# Appendix I. Model Details[1]

**1. The government maximizes household utility by choosing a change in structural primary balance to stabilize output fluctuations intertemporally under constraints.** The value function of the government is

$$V_t(d_{t-1}, gap_{t-1}, pb_{t-1}^{st}) = \max_{\Delta pb_t^{st}} E_t[u(c_t, L_t) + \beta V_{t+1}(d_t, gap_t, pb_t^{st})]$$

where $t$ is the year, $d_t$ is the gross government debt to potential GDP ratio, $gap_t$ is the output gap, $pb^{st}_t$ is the structural primary balance, $c_t$ is aggregate consumption[2], $L_t$ is labor, u(.,.) is the instantaneous utility function and $\boldsymbol{\beta}$ is the discount factor. The state of the economy is summarized by three variables: government debt, the output gap and the structural primary balance. The optimization is subject to the structure of the economy and the government budget constraint that takes the form of a risk to lose market access rising in debt (see below).

**2. The value function consists of the per-period utility function u(.) and the expected continuation value discounted by β.** The per-period utility function is:

$$u(c_t, L_t) = \frac{{c_t}^{1-\sigma}}{1-\sigma} - \xi y_t^{*1-\sigma} \frac{L_t^{1+\eta}}{1+\eta}$$

which is a standard constant relative risk aversion utility function in consumption and labor where $\boldsymbol{\rho}$ is the parameter of risk aversion. Households enjoy consumption, but also face labor disutility. Utility peaks at an equilibrium output for which the marginal income gain of work equates the marginal loss of utility due to labor. $\xi$ is calibrated so that utility peaks when output is equal to its potential. In other words, utility declines not only if output decreases below its potential, but also if output increases above potential, consistent with the view that positive output gap can be associated with costly distortions. This gives the government a motive to counter output deviations from this potential.

**3. The model features rising market pressure when debt is rising**. First, the interest rate increases in public debt, with a calibration in line with empirical evidence (Gruber and Kamin 2012; Poghosyan 2012; D'Agostino and Ehrmann 2014; Fall and Fournier 2015; Henao-Arbelaez and Sobrinho, 2017). This sensitivity of the interest rate to debt reflects a higher risk premium, it can be regarded as the consequence of an excess of supply of government bonds. Furthermore, the risk premium increases in the change in debt; investors are more likely to be concerned if debt is rising. Symmetrically, even at high debt level, risk premium may be moderate if the government shows its capacity to reduce it. Second, a risk to lose market access rules out unbounded debt paths. The probability to lose market access also depends on the level and the change of government debt:

[1] This appendix follows closely Fournier (2019).

[2] Public and private consumption are not distinguished, and hence assumed to provide the same utility.

$$P(lma) = [1 + exp(d_1(1 - d_t/\overline{d} - d_2(d_t - d_{t-1})))]^{-1}$$

where $d_1$ governs the debt limit uncertainty, $d_2$ governs the effect of a debt change on the risk to lose market access, and $\overline{d}$ is the debt level at which the probability to lose market access is 50 percent (given no change in the debt level). If the government loses market access, the government has to keep debt constant under an adverse scenario of a shock of $d_3\sigma$, where $\sigma$ is the standard deviation of economic shocks, to be explained below.

4. **The budget constraint of the government is governed by a standard debt accumulation dynamic,** with a deterministic stock-flow adjustment $sf_t$ that can capture planned one-offs:

$$d_t = \left(\frac{1 + r_0 + \alpha_1 d_{t-1} + \alpha_2 \Delta d_t)}{1 + g_t{}^*}\right) d_{t-1} - pb_t + sf_t$$

5. **Output is driven by a long-term exogenous potential growth and hysteresis costs in the long-run.** Output is produced by a standard linear production function in labor:

$$Y_t = A_t L_t$$

where $A_t$ is productivity and L is labor. Potential output $\bar{Y}_t$ is the output that would prevail if labor is at its equilibrium level $\bar{L}$:

$$\bar{Y}_t = A_t \overline{L}$$

6. **Productivity is affected by a permanent hysteresis effect of crisis**. If production is below its perceived potential, unemployed workers can see their skills, their network and their morale all decay (Blanchard and Summers, 1987, DeLong and Summers, 2012).

$$A_t = A \prod_{\tau=1}^{t} \left( (1 + g_\tau^*) \left( 1 + h.\left( min(gap_{\tau-1}, h^{th}) - h^{th} \right) \right) \right)$$

where $A = \overline{L} = 1$ and $g^*$ is potential growth that would prevail in the absence of hysteresis.

7. **The parameter $h \geq 0$ governs the size of hysteresis, a permanent loss of potential output level, and $h^{th}$ is a threshold below which hysteresis kicks in.** The calibrated effect on output level is in line with Mourougane (2017) who finds large hysteresis effects on potential GDP level but no effect on long-run potential growth.

8. **The output deviates from its potential because of a process of shocks $v_t$ and because of the primary balance**. The sensitivity of the output gap to the primary balance is its derivative with respect to the primary balance, which is set equal to a usual fiscal multiplier $m_1$ when the economy is at output equilibrium. This is consistent with the literature, which either defines the fiscal multiplier as the effect of level of primary balance (or tax, spending level) on a level of output (or

consumption, investment) as in Blanchard and Perotti (2002), or matches first differences on both sides (e.g., Alesina et al. 2015 in the empirical literature or Zubairy 2014 in the modeling literature). The fiscal multiplier depends on the output gap itself, reflecting recent empirical literature on larger multipliers in downturns (Baum et al, 2012; Auerbach and Gorodnichenko 2013), corroborated by modeling with financial frictions (Canzoneri et al., 2016). Indeed, when slack is large, a demand stimulus is more likely to boost output as there is spare production capacity. The additional term governed by coefficient $m_2$ magnifies the multiplier in downturns:

$$\frac{\partial gap(pb_t, v_t)}{\partial pb_t} = -m_1(1 - m_2 gap(pb_t, v_t))$$

**9.** **The primary balance is the sum of a cyclical component and of a structural component decided by the government:**

$$pb_t = pb_t^{st} + a.gap_t$$

where a is an automatic stabilizer coefficient. This defines a two-way relationship between the output gap and the primary balance. An increase in the structural primary balance is a fiscal tightening, this implies a decrease in the output gap. At the same time, a decrease in the output gap reduces tax revenue or increases means-tested transfers, and this implies a decrease in the primary balance. The equilibrium is solved analytically, and an approximation of the solution for small shocks shows that the effect of shocks and of changes in the primary balance are reduced by automatic stabilizers: [3]

$$gap_t \approx \frac{v_t - m_1 pb_t^{st}}{1 + m_1 a}$$

The structural balance that offsets the underlying shock process is $v_t/m_1$ in this approximation. It is larger when the fiscal multiplier is lower. It is worth noting that the parameter $m_1$ captures a causal effect of the primary balance on the output gap. Many authors regard the fiscal multiplier as the causal of a change in the structural primary on output, encompassing the mitigating effect of automatic stabilizers (as in Batini et al. 2014). The multiplier considered in such papers corresponds to $m_1/(1+m_1.a)$.

**10.** Finally, the aggregate resource constraint is:

$$c_t = y_t(1 - \chi(\Delta pb_t^{st})^2)$$

where $c_t$ denotes aggregate consumption (both private and public), and the last term represents some fiscal adjustment costs, which we model as direct resource costs. These adjustment costs can reflect implementation costs of changes in spending plans, costs associated with tax uncertainty (e.g. Skinner, 1988). This can also reflect the difficulty in reversing fiscal decisions (IMF, 2017). This adjustment cost is relative to output.

[3] This approximation is a simplified version of the actual formula used in the model. See the appendix and Fournier (2018) for more details.

**11. The calibration used for Belgium is reported in Table 1**. Most parameters are taken from Fournier and Lieberknecht (2020) who provide the information used for this purpose. Some parameters reflect cross-country empirical evidence that embed more information than country-specific estimates (e.g. the elasticity of debt to interest rate, or the risk aversion parameter). Some other parameters are specific to Belgium:

**Table 1. Belgium: Baseline Calibration**

| | |
|---|---|
| **Welfare function** | |
| Discount factor $\beta$ | 0.99 |
| Risk aversion $\sigma$ | 2 |
| Labor elasticity $\eta$ | 1/0.3 |
| Weight of labor $\xi$ | 1 |
| **Fiscal parameters** | |
| Fiscal multiplier when the gap is null $m_1$ | 0.50 |
| Fiscal multiplier sensitivity to shocks $m_2$ | 3 |
| Automatic stabilizers (primary balance semi-elasticity to the gap) $a$ | 0.66 |
| Adjustment cost $\chi$ | 3 |
| **Interest rate and debt parameters** | |
| Growth-adjusted interest rate when debt is 90 percent of GDP | 1.02% |
| Effect of debt level on the risk premium $\alpha_1$ | 1.5% |
| Effect of debt change on the risk premium $\alpha_2$ | 0.5% |
| Debt level at which the risk to lose market access is 50% $\bar{d}$ | 150% |
| Debt limit accuracy $d_1$ | 3 |
| Effect of debt change on the risk to lose market access is $d_2$ | 1 |
| Adverse scenario coefficient in case of loss of market access $d_3$ | -1% |
| **Economy parameters** | |
| Potential GDP per capita growth | 0.9% |
| Shock persistence $\rho$ | 0.70 |
| Shock size $\sigma$ | 1.8% |
| Hysteresis | 10% |
| Hysteresis threshold | -1% |

- The potential growth assumption is an average of WEO potential growth over 2017–21. The growth interest rate differential is calibrated with 20-year averages of historical data. Shock parameters (size σ and persistence ρ) are estimated with past shocks reflecting the output gap and the primary balance:

$$v_t = \left(gap_t - \frac{1}{m_2}\right) e^{-m_1 m_2 pb_t} + \frac{1}{m_2}$$

- The fiscal multiplier calibration is set to 0.5 as assumed in the recommended scenario in the staff report.

- The automatic stabilizer coefficient is taken from Price et al. (2015).

# References

Alesina, A., C. Favero, and F. Giavazzi, 2015, "The Output Effect of Fiscal Consolidation Plans," *Journal of International Economics*: S19–S42.

Auerbach, A. J. and Y. Gorodnichenko, 2013, "Fiscal Multipliers in Recession and Expansion," *Fiscal Policy after the Financial Crisis*, National Bureau of Economic Research, Inc.: 63–98.

Batini, N., L. Eyraud, L. Forni, and A. Weber, 2014, "Fiscal Multipliers: Size, Determinants, and Use in Macroeconomic Projections," IMF Technical Notes and Manuals 14/04.

Baum, A., M. Poplawski-Ribeiro, and A. Weber 2012, "Fiscal Multipliers and the State of the Economy," IMF Working Paper 12/286, International Monetary Fund, Washington, DC. Bohn, H. 1998. "The behavior of US public debt and deficits," the Quarterly Journal of economics, Volume 113, Issue 3, Pages 949–963.

Blanchard, O., J. and R. Perotti, 2002, "An Empirical Characterization of the Dynamic Effects of Changes in Government Spending and Taxes on Output," *Quarterly Journal of Economics*, 117: 1329–68.

Blanchard, O., J. and L. H. Summers, 1987, "Hysteresis in Unemployment," *European Economic Review* 31: 288–295.

Canzoneri, M., F. Collard, H, Dellas, and B. Diba, 2016, "Fiscal Multipliers in Recessions," *The Economic Journal*, 126: 75–108.

Carnot, N., 2014, "Evaluating Fiscal Policy. A Rule of Thumb," European Commission, Economic Papers 526.

D'Agostino, A. and M. Ehrmann. 2014. The pricing of G7 sovereign bond spreads – The times, they are a-changin, Journal of Banking and Finance, 47 (2014) 155–176.

Delong, J. B. and L. H. Summers, 2012, "Fiscal Policy in a Depressed Economy," Brookings Papers on Economic Activity.

Devries, P., J. Guajardo, D. Leigh, and A. Pescatori. 2011 A New Action-based Dataset of Fiscal Consolidation," IMF Working Paper, No. 11/128.

Fall, F. and J.M. Fournier. 2015. "Macroeconomic uncertainties, prudent debt targets and fiscal rules," OECD Economics Department Working Papers, No. 1230, OECD Publishing.

Fournier, J.-M. 2019. "A Buffer-Stock Model for The Government: Balancing Stability And Sustainability", IMF Working Paper, No 19/159.

Fournier, J.M. and P. Lieberknecht, 2020, “A Model-Based Fiscal Taylor Rule and a Toolkit to Assess the Fiscal Stance,” IMF Working paper, Forthcoming.

Gruber, J. W. and S. B. Kamin. 2012. “Fiscal Positions and Government Bond Yields in OECD

Countries, Journal of Money, Credit and Banking, 44(8): 1563–1587.

Henao-Arbelaez, C. and N. Sobrinho, 2017, “Government Financial Assets and Debt Sustainability,” IMF Working Paper 17/173, International Monetary Fund, Washington, DC.

International Monetary Fund (IMF), 2017, Fiscal Monitor 2017/1, International Monetary Fund,

Washington, DC.

Mauro, P., R. Romeu, A. Binder and A. Zaman. 2015. “A modern history of fiscal prudence and profligacy”, Journal of Monetary Economics, 76: 55–70.

Mourougane, A., 2017, “Crisis, Potential output and Hysteresis,” International Economics, Vol. 149, pp. 1–14.

Poghosyan, T. 2012. “Long-Run and Short-Run Determinants of Sovereign Bond Yields in Advanced Economies,” IMF Working Paper 12/271, International Monetary Fund, Washington, DC.

Price, R., T. Dang and J. Botev, 2015, "Adjusting fiscal balances for the business cycle: New tax and expenditure elasticity estimates for OECD countries," OECD Economics Department Working Papers, No. 1275, OECD Publishing, Paris.

Skinner, J., 1988, “The Welfare Cost of Uncertain Tax Policy,” Journal of Public Economics, 37(2), pp.129–145.

Zubairy, S., 2014, "On Fiscal Multipliers: Estimates from a Medium Scale DSGE Model," *International Economic Review*, vol. 55, pages 169–195.