The Adaptive Expectation Model of the Demand for Money: Some Further Results

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RECENT EMPIRICAL STUDIES of the demand for money have applied distributed lag models to specifications of monetary behavior. One such study by Joseph Adekunle [1] focused on the manner in which adaptive expectations affect portfolio behavior. The present paper is a further investigation into the adaptive expectation model of the demand for money.

One of the explicit properties of the adaptive expectation model is the (theoretical) presence of autocorrelated disturbances. An estimation procedure is utilized here to take into account autocorrelation of the residuals. In particular, an autoregressive mechanism for the serially correlated error term is postulated and estimated simultaneously.[2] As numerical examples, regression results are reported for the United States, Canada, Australia, and Norway.

I. Model A

Assume that the demand for real cash balances, m_t^d, is a function of expected or "permanent" real income, y_t^e, and the current rate of inflation, $p_t - p_{t-1}$, where all variables are in logarithms.

$$m_t^d = a_0 + a_1 y_t^e + a_2(p_t - p_{t-1}) + u_t \tag{1}$$

Here, a_1 is a constant income elasticity and a_2 is a constant inflation elasticity. It is assumed that

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[1] Joseph O. Adekunle, "The Demand for Money: Evidence from Developed and Less Developed Economies," *Staff Papers*, Vol. XV (1968), pp. 220–66.

[2] This procedure is superior to the Durbin-Watson test, which yields biased estimates of the autoregressive coefficient. If negative, the estimate is biased toward zero, in which case one might be misled into thinking that serial correlation is not present when in fact it is. A typical procedure to eliminate serial correlation is to use the first differences of the variables, but this assumes that the autoregressive coefficient is equal to 1.

$$E(u_t) = 0 \text{ for all } t;$$
$$E(u_t u_s) = 0 \text{ for } t \neq s;\ t,\ s = 1, 2, \ldots, n;$$
$$E(u_t u_s) = \sigma_u^2 \text{ for } t = s;\ t,\ s = 1, 2, \ldots, n.$$

Next, assume that expected or "permanent" real income is generated by a geometrically decaying weighted average of current and past values of actual real income.

(2) $$y_t^e = \lambda[y_t + (1-\lambda)y_{t-1} + (1-\lambda)^2 y_{t-2} + \ldots] \qquad 0<\lambda<1$$

The parameter λ is the elasticity of income expectations. To close the model, it is necessary to specify the supply of real cash balances and the demand/supply adjustment mechanism.

(3) $$m_t^s = m_t^o$$

(4) $$m_t^d = m_t^s$$

Equation (3) specifies that the supply of real cash balances is determined exogenously. Equation (4) assumes that in the short run the demand for money adjusts, without a lag, to a change in the money supply.[3]

By successive substitutions and use of the Koyck transformation,[4] the system of equations (1)–(4) reduces to

(5) $$m_t^o = b_0 + b_1 m_{t-1}^o + b_2 y_t + b_3(p_t - p_{t-1}) + b_4(p_{t-1} - p_{t-2}) + v_t$$

where

(6) $$v_t = u_t - (1-\lambda)u_{t-1}$$

and where the b vector is a function of the structural parameters of the model.

Take the mean, variance, and autocovariance of (6):

(7) $$E(v_t) = 0$$
$$E(v_t^2) = (2-2\lambda + \lambda^2)\sigma_u^2$$
$$E(v_t v_{t-1}) = -(1-\lambda)\sigma_u^2 \neq 0$$

[3] A more general demand/supply adjustment mechanism is

$$m_t^d = \gamma m_t^s + (1-\gamma)m_{t-1}^d$$

or, $$m_t^d = m_{t-1}^d + \gamma(m_t^s - m_{t-1}^d).$$

The parameter γ, which lies between 0 and 1, is the adjustment elasticity.

[4] See L.M. Koyck, *Distributed Lags and Investment Analysis* (Amsterdam, 1954).

Because $0<\lambda<1$ by assumption, the first autocovariance of v_t is necessarily nonzero. Model A is estimated and the results are presented in Section III.

II. Model B

Assume a first-order autoregressive process for v_t,[5]

$$v_t = \rho v_{t-1} + e_t \qquad |\rho| < 1 \tag{8}$$

where ρ is the autoregressive coefficient. It is assumed that

$$\begin{aligned} E(e_t) &= 0 \text{ for all } t; \\ E(e_t e_s) &= 0 \text{ for } t \neq s;\ t,\ s = 1, 2, \ldots, n; \\ E(e_t e_s) &= \sigma_e^2 \text{ for } t = s;\ t,\ s = 1, 2, \ldots, n. \end{aligned}$$

Combining (8) with the equations of Model A results in

$$\begin{aligned} \overset{o}{m}_t = {} & c_0 + c_1 \overset{o}{m}_{t-1} + c_2 \overset{o}{m}_{t-2} + c_3 y_t + c_4 y_{t-1} + c_5(p_t - p_{t-1}) \\ & + c_6(p_{t-1} - p_{t-2}) + c_7(p_{t-2} - p_{t-3}) + e_t. \end{aligned} \tag{9}$$

Again, the c vector is a function of the structural parameters.

III. Estimation of Models A and B

To isolate the possible loss of efficiency owing to the presence of autocorrelated disturbances, the parameters of both Models A and B were estimated by constrained nonlinear two-stage least squares.[6] A constrained nonlinear estimation technique is necessary because both reduced equations of the two models are overidentified; the two-stage procedure avoids simultaneous equation bias.

Of the 18 individual countries covered by Adekunle's study,[7] only 4—the United States, Canada, Australia, and Norway—have quarterly data on gross national product, money, and prices. The data, drawn from the IFS Data Fund, consist of 36 quarterly observations for the

[5] This is rather arbitrary; it is only a first approximation.

[6] The instrumental variables used were high-powered money, government expenditures, and exports. The nonlinear least-squares method was the BMDX85 program, based on a Gauss-Newton iterative procedure.

[7] Adekunle, *op. cit.*

United States, 47 for Canada, 42 for Australia, and 30 for Norway. The empirical results are reported in Tables 1 and 2.

TABLE 1. MODEL A: PARAMETERS ESTIMATED BY IMPOSING NONLINEAR CONSTRAINTS AND USING A TWO-STAGE LEAST-SQUARES TECHNIQUE [1]

	Parameter Estimated		
Country	Income elasticity	Inflation elasticity	Elasticity of income expectations
United States	0.26330 (0.05640)	0.28948 (0.90428)	0.75452 (0.01767)
Canada	1.08320 (0.07690)	0.46836 (0.64787)	0.35537 (0.12417)
Australia	0.35760 (0.05990)	−1.91630 (0.75940)	0.59788 (0.13418)
Norway	0.68532 (0.11070)	−1.37460 (1.11210)	0.90718 (0.19544)

[1] Asymptotic standard deviations of the parameters are in parentheses.

TABLE 2. MODEL B: PARAMETERS ESTIMATED BY IMPOSING NONLINEAR CONSTRAINTS AND USING A TWO-STAGE LEAST-SQUARES TECHNIQUE [1]

	Parameter Estimated			
Country	Income elasticity	Inflation elasticity	Elasticity of income expectations	Auto-regressive coefficient
United States	0.27117 (0.05133)	−0.04656 (1.00190)	0.69916 (0.01881)	−0.19189 (0.18515)
Canada	1.07600 (0.07340)	0.47596 (0.58743)	0.54892 (0.25725)	0.32637 (0.29418)
Australia	0.46499 (0.04098)	−1.87620 (0.51610)	0.93736 (0.12341)	0.70502 (0.11611)
Norway	0.67384 (0.09819)	−0.84551 (0.72902)	0.42847 (0.15550)	−0.86421 (0.11870)

[1] Asymptotic standard deviations of the parameters are in parentheses.

In the last two regressions (Australia and Norway), where first-order serial correlation in the residuals was incorporated in the estimation procedure and found to be highly significant, the asymptotic standard deviations of the estimated elasticities are reduced by about 8–35 per cent.

IV. Conclusion

The error term of the reduced equation is serially correlated under the adaptive expectation model even if the underlying disturbance terms are serially uncorrelated. This suggests that a more efficient test of the adaptive expectation model could be made by incorporating in the estimation procedure an autoregressive mechanism for the serially correlated disturbance term. Regressions were run using relatively large samples for several individual countries covered in Adekunle's study; Adekunle's method and the proposed technique were employed on the same sets of data. Considerable efficiency was gained, as theoretically expected, when the proposed method was used. Model B can be extended (1) to handle more explanatory expectation variables (expected interest rates, expected rates of inflation, etc.), (2) to allow a systematic lag between the supply of money and the demand for it, and (3) to cases where the error term of the reduced equation follows an autoregressive process of second, or higher, order. Naturally, computations would become quite cumbersome in these instances, but the gain would be an increase in the precision of the parameter estimates.

Modèle des anticipations de la demande de monnaie ajustées de période en période : quelques résultats additionnels

Résumé

Dans le modèle des anticipations de la demande de monnaie ajustées de période en période, il existe une autocorrélation entre les termes d'erreur de l'équation réduite, même s'il n'existe aucune corrélation sérielle entre les résidus des équations structurelles sous-jacents. Cette observation permet de penser que l'on pourrait effectuer un test plus précis de ce modèle si on introduisait dans la méthode d'estimation un processus d'autorégression pour les termes d'erreur entre lesquels existe une autocorrélation. Des régressions ont été calculées sur la base de larges échantillons de données trimestrielles relatives Etats-Unis, au Canada, à l'Australie et à la Norvège. Comme le prévoyait la théorie, l'utilisation de la méthode proposée a considérablement accru l'efficacité de la vérification.

El modelo de expectativas adaptantes para la demanda de dinero: resultados adicionales

Resumen

El término de error de la ecuación reducida está correlacionado en serie según el modelo de expectativas adaptantes, aun cuando los términos de perturbación básica no tengan correlación en serie. Ello sugiere que podría efectuarse una comprobación más eficaz del modelo de expectativas adaptantes, introduciendo en el procedimiento de cálculo un proceso autoregresivo para el término de error con correlación en serie. Las regresiones se efectuaron utilizando amplias muestras con datos trimestrales para Estados Unidos, Canadá, Australia, y Noruega. Como cabía esperar teóricamente, se ganó considerable eficiencia al aplicar el modelo propuesto.