

# To Demand or Not to Demand: On Quantifying the Future Appetite for CBDC

Marco Gross and Elisa Letizia

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**To Demand or Not to Demand: On Quantifying the Future Appetite for CBDC**

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**ABSTRACT:** We set up a model of banks, the central bank, the payment system, and the surrounding private sector economic environment. It is a structural, choice-theoretic model which is deeply rooted in data. We use the model to conduct a structural counterfactual that introduces a Central Bank Digital Currency (CBDC) which is optionally interest-bearing. The model can be used to provide estimates of the emerging CBDC-in-total-money shares, the drop of deposit rate spreads to policy rates, the impact on reserve needs, the implied rotation of profits away from banks toward central banks, and the extent to which monetary policy pass-through may become stronger. We obtain *upper bound* estimates for the CBDC-in-money shares of about 25 percent and 20 percent, respectively for the U.S. and euro area, when CBDC would be remunerated at the policy rates and be perceived as “deposit-like” by the public. Actual take-up may likely be below such upper bound estimates. The model codes—to replicate all results and to apply them to other countries—are made available along with the paper.

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## WORKING PAPERS

# To Demand or Not to Demand: On Quantifying the Future Appetite for CBDC

Prepared by Marco Gross and Elisa Letizia<sup>1</sup>

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<sup>1</sup> The paper has benefitted from valuable comments and suggestions from the participants of seminars at the IMF, with the European Central Bank, Bank of England, Bank of Canada, Reserve Bank of Australia, Board of Governors of the Federal Reserve System, Central Bank of Brazil, Bank of Korea, and Bank of Japan. For valuable discussions and comments, we thank Atai Agur, Christoph Bertsch, Martin Čihák, Shafiq Ghawi, Reimo Juks, Argyris Kahros, Inutu Lukonga, Sahra Sakha, Manmohan Singh, and Gabriel Soderberg. The views expressed in this paper are those of the authors and do not necessarily represent those of the International Monetary Fund (IMF).

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## 1. Introduction

**How would economic dynamics be affected when introducing a central bank digital currency (CBDC)?** This headline question ought to be addressed, amid the strongly rising interest in CBDC over recent years and the potentially substantive change it would imply for financial system architecture and macroeconomic dynamics at large. We ask five questions that we think are crucial underlying components: (1) What CBDC-in-total-money shares would emerge? (2) To what extent would bank deposit rate-to-policy rate spreads fall? (3) How much would bank reserve borrowing needs rise in lieu of falling deposit balances? (4) How would bank and central bank interest income and expenses (and net seigniorage) be affected via volume and price effects? (5) To what extent would monetary policy pass-through become stronger?

**We devise a novel model that can help answer these questions quantitatively.** It comprises banks, a central bank, and the private sector (individual “non-bank” agents). The model simulates their integrated balance sheets and financial flows and allows us to conduct CBDC counterfactual analyses. It combines elements of (1) choice theory (utility maximization; [Luce and Suppes 1965](#)), (2) game theory (Nash equilibria and stochastic games; [Nash 1951](#) and [Shapley 1953](#)), (3) industrial organization (pass-through pricing and competition; [Drechsler, Savov, and Schnabl 2017](#)), (4) reinforcement learning ([Sutton and Barto 1998](#)), and (5) stock-flow consistency ([Godley and Lavoie 2006](#)). The model is structural to be useful for assessing the introduction of a CBDC that has no precedent in history. Operating at *agent-level* for banks and non-banks was a model choice to let the need for reserves arise *structurally*, for banks to settle cross-bank payments across non-banks, between banks and back and forth from cash, and CBDC in the future.

**The model is parsimoniously parameterized and strongly rooted in macro-financial data.** It requires five inputs for calibration and estimation purposes: (1) the total money stock in an economy, (2) its subset of currency in circulation, (3) bank deposit rates, (4) policy rates, and (5) nominal GDP. Some of the model's parameters are aligned with empirical counterparts directly. Others are estimated to match, for instance, current cash-in-total-money ratios and deposit-to-policy rate spreads. This allows the model to capture country-specific initial conditions regarding competition and people's preference for central bank money pre-CBDC. The level of competition can lie anywhere between monopoly and maximal competition.

**The model and the simulations conducted with it let us draw four primary conclusions:**

- (1) CBDC-in-total-money shares depend on behavioral assumptions as to whether CBDC would be perceived as more “cash-like” or “deposit-like.”** They are estimated to range between 5-25 percent for the U.S. and 1-20 percent for the euro area. These represent likely lower and upper bounds. At the upper bounds, CBDC is assumed to be perceived as “deposit-like” and CBDC interest paid at the policy rate. A move to outside such ranges can be possible if a CBDC's design would imply notably lower or higher “base utility”, i.e., beyond what is implied by interest, relative to cash or deposits.
- (2) Bank deposit-policy rate spreads fall and monetary policy pass-through may strengthen.** Deposit rates rise, though only marginally so for the U.S. and euro area model calibration (for reasons that will be explained in Section 5). Monetary policy pass-through has nonetheless the potential to become stronger. For the U.S., it rises from an estimated 0.55 under the no-CBDC baseline to beyond 0.9 when assuming deposit-like CBDC with interest at the policy rate. It rises from a baseline at 0.85 to beyond 0.95 for the euro area. Again, these estimates represent the likely upper bounds.
- (3) Bank profitability declines.** Bank *deposit expenses* are driven by two opposing factors: deposit rates rise (assuming policy rates do not drop) while deposit balances fall. For the U.S. and euro area counterfactuals, the volume effects dominate and hence make deposit expenses fall. For other countries this balance might be different. Bank *reserve borrowing expenses* rise (assuming policy rates

do not drop) and outweigh the declining deposit expense, thereby making bank net income and dividend distribution fall on net.

**(4) Central bank seigniorage is a nonlinear function of CBDC interest rates.** With zero or low CBDC interest rates, seigniorage first rises due to higher interest income from reserve lending. As CBDC interest rates continue to rise, CBDC interest expense starts dominating and let seigniorage fall again. Besides, the model does not address the question whether designing CBDC as interest-bearing will be net beneficial but aims to offer quantitative input to inform such and related design choices.

**The paper is structured as follows.** The literature is summarized in Section 2. The model is presented in Section 3. The estimation method is laid out in Section 4. The simulation results are placed in Section 5. Section 6 concludes.

**The model codes are made available along with this paper.** This is meant to allow replicating the results that are shown in the paper and to apply the model to other countries.

## 2. Literature

**The idea of offering electronic central bank money to the public started surfacing between 2015-17 with a series of central bankers' speeches.** Examples include Bank of England (BoE)'s [Haldane \(2015\)](#), BoE's [Broadbent \(2016\)](#), European Central Bank (ECB)'s [Mersch \(2017\)](#), and Norges Bank's [Nicolaisen \(2017\)](#). They started voicing some first motives for CBDC, such as a warranted augmentation of the monetary policy toolkit post-global financial crisis (GFC; Haldane), the hope that CBDC may make financial systems safer (Broadbent), and the potential for strengthening monetary policy pass-through (Mersch).

**Various central bank reports started examining the rationale for introducing CBDCs, which we can summarize along six dimensions.** Examples include reports by the [Bank of Denmark \(2017\)](#), [Riksbank \(2017/18\)](#), [Bank of Israel \(2018\)](#), [Norges Bank \(2018/19\)](#), [Bank of Canada \(2020\)](#), [ECB \(2020\)](#), and [Bank of England \(2021\)](#).<sup>1</sup> The motives for CBDC that most reports address include (1) strengthening competition in banking systems and reducing monopolistic rents for e-money payment providers in concentrated markets; (2) expanding concentrated payment systems to reduce operational risks; (3) strengthening monetary policy pass-through; (4) fostering financial inclusion; (5) combating the illicit use of money; and (6) maintaining monetary sovereignty. The latter aspect received particular emphasis in [ECB \(2020\)](#). All reports note the importance of examining whether CBDC should bear interest, for influencing its future demand and strengthening monetary policy pass-through.<sup>2</sup> Discussions are provided in [Mancini-Griffoli et al. \(2018\)](#), various BIS papers ([BIS 2021a/b](#), [Boar and Wehrli 2021](#)), and [Soderberg et al. \(2022\)](#). A valuable discussion of the implications of interest-bearing CBDC for monetary policy transmission—conditional on different monetary regimes—can be found in [Das et al. \(2023\)](#).

**Model-based work to assess the economic implications of CBDC can be grouped into four categories: New Monetarist (NM), Overlapping Generations (OLG), Dynamic Stochastic General Equilibrium (DSGE) models, and other models.** NM models suggest that interest-bearing CBDCs can strengthen competition and hence monetary policy pass-through ([Chiu et al. 2019](#); [Jiang and Zhu 2021](#);

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<sup>1</sup> All reports convey a neutral stance and suggest conducting research and pilot work before net benefits can be judged and decisions be made to whether to introduce a CBDC. Bank of Denmark's reports are an exception. They suggest a CBDC's negative net benefit and recommend against its introduction.

<sup>2</sup> [Riksbank \(2018, p.29\)](#) states that the "analysis indicates that if the e-krona is universally available, it would be advantageous for it to be interest-bearing." The [ECB \(2020, p.14\)](#) notes that if a CBDC is "considered [...] for improving the transmission of monetary policy, the digital euro should be remunerated at interest rate(s) that the central bank can modify over time." The [Bank of England \(2021, p.13\)](#) suggests that "if new forms of digital money offered interest rates, [...] they could potentially enhance the transmission of monetary policy." African countries focus on monetary policy transmission-related arguments as well ([Alberola and Mattai 2022](#)).

Davoodalhosseini 2022; Keister and Sanches 2022). OLG models, likewise, focus on falling deposit-policy rate spreads due to CBDC-induced stronger competition, alongside the implied loss in profit for banks (Andolfatto 2021) and the expected positive effects on financial inclusion (Banet and Lebeau 2022).<sup>3</sup> DSGE models started assessing the expected real economic effects: an assumed 30 percent CBDC to nominal GDP share was estimated to lift U.S. real GDP by 3 percent through a reduction in real interest rates, distortionary taxes, and monetary transaction costs in Barrdear and Kumhof (2021). A CBDC renders cross-border spillovers stronger and exchange rates more volatile in Ferrari Minesso, Mehl, and Stracca (2022)’s DSGE model.<sup>4,5</sup> Among other models, Fernández-Villaverde et al. (2021) augment a Diamond-Dybvig micro model of banks and conclude that people may move 100 percent to CBDC once introduced. Agur et al. (2022) focus on network effects, i.e., on adoption depending endogenously upon the use by others. None of these models can answer our five questions. Li (2021) quantifies the expected CBDC take-up for Canada; while Whited et al. (2022) quantify the CBDC take-up and price and profit effects for the U.S.

**Almost all such models treat banks as intermediaries instead of as money creators.** This concern applies to all NM, OLG, and DSGE models referenced above, except for Barrdear and Kumhof (2021) and the survey data-based model of Li. OLG models are rooted in the micro bank models of Klein (1971) and Monti (1972).<sup>6</sup> The fact that banks are money creators instead is emphasized in several of the speeches cited above (e.g., Haldane 2015; Nicolaisen 2017) and further in McLeay et al. (2014a/b), Jakab and Kumhof (2015), and Gross and Siebenbrunner (2022). Various valuable CBDC-related analyses rightly see and model, if model-based, banks as money creators, as in our model: Juks (2018/20), Bindseil (2020), Kumhof and Noone (2021), Meaning et al. (2021), and Garratt et al. (2022).<sup>7</sup>

**Our work relates to a pre-CBDC literature on the relation between competition and monetary policy pass-through.** Empirical work based on U.S. data shows that higher concentration in the banking system implies a looser connection between deposit and policy rate changes, i.e., less competition reduces monetary policy pass-through (Hannan and Berger 1991; Neumark and Sharpe 1992; Drechsler, Savov and Schnabl 2017). We let our model replicate such features through two related means: an embedded Random Utility Model (RUM) structure following McFadden (1973/78/81), for non-banks’ choice of money holdings, and via the number of banks relative to the population of non-banks. It is a matter of calibration through such and related parameters to match any level of initial competition and deposit-policy rate spreads for a country that the model is to be applied to, before conducting CBDC counterfactual analyses.

**We embed in our model a reinforcement learning method to allow bank agents collectively find a Nash equilibrium for their deposit rates.** We resort to a method put forth by Thompson (1933) to that end. Sutton and Barto (1998) offer an introduction to reinforcement learning and Russo et al. (2018) a primer on Thompson Sampling (TS). The algorithm we develop is an additional methodological contribution.

<sup>3</sup> NM and OLG models tend to model bank deposit markets based on Cournot (volume-based) competition. Bertrand (price-based) competition is a more adequate assumption in our view for banking systems. Hence, we use that in the model put forth in this paper.

<sup>4</sup> Their model assumes that deposits are not money, while cash and CBDC are assumed to be.

<sup>5</sup> Other DSGE model applications in a CBDC context consider shocks to monetary policy, productivity, government spending, or demand, to assess the macroeconomic response with and without a CBDC (e.g., Gross and Schiller 2022, Moreno and Montañó 2022, Oh and Zhang 2022, Syarifuddin and Bakhtiar 2022). Their impulse response analyses include the reaction of CBDC volumes and some forms of interest rates (related to an extent to our first two of five questions) but express all such results as *deviations from steady state*. Thus, they do not quantify the expected demand, nor the effects for profits, reserve dynamics, etc., in an explicit way as we wish to do. Burlon et al. (2022) provides estimates of an optimal CBDC volume.

<sup>6</sup> Klein (1971, p. 218) calls his model a “microeconomic model of the banking firm.” Monti (1972, p. 432) speaks of his “microtheoretic model of bank behavior.” Devising micro models is fine but seeing and modeling banks as intermediaries is mistaken at the micro level just as at the macro level.

<sup>7</sup> The money creation understanding correlates with establishing references—in the CBDC context—to the Chicago plan of the 1930s, for example, in Broadbent (2016), Brunnermeier and Niepelt (2019), Niepelt (2020), and Gross and Siebenbrunner (2022). If demand for CBDC were to arise once introduced, it would make the system move to an extent toward a full reserve system, just as if physical cash use were to rise notably, or if reserve requirements for deposits were leveled up significantly.



### 3. The Model

The model has three defining features:

- (1) it is stock-flow consistent;
- (2) it is dynamic, having agents of several types which interact with each other over time, based on a set of predefined sector-dependent actions;
- (3) it involves a reinforcement learning algorithm which is used for bank agents to collectively converge to a Nash equilibrium regarding the rates and all bank deposit market shares and deposit- vs. central bank money shares.

The first two features are defining criteria of agent-based model (ABM) methodologies.<sup>8</sup> The time steps in the model have no meaning. They are not to be interpreted, e.g., as quarterly, annual, or any other frequency. The time it takes to transition from one equilibrium to another, for example when considering the CBDC counterfactuals later, has therefore no interpretation. Only the equilibria as such are of interest.

The model's stock-flow consistent basis is set forth in Section 3.1. The economics of the model and the learning algorithm (points 2 and 3 above) are laid out in Section 3.2.

#### 3.1 The Stock-Flow Consistent Grounding for the Model

The model comprises three agent types: non-banks ( $1, \dots, N$ ), banks ( $1, \dots, B$ ), and a central bank. Their balance sheets are integrated, i.e., each agent's asset is another one's liability (Table 1). The stocks for the non-bank and banking sector in Table 1 refer to the sum of the underlying individual agents. For example, the sum of loans as a liability for the non-bank sector,  $-L$ , is the sum of all  $N$  non-bank agents' loans.

The model's financial flows are summarized in Table 2. It does not show cross-non-bank agent spending flows; these will be flowing amongst non-banks and sum to zero at the non-bank sectoral level. Each agent's financial expense is another one's income. Hence, all financial flows sum to zero at the system level across agents.

**Table 1. Agents' Integrated Balance Sheets**

|  | Non-Banks | Banks  | Central Bank | $\Sigma$ |
|--|-----------|--------|--------------|----------|
| Loans  | $-L$      | $+L$   |              | 0        |
| Deposits   | $+D$      | $-D$   |              | 0        |
| Cash   | $+C$      |        | $-C$         | 0        |
| CBDC   | $+DC$     |        | $-DC$        | 0        |
| Reserve holding                                  |           | $+R$   | $-R$         | 0        |
| Reserve borrowing                                |           | $-B$   | $+B$         | 0        |
| <b><math>\Sigma</math> (financial net worth)</b> | $NW^{NB}$ | $NW^B$ | $NW^{CB}$    | 0        |

Note: "+" and "-" denote assets and liabilities, respectively.

<sup>8</sup> ABM methodologies were considered to analyze payment system dynamics in Galbiati and Soramäki (2011). See also the large-scale volumes on simulation models compiled in Bank of Finland (2005/09).



**Table 2. Financial Flows**

| Category   | Financial Flow                 | Non-Banks        | Banks         | Central Bank     | $\Sigma$ |
|--|--------------------------------|------------------|---------------|------------------|----------|
| Interest   | Loan interest                  | $- I^L$          | $+ I^L$       |                  | 0        |
|  | Deposit interest               | $+ I^D$          | $- I^D$       |                  | 0        |
|  | CBDC interest                  | $+ I^{DC}$       |               | $- I^{DC}$       | 0        |
|  | Interest on reserve holdings   |                  | $+ I^{RH}$    | $- I^{RH}$       | 0        |
|  | Interest for reserve borrowing |                  | $- I^{RB}$    | $+ I^{RB}$       | 0        |
| Dividends  | Banks' dividend payouts        | $+ DIV^B$        | $- DIV^B$     |                  | 0        |
|  | Central bank seigniorage       | $+ DIV^{CB}$     |               | $- DIV^{CB}$     | 0        |
| <b><math>\Sigma</math> (change in financial net worth)</b> |                                | $\Delta NW^{NB}$ | $\Delta NW^B$ | $\Delta NW^{CB}$ | 0        |

Note: "+" and "-" denote income and expense flows, respectively. Spending flows among non-bank agents are not included in the table.

CBDC in the model is understood as a *broad-based* form of money for all non-bank agents. It comprises both nonfinancial private sector agents' CBDC ("retail CBDC") and nonbank financial firms' use of CBDC ("wholesale CBDC"). All such agent types, alongside a sovereign, are subsumed under our "non-bank" agent category. The model remains agnostic about the operational procedures for CBDC to come into circulation and leaves it at assuming that CBDC will be a liability for a central bank.

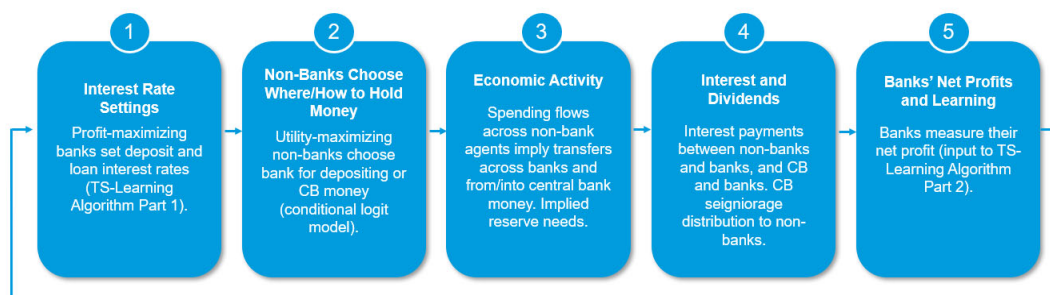
Total money is defined as the sum of non-bank deposits, cash, and CBDC. Banks' reserve assets are not money (this is why they are excluded from monetary aggregates such as M2). They rest in a segregated payment system layer. They cannot and are not being lent out to the private sector, not in the model and not in reality. They just accompany the flow of bank liabilities (deposits) across banks, upon their creation via loans, and are used to accomplish the exchange of deposit liabilities back and forth into cash, and CBDC once introduced.

Banks can obtain reserve assets in four ways: (1) "pulling" (borrowing) them from the central bank, (2) the central bank "pushing" them into the banking system via open market operations, i.e., via bond purchases, (3) attracting deposit liabilities from other banks as they would flow along with reserve assets, and (4) borrowing them through a money market from banks with surplus reserve holdings. Only options (1) and (2) change the total net reserves in the system. Options (3) and (4) are just redistributive. Options (1) and (3) are reflected in the model; while open market operations and a money market are not included yet. Banks seek to attract deposit liabilities in reality—as in the model—because they would flow along with reserve assets to the attracting bank, not because they would "on-lend" the deposits (they are a liability and cannot be lent out of course). Obtaining reserve assets via attracting deposits is cheaper than borrowing reserve assets from the central bank. The gap of that cost, the deposit-policy rate spread, is a function of the level of competition in the system, which is a central endogenous object in our model.

### 3.2 Sequence of Agent (Inter-) Actions in the Model

The model entails five steps per time increment (Figure 1). We start with the description of Steps 2-4 and then explain Steps 1 and 5 in conjunction, because these two together pertain to the learning mechanism embedded in the model.

**Figure 1. Sequence of Agent (Inter-) Actions—Five Steps Per Time Increment**



*Note:* The schematic summarizes the sequence of actions per time increment in the model. These represent the economics of the model.

### **Step 2: Non-Bank Agents' Choose How to Hold Their Money by Maximizing their Utility**

After the interest rates are set in Step 1, we let non-bank agents choose how to hold their money based on a **Random Utility Model** (RUM) structure. It is used widely in consumer choice theory following [McFadden \(1973\)](#) and has a structural interpretation of a demand function that is derived from stochastic utility ([McFadden 1981](#)). In our context, the utility,  $u_{nb}$ , that non-bank agent  $n$  derives from any of the  $B + 2$  options ( $B$  banks, and the additional two for cash and CBDC) is the sum of a contribution from observables,  $v_{nb}$ , and a random component,  $\varepsilon_{nb}$ :

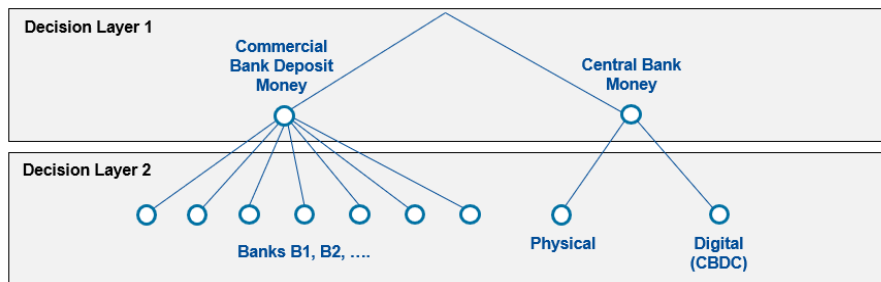
$$u_{nb} = \underbrace{\alpha_b + i_b \beta}_{\equiv v_b} + \varepsilon_{nb} \quad (1)$$

The deterministic component,  $v_b$ , is the sum of a “base utility,”  $\alpha_b$ , and the utility stemming from the interest income implied by the rates,  $i_b$ , and non-banks’ price sensitivity,  $\beta$ . The rates,  $i_b$ , are non-negative for the banks, zero for cash, and may be zero or positive for CBDC. The base utility,  $\alpha_b$ , can also be referred to as “market power,” which can be equal or heterogenous across banks, and different for the central bank’s provision of cash and CBDC. We keep the  $\alpha_b$  parameters exogenous. They may be endogenized later.

We choose a *nested* logit model structure, which implies a specific distributional form for the error term,  $\varepsilon_{nb}$ , in the model ([McFadden 1978](#); see also Annex I and II for details). It is a well-known means to address the Independence of Irrelevant Alternatives (IIA) characteristic pertaining to conventional logit models without nesting.

Figure 2 illustrates the meaning of the nested model structure in our context. It allows us to reflect two behavioral assumptions—at the ends of the spectrum or any position in between—for non-banks’ choices when introducing a CBDC. At one end, the availability of CBDC, with an assumed identical utility as for cash (i.e., with equal base utility and no interest), would crowd out the previous cash share and half that; or subtract more than half of it once CBDC interest is considered on top. At the other end, introducing a CBDC also subtracts from deposit holding shares and hence makes the total central bank money share rise. We argue that the positioning of non-banks between such two ends of a behavioral spectrum cannot be anticipated unambiguously *ex-ante*. Hence, we will consider different assumptions regarding such behavior for our CBDC counterfactuals later.

**Figure 2. Hierarchical Choice of Holding Money in a Nested Logit Model**



*Note:* This schematic illustrates the meaning of a nested logit structure in the context of the choice of forms of holding and transacting money—as embedded in the model.

The marginal choice probabilities of the  $b = B + 2$  options at the bottom of Figure 2 are:

$$P_b = P_{b|m} \cdot P_m, \quad (2)$$

where  $m = 1, 2$  denotes the nest in the upper layer (central bank money vs. deposit money) to which the choices from the lower layer belong. The probabilities of the choices within and conditional on a nest are:

$$P_{b|m} = \frac{e^{\mu_m v_b}}{\sum_{b' \in M_m} e^{\mu_m v_{b'}}}. \quad (3)$$

The probabilities of the choice of the nests at the upper level are:

$$P_m = \frac{e^{\Xi_m / \mu_m}}{\sum_{m'} e^{\Xi_{m'} / \mu_{m'}}}, \quad (4)$$

with  $\Xi_m$ <sup>9</sup> being defined as:

$$\Xi_m = \ln \sum_{b' \in M_m} e^{\mu_m v_{b'}}. \quad (5)$$

The parameters  $\mu_m$  should be larger/equal to 1 for the model to be consistent with utility maximization.<sup>10</sup> When it equals 1 for all nests, the nesting feature disappears and the model collapses to a conditional logit structure, i.e., Decision Layer 1 in Figure 2 disappears.

We denote by  $\hat{b}_{nt}$  the non-bank agent  $n$ 's choice at time  $t$  for its money allocation. Annex II explains how we obtain the choice signals during the simulation.

When the money holding choice changes from one period to the next, i.e., if  $\hat{b}_{nt} \neq \hat{b}_{n,t-1}$ , it implies a balance sheet transaction for the non-bank's bank and/or the central bank, the latter if involving the move to or away from cash/CBDC. A transfer of the deposit account to another bank will entail a corresponding reserve asset flow between the previous and new bank. Likewise, a non-bank's move from a bank to cash/CBDC will imply a reserve asset/borrowing need for the bank, which is used to let the non-bank agent swap its deposit holdings into cash/CBDC. The detailed transactions are protocoled in the model code.

<sup>9</sup> In the literature, this term is referred to as the "inclusive value," while the equation overall is typically called the "log-sum formula."

<sup>10</sup> Our  $\mu_m$  is defined and used in eqs. (3), (4), and (5) to lie on the  $[1, +\infty]$  interval, as in Louvière, Hensher, and Swait (2000). In papers such as those by McFadden, the inverse form is used, for the  $\mu_m$ 's to lie on the  $(0, 1]$  interval, in which multiplication and division as in our notation are just swapped. This is inconsequential for the model dynamics.

We combine the money holding transfer process with a *net* reserve settlement system. What matters for the system dynamics is thereby the *net* money holding migration of non-banks between bank deposit accounts/cash/CBDC. We deem this the relevant reference for later aligning the model with empirical data.

Finally, we allow the money holding choice to also affect the loan contract relationships between non-banks and banks. At the onset of the model simulation, each non-bank agent has a deposit at and a loan from the same bank (the loans created the deposits in the first instance; the loans are perpetual, continuously outstanding and hence resulting in a constant total money stock). In subsequent periods, the loan relationship may follow a deposit account move with probability  $\delta \in [0,1]$ . We examine the impact of different correlations as captured by this parameter on Nash equilibrium rates and money shares in Annex III. The transfer of loans also generates reserve asset flows, which we document in the model code.

Li (2021)'s model works with a similar nested logit model structure as ours. Whited et al. (2022) use a logit model without nesting, in a model that treats banks as intermediaries, not considering stock-flow consistency, not having an explicit payment system, and no structural reserve needs. Unlike our model, it has an endogenous lending rate, bank defaults, bank wholesale funding, and it distinguishes households and firms.

### Step 3: Economic Activity

“Economic activity” in the model means *spending flows* between randomly drawn pairs of non-bank agents. Each non-bank agent first randomly picks one spending partner from the other  $N - 1$  non-bank agents. It then spends a fraction,  $\gamma$ , of its money holding toward the spending partner. *Net* spending flows for any one non-bank agent can be positive or negative. Since the spending partner choice is random, the distribution of any one non-bank agent's money holdings is stationary over time.<sup>11</sup>

The spending flows represent the payment of wages from firms to households, with households spending such wage income back toward firms, and possibly subsequently for firms spending their excess profits in the form of dividends back to households. The distinction between firms and households underneath the non-bank agent category and the rationale for the various flows remain implicit. Fleshing such processes out is not required for what the purpose of the model is. While the price sensitivities of such two agent groups may be different empirically, it should not bias the system level results that we will obtain.

The reason for having a spending process in the model, albeit rudimentary, is that it implies a *structural* need for reserves and thereby a structural dependence on the central bank policy rate and the implied Nash equilibrating deposit rates.

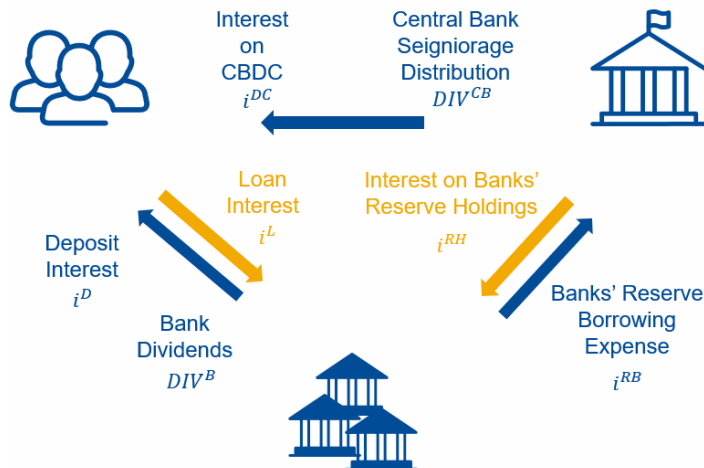
### Step 4: Interest and Dividend Payments

The interest and dividend payments as part of Step 4 comprise seven components. Going counterclockwise from the left in Figure 3: Banks settle the deposit and loan interest payments with non-bank agents. Reserve holding and reserve borrowing interest flows between banks and the central bank. The central bank pays interest on CBDC, if  $i^{DC}$  is set to be positive. It distributes its net profit (seigniorage), if positive, to non-banks. Finally, banks distribute their net profit in the form of dividends to non-banks.<sup>12</sup>

<sup>11</sup> There are no features anywhere in the model that would imply a source of persistent deviations or long-run divergence in non-bank agents' money holdings, neither from the perspective of individual agents nor regarding a cross-agent distribution.

<sup>12</sup> One can select a subset of non-banks to represent the shareholders of a certain bank. While easy to do that in our setup, such distributional aspects are not relevant for our purposes. Future extensions of the model can consider non-banks choosing between different types of assets (currently they only have deposits or central bank money), which may include bonds, equity, etc.

**Figure 3. Interest and Dividend Payment Flows**



Note: The schematic visualizes the interest and dividend flows (Step 4 in Figure 1 and Table 2) that are relevant in the model.

Letting the central bank do the latter is equivalent in the model to first transferring the positive seigniorage gain to a sovereign account and then letting the sovereign spend toward the non-banks.<sup>13</sup> The *ex-post* balance sheet positions will be the same and hence we choose the simpler direct distribution. The result is that bank balance sheets are expanded, by having simultaneously credited bank reserve accounts and non-bank deposit accounts. For the cash-holding portion of the population, bank balance sheets (reserves) are unaffected, and only the amount of cash on the central bank liability side is expanded. The result is that the central bank's net worth position stays at nil.

### Steps 1 and 5: Profit-Maximizing Banks Learn Nash-Equilibrating Deposit Rates

Banks set their deposit rates in a way that maximizes their pre-dividend profits,  $\pi_{bt}$ , which are defined as:

$$\pi_{bt} = \underbrace{i^L L_{bt}}_{=i_{bt}^L} + \underbrace{i^{RRH} \bar{R}_{bt}^R + i^{ERH} \bar{R}_{bt}^E}_{=i_{bt}^{RH}} - \underbrace{i_{bt}^D \bar{D}_{bt}}_{=i_{bt}^D} - \underbrace{i^{RB} \bar{B}_{bt}}_{=i_{bt}^{RB}} \quad (6)$$

The five interest rates are the lending rate,  $i^L$ , which in the base model version is set to the reserve borrowing rate,  $i^{RB}$ ; the two remuneration rates on required and excess reserve holdings,  $i^{RRH}$  and  $i^{ERH}$ ; the bank-specific and endogenous deposit rate,  $i_{bt}^D$ , which banks set to maximize their profit; and  $i^{RB}$ , the reserve borrowing rate. When computing the interest, stock variables (e.g., deposit balances) are computed as within-period averages, denoted by the bars. The averages are based on the balances just before Step 3 (spending) and the respective interest calculation steps under Step 4. Using such average balances reflects the assumption that spending in Step 3 happens *continuously* over the period while interest is implicitly paid at a higher (though unspecified) frequency. The deposit rates,  $i_{bt}^D$ , are set by banks in Step 1. Bank profits, as defined in eq. (6), are measured in Step 5.

<sup>13</sup> It is common practice for central banks to distribute their profit to their sovereign, whether to a reserve account at the central bank or sovereign deposit account at a bank. In the U.S., “[c]ommercial banks that are members of the Federal Reserve System hold stock in their District’s Reserve Bank. [...] the Reserve Banks are required by law to transfer net earnings to the U.S. Treasury, after providing for all necessary expenses of the Reserve Banks, legally required dividend payments, and maintaining a limited balance in a surplus fund” (see [The Fed - Who owns the Federal Reserve?](#)). Similarly for the Central Bank of Brazil (see [“Relationship with the Treasury”](#)): “The BCB’s result (profit or loss) is determined semiannually on an accrual basis. Assuming that its balance sheet presents a positive result, after constitution of reserves, the result becomes a liability to be settled through a transfer to the Treasury.” For the UK, see [“Who pays for the Bank of England? | Bank of England.”](#) For the EU, see [“Does the ECB make a profit?”](#)

We develop a **model-free reinforcement learning algorithm** to let banks learn their Nash equilibrium deposit rates and the implied bank deposit market shares, alongside the money shares, dynamically during the simulation. The short pseudo code in Table 3 summarizes the logic of the algorithm. It abstracts yet from further implementation details. These are described in detail in Annex I.

At the beginning of each iteration, an explorer bank is chosen to either increase or decrease its deposit rate based on a probabilistic draw (Step 1 in Figure 1 and lines 3-7 in Table 3). A discrete grid for the deposit rates is defined, on  $[0, i^{RB}]$ , with grid length  $G$ . The {up, down} choice refers to a 1-step move on that grid.<sup>14</sup> We let the two actions' success probabilities,  $\theta_k$ , for  $k = \{\text{up, down}\}$ , follow a beta distribution with shape parameters  $(\eta_{up}, \eta_{down})$  and  $(\kappa_{up}, \kappa_{down})$ .<sup>15</sup> The probability density of the probabilities,  $q(\theta_k)$ , is defined as:

$$q(\theta_k) = \frac{\Gamma(\eta_k + \kappa_k)}{\Gamma(\eta_k)\Gamma(\kappa_k)} \theta_k^{\eta_k - 1} (1 - \theta_k)^{\kappa_k - 1}. \quad (7)$$

The two distributions are initialized with their shape parameters all set to 1, so that they are initially uniform. The simulation results do not depend on this initialization; convergence would be achieved from any prior calibration.

In Step 5 (Figure 1), the explorer bank first measures its *reward*,  $r_t$ , which is 1 if its net income rose or 0 if it fell, relative to the previous period (Table 3, line 9):

$$r_t = \begin{cases} 1 & \text{if } \pi_{t,\text{explorer}} > \pi_{t-1,\text{explorer}} \\ 0 & \text{otherwise} \end{cases}. \quad (8)$$

The beta distributions are then updated (Table 3, line 10), by adding 1 to  $\eta_k$  in case of “success” and to  $\kappa_k$  in case of “failure” of the deposit rate move to imply a higher profit.

**Table 3. Pseudo Code—Thompson Sampling-Based Learning of Nash Equilibrium Deposit Rates**

|    |  |
|----|--|
| 1  | <b>for</b> $t = 1, 2, \dots$   |
| 2  | # sample   |
| 3  | sample $\hat{\theta}_{up} \sim \text{beta}(\eta_{up}, \kappa_{up})$                |
| 4  | sample $\hat{\theta}_{down} \sim \text{beta}(\eta_{down}, \kappa_{down})$          |
| 5  | # select and apply action  |
| 6  | $x_t \leftarrow \text{argmax}_k(\hat{\theta}_k)$                                   |
| 7  | apply $x_t$  |
| 8  | # observe reward and update beta distribution parameters                           |
| 9  | observe reward $r_t = 1, 0$  |
| 10 | $(\eta_{x_t}, \kappa_{x_t}) \leftarrow (\eta_{x_t} + r_t, \kappa_{x_t} + 1 - r_t)$ |
| 11 | <b>end</b>   |

Note: The table summarizes the high-level logic of the Thompson Sampling-based learning algorithm that is embedded in the ABM. Lines 2-7 correspond to Step 1 in Figure 1. Lines 8-10 correspond to Step 5 in Figure 1.

The environment under which the algorithm finds the Nash equilibrium can range from monopoly ( $B = 1$ ), duopoly ( $B = 2$ ), oligopoly with an intermediate number of banks, to increasingly strong competition with a

<sup>14</sup> For a discussion of the relevance of grid design in discrete learning problems see Munos and Moore (2002).

<sup>15</sup> The two shape parameters of a beta distribution are conventionally referred to as  $\alpha$  and  $\beta$ . We refer to them as  $\eta$  and  $\kappa$  to avoid confusion with the use of  $\alpha$ 's and  $\beta$ 's in other parts of the model.

further rising number of banks. When  $B > 1$ , the banks are assumed to not collude; i.e., it is a noncooperative game. We let the banks learn *jointly* based on one pair of beta distributions, because we assume that all banks have the same market power and hence the same Nash equilibrium rates. The algorithm can also be used to let banks learn their individual Nash rates in case of heterogeneous market power distributions across banks. The Nash equilibrium rates would in that case be bank-specific (Gross and Letizia, forthcoming).<sup>16</sup> Annex I demonstrates that the algorithm successfully identifies the Nash equilibrium, which, as a benchmark, can also be computed analytically in a simple model environment.

#### 4. Model Parameterization and Estimation

The model is flexible in capturing country-specific initial conditions regarding competition and people's preference for central bank money pre-CBDC, while requiring minimal inputs in terms of macro-financial data to that end. The model needs five inputs: the total money stock, currency in circulation, bank deposit rates, policy-related parameters (reserve borrowing and remuneration rates, reserve requirements), and nominal GDP. Some selected additional time series of macroeconomic variables may be needed as control variables for estimating the private sector's price sensitivity (e.g., price inflation, in our application). We outline in this section how to calibrate the model with such inputs, while providing more details in Annex IV.

The model comprises nine parameters. They are summarized in Table 4. The table also shows the parameterization for the U.S. and euro area. This section will explain how it was obtained.

When the number of banks or the price sensitivity rise (separately, *ceteris paribus*), it makes Nash equilibrium deposit rates rise and Nash cash ratios fall. When the base utility of cash rises, *ceteris paribus*, it does the opposite. These basic dependencies are shown in Annex I where the ability of our learning algorithm to identify the Nash equilibrium is demonstrated. The dependencies on the velocity of money, the loan transfer probability, and the reserve remuneration rates and reserve requirement are presented in Annex III.

**Table 4. Model Parameters**

| # | PARAMETER                            | SYMBOL                   | COMMENTS  | U.S. | EURO AREA |
|---|--------------------------------------|--------------------------|---|------|-----------|
| 1 | Number of banks                      | $B$                      | Number of bank agents in the model.   | 8    | 12        |
| 2 | Base utility of deposits, cash, CBDC | $\alpha_{b=1,\dots,B+2}$ | Normalized to zero for all $b = 1, \dots, B$ banks. $\alpha_{cash}$ used in conjunction with $B$ to estimate the model to match empirical deposit rate markdowns and cash ratios. | 0.5  | 2         |
| 3 | Price sensitivity                    | $\beta$                  | Non-bank propensity to switch banks/cash/CBDC in response to changing interest rates (deposit rates and CBDC rates).  | 50   | 150       |
| 4 | Velocity of money                    | $\gamma$                 | Non-bank share of beginning-of-period money holdings that they spend toward other non-banks during the period.  | 1.1  | 0.85      |

<sup>16</sup> One can let the bank agents learn individually, using their own learning space (beta distributions) in case of homogenous market power, but this would be less efficient, i.e., the algorithm would take a bit longer to find the Nash equilibrium (though it would be found successfully).



|   |                              |                      |   |              |                 |
|---|------------------------------|----------------------|---|--------------|-----------------|
| 5 | Loan transfer probability    | $\delta$             | Probability from [0,1] for a non-bank's deposit account move to imply loan transfer. 0 means that the loan contract relation between non-banks and banks stays constant at initial T0 assignment; 1 means 1-to-1 following. | 0.75         | 0.75            |
| 6 | Reserve requirement          | $\lambda$            | Required reserve ratio, required reserves over deposits.  | 0            | 0.01            |
| 7 | Reserve borrowing rate       | $i^{RB}$             | Rate that banks pay for period average reserve borrowing (RB) balances from the central bank.   | 0.035        | 0.02            |
| 8 | Interest on reserve holdings | $[i^{RRH}, i^{ERH}]$ | Interest on required and excess reserves, respectively.   | $[-, 0.035]$ | $[0.015, 0.01]$ |
| 9 | Interest on CBDC             | $i^{DC}$             | Rate that the central bank pays on non-bank period average holdings of CBDC.  | .            | .               |

Aligning the model with empirical data is accomplished in three steps. We refer to them as Steps A, B, and C in the following. The description here is brief. Details are provided in Annex IV.

#### Step A. Estimate non-banks' price sensitivity ( $\beta$ )

We estimate non-bank agents' price sensitivity based on a conditional logit model separate from the ABM. It involves the economy-aggregate historical cash ratios and aggregate deposit rates for the banking system. The price sensitivity estimate is computed by minimizing the deviation between the historically observed cash ratios and those implied by that model.

The point estimates obtained for the U.S. and the euro area were set to a rounded 50 and 150, respectively. They are surrounded by notable estimation uncertainty, in particular for the euro area. The uncertainty for the euro area likely stems from the aggregation of all euro zone countries, which may be characterized by heterogeneous price sensitivities and cash ratios, the latter being reflective of possibly different levels of preference of central bank money across countries. The estimation of the price sensitivity can be refined in various ways to enhance the estimation precision. Individual bank deposit rates can be brought in instead of banking system aggregates. Detailed micro account-level data may be used, when available. For the euro area, the estimation can be considered at the country level to capture the heterogeneities mentioned above. All such options can help increase the precision of the price sensitivity estimates for future country applications. Useful work to further inform the  $\beta$ -estimation step includes models involving micro data, where cross-bank interest rate differentials are found to be among the important drivers of switching (Gerritsen and Bikker 2020), and randomized trial-based analyses such as in Adams et al. (2021).

#### Step B. Calibrate other model parameters: Velocity of money ( $\gamma$ ), loan transfer probability ( $\delta$ ), and reserve-related parameters ( $\lambda$ , $i^{RB}$ , $i^{RRH}$ , $i^{ERH}$ )

The velocity of money in the model ( $\gamma$ ) is aligned with its empirical counterpart, annual nominal GDP over the money stock (M2). It amounts to a rounded 110% for the U.S. and 85% for the euro area, as of 2022Q2.

The probability of loan transfers conditional on non-banks' deposit account moves ( $\delta$ ) is the only parameter that is set judgmentally—to 0.75 for both the U.S. and the euro area. We are not aware of data that would be available to inform or estimate such a parameter. While the parameter influences the Nash equilibrium outcome of the model to an extent (Annex III), it has no notable impact on the outcome of our CBDC counterfactual analysis.

As of September 2022, the reserve requirement ( $\lambda$ ) is 0% and 1% for the U.S./FED and the euro area/ECB, respectively. They were set accordingly in the model.

The reserve borrowing rate,  $i^{RB}$ , and the remuneration rates for required and excess reserves,  $i^{RRH}$  and  $i^{ERH}$ , were set to recent, historical reference values. For the U.S., both the borrowing rate and the excess reserve remuneration rate are set to 3.5%. The rate on required reserves is irrelevant since the reserve requirement is 0%. For the euro area, the three rates [ $i^{RB}$ ,  $i^{RRH}$ ,  $i^{ERH}$ ] are set to [2%, 1.5%, 1%].

### Step C. Estimate the number of banks ( $B$ ) and the base utility of cash ( $\alpha_{cash}$ ) to let the ABM match a target cash ratio and deposit rate.

After the first set of parameters is set in Steps A and B, two parameters remain: the number of banks and the base utility of cash. They are estimated for the implied cash ratio and deposit rate—two of the model's endogenous variables—to match some self-defined target. The cash ratio targets are set to currently observed ones; a rounded 10% for both the U.S. and the euro area as of end-2022Q2. The deposit rate targets are set to be consistent with the central bank rate assumptions (reserve borrowing and remuneration rates).<sup>17</sup> They were set to 1% and 1.2% for the U.S. and the euro area, respectively.

The estimated number of banks ( $B$ ) is not expected to equate to a number of banks in a country to which the model is applied. One may compare such a parameter across countries though, in conjunction with other relevant parameters (e.g., the price sensitivity), to judge the level of competition in different jurisdictions.

A final calibration step entails the scaling of the total money stock that is initially distributed in the form of loans and the deposits created through these loans to non-banks to empirically observed M2. This parameter is not listed in Table 4 because it has no impact on outcomes such as Nash deposit rates, Nash money shares, or other metrics. It is a pure scaling parameter and just meant to let changes in stocks and flows be interpretable in relation to real world quantities. We set the money stocks for the U.S. and euro area to USD 22 trillion and EUR 15 trillion, respectively, the rounded values measured at end 2022Q2.

## 5. Counterfactual Simulations: Introducing a CBDC

We conduct our counterfactual analysis by running the model with different parameter configurations. Each simulation is run for 2,000 iterations. A “burn-in” period of 500 iterations is discarded at the beginning of each simulation to ensure that the equilibrium state is reached before examining the distributions of all balance sheet stocks and flow variables for all agent types over the remaining 1,500 periods per counterfactual setting. The settings can be summarized as follows:

1. **Baseline:** This is conducted based on the estimated model (see previous section), while having the base utility ( $\alpha_{CBDC}$ ) of CBDC at  $-\ln f$ , for no CBDC demand to arise. The results are shown in grey in the charts following in this section.
2. **CBDC “Cash-Like”:** We equate the base utility for CBDC with that of cash ( $\alpha_{CBDC} = \alpha_{cash}$ ). The nesting parameter is set to a high value that mimics “full nesting” ( $\mu \rightarrow \infty$ )<sup>18</sup>. “Full nesting” means that non-banks perceive CBDC as “cash-like.” We let the CBDC rate range between 0 and 100 percent of the policy rate.<sup>19</sup> The results related to this counterfactual are shown in yellow in the following charts.

<sup>17</sup> Empirical bridge equations were estimated to support the calibration step for the euro area. They were used to inform what the historical average deposit rate should be for the previously self-defined policy rates.

<sup>18</sup> A value of  $\mu = 100$  was used. This is numerically sufficient to approach the “full nesting” case.

<sup>19</sup> Negative interest rates on CBDC can be considered in the model.

3. **CBDC “Deposit-Like”:** CBDC’s base utility is set to that of banks ( $\alpha_{CBDC} = \alpha_{banks} = 0$ ). The nesting parameter set at  $\mu = 1$  means “no nesting” and implies that non-banks perceive CBDC as “deposit-like.” The CBDC interest rate ranges again between 0 and 100 percent of the policy rate. The results for this counterfactual setting are displayed in blue in the following chart collection.

**(1) What CBDC-in-total-money shares would emerge?**

For the U.S., the estimated CBDC adoption rate ranges between 5-25 percent depending on the behavioral assumptions and the CBDC interest rate (Figure 4, first row).<sup>20</sup> For the euro area, the take-up rate ranges between 1-20 percent (Figure 5, first row). The lower and upper bounds correspond to the “cash-like” and “deposit-like” end of the spectra. The choice of perceiving CBDC as closer substitutes to either cash or deposits is crucial for the estimated adoption: when CBDC is considered as cash, the initial cash-in-total-money ratio is effectively the upper bound to the joint share of cash and CBDC in money. Further, a higher interest rate on CBDC crowds out cash almost entirely, while keeping the share of deposits in total money about constant. When CBDC is perceived as deposits instead, CBDC crowds out both deposits and cash.

**(2) To what extent would bank deposit rate spreads to policy rates fall?**

For the U.S., the average bank deposit rate would rise by up to a marginal 20 bps when introducing a CBDC (Figure 4, third panel).<sup>21</sup> Similarly, for the euro area, the effect is not materially different from zero (Figure 5, third panel). A reason why the effect is small (not visible) for the euro area is that the level of competition is already stronger in the pre-CBDC setting, as reflected by higher deposit rates (smaller deposit-policy rate spreads) and a larger estimated number of banks to make the model outcome consistent with observed cash ratios and deposit rate spreads.

An examination of what determines the sensitivity of the equilibrium deposit rate response to the introduction of a CBDC suggests that the counterfactual impacts become more visible the smaller the price sensitivity, the lower the base utility for cash, the smaller the number of banks, and the more one approaches a “no nesting” assumption (i.e., allowing CBDC to crowd out previous deposit shares). All these dependences are to be understood as *ceteris paribus*. The first of these four dependencies, the price sensitivity effect, can be explained as follows: When non-banks’ price sensitivity is comparably high, the change in the deposit rate that banks will determine as optimal (profit maximizing) is smaller to thereby incentivize non-banks to keep preferring to hold their money in deposit form. This is part of the reason why an effect (albeit small) for the U.S. deposit rates is measured, while not so for the euro area. Overall, the parameterization for both country cases is such that the responsiveness of deposit rates is small. Considering different parameterizations, for other countries, with a view to the four parameter dependences summarized above, can make the deposit rate dependence more sizeable.

**(3) How much would bank reserve borrowing needs rise in lieu of falling deposit stocks?**

Non-banks adopting CBDC increases bank reserves borrowing needs between USD 1.1-4.8 trillion to counterbalance the falling deposit balances in the U.S. Deposit balances in the euro area would fall between EUR 0.2-2.8 trillion (first and second panels of the second rows in Figure 4 and Figure 5, for the U.S. and euro area, respectively). This effect is material when we assume that CBDC crowd out deposits rather than

<sup>20</sup> When assuming the CBDC interest rate to equal the Fed Funds rate, Whited et al. (2022) estimate the CBDC take-up for the U.S. at about 31 percent (ibid., Table 6).

<sup>21</sup> Similarly, Whited et al. (2022) estimate the drop in the deposit-policy interest spread shift under a CBDC counterfactual with CBDC interest at the policy rate to equal about 16 basis points (difference between baseline spread at 1.125 percent, ibid. Table 4, vs. counterfactual spread at 0.965 percent, ibid. Table 6).

cash, i.e., when we relax the nesting assumption. It also grows with the CBDC interest rate, due to more wide-spread adoption. Increased reserves borrowing has a parallel effect on bank funding cost (more on this below), and on their collateral needs.

The by-product of such an increase is the large expansion of the central bank balance sheet (mid panel of last rows in Figure 4 and Figure 5). This effect has been emphasized by [Bindseil \(2020\)](#), among others, and it is one of the primary aspects a central bank would want to assess when designing its CBDC, as it has implications for collateral availability and the central bank's collateral eligibility criteria.

Our setup does not give any indication on the effect of such a funding shift on credit supply by banks, as the money creation function of banks is not altered by their source and cost of funding. We would refine the analysis in this respect, however, once the lending rate would also be endogenized in the model later.

**(4) *How would bank and central bank interest income and expenses (net seigniorage) be affected through volume and price effects? How would net financial income of the private sector be affected in the sequel?***

Bank deposit interest rates are seen to increase only marginally (see question 2 above), while deposit volumes drop (question 3 above). The results for both the U.S. and the euro area suggest that the combined effect is negative, for deposit expenses to fall on net, while the absolute expense for central bank reserve borrowing increases (Figure 4 and Figure 5 bottom row). This assumes that central banks would not structurally lower the policy rate (which they may consider, however). Net income and hence dividend payouts for banks may fall: by about USD 141 billion (-30%) per annum in the U.S., and by EUR 22 billion (-20%) per annum in the euro area, at the upper bounds. The net negative effect is due to rising reserve borrowing costs outpacing the falling deposit expense.

For the return on assets (RoA) of the U.S. banking system, for instance, the upper bound estimate corresponds to a shift in RoA by about 70 basis points. An empirical RoA reference value at about 1.25 percent by mid-2022 reflects *all* income and expense components, relative to which a model-implied shift—despite not having model elements for all profit and loss items—can be indicative. The RoA for the U.S. would remain positive according to the upper bound estimates, implying no outright potential for capital ratios to fall.

On the central bank side (Figure 4 and Figure 5), we see their interest income from reserve lending rise, their interest expense for bank surplus reserve balances fall, and the interest expense for CBDC rise once we allow for CBDC interest to be paid. Regarding central bank net income, we observe a nonlinearity: it first rises when the CBDC interest rate is zero or small, due to higher reserve lending-related interest income. With further rising CBDC interest rates, the CBDC interest expense at some point starts dominating for net income to turn and fall again.

The falling seigniorage payout flow should not be interpreted as implying less sizeable financial flows to the private sector, however, because the interest for CBDC is an additional flow to be considered. In fact, and importantly, the net financial income for the private sector remains unaffected (see Annex V for the detailed panels including non-banks' P&L flow effects): Non-banks' interest income from deposits falls, bank dividend income drops, while interest income from CBDC increases and seigniorage payment flows from the central bank keep flowing, albeit at a smaller scale. All these flows balance to a *net zero net income effect* for the non-bank private sector under all scenarios.

Regarding the aforementioned rising net reserve deficit position of the banking system and associated higher reserve borrowing expenses: At the current juncture, banking systems such as in the U.S. are still characterized by sizeable surplus reserves—post-GFC and post-Covid—so that a CBDC introduction may not imply the need for outright additional reserve borrowing. The excess reserves resting in the system, however, may be drawn down when CBDC demand would arise (the surplus reserve position of the U.S. banking system at end-2022Q2 amounted to USD 2.1 trillion<sup>22</sup>). Such draw-down would imply *less reserve interest income* for banks; which is remunerated at about the Fed funds rate in the U.S. In that sense, we may argue that—despite not reflecting the surplus reserve position (alongside open market operations and bond markets, etc.) in the model yet—the effects for the rotation in profit and dividend flows should be indicative already now.

**(5) To what extent would monetary policy pass-through become stronger?**

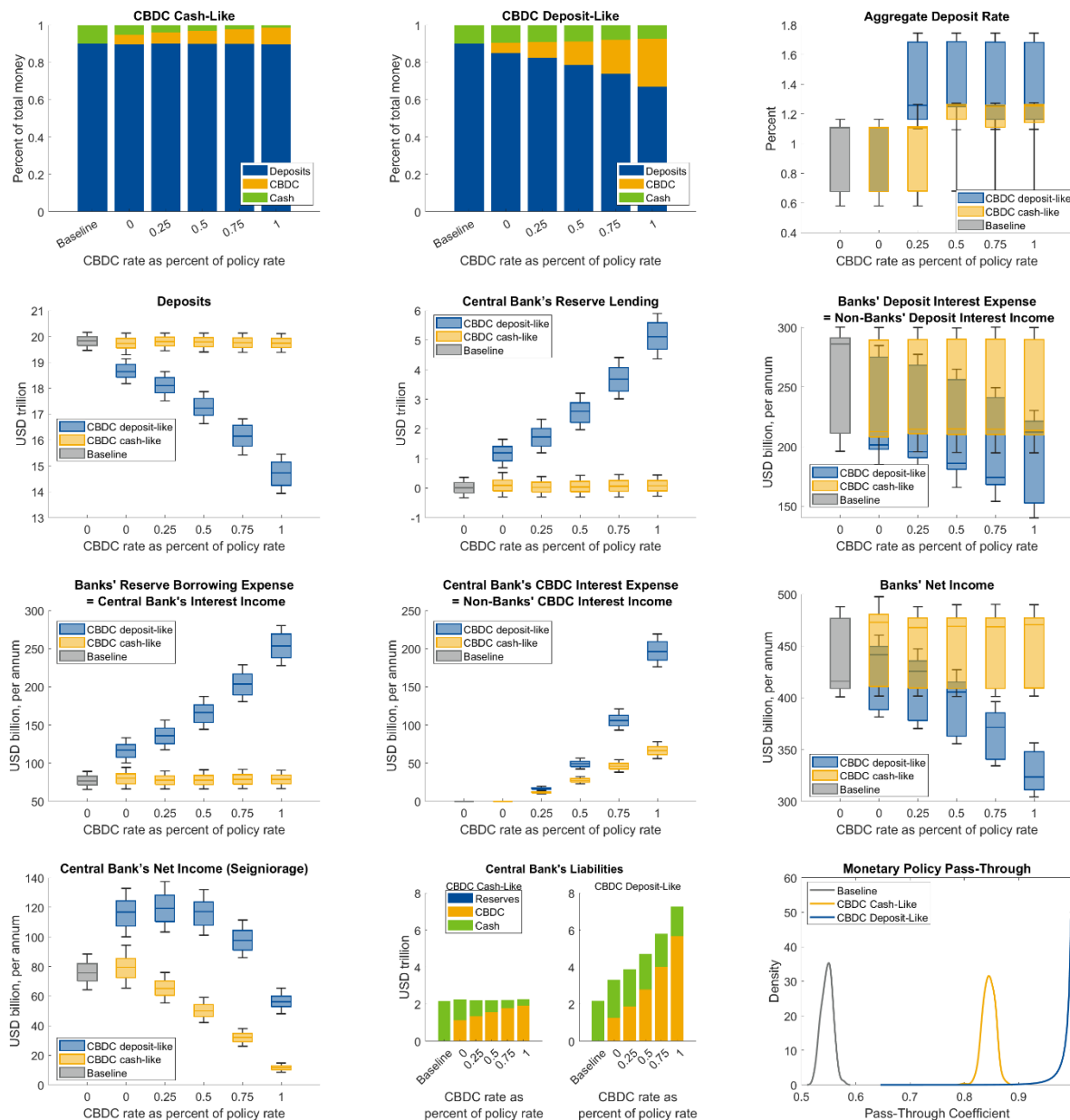
Monetary policy pass-through estimates were obtained by moving the policy rate, i.e., the reserve borrowing rate in the model, by 1 percentage point under different counterfactual regimes (baseline vs. CBDC). We then estimated a regression model that relate the Nash deposit rates to the policy rate, before and after the policy rate shift. A nonparametric residual bootstrap was employed to obtain coefficient distributions. The pass-through estimates were obtained for only the case of having the CBDC interest rate set equal to the policy rate. All intermediate cases can be conceived as falling in between zero and the estimated upper bound impact for the pass-through change.

The U.S. baseline pass-through estimate at 0.55 rises to 0.85 under the CBDC-cash-like scenario, and to beyond 0.95 under the CBDC-deposit-like scenario; in either case assuming CBDC interest at the policy rate (Figure 4). The baseline pass-through estimate for the U.S. corresponds well with an approximate range estimated in [Drechsler, Savov, and Schnabl \(2017\)](#). For the euro area, the baseline and two counterfactual pass-through parameters are estimated at 0.86, 0.89, and 0.97. As emphasized above, the respective latter two are upper bound estimates. The baseline pass-through estimate for the euro area exceeding the one for the U.S. is compatible with the observed, more sizeable deposit-policy rate spreads in the U.S. historically (i.e., lower deposit rates, for a given policy rate). It also aligns with the estimated larger number of banks ( $B$ , reported in Table 4) for the euro area. Overall, it suggests that the initial level of competition in the euro area is to an extent larger than in the U.S.

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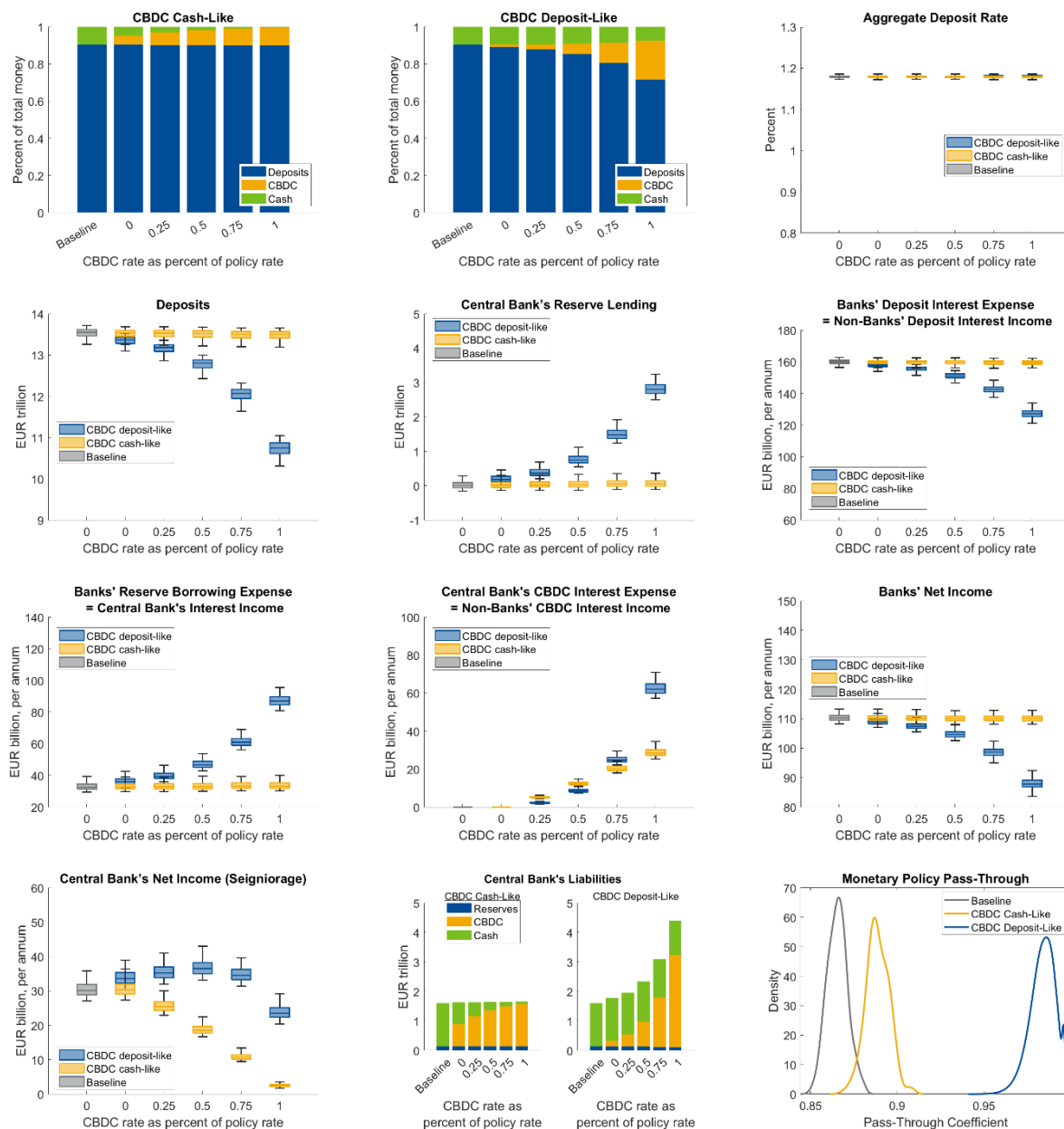
<sup>22</sup> Source: FRB Financial Accounts and own calculations.

**Figure 4. CBDC Counterfactual Results for the U.S.**



**Note:** The chart collection is a summary of the CBDC counterfactual analysis for the U.S. All stocks and flows represent their sum across individual agents (e.g., for individual banks). The CBDC-counterfactual estimates of the shifts in monetary policy pass-through (lower right corner) assume that the CBDC interest rate equals the policy rate. Annex V has the complete underlying chart panels. See text for details.

**Figure 5. CBDC Counterfactual Results for the Euro Area**



*Note:* The chart collection is a summary of the CBDC counterfactual analysis for the euro area. All stocks and flows represent their sum across individual agents (e.g., for individual banks). The CBDC-counterfactual estimates of the shifts in monetary policy pass-through (lower right corner) assume that the CBDC interest rate equals the policy rate. Annex V has the complete underlying chart panels. See text for details.



## 6. Discussion and Conclusions

**We have set up a model for simulating the integrated balance sheets of banks, non-bank agents, and the central bank.** The model combines elements of choice theory, game theory, industrial organization, reinforcement learning, and stock-flow consistency. One subset of the model's parameters is aligned with empirical counterparts. Another subset is estimated for the model's endogenous cash ratios and deposit rate-policy rate spreads to match some corresponding empirical targets, such as currently observed ones. This is how the model captures any initial level of competition in a banking system, alongside people's pre-CBDC preference for (utility derived from) central bank money. Counterfactuals such as the introduction of a CBDC can then be conducted.

**An analytical model variant of the simulation-based one reveals a close quantitative correspondence between both models, while the analytical one cannot answer all our questions, however.** The analytical model variant was useful to prove that our learning algorithm works as intended in identifying Nash equilibria, and for confirming the correspondence of the counterfactual results for the first two of our five questions; while our questions 3-5 cannot be addressed with the analytical variant. The ABM's primary avail therefore lies in answering the additional questions, regarding the profits for banks and the central bank, including all underlying drivers, reserve dynamics, monetary policy pass-through, etc.

**The CBDC counterfactual simulations suggest that the CBDC-in-total-money shares for the U.S. and the euro area may approach 25% and 20%, respectively, as upper bounds.** This assumes that interest on CBDC was equated to the central bank's reserve borrowing rate and that people perceive CBDC as close substitutes to bank deposits.

**Equilibrium deposit rate-policy rate spreads fall, though quantitatively not much for the applications for the U.S. and the euro area.** The deposit rate spreads are estimated to fall by up to 20 basis points for the U.S., while no significant variation is observed for the euro area. Monetary policy pass-through has nonetheless been found to have the potential to become notably stronger. The conditions under which equilibrium deposit rate spreads may be more reactive are discussed in the paper.

**The upper bound volume effects for deposits and reserve balances are economically significant.** Deposits may fall by USD 4.8 trillion in the U.S. (-25%) and by EUR 2.8 trillion in the euro area (-21%), at the upper bound.

**The net effect for banks' deposit expense has two opposing drivers—rising price (relative to policy rates) and falling balances—for the net effect to be either positive or negative.** The net effect for the U.S. and euro area application suggest that deposit volume effects dominate, so that banks' absolute deposit expense flows fall. This may be different for other jurisdictions, conditional on their initial conditions. Reserve borrowing expenses rise more than deposit expenses fall. Hence, bank net profits and dividend payouts drop. Central banks may structurally lower their policy rates to counter banks' otherwise rising reserve borrowing expense (an assessment of the macroeconomic consequences of which would necessitate an extended model with a real economy layer).

**Net profits for banks would fall as expected, while central bank seigniorage behaves nonlinearly as depending on the level of the CBDC interest rate.** Net income for the U.S. banking system may fall by about USD 141 billion (-30%) per annum and by EUR 22 billion (-20%) for the euro area (upper bounds). Central bank interest income through higher reserve lending would increase (or reserve remuneration

expenses drop through falling reserve holding surpluses). Their net income (seigniorage) would first—with zero or low CBDC interest—rise but tend to fall again with further rising CBDC interest rates.

**Falling net income for banks is accompanied by central banks' rising net income, interest payouts, and seigniorage redistribution, which let private sector financial income remain unaffected on net.** This result is revealed thanks to the system-wide stock-flow consistent structure of the model.

**All counterfactual estimates are surrounded by uncertainty stemming from various sources.** These include, first, various behavioral assumptions, such as whether people would perceive CBDC as a closer substitute to cash or deposits. This has a counterpart in the model with the “base utility” and “nesting” parameters in the conditional logit model component for people's choice of how to hold and transact money. Second, the price sensitivity parameter plays a decisive role for the counterfactual results, and is surrounded by estimation uncertainty, in particular for the euro area. Any application of the model to other countries should therefore be preceded by obtaining price sensitivity estimates with maximal precision. We have hinted to some options for doing so in the paper.

**Various factors may drive the utility of CBDC in comparative terms to cash and deposits.** People's understanding that a central bank cannot default may be one of the contributors to a higher base utility of CBDC relative to bank deposits (the role of deposit insurance would flank a discussion of this aspect because it contributes to non-banks perceiving bank deposits as closer to central bank money). Factors such as policy-induced holding or transactions flow limits, concerns over less anonymity, and over cyber risk-implied leakages or operational disruptions, may lower the base utility relative to cash, however. All such factors have a corresponding parameter in the model. This may not be easily quantified to reflect such factors but at least directionally be discussed and accompanied by sensitivity analyses with the model.

**Some policy-relevant model extensions can be considered.** Eight examples include:

- **Making bank reserve borrowing collateralized:** To address collateral scarcity and the need for relaxing central bank collateral eligibility criteria in case CBDC demand would rise strongly.
- **Introducing a sovereign agent and sovereign bond markets:** To model central bank open market operations through sovereign bond markets and to account for sizeable surplus reserves still resting in the banking system in many countries post-GFC and post-Covid.<sup>23</sup>
- **Introducing heterogeneous bank market power:** To align banks in the model with specific banks in a country (individual banks in the model are already needed now, despite them not being heterogeneous, to let reserve needs arise structurally and capture any initial level of competition).
- **Refining the liability (and asset) structure for banks:** To analyze the additional behavioral responses that can be expected when a CBDC is introduced, e.g., that banks may incentivize depositors to move from sight to term deposits through raising term deposit rates which implies a rising expense, though likely less so than through outright additional reserve borrowing from the central bank. Similarly, introducing wholesale funding and its endogenous use will be useful.
- **Endogenizing bank lending rates:** To complete the pass-through process and analysis and assess the impact on bank lending. While our current reinforcement learning method is built around one instrument

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<sup>23</sup> Sizeable excess reserves post-GFC/post-Covid (i.e., sizeable banking system reserve surpluses) mean that less outright reserve borrowing would be needed if a CBDC was introduced—and its demand rise strongly. Reserve borrowing expenses may therefore not rise. But when excess reserve assets would be drawn down, bank interest income on reserve holdings would fall and diminish banks' net income compared to the pre-CBDC starting point. This is why we think the model simulations are informative already now.

(bank deposit rates) and one target (profit), such an extension will imply a more challenging “two-instruments-one-target” environment and hence require augmenting our learning algorithm.

- **Caps on CBDC flows and/or balances:** Such options are discussed widely by central banks for constraining CBDC demand. They can be easily reflected in the model. The effects for all metrics in comparison to the CBDC counterfactuals without caps can then be assessed.
- **Multiple currency use and dollarization:** The model can easily be extended to feature one or multiple additional currencies, e.g., the U.S. dollar, to conduct the CBDC counterfactual analyses for dollarized economies. The money shares would be for four instead of three buckets (currently: local currency cash, deposits, and CBDC).
- **Integration with larger-scale macroeconomic models:** Macro model shells such as the one by [Barrdear and Kumhof \(2021\)](#) can be connected with our model. Barrdear and Kumhof consider exogenous assumptions for CBDC take-up rates, which a model as ours, alongside all aspects captured in our five questions, can endogenize. The currently exogenous policy rate in our model would be endogenized through a Taylor rule from the macro model block.

**Adequately reflecting the money creation feature of banks in a model is important in our view for accurately assessing the implications of a fundamental structural change as a CBDC introduction, as pertaining exactly to money and how it is created.** It would be instrumental to further reveal the qualitative and quantitative consequence of the model choice in this regard, following the example of [Jakab and Kumhof \(2015\)](#). Banks are implemented as money creators in our model, in a two-layer monetary system that features a segregated reserve-settlement structure in its second layer, in line with reality.

**Related, regarding terminology, we recommend not referring to the scenario of falling deposit balances and rising central bank reserve borrowing—both as a result of rising CBDC demand—as “disintermediation.”** Since banks are not intermediaries in the first place, speaking of disintermediation would be a misnomer. We define intermediation as any *non-banks’* on-lending of their pre-existing money asset, so that the total money stock (e.g., measured as M2) would *not* change. This can happen, for example, through peer-to-peer lending, or decentralized market-based, bond lending by non-bank financial or nonfinancial agents. In a scenario with positive CBDC demand, *banks*, however, would still create new money liabilities and hence increase (decrease) M2 through lending (principal repayment), just as with co-existing physical cash nowadays. Meanwhile, the difference would be that banks would increasingly generate *another agent’s money liabilities*—the central bank’s. Should we see banks as becoming *more of an intermediary* in this case? We would not suggest so, because banks would remain the actors responsible for influencing the money stock dynamics, i.e., they remain being money creators, while non-banks’ monetary savings would not be needed to create such new money, neither in a pre-CBDC nor a post-CBDC world.

**The model codes are available along with this paper.** This is meant to allow replicating the results that are shown in the paper and to apply the model to other jurisdictions. Cross-references to all equations in the paper are placed in the model codes.

## Annexes

### Annex I: Thompson Sampling (TS)-Based Reinforcement Learning of Nash Equilibrium Deposit Rates

This Annex first lays out the details of how our TS-based Nash learning algorithm is designed and embedded in the agent-based model (ABM). Second, it demonstrates that a Nash equilibrium computed dynamically based on the algorithm converges to an analytically derived Nash solution in a simple economic environment.

#### Background

The Thompson Sampling (TS) algorithm (Thompson 1933) was developed for so-called multi-armed bandit problems, to capture the trade-off between the acquisition cost of new information (exploration) and the generation of rewards based on existing information (exploitation). Thompson developed this concept in a drug testing context. Chapelle and Li (2011) find that TS is asymptotically optimal and outperforms so-called Upper Confidence Bound (UCB) methods. The advantages of TS over UCB methods are further explored in Russo and Van Roy (2014), who also establish a methodological connect between TS and UCB to thereby help convert regret bounds developed for UCB methods to Bayesian regret bounds for posterior sampling. The concept of a Nash equilibrium has not been considered in any of such papers. In the economics domain, TS was used only for dynamic pricing. Ferreira, Simchi-Levi, and Wang (2018), for example, implement a TS-based revenue optimization algorithm to solve a dynamic pricing problem for multiple items under finite horizon, involving inventory constraints and unknown demand functions. Such and other papers with dynamic pricing applications do not bridge to games and Nash equilibria, as we do here.

#### The Thompson Sampling Algorithm for Finding Nash Equilibria

Beyond the high-level logic of our algorithm (shown in Table 3 of the main text), its fuller description, beginning with Table A1, includes the details regarding deposit rate initialization and the sequence by which each iteration during the model simulation entails either a *sampling step* or a *learning step*.

**Table A1. Pseudo Code—TS-Based Learning of Nash Deposit Rate Equilibrium**

|  |    |   |
|--|----|---|
| <b>Step 0</b><br>Prepare objects<br>required for TS-<br>based learning | 1  | IG $\leftarrow$ 1xG interest rate grid vector, equally spaced on $[0, i^{RB}]$ interval ( $i^{RB}$ =policy rate)  |
|  | 2  | mA $\leftarrow$ 2xG matrix for holding the evolving posterior beta distribution's shape parameter $\eta_{up}$ and $\eta_{down}$ , initialized at ones     |
|  | 3  | mB $\leftarrow$ 2xG matrix for holding the evolving posterior beta distribution's shape parameter $\kappa_{up}$ and $\kappa_{down}$ , initialized at ones |
|  | 4  | TS $\leftarrow$ 0-1 indicator signaling whether sampling step in Step 1 is to be conducted (1) or not (0); initialize at 0                                |
| <b>Step 1</b><br>TS-based learning<br>Part 1: Sampling                 | 5  | For t=1...T   |
|  | 6  | If t=1  |
|  | 7  | RATES(t, :) $\leftarrow$ random grid position from IG to set first deposit rate for all banks   |
|  | 8  | TS $\leftarrow$ 0, keep TS still off in first period  |
|  | 9  | Elseif t=2  |
|  | 10 | RATES(t, :) $\leftarrow$ RATES(t-1, :)  |
|  | 11 | TS $\leftarrow$ 1   |
|  | 12 | Elseif t>2  |
|  | 13 | If TS==0 # previous period was TS=1 and "success" for explorer  |
|  | 14 | RATES(t, :) $\leftarrow$ RATES(t-1, EXPL), i.e., all banks adopt explorers' previous period rate  |
|  | 15 | Elseif TS==1 # previous period was TS=1 and "failure" for explorer  |
|  | 16 | RATES(t, :) $\leftarrow$ RATESLAST, i.e., deposit rate vector from last TS=0 period   |
|  | 17 | End   |
|  | 18 | End   |
|  | 19 | If TS==1 # Thompson Sampling Part I: Sampling   |

|                   |    |  |
|-------------------|----|--|
|                   | 20 | EXPL $\leftarrow$ Pick one random bank, the "explorer"   |
|                   | 21 | $g \leftarrow$ determine explorer's position in the rate grid IG   |
|                   | 22 | $d1 \leftarrow$ scalar random draw from posterior $\text{beta}(mA(1,g), mB(1,g))$                            |
|                   | 23 | $d2 \leftarrow$ scalar random draw from posterior $\text{beta}(mA(2,g), mB(2,g))$                            |
|                   | 24 | When $g=G$ , set $d1 = 0$ . When $g=1$ , set $d2 = 0$ . To prevent rates from moving outside the grid IG.    |
|                   | 25 | If $d1 > d2$   |
|                   | 26 | RATES( $t, \text{EXPL}$ ) $\leftarrow$ IG( $g+1$ )   |
|                   | 27 | MOVE $\leftarrow 1$ , indicate that move is up   |
|                   | 28 | Elseif $d1 < d2$   |
|                   | 29 | RATES( $t, \text{EXPL}$ ) $\leftarrow$ IG( $g-1$ )   |
|                   | 30 | MOVE $\leftarrow 2$ , indicate that move is down   |
|                   | 31 | End  |
|                   | 32 | $g^* \leftarrow$ explorer's new temporary grid position, either equal $g+1$ or $g-1$                         |
|                   | 33 | End  |
| Steps 2-4         | 34 | [...]  |
| Step 5:           | 35 | If TS==1 # Thompson Sampling Part II: Learning   |
| TS-Based Learning | 36 | If PROF( $t, \text{EXPL}$ ) > PROFREF( EXPL ) # explorer "success," i.e., net profit up                      |
| Part 2: Learning  | 37 | $mA(\text{MOVE}, g^*) \leftarrow mA(\text{MOVE}, g^*) + 1$   |
|                   | 38 | TS $\leftarrow 0$ , i.e., "success" followed by "no TS" period to obtain new profit reference                |
|                   | 39 | Else # explorer "failure," i.e., net profit fell   |
|                   | 40 | $mB(\text{MOVE}, g^*) \leftarrow mB(\text{MOVE}, g^*) + 1$   |
|                   | 41 | TS $\leftarrow 1$ , i.e., "failure" followed by TS period, but with possibly further lagged profit reference |
|                   | 42 | Elseif TS==0   |
|                   | 43 | PROFREF $\leftarrow$ PROF( $t, :$ ), i.e., track bank profits from the "no TS" period with common rates      |
|                   | 44 | RATESLAST $\leftarrow$ RATES( $t, :$ ), i.e., track deposits rates   |
|                   | 45 | TS $\leftarrow 1$  |
|                   | 46 | End  |
|                   | 47 | End # Loop over time   |

Note: Steps 1-5 as indicated in the first column correspond to Steps 1-5 in Figure 1.

The RATES vector referred to in Table A1 is of size  $1 \times B$  per point in time, containing only bank deposit rates. The rates on cash (0) and CBDC ( $i^{DC}$ ) are kept separate for their use in Step 2 (Figure 1, main text) as input to non-banks' choice drawings.

Two design features of the algorithm are worth emphasizing. First, whenever an explorer bank was deviating with its deposit rate and the step was found to be a "success" (i.e., its profit rose by the end of the period), in the subsequent period all banks follow suit to that same new rate. In that period, no exploration takes places, because a new "profit reference" will have to be measured (line 43 in Table A1). When, however, the explorer bank's move implied a fall in profit, then the subsequent period will right away be another exploring period, with the reference period for comparing to previous profits being from when there was no exploration the last time. This structure is reflected with the if-else structures in lines 13-17, 19-33, and 42-46 in Table A1. A less efficient alternative—which also converges but at lower speed—would be to force an alternating sequence of exploring and learning in one period and having a neutral reference period with equal rates across banks thereafter.

Second, the exploration and learning process happens simultaneously, with acting on what is learnt. A second option would in principle be to learn first for a pre-defined amount of time, while acting, i.e., choosing deposit rates, still randomly, and after that stop learning and start acting only. This would work but is not advisable because the length of the initial period would have to be set arbitrarily, while it would have to be made sure to be sufficiently long to ensure an adequate learning outcome. Determining whether such an outcome is sufficiently close to equilibrium can only be assessed by exactly intertwining the learning and

acting, as has been done in our algorithm. Hence, intertwining these steps from the beginning is more efficient.

### **Convergence of TS Algorithm to Analytical Nash Equilibrium in a Simple Economic Environment**

We consider now an economic environment that starts with the same random utility model for the agents' choice of banks as embedded in the ABM. That is, non-bank agents' utility is defined as in eq. (1).

The first way we simplify the economic environment is by employing a more stylized bank profit function (compare to eq. (6)):

$$\pi_b = D_b(r - i_b), \quad (9)$$

where  $D_b$  is the volume of deposits located at bank  $b$  and  $r$  is the policy rate. The function is simplistic in assuming, first, that the deposits residing in a bank imply the corresponding loan amount outstanding on the asset side of bank balance sheets, which generate the deposit liabilities; this can be interpreted as any deposit transfer implying a loan transfer from the deposit-losing to the deposit-receiving bank. Second, the interest earned on such loans is the policy rate for now, i.e., we abstract from dynamically moving spreads between lending rates and policy rates or bank funding costs. This simplifying assumption also applies to the ABM. Third, we abstract from all other sources of income and expenses for banks, e.g., loan losses, operational expenses, interest expense for central bank reserves to accomplish transfers of deposits or exchange of deposits to cash, and so on.

Further compared to the ABM, the simplified set-up does not involve any stock-flow consistent balance sheet tracking, no explicit interest and dividend flows, no non-bank consumption flows, and no tracking of non-bank-to-bank loan contract relationships. Doing the latter implies an assumption that any non-bank's deposit account move comes along with a loan contract transfer to the recipient bank; i.e., it is as if we had  $\delta = 1$  in the main model. All such extensions have an impact on the Nash equilibrium rates and market shares—in an intuitive manner regarding the impacts' sign—as is demonstrated in Annex III.

Banks set their deposit rates in a way that maximizes profits. The first order condition for each bank's profit maximization proceeding from eq. (9),  $\max_{i_b} \pi_b(i_b)$  is:

$$\frac{d\pi_b}{di_b} = \frac{ds_b}{di_b}(r - i_b) - s_b = D(\beta s_b(1 - s_b)(r - i_b) - s_b) = 0, \quad (10)$$

where  $D = \sum_b D_b$ , assuming the total amount of deposits is constant for now, and  $s_b$  is bank  $b$ 's market share, whose form results from the conditional logit model structure:

$$s_b = \frac{e^{\alpha_b + i_b \beta}}{\sum_{k=1}^B e^{\alpha_k + i_k \beta}}. \quad (11)$$

An analytical solution to any bank  $b$ 's rate,  $i_b^*$ , from eq. (11), conditional on all other banks' rates, is:

$$i_b^* = r - \frac{1}{\beta} \left( 1 + W \left( \frac{e^{\alpha_b + \beta r - 1}}{\sum_{k \neq b} (e^{\alpha_k + i_k \beta})} \right) \right), \quad (12)$$

where  $W$  is the Lambert function. Banks setting their rates based on this formula results in the Nash equilibrating rates after a few rounds (Bloechlinger and Maire 2010). When the market power of all banks

was identical, and without having outside options such as cash, the solution reduces to  $i_b^* = r - B/\beta(B-1)$ , as also shown in [Bloechlinger and Maire \(2010\)](#).

When we assume a nested logit form, recalling eqs. (2) to (6) in the main text, for any bank in nest  $m = 2$  and assuming  $\mu_2 = 1$ , the analytical solution that we obtain for any bank  $b$ 's rate,  $i_b^*$  is:

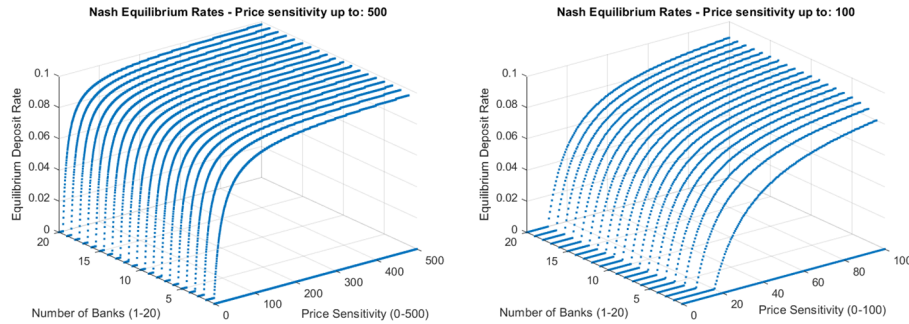
$$i_b^* = r - \frac{1}{\beta} \left( 1 + W \left( \frac{e^{\alpha_b + \beta r - 1}}{\sum_{k \neq b, m=2} (e^{\alpha_k + i_k \beta}) + \sum_{j, m=1} (e^{(\alpha_k + i_k \beta) \mu_1})^{1/\mu_1}} \right) \right). \quad (13)$$

The most noteworthy features of the Nash equilibrium and its drivers include:

1. A rising number of banks, all else equal, means higher deposit rates (smaller markdowns).
2. The number of banks falling to one makes the deposit rate fall to a minimum (maximal markdown), assuming that the base utility of outside options is sufficiently small.
3. A rising price sensitivity of depositors, all else equal, renders deposit rates higher (markdowns smaller). In a heterogeneous market power case, this holds for all individual banks' Nash rates as well as their deposit volume weighted average.
4. A price sensitivity of  $\beta = +\infty$  implies that deposit rates approach the upper bound, the policy rate, or converge to a minimum when  $\beta = 0$ , both regardless of the number of banks.

Figure A1 visualizes these basic dependencies based on the model with homogeneous market power across banks, without further outside options such as cash, and the policy rate set to  $r = 0.1$ .

**Figure A1. Nash Equilibrium Deposit Rates as a Function of the Number of Banks and Non-Banks' Price Sensitivity**



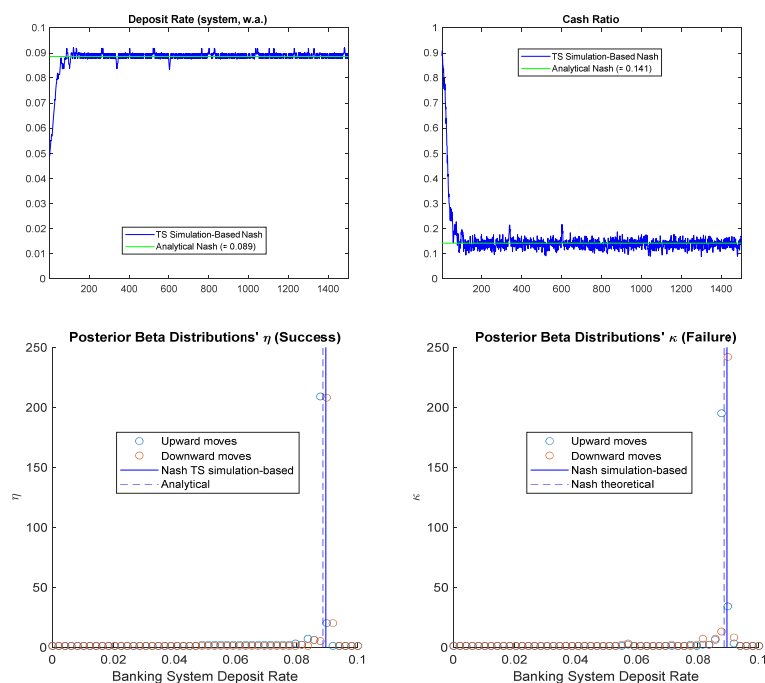
*Note:* The figures show the dependence of the Nash deposit rate on the number of banks and non-banks' price sensitivity. The estimates on the right side correspond to those on the left, with a zoom into the price sensitivity for a value of up to only 100.

Figure A2 shows how our TS learning algorithm successfully identifies the theoretical Nash equilibrium deposit rates, alongside the Nash market share of the banks and cash. The theoretical set-up could of course be extended to include a  $B+2^{\text{nd}}$  option pertaining to CBDC. The exemplary parameterization for the results in Figure A2 is  $r = 0.1$ ,  $B = 7$ ,  $\alpha_b = 0 \forall b = 1, \dots, B$ , and  $\alpha_{cash} = 9$ . The deposit rate grid length on the  $[0, r]$  interval was set to 50. The resulting analytical Nash equilibrium deposit rate and Nash cash ratio equal 8.9% and 14.1%, respectively. After about 150 periods, the equilibrium is closely identified by the algorithm. The upper row of Figure A2 shows the time series of deposit rates and cash ratios as evolving through the



simulation. The second row of Figure A2 visualizes the sum of additions to the shape parameters of the beta distributions during the simulation, confirming, equivalently with the first row, the successful identification of the analytical Nash equilibrium.

**Figure A2. Convergence to Nash Equilibrium Deposit Rates and Nash Cash Ratios from TS-Based Learning Algorithm Vs. Analytical Solution**



*Note:* The figures show that the TS simulation-based reinforcement learning algorithm successfully identifies the analytically computed Nash deposit rate and Nash cash ratio.

## Annex II: Sampling Choice Signals from Nested Conditional Logit Models

When considering a conditional logit model without nesting, i.e., when  $\mu_m = 1 \forall m = 1, 2$ , and if the utility function (eq. (1) in the main text) is parameterized with the  $\alpha_b$ 's and  $\beta$ , then a random draw of a choice is obtained as follows: first, a uniform random draw,  $p$  from  $[0, 1]$ , is generated and fed through the inverse cumulative GEV (Generalized Extreme Value) distribution,  $G^{-1}(p) = -\ln(-\ln(p))$ , and then summed with the deterministic part of utility,  $v_{tb}$ . That is, the  $b = 1, \dots, B + 2$  utility draws for all non-bank agents,  $u_{n,b,t}^*$ , are obtained as:

$$u_{n,b,t}^* = v_{tb} + G^{-1}(p), \forall b = 1, \dots, B + 2, \forall n = 1, \dots, N. \quad (14)$$

Second, the choice signal,  $\hat{b}_{nt}$ , then identifies the position of the maximum from the sampled utilities:

$$\hat{b}_{nt} = \underset{b}{\operatorname{argmax}} \{u_{t,n,b=1}^*, \dots, u_{t,n,b=B+2}^*\}. \quad (15)$$

When having a model with nesting, i.e., when  $\mu_m > 1$  for at least one nest, then the drawing works as follows: For the choice of the nest (Decision Layer 1 in Figure 2, main text), the scaled inclusive values are used to generate the utility draws for all non-bank agents and nests:

$$u_{n,m,t}^{*,\text{Layer 1}} = \frac{\Xi_m}{\mu_m} + G^{-1}(p), \forall m = \{1, 2\}, \forall n = 1, \dots, N. \quad (16)$$

The choice of the nest, for any one non-bank agent is:

$$\hat{b}_{nt}^{\text{Layer 1}} = \underset{m}{\operatorname{argmax}} \{u_{n,m=1,t}^{*,\text{Layer 1}}, u_{n,m=2,t}^{*,\text{Layer 1}}\}, \forall n = 1, \dots, N. \quad (17)$$

At the lower level (Decision Layer 2 in Figure 2, main text), the utility draws are computed as:

$$u_{n,b,t}^{*,\text{Layer 2}} = v_{tb} + \frac{G^{-1}(p)}{\mu_m}, \forall b = 1, \dots, B + 2, \forall n = 1, \dots, N, \quad (18)$$

where  $\mu_m$  in this equation is understood to be the one that is relevant for a Layer 2-choice  $b$ , that is,  $m = 1$  for  $b = 1, \dots, B$  (commercial bank deposit nest), and  $m = 2$  for  $b = B + 1, B + 2$  (central bank money nest). The choice at the lower level is then:

$$\hat{b}_{nt}^{\text{Layer 2}} = \underset{b}{\operatorname{argmax}} \{u_{n,b,t}^{*,\text{Layer 2}}, \dots\}, \forall n = 1, \dots, N. \quad (19)$$

Any one non-bank agent therefore makes two sequential decisions: first, the decision about the nest (eq. (17)), and then correspondingly the choice of the underlying bank, if in the bank deposit nest, or cash or CBDC, if in the central bank money nest (eq. (19)).

### Annex III: Dependence of the Model Dynamics on Model Parameters

This Annex presents the sensitivities of some primary model outcomes to selected model parameters. “Primary model outcomes” are chosen to be the equilibrium deposit rates, cash ratios, and the ratios of bank end-of-period reserve holdings over bank deposit balances. The model parameters are chosen to be four of the nine parameters listed in Table 4, excluding from that list the number of banks (#1), the base utilities (#2), the price sensitivity (#3), the reserve borrowing rate (#7), and the interest on CBDC (#9). The number of banks and the price sensitivity are excluded here because the dependence of the model dynamics on them was shown already in Annex I.

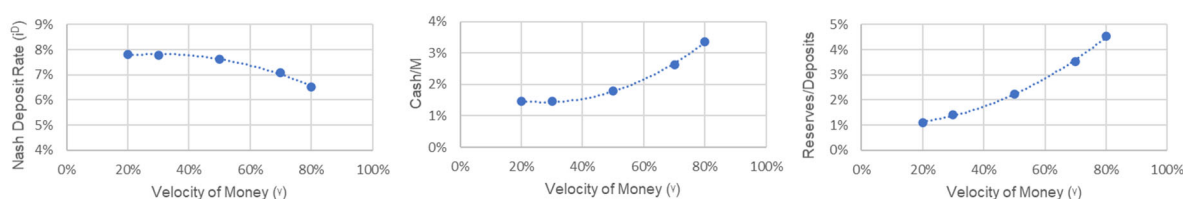
The base parameterization for the following sensitivity analyses is:  $B = 10$ ,  $\alpha_{b=1...B} = 0$ ,  $\alpha_{cash} = 50$ ,  $\alpha_{CBDC} = -\text{Inf}$ ,  $\beta = 60$ ,  $\gamma = 0.5$ ,  $\delta = 1$ ,  $\lambda = 0$ ,  $i^{RB} = 0.1$ ,  $i^{ERH} = 0$ . The rate on required reserve holdings is irrelevant because the required reserve ratio is set to zero here ( $\lambda = 0$ ), except when it will come to the related sensitivity analysis. The interest rate for CBDC is irrelevant because the base utility of CBDC ( $\alpha_{CBDC} = -\text{Inf}$ ) is set so that the CBDC share stays at zero.

#### (1) Velocity of Money ( $\gamma$ )

Five grid points for  $\gamma$  were considered:  $\gamma = [0.2, 0.3, 0.5, 0.7, 0.8]$ . Figure A3 shows the results for the three metrics of interest. With a rising money velocity, equilibrium deposit rates drop, while the cash ratio and the reserve-to-deposit ratio grow. Reserve needs increase as a direct result of the larger transaction flows (given an otherwise equal stock of money). Larger reserve needs in turn imply a higher interest expense for reserve borrowing, causing a drag on bank profits (eq. (6), main text). The falling deposit rates imply a rising share of cash because the relative utility of deposits (cash) decreases (increases).

The analytically computed Nash deposit rate without any dependence on the velocity of money (and other features such as the loan transfer process, etc.) equals 8.2% (Annex I has the formulas). The analytical cash ratio equals 1.5%. These are approached in Figure A3 for these two metrics on the left side, when the money velocity falls and thereby approaches the implicit zero money velocity in the analytical model variant.

**Figure A3. Sensitivity Analysis: Velocity of Money ( $\gamma$ )**



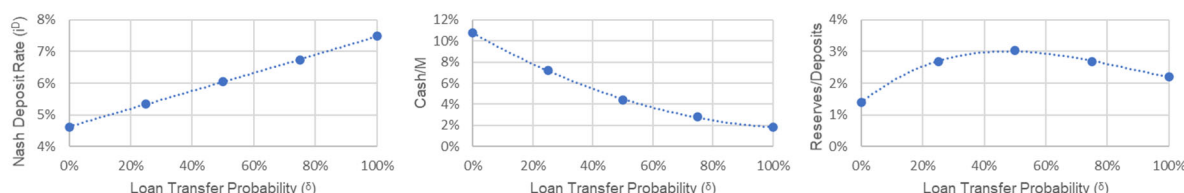
Note: The figure shows the dependence of some selected model outcomes to the velocity of money. “M” denotes monetary aggregate.

#### (2) Loan Transfer Probability ( $\delta$ )

Five grid points for  $\delta$  were considered:  $\delta = [0, 0.25, 0.5, 0.75, 1]$ . Figure A4 shows the results. The Nash deposit rates are an increasing function of the loan transfer probability, and hence the cash ratio a decreasing function thereof. This is because a bank incentivizing depositors with higher deposit rates to move to it is increasingly beneficial with a rising loan transfer probability, in turn because that generates more interest income when depositors also move their loan contract relationship to the bank. The reserve-to-deposit ratio is first an increasing and then a decreasing function of the transfer probability.

The analytically computed Nash deposit rates and cash ratios (8.2% and 1.5%, respectively, as before) are approached on the respective right ends of the grid we consider in Figure A4. This is due to the simpler analytical model variant having the built-in assumption of a 100% loan transfer probability when moving deposit accounts. The right ends still fall short of the analytical equilibrium because the velocity of money is assumed to still equal  $\gamma = 0.5$  (not 0).

**Figure A4. Sensitivity Analysis: Loan Transfer Probability ( $\delta$ )**



*Note:* The figure shows the dependence of some selected model outcomes to the loan transfer probability parameter.

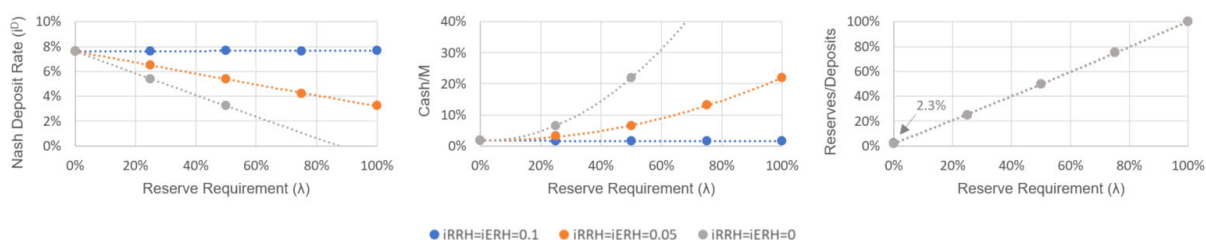
### (3) Reserve Requirement ( $\lambda$ ) and Reserve Remuneration Rates ( $i^{RRH}, i^{ERH}$ )

Five grid points for  $\lambda$  were considered:  $\lambda = [0, 0.25, 0.5, 0.75, 1]$ . The results for this grid were produced three times, for three reserve remuneration rates, treating the rates on required reserves and excess reserves in a symmetric manner here:  $i^{RRH} = i^{ERH} = [0.1, 0.05, 0]$ . We recall that the reserve borrowing rate (in short, “policy rate”) was set at  $i^{RB} = 0.1$ . Figure A5 shows the results.

When the reserve remuneration rates equal the policy rate, varying the reserve requirement does not influence the model outcomes. When lowering the remuneration rates to levels below the policy rate, the dependence of equilibrium outcomes on the reserve requirement starts being visible: a rising reserve requirement then means for Nash deposit rates to fall and cash ratios to rise, with an increasingly strong downward/upward slope the lower the reserve remuneration rates are.

When the reserve remuneration rates were sufficiently low, and the reserve requirement sufficiently high, the cash ratios at some point reach 100%. Deposit balances fall to zero and hence the deposit rates become inconsequential.

**Figure A5. Sensitivity Analysis: Reserve Requirement and Reserve Remuneration Rates**



*Note:* The figure shows the dependence of some selected model outcomes to the reserve requirements ratio and the reserve remuneration rates. With a reserve requirement at 0%, the simulated reserve-to-deposit ratio equals 2.3%, as on the right side of the third chart in Figure A4 (where  $\delta = 1$ ), and the middle of the third chart in Figure A3 (where  $\gamma = 0.5$ ).

## Annex IV: Empirical Estimation and Model Calibration

### Step A. Estimate non-banks' price sensitivity ( $\beta$ ).

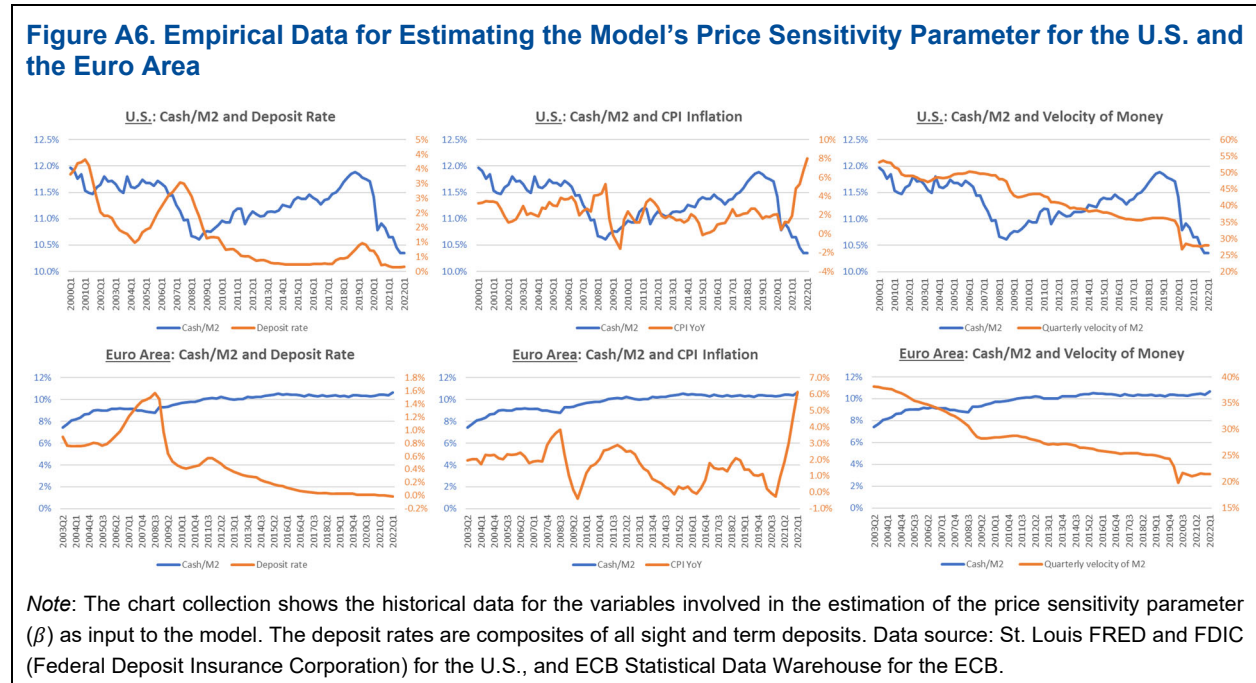
The price sensitivity,  $\beta$ , is estimated based on a multinomial logit structure of the kind as embedded in the ABM (eq. (1)). The utilities are here defined for deposits as an aggregate (while the ABM has individual banks), alongside cash, and hence operating at the upper decision layer in Figure 2, that is:

$$\begin{bmatrix} u_{nt}^{\text{CASH}} \\ u_{nt}^{\text{DEP}} \end{bmatrix} = \begin{bmatrix} \alpha + \mathbf{Z}_t \mathbf{\Gamma} \\ \beta i_t^{\text{DEP}} \end{bmatrix} + \begin{bmatrix} \varepsilon_{nt}^{\text{CASH}} \\ \varepsilon_{nt}^{\text{DEP}} \end{bmatrix}, \quad \varepsilon_{nt}^{\text{CASH/DEP}} \sim \text{GEV}(\text{Type-I}) \quad (20)$$

where  $\mathbf{Z}_t$  is a  $1 \times K$  vector of macro-financial control variables and  $\mathbf{\Gamma}$  is a  $K \times 1$  vector of corresponding loadings. Defining  $\tau_t(\alpha, \mathbf{\Gamma}) = e^{\alpha + \mathbf{Z}_t \mathbf{\Gamma}}$ , and denoting the parameter estimates with hats, the cash ratio resulting from eq. (20) is:

$$\widehat{\text{CR}}_t = \frac{\hat{\tau}_t(\hat{\alpha}, \hat{\mathbf{\Gamma}})}{\hat{\tau}_t(\hat{\alpha}, \hat{\mathbf{\Gamma}}) + e^{\hat{\beta} i_t^{\text{DEP}}}} \quad (21)$$

The required historical data inputs include a cash-in-total-money ratio (cash/M2),  $\text{CR}_t$ , a banking system deposit rate,  $i_t^{\text{DEP}}$ , and some additional controls,  $\mathbf{Z}_t$ . Figure A6 shows these required data for the U.S. and the euro area.



The parameter estimates for the utility of cash,  $\alpha$ , the price sensitivity,  $\beta$ , and the loadings on the controls,  $\mathbf{\Gamma}$ , are obtained by minimizing the sum of squared residuals of the cash ratios observed historically and those implied by eq. (21):

$$(\hat{\alpha}, \hat{\beta}, \hat{\mathbf{\Gamma}}) = \argmin_{\alpha, \beta, \mathbf{\Gamma}} \sum_t (\Delta \text{CR}_t - \Delta \widehat{\text{CR}}_t)^2 \quad (22)$$

The empirical performance of the estimation is notably superior when using quarter-on-quarter differences ( $\Delta$ ) of the cash ratios for the criterion in eq. (22). The estimates of  $\beta$  for the U.S. and euro area are reported in Table A2. The control vector  $\mathbf{Z}_t$  contained year-on-year growth of real GDP, year-on-year CPI (Consumer Price Index) inflation, and the quarterly velocity of money defined as quarterly nominal GDP divided by M2.

**Table A2. Price Sensitivity Estimates for the U.S. and the Euro Area**

|           | Price Sensitivity ( $\beta$ ) |       |       | R-square |       | DW   | obs. |
|-----------|-------------------------------|-------|-------|----------|-------|------|------|
|           | q=0.5                         | q=0.2 | q=0.8 | level    | diff. |      |      |
| U.S.      | 47.5                          | 2.7   | 57.5  | 0.941    | 0.438 | 2.01 | 88   |
| Euro Area | 153.6                         | 49.8  | 961.1 | 0.995    | 0.416 | 2.03 | 75   |

Note: DW = Durbin Watson. The table reports the estimates of the price sensitivity ( $\beta$ ), obtained from the multinomial logit model in eqs. (20)-(22) applied to aggregate cash-in-M2 ratios, deposit rates, and as additional controls price inflation and the velocity of money. The lower and upper bounds, q=0.2 and q=0.8, are the 20<sup>th</sup> and 80<sup>th</sup> percentiles of the bootstrap-based coefficient distributions.

Informed by these estimates, we assume the  $\beta$ 's at a rounded 50 for the U.S. and at 150 for the euro area; as recorded in Table 4.

The  $\hat{\alpha}$  and  $\hat{\Gamma}$  from Step A are discarded:  $\hat{\Gamma}$  is not needed because the control variables they refer to are either absent (e.g., real GDP growth) or exogenous (velocity of money) in the ABM. The base utility for cash,  $\hat{\alpha}$ , will be inferred in Step C—in a manner consistent with the ABM—as will be outlined in Step C below.

Individual bank data can be brought into the  $\beta$ -estimation step. The second line in eq. (20), currently an aggregate, would be replaced by bank-individual equations. Detailed micro data of consumer deposit accounts and their moves over time (alongside bank deposit rates and other controls) would be another option for estimating the price sensitivities. Such data are not available for any countries/banks in the public domain, however, to the best of our knowledge. Selected references were provided in the main text.

**Step B. Calibrate other model parameters: Velocity of money ( $\gamma$ ), loan transfer probability ( $\delta$ ), and reserve-related parameters ( $\lambda$ ,  $i^{RB}$ ,  $i^{RRH}$ ,  $i^{ERH}$ ).**

The velocity of money,  $\gamma$ , is aligned with a desired (e.g., recently observed) annual GDP/M2 ratio. For the U.S. and the euro area they stood at a rounded 110% and 85%, respectively, in 2022Q2.

The loan transfer probability,  $\delta$ , is set judgmentally, to a value of 0.75; as there are no micro data or related analyses in the literature that examine the dependence of loan contract relationships and moves, as a function of moving deposit account relations, to the best of our reading. A sensitivity analysis that we conducted confirmed that the CBDC counterfactual estimation results do not depend in any notable manner on this parameter.

As of September 2022, the reserve requirement ( $\lambda$ ) is 0% and 1% for the U.S./FED and the euro area/ECB, respectively.

The reserve borrowing rate,  $i^{RB}$ , and the remuneration rates for required and excess reserves,  $i^{RRH}$  and  $i^{ERH}$ , are set to recent, historical reference values. For the U.S., both the borrowing rate and the excess reserve remuneration rate are set to 3.5%; the rate on required reserves is irrelevant since the reserve requirement is 0%. For the euro area, the three rates, [ $i^{RB}$ ,  $i^{RRH}$ ,  $i^{ERH}$ ] are set to [2%, 1.5%, 1%]. The  $i^{RB}$  was assumed at the outset, which relates to the ECB's marginal lending facility (MLF). The rate on required

reserve holdings was informed by an estimate of where the ECB's main refinancing rate would stand for an MLF rate at 2%. The rate on excess reserve holdings relates to the ECB's rate on the deposit facility, whose value, also here, was informed by its spread to the MLF rate historically conditional on the MLF level.

**Step C. Estimate the number of banks ( $B$ ) and the base utility of cash ( $\alpha_{cash}$ ) to let the ABM match a target cash ratio and deposit rate.**

This third step entails three sub-steps.

First, the model is simulated for a base parameterization along two grids, for selected values of  $B$  and  $\alpha_{cash}$ , respectively. If the grids were of length 10 and 10, 100 simulations would be conducted, for the parameterization as set so far in Steps A and B, for  $T = 3000$  periods each. The average Nash equilibrium deposit rates and cash ratios are then recorded as a median over the last 1,000 periods. Two exemplary resulting matrices are shown in Figure A7.

**Figure A7. Input (Exemplary) to Estimation Step C: Dependence of Model-Implied Nash Deposit Rates and Cash Ratios on Grids of Number of Banks and Base Utility of Cash**

| Nash<br>Deposit Rate |    | $\alpha_{cash}$ |      |      |      |      |      |      |      |  |
|----------------------|----|-----------------|------|------|------|------|------|------|------|--|
|                      |    | -20             | -10  | -5   | 0    | 2.5  | 5    | 7.5  | 10   |  |
| $B$                  | 1  | 0.3%            | 0.3% | 0.5% | 0.8% | 4.9% | 6.1% | 6.1% | 6.1% |  |
|                      | 2  | 2.8%            | 2.8% | 2.8% | 3.1% | 4.9% | 6.1% | 6.1% | 6.1% |  |
|                      | 3  | 4.6%            | 4.6% | 4.6% | 4.7% | 5.4% | 6.1% | 6.1% | 6.2% |  |
|                      | 4  | 4.8%            | 5.0% | 5.2% | 5.2% | 5.5% | 6.1% | 6.2% | 6.2% |  |
|                      | 5  | 5.0%            | 5.0% | 5.2% | 5.2% | 5.5% | 6.1% | 6.2% | 6.2% |  |
|                      | 7  | 5.7%            | 5.7% | 5.8% | 5.8% | 5.8% | 6.1% | 6.3% | 6.3% |  |
|                      | 10 | 5.7%            | 5.7% | 5.8% | 5.8% | 5.8% | 6.1% | 6.3% | 6.3% |  |
|                      | 15 | 5.7%            | 5.8% | 5.8% | 5.8% | 5.8% | 6.1% | 6.3% | 6.3% |  |
|                      | 20 | 5.7%            | 6.1% | 6.1% | 6.1% | 6.1% | 6.1% | 6.3% | 6.3% |  |
|                      | 25 | 5.7%            | 6.1% | 6.2% | 6.2% | 6.2% | 6.2% | 6.3% | 6.3% |  |
|                      | 30 | 6.2%            | 6.2% | 6.2% | 6.2% | 6.2% | 6.2% | 6.3% | 6.3% |  |

| Cash Ratio |    | $\alpha_{cash}$ |      |       |       |       |       |       |        |  |
|------------|----|-----------------|------|-------|-------|-------|-------|-------|--------|--|
|            |    | -20             | -10  | -5    | 0     | 2.5   | 5     | 7.5   | 10     |  |
| $B$        | 1  | 5.3%            | 9.4% | 16.0% | 39.5% | 71.6% | 95.6% | 99.7% | 100.0% |  |
|            | 2  | 0.4%            | 0.4% | 0.5%  | 13.4% | 52.9% | 91.4% | 99.7% | 100.0% |  |
|            | 3  | 0.4%            | 0.4% | 0.4%  | 6.7%  | 41.2% | 88.0% | 99.0% | 100.0% |  |
|            | 4  | 0.4%            | 0.4% | 0.4%  | 4.7%  | 33.7% | 84.6% | 98.8% | 100.0% |  |
|            | 5  | 0.1%            | 0.4% | 0.4%  | 3.7%  | 28.2% | 81.2% | 98.3% | 100.0% |  |
|            | 7  | 0.1%            | 0.4% | 0.4%  | 2.3%  | 21.6% | 75.1% | 97.2% | 100.0% |  |
|            | 10 | 0.1%            | 0.4% | 0.4%  | 1.9%  | 15.6% | 67.9% | 96.7% | 100.0% |  |
|            | 15 | 0.1%            | 0.4% | 0.4%  | 1.6%  | 10.8% | 58.0% | 94.7% | 100.0% |  |
|            | 20 | 0.1%            | 0.3% | 0.4%  | 0.9%  | 8.2%  | 51.0% | 92.5% | 99.3%  |  |
|            | 25 | 0.1%            | 0.3% | 0.4%  | 0.8%  | 7.1%  | 45.7% | 91.2% | 99.3%  |  |
|            | 30 | 0.1%            | 0.3% | 0.3%  | 0.6%  | 5.6%  | 40.7% | 89.5% | 99.3%  |  |

Note: The two matrices are numerical examples, depicting the dependence of Nash deposit rates and cash ratios from the ABM on the number of banks ( $B$ ) and the base utility of cash ( $\alpha_{cash}$ ) as input to Step C. The matrices are conditional on the parameterization (of the parameters other than  $B$  and  $\alpha_{cash}$ ) obtained through Steps A and B.

Second, two polynomial equations for the two matrices are estimated: for the log of the Nash deposit rates,

$$f_{i^D} = \ln(i^D) = \varpi_{i^D,0} + \sum_{v=1}^V (\mu_{i^D,v} B^v + \pi_{i^D,v} \alpha_{cash}^v) + \sum_{v_1}^V \sum_{v_2}^V \omega_{i^D,v_1,v_2} B^{v_1} \alpha_{cash}^{v_2}, \quad (23)$$

and the logit of the cash ratio,

$$f_{CR} = \text{logit}(CR) = \varpi_{CR,0} + \sum_{v=1}^V (\mu_{CR,v} B^v + \pi_{CR,v} \alpha_{cash}^v) + \sum_{v_1}^V \sum_{v_2}^V \omega_{CR,v_1,v_2} B^{v_1} \alpha_{cash}^{v_2}. \quad (24)$$

Linear inequality constraints are imposed on the first derivatives of the equations to further “discipline” them: for the derivative regarding  $B$  to be negative (positive) in the cash ratio (Nash deposit rate) polynomial and for the derivative regarding  $\alpha_{cash}$  to be positive in both polynomials.

Third, the two polynomials are solved for  $\hat{B}$  and  $\hat{\alpha}_{cash}$  to match a target deposit rate,  $\hat{i}^D$ , and target cash ratio,  $\hat{CR}$ , using a trust-region-dogleg algorithm (Nocedal and Wright 2006) and equal weights for the two objectives.<sup>24</sup> The minimization problem is defined as:

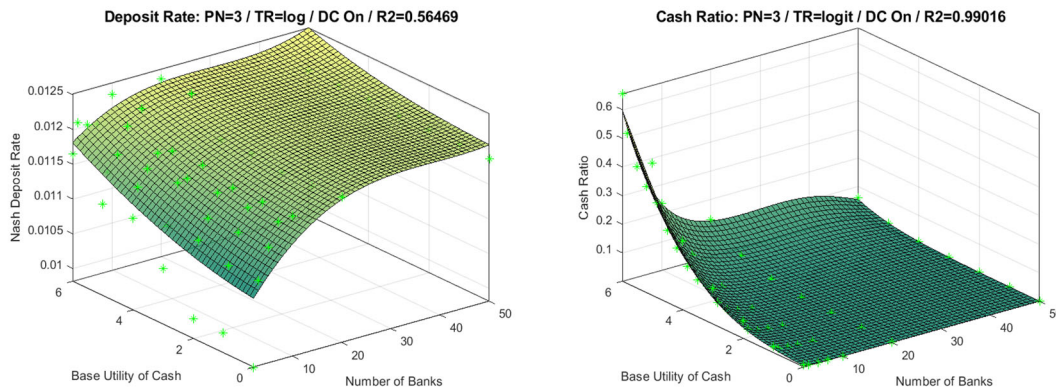
<sup>24</sup> In addition to employing such single-objective optimization algorithms that require rendering the multi-objective problems to single-objective ones by using weights *ex ante*, we also considered multi-objective optimization methods, such as a Genetic Algorithm (GA). The GA algorithm in conjunction with *ex post* weights for the two objectives resulted in the same quantitative outcomes as presented in this section using the trust-region-dogleg algorithm.



$$(\hat{B}, \hat{\alpha}_{cash}) = \underset{B, \alpha_{cash}}{\operatorname{argmin}} \left\{ \begin{aligned} & \left( \tilde{i}^D - f_{i^D}(B, \alpha_{cash}) \right)^2 \\ & \left( \widetilde{CR} - f_{CR}(B, \alpha_{cash}) \right)^2 \end{aligned} \right. \quad (25)$$

As for the policy rate in Step B, the deposit rate and cash ratio can correspond to a recent historical point in time, or to hypothetical alternative values for which a counterfactual CBDC simulation is to be conducted. Figure A8 visualizes the surfaces from which the  $(\hat{B}, \hat{\alpha}_{cash})$  coordinates—as recorded in Table 4—were obtained via the optimization in eq. (25).

**Figure A8. Polynomial Surfaces for Nash Deposit Rates and Nash Cash Ratios**

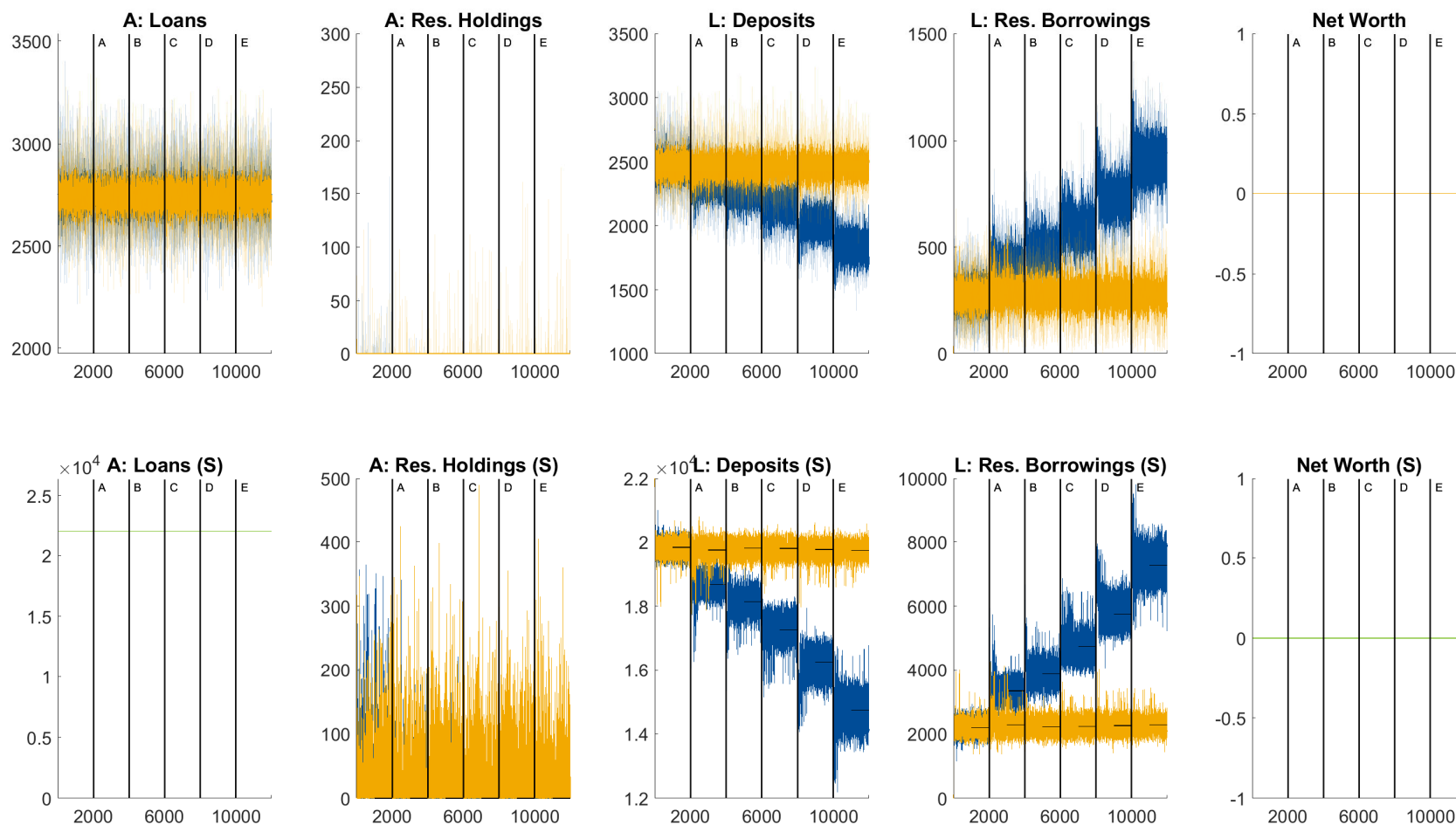


*Note:* The figures visualize the two polynomial functions that capture the relationship of the Nash deposit rate and the cash ratios as simulated from the ABM to the base utility of cash and the number of banks. The polynomial order (PN) was set to 3.

Is it an option to calibrate  $B$  first and then estimate  $\hat{\alpha}_{cash}$  alongside the price sensitivity,  $\beta$ , in the last third sub-step of Step C, instead of first setting  $\beta$  based on an empirical model, and then infer  $B$  and  $\hat{\alpha}_{cash}$  using the pair of polynomials? This is not an option because  $B$  cannot first be meaningfully calibrated with a reference to empirical data, such as the number of banks in a country. This is the case in turn because the non-bank population size  $N$  has no meaning in the model (note that  $N$  is not included in the model parameter list in Table 4).  $N$  should just be set sufficiently large in the model to imply that all banks and cash (and CBDC, once the model is accordingly parameterized) “are populated” by a sufficient number of non-bank agents for numerical stability; i.e., for no single bank to “run empty” of depositors for small deposit-to-total money ratios, unless they run all empty to 0% on purpose of a certain parameterization. Also, the analytical model to demonstrate the ability of our TS algorithm to identify the Nash equilibrium deposit rates in Annex I makes clear that there is no structural dependence on  $N$ , just like in the expanded ABM.

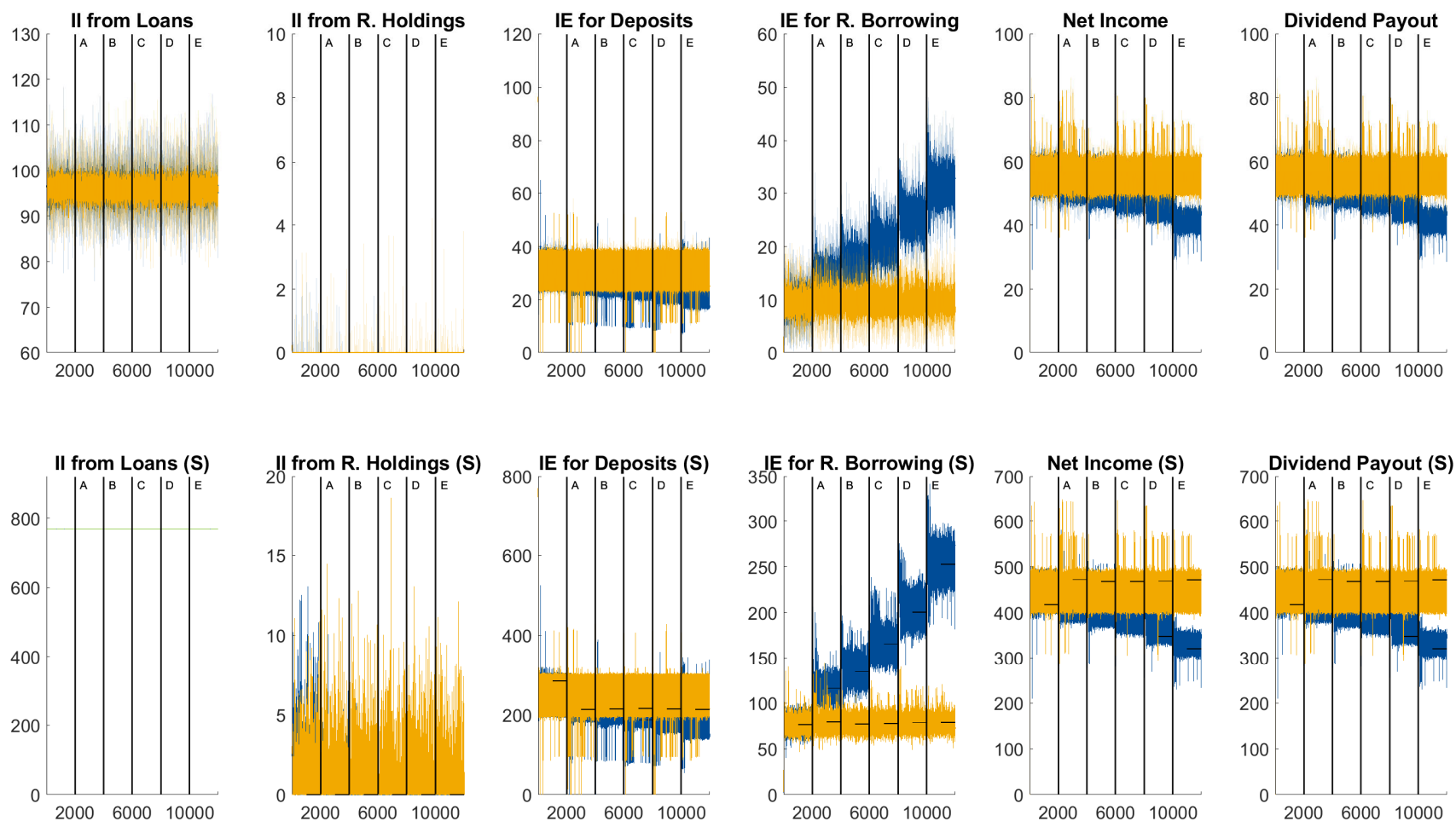
## Annex V: Detailed Counterfactual Simulation Results for the U.S. and the Euro Area

Figure A9. Bank Balance Sheets (USD billions)—Counterfactual Results for the U.S.



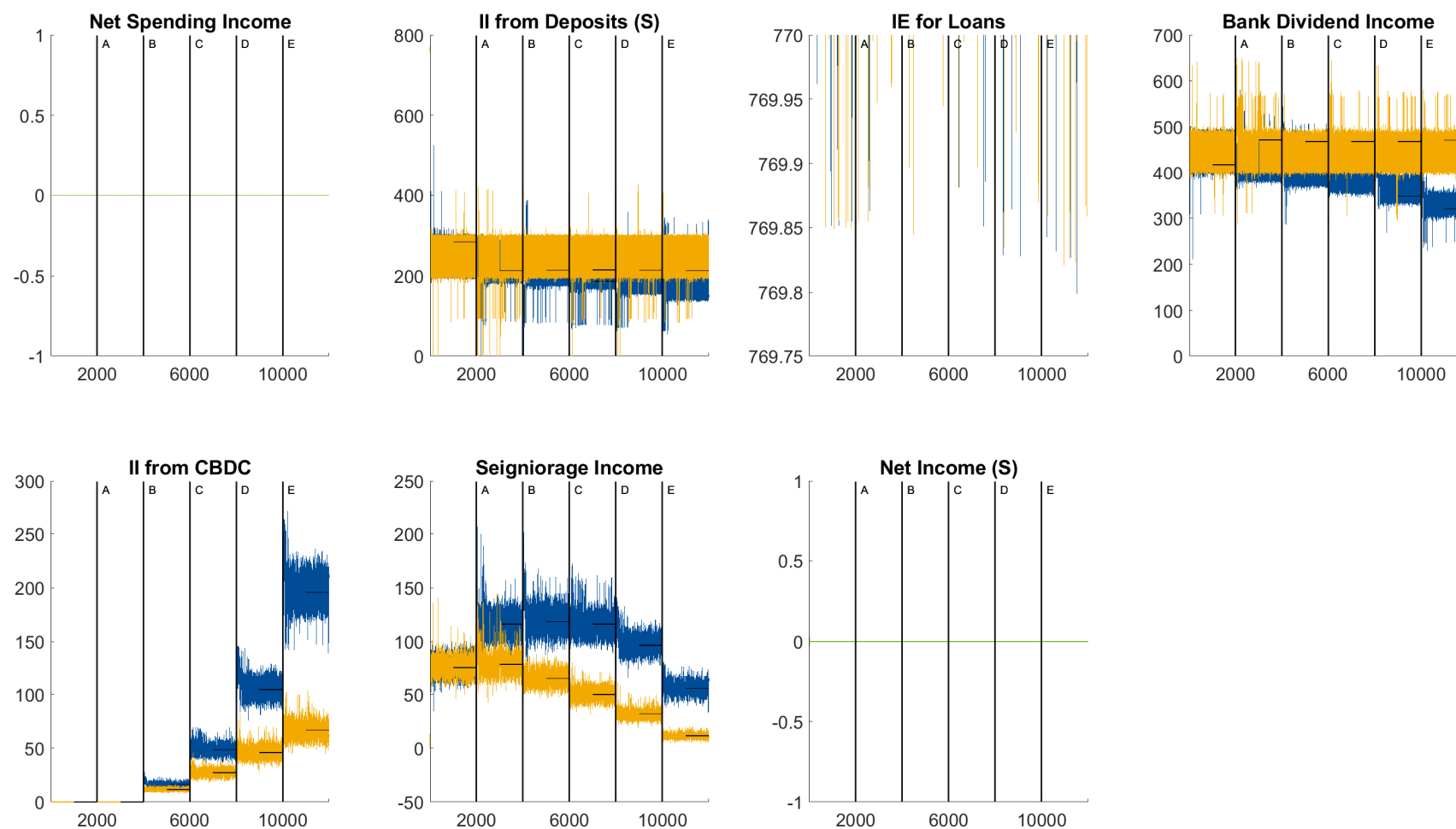
*Note:* The first of the six simulation windows corresponds to a base simulation without CBDC. The five counterfactual windows labeled A-E correspond to CBDC interest rates set to [0,25,50,75,100]% of the policy rate. The yellow and blue streams correspond to “full nesting” (cash-like CBDC) and “no nesting” (deposit-like CBDC). Charts in the first (second) row show the evolution of individual bank metrics (the sum across banks).

**Figure A10. Bank P&L Flows (USD billions, per annum)—Counterfactual Results for the U.S.**



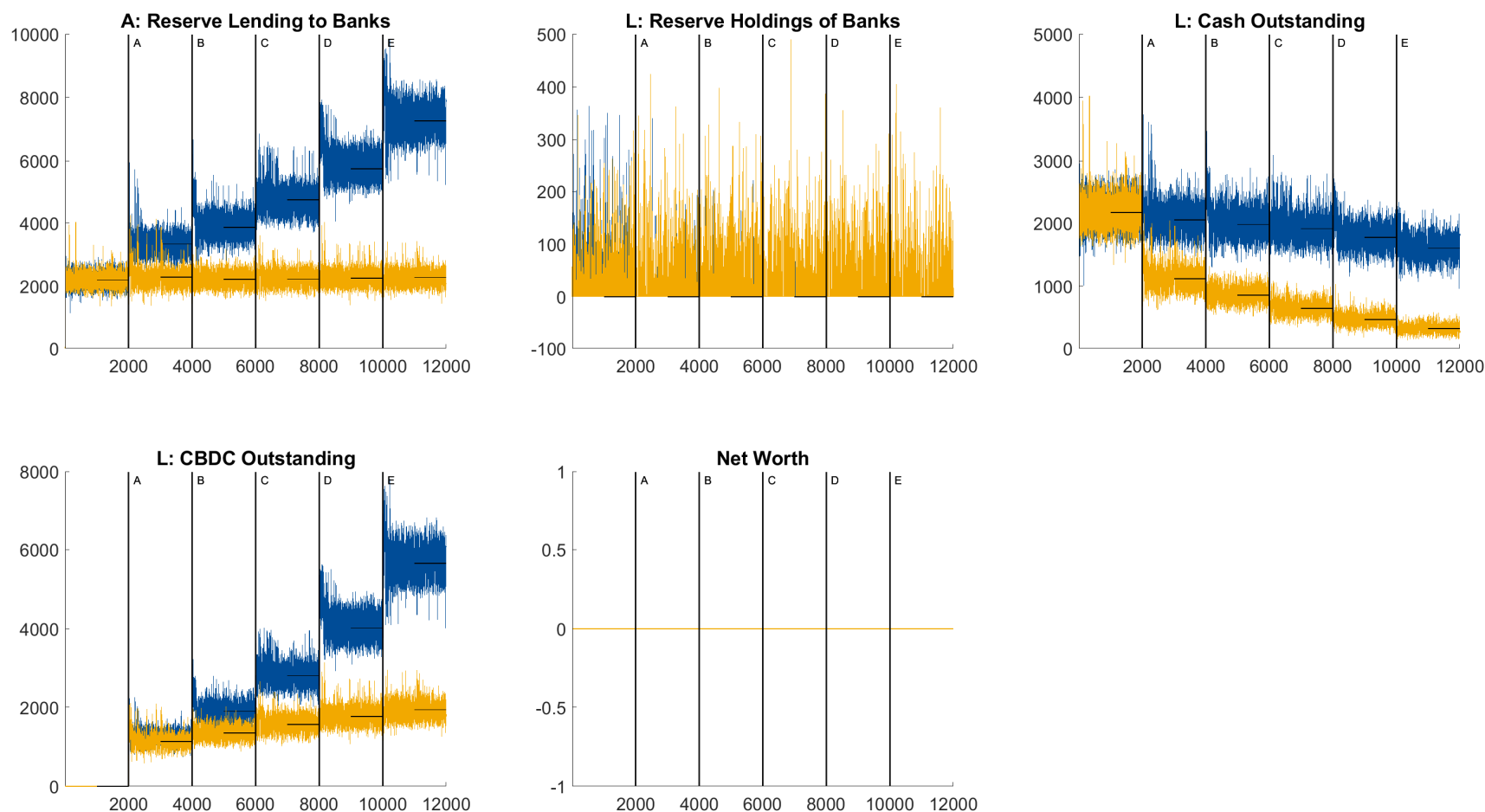
*Note:* The first of the six simulation windows corresponds to a base simulation without CBDC. The five counterfactual windows labeled A-E correspond to CBDC interest rates set to [0,25,50,75,100]% of the policy rate. The yellow and blue streams correspond to “full nesting” (cash-like CBDC) and “no nesting” (deposit-like CBDC). Charts in the first (second) row show the evolution of individual bank metrics (the sum across banks).

**Figure A11. Non-Bank Agents' P&L Flows (USD billions, per annum)—Counterfactual Results for the U.S.**



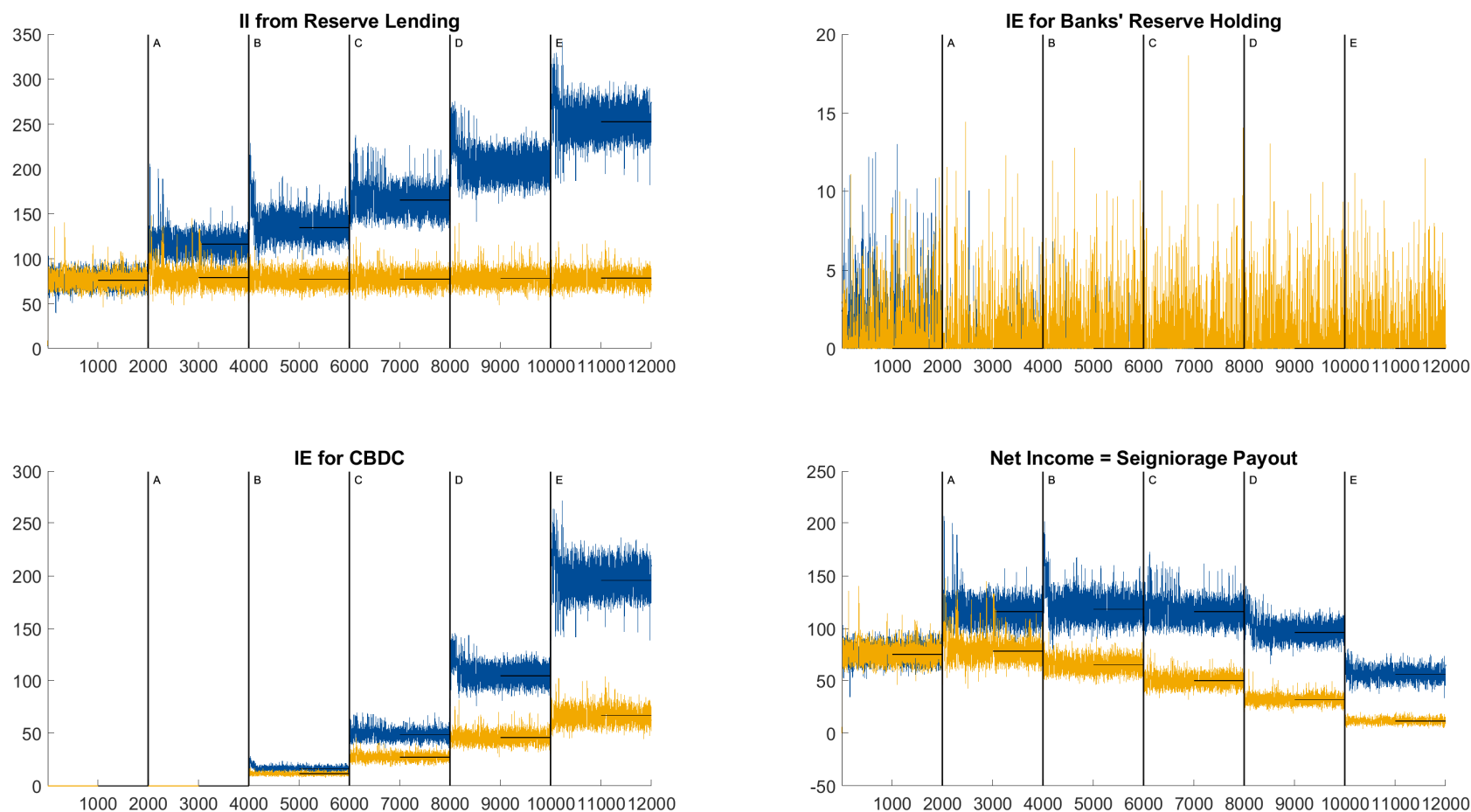
*Note:* The first of the six simulation windows corresponds to a base simulation without CBDC. The five counterfactual windows labeled A-E correspond to CBDC interest rates set to [0,25,50,75,100]% of the policy rate. The yellow and blue streams correspond to “full nesting” (cash-like CBDC) and “no nesting” (deposit-like CBDC). The charts all show the sum of the respective flow metrics across the underlying individual non-bank agents.

**Figure A12. Central Bank Balance Sheet (USD billions, end of period)—Counterfactual Results for the U.S.**



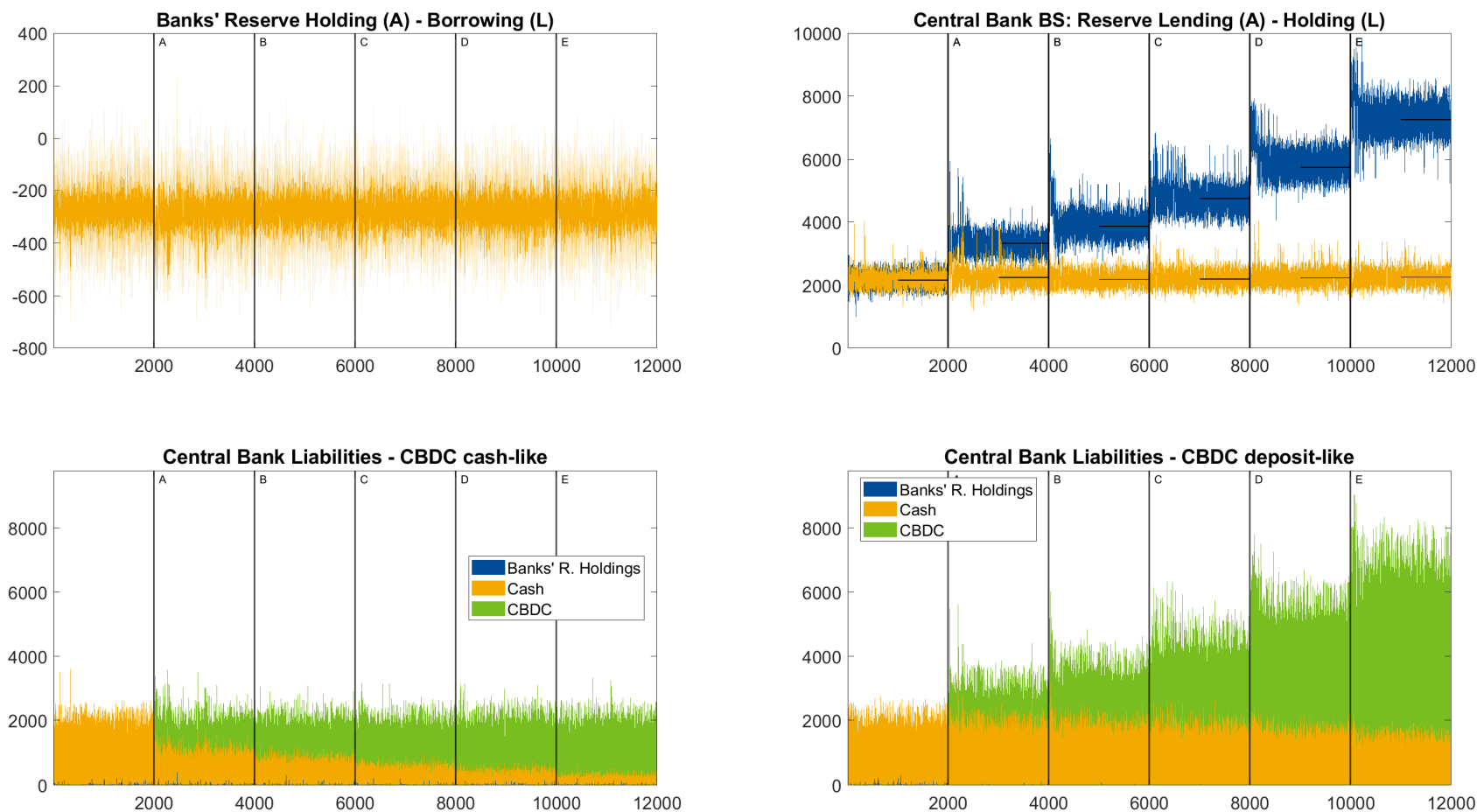
*Note:* The first of the six simulation windows corresponds to a base simulation without CBDC. The five counterfactual windows labeled A-E correspond to CBDC interest rates set to [0,25,50,75,100]% of the policy rate. The yellow and blue streams correspond to “full nesting” (cash-like CBDC) and “no nesting” (deposit-like CBDC).

**Figure A13. Central Bank P&L Flows (USD billions, per annum)—Counterfactual Results for the U.S.**



*Note:* The first of the six simulation windows corresponds to a base simulation without CBDC. The five counterfactual windows labeled A-E correspond to CBDC interest rates set to [0,25,50,75,100]% of the policy rate. The yellow and blue streams correspond to “full nesting” (cash-like CBDC) and “no nesting” (deposit-like CBDC).

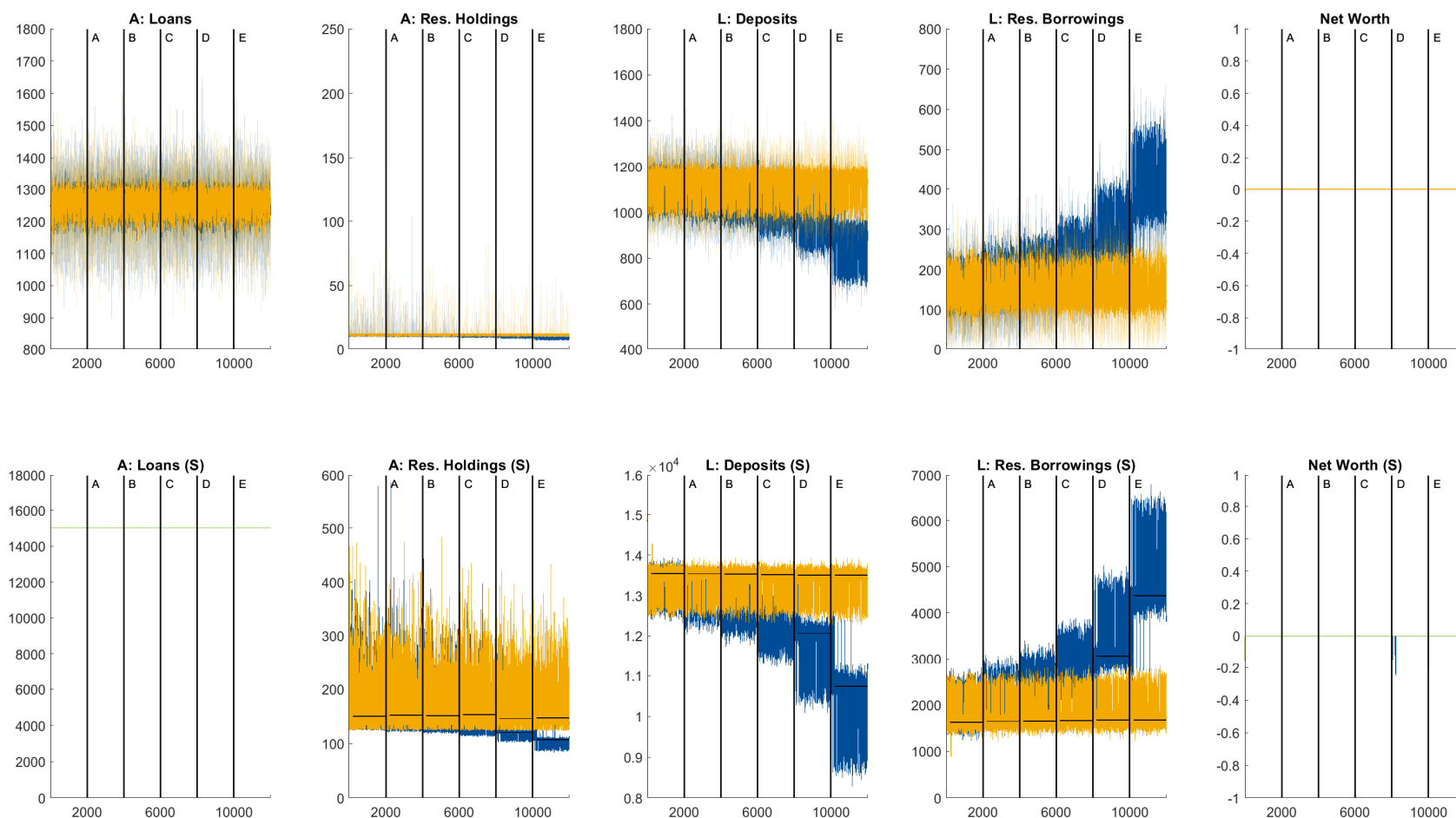
**Figure A14. Banks' and Central Bank Net Reserve Position (USD billions, end of period)—Counterfactual Results for the U.S.**



*Note:* The first of the six simulation windows corresponds to a base simulation without CBDC. The five counterfactual windows labeled A-E correspond to CBDC interest rates set to [0,25,50,75,100]% of the policy rate. The yellow and blue streams (in the first row) correspond to “full nesting” (cash-like CBDC) and “no nesting” (deposit-like CBDC).

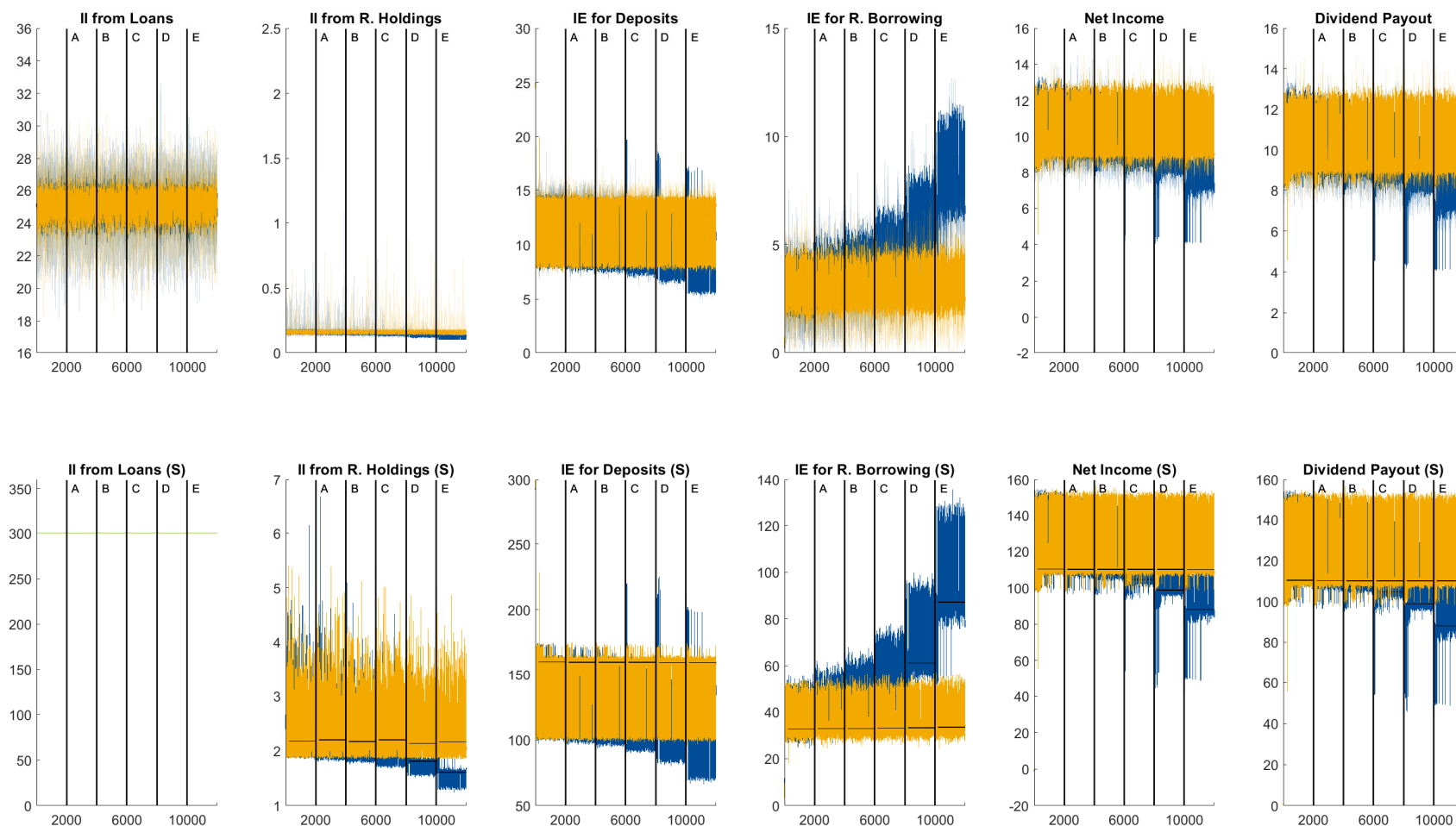


**Figure A15. Banks Balance Sheets (EUR billions)—Counterfactual Results for the Euro Area**



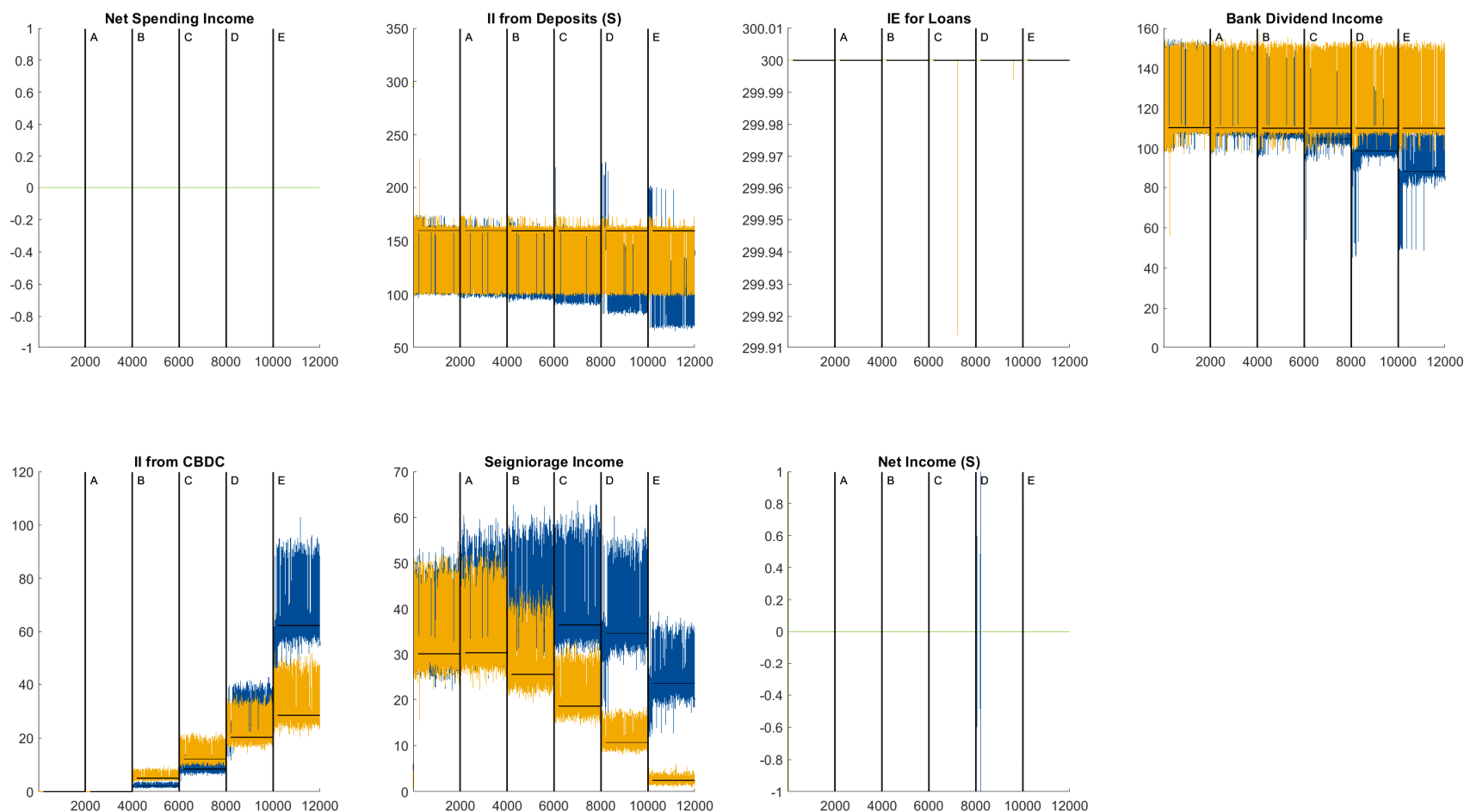
*Note:* The first of the six simulation windows corresponds to a base simulation without CBDC. The five counterfactual windows labeled A-E correspond to CBDC interest rates set to [0,25,50,75,100]% of the policy rate. The yellow and blue streams correspond to "full nesting" (cash-like CBDC) and "no nesting" (deposit-like CBDC). Charts in the first (second) row show the evolution of individual bank metrics (the sum across banks).

**Figure A16. Banks' P&L Flows (EUR billions, per annum)—Counterfactual Results for the Euro Area**



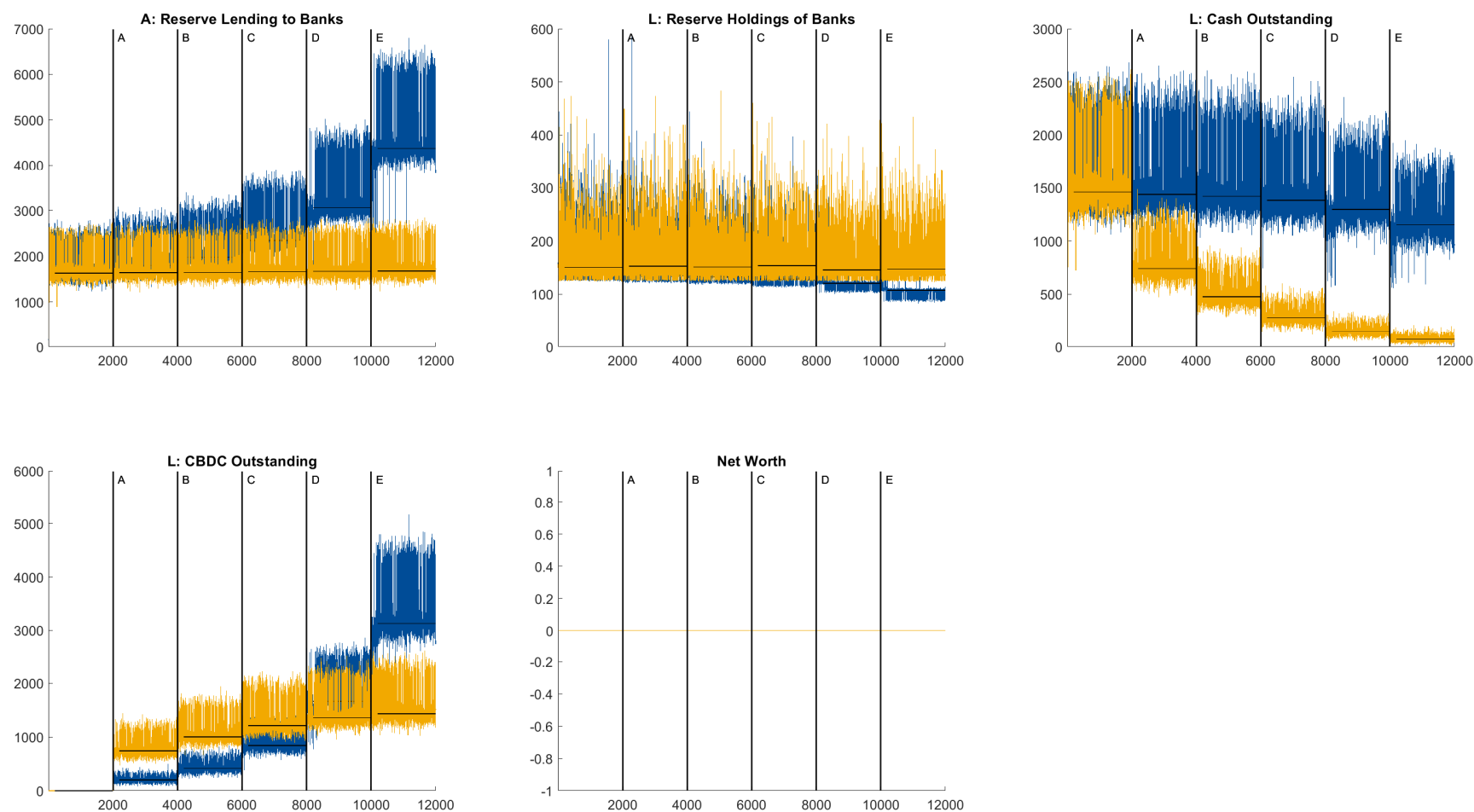
*Note:* The first of the six simulation windows corresponds to a base simulation without CBDC. The five counterfactual windows labeled A-E correspond to CBDC interest rates set to [0,25,50,75,100]% of the policy rate. The yellow and blue streams correspond to “full nesting” (cash-like CBDC) and “no nesting” (deposit-like CBDC). Charts in the first (second) row show the evolution of individual bank metrics (the sum across banks).

**Figure A17. Non-Bank Agents' P&L Flows (EUR billions, per annum)—Counterfactual Results for the Euro Area**



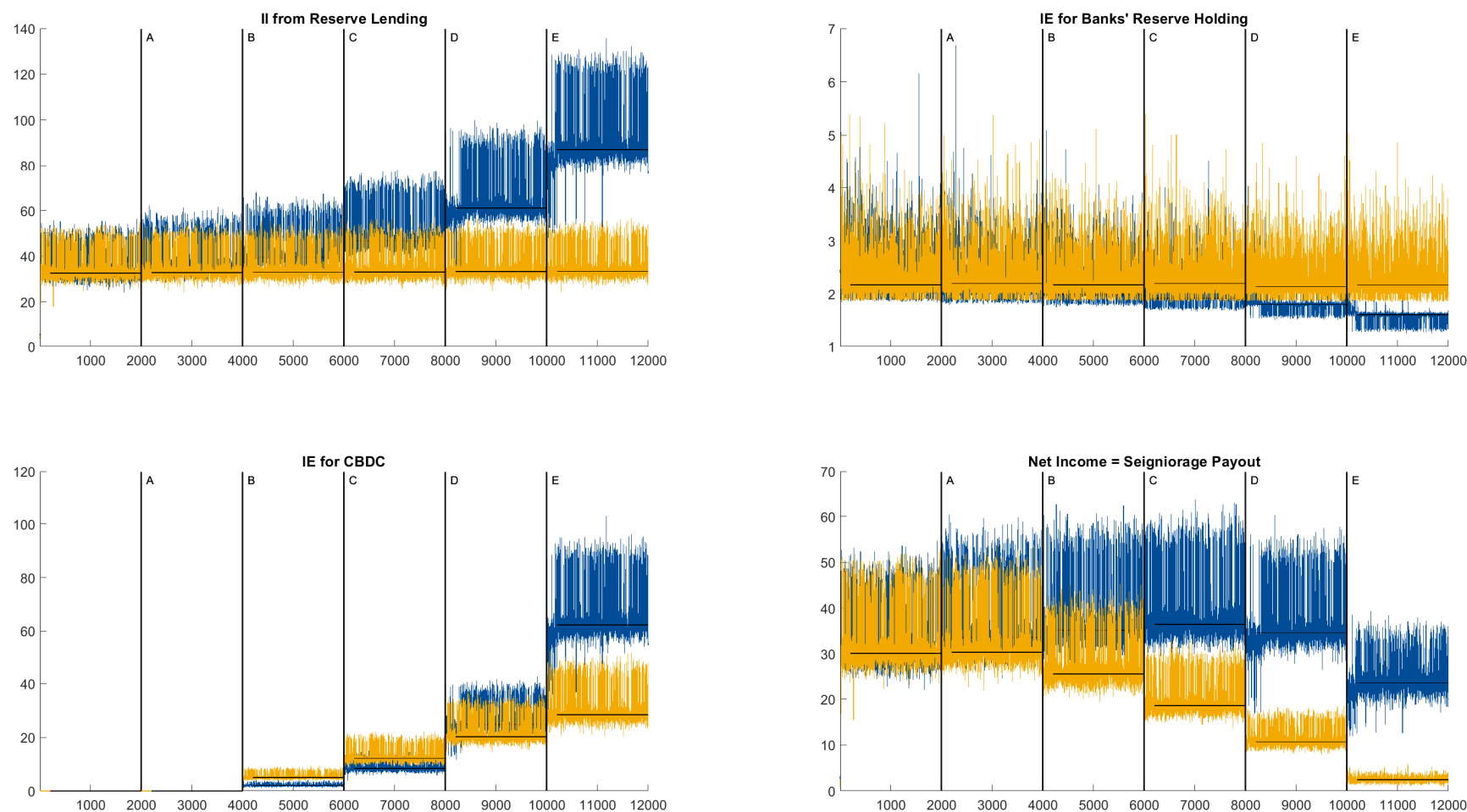
*Note:* The first of the six simulation windows corresponds to a base simulation without CBDC. The five counterfactual windows labeled A-E correspond to CBDC interest rates set to [0,25,50,75,100]% of the policy rate. The yellow and blue streams correspond to “full nesting” (cash-like CBDC) and “no nesting” (deposit-like CBDC). The charts all show the sum of the respective flow metrics across the underlying individual non-bank agents.

**Figure A18. Central Bank Balance Sheet (EUR billions, end of period)—Counterfactual Results for the Euro Area**



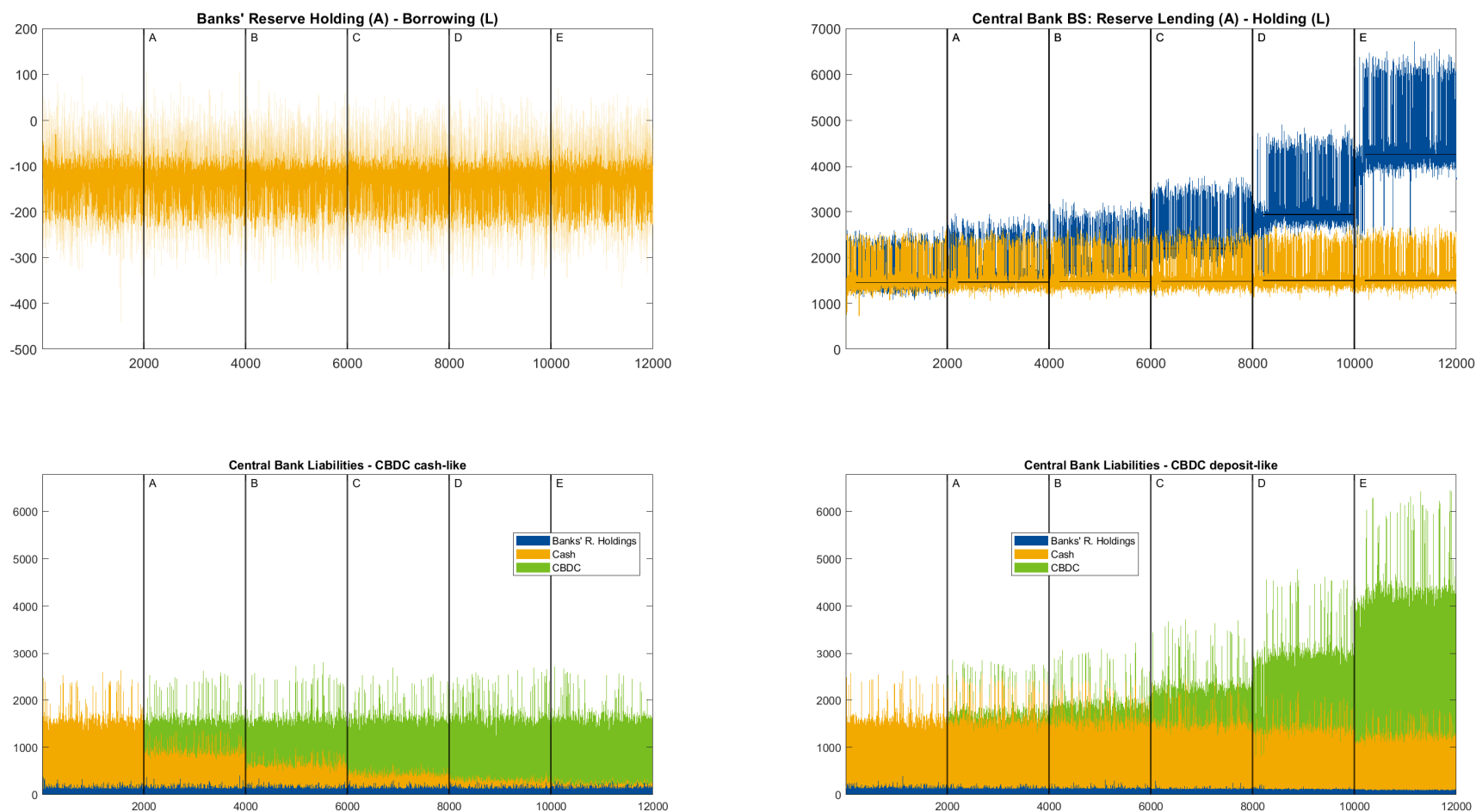
*Note:* The first of the six simulation windows corresponds to a base simulation without CBDC. The five counterfactual windows labeled A-E correspond to CBDC interest rates set to [0,25,50,75,100]% of the policy rate. The yellow and blue streams correspond to “full nesting” (cash-like CBDC) and “no nesting” (deposit-like CBDC).

**Figure A19. Central Bank P&L Flows (EUR billions, per annum)—Counterfactual Results for the Euro Area**



*Note:* The first of the six simulation windows corresponds to a base simulation without CBDC. The five counterfactual windows labeled A-E correspond to CBDC interest rates set to [0,25,50,75,100]% of the policy rate. The yellow and blue streams correspond to “full nesting” (cash-like CBDC) and “no nesting” (deposit-like CBDC).

**Figure A20. Banks' and Central Bank's Net Reserve Position (EUR billions, end of period)—Counterfactual Results for the Euro Area**



*Note:* The first of the six simulation windows corresponds to a base simulation without CBDC. The five counterfactual windows labeled A-E correspond to CBDC interest rates set to [0,25,50,75,100]% of the policy rate. The yellow and blue streams—in the first row of the figure collection—correspond to “full nesting” (cash-like CBDC) and “no nesting” (deposit-like CBDC).

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## PUBLICATIONS

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