_8
\end{bmatrix} \quad (8)
$$
where the \( v_i \) are normally distributed random errors with mean zero and standard errors equal to \( \sigma_{vt} \), where

\[
\sigma_{v_1} = \sigma_{v_2} = 0.05; \sigma_{v_4} = \sigma_{v_5} = 0.25; \sigma_{v_3} = 0.75
\]

A more difficult issue concerns the setting of a stochastic prior restriction on the adjustment coefficient \( \gamma \). Most of the recent theoretical work on the asset market approach to the exchange rate has stressed that asset markets adjust more rapidly than commodity or labor markets; an extreme version of this approach would argue that the adjustment coefficient is unity, so that the actual rate is always equal to the equilibrium rate. More generally, a testable specification of the concept of "rapid adjustment" is required. While recognizing its limitations, the concept employed in this paper will be that the monetary approach is consistent with a mean lag in the adjustment of the actual rate to the equilibrium rate that is uniformly distributed over the interval from 0 to 12 months. In effect, this restriction states that any mean lag that is less than 12 months is equally likely and is compatible with the asset market approach, while any mean lag that is greater than 12 months has a zero prior probability and is considered inconsistent with the predictions of the model. This statement can be translated into the following prior distribution on the coefficient on the lagged dependent variable:

\[
\beta_7 = 0.76 + v_6
\]

where \( v_6 \) is a normally distributed random variable with mean zero and standard error \( \sigma_{v_6} = 0.26 \).

Finally, no attempt is made to set a prior value on the coefficient of the time trend. This coefficient will presumably reflect developments that are specific to the experience of the two countries during the period under study. There is, consequently, little information from other studies of the demand for money that could be used to develop a prior distribution based upon information that is independent of the current experience of the two countries. In particular, the view that the demand for pounds has declined relative to the demand for deutsche mark is most often based upon an analysis of the two currencies over the last five years. Hence, it is not possible to construct an independent prior distribution on the value of this coefficient. Nor would it be useful to

---

15See Dornbusch (1976 b).

16Expressed in terms of the lagged dependent variable coefficient, the mean lag is equal to \( \beta_7/(1 - \beta_7) \). Allowing the mean lag to take values between 0 and 12, each having probability 1/13, the associated distribution of \( \beta_7 \) may be derived. This distribution has mean 0.76 and standard deviation 0.26.
do so, since the object of the test on the coefficients is to examine the
general validity of the approach for all countries, based upon the ex-
perience of the particular countries under examination. The test should
consequently be restricted to the coefficients that are assumed to be
common across countries.

The test on the coefficients can now be undertaken by testing whether
the prior information, contained in equations (8) and (9), is contradicted
by the sample evidence. Theil has constructed a test of the compatibility
of the prior and sample information, which is described in detail in
the Appendix. The logic behind Theil's test is as follows: since neither
the sample information nor the prior information provide complete
information about the values of the coefficients, the estimates from the
two sources are expected to differ from the true values and from each
other. Obviously, the difference between the prior and sample estimates
depends upon the covariance matrices of the two information sets. If,
however, after adjusting for the difference owing to this factor, the
squared difference between the sample and prior estimates is signifi-
cant, then the hypothesis that the two information sets are compatible
must be rejected. There are six restrictions on the coefficients. Under
the compatibility hypothesis, the normalized sum of squared differences
between sample and prior estimates is distributed according to the chi-
square ($\chi^2$) distribution with six degrees of freedom. Eighty per cent
of the observations of a sample drawn from this distribution will lie
within the range 2.204 to 10.645, while 90 per cent will lie within the
range 1.635 to 12.592. If the actual value of the statistic, estimated from
the sample and prior coefficients, falls outside this range, then the
hypothesis that the two information sets are compatible can be rejected,
because the incompatibility is an additional source of error between
the sample and prior estimates. The statistic, whose construction is
defined in the Appendix, has a value of 10.570, which lies within both
sets of confidence intervals. The test cannot, therefore, reject the hy-
pothesis that the sample and prior information are compatible at these
levels of significance. 18

An indication of the compatibility of the sample and prior information
may be obtained by comparing the restricted and unrestricted estimates
of equation (7). The restricted estimates are obtained by combining the
sample and prior information through the use of the Theil-Goldberger

---

18In the test, the estimated value of the coefficient on the lagged dependent variable
in the unrestricted regression was used to transform the covariance matrix of the prior
information. The unrestricted equation was also estimated subject to the assumption that
the residual was generated by a first-order autoregressive process.

©International Monetary Fund. Not for Redistribution
mixed estimation procedure. To allow for autocorrelation in the residuals, the procedure was implemented using the iterative Cochrane-Orcutt transformation. An additional estimation problem arises because, as can be seen in equation (8), the covariance matrix of the prior information depends upon the value of the lagged dependent variable coefficient. This covariance matrix is consequently also transformed after each iteration using the new value of the coefficient from the last iteration. The iterations were concluded when the estimates of the lagged dependent variable and the first-order autoregressive coefficients stabilized. The final estimates of the restricted and unrestricted equations are given in equations (10) and (11).

**Unrestricted—Sample Information Only**

\[
\ln(S_t) = \\
-1.1513 + 0.4656 \ln(M_t) + 0.0379 \ln(M_i^*) + 0.3452 (i - i^*) \\
(0.765) \quad (1.446) \quad (0.457) \quad (2.670) \\
-0.0316 \ln(y_t) - 0.1796 \ln(y_i^*) - 0.0075 t + 0.7090 \ln(S_{t-1}) \\
(0.2183) \quad (1.099) \quad (2.321) \quad (9.278) \\
+ u_t \quad (10)
\]

Standard error of regression = 0.0258 \quad \text{\(R\)-squared} = 0.9832

Estimate of first-order autoregressive coefficient = 0.2087 (1.943)

Durbin-Watson statistic = 1.9542

**Restricted—Mixed Sample and Prior Information**

\[
\ln(S_t) = \\
-0.2494 + 0.1882 \ln(M_t) - 0.1848 \ln(M_i^*) + 0.2600 (i - i^*) \\
(0.9007) \quad (5.172) \quad (5.172) \quad (2.741) \\
-0.1691 \ln(y_t) + 0.1911 \ln(y_i^*) - 0.0009 t + 0.8122 \ln(S_{t-1}) \\
(3.135) \quad (3.362) \quad (3.055) \quad (23.20) \\
+ u_t \quad (11)
\]

Standard error of regression = 0.0276 \quad \text{\(R\)-squared} = 0.9807

Estimate of first-order autocorrelation coefficient = 0.2496 (2.336)

Durbin-Watson statistic = 1.9707

A casual observation of equation (10) could lead to the conclusion that these results do not support the monetary model of exchange rate determination, based upon the fact that none of the coefficients of the

---

19 See Theil (1971), pp. 350-51. See the Appendix in this paper for details.

20 The numbers in parentheses are \(t\)-statistics.

21 For these estimates, the \(t\)-statistics are derived from the mixed estimation results. However, the summary statistics beneath the equation only refer to the sample information.
money supply or real income variables are statistically significant. The problem is, of course, that many of the variables on the right-hand side of the equation are highly collinear, so that the sample information cannot distinguish their influence on the exchange rate. Thus, when the prior information is included in the estimation in equation (11), all of the estimated coefficients become consistent with prior beliefs, while the predictive power of the model is not substantially altered, as can be seen by comparing the standard error of the two regressions.

The consistency of equation (11) with the predictions of the monetary approach may be best seen by deriving the associated expression for the equilibrium exchange rate. The equilibrium rate is derived by setting \( S_t \) equal to \( S_{t-1} \) in equation (11):

\[
ln(S_t) = -1.3280 + 1.0026 \ ln(M_t) - 0.9846 \ ln(M^{*}) + 1.3853 \ (i - i^{*})
\]

\[
(6.259) \hspace{1cm} (6.258) \hspace{1cm} (2.792)
\]

\[
- 0.9009 \ ln(Y_t) + 1.0183 \ ln(Y^{*}) - 0.0049 \ t
\]

\[
(3.341) \hspace{1cm} (3.623) \hspace{1cm} (3.247)
\]

Both of the money supply elasticities are extremely close to the expected value of unity, and the results indicate plausible income and interest rate elasticities of the demand for money. The interest rate elasticity, which is equal to the product of the regression coefficient and the interest rate, would be equal to \(-0.15\) if the rate of interest were 10.83 per cent. Interest elasticities of approximately \(-0.15\) have been reported by Goldfeld (1973), Laidler (1966), and Feige (1964). Equation (12) also indicates that the demand for pounds relative to deutsche mark has decreased at a rate of approximately 6 per cent per year over the sample period, leading to an exogenous trend rate of appreciation in the DM/pound rate.

In Chart 1, the change in the equilibrium exchange rate is decomposed into the changes owing to variation in four factors: relative money supply growth, relative real income growth, changes in the interest rate differential, and the trend in relative money demand. The trend factor, for example, is responsible for a 40 per cent depreciation of the pound over the sample period. This factor dominates the depreciation owing to relative money supply growth. Although relative money supply growth caused a 30 per cent depreciation of the pound over the period January 1972–May 1975, it has, if anything, led to an appreciation in the following period. During this later period, it appears that relative

\[22\] The \( t \)-statistics beneath the coefficients in this equation are approximations derived from a first-order Taylor's series expansion of the long-run elasticities.
real income growth has played a more important role than monetary policy in the depreciation of the pound relative to the DM.

**Chart 1. Sources of Change in Equilibrium DM/Pound Exchange Rate, April 1970–May 1977**

The most interesting aspect of Chart 1 is the attribution of almost all the short-term fluctuations in the exchange rate to the interest rate differential. If this differential is interpreted as a market estimate of the expected rate of depreciation, then the results offer strong support for the view that the short-term instability of the exchange rate has been owing to foreign exchange speculation. The sharp short-term fluctuations in the interest rate differential may imply that speculators do not have a clear view of the fundamental determinants of the exchange rate, so that slight shifts in opinion lead to dramatic changes in the expected rate of depreciation. These changes, in turn, lead to equally dramatic changes in the actual exchange rate that justify the diffuse expectations of speculators. Hence, although speculators may, on average, provide forecasts that are as accurate as available alternatives, a speculative "vicious circle" exists, in which diffuse expectations lead to erratic exchange rate movements, which, in turn, lead to more diffuse
expectations. McKinnon (1976) offers an extension to this argument based upon the belief that efficient speculators—notably private banks—are more risk averse than the inefficient speculators, so that exchange rate uncertainty leads to a greater weight being given to uninformed market participants in the determination of the forward rate. These arguments may be used to justify the use of forward or spot market intervention in order to smooth out short-term fluctuations in the exchange rate.

An alternative interpretation of this evidence is that the interest rate differential reflects a rational forecast of the impact of actual and anticipated changes in the exogenous variables on the current and expected future exchange rate. In this case, the stabilization of the interest rate differential requires stabilization of the underlying fundamental variables. The extent to which the monetary model is consistent with this "rational expectations" view of the interest rate differential will be examined in Section IV of the paper. In the next section, the predictive performance of the monetary model will be compared with the performance of two alternative empirical models of exchange rate determination.

III. Comparisons with Alternative Empirical Models

The alternative models are based upon the direct implementation of the purchasing-power-parity theory through the use of observable relative price variables. Before defining these models, it should be stressed that they are not alternative theoretical hypotheses, since the purchasing-power-parity condition is an integral part of the monetary approach. It may well be the case, however, that observable price indices are more closely related to the exchange rate than are the predicted values from the reduced-form monetary model. The choice between the two models rests upon two considerations. First, that observable price indices may not be accurate indicators of the true price index in the money demand function, and second, that the specified money demand function may not be an accurate specification of the true money demand function. The relative performance of the models depends upon the relative importance of these two factors.

The first purchasing-power-parity model simply relates the exchange rate to the ratio of consumer price indices,

\[ S_t = P_t/P^*_t \]  

(13)

where \( P = \) the consumer price index. A more sophisticated version of the purchasing-power-parity model states that the equilibrium exchange
rate is equal to the ratio of consumer price indices, while deviations from the equilibrium rate are owing primarily to short-term factors, particularly interest rate differentials. This model may be expressed as:

\[
\ln(S_t) - \ln(P_t/P_t^*) = \psi_0 + \psi_1(i^*-i) + \psi_2[\ln(S_{t-1}) - (\ln(P_t/P_t^*))]
\]

(14)

If the equilibrium is characterized by a stable exchange rate, then equation (14) states that the equilibrium rate will be equal to the ratio of consumer prices. In the short term, however, speculative factors, as represented by the interest rate differential, will influence the exchange rate more intensely than they will influence consumer prices, thus giving rise to short-run deviations from purchasing power parity. This model of exchange rate/price interaction may be justified by assuming that firms only change their prices in response to permanent changes in the exchange rate. 23

Estimating equation (14) over the period May 1970–May 1977 yielded the following results:

\[
\begin{align*}
\ln(S_t) - \ln(P_t/P_t^*) &= 0.1874 + 0.3490 (i^*-i) + 0.9127\ln(S_{t-1}) \\
(2.514) & \quad (3.432) \quad (26.300) \\
-\ln(P_t/P_t^*) & + \hat{\psi}_t
\end{align*}
\]

(15)

where a circumflex (') denotes the estimated value. These results compare favorably with the results from the monetary model. The standard error is, in fact, slightly smaller than the standard error of equation (11). The coefficient of the interest rate differential is significant and of the correct sign, and the estimate of the speed of adjustment—9 per cent per month—is plausible.

Unfortunately, statistical testing of alternative non-nested models is not an advanced econometric art. 24 Subject to some qualification, Theil suggests the residual-variance criterion as the basis for choosing between alternative specifications. 25 The residual-variance criterion is based upon the result that an incorrect specification will have, on

23It is extremely difficult to specify, within the single-equation framework, whether equation (14) is an equation determining prices, exchange rates, or interest rate differentials. It is, therefore, put forward as a simple empirical regularity without any strong presumption concerning causality.

24A nested model contains only a subset of the variables in a larger model. In contrast, a non-nested model includes at least one variable that is not present in the larger model.

average, a larger residual variance than a correct specification, so that the choice of the specification with the smallest residual variance will be, on average, correct. The test is complicated by the fact that two of the models include a lagged dependent variable that practically ensures a small residual variance and hence makes it very difficult to choose between the alternative models. For this reason, the tests are based upon a dynamic simulation of the models over the sample period, in which the lagged dependent variable is replaced by the previous period’s predicted value. In all of the models, the first-period value is set equal to the actual exchange rate.

A variety of summary statistics relating to the predictive performance of the three models are presented in Table 1. It is clear that the dynamic purchasing-power-parity (PPP) model is superior to both the monetary model and the simple PPP model as a predictor of the exchange rate; the predictions of the model are less biased and have a smaller standard error and range of errors than the alternatives. Second, it is clear that the monetary model is notably superior to the simple PPP model.

<table>
<thead>
<tr>
<th>Table 1. Summary Statistics from Dynamic Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(In per cent)</td>
</tr>
<tr>
<td>Statistic</td>
</tr>
<tr>
<td>Mean error</td>
</tr>
<tr>
<td>Standard error</td>
</tr>
<tr>
<td>Root-mean-square-error</td>
</tr>
<tr>
<td>Range of forecast error</td>
</tr>
<tr>
<td>Weight in composite forecast</td>
</tr>
<tr>
<td>Monetary Model</td>
</tr>
<tr>
<td>Mean error</td>
</tr>
<tr>
<td>Standard error</td>
</tr>
<tr>
<td>Root-mean-square-error</td>
</tr>
<tr>
<td>Range of forecast error</td>
</tr>
<tr>
<td>Weight in composite forecast</td>
</tr>
<tr>
<td>Monetary Model</td>
</tr>
<tr>
<td>(a) Weight in composite forecast</td>
</tr>
<tr>
<td>(b) Weight in composite forecast</td>
</tr>
</tbody>
</table>

1 Root-mean-square-error equals \( \left( \sum u_t^2 / T \right)^{1/2} \) where \( u_t \) is the forecast error for period \( t \), and \( T \) equals the number of time periods.

The last two rows in Table 1 give least-squares weights to the three forecasts. A composite forecast of the three models, chosen so as to minimize the sum of squared forecast errors, would give weights of 0.85 to the dynamic PPP model, 0.26 to the monetary model, and -0.12 to the simple PPP model. If the simple PPP model were eliminated from the composite forecast, the optimal combination would be a weight of two thirds on the dynamic PPP model, and one third on the monetary model.

The harsh truth is, consequently, that the monetary model does not improve upon a sophisticated PPP model as an exchange rate forecasting tool. This fact should not, however, be used to disguise the fact
that the monetary model also provides an accurate set of dynamic forecasts over the sample period, as can be seen in Chart 2, where the dynamic predictions of the two models are described. During the two periods when the predictions of the two models diverged, 1970–72 and 1974–75, the actual exchange rate lay between the predicted rates, thus supporting the idea that a composite forecast is appropriate.

The 1974–75 period is also the period during which the monetary model consistently underestimated the rate. One explanation for this result is that the flow of oil money into the London capital market during 1974 sharply increased the demand for sterling. Since this shift in demand is not accounted for in the relative money demand function, the underestimation of the actual exchange rate may be explained within a more general formulation of the monetary approach. The fact that the dynamic PPP model overestimates the exchange rate during this period may also be owing to the oil price increase, since U. K. prices responded more strongly and more rapidly than German (Fed. Rep.) prices to the increase in the price of oil.

Given that the predictive accuracy of the monetary model is not greatly inferior to that of the dynamic PPP model, the monetary model
may still be more useful as a rough guide to exchange rate developments. The main advantage of the monetary model is that it expresses the direct relationship between the exchange rate and the underlying instruments of government policy. By way of contrast, the dynamic PPP model is simply a relationship between three endogenous variables—prices, exchange rates, and interest rates—and it would be necessary to extend the simple framework in order to relate these variables to underlying exogenous factors.

This statement also points to the main weakness of the monetary model—the assumed exogeneity of the interest rate differential. In the next section of the paper, the relationship between the exchange rate and the relative levels of money and income will be discussed, under the assumption that the interest rate differential is equal to the rate of depreciation that is forecasted by the model itself. This assumption, known as the "rational" expectations hypothesis, is a useful way of testing the internal consistency of an econometric model. If the forecasts of the model are not consistent with market predictions of the exchange rate, it is likely that the econometric model is not fully capturing the determinants of the exchange rate. It is, of course, possible that the model could be correct, so that the inconsistency is owing to inefficient forecasting by speculators. However, it is extremely unlikely that such a situation could persist for any extended period of time, because this would imply the existence of profit opportunities for speculators who made use of the model's forecasts.

IV. Rational Expectations in a Partial Adjustment Model

In order to justify theoretically the inclusion of rational expectations in a partial adjustment model, it is necessary to distinguish between two sets of market participants. The first set consists of the vast majority of money holders, who do not engage in foreign exchange speculation and who base their decisions on interest rates, which they assume to be independent of their own asset preferences. The relative money demand function of this group is assumed to be represented by the partial adjustment mechanism, which is reproduced, in more compact notation, in equation (16),

$$\Delta s_t = \gamma [s_t + e x_t - s_{t-1}]$$ (16)

26The concept of "rational expectations" was introduced into economics by Muth (1961). Some tests of the forecasting efficiency of the forward exchange rate are presented in Bilson and Levich (1977); these authors find that the predictions of the forward rate are comparable, in terms of bias and standard error, to the predictions of simple time-series models.
where \( s_t = \ln(S_t); \ x_t = i - i^*; \) and \( \bar{s}_t = \) the equilibrium exchange rate.

The equilibrium exchange rate may be defined for both the monetary model and the dynamic PPP model,

**Monetary model:** \( \bar{s}_t = \ln(M/M^*) - \eta \ln(Y/Y^*) - \ln(k/k^*) \)  \( (16') \)

**Dynamic PPP model:** \( \bar{s}_t = \ln(P/P^*) \)  \( (16'') \)

The second group of market participants consists of the foreign exchange market speculators, who are assumed to purchase a quantity of forward exchange contracts sufficient to set the interest rate differential equal to the expected change in the exchange rate. The speculative function can therefore be written as,

\[ x_t = E \Delta s_{t+1} \]  \( (17) \)

where \( E \) is the expectations operator, conditional upon the information available at \( t \). This information set is assumed to include a knowledge of the behavioral function of the nonspeculative participants and of the process generating the equilibrium exchange rate. For simplicity, the equilibrium rate is assumed to follow a random walk, so that the current value is also the expected value for all future periods,

\[ E \bar{s}_{t+j} = \bar{s}_t; \quad j = 1 \text{ to } \infty \]  \( (18) \)

In this model, the current value of the spot rate depends upon all of the expected future changes in the rate. The solution to the model is found by beginning at some period \( J \) in which the expected change in the exchange rate is small enough to be ignored. Hence, at \( J \), equilibrium is reached,

\[ s_J = \bar{s}_t \]  \( (19) \)

The expected change in the exchange rate in period \( J-1 \) is then,

\[ E \Delta s_{J-1} = \gamma[\bar{s}_t + \epsilon(\bar{s}_t - E s_{J-1}) - E s_{J-2}] = \frac{\gamma(1+\epsilon)(\bar{s}_t - E s_{J-2})}{1+\epsilon \gamma} \]  \( (20) \)

Similar substitutions for earlier periods reveal that the expected rate of change in the spot rate may always be expressed as,

\[ E \Delta s_j = \alpha(j)[\bar{s}_t - E s_{j-1}] \]  \( (21) \)

The one-step-ahead forecast \( x_t \) may therefore be written as,

\[ x_t = \alpha(\bar{s}_t - s_t) \]  \( (22) \)

where the parameter \( \alpha \) is a function of the adjustment coefficient \( \gamma \) and
the interest rate semi-elasticity $\varepsilon$. The particular function is found by solving equations (16) and (22) for the first-period expected rate of change in the exchange rate,

$$x_t = E_s t+1 = \gamma[s_t + \varepsilon(1+s_t - E s_{t+1}) - s_t] = \frac{(1+\varepsilon\alpha)}{1+\gamma\varepsilon\alpha}[s_t - s_t]$$  \hspace{1cm} (23)

In order for equations (22) and (23) to be the same, $\alpha$ must equal $\gamma(1+\varepsilon\alpha)/(1+\gamma\varepsilon\alpha)$. This equality implies the following quadratic equation

$$\gamma - (1 - \varepsilon\gamma)\alpha - \varepsilon\gamma\alpha^2 = 0$$  \hspace{1cm} (24)

The solution for the positive root of this equation is,

$$\alpha = \frac{-(1-\varepsilon\gamma) + \sqrt{(1-\varepsilon\gamma)^2 + 4\varepsilon\gamma^2}}{2\varepsilon\gamma}$$  \hspace{1cm} (25)

Using the estimated values obtained above, the value of $\alpha$ for the monetary and dynamic PPP models can be estimated. For the monetary model, in which $\varepsilon$ is equal to 16.16 and $\gamma$ is equal to 0.1878, the estimated value of $\alpha$ is 0.758 per month. The same statistic for the dynamic PPP model, with $\varepsilon$ equal to 47.972 and $\gamma$ equal to 0.0873, is 0.787. The importance of these calculations for the relationship between the actual exchange rate and the equilibrium rate may be seen by noting that, from equations (16) and (23), the change in the current spot rate may be expressed as,

$$\Delta s_t = \alpha[s_t - s_{t-1}]$$  \hspace{1cm} (26)

Equation (26) demonstrates that the major consequence of the introduction of rational expectations is to increase significantly the adjustment to a change in the equilibrium exchange rate.

A mechanistic application of the model with exogenous interest rates would yield the prediction that a 10 per cent increase in the money supply would only depreciate the exchange rate by 1.9 per cent in the first month. By way of contrast, the rational expectations model predicts an immediate depreciation of 7.5 per cent. This sharp acceleration in the speed of adjustment is because of the induced movement in the expected rate of depreciation. Speculators anticipate the further depreciation of the rate as wealth holders move toward their new asset equilibrium position. As a consequence, the forward rate will depreciate by more than the spot rate, and nominal interest rates will increase. The rise in nominal interest rates will decrease the demand for money.

\footnote{The interest rate differential employed in the regression analysis is expressed as an annual percentage rate. It is therefore necessary to multiply the regression coefficient by 12 in order to arrive at the true monthly value of $\varepsilon$.}

©International Monetary Fund. Not for Redistribution
by asset holders and result in a further depreciation in the spot rate.

In Chart 3, the predictions of the rational expectations (R. E.) versions of the monetary and dynamic PPP models are plotted beside the actual exchange rate. These predictions are derived from a dynamic simulation over the sample period (Lagged dependent variables are replaced by the lagged predicted values.) with the starting value set equal to the actual exchange rate. It is immediately obvious that the R. E. version of the monetary model is more consistent with the sample data than the dynamic PPP model, with the possible exception of the first two years of the sample period. This implies that the PPP model possesses less internal consistency than the monetary model.

The preceding statement is not meant to imply that the performance of the monetary model is without fault. Large divergences between the actual and predicted rates have occurred in a number of instances: the sharp fall in the rate in April 1973 was not predicted by the model; and the errors are again large during 1974, presumably as a result of the oil


©International Monetary Fund. Not for Redistribution
revenues flowing into the London capital market. In both of these cases, however, the actual and predicted rates did not diverge for an extended period of time. The most interesting error in the monetary model is the failure to predict the depreciation of the pound during 1976. These errors have only shown a limited tendency toward self-correction during 1977, and the model suggests that the pound is undervalued relative to the DM at the present time, a suggestion that is consistent with the rapid accumulation of international reserves by the Bank of England during 1977. In the absence of this intervention, it is possible that the pound would have appreciated toward the rate predicted by the rational expectations monetary model.

It is, of course, possible that the model is incorrectly specified and that the errors in the last year of the sample period reflect this misspecification. There is, however, some evidence against this interpretation of the results. In particular, the model using market interest rate differentials predicts the depreciation of the pound during 1976–77 quite accurately, as can be seen in Chart 2. Consequently, the difference between the actual exchange rate and the predictions of the R.E. monetary model reflects a difference between the rate of depreciation expected by the market, as measured by the interest rate differential, and the rate predicted by the rational expectations model. In particular, the sharp depreciations of the pound between December 1975 and March 1976, and between June and August in 1976, were associated with sharp increases in the interest rate differential between the United Kingdom and the Federal Republic of Germany that cannot be related to changes in relative money supply or real income growth. This analysis suggests that the depreciation of the pound during 1976 was, to some extent, owing to irrational speculation—speculation not based upon a rational forecast of the effect of the exogenous variables on the future path of the exchange rate. However, the active intervention by the Bank of England during 1977 has probably contributed to the persistence of the difference between the actual and equilibrium exchange rates. This situation may not persist for an extended period of time, because the increase in the Bank of England’s stock of international reserves may eventually increase the money supply in the United Kingdom and induce a depreciation of the equilibrium rate.

There is a significant difference between the actual behavior of interest rates and the behavior predicted by the R. E. monetary model. These differences are summarized in Table 2. These statistics show that, while the mean rate of appreciation was substantially greater than the rate predicted by the R. E. model, the variance around the mean was substantially smaller. This is because of the fact that, in the rational
Table 2. Interest Rate Differential Between Deutsche Mark and Sterling Eurocurrency Deposits, April 1970–May 1977

(In per cent)

<table>
<thead>
<tr>
<th></th>
<th>Actual</th>
<th>Predicted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-5.289</td>
<td>-1.594</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>3.809</td>
<td>11.823</td>
</tr>
<tr>
<td>Range</td>
<td>-17.10 to 3.83</td>
<td>-36.73 to 26.87</td>
</tr>
</tbody>
</table>

Both the actual and predicted interest rate differentials are expressed in annual rates.

expectations model, interest rates react strongly to differences between the actual and equilibrium rates, and thereby rapidly eliminate any differential. Casual observation suggests that actual interest rates are less sensitive to exchange market disequilibrium, so that interest rate movements are less volatile, and differences between actual and equilibrium rates are eliminated more gradually. These results offer some support to McKinnon's conjecture that the problem with floating rates is related to insufficient speculation, in addition to irrational speculation.

V. Conclusion

The main body of this paper has been concerned with the description and testing of a simple monetary model of exchange rate determination. The results suggest that the actual behavior of the DM/pound rate during the period since 1970 is broadly consistent with the predictions of the monetary model. This conclusion is based upon three findings:

1. The estimated parameters of the model were consistent with estimates obtained from other studies of the demand for money.

2. In a dynamic simulation, the model predicted more accurately than a simple purchasing-power-parity model, and almost as accurately as a dynamic version of the purchasing-power-parity hypothesis, in which short-run deviations from purchasing power parity were caused by interest rate differentials.

3. In a rational expectations variant of the model, in which the expected rate of depreciation was set equal to the rate predicted by the model, the monetary model remained broadly consistent with the sample evidence.

If, on the basis of this evidence, the monetary model could be accepted as "true," it would be an extremely useful tool for exchange rate analysis. The most important contribution of the approach is the

28See McKinnon (1976).
simple specification of the equilibrium exchange rate. This concept is directly related to the underlying relative money supplies, so that it is a simple matter to calculate the monetary contraction or expansion required to set the equilibrium exchange rate equal to some target level. Second, the approach may be used as an indicator of the disequilibrium in the exchange rate. In this regard, the model's prediction that the pound has been undervalued since 1976 is particularly interesting. Finally, the monetary approach may be used to investigate whether private speculation is stabilizing or destabilizing, since, within this framework, the question may be usefully restated as asking whether movements in the interest rate differential are related to the predictions of the rational expectations model.

Unfortunately, the tests undertaken in the paper are not strong enough to yield an indisputable affirmative conclusion on the validity of the model. Of the many limitations, the failure to take account of the causal relationships between the variables in the money demand function appears to be the most serious. In particular, the interest rate differential was shown to be related to the spot exchange rate in a particular way if expectations were rational, but this interrelationship was not taken into account in the estimation.\footnote{The other major weakness of the present attempt is the failure to take account of (1) the slow adjustment of commodity prices, and (2) the influence of risk on the relative demand for the two currencies. These problems will be tackled in later work.}

Despite these limitations, the results suggest that the monetary approach, which has previously been exclusively applied to long-run analysis or to situations in which monetary expansion was the overwhelming cause of exchange rate depreciation,\footnote{See Bilson (1977) for a long-run study of the monetary approach. Frenkel (1976) applies a similar approach to the German hyperinflation of the early 1920s.} may also be useful in the analysis of short-run behavior and as a guide to intervention policies.

APPENDIX

In this Appendix, the test of the compatibility between the sample and prior information is explained. The sample information may be compactly stated in the relation,
\[
y = X \beta + u
\]
where \( y \) = the dependent variable
\( X \) = the matrix of independent variables
\( \beta \) = the vector of regression coefficients
and
\( u \) = the vector of residuals.

The residuals are assumed to be independently and identically distributed with variance \( \sigma^2_u \).

Similarly, the prior information may be stated as
\[
r = R \beta + v
\]
where \( r' = [1, -1, 1.5, -1, 1, 0.76] \)
\[
R = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 1 & 0 & 0 & 1.5 \\
0 & 0 & 0 & 0 & 1 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]
\[
v' = [(1-\beta_1)v_1, (1-\beta_2)v_2, (1-\beta_3)v_3, (1-\beta_4)v_4, (1-\beta_5)v_5, v_6]
\]

The compatibility test is a test of whether the regression coefficients \( \hat{\beta} \) estimated from the sample data are significantly different from the values specified in the prior information. The difference between the sample and prior estimates is,
\[
r - R\hat{\beta} = -R(\hat{\beta} - \beta) + r - R\beta
\]
where \( r \) is the point estimate of the regression coefficients from the prior information, while \( R\hat{\beta} \) is the point estimate from the sample data. Equation (29) indicates the two sources of difference between the sample and prior estimates that are consistent with the compatibility hypothesis: (1.) The sample estimate \( \hat{\beta} \) may not be equal to the true coefficient vector, and (2.) The prior estimates are also not known with certainty. For these reasons, the sample and prior estimates are expected to differ, even if the two information sets are compatible. It is therefore necessary to know the extent of the difference that is owing purely to the underlying distributions of the sample and prior errors. It will then be possible to test whether the sum of squared differences between the estimates is within the range predicted by these distributions.

The covariance matrix of the difference between the point estimates is,
\[
V(r-R\hat{\beta}) = \sigma^2_v R(X'X)^{-1} R' + \nu
\]
where \( \nu \) is the variance/covariance matrix of the prior information. The quadratic form,
\[
(r-R\hat{\beta})'\left(\sigma^2_v R(X'X)^{-1} R' + \nu\right)(r-R\hat{\beta})
\]
is consequently the sum of \( q \) squared standardized normal variables, which are distributed \( \chi^2(q) \), where \( q \) is the number of prior restrictions, under the compatibility hypothesis. Ninety per cent of the observations drawn from their distribution, with six degrees of freedom, will lie within the range from 1.635 to 12.592. If the actual value of the statistic lies outside of this range, then the hypothesis that the sample and prior information are compatible is rejected by the test at this level of significance. Since the actual value of the statistic was 10.570, which lies within the confidence interval, it was not possible to reject the hypothesis that the two information sets are compatible.
The two information sets were then combined into one equation, as specified in
\[
\begin{bmatrix}
y \\
R
\end{bmatrix} = \begin{bmatrix} X \end{bmatrix} \beta + \begin{bmatrix} u \\
y \end{bmatrix}
\] (32)

Since the two errors have differing variances, it is not possible to obtain efficient estimates by simply applying ordinary least squares to equation (32). Multiplying the prior information by \( \sigma_y/\sigma_u \) standardized the variance of the prior errors, so that they had the same variance as the sample errors. Application of ordinary least squares to the transformed equation yields the appropriate generalized least-squares estimate of the regression coefficients. This technique is discussed in detail in Theil (1971, pp. 350-51).

**BIBLIOGRAPHY**


———, and Richard M. Levich, "A Test of the Forecasting Efficiency of the Forward Exchange Rate" (mimeographed, New York University, Graduate School of Business Administration, 1977).


