What Caused the Beveridge Curve To Shift Higher in the United States During the Pandemic?

Gene Kindberg-Hanlon and Michael Girard

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ABSTRACT: The Beveridge curve shifted substantially higher in the United States following the start of the COVID pandemic. In 2022, vacancies reached record highs across all sectors while unemployment fell to pre-pandemic lows. At the same time, the pandemic has resulted in severe labor shortages, and we estimate that the labor force was approximately 2 million below trend at the start of 2023. We exploit state-level data in the United States to find that lower immigration, higher excess mortality due to COVID, and falling older-worker labor force participation were associated with larger upward shifts in the Beveridge curve. We also find that states that had a larger employment concentration in contact-intensive sectors had larger upward shifts in their Beveridge curve. While the effect of sectoral reallocation and rehiring has been shown in theoretical models to lift the Beveridge curve, we show that worker shortages also result in an upward shift in the Beveridge curve if they increase the marginal product of labor. This result holds in a search and matching model with on-the-job search, but does not hold without on-the-job search.

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1 Introduction

The Beveridge curve in the United States has shifted substantially higher since the start of the COVID-19 pandemic (Figure 1). The inverse relationship between unemployment and vacancies in the Beveridge curve reflects cyclical factors influencing the demand for labor and pool of unemployed workers, but shifts in the relationship can follow a range a structural changes. By the end of 2022, the unemployment rate had returned to its pre-pandemic trough of about 3 1/2% suggesting that the economy had largely cyclically recovered from the pandemic shock, while the vacancy rate increased from 4 1/2% prior to the pandemic to about 6 1/2%. Vacancies have risen across a broad range of sectors and increases have not been limited to those that were most affected by the pandemic (Figure 1). The shift in the Beveridge curve has important macroeconomic implications. For example, several studies have found a non-linear relationship between the ratio of vacancies to unemployment, a measure of labor market tightness, and the rate of inflation (Ball et al., 2022; Benigno and Eggertsson, 2023). The shift in the Beveridge curve since the pandemic implies a tighter labor market than the rate of unemployment would signal alone.

Figure 1: Beveridge curve shifts and sectoral trends

Beveridge curve pre- and post-COVID  
Sectoral vacancy and employment developments

Note: Vacancy rate refers to the total JOLTs-measured number of vacancies as a percentage of the labor force. Sectoral change in vacancies employment shows the change between the annual average in 2019 and 2022. Payroll concept of employment is used to measure employment by sector.

Using state-level data, we find support for the existing view that layoffs and reallocation driven by the pandemic has resulted in shifts in the Beveridge curve, as workers were re-hired.
into their previous roles or new sectors and locations (Blanchard et al., 2022; Figura and Waller, 2022). This factor was likely to be most important early in the recovery from the pandemic. We also find that a large labor shortage generated by the pandemic has driven intense competition for workers, leading to high hiring intensity (and thus vacancies), increased job-to-job flows, and lower layoffs (Figure 2). We estimate that the labor force was approximately 2 million below trend at the start of 2023 due to COVID-related mortality, lower older-worker participation rates, and lower immigration. This shortage of workers, alongside the large initial layoffs and reallocation effects driven by the pandemic, has also been a large contributor to the observed upward shift in the Beveridge curve. During 2021-22, the shortage of workers relative to trend has reached as high as 4 million, even as GDP returned to trend in late 2021. The effect of increased exogenous separations, such as reallocation between sectors as consumer spending was redirected by the pandemic, is already well-known to be associated with shifts in the Beveridge curve (Elsby et al., 2015). The effect of worker shortages on the Beveridge curve has yet to be studied to our knowledge.

We find that in a search and matching model, a shock that boosts the marginal product of labor will result in a vertical shift in the Beveridge curve. In isolation, a negative shock to labor supply will induce a rise in the marginal product of labor and excess demand in the economy (a positive output gap) (Galí et al., 2012). Higher returns from new hires incentivizes firms to bear higher costs of advertising more vacancies even when the possibility of securing a new firm-worker match is low. In addition, while a worker shortage would encourage worker hoarding, which should shift the Beveridge curve down by reducing separation rates, this is not the case in a model which incorporates on-the-job search (OJS henceforth). A worker shortage will encourage a larger proportion of workers to engage in OJS in the hope of gaining better employment terms. Increased competition for firm-worker matches from OJS workers crowds out unemployed workers from gaining employment for a given number of vacancies. Vacancies must be higher in equilibrium to maintain a particular unemployment rate. In a calibrated model, we find that under certain assumptions about the rise in the marginal product of labor during the pandemic, this mechanism could explain a positive 0.6 percentage point shift in the Beveridge curve at low levels of unemployment.

The paper proceeds as follows: we first outline the literature that has examined the post-COVID changes in the Beveridge curve and the literature on the importance of OJS in theoretical frameworks. We then examine the covariates of state-level heterogeneity in the scale of the post-
Figure 2: Job-to-job switches have been high, layoffs low

Labor market flows developments

Note: Hires, quits and layoffs obtained from the JOLTS survey. Labor force and employment data obtained from the BLS household survey.

COVID shift in the Beveridge curve. Finally, we outline the COVID-driven reduction in the labor force for the U.S. as a whole, and how this can cause an upward shift in the Beveridge curve in a calibrated search and matching model augmented with OJS, and also show how COVID-driven reallocation effects can shift the Beveridge curve higher in the same model.

2 Literature

The literature examining changes in the Beveridge curve post-COVID has focused on both the slope of the curve and shifts in the curve. Figura and Waller (2022) argue that the apparent upward vertical shift of the Beveridge curve is an illusion. Specifically, they suggest that the Beveridge curve is steep at low levels of unemployment and has shifted inwards as separation rates initially rose and then declined post COVID, reflecting a need for sectoral labor reallocation. In their framework, inward shifts of a steeply sloped curve give the illusion of vertical shifts. Notably, they do not account for OJS in their theoretical framework or parameter estimates of the Beveridge curve. Blanchard et al. (2022) suggest a vertical shift has occurred in the Beveridge curve and the curve remains relatively flat, implying a high tradeoff in terms of employment as vacancies fall. Their work is based on parameter estimates that include data on job-to-job hires reflecting OJS and also reinforced by historical experience. Bi et al. (2022) also
find historical evidence that the Beveridge curve is rarely steeply sloped.

Theoretical and empirical work has examined the drivers of outwards shifts in the Beveridge curve in earlier episodes. The Beveridge curve shifted substantially to the right in the 1970s, possibly driven by mismatches between growth in different regions of the U.S. and limited labor mobility, creating a mismatch of demand and supply of labor across U.S. states (Abraham, 1987). There are parallels between this analysis of structural changes in the 1970s and our analysis of post-COVID changes, in that we note that a shortage of labor is likely to be a key driver. The U.S. Beveridge curve also shifted outwards following the 2008 Global Financial Crisis. This was attributed to a deterioration in matching efficiency and search intensity (Davis et al., 2013; Elsby et al., 2010).

The most closely related work to our paper specifically incorporates OJS into search and matching models to explain post-COVID developments. Cheremukhin and Restrepo-Echavarria (2022) argue that in a dual-labor market setup where the unemployed and employed search for work in separate markets the Beveridge curve for unemployed workers has stayed broadly unchanged post-COVID, while the market for employed workers has steepened. Our framework is different, in that we assume that workers and the unemployed are competing in the same market for jobs, and therefore there is a crowding out effect for unemployed workers when more OJS takes place. This feature results in vertical shifts in the Beveridge curve. In a stylized framework, Barlevy et al. (2023) also note the importance of OJS in driving shifts in the Beveridge curve following the pandemic. In contrast to our work, they attribute the rise in OJS to workers reevaluating career choices and searching for improved work-life balances after the pandemic. Our mechanism for increasing OJS is different, with workers instead incentivized to search by the payrises available for job-movers during a labor force shortfall.

We note that the search and matching literature has found that OJS increases the ability of this class of models to match empirical moments of labor market data, but frequently these methods are not employed due to the unobservable nature of workers engaging in OJS. Approximately one-third of hiring involves job-to-job transitions in estimates using CPS microdata (Elsby et al., 2015; Fallick and Fleischman, 2004), similar to that suggested by the Job Openings and Labor Turnover Survey (JOLTS) quits and hiring data. Fujita and Ramey (2012) find that models with OJS better match U.S. labor market dynamics relative to simpler models with only transitions from unemployment to employment. Our model contains a mechanism first proposed in Pissarides (1994), which shows that in a model with OJS and tenure-related human capital
building, a rise in productivity of workers can lead to a vertical shift in the Beveridge curve. It is this feature of OJS models which we link to the post-COVID labor shortage to explain part of the upward shift in the Beveridge curve.

3 Assessing covariates of Beveridge curve shifts using U.S. state-level data

To assess the empirical support for our view that labor shortages and reallocation have been important in explaining structural shifts in the Beveridge curve, we exploit cross state variation in shifts in the Beveridge curve in the United States. We first estimate a reduced form Beveridge curve separately in each of the 50 states and the District of Columbia using the following specification:

\[ v_{i,t} = c + \beta_{i,1} D_{March2020} + \beta_{i,2} trend_t + \beta_{i,3} u_{i,t} + \beta_{i,4} u_{i,t}^2 + \epsilon_{i,t} \]  

(1)

Where \( t \) is the (monthly) time period, and \( i \) is the state. \( v \) is the vacancy rate, defined as the ratio of JOLTS-measured vacancies to the labor force, and \( u \) is the unemployment rate. The squared unemployment rate term allows for non-linearity in the relationship, and follows the specification used by Abraham (1987).\(^1\) The specification is estimated with data from 2001 to end-2022, the available range of the JOLTS data for vacancies. The specification includes a time trend to account for the rising drift of the curve in the U.S. since 2000. The dummy variable \( (D_{March2020}) \) identifies the shift in the intercept since the outbreak of the COVID pandemic and takes a value of 1 after February 2020. Our estimates show a vertical shift in each state and D.C. post-COVID, but with substantial variation, ranging from a 1.1 to a 3.4 percentage point increase.\(^2\)

To identify correlates of the U.S. Beveridge curve shift, we regress the shift in the intercept estimated above in each state \( (\beta_{i,1}) \) on a range of variables in the following specification:

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\(^1\)We note that state-level unemployment data published by the BLS is partly model-based rather than relying entirely on the CPS survey data (as the national unemployment estimate does).

\(^2\)The reduced form specification is liable to the critique that it may incorrectly identify vertical shifts in the curve that actually capture horizontal shifts of a steeply sloped curve (Figura and Waller, 2022). However, we note that once gross hires including job-to-job switches are included in estimates of the matching function, in the same specification used by Figura and Waller (2022), the curve looks similar to the reduced form specification with aggregate U.S. data.

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\[ D = c + \beta X + \epsilon \] (2)

Where \( D \) is a vector of the estimated Beveridge curve intercept shift in each state, \( X \) contains potential covariates, of the shift and \( \beta \) is a vector of coefficients. These include variables related to the sectoral disruption driven by the pandemic, as well as variables related to the impact of the pandemic on the labor force. Specifically, we include:

1. I-94 temporary worker visa admissions in each state, shortfall relative to a continuation of the 2019 fiscal year rate from 2020 until 2021. The data is sourced from the Department of Homeland Security and expressed as a proportion of the labor force. This data shows entrants to each state from holders of work-related visas and their families, and provides a proxy for the decline in work-related immigration to each state. As the data reflect instances of entering the country on a work-related visa, they can record the same individual multiple times per year, and also capture declining travel. The variable nevertheless can reflect the relative importance of immigrant workers in each state and the effect of COVID in curtailing immigrant movement to the state.

2. Share of workers in contact-intensive sectors in 2019 which were most vulnerable to disruption caused by the COVID-19 pandemic, the leisure & hospitality and retail trade sectors.

3. Census data on net domestic immigration between states to account for mass movement that occurred in the immediate aftermath of the pandemic.\(^3\) As in the case of visa issuance, this is calculated as the cumulative shortfall in 2020-21 relative to the 2019 rate.

4. Census net international migration, to account for a broader range of international migration than captured by purely worker-focused I-94 worker admissions and a more accurate reflection of those permanently moving to a state. In contrast, I-94 admittance can capture temporary visits or returns from travel (but is more specific about entrants for work rather than other reasons, such as for study and family reasons).

5. The over-65 participation rate change between 2019-21, to capture shortfalls in workers driven by that sharp decline in participation at this age group. This figure is derived from

\(^3\)See for example: https://www.census.gov/library/stories/2022/05/population-shifts-in-cities-and-towns-one-year-into-pandemic.html
the CPS microdata and is not available as a published statistic.

6. Excess deaths by state for individuals aged 15-64 between 2020-2021, calculated by the CDC’s National Center for Health Statistics.\(^4\)

7. Google mobility data for retail and recreation locations. Deviation by state from 2019 levels on average in 2021. This indicator may reflect the degree of changing consumer preferences in the state and the extent of behavioral change driven by the pandemic.

8. State domestic product growth, 2019-2021. State domestic product growth reflects the recovery in demand across states and controls for the extent to which other variables under consideration affect demand (i.e., higher net migration increase the supply of labor and demand for goods and services, but not necessarily to an equivalent extent in the timeframe under consideration).

Additional factors may be important in driving state-variation in the Beveridge curve shift. These include pandemic-induced changes in unemployment insurance, or variation in the desire to pursue changes to work-life balance. However, we are not aware of data sources covering cross-state variation in these factors.

**Endogeneity** Several of our chosen covariates could be endogenous with respect to the shift in the Beveridge curve or reflect the effects of omitted variables. For example, domestic and international migration rates in each state could partly depend on the exogenous fall in migration driven by government-imposed restrictions and voluntary social distancing, but also reflect labor market conditions in individual states. A tighter labor market may induce higher migration. The participation rate of over-65s also is likely to reflect labor market tightness as well as exogenous effects related to the pandemic. Given the small size of the dataset, its single period nature, and collinearity issues, we do not attempt to overcome these issues. However, in our theoretical discussion below, we attempt to demonstrate that the effects that variables associated with labor shortages and reallocations have are consistent with the direction of correlation in these regressions.

**Regression results** In many specifications, the I94 admittance shortfall is statistically significant and negatively signed, such that a shortfall is associated with an increase in the Beveridge

\(^4\)Available at: https://www.cdc.gov/nchs/nvss/vsrr/covid19/excess_deaths.htm#dashboard
Table 1: Covariates of the Beveridge Curve Shift in US States

<table>
<thead>
<tr>
<th></th>
<th>Dependent variable: State Vertical Beveridge Curve Shift</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I94 worker admittance shortfall (2020-21)</td>
<td>-0.143***</td>
<td>-0.076*</td>
<td>-0.075*</td>
<td>-0.151***</td>
<td>-0.080*</td>
<td>0.038</td>
</tr>
<tr>
<td>Share vulnerable sectors (2019)</td>
<td>0.030*</td>
<td>0.030**</td>
<td>0.028*</td>
<td>0.036**</td>
<td>0.033**</td>
<td>0.017</td>
</tr>
<tr>
<td>Change in 65+ participation (2019-21)</td>
<td>-0.090**</td>
<td>-0.078**</td>
<td>-0.081**</td>
<td>-0.079**</td>
<td>-0.094**</td>
<td>0.036</td>
</tr>
<tr>
<td>Excess deaths 15-64 (2020-2022)</td>
<td>0.128</td>
<td>0.172**</td>
<td>0.152*</td>
<td>0.161*</td>
<td>0.161*</td>
<td>0.089</td>
</tr>
<tr>
<td>Google mobility (retail &amp; rec., 2021)</td>
<td>-0.022***</td>
<td>-0.021**</td>
<td>-0.021**</td>
<td>-0.025***</td>
<td>0.008</td>
<td>0.008</td>
</tr>
<tr>
<td>State domestic product growth (2019-21)</td>
<td>-0.026</td>
<td>-0.035</td>
<td>-0.035</td>
<td>-0.035</td>
<td>-0.035</td>
<td>0.014</td>
</tr>
<tr>
<td>Domestic net migration change (2020-21)</td>
<td>-0.103</td>
<td>-0.213</td>
<td>-0.213</td>
<td>-0.213</td>
<td>-0.213</td>
<td>0.196</td>
</tr>
<tr>
<td>International net migration change (2020-21)</td>
<td>-0.103</td>
<td>-0.213</td>
<td>-0.213</td>
<td>-0.213</td>
<td>-0.213</td>
<td>0.196</td>
</tr>
<tr>
<td>Constant</td>
<td>0.754</td>
<td>0.721</td>
<td>0.821*</td>
<td>0.750</td>
<td>0.724</td>
<td>0.464</td>
</tr>
<tr>
<td>Observations</td>
<td>51</td>
<td>51</td>
<td>51</td>
<td>51</td>
<td>51</td>
<td>0.262</td>
</tr>
<tr>
<td>Adjusted R²</td>
<td>0.262</td>
<td>0.360</td>
<td>0.365</td>
<td>0.225</td>
<td>0.361</td>
<td>0.262</td>
</tr>
</tbody>
</table>

*Note:* *p<0.1; **p<0.05; ***p<0.01
Figure 3: Covariates of Beveridge curve shift across states

Note: Plots show the estimated Beveridge curve shift in each state and D.C. plotted against selected covariates included in the regression in Table 1. The change in participation rates of those 65+ is negatively correlated in each specification and statistically significant in most, such that states with fewer older workers saw larger rises in the Beveridge curve. This is despite the endogeneity of the variable (tight labor markets drawing more individuals into the labor force) working in the opposite direction to the coefficient. In every specification, a higher share of workers in COVID-19 vulnerable sectors prior to the pandemic was associated with a larger rise in the Beveridge curve. Lower Google mobility in 2021 was also associated with larger upward shifts in the curve. This provides support for the ‘reallocation’ effect theory, where rising worker separations to new industries caused by the pandemic through changing consumer preferences and contact avoidance drive the Beveridge curve intercept (Table 1).
eridge curve higher. Finally, excess deaths of those 16-64 (where participation rates are high) is positively correlated with larger positive Beveridge curve shifts, as predicted by our hypothesis that labor shortages have contributed to the Beveridge curve shift. We find less evidence that broader measures of international or domestic migration are drivers of the Beveridge curve shift, possibly because they cover large non-work related migrant flows. In contrast, the I94 admit-
tance is more likely to reflect work-related visa flows. We plot some simple correlations between the Beveridge curve shift and several of these covariates in Figure 3.

4 Factors driving the labor shortage

Our reduced-form analysis suggests that worker shortages can result in vertical shifts in the Beveridge curve. In this section, we attempt to quantify the scale of the labor force shortfall for the country as a whole and decompose it into structural drivers related to the pandemic that are unlikely to reflect the cyclical position of the economy.

Relative to projections by the Congressional Budget Office (CBO) in January 2020, there are approximately 1.8 million (1.1%) fewer individuals in the labor force at the start of 2023 (Figure 4). This estimate is below the Federal Reserve (Board of Governors of the Federal Reserve System, 2023) estimate of a 3.6 million shortfall at the end of 2022, but accounts for more up-to-date population adjustments from the Census and more recent participation developments. It is arguably a lower bound estimate, since some population changes in the latest revision were due to changes to pre-COVID population statistics. The shortfall is unlikely to reflect cyclical factors; GDP and consumption have broadly returned to trend, while the labor force gap has remained persistently wide. Moreover, the participation rate of workers aged 25-54 has exceeded the pre-COVID peak as of end-2022. The labor force shortfall was substantially larger in 2021-2022, ranging from 3 to 5 million, even as GDP had returned to trend in late 2021, suggesting a larger labor force constraint in those years.

We can decompose the latest shortfall into 3 primary structural factors: a) COVID-related

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5 This estimate assumes that all of the BLS’s population adjustment applied to the January 2022 estimates of the labor force stemmed from updated estimates of the population pre-2020. The Census population estimates for 2021 (the basis for the January 2022 population adjustment) methodology documents suggest that revisions were largely based on data sources from the 2020 Census at the start of the pandemic and pre-pandemic period sources rather than population flows post COVID. In contrast, the January 2021 population adjustment largely reflected updated births, deaths, and migration updates following the outbreak and so are not backdated. The January 2023 adjustment reflects new migration data and births and deaths after the pandemic began, but also reflects large changes to the population base before COVID. If this population adjustment is backdated before the start of 2023, the labor force shortfall rises to 2 1/2 million in Q1 2023, although this would be an upper bound estimate.
deaths b) a sharp fall in immigration and c) an increase in the proportion of those aged 65+ who are retired, which has caused a persistent fall in the overall participation rate.

Overall, these three factors explain more than 100 percent of the latest shortfall in the labor force relative to projections by the CBO in January 2020, just before the pandemic struck the U.S (Table 2). This partly reflects the potential underestimate of the labor force shortfall by treating the BLS’s January 2023 population adjustment entirely as a post-COVID revision, when it likely also reflected material pre-COVID population adjustments. In addition, this reflects offsetting cyclical factors: a shortfall of workers aged over 65 has been offset by younger-worker participation exceeding pre-COVID levels. Second, data on immigration is lagging, going only to mid-2022, whereas our latest estimate of the labor force shortfall extends to Q1 2023. It is likely that the gap in immigration relative to trend has been further reduced since mid-2022.

Table 2: Contributions to Labor Force Shortfall vs CBO Forecast

<table>
<thead>
<tr>
<th>Component</th>
<th>Approximate contribution (millions)</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excess retirements</td>
<td>1.2</td>
<td>Calculated using BLS data</td>
</tr>
<tr>
<td>Excess COVID mortality</td>
<td>0.6</td>
<td>Center for Disease Control</td>
</tr>
<tr>
<td>Immigration shortfall</td>
<td>0.6</td>
<td>Calculated using Census Bureau data</td>
</tr>
<tr>
<td>Other</td>
<td>-0.7</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>1.8</td>
<td></td>
</tr>
</tbody>
</table>

**Immigration**  Overseas immigration fell substantially in the immediate aftermath of the pandemic, reflecting outright bans on travel and certain types of visa, and voluntary avoidance of travel for social distancing purposes. Net migration for the country as a whole fell from about 900 thousand in the year to June 2019, to just 400 thousand in the year to June 2021 (Figure 5). The most recent estimate for net migration in the year to June 2022 is that it returned broadly to pre-COVID levels. However, relative to a continuation at 2019 levels, net migration is still cumulatively about 600 thousand below trend, although this shortfall could have fallen further since June 2022. The existing literature has noted that the slowdown even ahead of the pandemic, in part due to tightening immigration restrictions enacted after Jan 2017, contributed to tighter labor markets in the US (Duzhak, 2023; Giovanni and Zaiour, 2022).

Secondly, the net migration figures alone are an imperfect measure of the impact of migration on the labor force. Only about one-third of immigration to the US reflects directly work-related visa admittance in an average year. Much of the remaining inflow reflects humanitarian
Figure 4: Labor force shortfall

CBO projected and actual labor force

<table>
<thead>
<tr>
<th>Year</th>
<th>CBO Jan 2020</th>
<th>Actual Labor Force</th>
</tr>
</thead>
<tbody>
<tr>
<td>2017Q1</td>
<td>156</td>
<td>158</td>
</tr>
<tr>
<td>2017Q2</td>
<td>158</td>
<td>160</td>
</tr>
<tr>
<td>2017Q3</td>
<td>160</td>
<td>162</td>
</tr>
<tr>
<td>2017Q4</td>
<td>162</td>
<td>164</td>
</tr>
<tr>
<td>2018Q1</td>
<td>164</td>
<td>166</td>
</tr>
<tr>
<td>2018Q2</td>
<td>166</td>
<td>168</td>
</tr>
<tr>
<td>2018Q3</td>
<td>168</td>
<td>170</td>
</tr>
</tbody>
</table>

Note: To make estimates comparable, population control impacts on the labor force and participation rate of the BLS's January 2022 population adjustment exercise is backdated to Q1 2020 in both the CBO projection and actual labor force developments. Population controls are applied by the Bureau of Labor Statistics only in the year they are made, and are not backdated by the BLS, even though they frequently are made due to incorporation of historical data sources and revisions. The January 2022 population control largely reflected modifications to the population base, based on analysis of the 2020 Census data and other historical data sources. In contrast, the January 2021 population controls largely reflected Census analysis of the impact of COVID. The January 2023 population adjustment contained material pre- and post-COVID population changes but is not backdated in this analysis. This makes our shortfall estimates a lower bound.

admittance (often with work restrictions), student admittance, and family-related permanent resident visas which are not necessarily for the purpose of work. Homeland security data on I-94 admittance of those on work-related visas tells a more damaging story on the impact of falling migration on the labor force. In this case, admittance has not caught up with pre-COVID rate even in FY2022. I-94 admittance of those with temporary work-related visas, which include H1-Bs for specialty occupations where there are skill shortages, and even visas issued to artists and entertainers, is below 2019 levels for the third consecutive year. Cumulatively, there were 2.2 million fewer I-94 work-related visa admittances in 2020-22 relative to a continuation at 2019 levels. 878 thousand of this shortfall was due to reduced H1B visa-holder admittance for specialty occupations . These figures do not cleanly map into net migration and the labor force given they do not account for multiple entrances by the same visa holder. However, the shortfall appears to reflect more than reduced travel. State department data indicates that H1B visa issuance during FY2020-22 was cumulatively 172 thousand lower than if visa issuance had continued at its 2019 rate.
Retirements  The participation rate of individuals aged 65 and over fell sharply following the pandemic and has yet to recover (Figure 6). The fall stands in sharp contrast to the rising trend in participation pre-COVID and the rapid recovery of participation rates of younger workers. For those aged 25-64 (‘prime-age’), the participation rate has exceeded its pre-COVID rate as of Q1 2023. In contrast, the participation rate of those 65 and over remained about 2 percentage points below its pre-COVID peak at the start of 2023. Relative to the 2012-2019 trend, approximately 1.2 million more individuals were retired at the start of 2023. This estimate is similar to the analysis of post-pandemic excess retirements in Montes et al. (2022). Excess retirements alone would account for a fall in the participation rate of all workers by 0.7 percentage points, larger than the 0.4 percentage point shortfall relative to January 2020 CBO forecasts (Figure 4). The rapid recovery in younger worker participation, possibly for cyclical reasons, has reduced the aggregate participation gap.
COVID-related deaths The National Center for Health Statistics (part of the Center for Disease Control, CDC) provides an estimate for excess deaths that are directly or indirectly linked to COVID-19 (National Center for Health Statistics, 2023). Indirect causes may include a lack of access to preventative or acute healthcare, or socio-economic changes driven by the pandemic. While COVID was primarily associated with mortality in older age groups, excess mortality from February 2020 to Dec 2022 in those aged 15-64 was 695 thousand. Excess mortality in the 65-plus age group was 729 thousand. Applying participation rates in 2019 to both age brackets suggests that the labor force is approximately 650 thousand smaller due to COVID-related excess mortality. This is likely to be an upper bound however, as those vulnerable to COVID-related death may have had lower-than-average participation rates than the general population.

5 A search and matching model with OJS

To demonstrate how large labor shortages and pandemic-driven reallocation can have driven an upward shift in the Beveridge curve, we turn to a search and matching model. Here, we use
the standard search and matching model of Mortensen and Pissarides (1994), augmented with OJS. Secondly, we assume that worker shortages increase the marginal product of labor. The model is not general equilibrium, and we are assuming a labor shortage will induce a rise in the marginal product of labor that in turn results in changes to hiring and job search incentives. However, the link between a negative labor supply shock and the marginal product of labor is generally established in standard New Keynesian models such as Galí et al. (2012). In these models, a negative labor shock results in a rise in real wages and excess demand in the form of a positive output gap. Production functions in New Keynesian models almost universally assume decreasing returns to labor inputs, such that lower inputs will increase marginal product. In a search and matching model, as hiring and separation decisions depend on the output of the marginal worker, a labor shortage that increases the marginal product of workers will change the incentives for employers to seek out new hires, separate from existing matches, and also the incentives for workers to search for new roles that offer higher wages.

We show that a boost to the marginal product of labor reduces layoffs, as observed in the post-COVID data, but also increases voluntary worker quits to take up new roles, also observed in the data. The higher proportion of employed workers engaging in OJS results in a crowding out effect, requiring higher vacancies for a given level of unemployment to ensure that exits from the labor force to unemployment match inflows from unemployment to employment.

The model

We use a framework that is similar to Fujita and Ramey (2012) to incorporate OJS into a search and matching model. In contrast to previous work, our interest is not matching moments of the model to the data, but showing how a shock to labor supply and marginal productivity can shift the Beveridge curve higher. Therefore, unlike Fujita and Ramey (2012), we explicitly solve the model for its equilibrium and perform comparative statics in response to a rise in the marginal product of workers and separation rates to proxy for the COVID-induced labor market changes that have occurred since 2020.6 This section provides an overview of the model. Full details are presented in Appendix A.

6We treat search markets as homogeneous for unemployed and employed workers engaging in OJS for simplicity. A range of literature has explored segmented labor markets with on-the-job learning and high and low productivity roles where search is directed only for specific subgroups of searchers (Menzio and Shi, 2011; Krause and Lubik, 2010).
The matching process between firms and job-seekers takes the standard Cobb Douglas form,

\[ m(U_t + \Phi_t, V_t) = \kappa(U_t + \Phi_t)^\alpha V_t^{1-\alpha}, \]

where \( U + \Phi \) reflects total individuals searching for jobs, including the unemployed \( U \) and employed workers engaging in OJS \( \Phi \). \( V \) represents total vacancies. The ratio of vacancies to those searching for work is denoted \( \theta_t = \frac{V_t}{U_t + \Phi_t} \). The probability of a vacancy creating a match can be written as \( m(U_t + \Phi_t, V_t) = q(\theta) = \kappa \theta^{-\alpha} \). The probability of an unemployed individual or a worker undertaking on-the-job search being matched to a role is \( \frac{m(U_t + \Phi_t, V_t)}{U_t + \Phi_t} = \theta_t q(\theta_t) \).

**Firms** There is a continuum of firms, indexed by \( i \), with a function for the marginal product of labor specified as:

\[ y(i, x) = F'(N(i))x_t(i, m), \]

where \( F'(N(i)) \) is the production function differentiated by labor input, \( N \), and \( F''(N(i)) < 0 \) such that there are decreasing returns to labor inputs. Each firm-worker match \( m \) incorporates an idiosyncratic element to productivity, \( x \). \( x \) ranges between 0 and 1 and is drawn from the distribution \( G(x) \). New matches automatically begin at the top of the distribution, \( x = 1 \), but are redrawn through a Poisson process with probability \( \lambda \) each period.

**Individual and firm value functions** Workers can search for a new vacancy at a cost of \( s \) while receiving wages \( w \) dependent on their idiosyncratic productivity level. The value functions are discounted at rate \( r \). The value of being employed at productivity level \( x \) while engaging in OJS is:

\[ rE_s(x) = w_s(x) - s + \lambda \int_0^1 E(s)dG(s) + \lambda G(R)U + \theta q(\theta) E_{ns}(1) - (\lambda + \theta q(\theta))E_s(x) \quad (3) \]

There is a threshold, \( R \), for which the surplus value of a worker-firm match is not positive, and the job will be terminated. If \( x \) is redrawn with probability \( \lambda \), the match will only continue if \( x \geq R \). If productivity is not redrawn the match will continue at level \( x \). All transitions to new roles from unemployment or another job begin at productivity level \( x = 1 \), so that workers engaging in OJS will transition to \( E_{ns}(1) \) if their search is successful.
Alternatively, the value of being employed while not engaging in OJS is

\[ rE_{ns}(x) = w_{ns}(x) + \lambda \int_{R}^{1} E(s)dG(s) + \lambda G(R)U - \lambda E_{ns}(x) \]  

(4)

Employee search will occur if \( E_s(x) > E_{ns}(x) \), which occurs at a threshold \( x = R^* \), as in Fujita and Ramey (2012). Intuitively, because search is costly, and gains from a successful match are capped at a productivity draw of \( x = 1 \), workers with productivity \( x > R^* \) will not engage in search. \( \int_{R}^{1} E(s)dG(s) = \int_{R}^{R^*} E_s(s)dG(s) + \int_{R^*}^{1} E_{ns}(s)dG(s) \), reflects the possibility that a new productivity draw will result in the worker either searching or not searching for a new role.

The value of being unemployed is:

\[ rU = b + \theta t q(\theta t)(E_{ns}(1) - U) \]  

(5)

Where \( b \) is the benefit of being unemployed (due to unemployment insurance or leisure time). Workers automatically join the labor force at the highest productivity level if they are successfully matched.

For firms, the value of a filled job to a firm depends on both the idiosyncratic and common components of productivity. The marginal output of a filled role, and its value to the firm is therefore \( y(i, x) \). However, if the worker is searching for a new role, the match will cease if the worker successfully matches with a new role with a probability \( \theta t q(\theta t) \).

\[ rJ_s(x) = y(i, x) - w_s(x) + \lambda \int_{R}^{1} J(s)dG(s) - (\lambda + \theta t q(\theta t))J_s(x) \]  

(6)

In a match in which the worker is not actively searching for new roles, the value to the firm follows a similar specification:

\[ rJ_{ns}(x) = y(i, x) - w_{ns}(x) + \lambda \int_{R}^{1} J(s)dG(s) - \lambda J_{ns}(x) \]  

(7)

Which uses the simplification that the value of a vacancy is 0 in equilibrium. \( \int_{R}^{1} J(s)dG(s) = \int_{R}^{R^*} J_s(s)dG(s) + \int_{R^*}^{1} J_{ns}(s)dG(s) \) reflects the probability that a new productivity draw could induce the worker to search \((R < x < R^*)\) or not search \((x \geq R^*)\).

The Beveridge curve, flows into and out of employment  Flows into employment consist of those hired from the unemployed and those hired from the employed. Those separating from
matches consist of workers who separate into unemployment when the productivity draw is below a threshold $R$, drawn with probability $\lambda$ from the distribution $G(s)$, and those who transition between jobs. In equilibrium, flows out of employment match flows into employment:

$$\lambda G(R)(L - U) + \theta q(\theta)\Phi^* = \theta q(\theta)(U + \Phi^*)$$

(8)

Normalizing by the labor force, $L$, yields

$$\lambda G(R)(1 - u) = \theta q(\theta)u$$

Or,

$$u = \frac{\lambda G(R)}{\lambda G(R) + \theta q(\theta)}$$

An increase in the proportion of workers undertaking OJS will increase the number of firm worker matches by increasing the ease at which firms can hire, but lower the probability of unemployed individuals successfully matching with hiring firms (since $\theta$ is decreasing in $\phi$). Therefore, for a higher proportion of OJS workers ($\Phi$), vacancies must be higher in equilibrium at each level of unemployment. This gives rise to an upward shift in the Beveridge curve.

**Equilibrium** The value of the separation cutoff point, $R$, and the OJS threshold, $R^*$, are the key determinants explaining the number of workers engaging in OJS, $\Phi^*$. In addition, the determination of $R$ affects the number of separations in each period. Both factors determine shifts in the Beveridge curve. Several equations identify these parameters (details of the full model are in Appendix A):

The first is the job creation curve, which defines the level of $R$ at which firms are incentivized to create vacancies until the marginal value of a vacancy is 0 for a given degree of labor market tightness $\theta$. $R$ is decreasing in labor market tightness, $\theta$, as firms are more incentivized to create new roles when the probability of job destruction is lower.

$$R = 1 + \frac{1}{(1 - \beta)y(\iota)} \left( (1 - \beta)s - \theta q(\theta) - \frac{(r + \lambda)c}{q(\theta)} \right)$$

(9)

The second is a job destruction curve, which defines the value of $R$ at the point that the value of a filled job is zero and therefore no longer efficient to continue when productivity draws are below this level. $R$ is increasing in $\theta$ in the job destruction condition, as workers are more certain
of re-entering the labor force following a separation, so the opportunity cost of employment is higher. In conjunction with the job creation curve, the job destruction condition determines $R$ and labor market tightness, $\theta$.

$$0 = (1 - \beta)y(i)R - (1 - \beta)(s + b) + \lambda \bar{J}$$ \hspace{1cm} (10)

The next equation determines the productivity level where workers are indifferent between engaging in costly OJS and not engaging in OJS ($R^*$).

$$R^* = 1 - \frac{(r + \lambda)}{y(i)} \frac{1}{q(\theta)} \left( \frac{s}{\theta} + c \right)$$ \hspace{1cm} (11)

Using the values of $R$ derived from the first two equations and the value $R^*$ derived from the third, we can determine the number of workers (as a proportion of the labor force, $\phi^*$) engaging in OJS as the proportion of workers in steady state with productivity draws between those bounds:

$$\phi^* = (1 - u) \frac{G(R^*) - G(R)}{1 - G(R)}$$ \hspace{1cm} (12)

**Comparative statics from a rise in exogenous separations and rise in the marginal product of labor** To quantitatively assess the importance of different factors on the Beveridge curve, we parameterize the model with standard values from the literature (for monthly frequency models), although we adjust the matching function to be compatible with OJS. Specifically, while Fujita and Ramey (2012) use a value of $\alpha = 0.3$ and $\kappa = 0.095$, we find that these values are a poor fit when estimating the matching function on data including job-to-job hires. Instead, we use parameters that better fit the data, and are very close to those used by Blanchard et al. (2022), who also consider hires from the unemployed and OJS pool (Appendix B for further details). We choose a beta distribution for $G(s)$, which has the feature of peaking at 1, as in our model. The beta distribution is parameterized to yield a monthly endogenous separation rate ($\lambda G(s)$) of 1.3% in the baseline calibration, in line with the 2010-2019 average layoff rate in the JOLTS survey. There is a high degree of uncertainty about parameters for search and matching models in the literature. However, our goal is merely to demonstrate that our proposed mechanisms for shifting the Beveridge curve can occur with a plausible parameter set.
Table 3: Parameters used in baseline calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>β</td>
<td>Nash bargaining weight of workers</td>
<td>0.7</td>
</tr>
<tr>
<td>α</td>
<td>Cobb Douglas weight on job-searchers</td>
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</tr>
<tr>
<td>κ</td>
<td>Search efficiency</td>
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</tr>
<tr>
<td>s</td>
<td>Cost of search</td>
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</tr>
<tr>
<td>b</td>
<td>Unemployment benefit</td>
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</tr>
<tr>
<td>r</td>
<td>Discount rate</td>
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</tr>
<tr>
<td>c</td>
<td>Cost of posting a vacancy</td>
<td>0.17</td>
</tr>
<tr>
<td>λ</td>
<td>Frequency of new productivity draw</td>
<td>0.085</td>
</tr>
</tbody>
</table>

Rising exogenous separations  Initially, we show the effects on the Beveridge curve of a rise in separations and worker reallocation due to the initial stages of the pandemic. The literature has already shown that a rise in exogenous separations, such as a labor reallocation shock, will result in an outward shift in the Beveridge curve (Elsby et al., 2015). As shown earlier, worker separations and layoffs spiked in the immediate aftermath of the pandemic. This is likely to have shifted the Beveridge curve to be higher at the initially elevated levels of unemployment early in the pandemic. Augmenting our earlier-described Beveridge curve equation with an additional term for exogenous separations yields

\[
\begin{align*}
    u &= \frac{\lambda G(R) + \epsilon}{\lambda G'(R) + \epsilon + \theta q(\theta)}
\end{align*}
\]

Where \( \epsilon \) is an exogenous separation rate term. As this COVID-related separation rate is a one-off factor, we do not incorporate it into the worker and firm value functions. On average, the JOLTS layoff rate rose from 1.3% in 2019 to 2.3% in 2020. Applying this one percentage point exogenous shock to the separation rate leads to a substantial upward shift in the curve (Figure 7). At higher levels of unemployment that occurred soon after the layoffs began, the curve shifts two percentage points higher. Employment has only recently returned to pre-COVID levels, and remains below trend, suggesting that this layoff and re-hiring effect may still be in the data to a smaller degree.

Rising marginal product of labor due to labor shortages  Layoff rates fell below pre-COVID levels in 2021, while employment recovered to its pre-COVID level in early 2022, suggesting additional factors other than re-hiring and sectoral reallocation have been at work to
explain the persistently elevated Beveridge curve. We now provide a second example, outlining how a labor shortage may lead to an upward shift in the Beveridge curve, particularly at lower levels of unemployment. We assume that a labor shortage results in a rise in the marginal product of labor, and that the shortage occurs evenly across both the employed and unemployed, such that it has no other consequences for the matching process and incentives for firms.

First we shock marginal productivity by the amount suggested by a simple Cobb-Douglas production function given the earlier-identified labor shortfall relative to trend. This form of production function is almost universally assumed in a wide range of macro- and micro-economic fields, and embeds the assumption that the MPL is higher with lower levels of labor input. The change in output for a percentage point change in labor inputs can be written as:

$$\frac{\partial \ln Y}{\partial \ln N} = (\alpha - 1)d\ln N$$

The labor force shortfall during 2021-2023Q1 has ranged between 1-3% of the labor force, even as GDP returned to trend in late 2021. With a labor share of 0.6, a 2.5% fall in labor supply would result in a one percent rise in marginal productivity. However, this figure is potentially...
an understatement of the true rise in marginal worker productivity during the pandemic. The fall in the labor force in post-COVID was the largest on record since the data began in 1949 and it is possible that many firms faced non-linearities, with some firms struggling to maintain staffing for key operational roles. The National Federation of Independent Businesses survey of firms who said that they had few or no qualified applicants for roles rose to its highest ever level in 2021 at 62%. Secondly, rapid price rises in 2021-22 may have caused ‘money illusion’, with firms and workers viewing rising nominal profits as real gains. Third, average output per hour increased rapidly in 2020-21, reaching nearly 5% above trend based on average growth during 2015-19, suggesting that marginal productivity could have risen substantially more than implied by a production function approach. For this reason, we simulate a 5% shock to marginal productivity in addition to the 1% Cobb Douglas implied gain to account for the potential non-linearities of the shock.

We find that a 1% boost to marginal productivity would increase the vacancy rate only slightly by 0.1 percentage points at post-pandemic unemployment lows of 3 1/2% (Figure 8). A 5% boost to marginal productivity would boost the vacancy rate by about 3/4 percentage point at those same levels of unemployment, nearly half of the increase in vacancies observed at similar pre- and post-COVID unemployment rates. The mechanism is a substantial increase in OJS, which rises by about 4% of the labor force (a 50% increase vs the steady state value of nearly 8%) in the case of a 5% boost to marginal productivity. This substantial rise in workers engaging in OJS causes the Beveridge curve to shift higher, as the probability of being hired for a vacancy when unemployed falls with higher competition from OJS individuals. The upward shift in the Beveridge curve occurs even as the separation rate falls by about 0.2 percentage points (which lowers the curve, as fewer vacancies and hires are required to maintain a constant unemployment rate given smaller inflows to unemployment). The fall in separation rates in the 5% MPL-boost scenario most closely matches the 0.3 percentage point fall in layoff rates observed in the JOLTS data between the pre-COVID period and 2022 average rates. The OJS mechanism of crowding out unemployed workers from new hires dominates the effect of lower layoff rates as workers become more valuable to retain during a labor shortage.

The two mechanisms we have explored in this section, reallocation effects that drive exogenous separations higher and labor shortages that drive marginal labor productivity higher, are therefore able to explain a material proportion of the shift in the Beveridge curve under certain conditions in this stylized framework. We note that in a search and matching model without
Figure 8: Effect of rising marginal productivity on OJS, separation rates, and the Beveridge curve

Separation rate and OJS

Beveridge curve impact

Note: Equilibrium impact of rise in marginal product of labor above the baseline level.

OJS, a labor shortage would reduce separation rates (lower $R$), thus causing the Beveridge curve to shift down. The additional OJS mechanism is able to offset this effect, causing a positive vertical shift.

6 Conclusion and outlook

We have proposed several drivers of the shift in the Beveridge curve post-COVID, ranging from sectoral reallocation and rehiring driven by social distancing and changing consumer preferences, to the substantial fall in the labor force induced by the pandemic. Using state-level data in the U.S. we find that the above-mentioned factors are all statistically significant covariates of the variation in Beveridge curve shifts across states in the expected direction. We show that lower immigration, excess mortality, and falling older worker participation all contributed to a labor force shortfall that is conservatively estimated at 1.1% below the Congressional Budget Office’s pre-COVID projection at the start of 2023. The mechanism through which higher exogenous separations and sectoral reallocation can shift the Beveridge curve higher is already well known - we use a search and matching framework with OJS to show that a worker shortage that boosts the marginal product of labor also contributes to a rise in the Beveridge curve.

Many of the factors mentioned here will fade in importance as time goes on, suggesting that a progressive partial renormalization of the Beveridge curve could take place. The sectoral
reallocation effect will become less significant as changing consumption patterns converge to a new equilibrium. Excess deaths and reduced flows of migrants during 2020-21 are also likely to have a less material effect on the labor force in the future, particularly if immigration flows exceed pre-COVID rates for a period of time.
Appendix A  Model

This appendix outlines a search and matching model with OJS, and derives its equilibrium conditions. We proceed by outlining the matching process, whereby firms match with individuals searching for work by posting vacancies, before describing the value functions of firms and individuals in different circumstances. We describe how these value functions change when workers are engaging in OJS, and how equilibrium wages and surpluses are affected. Finally, we describe the equilibrium conditions for job-creation, job destruction, and the number of workers engaging in OJS.

The Matching Process

The worker-firm matching process takes the standard Cobb Douglass form, \( m(U_t + \Phi_s, V_t) = \kappa(U_t + \Phi_s)\theta V_t^{1-\alpha} \), where \( U \) and \( \Phi_s \) are total individuals searching for jobs and vacancies respectively. Individuals searching for jobs consist of the unemployed, \( U \) and employed workers that are also searching for new roles, \( \Phi_s \). The ratio of vacancies to those searching for work (labor market tightness) is denoted \( \theta_t = \frac{V_t}{U_t + \Phi_s} \).

The probability of a vacancy creating a match can be written as \( m(U_t + \Phi_s, V_t) \). The probability of an unemployed individual or a worker undertaking on-the-job search being matched to a role is \( m(U_t + \Phi_s, V_t) \).

Firms

There is a continuum of firms, indexed \( i = \{0, 1\} \), with the production function

\[ Y(i) = F(N(i))x(i, m), \]

where \( F''(N(i)) < 0 \) such that there are decreasing returns to labor input. Each firm-worker match incorporates an idiosyncratic element to productivity, \( x \). \( x \) ranges between 0 and 1 and is drawn from the distribution \( G(x) \). New matches automatically begin at the top of the distribution, \( x = 1 \), but are redrawn through a Poisson process with probability \( \lambda \) each period. The marginal output produced by each worker pair is thus:

\[ y(i, x) = y(i)x(i, m) = F'(N(i))x(i, m) \] (13)
The value of a filled job to a firm depends on both the idiosyncratic and common components of productivity. At lower levels of employment, the value of a marginal worker is higher. There is a threshold, $R$ for which the value of a filled role is not positive and the job will be terminated. If $x$ is redrawn with probability $\lambda$, the match will only continue if $x \geq R$. If productivity is not redrawn the match will continue at level $x$. However, if the worker is searching for a new role, the match will cease if the worker successfully matches with a new role through a Poisson process at rate $\theta q(\theta)$.

$$rJ_s(x) = y(i, x) - w_s(x) + \lambda \int_{R}^{1} J(s)dG(s) - (\lambda + \theta q(\theta))J_s(x)$$ (14)

In a match in which the worker is not actively searching for new roles, the value to the firm follows a similar specification:

$$rJ_{ns}(x) = y(i, x) - w_{ns}(x) + \lambda \int_{R}^{1} J(s)dG(s) - \lambda J_{ns}(x)$$ (15)

Both filled job value functions use the simplification that the value of a vacancy is 0 in equilibrium. $\int_{R}^{1} J(s)dG(s) = \int_{R}^{R^*} J_s(s)dG(s) + \int_{R}^{1} J_{ns}(s)dG(s)$ reflects the probability that a new productivity draw could induce the worker to search ($x < R^*$) or not search ($x \geq R^*$), where $R^*$ will be discussed later, and is the threshold at which workers choose to engage in OJS.

The value of a vacancy is

$$rV = -c + q(\theta)(J_{ns}(1) - V)$$ (16)

Posting a vacancy incurs cost $c$ until the role is filled. Immediately following a filled vacancy, there is no incentive to search, since new jobs start at the highest productivity level. Because the value of a vacancy is 0 in equilibrium (firms post vacancies until their marginal benefit is 0), we can determine an expression for the value of a new hire for the employer at equilibrium:

$$J_{ns}(1) = \frac{c}{q(\theta)}$$

Workers

Workers face the choice of searching for a new job at a cost of $s$. As new roles begin at the highest idiosyncratic productivity level, 1, workers are incentivized to search for a new role where the gap between their current productivity draw and the peak level is sufficiently large.
The value of being employed at productivity level \( x \) while searching for a new role is:

\[
rE_s(x) = w_s(x) - s + \lambda \int_{R}^{1} E(s)dG(s) + \lambda G(R)U + \theta q(\theta)E_{ns}(1) - (\lambda + \theta q(\theta))E_s(x) \tag{17}
\]

The value of being employed with no search is

\[
rE_{ns}(x) = w_{ns}(x) + \lambda \int_{R}^{1} E(s)dG(s) + \lambda G(R)U - \lambda E_{ns}(x) \tag{18}
\]

Employee search will occur if \( E_s(x) > E_{ns}(x) \), which occurs at a threshold \( x = R^* \).

\[
\int_{R}^{1} E(s)dG(s) = \int_{R}^{R^*} E_s(s)dG(s) + \int_{R^*}^{1} E_{ns}(s)dG(s),
\]
reflects the possibility that a new productivity and wage draw will result in the worker optimally searching or not searching for a new role.

The value of being unemployed is:

\[
rU = b + \theta q(\theta)(E_{ns}(1) - U) \tag{19}
\]

Where \( b \) is the value of being unemployed (due to unemployment insurance or leisure time) and workers automatically join the labor force at the highest productivity level when they secure a role.

**Match surplus allocation**

Wages are determined via Nash bargaining such that the total surplus value created by a new match is distributed among the worker with share \( \beta \) and the employer with share \( 1 - \beta \). For matches with on-the-job search,

\[
E_s(x) - U = \beta (E_s(x) - U + J_s), \tag{20}
\]

and for matches without on-the-job search

\[
E_{ns}(x) - U = \beta (E_{ns}(x) - U + J_{ns}), \tag{21}
\]

and for the expected future value of matches contingent on future probability draws:

\[
\bar{E} - U = \beta (\bar{E} - U + \bar{J}) \tag{22}
\]
Where $\bar{E} = \int_{R}^{1} E(s) dG(s) + G(R)U$ and $\bar{J} = \int_{R}^{1} J(s) dG(s)$ (since the value of a vacancy is 0).

Finally, a newly filled role at the peak of the productivity distribution must produce a surplus that is shared among workers and employers according to their Nash bargaining shares such that

$$E_{ns}(1) - U = \frac{\beta c}{1 - \beta q(\theta)}$$

Which made use of the earlier vacancies value equation for employers with $V = 0$ that produced the result that $J_{ns}(1) = \frac{c}{q(\theta)}$.

**Wages without OJS**

Using the Nash bargaining equations and value functions, the wage rate for matches with and without on-the-job search can be determined. The value function for employees and firms can be rewritten as

$$E_{ns}(x) = \frac{w_{ns}(x) + \lambda \bar{E}}{r + \lambda}$$

$$J_{ns}(x) = \frac{y(i, x) - w_{ns}(x) + \lambda \bar{J}}{r + \lambda}$$

Therefore,

$$\frac{w_{ns}(x) + \lambda \bar{E}}{r + \lambda} - U = \beta \left( \frac{w_{ns}(x) + \lambda \bar{E}}{r + \lambda} - U + \frac{y(i, x) - w_{ns}(x) + \lambda \bar{J}}{r + \lambda} \right)$$

Using $\bar{E} = \frac{\beta}{1 - \beta} \bar{J} + U$,

$$(1 - \beta) \left( w_{ns}(x) + \lambda \frac{\beta}{1 - \beta} \bar{J} - rU \right) = \beta (y(i, x) - w_{ns}(x) + \lambda \bar{J})$$

$$w_{ns}(x) = (\beta - 1)(\lambda \frac{\beta}{1 - \beta} \bar{J} - rU) + \beta(y(i, x) + \lambda \bar{J})$$

Which simplifies to

$$w_{ns}(x) = (1 - \beta) rU + \beta y(i, x)$$
We can substitute for $U$ using the value function for $U$ and equation 23

$$w_{ns}(x) = (1 - \beta)(b + \theta q(\theta) \frac{c}{1 - \beta q(\theta)}) + \beta y(i, x)$$

$$w_{ns}(x) = (1 - \beta)b + \beta c + \beta y(i, x)$$

Wages are therefore increasing in productivity of the match, the opportunity cost of unemployment ($b$) and the ratio of vacancies to job searchers (which can make re-hiring more difficult for firms and easier for workers, boosting wages).

**Wages with OJS**

The value function for employees and firms can be rewritten using the above equations and identities as

$$E_s(x) = \frac{w_s(x) - s + \lambda \bar{E} + \theta q(\theta)E_{ns}(1)}{r + \lambda + \theta q(\theta)}$$

$$J_s(x) = \frac{y(i, x) - w_s(x) + \lambda \bar{J}}{r + \lambda + \theta q(\theta)}$$

Therefore,

$$\frac{w_s(x) - s + \lambda \bar{E} + \theta q(\theta)E_{ns}(1)}{r + \lambda + \theta q(\theta)} - U = \beta \left( \frac{w_s(x) - s + \lambda \bar{E} + \theta q(\theta)E_{ns}(1)}{r + \lambda + \theta q(\theta)} - U + \frac{y(i, x) - w_s(x) + \lambda \bar{J}}{r + \lambda + \theta q(\theta)} \right)$$

Using $\bar{E} = \frac{\beta}{1 - \beta} \bar{J} + U$,

$$\frac{w_s(x) - s + \theta q(\theta)E_{ns}(1) + \lambda \bar{J}}{1 - \beta - (r + \theta q(\theta))U} = \beta (y(i, x) - w_s(x) + \lambda \bar{J})$$

$$w_s(x) = (\beta - 1)(\theta q(\theta)E_{ns}(1) - s + \lambda \frac{\beta}{1 - \beta} \bar{J} - (r + \theta q(\theta))U) + \beta (y(i, x) + \lambda \bar{J})$$
Which simplifies to

\[ w_s(x) = (1 - \beta)(s + (r + \theta q(\theta))U - \theta q(\theta)E_{ns}(1)) + \beta(y(i, x)) \]

Using equation 23 and 19, this further simplifies to

\[ w_s(x) = (1 - \beta)(s + b) + \beta y(i, x) \]

Wages are again increasing in the productivity of the match, and the opportunity cost of unemployment \((b)\). In addition, wages are also increasing in the cost of search. However, wages are no longer increasing in labor market tightness. In the no-search scenario, workers are paid for the opportunity cost of being unemployed and potentially gaining a high-paying new job. Now, this is offset by the firm needing to be compensated for the possibility that the match is destroyed by the worker finding a new higher-paying role. The Nash bargaining solution to surplus distribution requires that the surplus for workers from finding a high-wage match through searching must be distributed among both the current employer and employee. These two mechanisms effectively cancel out, eliminating labor market tightness from the wage equation with search.

**Deriving expression for \(R^*\)**

Here, we derive an expression for \(R^*\) and thus how the number of workers engaging in OJS will vary following shocks.

To establish the threshold at which workers do not undertake search, it must be the case that the expected value of search and not searching must be equal. Thus:\n
\[ E_s(R^*) - E_{ns}(R^*) = 0 \]

equation \(rE_s(R^*) - rE_{ns}(R^*) = 0 = w_s(R^*) - s - w_{ns}(R^*) + \theta q(\theta)E_{ns}(1) - \theta q(\theta)E_s(R^*)\)

From the earlier derived expressions for wages,

\[ w_s(R^*) - w_{ns}(R^*) = (1 - \beta)s - \beta c\theta t \]

So that

\[ \theta q(\theta)E_s(R^*) = -\beta s - \beta c\theta t + \theta q(\theta)E_{ns}(1) \]
Then using the fact that at $R^*$, $E_s(R^*) = E_{ns}(R^*)$,

$$E_{ns}(1) - E_{ns}(R^*) = \frac{\beta_s}{\theta q(\theta)} + \frac{\beta c}{q(\theta)}$$

$$w_{ns}(1) - w_{ns}(R^*) = \frac{\beta_s}{\theta q(\theta)} + \frac{\beta c}{q(\theta)}$$

$$\frac{\beta(y(i, 1) - y(i, R^*))}{r + \lambda} = \frac{\beta_s y_1 q(\theta)}{\theta q(\theta)} + \frac{\beta c y_1 q(\theta)}{q(\theta)}$$

$$(y(i, 1) - y(i, R^*)) = (r + \lambda)\left(\frac{s}{\theta q(\theta)} + \frac{c}{q(\theta)}\right)$$

$$R^* = 1 - \frac{(r + \lambda)}{y} \frac{1}{q(\theta_t)} \left(\frac{s}{\theta_t} + c\right)$$

Where $y = F'(N)$, reflecting the non-idiosyncratic component of the firm-worker match, which only reflects the labor input of the representative firm. $R^*$ increases as marginal productivity increases, as the costs of search and opportunity cost of unemployment decrease, and as labor market tightness increases, which increases the probability of securing a higher-productivity role ($\theta$).

**The job creation curve**

In this section, we subtract the value of a new job (at $x = 1$, providing a fixed threshold) from the value of a job at the destruction threshold, so that we can establish the minimum threshold of productivity for job creation for different levels of labor market tightness. At higher values of $R$, job destruction becomes more likely and firms are less incentivized to create new roles, due to an increased likelihood of endogenous separation. This is reflected as a negative relationship between $\theta$ and $R$. 

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\[(r + \lambda + \theta q(\theta))J_s(x) = y(i, x) - w_s(x) + \lambda \int_R^1 J(s)dG(s)\]
\[= y(i, x) - ((1 - \beta)(s + b) + \beta y(i, x)) + \lambda \bar{J}\]
\[= (1 - \beta)y(i, x) - (1 - \beta)(s + b) + \lambda \bar{J}\]

And,
\[(r + \lambda)J_{ns}(x) = y(i, x) - w_{ns}(x) + \lambda \bar{J}\]
\[= y(i, x) - ((1 - \beta)b + \beta c\theta + \beta y(i, x)) + \lambda \bar{J}\]
\[= (1 - \beta)y(i, x) - (1 - \beta)b - \beta c\theta + \lambda \bar{J}\]

Therefore, using \((r + \lambda)J_{ns}(1) - (r + \lambda + \theta q(\theta))J_s(R)\), and noting that \(J_s(R) = 0\)

\[
\begin{align*}
(r + \lambda)J_s(1) &= (1 - \beta)y(i)(1 - R) + (1 - \beta)s - \theta q(\theta) \\
\frac{(r + \lambda)c}{q(\theta)} &= (1 - \beta)y(i)(1 - R) + (1 - \beta)s - \theta q(\theta)
\end{align*}
\]

Resulting in the JC condition:
\[R = 1 + \frac{1}{(1 - \beta)y(i)} \left( (1 - \beta)s - \theta q(\theta) - \frac{(r + \lambda)c}{q(\theta)} \right) \quad (24)\]

**Deriving the job destruction curve**

Here, we can establish a separate threshold for \(R\), by solving the filled job productivity value that makes a filled job worth 0 and therefore not efficient to continue below this threshold. In contrast to the JC condition, the JD condition produces a positive relationship between labor market rightness and the separation threshold, \(R\). In this case, the opportunity cost of separation is lower for workers when labor market tightness is higher, as workers can re-enter the labor
market at a high wage following separation.

\[
(r + \lambda + \theta q(\theta))J_s(x) = y(i, x) - w_s(x) + \lambda \int_R^1 J(s)dG(s) \\
= y(i, x) - ((1 - \beta)(s + b) + \beta(y(i, x))) + \lambda J \\
= (1 - \beta)y(i, x) - (1 - \beta)(s + b) + \lambda J
\]

At R, \( J_s(R) = 0 \), producing the JD condition:

\[
0 = (1 - \beta)y(i, R) - (1 - \beta)(s + b) + \lambda J
\]  

(25)

To solve the model, we require an expression for \( \bar{J} \) in terms of the parameters of the model.

In the following, we derive and expression for the expected future value of a job match (\( \lambda \bar{J} \)) contingent on the probability of future productivity draws

\[
\lambda \bar{J} = \lambda \int_R^1 J(s)dG(s) = \lambda \int_{R^*}^1 J_{ns}(s)dG(s) + \lambda \int_R^{R^*} J_{s}(s)dG(s)
\]

The value of a vacancy with search is: \( (r + \lambda + \theta q(\theta))J_s(x) = y(i, x) - ((1 - \beta)(s + b) + \beta y(i, x)) + \lambda \int_R^1 J(s)dG(s) \) Which can be manipulated to:

\[
J_s(x) - J_s(R) = \frac{(1 - \beta)}{(r + \lambda + \theta q(\theta))} (y(i, x) - y(i, R))
\]

Since \( J_s(R) = 0 \):

\[
J_s(x) = \frac{(1 - \beta)}{(r + \lambda + \theta q(\theta))} (y(i, x) - y(i, R))
\]

The value of a filled job without search is

\[
rJ_{ns}(x) = y(i, x) - ((1 - \beta)b + \beta c\theta + \beta y(i, x)) + \lambda \int_R^1 J(s)dG(s) - (\lambda)J_{ns}(x)
\]

Subtracting the cutoff productivity level where the worker will engage in OJS:

\[
(r + \lambda)(J_{ns}(x) - J_{ns}(R^*)) = (1 - \beta)(y(i, x) - y(i, R^*))
\]
Noting that at $R^*$, $J_s(R^*) = J_{ns}(R^*)$:

$$J_{ns}(x) = \frac{(1 - \beta)}{(r + \lambda)} (y(i, x) - y(i, R^*)) + \frac{(1 - \beta)}{(r + \lambda + \theta q(\theta))} (y(i, R^*) - y(i, R))$$

Using the integration by parts formula, $\int udv = uv - \int vdu$, we can rewrite the first component of $\bar{J}$ for jobs without search as:

$$\int_{R^0}^{1} J_{ns}(s)dG(s) = J_{ns}(s)G(s)|_{R^0}^{1} - \int_{R^0}^{1} \frac{1 - \beta}{r + \lambda} y(i)G(s)ds$$

$$= \frac{1 - \beta}{r + \lambda} y(i)(1 - R^*) + (1 - G(R^*)) \frac{1 - \beta}{r + \lambda + \theta q(\theta)} y(i)(R^* - R) - \frac{1 - \beta}{r + \lambda} y(i) \int_{R^0}^{1} G(s)ds$$

$$= (1 - G(R^*)) \frac{1 - \beta}{r + \lambda + \theta q(\theta)} y(i)(R^* - R) + \frac{1 - \beta}{r + \lambda} y(i) \int_{R^0}^{1} (1 - G(s))ds$$

Where the fact that $\int_{R^0}^{1} ds = 1 - R^*$ is used.

Next, we use integration by parts for the term

$$\int_{R}^{R^*} J_s(s)dG(s) = J_s(s)G(s)|_{R}^{R^*} - \int_{R}^{R^*} \frac{1 - \beta}{r + \lambda + \theta q(\theta)} y(i)G(s)ds$$

$$= \frac{1 - \beta}{r + \lambda + \theta q(\theta)} y(i)(R^* - R)G(R^*) - \frac{1 - \beta}{r + \lambda + \theta q(\theta)} y(i) \int_{R}^{R^*} G(s)ds$$

Adding the $J_s$ to the $J_{ns}$ components:

$$\bar{J} = \frac{1 - \beta}{r + \lambda + \theta q(\theta)} y(R^* - R) + \frac{1 - \beta}{r + \lambda} y(i) \int_{R^0}^{1} (1 - G(s))ds - \frac{1 - \beta}{r + \lambda + \theta q(\theta)} y(i) \int_{R}^{R^*} G(s)ds$$

**The Beveridge Curve**

Flows into employment consist of those hired from the unemployed, $U\theta q(\theta)$ and those hired from the employed, $\Phi\theta q(\theta)$. Those separating from the workforce include those workers who endogenously separate when the productivity draw is below a threshold $R$, drawn with proba-
bility $\lambda$ from the distribution $G(s)$. In equilibrium, flows out of employment match flows into employment. Separations are equal to endogenous separations due to low productivity draws plus those undertaking OJS who successfully find a new role, while flows into jobs are equal to unemployed workers who find roles and OJS workers transitioning to new roles:

$$\lambda G(R)(L - U) + \theta q(\theta)\Phi^s = \theta q(\theta)U + \theta q(\theta)\Phi^s$$

(26)

Normalizing by the labor force, $L$, yields

$$\lambda G(R)(1 - u) = \theta q(\theta)u$$

Or,

$$u = \frac{\lambda G(R)}{\lambda G(R) + \theta q(\theta)}$$

An increase in the proportion of workers undertaking OJS will increase the number of firm-worker matches by increasing the ease at which firms can hire. However, an increase in OJS will also crowd-out unemployed workers who are searching for work by lowering the match probability for a given level of vacancies. The crowding out effect will result in an upward shift in the Beveridge curve.

Summary of equations of the model

The full system of equations has now been derived. The first is the Beveridge curve itself, which determines equilibrium between inflows and outflows from employment:

$$u = \frac{\lambda G(R)}{\lambda G(R) + \theta q(\theta)}$$

The second is the job creation curve:

$$R = 1 + \frac{1}{(1 - \beta)y} \left( (1 - \beta)s - \theta q(\theta) - \frac{(r + \lambda)c}{q(\theta)} \right)$$

(27)

The third is the job destruction curve:

$$0 = (1 - \beta)yR - (1 - \beta)(s + b) + \lambda \bar{J}$$

(28)
And the final equation is the determination of $R^*$, the threshold at which it makes sense for workers to engage in on-the-job search

$$R^* = 1 - \frac{(r + \lambda)}{y} \frac{1}{q(\theta_1)} (s \theta_1 + c) \quad (29)$$

And using integration by parts, an analytical derivation of the value of future jobs contingent on productivity draws was produced:

$$J = \frac{1 - \beta}{r + \lambda + \theta q(\theta)} y(R^* - R) + \frac{1 - \beta}{r + \lambda} y \int_{R^*}^{1} (1 - G(s)) ds - \frac{1 - \beta}{r + \lambda + \theta q(\theta)} y \int_{R}^{R^*} G(s) ds \quad (30)$$

In addition, we must have an accounting identity to determine the number of workers which engaged in on-the-job search given the productivity thresholds. Since workers engage in OJS when their productivity draw is above $R$ but less than $R^*$:

$$\phi^s = (1 - u) G(R^*) - G(R)$$

Where $\phi^s = \frac{4\theta}{\pi} \cdot \text{engaged in on-the-job search given the productivity thresholds.}$

Solution

To solve the model, the JC and JD schedules intersect at a unique solution to labor tightness ($\theta$) and the job separation threshold ($R$). The solution for labor market tightness ($\theta$) provides a solution for the OJS productivity threshold ($R^*$). Given a known separation threshold, OJS threshold, and labor market tightness, the equations for the Beveridge curve and share of OJS workers in the labor force determine the share of workers engaging in OJS and unemployment rate. The vacancy rate is plotted against a range of unemployment rates given the equilibrium OJS and separation rates in equilibrium for different productivity levels.

Appendix B  Estimating the matching function

In this section, we provide estimates of the matching function that are used to parameterize our calibrated search and marching model. The hiring rate is assumed to depend on a Cobb Douglas matching function, where the number of hires is a function of the number of posted vacancies and the number of those searching for work:
$$H_t = \kappa(U_t + \Phi_t)^\alpha V^{1-\alpha}$$

As we cannot directly observe the number of workers engaging in OJS (or predict them in a model for which we do not know the true parameters), we initially estimate this function using only the unemployed as those searching for work. For hires, $H_t$, we use gross hires from the JOLTS, available since 2001 for the U.S. on a monthly basis. We also take vacancies from the JOLTS survey and include a time trend, $t$. The sample runs until end-2022.

A regression in logarithmic form is estimated to identify the parameters $\alpha$ and $\kappa$:

$$\ln\left(\frac{H_t}{U_t}\right) = \ln(\kappa) + \beta D_{March2020} + \beta_1 t + (1 - \alpha)\ln\left(\frac{V_t}{U_t}\right) + \epsilon_t$$

This function controls for time trends in matching efficiency and post-COVID impacts. As we are interested in the marginal effects of labor shortages through their productivity impacts in this paper, we do not consider the change in matching efficiency post COVID (which are considered in detail in Blanchard et al. (2022)).

We also estimate a second specification, including an estimate of the number of OJS workers that are also searching for jobs. In our model shown below, we make the simple assumption that there is a pooled matching market between unemployed workers and OJS employed workers, $m(U_t + \Phi^s, V_t) = \kappa(U_t + \Phi^s)^\alpha V^{1-\alpha}$. This implies that the number of workers engaging in OJS is proportional to unemployment, specifically as a ratio of the number of new hires from existing jobs compared to total hires. We cannot observe the latter but proxy the number of hires from those already employed as the number of voluntary quits ($Q$) divided by the number of gross hires from the JOLTS survey. This proportion can then be applied to the number of unemployed workers as follows:

$$\Phi_t = \frac{Q_t}{H_t} U_t$$

We can then estimate an alternative matching function with a relatively crude proxy for OJS workers:

$$\ln\left(\frac{H_t}{U_t + \Phi_t}\right) = \ln(\kappa) + \beta D_{March2020} + \beta_1 t + (1 - \alpha)\ln\left(\frac{V_t}{U_t + \Phi_t}\right) + \epsilon_t$$

We summarize the estimates of $\kappa$ and $\alpha$ in Table 4.
Table 4: Matching parameter estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Without OJS Proxy</th>
<th>With OJS Proxy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa$</td>
<td>0.8</td>
<td>0.64</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.23</td>
<td>0.27</td>
</tr>
</tbody>
</table>

The results do not materially change between specifications. In the calibrated model in the main text, we use $\kappa = 0.7$ and $\alpha = 0.25$, broadly the midpoint between the two estimates. Note that these estimates are substantially different than those obtained when not taking OJS into account: Figura and Waller (2022) use $\kappa = 0.27$ and $\alpha = 0.7$, which produces a substantially steeper Beveridge curve.
References


