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# Agglomeration, Innovation, and Spatial Reallocation

## The Aggregate Effects of R&D Tax Credits

Alexandre Sollaci

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**Agglomeration, Innovation, and Spatial Reallocation: The Aggregate Effects of R&D Tax Credits**
**Prepared by Alexandre Sollaci**

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**ABSTRACT:** I investigate the aggregate effects of R&D tax credits in the US. Because it subsidizes R&D activity and because credit rates vary between states, this policy has both spatial and dynamic effects on the economy. To address this issue, I construct an endogenous growth model with spatial heterogeneity and agglomeration spillovers in innovation. Aggregate outcomes in this model are thus affected by the spatial distribution of the population in the economy, which is itself endogenous and reacts to policy. I use this framework to identify a set of local R&D subsidies that maximize aggregate welfare.

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WORKING PAPERS

# Agglomeration, Innovation, and Spatial Reallocation

## The Aggregate Effects of R&D Tax Credits

Prepared by Alexandre Sollaci*

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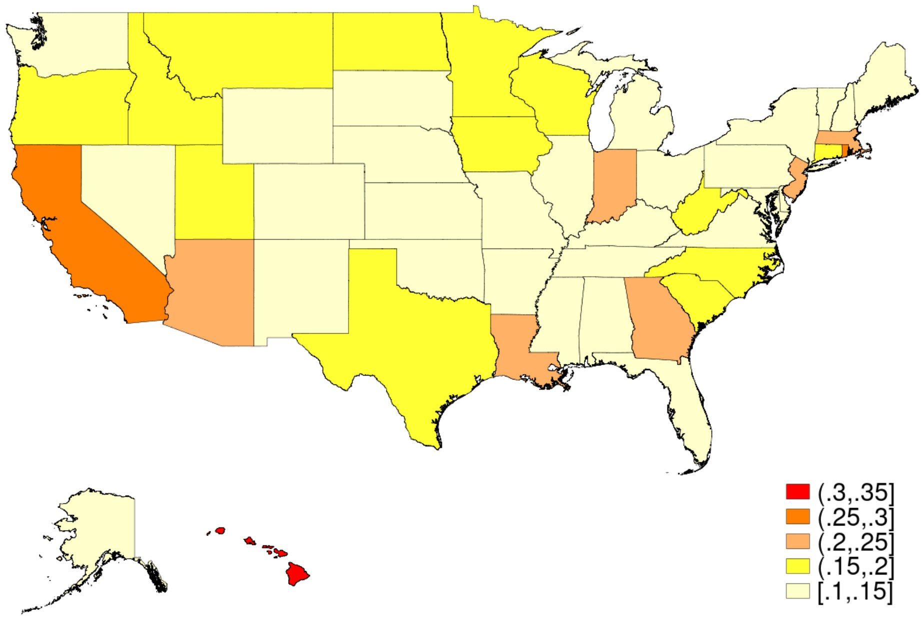
# 1 Introduction

Research and Development (R&D) tax credits are one of the most important policies that foster innovation in the US. Introduced in 1981 at the federal level, these credits apply to company-funded R&D expenditures above a pre-determined baseline level. It is estimated that R&D tax credits would amount to over US$ 9 billion in foregone revenues to the federal government in 2019 – which is about 20% more than the National Science Foundation's entire budget request for that year.[1] In addition to the federal credits, most states have adopted some form of subsidy to research activity as well (see appendix figures A.1 and A.2). As a result, the US features a wide spatial dispersion of R&D tax credits, shown in figure 1.

Figure 1: R&D Tax Credit Rates per State, 2005.




**Note**: The figure shows the *effective* R&D tax credit rates as computed by Wilson (2009). Statutory and effective credit rates differ based on how the base amount is defined and whether or not the credit is "recaptured" (i.e., considered taxable income).

In this paper, I investigate how the dispersion of R&D tax credits impacts the aggregate economy. R&D tax credits (or direct subsidies) are fundamentally different from most other policy instruments because they can impact both the location of workers of and firms, as well as their decisions over time (i.e., investments in innovation). In contrast, most of the existing studies that evaluate the impact of spatial policies do so in a static setting (Kline and Moretti, 2014; Ossa, 2015; Gaubert, 2018; Fajgelbaum and Gaubert, 2020). Similarly, studies that evaluate the long-run effects of R&D policies seldom account for local externalities or the spatial heterogeneity of subsidies to innovation.

The spatial effects of R&D tax credits are a product of the interaction between pre-existing spatial heterogeneity, agglomeration externalities, and the variation of the tax credit rate over space. Inventors and firms have incentives to move to places where taxes are relatively lower; thus,

[1]The cost of R&D tax credits for the federal government is estimated by the Joint Committee on Taxation (2010). The NSF's budget request can be found in `https://www.nsf.gov/about/budget/fy2019/pdf/01_fy2019.pdf`.

changes in the spatial distribution of tax credits can change the location of agents in the economy (Moretti and Wilson, 2014, 2017; Akcigit et al., 2016). This is relevant because innovative activity benefits from agglomeration spillovers: the productivity of individual inventors (and therefore of the firms that hire them) in producing patents increases when they are located in densely populated cities (Moretti, 2021). The benefits from agglomeration, however, must be weighted against congestion costs and the fact that local benefits might be offset by losses elsewhere.[2]

In addition, R&D tax credits have dynamic effects by their very nature, as they subsidize the cost of innovation for firms. Combining this with the effects described above, it is apparent that the way in which R&D tax credits are distributed over space can affect the aggregate rate of innovation – and therefore growth – of the country. Note that this creates an interaction between the spatial and dynamic aspects of R&D policy, which means that both of these dimensions must be jointly considered if one is interested in assessing this policy's impacts.

To evaluate the effects of counterfactual R&D policies, I build an endogenous growth model with local agglomeration externalities and spatial heterogeneity. This framework captures both the spatial and dynamic aspects of R&D tax credits by allowing the productivity of individual inventors to increase with the population density of the city where they live. In addition, the spatial distribution of the population in this model is itself endogenous, which means that changes in local R&D tax credits can affect the location of inventors/firms. I use this model to compare the welfare level under the current distribution of tax credits in the US with two other policies: a spatially neutral R&D subsidy (i.e., that does not vary over space) and a theoretical welfare maximizing distribution of R&D subsidies that is obtained by solving a social planner's problem. My findings suggest that removing the spatial variation of R&D policy hurts the economy, but that the current distribution of subsidies is far from being optimal.

In its essence, the framework I develop adds a spatial dimension to Schumpeterian models of endogenous growth (Aghion et al., 2014). The economy is composed of a system of cities $c \in \{0, 1, \ldots, C\}$, where $C \to \infty$. Each city is endowed with an amenity level, a stochastic productivity for innovation, and a R&D tax credit (modeled as a direct subsidy for simplicity). Cities are populated by inventors (who produce innovation), production workers (who produce goods), and by the firms that hire each of them. I assume that both inventors and production workers are freely mobile at all points in time, but can only be hired by firms located in the city where they live.

I also allow for agglomeration spillovers in the production of innovation. Specifically, inventors become more productive when they are living in cities that are densely populated by other inventors. As a result, firms that invest in R&D also benefit from locating in cities where there is a large population of inventors. One potential concern about this formulation is that communications technology has made actual physical proximity irrelevant for the efficient exchange of ideas, particularly after the Covid-19 pandemic, which would greatly diminish the benefits of agglomeration. While it is true that technology has made remote communication possible, I argue that it

[2]For example, Kline and Moretti (2014) find that the gains due to agglomeration from investments made by the Tennessee Valley Authority were largely neutralized by losses in the rest of the country. Glaeser and Gottlieb (2008) stress that aggregate gains can only be obtained from the redistribution of economic activity if the policy makers favor areas where the elasticity of output with respect to agglomeration is high.

is not able to replace the mechanisms that drive agglomeration spillovers. In particular, one important benefit from physical proximity are casual and unintended flows of information between workers (Puga, 2010), which cannot be replicated in virtual environments.[3] Indeed, as of the writing of this paper, most companies have reverted to (at least) a hybrid work model, emphasizing the benefits of having workers in close proximity, for at least for part of their time. Regardless of the reason for this decision, the fact that workers will continue to share the same space means that agglomeration benefits will remain relevant.

The remainder of the structure of the model regarding innovation follows a standard quality ladder framework, with a few modifications that will discussed in detail in the model section.[4] Firms innovate over intermediate goods, which exist in a continuum of varieties, each with a different quality level. After a successful innovation, a firm is able to produce the good over which it innovated at a higher quality than any other firm, becoming the technological leader over that product. The leader is able to drive all other competitors out of the market, producing the good under monopolistic competition.

Firms in this economy produce three types of goods. Intermediate goods are freely tradable and used as inputs in the production of a final (tradable) good. A second type of final good, which is non-tradable, is produced and consumed in each city (e.g., housing). Both types of final good are produced by a representative producer under perfect competition. I also assume the existence of a central planner (government) that fully taxes all land- and firm-owners to provide R&D subsidies and a public good. Consumers derive utility from the consumption of both final goods (tradable and non-tradable) and the public good. For ease of notation, I refer to the final tradable good as "final" and the final non-tradable good as "non-tradable." In line with Henderson (1974), the production of the non-tradable good displays decreasing returns to scale (as it requires the use of land, a fixed factor of production), which generates congestion costs and limits the size of cities.

The last ingredient of the model is the free entry of firms into any city in the economy. Along with the free mobility of inventors and production workers, this condition implies that the distribution of the population is endogenous and responds to changes in R&D policy. This is one of the key results in the model, as it allows the government/social planner to use R&D tax credits to influence the location of inventors and firms – and therefore leverage agglomeration spillovers to affect the productivity of investments in innovation.

Because the population of inventors determines the agglomeration spillovers in each city, all other variables in the model's equilibrium will be a function of how inventors are distributed across space. The rate of growth of the economy, for example, is directly affected by both the dispersion of the population of inventors and by their location: more inventors in more productive cities will generate a higher rate of growth. One of the appealing aspects of the model is therefore that the

[3] Davis and Dingel (2019) introduce a model where that the costly exchange of ideas is the driver of agglomeration, and are able to replicate a broad set of empirical facts about the cross section of cities.

[4] Most of the structure will take elements from Akcigit and Kerr (2018). There are, however, a few simplifications. First, the rate at which firms innovate does not scale with firm size (this is relaxed in one extension of the model in appendix G). Second, all innovation generates creative destruction: i.e. there is no "internal" innovation. Lastly, all innovation is homogeneous, in the sense that the improvement in quality generated by an innovation (step size) is fixed.

share of inventors in each city can be expressed with a closed form solution. Furthermore, this share is completely determined by city-specific features (R&D subsidies, amenity levels, and the expected productivity of innovation) and three parameters: the elasticity of agglomeration, the elasticity of congestion, and the share of expenditures on non-tradable goods.

The estimation of the parameters in the model is done in three steps. In the first step I calibrate the value of the set of parameters that can be directly matched to quantities in the data. The second step takes advantage of the model's structure, which suggests that the elasticity of agglomeration and the elasticity of congestion can be estimated through linear regressions. In each case, the estimation consists of regressing the relevant outcome on the population of inventors or the population of production workers in each city (in addition to other controls, including city and year fixed effects). To account for endogeneity in each city's population, I propose an instrument that is based on shift-share research designs (Adão et al., 2019; Borusyak et al., 2022; Goldsmith-Pinkham et al., 2020). Like shift-share designs, the instrument leverages industry-specific growth in the employment shares of inventors as exogenous shifters for the population in each city.

The third step recovers the remaining parameters by matching model predictions to moments in the data. I focus on moments related to the production of innovation, such as the share of the population of inventors and patents filed in each city. I verify the model's external validity by showing that it can accurately replicate the spatial distribution of variables that were not targeted for estimation: the correlations between distributions generated by the model and the data range from $0.5$ (patents per capita by city) to $0.97$ (share of firms by city). I also show that changes in R&D tax credits over time can explain a large part of the variation in the share of inventors and patents filed in each city since the 1970's.[5] This result suggests that R&D policy is indeed well suited to spatially reallocate innovation in the country.

Finally, I use the framework described above to analyze the welfare effects of alternative spatial distributions of R&D subsidies. Specifically, I address the following question: "Given the government's revenue, how much can we increase aggregate welfare[6] in the economy simply by reshaping the distribution of R&D subsidies among cities/states?" Heuristically, this exercise consists of fixing the average level of the subsidy but changing its value across locations. I consider the effect of two counterfactual policies: one that removes all spatial variation by implementing a common subsidy in the entire country, and one that implements an "optimal" distribution of subsidies (that is found by solving a central planner's problem).

Quantitatively, I find that removing the spatial variation of R&D tax credits in the US would generate a slightly less concentrated distribution of the population (the HHI index moves from $0.027$ to $0.025$) and reduce welfare by $0.77\%$. This suggests that the states that offer the largest credits are indeed to ones that are comparatively better at producing innovation. When comparing the current welfare level with the level attained by an optimal distribution of R&D tax credits, I find that the potential welfare gains are fairly high: if subsidies are allowed to vary by city, aggregate

[5]It should also be mentioned that changes in the state-specific R&D tax credit rate are largely uncorrelated with changes in states' corporate and labor income taxes. This mitigates the concern that the population dynamics are actually driven by alternative local policies.

[6]Because firm- and land-owners are fully taxed be the government, welfare in the context of the model means the sum of the utility of all inventors and production workers.

welfare increases by at least 6% under the optimal distribution. If subsidies are only allowed to vary by state, the gains are about half as large, at 3.2%. Both cases, however, imply that part of population of inventors in small and medium-sized cities should instead be working at large (and more productive) cities.

## 1.1 Relationship to the Literature

This paper is related to several strands of the economic literature. The main findings of the paper contribute to the literature on spatial misallocation and optimal spatial policies. In line with my findings, a number of other studies have found large potential gains from reallocating resources across space in the US.[7] Hsieh and Moretti (2019) argue that housing supply restrictions adopted in some of the most productive cities in the US significantly lowered the country's rate of growth between 1964 and 2009. Fajgelbaum et al. (2019) find that tax dispersion across states leads to aggregate losses because it distorts the spatial allocation of resources. Adopting a strategy closer to mine (although not necessarily focusing on policies that foster innovation), Gaubert (2018) and Fajgelbaum and Gaubert (2020) develop general quantitative frameworks that allow them to compute optimal local subsidies designed to attract workers and firms to each city. Similarly, Ossa (2015) explores the welfare effects of subsidy competition among states, while Kline and Moretti (2014) study the long run effects of the Tennessee Valley Authority development program.

While these are important contributions, the existing literature has analyzed the welfare effects of spatial policies primarily though static frameworks. As discussed above, R&D tax credits have both spatial and dynamic effects that interact with each other. A purely spatial model would incorrectly evaluate the welfare effects of spatially reallocating R&D subsidies because it would ignore the effects of a rise/fall in the rate creative destruction over the incentives of firms. Similarly, a purely dynamic model would predict no relationship between the location of subsidies and aggregate growth. Thus, to understand how the aggregate economy reacts to changes in the distribution of R&D subsidies, one must adopt a setting that captures both the dynamic and spatial effects of this policy.

The theoretical framework developed here is based on endogenous growth models where innovation is the main driving force of economic growth (e.g., Aghion and Howitt, 1992; Klette and Kortum, 2004; Akcigit and Kerr, 2018). It contributes to this literature by nesting a simple model of innovation through creative destruction into a spatial setting. The resulting model retains many of the features present in the growth literature, but also allows for spatial heterogeneity, agglomeration spillovers and an endogenous distribution of the population. Adding a spatial dimension to this class of models helps us to understand the linkages between innovation at the firm level, which is greatly impacted by the firm's location through agglomeration spillovers, and broader economic growth. It is also fundamental to the investigation of the impact of spatial policies over

[7] In light of the recent rise in economic inequality across the US, some economists have diverted their focus to policies designed to mitigate economic distress (Austin et al., 2018) or income inequality Farrokhi (2021). Related to the topic of this paper, Glaeser and Hausman (2019) discuss changes in innovation policies that focus on particularly impoverished areas as means of reducing joblessness. At the same time, they recognize that spatially reallocating innovative activity might reduce the overall production of innovation in the country, and so they suggest alternatives that minimize the spatial reallocation of R&D funding.

aggregate welfare and growth.

This framework also contributes to the literature on spatial and dynamic models. One closely related paper is Duranton (2007), who embeds the quality ladder model of Grossman and Helpman (1991) into an urban structure in order to study the city size distribution and the movement of industries across cities. The main advantage of the current setting is that agglomeration and congestion externalities are microfounded and endogenously generated in the model's equilibrium; in contrast, Duranton (2007) relies on a reduced form that captures the net effect of these externalities on the size of cities. Other papers developing spatial and dynamic models include Desmet and Rossi-Hansberg (2014), Desmet et al. (2018), and Caliendo et al. (2019). The models in those papers feature a realistic geography, which includes trade and moving costs, as well as locations that are ordered in space. In contrast, I ignore most of these geographical frictions, which enables me to focus on the dynamics of innovation for individual firms. This produces a tractable model that has a natural link between innovation at the firm level and patent data.

The remainder of this paper is organized as follows. Section 2 develops a formal endogenous growth model that replicates the linkages between city-specific R&D subsidies and aggregate growth. Section 3 solves for the model's equilibrium in a Balanced Growth Path. Section 4 shows how to map the model's equilibrium to the US data and estimate its parameters. In section 5, I use this framework to measure the effects of the adoption of alternative R&D policies. Section 6 concludes.

## 2 A Dynamic Spatial Model of Innovation and Growth

In this section I describe in detail the environment of the model; section 3 then defines and solves for its equilibrium. The main contribution of this model is to allow for spatial heterogeneity and agglomeration externalities to affect the productivity of R&D investments by firms. Not only does this imply that the spatial distribution of population in the economy matters for growth, but, because firms and workers choose where to locate, this distribution is also endogenously determined. Since the goal of the model is to capture the aggregate effects of R&D subsidies, there are no agglomeration effects in the production of goods. I also introduce local shocks to the productivity of R&D investments in each city. Assuming that the number of cities is large, this structure allows for the existence of a Balanced Growth Path where the aggregate growth rate of the economy is constant, but the growth rate and population in each individual city can still fluctuate over time.

### 2.1 Cities

There are $C+1$ cities, indexed by $c \in \{0, 1, \ldots, C\}$. I also assume that there is large number of cities, i.e., $C \to \infty$. Cities differ from each other by their level of amenities, $\alpha_c$, the amount of land they posses, $\bar{m}_c$, and a stochastic (time-varying) city-specific productivity in the production of innovation, $\chi_c(t)$. Amenities reflect features that improve the quality of life in a particular city, making it more attractive for workers to live there. These features can be related to the city's geographical location, such as climate or proximity to an ocean, or to the city's history, customs, etc. Importantly, amenities are fixed over time. Land is a fixed factor in each city and is owned

by absentee land owners, who are fully taxed by the government. Without loss of generality, I normalize $\bar{m}_0 = 1$ and $\bar{m}_c = 1/C$ for all $c \geq 1$ (a discussion on why city 0 is different from the others will follow below).[8]

The city-specific productivity for innovation reflects the fact that some cities are "hubs" of innovation. These cities are usually the home to more venture capitalists, who favor local investments (Gompers and Lerner, 2001; Kolympiris et al., 2011), or have a "culture" that is favorable to entrepreneurship and innovation.[9] Unlike amenities or land, this productivity is not fixed over time. Specifically, I assume that it evolves stochastically according to

$$\chi_c(t) = \bar{\chi}_c e^{z_c(t)},$$

where $\bar{\chi}_c$ is a constant, capturing permanent differences in the productivity of investments in R&D between cities, and $z_c(t)$ follows an Ornstein-Uhlenbeck (O-U) process: $dz_c(t) = \phi(\mu - z_c(t))dt + \sigma dW_c(t)$, where $W_c(t)$ is a (city-specific) Wiener process.[10] I assume $\mu = -\sigma^2/4\phi$, so that, under the process' stationary distribution, $\mathbb{E}[e^{z_c}] = 1$.

All cities are populated by workers and firms. Workers are separated into two types: inventors and production workers. Inventors are hired by firms who wish to invest in R&D, while production workers are used to produce final goods. The total population of inventors in the economy is given by $I$, while the total population of production workers is given by $L$, both of which are kept constant over time. Note that this assumption excludes the possibility that workers adjust their occupation based on the amount of R&D subsidy offered. This is not far from reality, however, as the supply of inventors is very inelastic (Goolsbee, 1998).

Both types of workers are freely mobile between cities, but are hired locally. Free mobility of labor is of course a simplification of reality, but in line with empirical findings that individuals react quite strongly to tax differentials between regions (Moretti and Wilson, 2017). In contrast, I assume that firms are able to enter any city they wish, but are not allowed to move once their location is set (a common assumption in spatial models; see e.g., Behrens et al., 2014). The requirement that workers are hired locally thus guarantees that firms are located in the same city where their employees live. Finally, goods can also be subdivided into three types: a final non-tradable good, a final tradable good and intermediate goods. Both the tradable final good and intermediate goods can be traded at no cost. Firms, like land, are owned by absentee firm owners who are fully taxed

[8]This normalization stresses the fact that when the number of cities is large, each city represents a small share of the country – an argument that will be relevant for the application of the Law of Large Numbers when constructing the equilibrium of the model. Furthermore, the normalization is without loss of generality because differences in the land mass between these cities cannot be differentiated from differences in $\alpha_c$ or $\chi_c$.

[9]For example, Manso (2011) stresses the need for tolerance for failure in innovation and compensation practices. This idea is embodied in the Silicon Valley mantra "fail fast, fail often", while in some places the legal or reputational costs of an early failure are so high that entrepreneurs essentially have one shot. Saxenian (1994) describes strong cultural differences between Boston's Route 128 and the Silicon Valley in her arguments for why the semiconductor industry grew at a faster pace in the latter. See also the survey by Carlino and Kerr (2015) for a broader discussion on the links between innovation and the location of scientists and firms.

[10]The O-U process has several properties that make it ideal for modeling the evolution of the city-specific shock. In particular, it admits a stationary distribution (which is also its limiting distribution as $t \to \infty$) with finite mean and variance (see Stokey, 2008) – meaning that the Law of Large Numbers applies to city-specific outcomes in the model. As a result, the model allows for uncertainty on the city level, but no aggregate uncertainty when the number of cities is large.

by the social planner/government.

## 2.2 Preferences

Consumers in this model are inventors and production workers. All consumers have the same utility function, but might differ in their wages. Recall that both types of worker are free to move into any city they wish, so that a worker of type $h \in \{i, \ell\}$ (i.e., inventor and production worker, respectively) has utility given by

$$U^h(t_0) = \int_{t_0}^{\infty} e^{-\rho t} \max_{c(t)} \{u^h_{c(t)}(t) G(t)\} dt,$$

where $c(t)$ indicates the city where the worker lives during time $t$, $G(t)$ is the amount of public good consumed in period $t$ and $u^h_{c(t)}(t)$ is the highest utility that a worker of type $h$ living in city $c$ during time $t$ can attain from the consumption of the final goods. Formally, for each $c$

$$u^h_c(t) = \max_{n(t), y(t)} [\alpha_c n(t)]^{\theta} y(t)^{1-\theta} \quad \text{s.t. } y(t) + p_{n,c}(t) n(t) \leq w^h_c(t).$$

In the expression above, $n(t)$ is the amount of the non-tradable final good consumed by the worker in time $t$ and $y(t)$ is the amount of the tradable final good consumed in the same period. For simplicity of notation, I will refer to the non-tradable final good simple as "non-tradable good" and to the tradable final good as "final good" (to differentiate it from intermediate goods). All workers inelastically supply one unit of labor per period, so their income is equal to their wage, $w^h_c(t)$. To keep the model tractable, I assume that all consumers are "hand-to-mouth", in the sense that they cannot borrow or save.[11]

## 2.3 Technology

### 2.3.1 Non-tradable Good

Non-tradable goods are locally produced by a competitive firm using land and the labor of production workers. The representative non-tradable goods producer in each city chooses how many production workers to hire and how much land to rent in order to solve

$$\max_{\ell_{n,c}(t), m_c(t)} p_{n,c}(t) n(t) - w^{\ell}_c(t) \ell_{n,c}(t) - p_{m,c}(t) m_c(t) \quad \text{s.t. } n(t) = \ell_{n,c}(t)^{\beta} m_c(t)^{1-\beta},$$

where $p_{n,c}(t)$ is the price of the non-tradable good, $p_{m,c}(t)$ is the price of land, and $w^{\ell}_c(t)$ is the wage received by production workers in city $c$.

Land is a fixed factor in each city, which means that the production of the non-tradable good

[11] This assumption simplifies the worker's problem so that their demand functions can be easily computed regardless of the worker's location history – because wages vary between cities, a consumer's wealth would depend on the cities where he or she lived if savings were allowed. However, it also implies that the consumer's problem is essentially static. As a result, all of the dynamics in this model are generated in the production side, as firms invest in R&D and innovate upon goods. In addition, note that consumers cannot hold assets, so the redistribution of profits cannot be done through the ownership of equity by workers. Instead, profits will be redistributed by the government/social planner by taxing firms to provide the public good.

will have decreasing returns to scale in equilibrium. This is what generates congestion costs in cities: as the population of a city increases, so does the demand for the non-tradable good. Because of decreasing returns to scale, the higher demand will push the price $p_{n,c}$ up, making it less attractive to live in that city.[12]

### 2.3.2 Final Goods

The representative final good producer uses the labor of production workers and all intermediate goods as inputs to competitively produce the final good. Like all other firms, the final good producer is free to choose where to locate. If production occurs in city $c^*$, the final good producer solves

$$\max_{\ell_{y,c^*}(t),\{k_j(t)\}_{j\in\mathcal{J}}} Y(t) - \int_{\mathcal{J}} p_j(t)k_j(t)dj - w^{\ell}_{c^*}(t)\ell_{y,c^*}(t) \quad \text{s.t.} \quad Y(t) = \frac{\ell_{y,c^*}(t)^{\varepsilon}}{1-\varepsilon}\int_{\mathcal{J}} k_j(t)^{1-\varepsilon}q_j(t)^{\varepsilon}dj,$$

where $\ell_{y,c^*}$ is the number of production workers hired by the final good producer, and $k_j$ and $q_j$ are the quantity and quality of intermediate good $j$ used in production, respectively. Each intermediate good is sold at a price $p_j$ to the final good producer and the price of the final good is normalized to 1. Recall that all intermediate goods and the final good are freely tradable across cities, so the location of their production does affect their price.

### 2.3.3 Intermediate Goods

Intermediate goods exist in a continuum of varieties $j \in \mathcal{J} \equiv [0, 1]$. Intermediate good producers (or simply "firms") can produce any number of varieties. The set of products that each firm is able to produce – and the quality at which it can produce them – is given by the set of product over which that firm has innovated in the past. For now, I take this set as given. Following Akcigit and Kerr (2018), firms compete over the production of each variety $j$ by choosing prices according to the following.

**Assumption 1.** *All firms able to produce intermediate good $j$ enter a two-stage pricing game. In the first stage, firms decide whether to pay a small fee to be able to announce their price. In the second stage, they choose the price at which they propose to sell good $j$, given the set of firms who paid the fee in the first stage.*

A direct result of assumption 1 is that only the firm who can produce good $j$ with highest quality (i.e., the technical leader) pays the fee and enters the second stage of the game. As it will be shown in the equilibrium description below, the demand for intermediate goods increases with their quality, so technical laggards in the production of good $j$ can never recover their fee if they choose to enter the second stage. The technical leader can therefore choose its price as if it were a monopolist in the production of good $j$. One crucial condition for this argument to hold is that

[12] The production function shown above also implies that, unlike in Hsieh and Moretti (2019), the elasticity of the supply of the non-tradable good with respect to the population is the same for all cities. Note, however, that the supply function itself responds to differences in land mass and amenities (through their effect of wages). One way to interpret this result is that this production function abstracts from differences in land regulation that might induce the non-tradable good producer to have different responses to the amount of land in each city. While this obviously imposes some restriction to the model, it is also important to keep its tractability.

all firms have the same cost of production, regardless of their location (this excludes a production function where labor is used to produce intermediate goods, since wages vary by city). If the location of a firm affects its cost of production, the technical leader in the production of a good $j$ might not be able to drive low quality firms off the market. For example, firms who produce the good at a lower quality but who also face lower costs of production could remain in the market by selling that good at a lower price than the leader is willing to.[13]

I follow Akcigit et al. (2018) and assume that the final good is used a factor of production for intermediate goods. Since the final good is freely tradable, all firms face the same marginal cost $\nu > 0$ and the technical leader in each variety chooses the amount of good to produce by solving

$$\max_{k_j(t)} p_j(k_j(t); q_j(t))k_j(t) - \nu k_j(t).$$

where $p_j(k_j; q_j)$ is the inverse demand function for intermediate good $j$ the price of the final good is normalized to 1 in all periods. Also note that there is nothing that is city-specific in the production of intermediate goods. The only incentive that firms have to locate in specific cities comes from their investments in R&D, which ultimately determines the set of products that firms produce.

#### 2.3.4 Research and Development

Innovation serves two purposes in this model. First, it increases the quality of intermediate goods, which is the driving force for economic growth. Second, it adds products into a firm's portfolio by making it the quality leader for a given variety of intermediate good. Whenever a firm $f$ innovates over a product line $j$, it is immediately able to produce that good with quality $(1 + \lambda)q_j(t)$, where $\lambda > 0$ is the quality improvement of the innovation (or the *step size* in the quality ladder) and $q_j(t)$ is the highest quality at which good $j$ is currently produced by any firm. Note that the quality improvement makes the innovating firm the new technical leader on the production of good $j$, "stealing" this product from whichever firm currently produces it.

To produce innovation, firms must invest in R&D. For any given investment, the number of innovations realized in each period is stochastic and follows a Poisson distribution. In continuous time, this means that each firm produces at most one innovation per period.[14] I also assume that firms are not able to target any specific product line with their R&D investments, so the resulting innovation is realized over any product $j \in \mathcal{J}$ with equal probability. There are two important consequences of this assumption. First, there is no strategic interaction between firms in their R&D investment decisions, as they cannot target each others' products. Second, since all firms start with no product lines in their portfolio and can only add at most one product per period, the number of products in any firm's portfolio is always countable. Because there is a continuum of product lines in the economy, the probability that a firm innovates over one of its own products is zero. A firm

[13]The same issue arises if there are transportation costs for intermediate goods, where technological laggards are still able to make profits by selling a good to nearby cities. Eaton and Kortum (2002) highlight a similar mechanism, where the most productive firms are not necessarily the only ones who sell goods in the market, since geographic barriers also play a role in determining prices.

[14]Fix $x$ as the arrival rate of an innovation for an arbitrary firm. The number of arrivals $N$ in a period of length $\Delta$ is distributed according to $\mathbb{P}(N = k) = e^{-\Delta x}(\Delta x)^k/k!$. By Taylor expansion of $e^{\Delta x}$, it is straightforward to show that $\mathbb{P}(N = 0) = 1 - \Delta x + o(\Delta)$, $\mathbb{P}(N = 1) = \Delta x + o(\Delta)$ and $\mathbb{P}(N \geq 2) = o(\Delta)$.

does, however, need to consider the probability that another firm will innovate over a product it is currently producing. In this case, the product is stolen and the incumbent producer removes it from its portfolio.

The arrival rate of an innovation is determined by the firm's location and its investment in R&D. This investment requires the labor of inventors, who benefit from agglomeration externalities in the city where they work. Specifically, let $\tilde{I}_c$ be the population of inventors per unit of land in city $c$. The productivity of each individual inventor who resides in $c$ on the production of innovation is proportional to $\tilde{I}_c^\eta$, for $\eta \geq 0$. Hence, a firm $f$ located in city $c$ who hires $i_{f,c}$ inventors will produce an innovation with arrival rate

$$x_{f,c}(t) = \chi_c(t) \left( \tilde{I}_c(t)^\eta i_{f,c}(t) \right)^\psi . \tag{1}$$

The "strength" of the agglomeration externality is controlled by the parameter $\eta$; in particular this model nests the case where there are no agglomeration externalities by setting $\eta = 0$.

Investment in R&D also involves a fixed cost, paid in terms of the labor of inventors: firms need to hire $\kappa > 0$ inventors to cover their fixed cost. This cost is paid in every period that the firm decides to invest in R&D, and reflects managerial and maintenance costs associated with research, such as cleaning and repairing laboratory equipment and the management of inventors. Firms that decide not invest in R&D in a given period do not need to pay the cost in that period; but will need to pay it in case they decide to invest in the future. In terms of the model, this fixed cost acts as a benchmark for the value of entry – so that free entry drives the expected value of entrant firms to zero. Including a fixed cost instead of an entry cost is convenient in this model because it makes entrants and incumbents symmetric.

There are several assumptions implicit in the functional form of the production of innovation (1). The first assumption is that the population of inventors is the relevant measure of agglomeration. This is empirically tested and confirmed in appendix C.3.3. Second, innovation does not scale with firm size, so that all firms in the same city innovate at the same rate. Appendix G presents an extension of the model that allows for the scaling of innovation and the fixed cost with firm size, and shows that the aggregate predictions of the model remain unchanged.

Third, agglomeration spillovers are the same for all inhabitants of the same city, and non-existent outside the borders of the city. In practice, agglomeration externalities are likely to change continuously with the distance between agents (Rosenthal and Strange, 2001; Carlino et al., 2012). Under this interpretation, the elasticity of agglomeration defined in this paper is in fact the aggregate effect of agglomeration within a distance that corresponds to the average size of a city. With that said, an analysis that explicitly takes into account the distance between inventors would be both impossible to conduct given the available data and most likely not add much intuition to the model. Recall also that all policies/regulations are defined over specific geographical boundaries, so analyzing the effect of agglomeration over those boundaries is consistent with the purpose of this paper.

Fourth, there is no sorting into cities because inventors and firms are homogeneous. If there is sorting in reality, the *ex-post* differences between the productivity of workers in different cities will

be captured by $\bar{\chi}_c$, although this does not change the fact that sorting is endogenous and might respond to policy changes. Furthermore, it is unclear how the introduction of sorting would affect the optimal distribution of R&D subsidies. If sorting and agglomeration are complements (i.e., inventors with high productivity also benefit more from agglomeration), then it is likely that the economy would benefit from a higher spatial concentration of inventors. Nevertheless, the infra-marginal inventors moving in to larger cities would also be less productive, and reduce the average productivity in those places.

Finally, firms can't target innovations at their own goods, so that all innovation generates creative destruction. Intuitively, this means that firms do not benefit from their own past investments to increase the quality of goods that they own. Since firms located in more productive cities innovate more frequently, the absence of "internal" innovation reduces the incentives for firms to locate in those places. Both assumptions (lack of sorting and of internal innovation), however, are crucial for the tractability of the model and are thus kept throughout the paper. The counterfactual results in section 5 should be interpreted with those caveats in mind. With that said, including both sorting and internal innovation is likely to only strengthen those results, which already imply that a higher spatial concentration of inventors is welfare enhancing.

**City 0.** To reflect the fact that some cities in the US have never produced a single patent, I allow for the existence of one city where $\bar{\chi}_0 = 0$. Intuitively, city $0$ is representative of all cities that do not have minimal necessary conditions for investment in R&D (and therefore do not innovate) in the country. Firms located in city $0$, however, can still engage in the production of goods.

### 2.4 Local Policies and the Government

The policy of interest in this paper are local R&D subsidies. These subsidies transfer a share of each firm's R&D cost back to the firm. They are local in the sense that the subsidy rate, $s_c$, varies according to the city where the firm is located. On top of R&D subsidies, the government also provides a public good, $G$. The public good is nationally available and is consumed by all workers.

I assume that the government fully taxes all firm- and land-owners to finance its expenditures. In this setting, the provision of the public good plays two important roles. First, it is a simple device for redistributing profits and land rents back to workers. Second, expenditures on on the provision of this good are used to balance the government's budget constraint, given that the values of the R&D subsidy rates are taken directly from the data. Throughout the model and in all counterfactuals, the amount of public good provided in each period is fixed at $G(t) = \bar{G}$ for all $t$.

**Corporate and labor income taxes.** Local governments also have in place several other taxes/subsidies that are not explicitly included in the model. Two important taxes in this category are corporate income and labor income taxes. For the purposes of the model, differences in either of those taxes across regions would mainly change how attractive a city is to workers or firms. In that regard, any effect of the corporate income tax would be reflected in $\bar{\chi}_c$, while the effect of a labor income

tax would be reflected in $\alpha_c$.[15]

A different and important issue arises in the estimation of the model if corporate/labor income taxes are changing over time in a way that is correlated with how local R&D subsidies evolve. This would lead to an overestimation of the effect of those subsidies in the data. I explicitly address this issue in section 4.4 by showing that R&D tax credits have evolved in way that is largely uncorrelated with other local taxes.

# 3 The Balanced Growth Path Equilibrium

In this section, I explicitly pose all problems faced by consumers and firms and solve for the model's equilibrium. In what follows, I will impose two restrictions on the equilibrium. First, I will solve for a Balanced Growth Path, in which all aggregate variables grow a constant rate. Second, I require that each one of the local shocks $z_c(t)$ follows its stationary distribution. Not surprisingly, I refer to this equilibrium as a Stationary Balance Growth Path (SBGP), formally defined below.

**Definition 1.** *Given initial product quality levels $\{q_j(0) > 0\}_{j\in\mathcal{J}}$ and a set of values for the R&D subsidy, amenity level and expected productivity of innovation in each city $\{s_c, \alpha_c, \bar{\chi}_c\}_{c=0}^{C}$ such that $\chi_c(t) = \bar{\chi}_c e^{z_c(t)}$, $\bar{\chi}_0 = 0$ and $dz_c = \psi(\mu - z_c) + \sigma dW_c$, a **Stationary Balanced Growth Path Equilibrium** of the model consists of, for all periods $t \geq 0$: $(a)$ an allocation of goods $\left(Y(t), \{n_c(t)\}_{c=0}^{C}, \{k_j(t)\}_{j\in\mathcal{J}}\right)$, $(b)$ a spatial distribution of inventors, production workers and firms $\{I_c(t), L_c(t), N_c(t)\}_{c=0}^{C}$ and $(c)$ prices $\{w_c^i(t), w_c^\ell(t), p_{n,c}(t)\}_{c=0}^{C}$ such that*

*$(i)$ $z_c(t)$ is normally distributed with mean $\mu$ and variance $\sigma^2$ for all $t$ and all $c$.*

*$(ii)$ The production of the final good $Y(t)$, the average quality of intermediate goods in the economy $Q(t) = \int_{j\in\mathcal{J}} q_j(t)dj$, "baseline" wages (i.e., congestion-adjusted), and the utility of consumers grows at a constant rate.*

*$(iii)$ All workers are freely mobile and maximize their utility over the consumption of final and non-tradable goods, as well as over the city where they live.*

*$(iv)$ Final and non-tradable good producers maximize profits taking prices as given. Intermediate good producers (firms) operate under monopolistic competition in the production of each product line $j$.*

*$(v)$ Incumbent firms take their location as given and choose R&D investments to maximize their discounted stream of profits. There is free entry to all cities and a large mass of potential entrants.*

*$(vi)$ All labor and goods markets clear, and the amount of public good produced, $\bar{G}$, balances the government's budget constraint.*

[15] In addition to making cities more/less attractive to workers, a labor income tax would also distort the prices of tradable and non-tradable goods. Non-tradable goods are produced and consumed locally, which means that their prices will adjust, in equilibrium, to specific local conditions. In contrast, tradable goods are consumed across all regions, meaning that their equilibrium prices adjust to aggregate conditions in the economy. As a result, local income taxes can distort the relative prices between those two types of goods, which would unnecessarily complicate the model.

### 3.1 The Firm's Static Problem

Each firm's problem can be broken down into a static problem and a dynamic problem. In the static problem the firm chooses how much to produce of each good, taking as given the set of products it is able to produce and their respective qualities. In the dynamic problem, the firm chooses how much to invest in R&D after observing the local productivity shock.

**Final goods production.** The final good producer is free to choose in which city to locate. Because agglomeration externalities do not have any effect over the production of the final good, production will take place in whichever city has the lowest wages (least congestion): city $0$. Since $\chi_0(t) = 0$ for all $t$, no firms investing in R&D will locate there, which means that there no inventors living city $0$ as well. As a result, the only workers in city $0$ are the ones hired by the final good producer and those who produce the non-tradable good. In each period, the final good producer's profit maximization problem is therefore (I drop time from the notation as it causes no confusion)

$$\max_{\ell_{y,0},\{k_j\}_{j\in\mathcal{J}}} \frac{\ell_{y,0}^{\varepsilon}}{1-\varepsilon}\int_{\mathcal{J}} k_j^{1-\varepsilon} q_j^{\varepsilon} dj - \int_{\mathcal{J}} p_j k_j dj - w_0^{\ell}\ell_{y,0}.$$

The first-order conditions define the wage in city $0$ and the demand for intermediate goods:

$$[\ell_{y,0}]: \quad \frac{\varepsilon}{1-\varepsilon}\ell_{y,0}^{\varepsilon-1}\int_{\mathcal{J}} k_j^{1-\varepsilon} q_j^{\varepsilon} dj = w_0^{\ell}; \tag{2}$$

$$[k_j]: \quad p_j = \left(\ell_{y,0}\frac{q_j}{k_j}\right)^{\varepsilon}, \quad \forall j \in \mathcal{J}. \tag{3}$$

**Intermediate goods production.** Given the demand function (3), each firm chooses $k_j$ to

$$\max_{k_j} p_j(k_j; q_j)k_j - \nu k_j.$$

Since there are no transport costs and the marginal cost of production does not vary between cities, the firm's production decision is completely independent from the city where it is located. Solving the problem above gives the quantity of good $j$ produced

$$k_j = q_j\left(\frac{1-\varepsilon}{\nu}\right)^{\frac{1}{\varepsilon}}\ell_{y,0}. \tag{4}$$

The profit made with product line $j$ is $\pi_j = \ell_{y,0}\left(\frac{1-\varepsilon}{\nu}\right)^{\frac{1-\varepsilon}{\varepsilon}}\varepsilon q_j$.

### 3.2 Local Wages and Congestion Costs

Before getting into the dynamics of firm decisions, it is useful to understand how local wages respond to the city's population. Wage variation between cities is driven by differences in city-specific characteristics (i.e., amenities and productivity) and congestion costs among cities and, crucially, the fact that workers are freely mobile. Because of free mobility, it follows that $u_c^h(t) = u^h(t)$ for all $c, t$ and $h \in \{i, \ell\}$ in equilibrium – that is, the utility level of workers must be the same

in all cities and in all periods.[16] Given this condition, wages must adjust to compensate workers for any variation in utility caused by different levels of amenities or the price of the non-tradable good between cities. Wages between inventors and production workers differ due to differences in the supply and demand for each type of worker.

Congestion costs in this model arise from the fact that the production of the non-tradable good involves a fixed factor (land), and thus displays decreasing returns to scale in equilibrium. Combined with the fact that the demand for non-tradable goods increases with the population of workers in each city, DRS in the production of non-tradable goods implies that cities with a larger population will also have more expensive non-tradable goods – generating a congestion cost. Note that, unlike agglomeration, the congestion force in this model is not an externality, as its effects occur through prices.[17] It does, however, limit the size of cities: as the population of a city increases, so does the cost of living/producing there, so firms will start to favor locating in other places. Lemma 1 describes how wages respond to the city's population. In this lemma and throughout the paper, I use the "tilde" notation to refer to variables per unit of land: if $I_c$ is the population of inventors in city $c$, then $\tilde{I}_c = I_c/\bar{m}_c$.

**Lemma 1.** *Define $I_c$ and $L_c$, respectively, as the population of inventors and production workers in each city (with $I_0 = 0$). Under the free mobility of workers, wages can be expressed as the product of a "baseline" wage and a term that adjusts for congestion costs in each city. The wages of inventors in cities $c \in \{1, \dots, C\}$ is given by*

$$w_c^i = w^i \left( \frac{\tilde{I}_c^{1-\beta}}{\alpha_c} \right)^{\frac{\theta}{1-\theta}} \quad \textit{where} \quad w^i = \frac{1}{1-\theta} \left[ \frac{u^i}{[(1-\theta\beta)]^\theta} \right]^{\frac{1}{1-\theta}} \left( \frac{I}{L - L_0} \right)^{\frac{\theta\beta}{1-\theta}}, \tag{5}$$

*while the wages of production workers are*

$$w_c^\ell = w^\ell \left( \frac{\tilde{L}_c^{1-\beta}}{\alpha_c} \right)^{\frac{\theta}{1-\theta}} \quad \textit{where} \quad w^\ell = \frac{1}{1-\theta} \left[ \frac{u^\ell}{(\theta\beta)^\theta} \right]^{\frac{1}{1-\theta}}, \tag{6}$$

*where $u^i$ and $u^\ell$ are the utility levels of inventors and production workers, respectively. Land rents in each of those cities is given by*

$$p_{m,c}\bar{m}_c = \frac{(1-\beta)\theta}{1-\theta\beta} w_c^i I_c.$$

*In city 0, the number of production workers hired to produce non-tradable goods is $\ell_{n,0} = \theta\beta L_0$, and the number of workers hired to produce the final good is $\ell_{y,0} = (1-\theta\beta)L_0$. The wage of production workers is given by*

$$w_0^\ell = w^\ell \left( \frac{\tilde{L}_0^{1-\beta}}{\alpha_0} \right)^{\frac{\theta}{1-\theta}} (\theta\beta)^{\frac{(1-\beta)\theta}{1-\theta}} \tag{7}$$

[16] With the exception that $u_0^i$ is not defined, as there are no inventors in city 0. The same caveat applies to any other city that is not populated by some type of worker – although cities $1, \dots, C$ are populated by both inventors and production workers with probability 1 in the equilibrium.

[17] For this statement, I use a more restrictive definition of externality which only includes effects that are not reflected on prices. Alternatively, one could classify agglomeration as a non-pecuniary externality and congestion as a pecuniary externality.

*and the total land rent is* $p_{m,0}\bar{m}_0 = (1-\beta)\theta w_0^\ell L_0$.

See appendix B for the proof. The equations in lemma 1 also imply that the number of inventors and production workers cities where there is innovation is proportional: $I_c/I = L_c/(L - L_0)$. As a result, the population of production workers is sufficient to characterize the population of inventors in each city, and vice-versa.[18] Finally, note that the wage of production workers in city 0 can be written in an alternative form by plugging (4) into the F.O.C. of the final good producer, equation (2):

$$w_0^\ell = \frac{\varepsilon}{1-\varepsilon}\left(\frac{1-\varepsilon}{\nu}\right)^{\frac{1-\varepsilon}{\varepsilon}} Q, \tag{8}$$

where $Q = \int_{\mathcal{J}} q_j dj$ is the average quality of all intermediate goods produced in the economy.

### 3.3 The Firm's Dynamic Problem

Firms can either be entrant or incumbent. In each period firms choose how much to invest in innovation, given the set of product varieties they own and the optimal production decisions described above. Incumbent firms take their location as given, and are not allowed to move. Entrant firms are free to choose which city to locate in. The timing of decisions in each period is as follows:

(i) The shock $z_c(t)$ is realized and observed in all cities.

(ii) Potential entrants decide whether or not to enter and in which city to locate.

(iii) Entrants and incumbents decide how many inventors to hire.

(iv) Innovations are realized (based on the arrival rates $x_{f,c}$) and production takes place.

**Incumbents** The dynamic problem faced by an incumbent firm located in city $c$ can be described by the Hamilton-Jacobi-Bellman (HJB) Equation in lemma 2 below. To make the exposition clearer, first define $\mathbf{q}_f$ to be the multiset of the qualities of products that are currently being produced by the firm[19] and $D$ to be the rate of creative destruction of the economy – or equivalently, the aggregate rate of innovation. Because the measure of intermediate good varieties in the economy is one, $D$ also coincides with the probability than any single product line will be "stolen" at any point in time.

Let $r$ be the (exogenous) interest rate and $A = (Q, w^i, D, L_0)$ denote the "aggregate state" of the economy, where $Q$ is the average quality of all intermediate goods produced, $w^i$ is the "baseline" wage of inventors, $D$ is the rate of creative destruction, and $L_0$ is the population of production workers in city 0 (which affects the flow of profits for firms). For notational convenience, I also

[18] While greatly simplifying the model, this proportionality also means that the model fails to capture any specialization of cities into activities that are more or less intensive in innovation. One potential extension that addresses this issue is to allow for heterogeneity at the city-industry level, so that different industries, who might produce innovation with varying intensity, self-select into different cities. Being that as it may, this model can still reasonably replicate both the spatial distribution of inventors and of the general population in the data (see section 4.4).

[19] A multiset is a generalization of a set that allows for multiple instances of each of its elements. The notation $\cup_+$ indicates the multiset union operator, such that $\{a, b\} \cup_+ \{b\} = \{a, b, b\}$. Similarly, the notation $\setminus_-$ indicates the multiset difference operator, such that $\{a, b, b\} \setminus_- \{b\} = \{a, b\}$. The reason why $\mathbf{q}_f$ is a multiset is that a firm can have multiple products that have the same quality.

define $\bar{\pi} = (1-\theta\beta)\left(\frac{1-\varepsilon}{\nu}\right)^{\frac{1-\varepsilon}{\varepsilon}}\varepsilon$, so that the per-period profit of each firm is $\pi_j = \bar{\pi}L_0 q_j$, and $Z_c = e^{z_c}$, so that the city-specific productivity in the production on innovation is $\chi_c = \bar{\chi}_c Z_c$.

**Lemma 2.** *The HJB equation that describes the problem faced by an incumbent firm located in city $c \in \{1, \ldots, C\}$ is*

$$rV_c(\mathbf{q}_f, \tilde{I}_c, Z_c, A) - \frac{\partial V_c(\mathbf{q}_f, \tilde{I}_c, Z_c, A)}{\partial A}\frac{\partial A}{\partial t} =$$

$$\max_{x_{f,c}} \left\{ \begin{array}{l} \sum_{q_j \in \mathbf{q}_f} \bar{\pi}L_0 q_j + x_{f,c}\mathbb{E}_j[V_c(\mathbf{q}_f \cup_+ \{(1+\lambda)q_j\}, \tilde{I}_c, Z_c, A) - V_c(\mathbf{q}_f, \tilde{I}_c, Z_c, A)] \\ \\ -(1-s_c)w_c^i(i_{f,c}+\kappa) - D\sum_{q_j \in \mathbf{q}_f}[V_c(\mathbf{q}_f, \tilde{I}_c, Z_c, A) - V_c(\mathbf{q}_f \backslash_- \{q_j\}, \tilde{I}_c, Z_c, A)] + R_c(\mathbf{q}_f, \tilde{I}_c, Z_c, A) \end{array} \right\}$$

$$x_{f,c} = \bar{\chi}_c Z_c(\tilde{I}_c^\eta i_{f,c})^\psi$$

The derivation of this equation can be found in appendix B. There are three groups of state variables in the firm's problem: the first is firm-specific, the second is city-specific and the third is common across all firms in the economy. The first group includes $\mathbf{q}_f$, the set of qualities of the product lines that are currently produced by the firm; the second group has $\tilde{I}_c$, the population of inventors per unit of land in city $c$ (which determines agglomeration spillovers), and $Z_c$, the productivity shock in city $c$; finally, the third group has the aggregate state $A$, which includes the rate of creative destruction (i.e., the probability that one of the firm's product lines will be "stolen"), the "baseline" wage of inventors (which along with $\tilde{I}_c$ determines the wage $w_c^i$) and the average quality of all intermediate goods produced in the economy.

The first term inside the curly brackets is the profit made through the production and sale of goods. It is followed by the expected gain from one more innovation, which introduces a new product into $\mathbf{q}_f$ (recall that the number of innovations per period follows a Poisson distribution, so that in continuous time the probability that two or more arrivals occur can be ignored). The first term in the second line is the cost of investment in R&D (both variable and fixed), subsidized at rate $s_c$. The second term in the second line is the expected cost from the loss of a product line due to creative destruction. The remaining term, $R_c(\mathbf{q}_f, \tilde{I}_c, Z_c, A)$, captures the risk that firms in city $c$ face due to the productivity shock.

As a final note, recall from section 2 that one of the sources of revenue for the government are taxes on firm owners. Taxing firm owners in this model is the same as taxing firm's profits; yet no corporate income taxes appear on the HJB equation above. As shown in the proof of lemma 2, this can be done because a tax on a firm's profit will not affect any of the firm's decisions, as long as the "profit" also includes R&D expenditures. As a result, the corporate income tax only shifts the share of the firm's value that is accrued to the firm owner, without having any effect on the allocation of resources in the economy. By fully taxing firm owners, I study the limiting case where corporate taxes are arbitrarily close to one, but where firms still behave as profit maximizers (i.e. they behave as if their value function was $V_c$, as in lemma 2).

**Entrants** Entrants behave in the same way that incumbents do, with two exceptions: entrants do not yet have any product lines of their own and are able to choose where to locate. As before, define the aggregate state of the economy as $A = (Q, w^i, D, L_0)$. The entrant firm's problem can be solved in two steps:

**Step 1:** Choose which city to locate in after observing all shocks $\{Z_c\}_{c=1}^C$:

$$V^e(A) = \max_c V_c^e(\tilde{I}_c, Z_c, A).$$

**Step 2:** Choose the level of innovation subject to being in city $c$.

$$rV_c^e(\tilde{I}_c, Z_c, A) - \frac{\partial V_c^e(\tilde{I}_c, Z_c, A)}{\partial A}\frac{\partial A}{\partial t} =$$

$$\max_{x_{f,c}} \Big\{ \; x_{f,c}\mathbb{E}_j[V_c(q_j, \tilde{I}_c, Z_c, A) - V_c^e(\tilde{I}_c, Z_c, A)] - (1 - s_c)w_c^i(i_{f,c} + \kappa) + R_c^e(\tilde{I}_c, Z_c, A) \; \Big\}$$

$$\text{s.t. } x_{f,c} = \bar{\chi}_c Z_c(\tilde{I}_c^\eta i_{f,c})^\psi$$

The HJB equation for the second stage of the entrant's problem is exactly analogous to the incumbent's problem, but does not include the flow of profits from current production or the risk of loosing products to other firms by creative destruction. Proposition 1 describes the value function for entrants and incumbents (see appendix B for the proof).

**Proposition 1.** *In a Stationary Balanced Growth Path Equilibrium where the production of final goods $Y$ grows at rate $g < r$, the value function of an incumbent firm located in city $c \geq 1$ and whose portfolio of products is $\mathbf{q}_f$ is*

$$V_c(\mathbf{q}_f, \tilde{I}_c, Z_c, A) = F(D, L_0) \sum_{q_j \in \mathbf{q}_f} q_j + \max\left\{0, E_c(\tilde{I}_c, Z_c, w^i/Q, D, L_0)Q\right\},$$

*where $F(D, L_0) = \bar{\pi}L_0/(r + D)$ is the "franchise value" of adding a new product to the portfolio and $E_c$ is the entry value for firms city $c$ (see the proof for a complete characterization).*

*In addition, the second stage value function of an entrant firm who is located in city $c$ is*

$$V_c^e(\tilde{I}_c, Z_c, A) = \max\left\{0, E_c(\tilde{I}_c, Z_c, w^i/Q, D, L_0)Q\right\}.$$

Intuitively, $F$ can be interpreted as the quality-adjusted franchise value of adding a new product to the firm's portfolio, while $E_cQ$ is the value at entry for all firms in city $c$. Note that $E_c$ does not depend on the firm's portfolio of products, so it does not vary across firms in the same city. The term $\max\{0, E_cQ\}$ in the firm's value function reflects the fact that each firm has the choice to invest in R&D or not, and as such can be interpreted as the option value of investments in innovation. When investing in R&D is optimal, $E_c \geq 0$ and the arrival $x_{f,c}$ is strictly positive. However, firms can also choose not to invest in R&D in any given period – for example if the realized value of the shock $Z_c$ is too low. In this case, the firm does not hire inventors $i_{f,c} = 0$, produces no innovation $x_{f,c} = 0$, and does not need to pay the fixed cost $w_c^i\kappa$.

The assumption that $g < r$ is a technical requirement for the present discounted value of firms to be finite. From an intuitive point of view, note that the firm's value from investing in R&D grows at rate $g$ (so long as $E_c \geq 0$). Thus, if $g > r$, the optimal strategy for any given firm will be to invest as much as possible on R&D (for example, by borrowing capital at rate $r$). This will generate an expected growth in the firm's value that is larger than the firm's discount factor and thus the PDV diverges. The assumption that the rate of growth is smaller than the real interest rate mitigates those concerns.

**Free Entry.** The first stage of the entrant's problem specifies that entrants are free to locate in any city they wish. Since there is a large mass of potential entrants to every city in the economy, free entry implies that, in equilibrium

$$V_c^e(\tilde{I}_c, Z_c, A) = 0 \text{ for all } c \in \{1, \ldots, C\} \text{ and all } t.$$

This condition says that the value of entry must be zero for all cities and at all times. The intuition behind it simple: if the entry value were positive, firms would keep entering the city, which increases congestion and eventually pushes the value of entry to zero. If the entry value were negative (i.e., $E_c < 0$), two things happen. First, no firms will enter the city. Second, incumbents will refrain from investing in R&D (see the discussion above). This pushes down the demand for inventors in the city, which reduces congestion costs and the value of entry grows back to zero.

One alternative way to read the free entry condition is that $V_c^e(\tilde{I}_c, Z_c, A) = 0$ for all cities and for *any* value of the shock $Z_c$ and the aggregate state $A$. In other words, the population of inventors must adjust so that the value of entry is zero in all cities. From proposition 1, this implies that $E_c = 0$ regardless of the value of the state variables. This interpretation of the free entry condition, proposition 2 determines the population of inventors in each city (once again the proof can be found in appendix B).

**Proposition 2.** *Imposing (1) free entry, (2) labor market clearing for both inventors and production workers, and (3) assuming a large number of cities $C \to \infty$ (so that the Law of Large Number applies and the average of city-specific shocks converges to its mean), the population of inventors in each city is given by*

$$I_c = I \times \frac{\left(\frac{\bar{\chi}_c}{1-s_c}\right)^{\frac{1-\theta}{\Theta}} \alpha_c^{\frac{\theta}{\Theta}}}{\sum_{c=1}^{C} \left(\frac{\bar{\chi}_c}{1-s_c}\right)^{\frac{1-\theta}{\Theta}} \alpha_c^{\frac{\theta}{\Theta}}} \times \frac{Z_c^{\frac{1-\theta}{\Theta}}}{e^{\frac{1-\theta}{\Theta}\left(\frac{1-\theta}{\Theta}-1\right)\frac{\sigma^2}{4\phi}}} \tag{9}$$

*where $\Theta = (1-\beta)\theta - \psi\eta(1-\theta)$. Moreover, the arrival rate of an innovation for a firm $f$ located in city $c$ is*

$$x_{f,c} = \left(\kappa \frac{\psi}{1-\psi}\right)^{\psi} \bar{\chi}_c \tilde{I}_c^{\psi\eta} Z_c, \tag{10}$$

*if the firm chooses to invest in R&D and zero otherwise. Similarly, the number of inventors hired by each firm in city $c$ is $i_{f,c} = \frac{\psi}{1-\psi}\kappa$ in case of positive investment and zero otherwise. The number of firms located*

*in city $c$ who invest in R&D in each period is*

$$N_c = \left(\frac{1-\psi}{\kappa}\right) I_c \tag{11}$$

*Finally, it can also be shown that the population of production workers in city $0$ is proportional to $L$ (i.e., $L_0$ does not vary over time), and that $w^i$ is not affected by the city-specific productivity shocks*

$$\frac{w^i}{Q} \propto \frac{\bar{\pi} L_0}{r+D} \left\{ \frac{1}{C} \sum_{c=1}^{C} \left(\frac{\bar{\chi}_c}{1-s_c}\right)^{\frac{1-\theta}{\Theta}} \alpha_c^{\frac{\theta}{\Theta}} \right\}^{\frac{\Theta}{1-\theta}} . \tag{12}$$

Proposition 2 has three important results. First, it shows that the population of inventors in each city has a closed form solution in equation (9), with relatively simple terms. Cities with higher amenities, higher productivity for innovation, and higher R&D subsidies will have more inventors (note that $\Theta > 0$ using the parameter values from section 4). The population of inventors in each city also reacts to the productivity shock $Z_c$, increasing in periods where the shock is larger. The composite parameter $\Theta$ can be interpreted as the "net elasticity" of congestion: $(1-\beta)$ captures the elasticity of congestion with respect to a city's population, while $\psi\eta$ is the elasticity of the production of innovation with respect to the population of inventors. $\Theta$ is then defined as the weighted difference between these two elasticities, where the weights are determined by the share of expenditure on the tradable good, $\theta$.

Second, proposition 2 says that the *baseline* wage of inventors – and therefore of production workers as well (see equation B.4) – does not react to any of the city-specific productivity shocks. Put differently, even though each city is subject to an idiosyncratic productivity shock, these shocks "average out" on the aggregate, so the economy can still operate on a Balanced Growth Path where there is no aggregate uncertainty.

Third, this proposition shows that the optimal arrival rate of innovation $x_{f,c}$ is uniform across firms located in the same city and who make positive investments in R&D. Note, however, that the expected value of investing in R&D is null because of the free entry condition. As a result, it is possible that some firms in the city choose to invest in R&D and some choose not to. Indeed, the number of inventors hired by firms who make positive investments does not respond to productivity shocks. Instead, the adjustment to those shocks is done entirely on the extensive margin – both by incumbents who decide whether or not to hire inventors and by the entry and exit of firms.[20]

### 3.4 Determining the Growth Rate

In this section, I finish the characterization of the SBGP equilibrium of the model by determining the rate of growth of the economy. To be able to do this, I first must determine the aggregate rate of creative destruction of the economy, $D$. Note that, because all firms located in the same city

[20]This is seldom the case in reality, as firms face a number of frictions (which are not included in the model) that discourage them to, for example, periodically shut down their entire R&D division due to a bad transitory shock. Having said that, this counterfactual conclusion for individual firms need not be an issue for this model, since its purpose is not capture firms' responses to local shocks, but to reproduce the link between city- and nation-wide economic activity.

will make the same investment in R&D, $D = \sum_{c=1}^{C} N_c x_{f,c}$. Corollary 1 shows that this rate can be expressed as a function of the spatial distribution of the population of inventors in the economy (see appendix B for the proof).

**Corollary 1.** *Define* $\bar{I}_c = I \left(\frac{\bar{\chi}_c}{1-s_c}\right)^{\frac{1-\theta}{\Theta}} \alpha_c^{\frac{\theta}{\Theta}} \left\{\sum_{c=1}^{C} \left(\frac{\bar{\chi}_c}{1-s_c}\right)^{\frac{1-\theta}{\Theta}} \alpha_c^{\frac{\theta}{\Theta}}\right\}^{-1}$ *to be the expected value of* $I_c$ *w.r.t. the local productivity shocks. Similarly,* $\bar{\tilde{I}}_c$ *is the expected density of inventors in city* $c$*. The aggregate rate of creative destruction is then*

$$D \propto \frac{1}{C}\sum_{c=1}^{C} \bar{\chi}_c \bar{\tilde{I}}_c^{1+\psi\eta}, \tag{13}$$

Corollary 1 shows one of the key takeaways from this model: the aggregate rate of innovation in the economy depends not only on the amount of inventors in the country, but also on how inventors are geographically distributed. Since local R&D subsidies can change the spatial distribution of inventors by attracting more firms to a given location, it follows that changes in the spatial distribution of those subsidies affect the aggregate rate of innovation in the economy – even if the average subsidy rate (or total expenditure) is kept constant. The corollary also proves, as alluded to before, that the rate of creative destruction is indeed constant on the SBGP equilibrium (as both $\bar{\chi}_c$ and $\bar{\tilde{I}}_c$ are fixed over time).

Proposition 3 now determines the value of the rate of growth of the economy in the SBGP equilibrium, defined by $g = \dot{Y}/Y$. The proof can be found in appendix B.

**Proposition 3.** *In a Stationary Balanced Growth Path equilibrium where the production of the final good in the economy grows at rate* $g$*, the following are true:*

(1) *The average quality of intermediate goods,* $Q$*, and the baseline wage for both types of workers,* $w^i$ *and* $w^\ell$*, all grow at rate* $g$*. The utility level of workers,* $u^i$ *and* $u^\ell$*, grows at rate* $(1-\theta)g$*.*

(2) *The value of the rate of growth is* $g = \lambda D$*.*

(3) *Let* $\mathcal{J}_c(t)$ *be the set of intermediate goods that are produced in city* $c$ *at time* $t$*,* $Q_c(t) = \int_{\mathcal{J}_c(t)} q_j(t)dj$ *their aggregate quality, and* $g_c(t) = \dot{Q}_c(t)/Q_c(t)$ *the rate of growth of* $Q_c$ *in time* $t$*. For large values of* $t$ *(i.e., as* $t$ *grows to infinity), we have that*

$$\frac{\mathbb{E}[\dot{Q}_c(t)]}{\mathbb{E}[Q_c(t)]} = g \quad \textit{or, equivalently,} \quad \mathbb{E}[g_c(t)] = g - \frac{\mathrm{Cov}(g_c(t), Q_c(t))}{\mathbb{E}[Q_c(t)]}.$$

Part (1) of proposition 3 describes the rate of growth of aggregate variables in the SBGP equilibrium of the model. Part (2) then shows that the rate of growth of the economy is equal to the rate of creative destruction multiplied by the innovation step-size. Part (3) shows that the ratio between the expected variation in the aggregate quality of goods produced in city $c$ and the expected quality of these goods converges to the national growth rate with time. Intuitively, this result is a consequence of innovation by creative destruction: cities that produce goods with higher-than-average quality will innovate over products whose qualities are, on average, lower than the goods already produced in the city – and vice-versa. Through this mechanism, creative destruction acts

as a mean reverting force for the quality and the value of goods produced in each city, precluding all economic activity from concentrating in a single city.[21,22] Lastly, note that none of the results in the proposition are predicated upon the initial spatial distribution of economic activity: as long as no city is large enough to drive the evolution of aggregate variables by itself, proposition 3 holds regardless of where the economy starts from.

### 3.5 Existence and Uniqueness

The existence of a solution for the SBGP equilibrium relies in two conditions. First, it requires that $r > g$ so that present discounted values of profits are finite (see the discussion following proposition 1). Second, the number of cities must large enough so that the local shocks do not generate aggregate uncertainty in the economy.[23] Uniqueness comes from the unique spatial distribution of inventors defined by equation (9). One important caveat here is that this distribution is only unique if the net elasticity of congestion $\Theta > 0$. If this is not the case, the agglomeration force is always larger than congestion and therefore it is always profitable for all firms to locate in the same place. This generates multiple equilibria since the initial distribution of population can affect the entire path of the economy – for example, if the entire population is located in city $\hat{c}$ in the initial period, there is no incentive for entrants to go anywhere else.

## 4 Identification and Estimation of the Model

The identification and subsequent estimation of the parameters in the model proceeds in three steps. In the first step (section 4.1), I calibrate a set of parameters that can be directly matched to quantities in data or reliably found in other studies in the literature. The second step (section 4.2) uses linear regressions to estimate the elasticity of the agglomeration spillover with respect to the population of inventors, as well as the elasticity of congestion with respect to the population of production workers in each city. Finally, the third step (section 4.3) identifies the remaining parameters by matching moments in the model to moments in the data. This step-by-step structure simplifies identification of the model by allowing each step of the process to take as given the values of parameters identified in previous steps.

To estimate the model, I use data on patent filings and economic activity within cities. Data on patents filed is available on the USPTO Patent Dataset, which records all patents registered in the US. It also provides data on the patent's inventors and assignees (owners) and, importantly, their location. In addition, I use the County Business Patterns Dataset (CBP), which provides information on the demography and economic activity in each county in the US. To approximate the prices of non-tradable goods I use the Zillow Rent Index (ZRI), which estimates the median

[21] The rate of growth of $Q_c$ is also a measure of the rate of growth of the value of tradable goods produced in each city, since $\int_{J_c(t)} p_j(t)k_j(t)dj \propto Q_c(t)$.

[22] Another way to see this result is to note that the covariance between the rate of growth of a city and the quality of goods produced there is proportional to $g - \mathbb{E}[g_c]$. Hence, if the expected rate of growth of a city is larger than $g$, this city tends to grow faster when the quality of goods produced there is lower.

[23] An equilibrium could still exist if this condition is violated, but it would not be a balanced growth path.

rental value per square foot across counties.[24] Both the CBP and the ZRI are aggregated to the city level, whose empirical counterpart is a CBSA. CBSAs, or core-based statistical areas, are geographic areas defined by the US Office of Management and Budget that consist of one or more counties (or equivalents) anchored by an urban center of at least 10,000 people plus adjacent counties that are socioeconomically tied to the urban center by commuting. I use the most recent definitions of CBSAs, based on the 2010 Census standards. Appendix C.1 describes in detail the construction of the dataset. For most of the estimation procedure, I focus on a panel of firms (and their respective locations) who have filed patents between 1998 and 2016.

### 4.1 Calibration

A subset of the parameters in the model, shown in table 1, can be matched to the data in a straightforward way. In this section, I detail how to calibrate these parameters. I start by setting the rate of growth of the economy to $g = 2\%$, which corresponds to the annualized historic rate of growth in the US. Similarly, the discount rate of consumers is also set to $\rho = 2\%$, following a common practice in the growth literature (e.g., Acemoglu et al., 2018). The real interest rate is fixed at $r = 3.8\%$, which corresponds to the average interest rate in the US between 1961 and 2017 according to the World Bank.[25]

**Innovation.** I rely on the economic literature to determine the value of two parameters in the innovation process: the curvature of the innovation production function, $\psi$, and the innovation quality multiplier (or step-size), $\lambda$. I set $\psi = 0.5$ following several studies that agree on this number. A series of papers identify $\psi$ as the elasticity of patents with respect to R&D expenditure. Blundell et al. (2002) estimates this elasticity to be $0.5$ using count data models. Griliches (1990) and Hall and Ziedonis (2001) find similar values. Other researchers identify $\psi$ using the elasticity of R&D expenditure with respect to taxes/subsidies, which is equal to $\psi/(1-\psi)$.[26] In a survey, Hall and Van Reenen (2000) conclude that this elasticity is around unity. Similar estimates have been found in more recent papers as well (Bloom et al., 2002; Wilson, 2009). Akcigit et al. (2020) add interesting nuances to this result, differentiating between the impact of taxes on inventors/firms (micro level) and on states over time (macro level). Consistent with the discussion here, they find that "a one percentage point increase in the personal tax rate leads to a 1.1 percent decline in the number of patents" at the inventor level. Note that a unit elasticity of patents relative to taxes also implies $\psi = 0.5$. Finally, Acemoglu et al. (2018) arrive at the same conclusion when computing the elasticity of R&D expenditures with respect to scientists' wages using firm level data from the Census Bureau.

[24] One drawback of this series is that it is only available after 2010. As an alternative, I also employ the Zillow Home Value Index (ZHVI) as a measure of the price of non-tradable goods. The ZHVI has the median house price per square foot in each county in the US, starting in 1996.

[25] See `https://data.worldbank.org/indicator/FR.INR.RINR?locations=US&view=chart`.

[26] The mapping is straightforward: the FOC from the firm's problem (proposition 1) yields $x_{f,c} = (\bar{\chi}_c Z_c)^{\frac{1}{1-\psi}} [\psi F(1+\lambda) Q \tilde{I}_c^{\eta} (w_c^i(1-s_c))^{-1}]^{\frac{\psi}{1-\psi}}$. Using $i_{f,c} = x_{f,c}^{\frac{1}{\psi}} / ((\bar{\chi}_c Z_c)^{\frac{1}{\psi}} \tilde{I}_c^{\eta})$ and the fact that R&D spending (minus the fixed cost) is $(1-s_c) w_c^i i_{f,c}$, The FOC gives $(1-s_c) w_c^i i_{f,c} \propto (\frac{1}{(1-s_c) w_c^i})^{\frac{\psi}{1-\psi}}$. The elasticity of R&D spending w.r.t. to the subsidy (tax) is therefore $\psi/(1-\psi)$.

The innovation step-size is $\lambda = 0.132$. This is the value estimated by Acemoglu et al. (2018) in a setting close to the one presented here. Akcigit and Kerr (2018) find a similar value for the step size of "external" innovations (i.e., innovations that do not target a firm's own products). In both cases, the estimation of the step size is achieved through a simulated method of moments procedure that targets, among others, firms' sales and R&D costs. Intuitively, these data identify the innovation step-size because the increase in the quality of goods after an innovation is reflected on the sales to R&D cost ratio of firms.[27]

**R&D Subsidies.** R&D subsidies are equivalent to R&D tax credits in the model, to which I assign the values of existing credits in each state (i.e., the sum of federal and state-specific credits). Because of differences in the tax code for each state, the *statutory* credit rates can be different from the *effective* credit rates (i.e., the rates that are actually applicable as subsidies to firms). For example, R&D tax credits are only applicable to R&D investments over and above a given base value, which can differ across states. In addition, some states consider the federal tax credit as taxable income, thus "recapturing" part of the credit. In all that follows, I assign $s_c$ to be equal to effective R&D tax credit rate that applies to the highest tier of R&D investments in each state, as computed by Wilson (2009), who has made this data publicly available[28].

Nevertheless, there are a couple of details that require attention in this data. First, the value of R&D credits changes over time, while the model assumes that $s_c$ is fixed. Most of these changes happen before 1995, but there are still some cases where there is variation in the credit rate after this period. I therefore use the average credit rate between 1998 and 2006 (the last year available in the data) as a measure of the R&D subsidy in each state. Second, the model is written in terms of cities (CBSAs), which is some instances do not fall into the geographical boundaries of states. In those cases, I match the CBSA to the state of its largest urban center and assume that the R&D credit rate of that state applies to the entire CBSA. For example, the New York-Newark-Jersey City, NY-NJ-PA MSA is matched to the state of New York, the Chicago-Naperville-Elgin, IL-IN-WI MSA is matched to Illinois, the Boston-Cambridge-Newton, MA-NH MSA is matched to Massachusetts, and so on.

**Production and Preferences.** The elasticity of quality in the production of the final good, $\varepsilon$, coincides with the profit/sales ratio for intermediate good producers, which can be observed in the National Income and Product Accounts tables published by the Bureau of Economic Analysis (BEA). In addition, the preference parameter $\theta$ is the share of expenditure on non-tradable goods by consumers. I set this parameter to $0.6$, which is roughly the share of expenditure in housing and transportation found in the Consumer Expenditure Survey, published by the Bureau of Labor Statistics (BLS) and the share of aggregate investment on non-tradable goods found by Bems (2008).

[27]This information, however, cannot be found in widely available datasets. Compustat contains this data for some firms, but they represent a very small fraction of the universe of firms that file patents.

[28]See `https://www.frbsf.org/economic-research/economists/daniel-wilson/`.

**Population.** The population of inventors in the economy, $I$, can be found through the unique inventor ID in the USPTO patent data. For each year, I compute the total number of inventors in the US – let $I^{pat}$ be the average of this series over time.[29] Since only inventors that have authored patents are identified in the data, $I^{pat} = \psi I$, which is the total number of inventors hired by firms to produce patents ($\sum_c N_c i_{f,c}$), not including those hired to cover fixed costs. A simple adjustment gives $I = I^{pat}/\psi$.

The population of production workers or "non-inventors" in the economy is set so that $L + I$ matches the total employed population in the CBP data. Finally, $L_0$ is the total employed population in CBSAs that did not file any patents throughout the sample (once again averaged over time). Since the size of the economy does not affect the results in the quantitative exercises, I normalize the population so that the total number of inventors in the economy is 1.

The number of cities is also chosen to match the data. There are 917 CBSAs in the US (not counting Puerto Rico because it is not included in the CBP), of which 860 have filed at least one patent between 1998 and 2016. Given that the number of patents filed is my measure of innovation, I assume that the remaining CBSAs have not produced any innovation over my sample. As a result, $C = 860$ (the cities who have a positive expected productivity in innovation) and city 0 is representative of the remaining 57 CBSAs.

Table 1: Calibrated Parameters

| Parameter | Value | Description | Matches |
|---|---|---|---|
| $\psi$ | 0.5 | Elast. innovation wrt R&D | Literature |
| $\lambda$ | 0.132 | Innovation step size | Acemoglu et al. (2018) |
| $s_c$ | $[0.13, 0.30]$ | Effective R&D credit rate | Wilson (2009) |
| $\rho$ | 0.02 | Discount rate | Literature |
| $g$ | 0.02 | Growth rate | Annualized growth rate |
| $r$ | 0.038 | Real interest rate | Avg. real interest rate |
| $\varepsilon$ | 0.15 | Elast. quality in final goods | Profit/sales ratio (BEA) |
| $\theta$ | 0.6 | Preference parameter | Share of expenditure in non-tradables (BLS; Bems, 2008) |
| $I$ | 1 | Population of inventors | Avg. number of inventors residing in CBSA's (1998 - 2016) |
| $L$ | 175 | Population of production workers | Avg. employed population residing in CBSA's (1998 - 2016). |
| $L_0$ | 0.85 | Population of production workers in CBSA's that do not innovate | Avg. employed population residing in CBSA's w/ no patents filed (1998 - 2016). |

## 4.2 Linear Regressions

In this section, I show how to identify and estimate $\eta$ and $\beta$, which help to determine the elasticities of agglomeration and congestion, respectively. In both cases, these parameters can be estimated using linear regressions based on relationships predicted by the model.

[29] The superscript "pat" alludes to the fact that this is the total number of inventors who have authored patents.

### 4.2.1 The Elasticity of Agglomeration

The functional form of the production function for innovation, equation (1), leads to a log-linear relationship between the arrival rate of innovation, the number of inventors hired, and population of inventors in the city for each firm located in city $c$ – which suggests that the parameters of this function can be estimated by linear regression. However, the model presented in section 2 is written continuous time, while the data is only available at a yearly rate. Therefore, we must first transform equation (1) to reflect the same frequency as the data. This transformation is straightforward and is described in detail in appendix C.2. It leads to the following regression model

$$\log(x_{f,c,t}) = \psi \log(i_{f,c,t}) + \psi\eta \log(I_{c,t}) + \delta_c + z_{f,c,t}, \tag{14}$$

where, $x_{f,c,t}$ is the number of innovations produced by firm $f$, located in city $c$, during year $t$; $i_{f,c,t}$ is the number of inventors hired by firm $f$ during year $t$; $I_{c,t}$ is the population of inventors in city $c$ during year $t$; $\delta_c$ is a city fixed-effect; and $z_{f,c,t}$ is a function off the city-specific productivity shocks.

To proxy for the production of innovation in each year, I use the number of patents filed by a firm in that same year. This is a fairly common practice, but it does have some caveats. First of all, not all innovations are patented. Possible reasons for that include firms who decide to protect their intellectual property by other means (for example with trade secrets) or the fact that some innovations are not "patentable" (e.g., new managerial practices or marketing strategies).[30] Second, not all patents represent an innovation over a product. Examples include defensive patenting and patent trolls.

To reduce the potential for a mismatch between patents filed and the production of innovation by firms, I include two controls into the regression above. The first is the total number of citations that the patents filed by each firm jointly receive. Patents whose main goal is not to generate an innovation to increase the quality of a product are less likely to be cited by future patents – so including the number of citations as a control helps to separate innovations over products from other types of patents. On top of that, more recent patents mechanically receive fewer citations (Hall et al., 2001), so I interact the number of citations with a dummy for the year in which the patent applications were filed.[31] The second variable included is the firm's industry, which controls for the possibility that some industries are more prone to patent innovations than others. In addition, I also add a year fixed effect into the regression to capture aggregate variations over time (for example population growth).

One important empirical prediction of the model regarding the regression model (14) is that both $i_{f,c,t}$ ad $I_{c,t}$ are correlated with the local shock $z_{f,c,t}$, so that estimating the coefficients in that regression via OLS would recover neither $\psi$ or $\eta$. To see why the model predicts that $i_{f,c,t}$ is correlated with $z_{f,c,t}$, recall that the value of the productivity shock changes the number of firms

[30] Argente et al. (2019) argue that a substantial amount of product innovation comes from firms who do not patent. However, patenting is positively associated with product innovation both at the intensive and extensive (when firms switch to patents) margins.

[31] One alternative is to use citation-weighted patent counts as a measure of the production of innovation. While this is likely a better measure of the impact of patents produced by a firm, it is not clear how much control firms have over the quality of the patents that result from their investments. Because of that, I use patent counts as a measure of the output of a firm's R&D investment.

investing in R&D – so that a particularly low shock could induce some firms not to invest in R&D in a particular period, while a high value of the shock would induce more firms to invest (along with a larger number of entrants). Since $z_{f,c,t}$ is a function of the local productivity shocks, the correlation follows. In the case of $I_{c,t}$, this correlation is easily seen from equation (9), where the population of inventors in each city is a function of the city's productivity in each period (see appendix C.2 for more details). In practice what this means is that estimating the coefficients on regression (14) via OLS would not recover neither $\psi$ or $\eta$.

Note, however, that the regression (14) can be rearranged as

$$\log(\text{patents}_{f,c,t}) - \psi\log(i_{f,c,t}) = \psi\eta\log(I_{c,t}) + X'_{f,c,t}\Gamma + \delta_c + \delta_t + z_{f,c,t}$$

where $X_{f,c,t}$ includes the controls mentioned above, and $\delta_t$ is a year fixed effect. Note that, since the value of $\psi$ is known from the previous literature, the left-hand side of the equation above can be constructed in the data. Furthermore, because my the goal is to estimate $\eta$, this equation can be aggregated to the city level

$$\frac{1}{N_{c,t}}\sum_{f=1}^{N_{c,t}}\left[\log(\text{patents}_{f,c,t}) - \psi\log(i_{f,c,t})\right] = \psi\eta\log(I_{c,t}) + X'_{c,t}\Gamma + \delta_c + \delta_t + z_{c,t} \tag{15}$$

where $N_{c,t}$ be the number of firms in city $c$ during year $t$ and $z_{c,t}$ is the average shock inside each city. The control vector $X_{c,t}$ includes the average number of citations received by patents filed by firms in city $c$ (interacted with a year dummy) and the employment shares in each NAICS 2-digit industry in city $c$.[32] The dependent variable in this model is the average log production of patents per inventor in each firm, where number of inventors per firm is transformed by raising it to the elasticity of labor in the production of innovation.

The regression model (15) simplifies the analysis by removing one of the endogenous variables from the RHS of the equation. I account for the endogeneity of the population of inventors $I_{c,t}$ by constructing an instrument that leverages exogenous shocks to industries. I partition the set of products $J$ into $K$ industries, so that each product $j$ can be assigned to a single industry $k$. Note that the population of inventors in city $c$ can then be written as

$$I_{c,t} = \sum_{k=1}^{K} I_{k,c,t} = \sum_{k=1}^{K} I_{k,c,t-l}(1+\gamma_{k,c,t-l\to t})$$

where $I_{k,c,t}$ is the number of inventors in industry $k$ living at city $c$ during time $t$ and $\gamma_{k,c,t-l\to t}$ is the rate of growth of $I_{k,c,t}$ between periods $t-l$ and $t$.[33] Based on this identity, I construct the

[32] I use the employment shares (instead of a simple firm count) to measure the industry composition in each city because it accounts for differences in firm size across industries. Furthermore, industry shares are computed including all firms in each CBSA (not only those who innovate). This is done to avoid having a correlation between the industry employment share and the population of inventors in the city, which is endogenous.

[33] Since the model does not include industries, it does not predict that different industries would experience different growth in employment. I therefore deviate from the structure of the model by leveraging industry-level variation to estimate $\eta$. It should be noted, however, that equation (15) is derived from the theoretical model regardless of the instrument that is used to estimate the parameters in it. As a result, the estimated parameters also retain their interpretation in the model regardless of the estimation procedure.

following instrument for $I_{c,t}$:

$$\mathcal{I}_{c,t,l} = \sum_{k=1}^{K} I_{k,c,t-l}(1 + \gamma_{k,t-l\to t}). \tag{16}$$

where $\gamma_{k,t-l\to t}$ is the overall growth rate of employment in industry $k$ from year $t-l$ to year $t$. To avoid picking up variation in the total number of inventors in each year, the industry growth rate is computed using the shares of employment in each industry relative to the total population of inventors. Slightly abusing notation to define $I_{k,t}$ as the number of inventors in industry $k$ during year $t$, $\gamma_{k,t-l\to t} = \frac{I_{k,t}/I_t - I_{k,t-l}/I_{t-l}}{I_{k,t-l}/I_{t-l}}$.

To ensure the exogeneity of the instrument, I follow Autor et al. (2013) – among many others –, and compute the industry growth rates $\gamma_{k,t-l\to t}$ using inventors residing outside of the US who have registered patents with the USPTO (who are responsible for about $50\%$ of all registered patents during the period of my sample). Industries are defined based on NBER's patent subcategories (which add up to 38 different industries) and inventors are assigned to an industry based on the modal sub-class of the patents he or she filed (see the data appendix C.1 for more details).

The instrument $\mathcal{I}_{c,t,l}$ has a structure that resembles the commonly used "shift-share" research design (Adão et al., 2019; Borusyak et al., 2022; Goldsmith-Pinkham et al., 2020), with the obvious difference that $I_{c,k,t-l}$ is the population level, not a share (see appendix C.3.1 for more details). The economic content is preserved, however: the growth rate of employment in each industry acts as an exogenous shock or "shifter" and the population level $I_{k,c,t-l}$ measures the city's exposure to industry shocks. However, the shift-share structure can also affect the across-region correlation of the regression residuals (Adão et al., 2019). Intuitively, regions that have a similar industry composition in their population of inventors will also have a similar exposure to the shifters $\gamma_{k,t-l\to t}$, and therefore will tend to have similar residuals as well. As a result, I include two sets of standard errors when the IV strategy is used: one that is clustered across regions and one that is adjusted using the methods developed by Adão et al. (2019).[34]

The results of the estimation are in table 2. The top panel shows the first stage of the IV estimation, and the bottom panel shows the second stage. Column (1) shows the OLS estimates. Columns (2) - (4) show the IV estimates with lags $l$ between $5$ and $10$ years (specified at the bottom of the table). Column (5) has the IV estimates for $l = t - t_{90-95}$ – that is, $I_{i,c,t-l} = I_{i,c,t_{90-95}}$ is fixed at its average level between 1990 and 1995, and $\gamma_{k,t_{90-95}\to t}$ is computed using average industry shares in the same period as the base value (the first year of the estimation period is 1998). All regressions are weighted by the number of firms in each city, to account for the fact that the data consists of averages over firms. Standard errors are clustered at the city level to allow for serial correlation of shocks within each city. In the second stage, AKM SE indicates adjusted standard errors that account for the shift-share structure of the instrument.

[34]For the computation of those standard errors, I use the `ivreg_ss` packages in Stata. See `https://github.com/zhangxiang0822/ShiftShareSEStata`.

Table 2: Estimation of the elasticity of agglomeration, $\psi\eta$.

First Stage

| | (1) | (2) | (3) | (4) | (5) |
|---|---|---|---|---|---|
| $\log(\mathcal{I}_{c,t,l})$ | | $0.546^{***}$ | $0.481^{***}$ | $0.398^{***}$ | $0.244^{***}$ |
| | | (0.031) | (0.033) | (0.035) | (0.042) |
| F-stat. excluded inst. | | 315.10 | 214.79 | 130.93 | 33.94 |

Second Stage

| | (1) | (2) | (3) | (4) | (5) |
|---|---|---|---|---|---|
| log(Inventors in City) | $0.070^{***}$ | $0.104^{***}$ | $0.098^{***}$ | $0.105^{***}$ | $0.104^{*}$ |
| | (0.014) | (0.019) | (0.020) | (0.023) | (0.060) |
| AKM SE | | (0.014) | (0.017) | (0.016) | (0.001) |
| Method | OLS | IV ($l = 5$) | IV ($l = 7$) | IV ($l = 10$) | IV ($l = t - t_{90-95}$) |
| Observations | 11279 | 11231 | 11220 | 11201 | 11210 |
| Implied $\eta$ | 0.140 | 0.208 | 0.196 | 0.210 | 0.208 |

Standard errors are clustered at the CBSA level and shown in parenthesis. $^*$, $^{**}$, and $^{***}$ indicate that the coefficient is statistically different from 0 at the 10%, 5%, and 1% levels, respectively. AKM SE indicates alternative standard errors, calculated according to Adão et al. (2019). All specifications control for patent quality and city industry composition, as well as CBSA and year fixed effects.

By and large, the estimated coefficients are highly significant and vary between 0.07 and 0.10.[35] Those numbers do not change much in most of the robustness checks – and when they do, the value of the agglomeration elasticity tends to be higher. If compared to the other estimates of the elasticity of agglomeration (which usually do not focus on innovation), the values in table 2 are quite large. Duranton and Puga (2014), for example, state that most studies have found this elasticity to be between 0.02 and 0.05.[36]

Innovation can, however, be more responsive to agglomeration spillovers than the production of goods. In an exercise similar to mine, Carlino et al. (2007) compute the elasticity between the number of patents per capita and employment density in metropolitan/urban areas in the US. In their baseline specification, they find this elasticity to be approximately 0.19, which is quite larger than the values shown in table 2. The main difference between that study and regression (15) is that the amount of patents per capita does not account for differences in firm size that arise in different cities – and cities with higher agglomeration will also have more and larger firms, so they naturally produce more patents.

**Identification Conditions and Robustness Checks.** There are two ways to interpret the orthogonality condition for shift-share instruments, and therefore for the instrument proposed here as

[35] Note that the IV estimates tend to be slightly larger than their OLS counterpart, which is counter-intuitive since the OLS estimate is likely biased up due to a positive correlation with the error term. However, recall that the dependent variable of the regression includes $-\sum \log(i_{f,c,t})$, and that $I_{c,t} = \sum_f i_{f,c,t}$. Thus, if there is any measurement error in $i_{f,c,t}$, it will also be present in $I_{c,t}$ and could bias the OLS coefficient downwards.

[36] There are at least two broad methods to identify this elasticity. First, one can estimate the response of firms' outputs with respect to some measure of agglomeration, such as Greenstone et al. (2010) or Gaubert (2018). A second strand of the literature explores variation on workers' wages to learn about agglomeration spillovers. Examples include Glaeser and Maré (2001), Combes et al. (2008) or De La Roca and Puga (2016).

well. The first one, discussed at length by Goldsmith-Pinkham et al. (2020), is that it requires that the exposures $I_{k,c,t-l}$ to be uncorrelated with the local shock $z_{c,t}$. This is unlikely to be true for small values of the lag $l$, as local shocks can differentially affect industry employment shares depending on the city's industrial composition (and be correlated with the lagged share if shocks are serially correlated). However, this condition becomes plausible when lags are larger, for example in column (4) that uses a 10-year lag, or when the industry employment levels are fixed at a period that predates the estimation sample, as in column (5).[37]

The second interpretation, suggested by Borusyak et al. (2022), assumes that the industry growth rates $\gamma_{k,t-l\to t}$ are asymptotically uncorrelated with the industry-specific average of local shocks, $\mathbb{E}[I_{k,c,t-l}z_{c,t}]$ (where the expectation is taken over $c$). Measuring growth rates $\gamma_{k,t-l\to t}$ outside of the US addresses many of the issues that could be raised about the plausibility of this assumption. One concern that remains is that some industries might be highly concentrated in one single city – enough that the city's local shocks are able to affect global trends in that industry (Silicon Valley may come to mind). To address this issue, I re-run the regressions in table 2 with a slightly different instrument that excludes industries whose employment share in a single city exceeds $15\%$ in any point in time (varying this threshold between 10 and 25% produces comparable results). Once again, those estimates are in line ones presented here and can be seen in appendix C.3.2.

Another argument that might call into question the validity of the estimates above is that the log-log specification of the regression discards observations in which firms have not produced a patent. Since innovation is stochastic, this specification would introduce bias by selecting firms located in larger cities or firms located in cities who by chance experienced mostly positive shocks (both of which increase the arrival rate of innovation). To put those concerns to rest, I slightly modify (15) to interpret it as a count-data (Poisson regression) model, which allows for firms to produce zero patents in any given year. Appendix C.3.3 describes this regression model in detail and shows the estimated coefficients. The resulting elasticity of agglomeration is even slightly higher than above, estimated at approximately $0.13 - 0.15$.

One last robustness check, also described in appendix C.3.3, tests the hypothesis inventors/firms might benefit from other sources agglomeration. For example, firms could benefit from being close to other firms that they can observe and learn from; alternatively, inventors could benefit simply from living in densely populated areas, not necessarily by other inventors. The results I find suggest otherwise: the coefficients on the number of firms (investing in R&D), total employment and total establishments in each city are either negative or not statistically significant (after accounting for the population of inventors).

**The value of $\eta$.** Taking into account the results in table 2, as well as the robustness checks in appendix C.3, I use $\eta = 0.20$ as the baseline value to compute the optimal distribution of R&D subsidies in section 5. In appendix F, I perform sensitivity analyses of my results using $\eta = 0.15$ and $\eta = 0.25$, which roughly spans the range of estimated coefficients found in all specifications.

[37]There is, however, a tradeoff when fixing shares in some pre-period, as the instrument becomes less predictive of $I_{c,t}$ in the first stage regression.

I also estimate equation 15 using different values of $\psi$ – namely, $\psi = 0.4$ and $\psi = 0.6$. When $\psi = 0.4$, the estimated coefficient $\psi\eta$ revolves around $0.11$ ($0.09$ if estimated via OLS), which implies $\eta = 0.275$. Conversely, when $\psi = 0.6$, the estimated coefficient $\psi\eta$ fluctuates around $0.09$ ($0.05$ if estimated via OLS), which implies $\eta = 0.15$. In all of the IV specifications, coefficients are statistically significant at the usual levels, and the implied value for $\eta$ falls in the range used for sensitivity analyses in appendix F.

### 4.2.2 The Elasticity of Congestion

Given the share of consumer expenditures on the non-tradable good, the parameter that determines the elasticity of congestion in the model is the return to scale on the production of that good, $\beta$. With constant returns to scale ($\beta = 1$) there is no congestion force, as the production of the non-tradable good scales up with the city size. As $\beta$ becomes closer to zero, congestion costs become more and more relevant – up to the point where the supply of the non-tradable good is fixed and all variation in city size is absorbed into prices. This intuition offers some insight into how best to identify $\beta$. The first-order condition of the non-tradable good producer's problem gives $p_{n,c} \propto w_c^{\ell} \left(\frac{L_c}{\bar{m}_c}\right)^{1-\beta}$. Using equation (6) to substitute for wages and approximating the price of the non-tradable good by the median rental value in each city, the model implies the following empirical relationship (details in appendix C.4)

$$\log(p_{c,t}^h) = \left(\frac{1-\beta}{1-\theta}\right) \log(L_{c,t}) + \delta_c + \delta_t + z_{c,t}^h \tag{17}$$

where $p_{c,t}^h$ is the median rental value per square foot of housing units in city $c$ during year $t$, $L_{c,t}$ is the population of non-inventors in city $c$ during year $t$, $\delta_c$ is a city fixed effect that accounts for variation in amenities and land availability, $\delta_t$ is a year fixed effect that accounts for the growth in wages/prices and $z_{c,t}^h$ is a city-specific shock (again a function of the productivity shock $z_c(t)$).

Given $\theta = 0.6$, $\beta$ is identified by the coefficient on $L_{c,t}$ in the regression above. However, like the population of inventors in each city, the model also predicts that the population of production workers is correlated with $z_{c,t}^h$. As a result, estimating this regression via OLS will not recover the value of $\beta$. Notwithstanding, given that the population of production workers and the population of inventors in each city is highly correlated (the model predicts that they are proportional), $\mathcal{I}_{c,t,l}$ also serves as an instrument for the population of production workers.

Table 3 displays the estimation results from (17) using $\mathcal{I}_{c,t,l}$ as an instrument for $L_{c,t}$. As one would expect, the instrument in this case has a much lower predictive value in first stage, although the F-statistic generally remains above commonly used thresholds. The values of the elasticity of rental prices with respect to each city's population are also quite large. For comparison, Behrens et al. (2014) find this elasticity to be between 0.08 and 0.09. This difference is due to the inclusion of city fixed effects in my model.[38] Those fixed effects reflect in part the fact that cities have different amenities, which affect the utility of consumers. Leaving them out of the regression can therefore severely bias the elasticity of prices with respect to the population, since individuals are willing to

[38] Running the regression in my data without the city fixed effects produces coefficients in the range of 0.074 to 0.078, depending on the lag of the instrument.

Table 3: Estimation of the elasticity of congestion, $(1-\beta)/(1-\theta)$.

| | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| **First Stage** | | | | |
| $\log(\mathcal{I}_{c,t,l})$ | 0.020*** | 0.023*** | 0.018*** | 0.023*** |
| | (0.005) | (0.008) | (0.005) | (0.008) |
| F-stat. excluded inst. | 14.58 | 7.47 | 12.95 | 8.89 |
| **Second Stage** | | | | |
| log(Prod. Workers in City) | 0.981*** | 1.013*** | 1.267*** | 0.902*** |
| | (0.326) | (0.394) | (0.392) | (0.337) |
| AKM SE | (0.026) | (0.023) | (0.033) | (0.013) |
| Method | IV ($l=5$) | IV ($l=7$) | IV ($l=10$) | IV ($l=t-t_{90-95}$) |
| Observations | 2855 | 2849 | 2845 | 2846 |
| Implied $\beta$ | 0.608 | 0.595 | 0.493 | 0.639 |

Standard errors are clustered at the CBSA level and shown in parenthesis. $^{*}$, $^{**}$, and $^{***}$ indicate that the coefficient is statistically different from 0 at the 10%, 5%, and 1% levels, respectively. AKM SE indicates alternative standard errors, calculated according to Adão et al. (2019). All specifications control for CBSA and year fixed effects.

pay higher prices to live in cities where amenities are higher.

The different specifications shown in the table all produce similar results, with $\beta$ around $0.5-0.6$. As was the case with the elasticity of agglomeration, I also run this regression using an instrument that is lagged up to 12 years and when excluding industries that are highly concentrated in one single place. Those results can be found in appendix C.4.

**Identification Conditions and Robustness Checks.** The structural error in equation (17) is a function of the same productivity shock that determines the residual in the previous section. Hence, the conditions for the orthogonality of the instrument in both cases are also the same. Table 3 shows the estimated elasticity of congestion when the lag $l$ used to construct the instrument varies from 5 to 10 years and when industry employment shares in each city are fixed at their average level between 1990 and 1995 – leveraging the fact that for large $l$, $I_{c,k,t-l}$ and $z^h_{c,t}$ are likely to be uncorrelated. Appendix C.4 presents these same results when using an instrument that excludes industries whose employment share (of inventors) in any single city share exceed 15% at any point in time.

Rental values are only available in the ZRI database after 2010, which explain the small number of observations in table 2. Because of that, I also an alternative version of (17), where $p^h_{c,t}$ is approximated by the median housing *price* in each city (a series that goes back to 1996)[39]. This regression implies a higher value or $\beta$, around $0.8$, which reflects the fact that housing prices tend to be less elastic to the population than rental prices.

[39]Using housing prices to approximate $p^h_{c,t}$ presents its own issues, as houses can also be thought of assets, whose prices might reflect agents' expectations about the future of the economy.

**The value of $\beta$.** Combining the estimation results in table 3 and in appendix C.4, I adopt $\beta = 0.6$ as the baseline value to compute the optimal distribution of R&D subsidies in section 5. I also perform sensitivity analyses using $\beta = 0.5$ and $\beta = 0.8$, which spans the range of estimated coefficients found in all specifications.

### 4.3 Moment Matching

**Fixed Cost of Innovation.** To estimate size of the fixed cost of innovation, $\kappa$, I use equation (11), which relates the number of firms in each city to the number of inventors in the city. Summing both sides of that equation over cities and rearranging gives

$$\kappa = (1-\psi)\frac{I}{N}$$

where $N$ is the total number of firms in the economy. Given that the average number of inventors per firm in the data is $I/N \approx 21.07$ and $\psi = 0.5$, this relationship gives $\kappa = 10.53$.

**City-Specific Parameters.** Next, I turn to the set of city amenities $\alpha_c$ and mean productivities $\bar{\chi}_c$ in each city. For cities $c \in \{1, \ldots, C\}$, these parameters can be identified off the average share of inventors and patents filed by cities over time. Specifically, the average share of inventors in city $c$ over time is

$$\frac{1}{T}\int_0^T \frac{I_c(t)}{I}dt = \frac{\left(\frac{\bar{\chi}_c}{1-s_c}\right)^{\frac{1-\theta}{\Theta}} \alpha_c^{\frac{\theta}{\Theta}}}{\sum_{c=1}^{C}\left(\frac{\bar{\chi}_c}{1-s_c}\right)^{\frac{1-\theta}{\Theta}} \alpha_c^{\frac{\theta}{\Theta}}} \times \frac{1}{T}\int_0^T \frac{Z_c(t)^{\frac{1-\theta}{\Theta}}}{\exp\left(\frac{1-\theta}{\Theta}\left(\frac{1-\theta}{\Theta}-1\right)\frac{\sigma^2}{4\phi}\right)}dt.$$

Given the assumptions on the evolution of $Z_c(t)$, it is not hard to show that it is an ergodic process. As such, the ergodic theorem applies (see Bergelson et al. (2012) for more on continuous-time ergodic theorems) and the integral in the expression above converges to an expected value when $T \to \infty$. Assuming that the number of periods available in the data is large enough so that this result approximately holds, the average share of inventors in each city is given by

$$(\text{avg. share of inventors})_c = \frac{\left(\frac{\bar{\chi}_c}{1-s_c}\right)^{\frac{1-\theta}{\Theta}} \alpha_c^{\frac{\theta}{\Theta}}}{\sum_{c=1}^{C}\left(\frac{\bar{\chi}_c}{1-s_c}\right)^{\frac{1-\theta}{\Theta}} \alpha_c^{\frac{\theta}{\Theta}}} \equiv \frac{\bar{I}_c}{I}.$$

Similarly, the average share of patents filed in each city is

$$(\text{avg. share of patents filed})_c = \frac{1}{T}\int_0^T \frac{N_c(t)x_{f,c}(t)}{\sum_{c=1}^{C} N_c(t)x_{f,c}(t)}dt = \frac{\bar{\chi}_c \bar{I}_c^{1+\psi\eta}}{\sum_{c=1}^{C}\bar{\chi}_c \bar{I}_c^{1+\psi\eta}}.$$

The two sets of equations above identify $\alpha_c$ and $\bar{\chi}_c$ for every $c$ up to a constant. Since $\alpha_c$ is a preference parameter, its level does not have much meaning and I normalize $\mathbb{E}_c[\alpha_c] = 1$. The scale of $\bar{\chi}_c$ can be identified off equation (13) by imposing that the rate of growth of the model $g = \lambda D$ equals 2%, the historic annualized rate of growth in the US. The amenity in city 0 can be found by

requiring that the share of population in city $0$, $L_0/(I + L)$, matches this share in the data. Both of these procedures are described in detail in appendix D.1. Before doing all of that, however, the value of $\sigma^2/4\phi$ must be known.

**Law of Motion of the Productivity Shock** Lastly, I describe the identification of shock distribution parameters $\sigma$ and $\phi$. Since only the ratio $\sigma^2/\phi$ matters for the equilibrium of the model, I set $\phi = 1$. Next, $\sigma$ can be found by matching the model-generated cross-sectional variance of the population of inventors between cities with the same moment in the data. Appendix D.2 derives the expression for this variance in the model and shows how to identify $\sigma$.

### 4.4 Comparison to Untargeted Moments

I assess the model's external validity by measuring how well it can fit the spatial distribution of variables that were not targeted for estimation. Figure A.3 plots the model's predictions against the data four untargeted variables: the share of firms per city (panel a), the average number of patents per firm in each city (panel b), the share of total employed population per city (panel c) and the spatial distribution of patents per capita (panel d). In general the model matches those distribution quite well – the correlation between the share of firms per city in the model and data is particularly high, at about $0.97$. The distribution of patents per firm is harder to match, as there are many cities that have on average one patent per firm. This pattern holds for cities with widely different sizes and production of patents (see the vertical alignment of points in panel (b) of figure A.3).

Panels (c) and (d) of figure A.3 show the match between the total employed population and patents per capita between model and data. As a general rule, the model tends to underestimate the total population in cities where there is a small number of inventors and overestimate the population of cities where many inventors live – recall that the model predicts that the population of inventors and production workers is proportional; in practice, however, cities tend to specialize to some degree in innovation or in production. As a result, the model-predicted total population and model-predicted patents per capita tend to be off in each end of the city size distribution. Nevertheless, the match between model and data is not bad, with the correlations shown in panel A of table 4.

Panel B in table 4 compares the outcomes in the model and data at different sections of the city size distribution. Specifically, it ranks cities based on their average population of inventors between 1998 and 2016 and divides them into five bins with an equal number of cities. It then compares the share of firms and the average number of patents per firm in each of those quintiles separately.[40] For reference, I also include share of inventors and patents in each quintile of the city size distribution (there is no comparison between model and data in those cases, as the match is one-to-one).

[40] I focus on these two variables since they are the ones more closely related to the spatial distribution of innovation, which is what the model is designed to capture.

Table 4: Spatial Distribution of Untargeted Variables

Panel A: Correlations Between Untargeted Variables: Model and Data

| Number of Firms | Patents per Firm | Employed Population | Patents per Capita |
|---|---|---|---|
| 0.97 | 0.58 | 0.81 | 0.49 |

Panel B: Spatial Distribution of Untargeted Variables

| Bin | Share of Inventors | Share of Patents | Share of Firms (model) | Share of Firms (data) | Avg. Patents/Firm (model) | Avg. Patents/Firm (data) |
|---|---|---|---|---|---|---|
| 1 | 0.002 | 0.001 | 0.002 | 0.003 | 1.052 | 0.81 |
| 2 | 0.006 | 0.002 | 0.006 | 0.007 | 1.006 | 0.875 |
| 3 | 0.013 | 0.006 | 0.013 | 0.017 | 1.234 | 0.993 |
| 4 | 0.04 | 0.018 | 0.04 | 0.045 | 1.271 | 1.171 |
| 5 | 0.939 | 0.973 | 0.939 | 0.927 | 2.181 | 2.894 |

#### 4.4.1 Can R&D Tax Credits Shift the Spatial Distribution of the Economy?

Another question addressed in this section is the extent to which R&D tax credits can influence the location of firms. Moretti and Wilson (2017) offer evidence that inventors are very sensitive to state taxes, but R&D tax credits tend to have a smaller effect than other forms of taxation. Similarly, Slattery (2019) finds that state-level subsidies have an important effect over firms' locations, but this effect includes all discretionary state subsidies and cannot be attributed to R&D tax credits alone. Understanding the extent to which R&D tax credits can change the spatial distribution of the economy is relevant for interpreting the counterfactual results in the next section, which assess the welfare effects of alternative spatial configurations of those credits. It is also relevant for policy-makers who wish to quantify the effects of R&D policy.

To answer this question, I leverage the variation of R&D tax credits over time and measure how well the model can predict the spatial dispersion of the economy in the years when the spatial distribution of R&D tax credits differed from what it is today. I focus on the spatial distribution of the population of inventors and of patents filed in the decades of 1970-1979, 1989-1989 and 1990-1999. Using averages across longer periods has two advantages. First, the equilibrium of the model assumes a BGP, so its predictions do not apply to short-term variations. Second, these three decades roughly coincide with broad trends in the adoption of R&D subsidies: in the 1970's, there were no subsidies; in the 1980's, there was a spatially uniform federal subsidy, plus a few states offering subsidies of their own; in the 1990's, this policy had already been adopted by most states.

I construct the model-implied distribution of inventors and patents per city in any given year by simply providing the model with the value of the R&D tax credit rates in that year (keeping all other parameters fixed). Using those credit rates and the parameters estimated above, I construct the share of inventors and patents produced in each city for all years, then calculate their averages for each decade. Panel A of table 5 shows the correlations between model outcomes and the data for each decade. For better visualization, I again aggregate cities according to quintiles of the city size distribution (where cities are ordered according to their population of inventors) and report the model-predicted and observed share of inventors and patents in each of those bins.

Table 5: Spatial Distribution of Inventors and Patents Over Time

PANEL A: COMPARISON IN LEVELS

| Bin | Share of Inventors | | | | | | Share of Patents | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1970's | | 1980's | | 1990's | | 1970's | | 1980's | | 1900's | |
| | (model) | (data) | (model) | (data) | (model) | (data) | (model) | (data) | (model) | (data) | (model) | (data) |
| 1 | 0.002 | 0.004 | 0.002 | 0.004 | 0.002 | 0.004 | 0.001 | 0.003 | 0.001 | 0.002 | 0.001 | 0.002 |
| 2 | 0.006 | 0.01 | 0.006 | 0.01 | 0.006 | 0.009 | 0.002 | 0.004 | 0.002 | 0.004 | 0.002 | 0.003 |
| 3 | 0.014 | 0.019 | 0.014 | 0.021 | 0.013 | 0.018 | 0.006 | 0.012 | 0.006 | 0.011 | 0.006 | 0.009 |
| 4 | 0.041 | 0.051 | 0.042 | 0.054 | 0.041 | 0.05 | 0.02 | 0.027 | 0.02 | 0.031 | 0.019 | 0.026 |
| 5 | 0.936 | 0.916 | 0.936 | 0.91 | 0.938 | 0.92 | 0.971 | 0.954 | 0.971 | 0.952 | 0.972 | 0.96 |
| Corr. | 0.86 | | 0.92 | | 0.96 | | 0.76 | | 0.84 | | 0.95 | |

PANEL B: REGRESSING DIFFERENCES IN THE DATA ON DIFFERENCES IN THE MODEL

| | $\Delta$ Share of Inventors | | | $\Delta$ Share of Patents | | |
|---|---|---|---|---|---|---|
| | 1970's | 1980's | 1990's | 1970's | 1980's | 1990's |
| Coefficient | 2.27 | 1.78 | 2.15 | 3.27 | 2.77 | 3.16 |
| Std. Error | 0.09 | 0.06 | 0.08 | 0.08 | 0.06 | 0.08 |
| R-squared | 0.39 | 0.45 | 0.44 | 0.64 | 0.68 | 0.64 |
| Observations | 860 | 860 | 860 | 860 | 860 | 860 |

To account for persistence in city size, I also analyze the model- and data-implied *changes* in the shares of inventors and patents in each city across time. To this end, I again compute the average share of inventors and patents produced in each city during the 1970's, 1980's, 1990's and 2000's. Next, I find the difference between those shares in the decades of 1970, 1980 and 1990 relative to their value in 2000. Figure A.4 shows the correlation between model and data outcomes in each decade, which hover around $0.6$ for changes in the share of inventors and $0.8$ for changes in the share of patents filed. Panel B of table 5 displays the output of regressing the changes in shares observed in the data on its counterpart in the model, where only the value of the R&D tax credit is allowed to vary.

My results suggest that R&D tax credits are quite relevant for the location decisions of inventors/firms and the production of innovation. The R-squared of the regressions in table 5 show that changes in the R&D tax credit rate can explain about $40\%$ of the variation of changes in population shares over time and over $60\%$ of the variation of changes in the production of patents. The correlations in Panel A of that same table indicate that those changes often go in the direction predicted by the model.[41]

**R&D Tax Credits vs Corporate and Labor Income Taxes.** Previous research has linked changes in corporate and labor income taxes to changes in the quantity and location of innovation (Akcigit et al., 2020). As a result, it is important to check whether R&D tax credit rates have moved in tandem with those taxes – which would mean that the results above might not be related to R&D

[41] Keep in mind that I analyze the effects of *effective* R&D tax credit rates, which can be affected by changes in corporate income taxes through the recapturing of tax credits (see Wilson, 2009).

tax credits themselves, but the effects of changes in other local policies.

To test this possibility, I calculate, for each state in the US, the year-over-year change in (1) the statutory R&D tax credit rate, (2) the state corporate income tax rate, and (3) the state labor income tax rate for the top income bracket.[42] Data for all series is available between 1977 and 2006. Across all states, the correlation between changes in the R&D tax credit rate and either corporate or labor income taxes is very small ($-0.01$ and $-0.03$, respectively), and not statistically significant at the $10\%$ level. Different specification of a regression of R&D tax credits on both corporate and labor income taxes all yield the same result: coefficients that are small and not statistically significant.[43]

## 5 The Welfare Effects of Spatial Policies

I now turn to the main question motivating this study: can a redistribution of local R&D subsidies increase aggregate welfare in the economy? The answer to this question has two parts. First, I compare the current spatial distribution of R&D subsidies in the US with a spatially homogeneous subsidy that is implemented with the same amount or resources. In practice, each state is able to choose its own tax credit level, so the spatial distribution of R&D subsidies US can be understood as the outcome of a non-cooperative game played by policy makers in each state. Comparing this decentralized outcome with a spatially neutral subsidy informs us about the welfare gains of allowing states to compete by choosing R&D policy.

Second, I compare the aggregate welfare level under the current distribution of R&D subsidies with the theoretical maximum welfare level that is obtained by solving a central planner's problem. Specifically, I assume the existence of a government that is able to choose the value of all local R&D subsidies in order to maximize welfare. The government's problem highlights some of the main tradeoffs associated with changing the spatial distribution of agents in the economy. I compute an approximate solution for this problem, which produces a set of "optimal" R&D subsidies. This approximate solution is then used to measure the potential welfare gains from the redistribution of local R&D subsidies in the US and to inform us about which places should benefit from R&D policy.

### 5.1 The Government's Problem

Aggregate welfare in this model is measured as the sum of the utility of all workers in the economy, since all firm- and land-owners are fully taxed. For convenience, I assume that the cost of producing $\bar{G}$ units of the public good is $\gamma(\bar{G}) = \bar{\pi}\bar{G}Q(t)$, and that the production of this good is fixed throughout all counterfactuals. I also define $\Pi(t)$ be the aggregate flow of profits by all firms in the economy in period $t$. The government's problem is

[42] Data for the first two series is available through the replication package for Wilson (2009), while state income taxes are obtained through NBER's TAXSIM model (`https://users.nber.org/~taxsim/state-rates/`).

[43] I have tried regressing changes in the statutory R&D tax credit on changes on corporate/labor income taxes (both individually and jointly), including lags and leads of each tax, and specifications using levels (instead of differences) while including different combinations of year and state fixed-effects. I have also repeated those regressions using adjusted (for recapture) R&D tax credit rates, instead of statutory tax credit rates. All regressions point to the same conclusion.

$$\max_{\{s_c\}_{c=1}^{C}} \quad \int_0^\infty e^{-\rho t}\left\{\sum_{c=0}^{C}\left[L_c(t)u^\ell(t)+I_c(t)u^i(t)\right]\right\}\bar{G}dt$$

$$\text{s.t.} \quad \int_0^\infty e^{-rt}\left[\sum_{c=1}^{C} s_c w_c^i(t)I_c(t)+\gamma(\bar{G})\right]dt=\int_0^\infty e^{-rt}\left[p_{m,0}(t)\bar{m}_0+\sum_{c=1}^{C}p_{m,c}(t)\bar{m}_c+\Pi(t)\right]dt.$$

Note that the population of inventors and production workers, as well as their utility, wages, the rate of creative destruction, land prices and profits are all endogenously defined in the model. Using the expressions for these variables obtained in the model's equilibrium and defining $\bar{w}^i = w^i(t)/[\bar{\pi}Q(t)]$ to be the normalized (static) baseline wage of inventors, the government's problem can be reduced to a static one (see appendix E):

$$\max_{\{s_c\}_{c=1}^{C}} \quad (L-L_0(\mathbf{s}))^{\theta\beta}\left(1+\theta\beta\frac{L_0(\mathbf{s})}{L-L_0(\mathbf{s})}\right)\frac{\bar{w}^i(\mathbf{s})^{1-\theta}}{\rho-(1-\theta)\lambda D(\mathbf{s})}$$

$$\text{s.t.} \quad e^{\frac{1-\theta\beta}{\Theta}\frac{(1-\beta)\theta}{\Theta}\frac{\sigma^2}{4\phi}}\frac{1}{C}\sum_{c=1}^{C}s_c\left(\frac{1}{\alpha_c}\right)^{\frac{\theta}{1-\theta}}\tilde{\bar{I}}_c(\mathbf{s})^{\frac{1-\theta\beta}{1-\theta}}+\frac{\bar{G}}{\bar{w}^i(\mathbf{s})}=\frac{r-\lambda D(\mathbf{s})}{r+D(\mathbf{s})}\frac{L_0(\mathbf{s})}{\bar{w}^i(\mathbf{s})}+\frac{(1-\beta)\theta}{1-\theta\beta}\times \tag{18}$$

$$\left[\left(\frac{1}{\alpha_0}\right)^{\frac{\theta}{1-\theta}}\left(\theta\beta I\frac{L_0(\mathbf{s})}{L-L_0(\mathbf{s})}\right)^{\frac{1-\theta\beta}{1-\theta}}+e^{\frac{1-\theta\beta}{\Theta}\frac{(1-\beta)\theta}{\Theta}\frac{\sigma^2}{4\phi}}\frac{1}{C}\sum_{c=1}^{C}\left(\frac{1}{\alpha_c}\right)^{\frac{\theta}{1-\theta}}\tilde{\bar{I}}_c(\mathbf{s})^{\frac{1-\theta\beta}{1-\theta}}\right],$$

where I use the notation $\bar{w}^i(\mathbf{s})$ to indicate that wages are a function of the vector of R&D subsidies $\mathbf{s}=(s_1,\ldots,s_C)$ – the same applies to $L_0(\mathbf{s})$, $\tilde{\bar{I}}_c(\mathbf{s})$ and $D(\mathbf{s})$. In the version of the problem shown in (18), it becomes clear that changing the value of the R&D subsidy affects the government's problem through its effect on (1) the population distribution, $\tilde{\bar{I}}_c$ and $L_0$; (2) wages, $\bar{w}^i$; (3) the rate of creative destruction, $D$ (and the rate of growth, $\lambda D$); and (4) the direct effect over expenditures.

The term $\frac{\bar{w}^i(\mathbf{s})^{1-\theta}}{\rho-(1-\theta)\lambda D(\mathbf{s})}$ highlights one important tradeoff in the government's problem. On the one hand, this term increases with the rate of creative destruction, $D$: a higher rate of creative destruction means that the economy grows at a higher rate, which therefore implies a higher present value of welfare. Furthermore, from corollary 1, $D \propto \frac{1}{C}\sum_{c=1}^{C}\bar{\chi}_c\tilde{\bar{I}}_c^{1+\psi\eta}$ which means that a more spatially concentrated population leads to a higher rate of innovation – especially if the population is concentrated on cities with a large $\bar{\chi}_c$. On the other hand, the normalized wage $\bar{w}^i$ decreases with the rate of creative destruction (see equation 12 in proposition 2). Intuitively, when the rate of creative destruction increases, so does the rate at which firms discount the future, $r+D$, because the probability that any of the firm's product lines will be stolen by a competitor increases. This leads firms to decrease investments in R&D, which reduces the demand for inventors and pushes their wages down (the same happens for production workers through general equilibrium effects).[44] Lower wages then result in lower welfare.

[44]Note that it is the *baseline* wage of inventors that falls. This statement is not necessarily true for the actual wage that inventors receive, as it also depends on congestion costs in the city where each inventor lives.

## 5.2 A Spatially Homogeneous Subsidy

The current spatial distribution of R&D subsidies in the US can be understood as the outcome of the competition among states to attract innovative firms and inventors into their jurisdiction. To evaluate the effects of this competition, consider a counterfactual economy where states are not allowed to compete, so that R&D subsidies are fixed over space $s_c \equiv \bar{s}$ for all $c$. Since taxes and other government expenditures are kept constant throughout all counterfactuals, $\bar{s}$ is fully determined by the government's budget constraint. Under the parameter values found in the previous section, this subsidy rate is close to $19\%$ (the average subsidy rate under the current distribution is about $16\%$).

Moving to a spatially homogeneous subsidy spreads the population of inventors more evenly over space: the HHI index of the city population shares moves from $0.027$ to $0.025$. Under this alternative population distribution, aggregate welfare falls by $0.77\%$ due to a decrease in the growth rate of the economy of approximately $0.03$ percentage points. In contrast, the static baseline wage $\bar{w}^i$ increases by $0.91\%$. In words, the decentralized adoption of R&D subsidies by states has led to a higher welfare level than what would be attained under a spatially neutral subsidy that is implemented using the same amount of resources. This suggests that the states that offer the largest R&D tax credits are indeed to ones that are comparatively better at producing innovation (leading to a higher growth rate). In the next section, I ask whether we can do even better by allowing a social planner to choose the value of all local subsidies.

## 5.3 Approximating the Optimal R&D Subsidies

Finding the exact optimal subsidies that solve (18) can be computationally challenging, as this a non-convex problem with 860 choice variables (cities). Therefore, I compute an approximate solution by imposing a functional form to $s_c$:

$$s_c = \begin{cases} \zeta \alpha_c^{\xi} \bar{\chi}_c^{\omega}, & \text{if } \zeta \alpha_c^{\xi} \bar{\chi}_c^{\omega} \leq \tau; \\ \tau, & \text{if } \zeta \alpha_c^{\xi} \bar{\chi}_c^{\omega} > \tau. \end{cases}$$

This functional form is motivated by the fact that cities only differ from each other because of either $\alpha_c$ or $\bar{\chi}_c$ – and therefore any differences in the optimal subsidy across cities will necessarily be driven by differences in these two parameters. Imposing this functional form, however, reduces the government's problem to the choice of a few parameters, instead of the full distribution of subsidies.

I consider three different values for the subsidy cap: $\tau \in \{0.3, 0.4, 0.5\}$. The highest credit rate currently offered in the data (combining state and federal tax credits) coincides with the lowest value of the cap, at about $30\%$. In each case, the parameters $\xi$ and $\omega$ are chosen in the interval $[-5, 10]$ to maximize aggregate welfare. The scale parameter $\zeta$ ensures that the budget constraint is satisfied.

Figure A.5 plots aggregate welfare as a function of $\xi$ and $\omega$ (fixing $\tau = 0.4$) and the resulting optimal subsidy as a function of amenities and local productivity. It is clear from panel (b) that the welfare is maximized when innovation is concentrated in cities with high amenities and high

productivity, so the optimal subsidy rates should move the economy in this direction. This result is intuitive: cities with high productivity produce more innovation per worker, so moving the population to those cities will generate a higher growth rate. Alternatively, workers living in cities with high amenities will accept relatively lower wages, so firms in those cities experience less congestion costs, all else equal.

Figure 2: Optimal R&D Tax Credit Rates per City

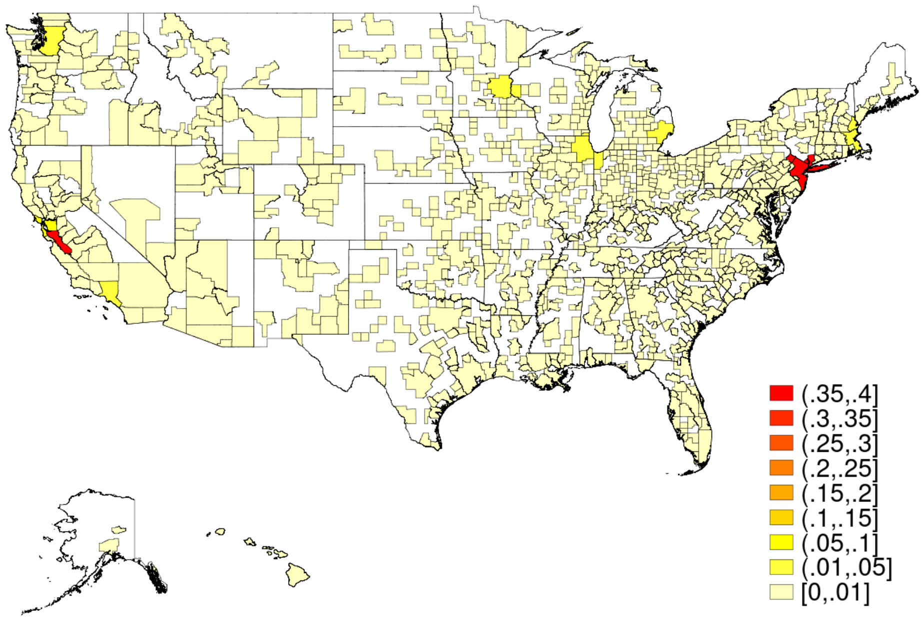



Figure 2 maps the optimal R&D tax credits for each city in the US. In accordance to the discussion above, there are two areas that are heavily subsidized under the optimal policy: the Silicon Valley (San Jose) and New York City, which are already the two largest producers of patents in the country. Figure A.6 shows how the optimal R&D subsidies affect the geographical distribution of inventors and patents produced in each city, relative their current values. Note that a big part of the effect of the optimal subsidies is to move the population from mid-sized cities to a few high productivity/high amenity cities (e.g. San Jose and NYC), dramatically increasing their share of the population and innovation.

The welfare gains from the spatial reallocation of the population caused by the optimal distribution of R&D subsidies is shown in table 6. When capping the city-level subsidy at $50\%$, the model predicts that total welfare would grow by at least $6\%$ if the optimal distribution of R&D subsidies was adopted. This gain is generated in part by an increase of $0.26$ percentage points in the rate of growth of the economy. However, as mentioned above, baseline wages also fall by over $7\%$, indicating that the higher rate of creative destruction has lowered the demand for labor by innovative firms.

Table 6: Gains from adopting optimal subsidies.

Panel A: City-level Subsidies

| Δ Welfare | Δ Baseline Wage | Δ Creative Destruction | Δ Rate of Growth | $\tau$ |
|---|---|---|---|---|
| 2.95% | −3.60% | 0.97 p.p. | 0.13 p.p. | 0.3 |
| 5.23% | −6.38% | 1.70 p.p. | 0.22 p.p. | 0.4 |
| 6.15% | −7.75% | 2.00 p.p. | 0.26 p.p. | 0.5 |

Panel B: State-level Subsidies

| Δ Welfare | Δ Baseline Wage | Δ Creative Destruction | Δ Rate of Growth | $\tau$ |
|---|---|---|---|---|
| 2.50% | −3.65% | 0.88 p.p. | 0.12 p.p. | 0.3 |
| 3.06% | −4.36% | 1.07 p.p. | 0.14 p.p. | 0.4 |
| 3.23% | −4.71% | 1.13 p.p. | 0.15 p.p. | 0.5 |

### 5.3.1 Subsidies by State

The results described above are predicated on the assumption that R&D subsidies can vary by city.[45] In practice, however, these subsidies are chosen at the state level. Taking the geographical scope of the policy as given, I re-run the exercise above while constraining subsidies to be constant within states. To do that, let $c(\mathcal{S})$ indicate a city $c$ that is located in state $\mathcal{S}$. Denote by $C(\mathcal{S})$ the total number of cities in each state and the approximate optimal subsidy by

$$s_{c(\mathcal{S})} = \begin{cases} \zeta \frac{1}{C(\mathcal{S})} \sum_{c=1}^{C(\mathcal{S})} \alpha_c^{\xi} \bar{\chi}_c^{\omega}, & \text{if } \zeta \frac{1}{C(\mathcal{S})} \sum_{c=1}^{C(\mathcal{S})} \alpha_c^{\xi} \bar{\chi}_c^{\omega} \leq \tau; \\ \tau, & \text{if } \zeta \frac{1}{C(\mathcal{S})} \sum_{c=1}^{C(\mathcal{S})} \alpha_c^{\xi} \bar{\chi}_c^{\omega} > \tau. \end{cases}$$

Once again, $\tau \in \{0.3, 0.4, 0.5\}$ and the parameters $\xi$ and $\omega$ are chosen to maximize total welfare in (18). The parameter $\zeta$ ensures the the government's budget constraint is satisfied. Panel B of table 6 shows the welfare effects of the optimal distribution of R&D subsidies across states. Note that the distribution follows the same pattern as above, where a higher spatial concentration of innovation leads to gains in welfare. Those gains are smaller, however, as there are more constraints on the value of the subsidy. Finally, note that this pattern is again robust to different values of the agglomeration and congestion elasticities, as shown by the sensitivity analyzes in appendix F.

Figure 3 shows the optimal value of the subsidy in each state. It is interesting to see that the states that should be subsidized the most are not California and New York – as would be suggested from the results in the previous section–, but California and Idaho! The reason for this difference is that the state of New York has a number of smaller cities that combined produce a sizable share of innovation in the state. Therefore, subsidizing New York state would bring more inventors to NYC, but also to all those smaller cities, not achieving a large concentration of the population.

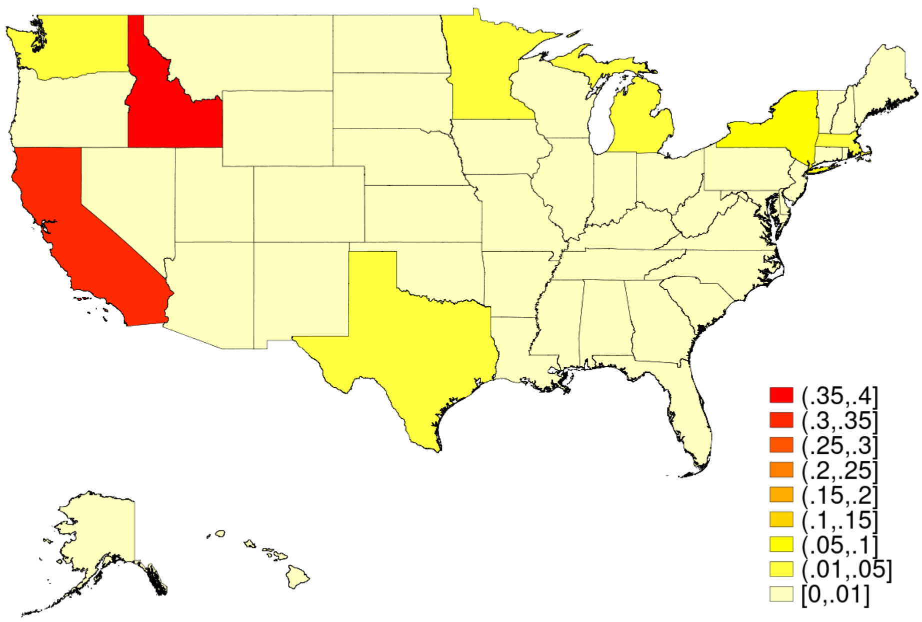
In contrast, innovation in Idaho is much more concentrated in Boise (which is itself an innovation "hub", being the headquarters of companies such as Micron and Hewlett-Packard).[46] As

[45] This is not necessarily an unrealistic assumption. Indeed, the US federal government (as well as state governments) has implemented a number of policies targeted at specific states (e.g., the Tennessee Valley Authority), cities and even neighborhoods (e.g., Urban Enterprise Zones).

[46] In fact, both NYC and Boise are home to about half of the population in their respective states; however, NYC only produces around 5% of patents in the state of New York, while Boise concentrates almost 60% of the patents in Idaho.

a result, the effect of an R&D subsidy in Idaho would be highly focused in that city, leading to a larger concentration of the population in one single place. One important conclusion from this discussion is that the spatial distribution of the optimal R&D subsidy can drastically change depending on the geographical scope of the policy.

Figure 3: Optimal R&D Tax Credit Rates per State




### 5.4 Discussion

There are a few important points to keep in mind when interpreting the results found in this section. First, the welfare gains reported here are the product of a pure redistribution of R&D subsidies over space. Expenditures on those subsidies are kept constant throughout all counterfactual exercises, with the potential exception of endogenous changes in the government's revenue caused by the reallocation of the population. As such, the optimal subsidy rates computed here require no changes in taxation by the government. Second, the gains reported in table 6 are only a lower bound for the increase in welfare that can be obtained by the redistribution of R&D subsidies. This is a direct consequence of imposing a functional form to approximate the optimal subsidies, which does not necessarily describe the policy that maximizes the government's problem.

A third point concerns the limitations of the model used to compute the optimal policies and aggregate welfare. The introduction of moving costs, for example, can have relevant effects on welfare and on the the distribution of the optimal R&D tax credits. In a similar note, short-run adjustment costs (e.g., in investments in R&D) are also ignored in the model, so the results found here should be thought of as long-run responses to changes in policy. Finally, there are many other issues that are relevant for policy makers and can be affected by changes in the spatial distribution of agents in the economy (e.g., income inequality, joblessness); this paper does not address those concerns, as they are outside of the scope its research question.

## 6 Conclusion

This paper assesses whether there are welfare gains from the spatial reallocation of R&D tax credits in the US. As a framework to analyze counterfactual spatial distributions of the tax credit, I construct an endogenous growth model with spatial heterogeneity and agglomeration economies in the production of innovation. This framework contributes to the literature on endogenous growth by nesting a model of growth through creative destruction into a spatial setting. It also contributes to the literature on spatial and dynamic models by developing a tractable model that can be easily matched to micro data.

Qualitatively, I identify an important tradeoff that must be addressed when computing the optimal spatial distribution of R&D subsidies: increasing the geographical concentration innovation in highly productive cities will increase the rate of growth of the economy, but it also increases the rate at which firms discount the future due to a higher rate of creative destruction. This reduces individual firms' investments in R&D, which puts downward pressure on the wages of inventors and decreases aggregate welfare. Quantitatively, I find that concentrating the population of inventors in cities with high amenities and high productivity has positive and potentially large impacts on aggregate welfare. Furthermore, those gains are achieved through a pure redistribution of the R&D subsidy over space, keeping all taxes and other government expenditures constant.

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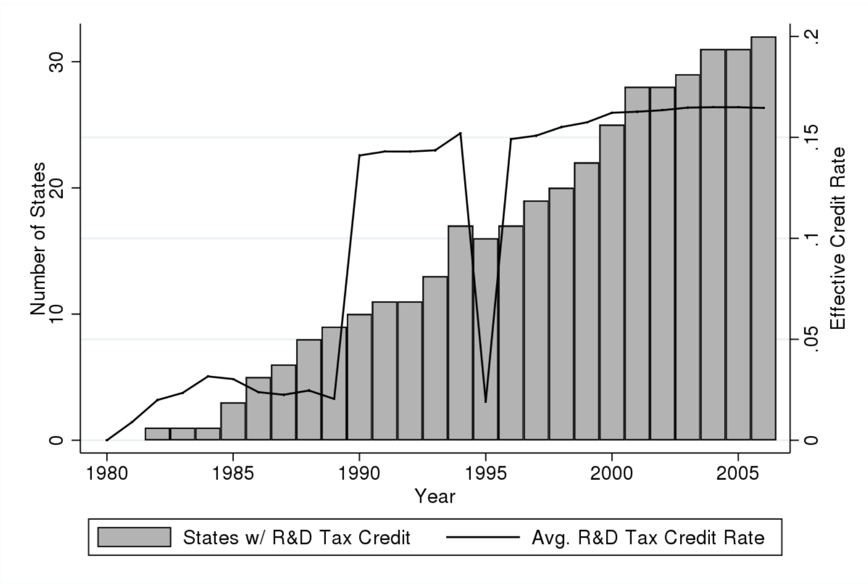
# Appendix

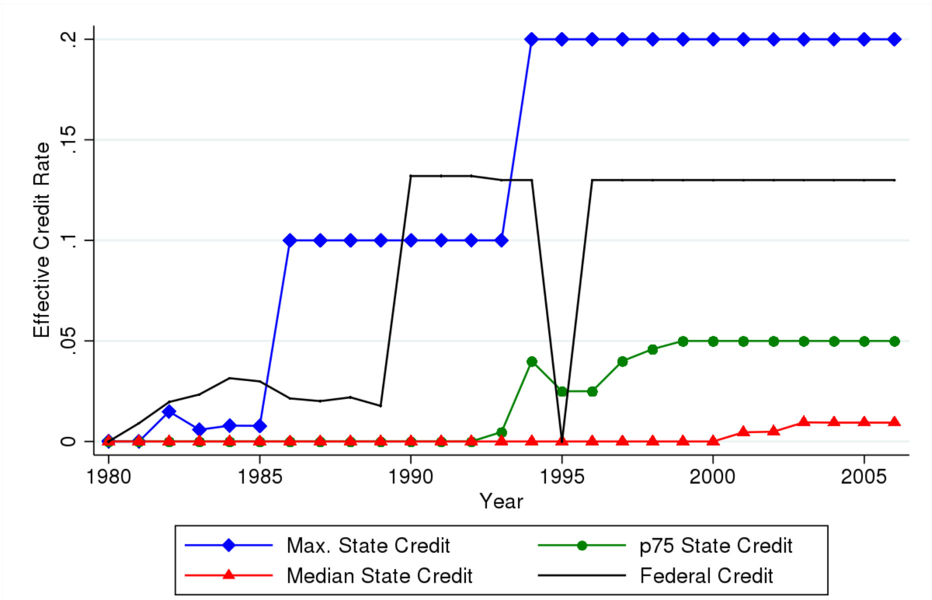
## A Figures

Figure A.1: Spatial Distribution of R&D Tax Credit Rates.



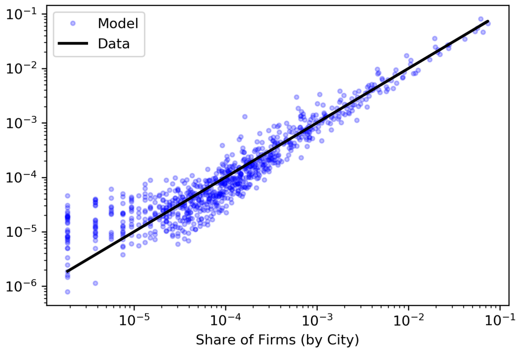

Note: figure shows the average *effective* R&D tax credit rate in the US. See the note in figure 1.

Figure A.2: Spatial Distribution of R&D Tax Credit Rates.

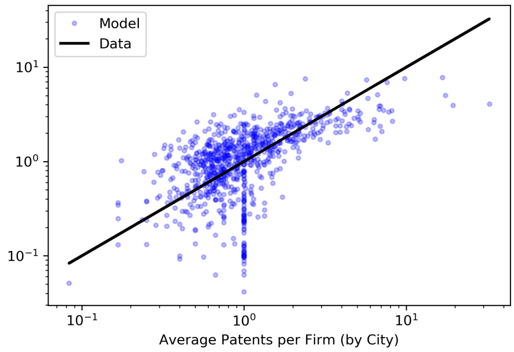



**Note**: figure shows the *effective* R&D tax credit rates in the US. See the note in figure 1. The two discontinuities in the federal credit rate are due to (1) a change in the method for computing the federal base level in 1991 and (2) the fact that there were no federal credits in 1995.

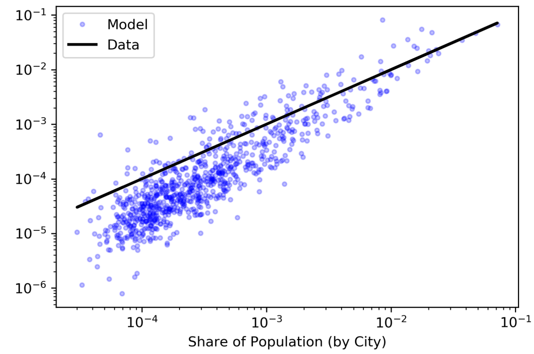
Figure A.3: Distributions in the Model and the Data



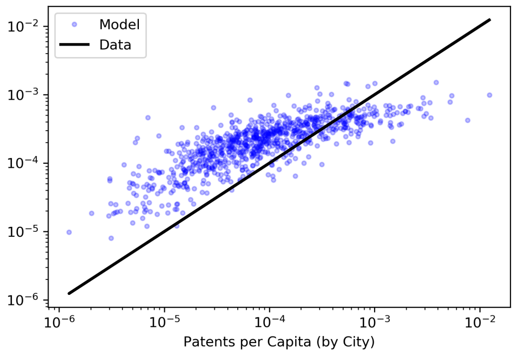

(a) Spatial distribution of firms




(b) Distribution of patents per firm

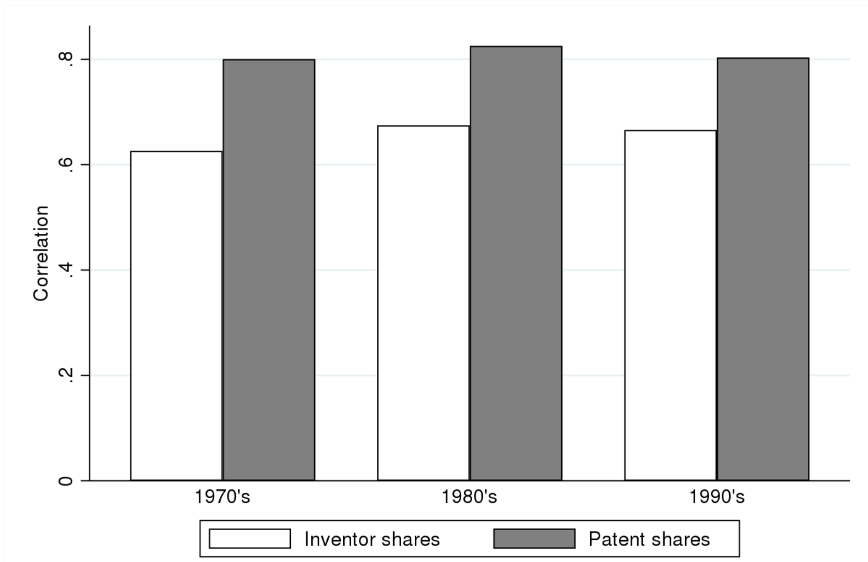



(c) Spatial distribution of employed population



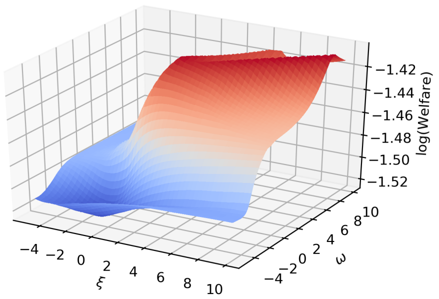

(d) Distribution of patents per capita

Figure A.4: Correlation Between Changes City Population: Model vs Data

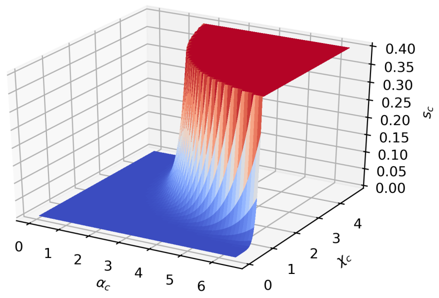



**Note:** The change in city population is defined as the difference between the average population share of the city in a given decade minus the average population share of the same city between 2000 and 2006.

Figure A.5: Results from Welfare Maximization

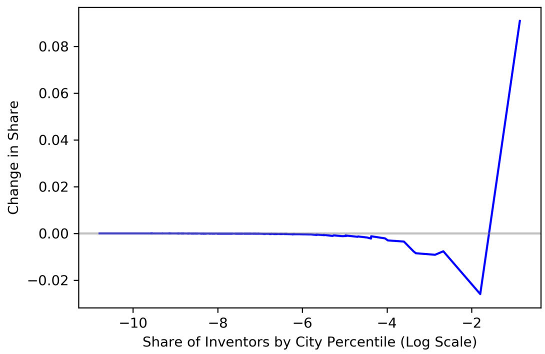



(a) Welfare as a function of $\xi$ and $\omega$.

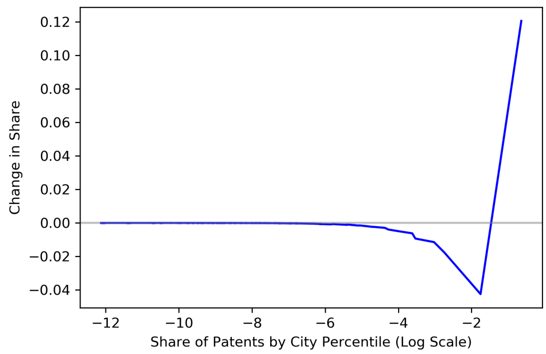



(b) Optimal R&D tax credits/subsidies.

Figure A.6: Changes in the spatial distribution of inventors and innovation




(a) Changes in the distribution of inventors




(b) Changes in the distribution of innovation

The x-axis shows the share of inventors and patents in each percentile of the city distribution (in log scale), where cities are ordered by their current share of inventors (panel a) or patents filed (panel b). The y-axis plots the expected change in those shares should the economy move to the optimal subsidy scheme. For better visualization, each plot aggregates cities into percentiles.

# B Proofs and Derivations

## Proof of Lemma 1

**City 0.** To derive the relationship between population and wages, it is useful to separate city $0$ from cities $1, \ldots, C$. The non-tradable good producer in city $0$ solves the problem

$$\max_{\ell_{n,0},m} p_{n,0}n - w_0^\ell \ell_{n,0} - p_{m,0}m \quad \text{s.t. } n = \ell_{n,0}^\beta m^{1-\beta},$$

where $w_0^\ell$ is determined in equation (8) using the final good producer's problem. The first-order conditions are

$$\begin{aligned}[\ell_{n,0}]: \quad & \beta p_{n,0}\left(\frac{m}{\ell_{n,0}}\right)^{1-\beta} = w_0^\ell \\ [m]: \quad & (1-\beta)p_{n,0}\left(\frac{\ell_{n,0}}{m}\right)^{\beta} = p_{m,0}.\end{aligned}$$

There are three local market clearing conditions (in the sense that they hold inside city $0$). The land market clearing condition is

$$m = m_0$$

since land is a fixed factor. The labor market clearing condition is

$$L_0 = \ell_{y,0} + \ell_{n,0},$$

where $L_0$ is defined as the total population of production workers in city 0. And finally the non-tradable good market clearing condition is

$$\theta\left[\ell_{y,0}\frac{w_0^\ell}{p_{n,0}} + \ell_{n,0}\frac{w_0^\ell}{p_{n,0}}\right] = n$$

where the demand for non-tradable good from each worker is $\theta w_0^\ell/p_{n,0}$, given the familiar Cobb-Douglas utility function of workers.

Using the F.O.C. $[\ell_{n,0}]$ from the non-tradable good producer's problem, the supply of the non-tradable good is

$$n = \frac{w_0^\ell}{p_{n,0}}\frac{\ell_{n,0}}{\beta}.$$

Plugging this into the non-tradable good market clearing condition,

$$\theta\beta[\ell_{y,0} + \ell_{n,0}] = \ell_{n,0} \quad \Longrightarrow \quad \ell_{n,0} = \theta\beta L_0 \quad \text{and} \quad \ell_{y,0} = (1-\theta\beta)L_0.$$

We can also compute the land rent in city $0$ by using the F.O.C. $[m]$ and the land market clearing condition:

$$p_{m,0} = (1-\beta)p_{n,0}\left(\frac{\ell_{n,0}}{m_0}\right)^{\beta}.$$

Plug in $p_{n,0}$ from the F.O.C. $[\ell_{n,0}]$ to find

$$p_{m,0}m_0 = (1-\beta)\theta w_0^\ell L_0.$$

Now turn to the free mobility condition $u_0^\ell = u^\ell$. A production worker's utility is

$$u^\ell = \left[\alpha_0\left(\theta\frac{w_0^\ell}{p_{n,c}}\right)\right]^\theta [(1-\theta)w_0^\ell]^{1-\theta}.$$

Once again, we can plug in the F.O.C. $[\ell_{n,0}]$ and the labor market clearing condition above to find,

$$u^\ell = \left[\alpha_0\theta\beta\left(\frac{m_0}{\theta\beta L_0}\right)^{1-\beta}\right]^\theta [(1-\theta)w_0^\ell]^{1-\theta}.$$

Define $\tilde{L}_0 = L_0/m_0$ as the population per unit of land in city 0. Rearranging the expression above,

$$w_0^\ell = \frac{1}{1-\theta}\left[\frac{u^\ell}{(\theta\beta)^{\theta\beta}}\right]^{\frac{1}{1-\theta}}\left(\frac{\tilde{L}_0^{1-\beta}}{\alpha_0}\right)^{\frac{\theta}{1-\theta}},$$

as desired.

**Cities $1,\ldots,C$.** The process for cities 1 through $C$ is very similar, with the exception that these cities also have a population of inventors. In each city, the non-tradable good producer solves

$$\max_{\ell_{n,c},m_c} p_{n,c}n - w_c^\ell\ell_{n,c} - p_{m,c}m_c \quad \text{s.t. } n = \ell_{n,c}^\beta m_c^{1-\beta}.$$

The first-order conditions are

$$[\ell_{n,c}]: \quad \beta p_{n,c}\left(\frac{m_c}{\ell_{n,c}}\right)^{1-\beta} = w_c^\ell$$
$$[m_c]: \quad (1-\beta)p_{n,c}\left(\frac{\ell_{n,c}}{m_c}\right)^{\beta} = p_{m,c}.$$

Again, there are three local market clearing conditions that must hold in equilibrium. For all $c \in \{1,\ldots,C\}$, the land market clearing condition is

$$m_c = \bar{m}_c;$$

the labor market clearing condition is

$$\ell_{n,c} = L_c;$$

and the goods market clearing condition is

$$\theta\left[L_c\frac{w_c^\ell}{p_{n,c}} + I_c\frac{w_c^i}{p_{n,c}}\right] = n$$

where $L_c$ and $I_c$ are, respectively, the population of production workers and the population of

inventors in city $c$.

Using the F.O.C. $[\ell_{n,c}]$ and the two latter market clearing conditions, we get

$$\frac{L_c}{\bar{m}_c} = \left(\beta \frac{p_{n,c}}{w_c^\ell}\right)^{\frac{1}{1-\beta}}$$

and

$$\theta \left[L_c \frac{w_c^\ell}{p_{n,c}} + I_c \frac{w_c^i}{p_{n,c}}\right] = \frac{w_c^\ell}{p_{n,c}} \frac{L_c}{\beta}.$$

This second equation simplifies to

$$L_c w_c^\ell = \frac{\theta\beta}{1-\theta\beta} I_c w_c^i. \tag{B.1}$$

The utility level for production workers is therefore

$$\begin{aligned} u^\ell &= \left[\alpha_c \theta \frac{w_c^\ell}{p_{n,c}}\right]^\theta \left[(1-\theta) w_c^\ell\right]^{1-\theta} \\ &= \left[\alpha_c \theta \beta \left(\frac{\bar{m}_c}{L_c}\right)^{1-\beta}\right]^\theta \left[(1-\theta) w_c^\ell\right]^{1-\theta}. \end{aligned}$$

Rearranging this expression and using the "tilde" to denote variables expressed by units of land ($\tilde{L}_c = L_c/\bar{m}_c$), production worker's wages are

$$w_c^\ell = w^\ell \left(\frac{\tilde{L}_c^{1-\beta}}{\alpha_c}\right)^{\frac{\theta}{1-\theta}} \quad \text{where} \quad w^\ell = \frac{1}{1-\theta}\left[\frac{u^\ell}{(\theta\beta)^\theta}\right]^{\frac{1}{1-\theta}}. \tag{B.2}$$

To find $w_c^i$, and rewrite equation (B.1) as (recall that "tildes" indicate variables per unit of land, $\tilde{I}_c = I_c/\bar{m}_c$)

$$\tilde{L}_c \frac{w_c^\ell}{p_{n,c}} = \frac{\theta\beta}{1-\theta\beta} \tilde{I}_c \frac{w_c^i}{p_{n,c}}.$$

Using $\frac{p_{n,c}}{w_c^\ell} = \frac{\tilde{L}_c^{1-\beta}}{\beta}$, we get

$$\tilde{L}_c = \left[\frac{\theta}{1-\theta\beta} \tilde{I}_c \frac{w_c^i}{p_{n,c}}\right]^{\frac{1}{\beta}}.$$

Now plug this and (B.2) into (B.1) to find

$$\frac{w_c^i}{p_{n,c}} = \frac{1-\theta\beta}{\theta} \left(\frac{1}{w^\ell} \frac{\theta\beta}{1-\theta\beta} w_c^i\right)^{\frac{\beta(1-\theta)}{1-\theta\beta}} \alpha_c^{\frac{\theta\beta}{1-\theta\beta}} \tilde{I}_c^{-\frac{1-\beta}{1-\theta\beta}}.$$

The utility of inventors is thus

$$\begin{aligned} u^i &= \left[\alpha_c \theta \frac{w_c^i}{p_{n,c}}\right]^\theta \left[(1-\theta) w_c^i\right]^{1-\theta} \\ &= \left[\alpha_c (1-\theta\beta) \left(\frac{1}{w^\ell} \frac{\theta\beta}{1-\theta\beta} w_c^i\right)^{\frac{\beta(1-\theta)}{1-\theta\beta}} \alpha_c^{\frac{\theta\beta}{1-\theta\beta}} \tilde{I}_c^{-\frac{1-\beta}{1-\theta\beta}}\right]^\theta \left[(1-\theta) w_c^i\right]^{1-\theta}. \end{aligned}$$

Rearranging,

$$w_c^i = w^i \left(\frac{\tilde{I}_c^{1-\beta}}{\alpha_c}\right)^{\frac{\theta}{1-\theta}} \quad \text{where} \quad w^i = \left\{\frac{u^i}{(1-\theta)^{1-\theta}\left[(1-\theta\beta)\left(\frac{1}{w^\ell}\frac{\theta\beta}{1-\theta\beta}\right)^{\frac{\beta(1-\theta)}{1-\theta\beta}}\right]^\theta}\right\}^{\frac{1-\theta\beta}{1-\theta}}. \tag{B.3}$$

Finally, going back to (B.1) and plugging in (B.2) and (B.3), we get

$$L_c = \left(\frac{\theta\beta}{1-\theta\beta}\frac{w^i}{w^\ell}\right)^{\frac{1-\theta}{1-\theta\beta}} I_c$$

for cities $c \in \{1, \ldots, C\}$. Summing over cities where there is innovation and using that $I = \sum_{c=1}^{C} I_c$ and $L = L_0 + \sum_{c=1}^{C} L_c$, it follows that

$$w^\ell = \left(\frac{I}{L-L_0}\right)^{\frac{1-\theta\beta}{1-\theta}} \frac{\theta\beta}{1-\theta\beta} w^i. \tag{B.4}$$

Finally, plug (B.4) into (B.3) to find

$$w^i = \frac{1}{1-\theta}\left[\frac{u^i}{[(1-\theta\beta)]^\theta}\right]^{\frac{1}{1-\theta}} \left(\frac{I}{L-L_0}\right)^{\frac{\theta\beta}{1-\theta}}.$$

Also note that plugging (B.2) and (B.3) into (B.1) and summing over $c \in \{1, \ldots, C\}$ implies that the number of inventors and production workers is proportional in those cities:

$$\frac{I_c}{I} = \frac{L_c}{L-L_0}.$$

Finally, we can find land rents in each city by plugging in the land market clearing condition and the F.O.C. $[\ell_{n,c}]$ into the F.O.C. $[m_c]$:

$$p_{m,c}\bar{m}_c = \frac{1-\beta}{\beta} w_c^\ell L_c.$$

It is convenient to write this expression in terms of the population and wage of inventors in each

city. Using equation (B.1), we have

$$p_{m,c}\bar{m}_c = \frac{(1-\beta)\theta}{1-\theta\beta} w_c^i I_c.$$

$\square$

**Proof of Lemma 2**

As described in the main text, the firm's HJB equation is

$$rV_c(\mathbf{q}_f, \tilde{I}_c, Z_c, A) = \max_{x_{f,c}} \left\{ \begin{array}{l} \sum_{q_j \in \mathbf{q}_f} \bar{\pi} L_0 q_j + x_{f,c}\mathbb{E}_j[V_c(\mathbf{q}_f \cup_+ \{(1+\lambda)q_j\}, \tilde{I}_c, Z_c, A) - V_c(\mathbf{q}_f, \tilde{I}_c, Z_c, A)] \\ \\ -D \sum_{q_j \in \mathbf{q}_f} [V_c(\mathbf{q}_f, \tilde{I}_c, Z_c, A) - V_c(\mathbf{q}_f \backslash_- \{q_j\}, \tilde{I}_c, Z_c, A)] \\ \\ -(1-s_c)w_c^i(i_{f,c}+\kappa) + \dfrac{\mathbb{E}[dV_c(\mathbf{q}_f, \tilde{I}_c, Z_c, A)]}{dt} \end{array} \right\}$$

$$\text{s.t. } x_{f,c} = \bar{\chi}_c Z_c (\tilde{I}_c^{\eta} i_{f,c})^{\psi}$$

where I have defined $Z_c = e^{z_c}$ as the local productivity shock. Since $z_c$ is an Ornstein-Uhlenbeck process with law of motion $dz_c = \phi(\mu - z_c)dt + \sigma dW_c(t)$, it follows that

$$dZ_c = \phi\left(\frac{\sigma^2}{4\phi} - \ln(Z_c)\right) Z_c dt + \sigma Z_c dW_c(t)$$

by application of Itô's lemma and using $\mu = -\frac{\sigma^2}{4\phi}$.

To prove lemma 2, we only need to determine $dV_c(\mathbf{q}_f, \tilde{I}_c, Z_c, A)$. This can be done by applying Itô's lemma to the the firm's value function $V_c$, while taking into account that one of the state variables – the population of inventors per land in the city $\tilde{I}_c$ – is a function of the shock $Z_c$. For each city $c \geq 1$, define a function $h_c : \mathbb{R}_+ \times \mathbb{R}_+^2 \times [0,1] \times [0,L] \to [0, I/\bar{m}_c]$ such that $\tilde{I}_c = h_c(Z_c; A)$ (recall that $A = (Q, w^i, D, L_0) \in \mathbb{R}_+^2 \times [0,1] \times [0,L]$). Itô's lemma implies that

$$dh_c = \frac{\partial h_c}{\partial A}\frac{\partial A}{\partial t}dt + \left[\phi\left(\frac{\sigma^2}{4\phi} - \ln(Z_c)\right) Z_c \frac{\partial h_c}{\partial Z_c} + \frac{(\sigma Z_c)^2}{2}\frac{\partial^2 h_c}{\partial Z_c^2}\right] dt + \sigma Z_c \frac{\partial h_c}{\partial Z_c} dW_c(t),$$

where the arguments of the function are suppressed for convenience in the notation. Note that the first term in the equation above is the regular derivative of $h_c$ w.r.t. the vector representing the aggregate state of the economy, while the remaining two terms involve differentiating w.r.t. the stochastic process $Z_c$.

Given the process for $h_c$, we use Itô's lemma once again to differentiate $V_c(\mathbf{q}_f, h_c(Z_c; A), Z_c, A)$

with respect to time:

$$
\begin{aligned}
dV_c &= \frac{\partial V_c}{\partial A}\frac{\partial A}{\partial t}dt + \frac{\partial V_c}{\partial Z_c}dZ_c + \frac{\partial V_c}{\partial h_c}dh_c + \frac{1}{2}\left[\frac{\partial^2 V_c}{\partial Z_c^2}(dZ_c)^2 + \frac{\partial^2 V_c}{\partial h_c^2}(dh_c)^2 + 2\frac{\partial^2 V_c}{\partial Z_c \partial h_c}dZ_c dh_c\right] \\
&= \frac{\partial V_c}{\partial A}\frac{\partial A}{\partial t}dt + \frac{\partial V_c}{\partial Z_c}\left[\phi\left(\frac{\sigma^2}{4\phi} - \ln(Z_c)\right)Z_c dt + \sigma Z_c dW_c(t)\right] \\
&\quad + \frac{\partial V_c}{\partial h_c}\left\{\left[\phi\left(\frac{\sigma^2}{4\phi} - \ln(Z_c)\right)Z_c\frac{\partial h_c}{\partial Z_c} + \frac{(\sigma Z_c)^2}{2}\frac{\partial^2 h_c}{\partial Z_c^2}\right]dt + \sigma Z_c\frac{\partial h_c}{\partial Z_c}dW_c(t) + \frac{\partial h_c}{\partial A}\frac{\partial A}{\partial t}dt\right\} \\
&\quad + \frac{1}{2}\left[\frac{\partial^2 V_c}{\partial Z_c^2}(\sigma Z_c)^2 + \frac{\partial^2 V_c}{\partial h_c^2}\left(\sigma Z_c\frac{\partial h_c}{\partial Z_c}\right)^2 + 2\frac{\partial^2 V_c}{\partial Z_c \partial h_c}(\sigma Z_c)^2\frac{\partial h_c}{\partial Z_c}\right]dt.
\end{aligned}
$$

Recall that $\mathbb{E}[dW] = 0$ and note that $\mathbb{E}[\frac{\partial h_c}{\partial A}\frac{\partial A}{\partial t}] = 0$ because the total number of inventors in the economy is fixed. Thus, taking the expectation and replacing $h_c(Z_c; A) = \tilde{I}_c$,

$$
\begin{aligned}
\frac{\mathbb{E}[dV_c]}{dt} &= \frac{\partial V_c}{\partial A}\frac{\partial A}{\partial t} + \phi\left(\frac{\sigma^2}{4\phi} - \ln(Z_c)\right)Z_c\left[\frac{\partial V_c}{\partial Z_c} + \frac{\partial V_c}{\partial \tilde{I}_c}\frac{\partial \tilde{I}_c}{\partial Z_c}\right] \\
&\quad + \frac{(\sigma Z_c)^2}{2}\left[\frac{\partial^2 V_c}{\partial Z_c^2} + \frac{\partial^2 V_c}{\partial \tilde{I}_c^2}\left(\frac{\partial \tilde{I}_c}{\partial Z_c}\right)^2 + \frac{\partial V_c}{\partial \tilde{I}_c}\frac{\partial^2 \tilde{I}_c}{\partial Z_c^2} + 2\frac{\partial^2 V_c}{\partial Z_c \partial \tilde{I}_c}\frac{\partial \tilde{I}_c}{\partial Z_c}\right].
\end{aligned}
$$

Finally, define $R_c(\mathbf{q}_f, \tilde{I}_c, Z_c, A) = \frac{\mathbb{E}[dV_c]}{dt} - \frac{\partial V_c}{\partial A}\frac{\partial A}{\partial t}$ to be the risk that firms face due to the productivity shock $Z_c$. This concludes the proof.

**A note on corporate income taxes** The HJB equation above does not include any corporate income taxes, even though these taxes change between locations in the data and are one of the sources of revenue for the government (see section 2). The reason for doing this is that taxing a firm's profits will not change any of its decisions in this model, as long as "profits" include the expenditure on R&D.

To see why, note that the firm's HJB equation including corporate taxes $\tau_c^\pi$ is

$$
rv_c(\mathbf{q}_f, \tilde{I}_c, Z_c, A) = \max_{x_{f,c}}
\left\{
\begin{array}{l}
(1-\tau_c^\pi)\left[\sum_{q_j \in \mathbf{q}_f} \bar{\pi} L_0 q_j - (1-s_c)w_c^i(i_{f,c}+\kappa)\right] \\
+x_{f,c}\mathbb{E}_j[v_c(\mathbf{q}_f \cup_+ \{(1+\lambda)q_j\}, \tilde{I}_c, Z_c, A) - v_c(\mathbf{q}_f, \tilde{I}_c, Z_c, A)] \\
-D\sum_{q_j \in \mathbf{q}_f}[v_c(\mathbf{q}_f, \tilde{I}_c, Z_c, A) - v_c(\mathbf{q}_f \backslash_- \{q_j\}, \tilde{I}_c, Z_c, A)] + \dfrac{\mathbb{E}[dv_c(\mathbf{q}_f, \tilde{I}_c, Z_c, A)]}{dt}
\end{array}
\right\}
$$

$$
\text{s.t. } x_{f,c} = \bar{\chi}_c Z_c (\tilde{I}_c^\eta i_{f,c})^\psi.
$$

For any function $v_c$ that satisfies this equation, we can define $V_c = v_c/(1-\tau_c^\pi)$, where $V_c$ is the

solution to the HJB equation shown in the beginning of this proof. The firm's decision of $x_{f,c}$ will thus only depend on the value $V_c$, which means that firms will choose the same amount of investment in R&D regardless of the corporate income tax rate. In addition, the location decisions of firms will also be independent of this tax, since free entry drives the value of entrants to zero (see the discussion preceding proposition 2).[47]

□

### Proof of Proposition 1

I start the proof by solving for the incumbent's value function. To do that, I use a guess ad verify argument. The guess is

$$V_c(\mathbf{q}_f, \tilde{I}_c, Z_c, A) = F \sum_{q_j \in \mathbf{q}_f} q_j + E_c Q,$$

where $F$ and $E_c$ are both functions of the state $(\tilde{I}_c, Z_c, A)$ and $\frac{dF}{dt} = 0$. Substituting this guess along with the constraint into the HJB equation, we find

$$rF \sum_{q_j \in \mathbf{q}_f} q_j + rE_c Q - \frac{\partial E_c}{\partial A}\frac{\partial A}{\partial t} Q - E_c \dot{Q} =$$

$$\max_{x_{f,c}} \left\{ \begin{array}{l} \bar{\pi} L_0 \sum_{q_j \in \mathbf{q}_f} q_j + x_{f,c} F(1+\lambda) Q - D \sum_{q_j \in \mathbf{q}_f} F q_j \\[2ex] -(1-s_c) w_c^i \left( \dfrac{x_{f,c}^{\frac{1}{\psi}}}{(\bar{\chi}_c Z_c)^{\frac{1}{\psi}} \tilde{I}_c^{\eta}} + \kappa \right) \\[2ex] +\phi \left( \dfrac{\sigma^2}{4\phi} - \ln(Z_c) \right) Z_c \left[ \dfrac{\partial E_c}{\partial Z_c} + \dfrac{\partial E_c}{\partial \tilde{I}_c} \dfrac{\partial \tilde{I}_c}{\partial Z_c} \right] Q + \dfrac{(\sigma Z_c)^2}{2} \left[ \dfrac{\partial^2 E_c}{\partial Z_c^2} \right] Q \\[2ex] + \dfrac{(\sigma Z_c)^2}{2} \left[ \dfrac{\partial E_c}{\partial \tilde{I}_c} \dfrac{\partial^2 \tilde{I}_c}{\partial Z_c^2} + \dfrac{\partial^2 E_c}{\partial \tilde{I}_c^2} \left( \dfrac{\partial \tilde{I}_c}{\partial Z_c} \right)^2 + 2 \dfrac{\partial^2 E_c}{\partial \tilde{I}_c \partial Z_c} \dfrac{\partial \tilde{I}_c}{\partial Z_c} \right] Q \end{array} \right\}$$

The first-order condition is

$$[x_{f,c}] : \quad F(1+\lambda)Q - \frac{1}{\psi} \frac{(1-s_c) w_c^i x_{f,c}^{\frac{1-\psi}{\psi}}}{(\bar{\chi}_c Z_c)^{\frac{1}{\psi}} \tilde{I}_c^{\eta}} \leq 0.$$

If the optimal solution is interior,

$$x_{f,c} = \bar{\chi}_c^{\frac{1}{1-\psi}} \left\{ \psi F(1+\lambda) \frac{Q}{w^i} \frac{\alpha_c^{\frac{\theta}{1-\theta}}}{1-s_c} \tilde{I}_c^{\eta - \frac{(1-\beta)\theta}{1-\theta}} \right\}^{\frac{\psi}{1-\psi}} Z_c^{\frac{1}{1-\psi}}$$

where I have used equation (5) to substitute for $w_c^i$. However, firms might prefer not to invest in

[47] Note that an incumbent firm's value will still depend on the corporate tax rate, but the entrant firm's value won't (as it equals zero for all cities).

R&D at all, particularly in periods where $Z_c$ is low. In those cases, the firm can choose a corner solution $x_{f,c} = 0$, which means that $i_{f,c} = 0$ and it does not need to pay the fixed cost $w_c^i \kappa$ as well. It is useful to analyze each case separately.

<u>Interior Solution.</u> I will start by considering an interior solution. Plugging $x_{f,c}$ in the HJB equation, we find

$$rF \sum_{q_j \in \mathbf{q}_f} q_j + rE_c Q - \frac{\partial E_c}{\partial A}\frac{\partial A}{\partial t} Q - E_c \dot{Q} =$$

$$\left\{ \begin{aligned} & \bar{\pi} L_0 \sum_{q_j \in \mathbf{q}_f} q_j - D \sum_{q_j \in \mathbf{q}_f} F q_j + (1-\psi) x_{f,c} F (1+\lambda) Q - (1-s_c)\kappa w^i \left(\frac{\tilde{I}_c^{1-\beta}}{\alpha_c}\right)^{\frac{\theta}{1-\theta}} \\ & +\phi\left(\frac{\sigma^2}{4\phi} - \ln(Z_c)\right) Z_c \left[\frac{\partial E_c}{\partial Z_c} + \frac{\partial E_c}{\partial \tilde{I}_c}\frac{\partial \tilde{I}_c}{\partial Z_c}\right] Q + \frac{(\sigma Z_c)^2}{2}\left[\frac{\partial^2 E_c}{\partial Z_c^2}\right] Q \\ & + \frac{(\sigma Z_c)^2}{2}\left[\frac{\partial E_c}{\partial \tilde{I}_c}\frac{\partial^2 \tilde{I}_c}{\partial Z_c^2} + \frac{\partial^2 E_c}{\partial \tilde{I}_c^2}\left(\frac{\partial \tilde{I}_c}{\partial Z_c}\right)^2 + 2\frac{\partial^2 E_c}{\partial \tilde{I}_c \partial Z_c}\frac{\partial \tilde{I}_c}{\partial Z_c}\right] Q, \end{aligned} \right\}$$

Collecting terms with and without $\sum_{q_j \in \mathbf{q}_f} q_j$,

$$rF = \bar{\pi} L_0 - DF, \tag{B.5}$$

and

$$\begin{aligned} E_c\left(r - \frac{\dot{Q}}{Q}\right) - \frac{\partial E_c}{\partial A}\frac{\partial A}{\partial t} &= (1-\psi) x_{f,c} F (1+\lambda) - (1-s_c)\kappa \frac{w^i}{Q}\left(\frac{\tilde{I}_c^{1-\beta}}{\alpha_c}\right)^{\frac{\theta}{1-\theta}} \\ & +\phi\left(\frac{\sigma^2}{4\phi} - \ln(Z_c)\right) Z_c \left[\frac{\partial E_c}{\partial Z_c} + \frac{\partial E_c}{\partial \tilde{I}_c}\frac{\partial \tilde{I}_c}{\partial Z_c}\right] + \frac{(\sigma Z_c)^2}{2}\left[\frac{\partial^2 E_c}{\partial Z_c^2}\right] \\ & + \frac{(\sigma Z_c)^2}{2}\left[\frac{\partial E_c}{\partial \tilde{I}_c}\frac{\partial^2 \tilde{I}_c}{\partial Z_c^2} + \frac{\partial^2 E_c}{\partial \tilde{I}_c^2}\left(\frac{\partial \tilde{I}_c}{\partial Z_c}\right)^2 + 2\frac{\partial^2 E_c}{\partial \tilde{I}_c \partial Z_c}\frac{\partial \tilde{I}_c}{\partial Z_c}\right]. \end{aligned} \tag{B.6}$$

Equation (B.5) immediately gives us

$$F(D, L_0) = \frac{\bar{\pi} L_0}{r + D}. \tag{B.5′}$$

To show that $\frac{dF}{dt} = 0$, it suffices to prove that $\dot{L}_0 = 0$ and $\dot{D} = 0$. Going back to the final good producer's problem and plugging in the quantity of each intermediate good produced (equation 4) and the number of production workers used in the production of the final good ($\ell_{y,0} = (1 -$

$\theta\beta)L_0)$, we find that

$$Y = \frac{1-\theta\beta}{1-\varepsilon}\left(\frac{1-\varepsilon}{\nu}\right)^{\frac{1-\varepsilon}{\varepsilon}} QL_0 \quad \Longrightarrow \quad \frac{\dot{Y}}{Y} = \frac{\dot{Q}}{Q} + \frac{\dot{L}_0}{L_0}.$$

By definition, $Y$ and $Q$ grow at a constant rate in a SBGP. From the equation above, this implies that $L_0$ must either be constant ($\dot{L}_0 = 0$) or grow at a constant rate as well. However, given that the population of production workers is fixed at $L$, it follows that the only possible rate of growth for $L_0$ is 0. This argument also implies that $Y$ and $Q$ grow at the same rate, so that $\dot{Y}/Y = \dot{Q}/Q = g$. Furthermore, it will be shown in proposition 3 that the rate of growth of the economy in the SBGP is $g = \lambda D$. Since both $g$ and $\lambda$ are constant, it follows that $D$ must be constant as well. In conclusion, both $L_0$ and $D$ are fixed over time, and therefore it follows that the same is true for the franchise value $F(D, L_0)$.

Using $\dot{Q}/Q = g$ and plugging in $x_{f,c}$ from the F.O.C. of the incumbent's problem, (B.6) can be rewritten as

$$\begin{aligned}
(r-g)E_c - \frac{\partial E_c}{\partial A}\frac{\partial A}{\partial t} \quad &= (1-\psi)\bar{\chi}_c[F(1+\lambda)]^{\frac{1}{1-\psi}}\left[\psi\frac{Q}{w^i}\frac{\alpha_c^{\frac{\theta}{1-\theta}}}{1-s_c}\tilde{I}_c^{\frac{\eta(1-\theta)-(1-\beta)\theta}{1-\theta}}\right]^{\frac{\psi}{1-\psi}} Z_c^{\frac{1}{1-\psi}} \\
&\quad -(1-s_c)\kappa\frac{w^i}{Q}\left(\frac{\tilde{I}_c^{1-\beta}}{\alpha_c}\right)^{\frac{\theta}{1-\theta}} + \phi\left(\frac{\sigma^2}{4\phi} - \ln(Z_c)\right)Z_c\left[\frac{\partial E_c}{\partial Z_c} + \frac{\partial E_c}{\partial \tilde{I}_c}\frac{\partial \tilde{I}_c}{\partial Z_c}\right] \\
&\quad + \frac{(\sigma Z_c)^2}{2}\left[\frac{\partial^2 E_c}{\partial Z_c^2} + \frac{\partial E_c}{\partial \tilde{I}_c}\frac{\partial^2 \tilde{I}_c}{\partial Z_c^2} + \frac{\partial^2 E_c}{\partial \tilde{I}_c^2}\left(\frac{\partial \tilde{I}_c}{\partial Z_c}\right)^2 + 2\frac{\partial^2 E_c}{\partial \tilde{I}_c \partial Z_c}\frac{\partial \tilde{I}_c}{\partial Z_c}\right],
\end{aligned} \tag{B.6′}$$

which implicitly defines $E_c = E_c(\tilde{I}_c, Z_c, w^i/Q, D, L_0)$. To understand the requirement that $r > g$, assume for a moment that $Z_c = 1$ for all cities and in all periods. In this case, equation (B.6′) becomes an ordinary differential equation and has an explicit solution:

$$\begin{aligned}
E_c(\tilde{I}_c(t), w^i(t)/Q(t), D, L_0) \quad &- \lim_{\hat{t}\to\infty} E_c(\tilde{I}_c(\hat{t}), w^i(\hat{t})/Q(\hat{t}), D, L_0)e^{-(r-g)\hat{t}} = \\
&\int_t^{\infty} e^{-(r-g)(s-t)}\left[\mathcal{K}_c(s)\tilde{I}_c(s)^{\frac{\eta(1-\theta)-(1-\beta)\theta}{1-\theta}\frac{\psi}{1-\psi}} - \mathcal{M}_c(s)\tilde{I}_c(s)^{\frac{(1-\beta)\theta}{1-\theta}}\right] ds,
\end{aligned}$$

where $\mathcal{K}_c$ and $\mathcal{M}_c$ are the collection of terms multiplying $\tilde{I}_c$ on the first and second lines of (B.6′), respectively. Since the total population of workers is fixed and $w^i(t)/Q(t)$ is stationary (see proposition 3), the requirement that $r > g$ is sufficient for the limit in the equation above to be zero and for the value of the integral to be well defined. Intuitively, if $g > r$, firms will always find it profitable to invest as much as possible in R&D (by, for example, borrowing capital at rate $r$), as the value of doing so grows at rate $g$. This intuitive argument applies to the case with local shocks as well.

<u>Corner Solution.</u> Now let us consider the case of a corner solution. The argument for used to

derive the expression for $F$ remains unchanged; however, equation (B.6) now becomes

$$rE_c - \frac{\partial E_c}{\partial A}\frac{\partial A}{\partial t} - E_c\frac{\dot{Q}}{Q} = \phi\left(\frac{\sigma^2}{4\phi} - \ln(Z_c)\right) Z_c \left[\frac{\partial E_c}{\partial Z_c} + \frac{\partial E_c}{\partial \tilde{I}_c}\frac{\partial \tilde{I}_c}{\partial Z_c}\right] + \frac{(\sigma Z_c)^2}{2}\left[\frac{\partial^2 E_c}{\partial Z_c^2}\right]$$
$$+\frac{(\sigma Z_c)^2}{2}\left[\frac{\partial E_c}{\partial \tilde{I}_c}\frac{\partial^2 \tilde{I}_c}{\partial Z_c^2} + \frac{\partial^2 E_c}{\partial \tilde{I}_c^2}\left(\frac{\partial \tilde{I}_c}{\partial Z_c}\right)^2 + 2\frac{\partial^2 E_c}{\partial \tilde{I}_c \partial Z_c}\frac{\partial \tilde{I}_c}{\partial Z_c}\right]$$

since $x_{f,c} = 0$ and the firm does not have to pay the fixed cost $w_c^i\kappa$. Note that a trivial solution for this equation is $E_c = 0$.

Summarizing both cases, we can conclude that

$$V_c(\mathbf{q}_f, \tilde{I}_c, Z_c, A) = F(D, L_0)\sum_{q_j \in \mathbf{q}_f} q_j + \max\left\{0, E_c(\tilde{I}_c, Z_c, w^i/Q, D, L_0)Q\right\},$$

where $F$ is given by equation (B.5′) and $E_c$ is given by (B.6′).

**Entrant's Problem.** The second stage of the entrant's problem can be solved in the same way, and in particular note that the guess $V_c^e(\tilde{I}_c, Z_c, A) = E_c Q$ implies

$$rE_cQ - \frac{\partial E_c}{\partial A}\frac{\partial A}{\partial t}Q - E_c\dot{Q} =$$
$$\max_{x_{f,c}}\left\{\begin{array}{l} x_{f,c}F(1+\lambda)Q - (1-s_c)w_c^i\left(\dfrac{x_{f,c}^{\frac{1}{\psi}}}{(\bar{\chi}_c Z_c)^{\frac{1}{\psi}}\tilde{I}_c^{\eta}} + \kappa\right) \\ +\phi\left(\dfrac{\sigma^2}{4\phi} - \ln(Z_c)\right) Z_c\left[\dfrac{\partial E_c}{\partial Z_c} + \dfrac{\partial E_c}{\partial \tilde{I}_c}\dfrac{\partial \tilde{I}_c}{\partial Z_c}\right]Q + \dfrac{(\sigma Z_c)^2}{2}\left[\dfrac{\partial^2 E_c}{\partial Z_c^2}\right]Q \\ +\dfrac{(\sigma Z_c)^2}{2}\left[\dfrac{\partial E_c}{\partial \tilde{I}_c}\dfrac{\partial^2 \tilde{I}_c}{\partial Z_c^2} + \dfrac{\partial^2 E_c}{\partial \tilde{I}_c^2}\left(\dfrac{\partial \tilde{I}_c}{\partial Z_c}\right)^2 + 2\dfrac{\partial^2 E_c}{\partial \tilde{I}_c\partial Z_c}\dfrac{\partial \tilde{I}_c}{\partial Z_c}\right]Q \end{array}\right\}$$

The first-order condition for this problem is exactly the same as in the incumbent's problem, so the arrival rates of innovation per product line of all firms is the same. Following the same steps as above, it is straightforward to show that the entrant's HJB equation will satisfy equation (B.6′), verifying the guess. The same holds true in case of a corner solution, where the firm hires no inventors and again $E_c = 0$. Collecting both cases, we have that $V_c^e(\tilde{I}_c, Z_c, A) = \max\left\{0, E_c(\tilde{I}_c, Z_c, w^i/Q, D, L_0)Q\right\}$, with $E_c$ defined exactly as it was for the incumbent firm. □

### Proof of Proposition 2

There are several claims made in proposition 2, so I proceed in order. I start by proving that the ratio $w^i/Q$ does not depend on any of the local shocks $Z_c$, which readily delivers the population of inventors in each city (equation 9). Then, using those results, I derive the expressions for the

number of inventors hired by each firm, the optimal arrival rates of innovation and the number of active firms in each city. Finally, I demonstrate how to find the population of production workers in city 0.

**Wages and the Population of Inventors.** Imposing the free entry condition ($E_c = 0 \; \forall c, t$) into equation (B.6′) gives us

$$0 = (1-\psi)\left[\bar{\chi}_c F(1+\lambda)\right]^{\frac{1}{1-\psi}} \left[\psi \frac{Q}{w^i} \frac{\alpha_c^{\frac{\theta}{1-\theta}}}{1-s_c} \tilde{I}_c^{\frac{\eta(1-\theta)-(1-\beta)\theta}{1-\theta}}\right]^{\frac{\psi}{1-\psi}} Z_c^{\frac{1}{1-\psi}} - (1-s_c)\kappa \frac{w^i}{Q}\left(\frac{\tilde{I}_c^{1-\beta}}{\alpha_c}\right)^{\frac{\theta}{1-\theta}}.$$

Define $\Theta = (1-\beta)\theta - \psi\eta(1-\theta)$ to be the net elasticity of congestion. Solving for $\tilde{I}_c$ in the equation above yields

$$\tilde{I}_c = \left\{\psi^{\psi}\left(\frac{1-\psi}{\kappa}\right)^{1-\psi} \frac{\bar{\chi}_c(1+\lambda)\bar{\pi}L_0}{r+D} \frac{Q}{w^i} \frac{\alpha_c^{\frac{\theta}{1-\theta}}}{1-s_c} Z_c\right\}^{\frac{1-\theta}{\Theta}}. \tag{B.7}$$

Next, I impose that labor markets clear in all periods. The population of inventors in the economy is fixed and equal to $I$; similarly, the population of production workers equals $L$. Since workers are freely mobile and supply one unit of labor inelastically, the labor market clearing conditions are (since there are no inventors in city 0)

$$I = \sum_{c=1}^{C} I_c \quad \text{and} \quad L = L_0 + \sum_{c=1}^{C} L_c.$$

Recall that $\tilde{I}_c = I_c/\bar{m}_c$ and that the land mass in cities $c \in \{1, \dots, C\}$ has been normalized to $\bar{m}_c = 1/C$. Multiplying both sides of equation (B.7) by $\bar{m}_c$ and summing over $c$ yields (after rearranging)

$$\frac{w^i}{Q} = \frac{1}{I^{\frac{\Theta}{1-\theta}}}\psi^{\psi}\left(\frac{1-\psi}{\kappa}\right)^{1-\psi} \frac{(1+\lambda)\bar{\pi}L_0}{r+D}\left\{\frac{1}{C}\sum_{c=1}^{C}\left(\frac{\bar{\chi}_c \alpha_c^{\frac{\theta}{1-\theta}}}{1-s_c} Z_c\right)^{\frac{1-\theta}{\Theta}}\right\}^{\frac{\Theta}{1-\theta}}.$$

Using the expression above, my goal is now to demonstrate that $\frac{w^i}{Q}$ is not a function of the local shocks $z_c$. For each $t$, define $\xi_c(t) = \left[\frac{\bar{\chi}_c \alpha_c^{\frac{\theta}{1-\theta}}}{1-s_c} Z_c(t)\right]^{\frac{1-\theta}{\Theta}}$. Recall that $z_c$ follows an Ornstein-Uhlenbeck process with stationary distribution $z_c \sim \mathcal{N}\left(\mu, \frac{\sigma^2}{2\phi}\right)$. Since $Z_c = e^{z_c}$ and $\mu = -\sigma^2/4\phi$, it follows that $Z_c$ is log-normally distributed with mean 1 and variance $\exp\left(\frac{\sigma^2}{2\phi}\right) - 1$. As a result, $\{\xi_c(t)\}_{c=1}^{C}$ is a sequence of independent random variables, each with finite first and second moments, for all $t \geq 0$. Applying Kolmogorov's Strong Law of Large Numbers (see Shiryaev, 1996, Ch. 4, §3),

$$\frac{1}{C}\sum_{c=1}^{C}\xi_c - \frac{1}{C}\mathbb{E}\left[\sum_{c=1}^{C}\xi_c\right] \xrightarrow{a.s.} 0.$$

To compute the expectation above, note that $(\bar{\chi}_c, \alpha_c, s_c)$ are fixed and $Z_c$ is independent and identically distributed across all $c$. Furthermore, for any $K \neq 0$, we have

$$\mathbb{E}\left[Z_c^K\right] = \mathbb{E}\left[e^{Kz_c}\right] = \frac{1}{\sqrt{\pi\sigma^2/\phi}} \int_{-\infty}^{\infty} e^{Kz - \frac{1}{2}(z-\mu)^2/\left(\frac{\sigma^2}{2\phi}\right)} dz = e^{K\left(\mu + K\frac{\sigma^2}{4\phi}\right)}.$$

Plugging in $K = (1-\theta)/\Theta$ and $\mu = -\sigma^2/4\phi$, we get $\mathbb{E}\left(Z_c^{\frac{1-\theta}{\Theta}}\right) = \exp\left(\frac{1-\theta}{\Theta}\left(\frac{1-\theta}{\Theta} - 1\right)\frac{\sigma^2}{4\phi}\right)$.

Since $C \to \infty$, it follows that wages in each period are, with probability 1,

$$\frac{w^i}{Q} = \frac{1}{I^{\frac{\Theta}{1-\theta}}} \psi^\psi \left(\frac{1-\psi}{\kappa}\right)^{1-\psi} \frac{(1+\lambda)\bar{\pi}L_0}{r+D} \left\{\frac{1}{C}\sum_{c=1}^{C} \left(\frac{\bar{\chi}_c}{1-s_c}\right)^{\frac{1-\theta}{\Theta}} \alpha_c^{\frac{\theta}{\Theta}} e^{\frac{1-\theta}{\Theta}\left(\frac{1-\theta}{\Theta}-1\right)\frac{\sigma^2}{4\phi}}\right\}^{\frac{\Theta}{1-\theta}}. \tag{B.8}$$

In words, the equation above shows that $w^i/Q$ does not depend on the individual realizations of the local shocks. It also implies equation (12) in the proposition's statement. Plugging (B.8) into (B.7), the population of inventors in each city is

$$\tilde{I}_c = I \times \frac{\left(\frac{\bar{\chi}_c}{1-s_c}\right)^{\frac{1-\theta}{\Theta}} \alpha_c^{\frac{\theta}{\Theta}}}{\frac{1}{C}\sum_{c=1}^{C}\left(\frac{\bar{\chi}_c}{1-s_c}\right)^{\frac{1-\theta}{\Theta}} \alpha_c^{\frac{\theta}{\Theta}}} \times \frac{Z_c^{\frac{1-\theta}{\Theta}}}{e^{\frac{1-\theta}{\Theta}\left(\frac{1-\theta}{\Theta}-1\right)\frac{\sigma^2}{4\phi}}}, \tag{B.9}$$

which is equivalent to equation (9).

**Firms, Inventors per Firm and the Arrival Rate of Innovation.** Once wages and the population of inventors is determined in each city, the remaining variables of the model can be readily computed. To begin, simply plug in equation (B.8) into the expression for the optimal arrival rate of innovation (see the proof of proposition 1) to find

$$x_{f,c} = \left[I \frac{\left(\frac{\bar{\chi}_c}{1-s_c}\right)^{\frac{1-\theta}{\Theta}} \alpha_c^{\frac{\theta}{\Theta}}}{\frac{1}{C}\sum_{c=1}^{C}\left(\frac{\bar{\chi}_c}{1-s_c}\right)^{\frac{1-\theta}{\Theta}} \alpha_c^{\frac{\theta}{\Theta}}} \frac{Z_c^{\frac{1-\theta}{\Theta}}}{e^{\frac{1-\theta}{\Theta}\left(\frac{1-\theta}{\Theta}-1\right)\frac{\sigma^2}{4\phi}}}\right]^{\frac{\Theta}{1-\theta}\frac{\psi}{1-\psi}} \psi^\psi \left(\frac{\kappa}{1-\psi}\right)^\psi \bar{\chi}_c Z_c \tilde{I}_c^{\frac{\eta(1-\theta)-(1-\beta)\theta}{1-\theta}\frac{\psi}{1-\psi}}.$$

Now use (B.9) to get

$$x_{f,c} = \psi^\psi \left(\frac{\kappa}{1-\psi}\right)^\psi \bar{\chi}_c Z_c \tilde{I}_c^{\psi\eta}.$$

The number of inventors hired by each firm is found by substituting the expression for $x_{f,c}$ above into the production function of innovation (1):

$$i_{f,c} = \frac{\psi}{1-\psi}\kappa.$$

Finally, we must also require that all inventors living in the city are employed by some firm. Let $N_c(t)$ be the number of firms located in city $c$ who invest in R&D during period $t$. Since all firms located in the same city will hire the same number of inventors (in case they decide to invest in

R&D), we have that

$$I_c = N_c(i_{f,c} + \kappa).$$

Using the expression for $i_{f,c}$, this expression becomes

$$I_c = \left(\frac{\kappa}{1-\psi}\right) N_c,$$

which equation proves (11) after rearranging.

**Population in City 0.** Finally, we can now determined the population of city $0$. To find $L_0$, note that equation (B.8) implies

$$w^i = Q L_0 \underbrace{\frac{1}{I^{\frac{\Theta}{1-\theta}}} \psi^\psi \left(\frac{1-\psi}{\kappa}\right)^{1-\psi} \frac{(1+\lambda)\bar{\pi}}{r+D} \left\{\frac{1}{C}\sum_{c=1}^{C}\left(\frac{\bar{\chi}_c}{1-s_c}\right)^{\frac{1-\theta}{\Theta}} \alpha_c^{\frac{\theta}{\Theta}} e^{\frac{1-\theta}{\Theta}\left(\frac{1-\theta}{\Theta}-1\right)\frac{\sigma^2}{4\phi}}\right\}^{\frac{\Theta}{1-\theta}}}_{\overline{W}^i}.$$

Also, recall that

$$w^\ell = \left(\frac{I}{L-L_0}\right)^{\frac{1-\theta\beta}{1-\theta}} \frac{\theta\beta}{1-\theta\beta} w^i \tag{B.4}$$

and

$$w_0^\ell = w^\ell \left(\frac{\tilde{L}_0^{1-\beta}}{\alpha_0}\right)^{\frac{\theta}{1-\theta}} (\theta\beta)^{\frac{(1-\beta)\theta}{1-\theta}}, \tag{7}$$

where $w_0^\ell = \frac{\varepsilon}{1-\varepsilon}\left(\frac{1-\varepsilon}{\nu}\right)^{\frac{1-\varepsilon}{\varepsilon}} Q$ from equation (8). Combining all of those results,

$$\begin{aligned}
\frac{\varepsilon}{1-\varepsilon}\left(\frac{1-\varepsilon}{\nu}\right)^{\frac{1-\varepsilon}{\varepsilon}} Q &= w_0^\ell \\
&= (\theta\beta)^{\frac{(1-\beta)\theta}{1-\theta}} w^\ell \left(\frac{\tilde{L}_0^{1-\beta}}{\alpha_0}\right)^{\frac{\theta}{1-\theta}} \\
&= (\theta\beta)^{\frac{(1-\beta)\theta}{1-\theta}} \left(\frac{I}{L-L_0}\right)^{\frac{1-\theta\beta}{1-\theta}} \frac{\theta\beta}{1-\theta\beta} w^i \left(\frac{\tilde{L}_0^{1-\beta}}{\alpha_0}\right)^{\frac{\theta}{1-\theta}} \\
&= \left(\frac{\theta\beta I}{L-L_0}\right)^{\frac{1-\theta\beta}{1-\theta}} \frac{1}{1-\theta\beta} Q \overline{W}^i L_0 \left(\frac{\tilde{L}_0^{1-\beta}}{\alpha_0}\right)^{\frac{\theta}{1-\theta}}.
\end{aligned}$$

Using the normalization $\bar{m}_0 = 1$ (so that $\tilde{L}_0 = L_0$) and rearranging this equation gives

$$\begin{aligned}
\left(\tfrac{L_0}{L-L_0}\right)^{\frac{1-\theta\beta}{1-\theta}} &= \frac{\varepsilon}{1-\varepsilon}\left(\frac{1-\varepsilon}{\nu}\right)^{\frac{1-\varepsilon}{\varepsilon}}(1-\theta\beta)\alpha_0^{\frac{\theta}{1-\theta}}\left\{(\theta\beta I)^{\frac{1-\theta\beta}{1-\theta}}\overline{W}^i\right\}^{-1} \\
&= \underbrace{\frac{\alpha_0^{\frac{\theta}{1-\theta}}}{(1-\varepsilon)\psi^\psi\left(\frac{1-\psi}{\kappa}\right)^{1-\psi}\frac{1+\lambda}{r+D}e^{\frac{\sigma^2}{4\phi}\left(\frac{1-\theta}{\Theta}-1\right)}I^{1+\psi\eta}(\theta\beta)^{\frac{1-\theta\beta}{1-\theta}}\left[\frac{1}{C}\sum_{c=1}^{C}\left(\frac{\bar{\chi}_c}{1-s_c}\right)^{\frac{1-\theta}{\Theta}}\alpha_c^{\frac{\theta}{\Theta}}\right]^{\frac{\Theta}{1-\theta}}}}_{\equiv \Lambda^{\frac{1-\theta\beta}{1-\theta}}}
\end{aligned} \tag{B.10}$$

and therefore

$$L_0 = \frac{\Lambda}{1+\Lambda}L.$$

Note that this equation implies that if the rate of creative destruction, $D$, is constant over time, then so is $\Lambda$ and therefore so is $L_0$. Since $D$ is constant in the Balanced Growth Path, it follows that the population in city $L_0$ is fixed and does not react to any of the shocks in other cities as well.

□

### Proof of Corollary 1

To prove this corollary, start with the definition of the rate of creative destruction and plug in the expressions for $N_c$ and $x_{f,c}$ found in proposition 2.

$$\begin{aligned}
D &= \sum_{c=1}^{C} N_c x_{f,c} \\
&= \left(\frac{1-\psi}{\kappa}\right)\sum_{c=1}^{C} I_c x_{f,c} \\
&= \left(\frac{1-\psi}{\kappa}\right)\frac{1}{C}\sum_{c=1}^{C} \tilde{I}_c x_{f,c} \\
&= \psi^\psi\left(\frac{1-\psi}{\kappa}\right)^{1-\psi}\frac{1}{C}\sum_{c=1}^{C}\bar{\chi}_c\tilde{I}_c^{1+\psi\eta}Z_c \\
&= \psi^\psi\left(\frac{1-\psi}{\kappa}\right)^{1-\psi}\frac{1}{C}\sum_{c=1}^{C}\bar{\chi}_c\tilde{\tilde{I}}_c^{1+\psi\eta}\frac{Z_c^{1+(1+\psi\eta)\frac{1-\theta}{\Theta}}}{e^{(1+\psi\eta)\frac{1-\theta}{\Theta}\left(\frac{1-\theta}{\Theta}-1\right)\frac{\sigma^2}{4\phi}}}
\end{aligned}$$

where the third equality uses the fact that $\tilde{I}_c/C = I_c$ and the last equality uses the expression for $\tilde{I}_c$.

Once again, since the number of cities is large ($C \to \infty$), the Law of Large Numbers applies and the average above converges almost surely to its expected value. Therefore, with probability one,

$$D = \psi^\psi\left(\frac{1-\psi}{\kappa}\right)^{1-\psi} e^{\frac{(1-\theta)(1+\psi\eta)}{\Theta}\left(\frac{(1-\beta)\theta}{\Theta}+1\right)\frac{\sigma^2}{4\phi}}\frac{1}{C}\sum_{c=1}^{C}\bar{\chi}_c\tilde{\tilde{I}}_c^{1+\psi\eta}$$

where the expectation of $Z_c^{1+(1+\psi\eta)\frac{1-\theta}{\Theta}}$ is found using the relationship $\mathbb{E}\left[Z_c^K\right] = e^{K\left(\mu+K\frac{\sigma^2}{4\phi}\right)}$ found

in proposition 2, using $K = 1 + (1+\psi\eta)\frac{1-\theta}{\Theta} = \frac{1-\theta\beta}{\Theta}$. □

**Proof of Proposition 3**

(1) Before I begin the proof, recall from corollary 1 that the rate of creative destruction is fixed over time. As a result, $L_0 = L\Lambda/(1+\Lambda)$ is also constant, since $\Lambda$ does not change between periods (see proposition 2).

The total production of final goods is given by

$$Y = \frac{\ell_{y,0}^{\varepsilon}}{1-\varepsilon}\int_{\mathcal{J}} q_j^{\varepsilon} k_j^{1-\varepsilon} dj$$

where $\ell_{y,0} = (1-\theta\beta)L_0$. By plugging in $k_j$ from equation (4) and doing some algebra,

$$Y = \frac{1-\theta\beta}{1-\varepsilon}\left(\frac{1-\varepsilon}{\nu}\right)^{\frac{1-\varepsilon}{\varepsilon}} L_0 Q.$$

Thus,

$$\frac{\dot{Y}}{Y} = \frac{\dot{Q}}{Q}.$$

Since, by definition, $\dot{Y}/Y = g$, it follows that $\dot{Q}/Q = g$.

To prove that $Q$ and $w^i$ grow at the same rate, it suffices to look at equation (B.8) and realize that the RHS of that equation is fixed over time. Thus

$$d\left(\frac{w^i}{Q}\right)/dt = 0 \Longrightarrow \frac{\dot{w}^i}{w^i} = \frac{\dot{Q}}{Q}.$$

Furthermore, from equation (B.4) it is evident that $w^\ell$ is proportional to $w^i$, which means that $w^\ell$ must also grow at rate $g$. Finally, equations (6) and (5) show that $u^\ell \propto (w^\ell)^{1-\theta}$ and $u^i \propto (w^i)^{1-\theta}$. From the results above, it follows that both $u^\ell$ and $u^i$ grow at rate $(1-\theta)g$.

(2) To show that $g = \lambda D$, one can analyze the dynamics of $Q$. For a small interval of time $\Delta$,

$$Q(t+\Delta) = Q(t) + \sum_{c=1}^{C} N_c(t)(\Delta x_{f,c}(t)) \times \lambda Q(t)$$

where $\Delta x_{f,c}(t)$ is the probability that an innovation will be produced by firm $f$ in city $c$ between $t$ and $t+\Delta$ and $\lambda Q(t)$ is the aggregate expected quality gain from that innovation. Rearranging and taking the limit as $\Delta \to 0$, we get

$$\lim_{\Delta\to 0}\frac{Q(t+\Delta)-Q(t)}{\Delta} = \lambda Q(t)\sum_{c=1}^{C} N_c(t) x_{f,c}(t)$$

$$\frac{\dot{Q}}{Q} = \lambda \sum_{c=1}^{C} N_c x_{f,c}.$$

Since $\dot{Q}/Q = g$ and $\sum_{c=1}^{C} N_c x_{f,c} = D$, the result follows. Note that because $D$ and $\lambda$ are constant, the rate of growth $g$ must also be constant over time.

(3) Once again, this claim can be proved by analyzing the dynamics of $Q_c$. For a small interval of time $\Delta$,

$$Q_c(t+\Delta) = Q_c(t) + N_c(t)(\Delta x_{f,c}(t)) \times (1+\lambda)Q - (\Delta D)Q_c(t).$$

Taking the limit as $\Delta \to 0$,

$$\dot{Q}_c = (1+\lambda)QN_c x_{f,c} - DQ_c.$$

Rearranging this expression and multiplying both sides by $e^{Dt}$, we have that

$$\frac{d}{dt}e^{Dt}Q_c = (1+\lambda)e^{Dt}QN_c x_{f,c}.$$

Integrating both sides in $t \in [0,T]$,

$$e^{DT}Q_c(T) - Q_c(0) = (1+\lambda)\int_0^T e^{Dt}Q(t)N_c(t)x_{f,c}(t)dt$$

for a given initial condition $Q_c(0)$.

Under the SBGP equilibrium, $Q$ grows at a constant rate so that $Q(t) = Q(0)e^{gt}$. In addition, $N_c(t)x_{f,c}(t)$ has a stationary distribution so its mean is constant over time. Therefore, taking the expectation on both sides results in

$$e^{DT}\mathbb{E}[Q_c(T)] - Q_c(0) = (1+\lambda)Q(0)\mathbb{E}\left[N_c x_{f,c}\right]\frac{1}{D+g}\left[e^{(D+g)T} - 1\right].$$

Now divide both sides by $e^{DT}$ and take the limit as $T \to \infty$ to find[48]

$$\lim_{t\to\infty}\mathbb{E}[Q_c(t)] = \lim_{t\to\infty}\frac{(1+\lambda)}{D+g}\mathbb{E}\left[N_c x_{f,c}\right]Q(t).$$

Finally, using $g = \lambda D$ we have that

$$\lim_{t\to\infty}\mathbb{E}[Q_c(t)] = \lim_{t\to\infty}\frac{\mathbb{E}\left[N_c x_{f,c}\right]}{D}Q(t). \tag{B.11}$$

Moving back to the law of motion for $Q_c$, take the expectation on both sides to find

$$\mathbb{E}[\dot{Q}_c(t)] = (1+\lambda)Q(t)\mathbb{E}\left[N_c x_{f,c}\right] - D\mathbb{E}[Q_c(t)].$$

Once again take the limit as $t \to \infty$ and use equation (B.11) to find

$$\lim_{t\to\infty}\mathbb{E}[\dot{Q}_c(t)] = (1+\lambda)D\lim_{t\to\infty}\mathbb{E}[Q_c(t)] - D\lim_{t\to\infty}\mathbb{E}[Q_c(t)]$$

[48] Alternatively, one can impose $Q_c(0) = \frac{(1+\lambda)Q(0)}{D+g}\mathbb{E}\left[N_c x_{f,c}\right]$ to find the same result for all $t$.

or

$$\lim_{t\to\infty} \mathbb{E}[\dot{Q}_c(t)] = g \lim_{t\to\infty} \mathbb{E}[Q_c(t)].$$

To avoid carrying limits in the notation, let $\mathsf{T}$ be a time period such that $\forall t > \mathsf{T}$ the relationship above is approximately true. Define the rate of growth of $Q_c(t)$ as $g_c(t) = \dot{Q}_c(t)/Q_c(t)$. Then, for $t > \mathsf{T}$

$$\mathbb{E}[g_c(t)Q_c(t)] = g\mathbb{E}[Q_c(t)]$$

$$\mathbb{E}[g_c(t)]\mathbb{E}[Q_c(t)] + \mathrm{Cov}(g_c(t), Q_c(t)) = g\mathbb{E}[Q_c(t)]$$

$$\mathbb{E}[g_c(t)] = g - \frac{\mathrm{Cov}(g_c(t), Q_c(t))}{\mathbb{E}[Q_c(t)]}.$$

□

# C Linear Regressions

## C.1 Data

The main dataset I use to derive the empirical results in section 4.2 is the patent data published by the United States Patent and Trademark Office (USPTO). Through the PatentsView platform,[49] the USPTO provides data on the universe of patents registered in the US, including citations made and received by each patent, their industry classification, who are their inventors and who those patents were assigned to (i.e., who owns the patent). I assume the year in which each patent was "produced" is the year when the patent application was filed. As a baseline quality cutoff, I drop all patents who were never granted.

The focus of this paper is on innovation led by firms, so I drop any patent assignee who is not labeled as a corporation from the data. Throughout the text, I use the words assignee and firm interchangeably to refer to the owner of a patent. I attribute a location to each patent by matching it to its assigned owner and assume that the patent was "produced" at the CBSA where the assignee is located. In case a patent has multiple owners, I split that patent into equal shares between each of them. To minimize double counting when an assignee has multiple addresses, I use the patent's inventors locations to select which address is currently used by that firm.

I also adjust the data for the fact that I only observe inventors and firms when they are successful in producing patents. For example, if a firm files a patent 2009 and in 2011 – but not in 2010 – this firm will not be included in the data in 2010. To deal with this selection problem, I "complete" the dataset by adding back the missing inventors and firms in each year. In the case of firms, I do that by determining an entry and exit year for each firm in every CBSA that they appear in the data. Then I create a new observation whenever a year is missing between entry and exit for each CBSA. In this new observation, the firm is assigned zero patents. In the case of inventors, the procedure is similar, but I also match them to the firms they are "working for" (i.e., the assignees of the patents created by that inventor). As was the case for firms, whenever an inventor has multiple addresses, I use the location of the assignee of the most recent patents he or

[49] See `http://www.patentsview.org/download/`.

she created to determine which address is currently used. If this process still results in multiple addresses, I assume that the inventor spends an equal fraction of his or her time in each of those places. As a result, the number of inventors hired by a firm and the population of inventors in a city can be non-integer.

The USPTO data also classifies each patent in one of 6 broad categories and 37 subcategories. For each firm and each inventor, I identify the most common subcategory among all patents produced and assign this subcategory as the firm/inventor's industry. In cases when the mode is not unique, I assign that inventor or firm into a separate industry subcategory, "industry 0." This happens for about $10\%$ of all inventors and firms, but this effect is concentrated on inventors/firms who have produced a small number of patents (for example, an inventor who produces a total of two patents, but in different subcategories, will by definition not have a unique subcategory mode). Weighting by patents produced, only approximately $7\%$ of inventors and $2\%$ of firms are assigned to "industry 0."

I also employ the County Business Patterns (CBP) data, published by the Census Bureau. It contains information on the demography and economic activity (employed population, number of establishments, industry classification, etc.) inside every county in the US. Similarly, I use the Zillow Rent Index (ZRI) and the Zillow Home Value Index (ZHVI), both published by Zillow Research, for data on the median rental value and housing price by square foot in each county (respectively).[50] I aggregate this information to the CBSA level using NBER's county to CBSA crosswalk.[51] In the data used for estimation, I focus on the years after 1998, when the CBP switched its industry classification system from SIC to NAICS. However, I occasionally use data that goes further back in time as well – for example in the construction of the instrument in section 4.2.1, which involves lags of employment shares. The two datasets combined contain information on 2,217,577 patents, 1,191,418 inventors and 136,124 firms (assignees), spread out over 860 CBSAs (not all CBSAs include firms/inventors who produced patents) between the years of 1998 and 2016.

## C.2 Going From the Model to the Data

The procedure to transform the continuous time flow variables in the model to quantities that are observed in the data is exactly the same in all cases. Here, I detail how to perform this transformation using the production function of innovation, given by equation (1). Normalizing one year to be equal to a time interval with measure one, the expected number of innovations produced by a firm $f$ located in city $c$ during year $T$, given the sequence of shocks $\{Z_c(t)\}_{T-1}^{T}$, is

$$\int_{T-1}^{T} x_{f,c}(t)dt = \bar{\chi}_c \int_{T-1}^{T} \left[ \left( \frac{I_c(t)}{\bar{m}_c} \right)^{\eta} i_{f,c}(t) \right]^{\psi} Z_c(t)dt.$$

[50]See `https://www.zillow.com/research/data/` for more details on the Zillow datasets.

[51]See `https://www.nber.org/data/cbsa-fips-county-crosswalk.html`. Out of the 929 CBSAs in the US, 917 can be merged with the CBP data. The remaining 12 are in Puerto Rico, which is not included in the CBP.

Taking logs

$$\log \int_{T-1}^{T} x_{f,c}(t)dt = \psi \log \left( \kappa \frac{\psi}{1-\psi} \right) + \psi\eta \log(\bar{I}_c) + \log \left( \frac{\bar{\chi}_c}{\bar{m}_c^{\psi\eta}} \right)$$
$$+ \log \int_{T-1}^{T} \mathbf{1}\{i_{f,c}(t) > 0 | Z_c(t)\} \left( \frac{Z_c(t)^{\frac{1-\theta}{\Theta}}}{e^{\frac{1-\theta}{\Theta}\left(\frac{1-\theta}{\Theta}-1\right)\frac{\sigma^2}{4\phi}}} \right)^{\psi\eta} Z_c(t)dt,$$

where I have used that $i_{f,c}(t) = \kappa \frac{\psi}{1-\psi}$ in the periods when firm $f$ decides to invest in R&D (indicated by $\mathbf{1}\{i_{f,c}(t) > 0 | Z_c(t)\}$) and defined $\bar{I}_c$ as in corollary 1. Once again using those results, we find that

$$\log \int_{T-1}^{T} i_{f,c}(t)dt = \log \left( \kappa \frac{\psi}{1-\psi} \right) + \log \int_{T-1}^{T} \mathbf{1}\{i_{f,c}(t) > 0 | Z_c(t)\}dt$$

and

$$\log \int_{T-1}^{T} I_c(t)dt = \log(\bar{I}_c) + \log \int_{T-1}^{T} \frac{Z_c(t)^{\frac{1-\theta}{\Theta}}}{e^{\frac{1-\theta}{\Theta}\left(\frac{1-\theta}{\Theta}-1\right)\frac{\sigma^2}{4\phi}}} dt.$$

As a result,

$$\log \int_{T-1}^{T} x_{f,c}(t)dt = \psi \log \int_{T-1}^{T} i_{f,c}(t)dt + \psi\eta \log \int_{T-1}^{T} I_c(t)dt + \log \left( \frac{\bar{\chi}_c}{\bar{m}_c^{\psi\eta}} \right)$$
$$+ \log \int_{T-1}^{T} \mathbf{1}\{i_{f,c}(t) > 0 | Z_c(t)\} \left( \frac{Z_c(t)^{\frac{1-\theta}{\Theta}}}{e^{\frac{1-\theta}{\Theta}\left(\frac{1-\theta}{\Theta}-1\right)\frac{\sigma^2}{4\phi}}} \right)^{\psi\eta} Z_c(t)dt$$
$$- \psi \log \int_{T-1}^{T} \mathbf{1}\{i_{f,c}(t) > 0 | Z_c(t)\}dt - \psi\eta \log \int_{T-1}^{T} \frac{Z_c(t)^{\frac{1-\theta}{\Theta}}}{e^{\frac{1-\theta}{\Theta}\left(\frac{1-\theta}{\Theta}-1\right)\frac{\sigma^2}{4\phi}}} dt.$$

This last equation leads to the regression model

$$\log(x_{f,c,t}) = \psi \log(i_{f,c,t}) + \psi\eta \log(I_{c,t}) + \delta_c + z_{f,c,t},$$

where, slightly abusing notation so that $t$ now represents a year instead of an infinitesimal period, $x_{f,c,t}$ is the number of innovations produced by a firm $f$ located in city $c$ during year $t$; $i_{f,c,t}$ is the number of inventors hired by firm $f$ during year $t$ (i.e., the average number of inventors hired per period); $I_{c,t}$ is the population of inventors in city $c$ during year $t$ (i.e., the average population of inventors in city $c$ per period); $\delta_c$ is a city fixed-effect that captures variation in $\bar{\chi}_c$ and $\bar{m}_c$; and $z_{f,c,t}$ is a shock aggregating the two bottom lines in the previous equation.

## C.3 Details on the Estimation of the Elasticity of Agglomeration

### C.3.1 Shift-Share Research Designs

This section provides a brief comparison between the "traditional" shift-share research design and the instrument proposed in section 4.2.1. Following the more common shifft-share approach, one

can take difference between variables in (15) relative to their values in period $t_0$ to find

$$y_{c,t} - y_{c,t_0} = \psi\eta[\log(I_{c,t}) - \log(I_{c,t_0})] + (X'_{c,t} - X'_{c,t_0})\Gamma + (\delta_t - \delta_{t_0}) + (z_{c,t} - z_{c,t_0}),$$

where $y_{c,t}$ is the outcome variable in the regression and $t_0$ is some pre-period, usually many years before year $t$ so that there is no correlation between $z_{c,t}$ and $I_{c,t_0}$. An instrument for $[\log(I_{c,t}) - \log(I_{c,t_0})]$ could then be defined as

$$\log(\mathcal{I}^{diff}_{c,t,t_0}) = \log \sum_{k=1}^{K} \frac{I_{k,c,t_0}}{I_{c,t_0}}(1 + \gamma_{k,t_0 \to t}).$$

Recall from the definition of $\mathcal{I}_{c,t,l}$ in equation (16) that this is exactly what we would get if we computed the difference $\log(\mathcal{I}_{c,t,l})\big|_{l=t-t_0} - \log(I_{c,t_0})$.

Alternatively, the fixed-effects regression model I run is equivalent to de-meaning the variables in (15):

$$y_{c,t} - \overline{y_c} = \psi\eta[\log(I_{c,t}) - \overline{\log(I_c)}] + (X'_{c,t} - \overline{X'_c})\Gamma + (\delta_t - \overline{\delta}) + (z_{c,t} - \overline{z_c}),$$

where the overlines indicate averages over time. Given the definition of $\mathcal{I}_{c,t,l}$ (eq. 16), the instrument for $[\log(I_{c,t}) - \overline{\log(I_c)}]$ is

$$\log(\mathcal{I}_{c,t,l})\big|_{l=t-t_0} - \overline{\log(I_c)} = \log \sum_{k=1}^{K} \frac{I_{k,c,t_0}}{\hat{I}_c}(1 + \gamma_{k,t_0 \to t}),$$

where $\log(\hat{I}_c) = \overline{\log(I_c)}$. The difference between the shift-share design and the fixed effects specification therefore resides only on the denominator used to compute the "share." As shown by Adão et al. (2019) and Borusyak et al. (2022), the econometric properties of the estimators using this type of instrument do not depend on the exact definition of these shares. Furthermore, by using the fixed-effects specification, data on the outcome $y_{c,t}$ and the control variables $X_{c,t}$ during period $t_0$ is not required to estimate the parameters in the regression.

### C.3.2 Threats to Identification

The exogeneity condition for the instrument defined in section 4.2.1 is $\mathbb{E}_c[\mathcal{I}_{c,t,l} z_{c,t}] = 0$. Following the argument made by Borusyak et al. (2022), this condition can be written as

$$\sum_{k=1}^{K} I_{k,t-l}\omega_{k,t}(1 + \gamma_{k,t-l \to t}) = 0,$$

where $I_{k,t-l} = \mathbb{E}_c[I_{k,c,t-l}]$ and $\omega_{k,t} = \mathbb{E}_c[I_{k,c,t-l} z_{c,t}]/\mathbb{E}_c[I_{k,c,t-l}]$. This equation provides two different interpretations for the conditions that are required for instrument exogeneity. The first one is that the lagged industry employment levels $I_{k,c,t-l}$ are uncorrelated with the current shock, so $\omega_{k,t} = 0$. As mentioned in the main text, this condition is unlikely to hold for contemporary shares/shocks. However, even is shocks are serially correlated, this condition will hold if lags are long enough. The estimates shown in table 2 cover this case, showing the results when the in-

strument is lagged for up to 10 years and when industry employment is fixed at its average level between 1990 and 1995.

The second interpretation for instrument endogeneity requires that

$$\sum_{k=1}^{K} I_{k,t-l}\omega_{k,t}(1+\gamma_{k,t-l\to t}) \longrightarrow 0$$

as the number of industries increases. In words, the industry-specific growth rate of employment in countries other than the US, $\gamma_{k,t-l\to t}$, is asymptotically uncorrelated with the industry-specific average of unobserved factors affecting the employment level in locations specializing in each industry, $\omega_{k,t}$. One concern that arises in this case are industries that are simultaneously highly concentrated in specific cities and large enough to drive international trends in employment shares. To avoid this issue, I slightly modify the instrument to exclude any industry whose employment share in any single city exceeds $15\%$ at any point in time.[52] This excludes 6 of the original 38 industries (information storage, drugs, semiconductor devices, motors and engines/parts, apparel and textile, earth working and wells). The estimation results can be seen in table C.1. While 32 industries might be a low number to claim asymptotic lack of correlation, the fact that the estimates below are similar to the ones found before indicates that no city is likely to be driving $\gamma_{k,t-l\to t}$ on its own.

Table C.1: Estimation of the elasticity of agglomeration – excludes spatially concentrated industries.

First Stage

| | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| $\log(\mathcal{I}_{c,t,l})$ | 0.532*** | 0.467*** | 0.389*** | 0.260*** |
| | (0.038) | (0.040) | (0.039) | (0.041) |
| F-stat. excluded inst. | 196.20 | 137.93 | 97.70 | 40.71 |

Second Stage

| | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| log(Inventors in City) | 0.109*** | 0.103*** | 0.092*** | 0.090 |
| | (0.021) | (0.022) | (0.026) | (0.057) |
| AKM SE | (0.009) | (0.011) | (0.011) | (0.001) |
| Method | IV ($l=5$) | IV ($l=7$) | IV ($l=10$) | IV ($l=t-t_{90-95}$) |
| Observations | 11184 | 11165 | 11119 | 11162 |
| Implied $\eta$ | 0.218 | 0.206 | 0.184 | 0.180 |

Standard errors are clustered at the CBSA level and shown in parenthesis. *, **, and *** indicate that the coefficient is statistically different from 0 at the 10%, 5%, and 1% levels, respectively. AKM SE indicates alternative standard errors, calculated according to Adão et al. (2019). All specifications control for CBSA and year fixed effects.

[52]The value of the threshold is arbitrary, but variations around it (e.g., 10, 20 or 25%) generate comparable results.

### C.3.3 Robustness Checks

**Including Cities with Zero Patents.** Recall from equation (14) that the theoretical model presented in this paper implies the following relationship in the data:

$$\log(\text{patents}_{f,c,t}) - \psi \log(i_{f,c,t}) = \psi\eta \log(I_{c,t}) + X'_{f,c,t}\Gamma + \delta_c + \delta_t + z_{f,c,t}.$$

One issue with this equation is that many firms, especially those in smaller cities, do not produce any patents over the course of one year. If this is the case, running the log-log regression above simply discards those observations, potentially biasing the estimation. To avoid this problem, we can instead run this regression in the form

$$Y_{f,c,t} = \exp\left(\psi\eta \log(I_{c,t}) + X'_{f,c,t}\Gamma + \delta_c + \delta_t + z_{f,c,t}\right),$$

where

$$Y_{f,c,t} = \begin{cases} \text{patents}_{f,c,t}/i^{\psi}_{f,c,t}, & \text{if } i_{f,c,t} > 0 \\ 0, & \text{otherwise.} \end{cases}$$

Once again, the goal is to estimate the elasticity of agglomeration $\psi\eta$, so we can aggregate the data by taking averages over cities.

$$Y_{c,t} = \exp\left(\psi\eta \log(I_{c,t}) + X'_{c,t}\Gamma + \delta_c + \delta_t + z_{c,t}\right)$$

where $Y_{c,t} = \dfrac{1}{N_{c,t}}\displaystyle\sum_{f=1}^{N_{c,t}} Y_{f,c,t}$ is the average of outcomes $Y_{f,c,t}$ and $\exp(z_{c,t}) = \dfrac{1}{N_{c,t}}\displaystyle\sum_{f=1}^{N_{c,t}} \exp(z_{f,c,t})$ aggregates the residuals. For consistency with the other empirical models run so far, $X_{c,t}$ includes the average number of citations that all firms in city $c$ receive in year $t$ and the industry employment shares in city $c$ during year $t$.

I control for the endogeneity of $\log(I_{c,t})$, by following the method proposed by Wooldridge (2010, Ch.19), which approaches the endogeneity problem from an omitted variables perspective. Specifically, let

$$\log(I_{c,t}) = [\log(\mathcal{I}_{c,t,l}), X'_{c,t}, \delta_c, \delta_t] \cdot \Pi + v_{c,t} \tag{C.1}$$

where $\Pi$ is vector of reduced-form parameters and $v_{c,t}$ is a residual. Furthermore, I assume that $(z_{c,t}, v_{c,t})$ are independent of $[\log(\mathcal{I}_{c,t.l}), X'_{c,t}, \delta_c, \delta_t]$ and that

$$z_{c,t} = \rho v_{c,t} + e_{c,t}$$

where $e_{c,t}$ is independent of $v_{c,t}$.

Under those conditions, and assuming that $Y_{c,t}$ has a Poisson distribution, it follows that (slightly abusing notation so that the fixed effects absorb constant terms)

$$\log\left(\mathbb{E}\left[Y_{c,t} \middle| I_{c,t}, X_{c,t}, \delta_c, \delta_t, v_{c,t}\right]\right) = \psi\eta \log(I_{c,t}) + X'_{c,t}\Gamma + \delta_c + \delta_t + \rho v_{c,t}. \tag{C.2}$$

Intuitively, the term $\rho v_{c,t}$ controls for the endogeneity of $\log(I_{c,t})$. The residuals $v_{c,t}$ can be obtained

from the first-stage regression (C.1) and the model (C.2) is estimated via pseudo-maximum likelihood with the help the `ppmlhdfe` command in Stata, developed by Correia et al. (2019). Once again, each observation is weighted by the number of firms in each city to reflect the fact that the aggregated data is comprised of means over these firms.

The estimated coefficients are in table C.2. Column (1) shows the coefficient estimate without the use of an instrument ($\rho = 0$), columns (2) - (4) show the estimates obtained when using different lags $l$ to compute the instrument and column (5) fixes $I_{k,c,t_{90-95}}$ at its average value between 1990 and 1995. The Poisson model produces larger point estimates than the linear model, potentially because the effect of spillovers at the extensive margin (and on firms who do not continuously innovate) is larger than for other incumbents. However, the standard errors are also quite big, leading values that are not statistically different from zero in columns (4) and (5).[53]

Table C.2: Estimation of the elasticity of agglomeration – Poisson regression.

| | (1) | (2) | (3) | (4) | (5) |
|---|---|---|---|---|---|
| **First Stage** | | | | | |
| $\log(\mathcal{I}_{c,t,l})$ | | 0.536*** | 0.469*** | 0.388*** | 0.248*** |
| | | (0.031) | (0.033) | (0.034) | (0.039) |
| F-stat. excluded inst. | | 308.28 | 206.71 | 129.93 | 40.02 |
| **Second Stage** | | | | | |
| log(Inventors in City) | 0.197*** | 0.122*** | 0.095** | 0.029 | -0.001 |
| | (0.030) | (0.039) | (0.043) | (0.052) | (0.120) |
| Method | OLS | IV ($l = 5$) | IV ($l = 7$) | IV ($l = 10$) | IV ($l = t - t_{90-95}$) |
| Observations | 12889 | 12827 | 12808 | 12766 | 12792 |
| Implied $\eta$ | 0.394 | 0.244 | 0.190 | 0.058 | -0.002 |

Standard errors are clustered at the CBSA level and shown in parenthesis. *, **, and *** indicate that the coefficient is statistically different from 0 at the 10%, 5%, and 1% levels, respectively. All specifications control for patent quality and city industry composition, as well as CBSA and year fixed effects.

**Other Sources of Externality.** The population of inventors in a city might not be the only source of agglomeration for firms investing in innovation. For example, an individual firm might benefit from locating near other companies, as this provides opportunities for the firm to learn from its competitors and improve the quality of its own investments. Similarly, the agglomeration spillover, as assumed by a number of papers in the literature, might be related to overall population density instead of the density of inventors.

To test these hypotheses, I include various sources of agglomeration into the regression equation (15): the total population of inventors, the total number of firms who invest in R&D, the total employed population and the total number of (overall) establishments in each city. I focus on simple correlations in this case, so no instrumental variable specification is shown in table C.3 below.

[53]Note that the AKM standard errors are not shown in this case, as the method developed by Adão et al. (2019) only applies to linear regressions.

Column (1) shows the coefficients from the log-linear model, and column (2) shows the coefficients from the Poisson counting model. As shown in the table, only the number of inventors in the city has a positive (and significant) effect over the average production of patents in each city. This result also holds in the disaggregated model (14), in different geographic levels of aggregation (e.g., counties), and after including non-linear functions of each of the potential sources of agglomeration into the regression model.

Table C.3: Estimation of the elasticity of agglomeration – multiple sources of agglomeration.

| | (1) | (2) |
|---|---|---|
| log(Inventors in City) | 0.064*** | 0.203*** |
| | (0.017) | (0.034) |
| log(Innov. Firms in City) | 0.007 | -0.002 |
| | (0.014) | (0.029) |
| log(Employment in City) | -0.078 | -0.124 |
| | (0.076) | (0.146) |
| log(Establishments in City) | 0.106* | -0.004 |
| | (0.062) | (0.151) |
| Model | Log-linear (OLS) | Poisson |
| Observations | 11279 | 12889 |

Standard errors are clustered at the CBSA level and shown in parenthesis. *, **, and *** indicate that the coefficient is statistically different from 0 at the 10%, 5%, and 1% levels, respectively. All specifications control for patent quality and city industry composition, as well as CBSA and year fixed effects.

## C.4 Details on the Estimation of the Elasticity of Congestion

To arrive at the regression model (17) from the equations in the model, I basically follow the argument described in section C.2 of this appendix. Plugging in the value of production workers' wages (eq. 6) into to intermediate good producer's FOC, we find

$$p_{n,c} = \frac{w^\ell}{\beta\alpha_c^{\frac{\theta}{1-\theta}}}\left(\frac{L_c}{\bar{m}_c}\right)^{\frac{1-\beta}{1-\theta}}$$

for cities $c \geq 1$. In city 0, a similar equation holds, but the RHS is multiplied by $(\theta\beta)^{\frac{(1-\beta)\theta}{1-\theta}}$. The equation above reflects the flows of continuous variables. To match them to the data, I integrate them over the period of one year:

$$\int_{T-1}^{T} p_{n,c}(t)dt = \left[\beta\alpha_c^{\frac{\theta}{1-\theta}}\bar{m}_c^{\frac{1-\beta}{1-\theta}}\right]^{-1}\int_{T-1}^{T} w^\ell(t)L_c(t)^{\frac{1-\beta}{1-\theta}}dt.$$

In city 0, the population of inventors is constant, so the integral reduces to $\int_{T-1}^{T} w^\ell(t)dt$. In cities $c \in \{1, \dots, C\}$, the population of production workers is proportional to the population of

inventors (see lemma 1). As a result,

$$L_c(t) = \frac{L - L_0}{I} I_c(t) = \frac{L - L_0}{I} \bar{I}_c \frac{Z_c(t)^{\frac{1-\theta}{\Theta}}}{e^{\frac{1-\theta}{\Theta}\left(\frac{1-\theta}{\Theta}-1\right)\frac{\sigma^2}{4\phi}}},$$

where $\bar{I}_c$ is constant over time. Define $\bar{L}_c = \frac{L-L_0}{I}\bar{I}_c$ so that

$$\int_{T-1}^{T} p_{n,c}(t)dt = \left[\beta\alpha_c^{\frac{\theta}{1-\theta}} \bar{m}_c^{\frac{1-\beta}{1-\theta}}\right]^{-1} \bar{L}_c^{\frac{1-\beta}{1-\theta}} \int_{T-1}^{T} w^\ell(t) \frac{Z_c(t)^{\frac{1-\beta}{\Theta}}}{e^{\frac{1-\beta}{\Theta}\left(\frac{1-\theta}{\Theta}-1\right)\frac{\sigma^2}{4\phi}}} dt.$$

Take logs to find

$$\log \int_{T-1}^{T} p_{n,c}(t)dt = \left(\frac{1-\beta}{1-\theta}\right)\log(\bar{L}_c) - \log\left(\beta\alpha_c^{\frac{\theta}{1-\theta}} \bar{m}_c^{\frac{1-\beta}{1-\theta}}\right) + \log \int_{T-1}^{T} w^\ell(t) \frac{Z_c(t)^{\frac{1-\beta}{\Theta}}}{e^{\frac{1-\beta}{\Theta}\left(\frac{1-\theta}{\Theta}-1\right)\frac{\sigma^2}{4\phi}}} dt.$$

Finally, use the fact that $\int_{T-1}^{T} L_c(t)dt = \bar{L}_c \int_{T-1}^{T} \frac{Z_c(t)^{\frac{1-\theta}{\Theta}}}{e^{\frac{1-\theta}{\Theta}\left(\frac{1-\theta}{\Theta}-1\right)\frac{\sigma^2}{4\phi}}} dt$ to replace $\bar{L}_c$ in the equation above:

$$\begin{aligned}\log \int_{T-1}^{T} p_{n,c}(t)dt = \; & \left(\frac{1-\beta}{1-\theta}\right)\log \int_{T-1}^{T} L_c(t)dt - \log\left(\beta\alpha_c^{\frac{\theta}{1-\theta}} \bar{m}_c^{\frac{1-\beta}{1-\theta}}\right) \\ & + \log \int_{T-1}^{T} w^\ell(t) \frac{Z_c(t)^{\frac{1-\beta}{\Theta}}}{e^{\frac{1-\beta}{\Theta}\left(\frac{1-\theta}{\Theta}-1\right)\frac{\sigma^2}{4\phi}}} dt - \left(\frac{1-\beta}{1-\theta}\right)\log \int_{T-1}^{T} \frac{Z_c(t)^{\frac{1-\theta}{\Theta}}}{e^{\frac{1-\theta}{\Theta}\left(\frac{1-\theta}{\Theta}-1\right)\frac{\sigma^2}{4\phi}}} dt,\end{aligned}$$

Inn the expression above, $\int_{T-1}^{T} p_{n,c}(t)dt$ is the average price charged for the non-tradable good in city $c$ during year $T$, which I approximate by the median rent value per square foot of housing units in city $c$, $p^h_{c,t}$. Similarly, $\int_{T-1}^{T} L_c(t)dt$ is the average population of production workers living in city $c$ during year $T$. This is matched to the total employed population minus the number of inventors in each CBSA in the data. The other terms in the equation are captured by a city fixed effect, which accounts for differences in land mass and amenities in each city; a year fixed effect, controlling for growth in wages $w^\ell(t)$; and a local shock $z^h_{c,t}$ that combines the variation in the integrals containing $Z_c(t)$. This produces the model (17),

$$\log(p^h_{c,t}) = \left(\frac{1-\beta}{1-\theta}\right)\log(L_{c,t}) + \delta_c + \delta_t + z^h_{c,t}.$$

Note that $L_{c,t}$ is correlated with the shock $z^h_{c,t}$, since $L_c(t)$ varies with the shock $z_c(t)$.

### C.4.1 Threats to Identification

Following the discussion in sections 4.2.2 and C.3.2, table C.4 shows the estimated elasticity of rental prices with respect to the city's population of production workers. For the estimation of those elasticities, the instrument $\mathcal{I}_{c,t,l}$ is computed after excluding industries whose employment share in any single city exceeds 15% in any point in time. The implied value of $\beta$ does not show any significant change when compared to table 3.

### C.4.2 Alternative Specification

Rental value data is only available in the Zillow Rent Index starting late in 2010. To take advantage of a larger dataset, I run an alternative regression that approximates the price of the non-tradable good in each city by the median price per square foot of housing units. This series is available since 1996 in the Zillow Home Value Index database. The main issue with using housing prices as an approximation for the price of non-tradable goods (whose consumption is modeled as a flow) is that houses are long-term assets, and therefore their prices could be influenced by agents' expectations about the future. Having that in mind, housing prices can still provide useful information on congestion costs in each city.

Table C.4: Estimation of the elasticity of congestion – excludes spatially concentrated industries.

First Stage

| | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| $\log(\mathcal{I}_{c,t,l})$ | 0.013*** | 0.017*** | 0.013*** | 0.022*** |
| | (0.004) | (0.007) | (0.005) | (0.007) |
| F-stat. excluded inst. | 11.20 | 5.67 | 8.31 | 9.56 |

Second Stage

| | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| log(Prod. Workers in City) | 1.336*** | 1.063** | 1.384*** | 1.033*** |
| | (0.413) | (0.459) | (0.479) | (0.368) |
| AKM SE | (0.019) | (0.021) | (0.032) | (0.006) |
| Method | IV ($l = 5$) | IV ($l = 7$) | IV ($l = 10$) | IV ($l = t - t_{90-95}$) |
| Observations | 2843 | 2833 | 2822 | 2825 |
| Implied $\beta$ | 0.466 | 0.575 | 0.446 | 0.587 |

Standard errors are clustered at the CBSA level and shown in parenthesis. *, **, and *** indicate that the coefficient is statistically different from 0 at the 10%, 5%, and 1% levels, respectively. AKM SE indicates alternative standard errors, calculated according to Adão et al. (2019). All specifications control for CBSA and year fixed effects.

Table C.5 displays the estimates elasticity of housing prices with respect to the city's population of production workers. The estimated coefficients are quite smaller than in table 3, and imply a value of $\beta$ closer to $0.8$. These numbers reflect a lower elasticity of housing prices (relative to rental prices) to the city's population, most likely because those prices only reflect permanent changes in congestion costs in each city. The value of $\beta$ found using this alternative specification is used as an upper bound in sensitivity analyses of the counterfactual results in this paper.

# D Matching Moments

## D.1 Identifying $\alpha_0$ and the Scale of $\chi_c$

Identifying the scale of the mean productivity in each city and the amenity level in city $0$ is straightforward once $\sigma^2/4\phi$ (along with the remaining parameters in the model) is known. For now, assume that this is the case. Define the scale of productivity as $\chi^s$, where $\bar{\chi}_c = \chi^s \hat{\chi}_c$ and $\mathbb{E}[\hat{\chi}_c] = 1$.

Table C.5: Estimation of the elasticity of congestion – housing prices.

| First Stage | | | | |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4) |
| $\log(\mathcal{I}_{c,t,l})$ | 0.039*** | 0.037*** | 0.035*** | 0.032*** |
| | (0.006) | (0.006) | (0.006) | (0.007) |
| F-stat. excluded inst. | 42.94 | 36.00 | 40.04 | 22.42 |
| **Second Stage** | | | | |
| log(Prod. Workers in City) | 0.335* | 0.322* | 0.418** | 0.426** |
| | (0.183) | (0.179) | (0.166) | (0.208) |
| AKM SE | (0.067) | (0.060) | (0.059) | (0.002) |
| Method | IV ($l = 5$) | IV ($l = 7$) | IV ($l = 10$) | IV ($l = t - t_{90-95}$) |
| Observations | 9900 | 9891 | 9874 | 9876 |
| Implied $\beta$ | 0.866 | 0.871 | 0.833 | 0.830 |

Standard errors are clustered at the CBSA level and shown in parenthesis. *, **, and *** indicate that the coefficient is statistically different from 0 at the 10%, 5%, and 1% levels, respectively. AKM SE indicates alternative standard errors, calculated according to Adão et al. (2019). All specifications control for CBSA and year fixed effects.

Then, from corollary 1, we have that

$$D = \chi^s \psi^\psi \left(\frac{1-\psi}{\kappa}\right)^{1-\psi} e^{\frac{(1-\theta)(1+\psi\eta)}{\Theta}\left(\frac{(1-\beta)\theta}{\Theta}+1\right)\frac{\sigma^2}{4\phi}} \frac{1}{C}\sum_{c=1}^{C} \hat{\chi}_c \tilde{\bar{I}}_c^{1+\psi\eta}.$$

Given that $g = \lambda D$, it follows that

$$\chi^s = \frac{g}{\lambda}\left\{\psi^\psi \left(\frac{1-\psi}{\kappa}\right)^{1-\psi} e^{\frac{(1-\theta)(1+\psi\eta)}{\Theta}\left(\frac{(1-\beta)\theta}{\Theta}+1\right)\frac{\sigma^2}{4\phi}} \frac{1}{C}\sum_{c=1}^{C} \hat{\chi}_c \tilde{\bar{I}}_c^{1+\psi\eta}\right\}^{-1}.$$

To identify $\alpha_0$, recall from the proof of proposition 2 that

$$\left(\frac{L_0}{L-L_0}\right)^{\frac{1-\theta\beta}{1-\theta}} = \frac{\alpha_0^{\frac{\theta}{1-\theta}}}{(1-\varepsilon)\psi^\psi\left(\frac{1-\psi}{\kappa}\right)^{1-\psi}\frac{1+\lambda}{r+D}e^{\frac{\sigma^2}{4\phi}\left(\frac{1-\theta}{\Theta}-1\right)}I^{1+\psi\eta}(\theta\beta)^{\frac{1-\theta\beta}{1-\theta}}\left[\frac{1}{C}\sum_{c=1}^{C}\left(\frac{\bar{\chi}_c}{1-s_c}\right)^{\frac{1-\theta}{\Theta}}\alpha_c^{\frac{\theta}{\Theta}}\right]^{\frac{\Theta}{1-\theta}}}.$$

The LHS of the equation above can be constructed using the data and values of the parameters already estimated. The same is true for the denominator in the RHS, since $\bar{\chi}_c$ is now fully determined. The equation above therefore identifies $\alpha_0$, given the values of parameters estimated in the previous steps.

## D.2 Law of Motion of the Productivity Shock

Finally, in this section I describe the identification of the parameters in the stochastic process of the productivity shock, $Z_c$. As mentioned in the main text, only the ratio $\sigma^2/\psi$ influences the values

of the variables of interest in the SBGP equilibrium, so I set $\phi = 1$. The remaining parameter, $\sigma$, is then identified by the cross-sectional variance of the population of inventors across cities. Recall that quantities in the model must be integrated over time to match the same frequency as the data. The variance of the population of inventors across cities in year $T$ is therefore

$$\begin{aligned}
\text{Var}(I_{c,T}) &= \frac{1}{C}\sum_{c=1}^{C}\mathbb{E}\left[\left(\int_{T-1}^{T} I_c(t)dt\right)^2\right] - \mathbb{E}\left[\int_{T-1}^{T} I_c(t)dt\right]^2 \\
&= \frac{1}{C}\sum_{c=1}^{C}\mathbb{E}\left[\int_{T-1}^{T} I_c(t)dt \int_{T-1}^{T} I_c(s)ds\right] - \mathbb{E}\left[\int_{T-1}^{T} I_c(t)dt\right]\mathbb{E}\left[\int_{T-1}^{T} I_c(s)ds\right] \\
&= \frac{1}{C}\sum_{c=1}^{C}\int_{T-1}^{T}\int_{T-1}^{T}\left\{\mathbb{E}[I_c(t)I_c(s)] - \mathbb{E}[I_c(t)]\mathbb{E}[I_c(s)]\right\} dtds \\
&= \frac{1}{C}\sum_{c=1}^{C}\left(\frac{\bar{I}_c}{\exp\left(\frac{1-\theta}{\Theta}\left(\frac{1-\theta}{\Theta}-1\right)\frac{\sigma^2}{4\phi}\right)}\right)^2\int_{T-1}^{T}\int_{T-1}^{T}\text{Cov}\left(Z_c(t)^{\frac{1-\theta}{\Theta}}, Z_c(s)^{\frac{1-\theta}{\Theta}}\right) dtds
\end{aligned}$$

To compute this covariance, I start by noting that the limiting (stationary) distribution of the Ornstein-Uhlenbeck process is such that $\text{Cov}(z_c(t), z_c(s)) = \frac{\sigma^2}{2\phi}e^{-\phi|t-s|}$. Using the properties of the multi-variate log-normal distribution, it follows that $\text{Cov}\left(Z_c(t)^{\frac{1-\theta}{\Theta}}, Z_c(s)^{\frac{1-\theta}{\Theta}}\right) = \exp\left(\frac{1-\theta}{\Theta}\left(\frac{1-\theta}{\Theta}-1\right)\frac{\sigma^2}{2\phi}\right)\left[\exp\left(1\right.\right.$
Plugging into the equation above,

$$\text{Var}(I_{c,T}) = \frac{1}{C}\sum_{c=1}^{C}\bar{I}_c^2\left[\int_0^1\int_0^1\exp\left(\frac{1-\theta}{\Theta}\frac{\sigma^2}{2\phi}\exp(-\phi|s-t|)\right)dtds - 1\right],$$

where a simple change in variables switches the region of integration to $[0, 1]$.[54] The LHS of the equation above is the average cross-sectional variance of the population of inventors across cities, which can be computed in the data. Given $\phi = 1$ and the values of the other parameters already identified, this equation identifies $\sigma$.

# E The Government's Problem

The government's problem, as stated in section 5, is

$$\max_{\{s_c\}_{c=1}^{C}} \quad \int_0^\infty e^{-\rho t}\left\{\sum_{c=0}^{C}\left[L_c(t)u^\ell(t) + I_c(t)u^i(t)\right]\right\}G(t)dt$$

$$\text{s.t.} \quad \int_0^\infty e^{-rt}\left[\sum_{c=1}^{C}s_c w_c^i(t)I_c(t) + \gamma(G(t))\right]dt = \int_0^\infty e^{-rt}\left[p_{m,0}(t)\bar{m}_0 + \sum_{c=1}^{C}p_{m,c}(t)\bar{m}_c + \Pi(t)\right]dt$$

[54]The integral in this equation does not have a closed form solution, but it can be simplified to $2\int_{e^{-\phi}}^{1}\int_{1}^{1/y}\exp\left(\frac{1-\theta}{\Theta}\frac{\sigma^2}{2\phi}xy\right)\frac{1}{\phi^2 xy}dxdy$, which makes the numerical integration easier.

The sum in the objective function reduces to $Lu^\ell(t) + Iu^i(t)$ because market clearing must hold in all periods. Using equations (5), (6) and (B.4), this term can be rewritten as

$$(L-L_0)^{\theta\beta} I^{1-\theta\beta}\left(1+\theta\beta\frac{L_0}{L-L_0}\right)\left(\frac{1-\theta}{1-\theta\beta}w^i(t)\right)^{1-\theta}.$$

Using the results in lemma 1, the term $p_{m,0}(t)\bar{m}_0 + \sum_{c=1}^{C} p_{m,c}(t)\bar{m}_c$ in the budget constraint is

$$p_{m,0}(t)\bar{m}_0 + \sum_{c=1}^{C} p_{m,c}(t)\bar{m}_c = (1-\beta)\theta w_0^\ell(t)L_0 + \sum_{c=1}^{C}\frac{(1-\beta)\theta}{1-\theta\beta}w_c^i(t)I_c(t).$$

From equations (7) and (B.4), the first part of the expression above is

$$\begin{aligned}(1-\beta)\theta w_0^\ell(t)L_0 &\overset{(7)}{=} (1-\beta)\beta^{\frac{(1-\beta)\theta}{1-\theta}}\left(\frac{1}{\alpha_0}\right)^{\frac{\theta}{1-\theta}}(\theta L_0)^{\frac{1-\theta\beta}{1-\theta}}w^\ell(t)\\ &\overset{(B.4)}{=} \frac{(1-\beta)\theta}{1-\theta\beta}\left(\frac{1}{\alpha_0}\right)^{\frac{\theta}{1-\theta}}\left(\theta\beta\frac{L_0}{L-L_0}\right)^{\frac{1-\theta\beta}{1-\theta}} I^{\frac{1-\theta\beta}{1-\theta}}w^i(t).\end{aligned}$$

Equation (5) and the definition $I_c = \tilde{I}_c/C$ yield

$$\sum_{c=1}^{C}\frac{(1-\beta)\theta}{1-\theta\beta}w_c^i(t)I_c(t) = \frac{(1-\beta)\theta}{1-\theta\beta}w^i(t)\frac{1}{C}\sum_{c=1}^{C}\left(\frac{1}{\alpha_c}\right)^{\frac{\theta}{1-\theta}}\tilde{I}_c(t)^{\frac{1-\theta\beta}{1-\theta}}.$$

The last term in the government's budget constraint is the integral $\int_0^\infty e^{-rt}\Pi(t)dt$. Recall that $\Pi(t)$ is defined as the aggregate flow of income net of costs for all firms in the economy in period $t$. For any individual firm, the discounted present value of this flow is equal to the firm's value at $t=0$. Furthermore, since neither the final or non-tradable good producers make profits, the discounted present value of $\Pi(t)$ starting in $t=0$ must be equal to

$$\int_0^\infty e^{-rt}\Pi(t)dt = F(D, L_0)Q(0).$$

Collecting the results above and using that $G(t)=\bar{G}$ for all $t$, the government's problem can be reduced to

$$\max_{\{s_c\}_{c=1}^{C}} \int_0^\infty e^{-\rho t}\left\{(L-L_0)^{\theta\beta}I^{1-\theta\beta}\left(1+\theta\beta\frac{L_0}{L-L_0}\right)\left(\frac{1-\theta}{1-\theta\beta}w^i(t)\right)^{1-\theta}\right\}dt$$

$$\text{s.t.}\quad \int_0^\infty e^{-rt}\left\{w^i(t)\frac{1}{C}\sum_{c=1}^{C}s_c\left(\frac{1}{\alpha_c}\right)^{\frac{\theta}{1-\theta}}\tilde{I}_c(t)^{\frac{1-\theta\beta}{1-\theta}} + \gamma(\bar{G})\right\}dt = F(D,L_0)Q(0)$$

$$+\frac{(1-\beta)\theta}{1-\theta\beta}\int_0^\infty e^{-rt}w^i(t)\left\{\left(\frac{1}{\alpha_0}\right)^{\frac{\theta}{1-\theta}}\left(\theta\beta I\frac{L_0}{L-L_0}\right)^{\frac{1-\theta\beta}{1-\theta}} + \frac{1}{C}\sum_{c=1}^{C}\left(\frac{1}{\alpha_c}\right)^{\frac{\theta}{1-\theta}}\tilde{I}_c(t)^{\frac{1-\theta\beta}{1-\theta}}\right\}dt.$$

This expression can be further simplified by using the expression for $\tilde{\tilde{I}}_c$, which gives

$$\frac{1}{C}\sum_{c=1}^{C} k_c \left(\frac{1}{\alpha_c}\right)^{\frac{\theta}{1-\theta}} \tilde{I}_c(t)^{\frac{1-\theta\beta}{1-\theta}} = \frac{1}{C}\sum_{c=1}^{C} k_c \left(\frac{1}{\alpha_c}\right)^{\frac{\theta}{1-\theta}} \tilde{\tilde{I}}_c^{\frac{1-\theta\beta}{1-\theta}} \times \frac{Z_c(t)^{\frac{1-\theta\beta}{\Theta}}}{e^{\frac{1-\theta\beta}{\Theta}\left(\frac{1-\theta}{\Theta}-1\right)\frac{\sigma^2}{4\phi}}}$$

where $k_c$ equals $s_c$ for the first term in the budget constraint and equals 1 in the last. Given that $C \to \infty$, we can once again apply the law of large numbers to find

$$\frac{1}{C}\sum_{c=1}^{C} k_c \left(\frac{1}{\alpha_c}\right)^{\frac{\theta}{1-\theta}} \tilde{I}_c(t)^{\frac{1-\theta\beta}{1-\theta}} = e^{\frac{1-\theta\beta}{\Theta}\frac{(1-\beta)\theta}{\Theta}\frac{\sigma^2}{4\phi}} \frac{1}{C}\sum_{c=1}^{C} k_c \left(\frac{1}{\alpha_c}\right)^{\frac{\theta}{1-\theta}} \tilde{\tilde{I}}_c^{\frac{1-\theta\beta}{1-\theta}}.$$

For convenience, define the public good production cost as $\gamma(\bar{G}) = \bar{\pi}\bar{G}Q(t)$ and let

$$\bar{w}^i = \frac{1}{\bar{\pi}}\frac{w^i(t)}{Q(t)} = \frac{1}{I^{\frac{\Theta}{1-\theta}}}\psi^{\psi}\left(\frac{1-\psi}{\kappa}\right)^{1-\psi} \frac{(1+\lambda)L_0}{r+D} e^{\left(\frac{1-\theta}{\Theta}-1\right)\frac{\sigma^2}{4\phi}} \left\{\frac{1}{C}\sum_{c=1}^{C}\left(\frac{\bar{\chi}_c}{1-s_c}\right)^{\frac{1-\theta}{\Theta}} \alpha_c^{\frac{\theta}{\Theta}}\right\}^{\frac{\Theta}{1-\theta}}.$$

Note that $\bar{w}^i$ does not vary with time, which reduces the government's problem to

$$\max_{\{s_c\}_{c=1}^{C}} \quad (L-L_0)^{\theta\beta}\left(1+\theta\beta\frac{L_0}{L-L_0}\right)\left(\frac{1-\theta}{1-\theta\beta}\bar{w}^i\right)^{1-\theta} \int_0^{\infty} e^{-\rho t} Q(t)^{1-\theta} dt$$

$$\text{s.t.} \qquad e^{\frac{1-\theta\beta}{\Theta}\frac{(1-\beta)\theta}{\Theta}\frac{\sigma^2}{4\phi}} \frac{1}{C}\sum_{c=1}^{C} s_c \left(\frac{1}{\alpha_c}\right)^{\frac{\theta}{1-\theta}} \tilde{\tilde{I}}_c^{\frac{1-\theta\beta}{1-\theta}} + \frac{\bar{G}}{\bar{w}^i} = \frac{L_0}{\bar{w}^i(r+D)}\frac{Q(0)}{\int_0^{\infty} e^{-rt}Q(t)dt}$$

$$+\frac{(1-\beta)\theta}{1-\theta\beta}\left\{\left(\frac{1}{\alpha_0}\right)^{\frac{\theta}{1-\theta}}\left(\theta\beta I\frac{L_0}{L-L_0}\right)^{\frac{1-\theta\beta}{1-\theta}} + e^{\frac{1-\theta\beta}{\Theta}\frac{(1-\beta)\theta}{\Theta}\frac{\sigma^2}{4\phi}} \frac{1}{C}\sum_{c=1}^{C}\left(\frac{1}{\alpha_c}\right)^{\frac{\theta}{1-\theta}} \tilde{\tilde{I}}_c^{\frac{1-\theta\beta}{1-\theta}}\right\}.$$

Since $Q(t)$ grows at a constant rate $g$, we can write $Q(t) = Q(0)e^{gt}$ for a given initial condition $Q(0)$. Assuming that $\rho > (1-\theta)g$ (otherwise the welfare function is not well defined),

$$\int_0^{\infty} e^{-\rho t} Q(t)^{1-\theta} dt = Q(0)^{1-\theta}\int_0^{\infty} e^{((1-\theta)g-\rho)t} dt = \frac{Q(0)^{1-\theta}}{r-(1-\theta)g}.$$

After plugging in this result into the government's objective function, equation (18) follows.

# F Sensitivity Analyzes

The different specifications used to estimate the elasticities of agglomeration and congestion in section 4 produced different point estimates, sometimes significantly different from one another. To account for this variation, I redo step three of the model's estimation procedure (section 4.3) and re-run the counterfactual experiments (section 5) using different values for these elasticities. To span the full range of estimated elasticities, I consider all possible combinations of $\eta \in \{0.15, 0.20, 0.25\}$ and $\beta \in \{0.5, 0.6, 0.8\}$. The counterfactual results are shown in table F.6, which

fixes the subsidy cap at 50%.

Note from table F.6 that, despite the wide variation in the values of those two parameters, the optimal subsidies all follow the same pattern: they increase the spatial concentration of the population, increasing the rate of creative destruction/growth, but decreasing the baseline wage of workers. Intuitively, higher values of $\eta$ and $\beta$ increase the gains from spatially concentrating the population: a high $\eta$ means a high elasticity of agglomeration and a high $\beta$ means a low elasticity of congestion.

Table F.6: Gains from adopting optimal subsidies, $\tau = 0.5$.

City-level Subsidies

| $\eta$ | $\beta$ | $\Delta$ Welfare | $\Delta$ Baseline Wage | $\Delta$ Creative Destruction | $\Delta$ Rate of Growth |
|---|---|---|---|---|---|
| 0.15 | 0.5 | 3.97% | −7.29% | 1.33 p.p. | 0.18 p.p. |
| 0.15 | 0.6 | 5.45% | −7.13% | 1.80 p.p. | 0.24 p.p. |
| 0.15 | 0.8 | 15.23% | −6.56% | 3.53 p.p. | 0.47 p.p. |
| 0.20 | 0.5 | 4.12% | −7.20% | 1.54 p.p. | 0.20 p.p. |
| 0.20 | 0.6 | 6.15% | −7.75% | 2.00 p.p. | 0.26 p.p. |
| 0.20 | 0.8 | 18.42% | −7.05% | 4.09 p.p. | 0.54 p.p. |
| 0.25 | 0.5 | 4.61% | −7.60% | 1.68 p.p. | 0.22 p.p. |
| 0.25 | 0.6 | 6.97% | −8.05% | 2.18 p.p. | 0.29 p.p. |
| 0.25 | 0.8 | 22.58% | −7.59% | 4.76 p.p. | 0.63 p.p. |

State-level Subsidies

| $\eta$ | $\beta$ | $\Delta$ Welfare | $\Delta$ Baseline Wage | $\Delta$ Creative Destruction | $\Delta$ Rate of Growth |
|---|---|---|---|---|---|
| 0.15 | 0.5 | 1.91% | −4.34% | 0.82 p.p. | 0.11 p.p. |
| 0.15 | 0.6 | 2.86% | −4.48% | 1.03 p.p. | 0.14 p.p. |
| 0.15 | 0.8 | 8.67% | −5.01% | 2.24 p.p. | 0.30 p.p. |
| 0.20 | 0.5 | 2.14% | −4.50% | 0.88 p.p. | 0.12 p.p. |
| 0.20 | 0.6 | 3.23% | −4.71% | 1.13 p.p. | 0.15 p.p. |
| 0.20 | 0.8 | 10.68% | −5.83% | 2.68 p.p. | 0.35 p.p. |
| 0.25 | 0.5 | 2.39% | −4.66% | 0.95 p.p. | 0.13 p.p. |
| 0.25 | 0.6 | 3.65% | −4.83% | 1.23 p.p. | 0.16 p.p. |
| 0.25 | 0.8 | 13.79% | −6.70% | 3.30 p.p. | 0.44 p.p. |

# G Extensions of the Model

The model presented in sections 2 and 3 predicts that all firms located in the same city, regardless of their size, will have the same arrival rate of innovation. Translating to the data, this means that big and small firms produce the same expected number of patents over any given year, which is course not true. In this section, I extend the model to allow the production of innovation to scale with firm size in the spirit of Klette and Kortum (2004). Keeping with the interpretation of maintenance and managerial costs, I assume that each firm's fixed costs also scale up with their size.

In this extended model, firms that own more product lines will also hire more inventors and produce more patents, but the innovation *intensity* of firms (patents per worker) will be constant

within a city. I will show that the predictions of this extended model concerning the spatial distribution of inventors and innovation and its relationship with economic growth are the exactly the same as the simplified model shown in the main text.

## G.1 The Extended Model

I start by redefining the innovation production function for a firm $f$ located in city $c$ as

$$x_{f,c}(t) = \chi_c(t)\left(\tilde{I}_c(t)^{\eta} i_{f,c}(t)\right)^{\psi} p_f(t)^{1-\psi} \tag{G.1}$$

where $p_f(t) = 1 + |\mathbf{q}_f(t)|$ and $\mathbf{q}_f(t)$ is the set of product lines owned by the firm. I define $p_f$ as the number of product lines plus one to maintain symmetry between entrant and incumbent firms (note that entrant firms do not own any product lines, so $\mathbf{q}_f = \varnothing$). I also assume that the fixed cost of innovation scales in the same way, so that firms must hire $\kappa p_f$ inventors to cover their fixed costs of innovation.

All of the remaining assumptions of the model in section 2 are kept, and any results not explicitly shown to be different will still apply to this extension (e.g., lemma 1). The HJB equation for an incumbent is thus given in lemma G.1, which is stated without proof (the proof is exactly analogous to lemma 2.

**Lemma G.1.** *The HJB equation that describes the problem faced by an incumbent firm located in city $c \in \{1, \ldots, C\}$ is*

$$rV_c(\mathbf{q}_f, \tilde{I}_c, Z_c, A) - \frac{\partial V_c(\mathbf{q}_f, \tilde{I}_c, Z_c, A)}{\partial A}\frac{\partial A}{\partial t} =$$

$$\max_{x_{f,c}} \left\{ \begin{array}{l} \displaystyle\sum_{q_j \in \mathbf{q}_f} \bar{\pi} L_0 q_j + x_{f,c}\mathbb{E}_j[V_c(\mathbf{q}_f \cup_+ \{(1+\lambda)q_j\}, \tilde{I}_c, Z_c, A) - V_c(\mathbf{q}_f, \tilde{I}_c, Z_c, A)] \\ \displaystyle -(1-s_c)w_c^i(i_{f,c} + \kappa p_f) - D\sum_{q_j \in \mathbf{q}_f}[V_c(\mathbf{q}_f, \tilde{I}_c, Z_c, A) - V_c(\mathbf{q}_f \backslash_- \{q_j\}, \tilde{I}_c, Z_c, A)] + R_c(\mathbf{q}_f, \tilde{I}_c, Z_c, A) \end{array} \right\}$$

$$x_{f,c} = \bar{\chi}_c Z_c(\tilde{I}_c^{\eta} i_{f,c})^{\psi} p_f^{1-\psi}$$

The HJB function of an entrant firm will be exactly analogous, with the exception that it own no product lines of its own. Building on lemma G.1, proposition G.1 presents the solution of the firm's problem.

**Proposition G.1.** *In a Stationary Balanced Growth Path Equilibrium where the total production of final goods $Y$ grows at rate $g < r$, the value function of an incumbent firm located in city $c \geq 1$ and whose portfolio of products is $\mathbf{q}_f$ is*

$$V_c(\mathbf{q}_f, \tilde{I}_c, Z_c, A) = F(D, L_0)\sum_{q_j \in \mathbf{q}_f} q_j + \max\left\{0, E_c(\tilde{I}_c, Z_c, w^i/Q, D, L_0)p_f Q\right\},$$

*where $F(D, L_0) = \bar{\pi}L_0/(r+D)$ is the "franchise value" of adding a new product to the portfolio and $E_c$ is the entry value for firms city $c$ (see the proof for a complete characterization).*

*In addition, the second stage value function of an entrant firm who is located in city $c$ is*

$$V_c^e(\tilde{I}_c, Z_c, A) = \max\left\{0, E_c(\tilde{I}_c, Z_c, w^i/Q, D, L_0)Q\right\}.$$

*Proof.* The proof of this proposition mimics the proof of proposition 1. I start by guessing the solution

$$V_c(\mathbf{q}_f, \tilde{I}_c, Z_c, A) = F\sum_{q_j\in\mathbf{q}_f} q_j + E_c p_f Q.$$

Substitute this into the firm's HJB to find

$$rF\sum_{q_j\in\mathbf{q}_f} q_j + rE_c p_f Q - \frac{\partial E_c}{\partial A}\frac{\partial A}{\partial t}p_f Q - E_c p_f \dot{Q} =$$

$$\max_{x_{f,c}}\left\{\begin{array}{l} \bar{\pi}L_0\sum_{q_j\in\mathbf{q}_f} q_j + x_{f,c}[F(1+\lambda)Q + E_c Q] - D\sum_{q_j\in\mathbf{q}_f} Fq_j \\ \\ -(1-s_c)w_c^i\left(\dfrac{x_{f,c}^{\frac{1}{\psi}}}{(\bar{\chi}_c Z_c)^{\frac{1}{\psi}}\tilde{I}_c^{\eta}p_f^{\frac{1-\psi}{\psi}}} + \kappa p_f\right) \\ \\ +\phi\left(\dfrac{\sigma^2}{4\phi} - \ln(Z_c)\right)Z_c\left[\dfrac{\partial E_c}{\partial Z_c} + \dfrac{\partial E_c}{\partial \tilde{I}_c}\dfrac{\partial \tilde{I}_c}{\partial Z_c}\right]p_f Q + \dfrac{(\sigma Z_c)^2}{2}\left[\dfrac{\partial^2 E_c}{\partial Z_c^2}\right]p_f Q \\ \\ +\dfrac{(\sigma Z_c)^2}{2}\left[\dfrac{\partial E_c}{\partial \tilde{I}_c}\dfrac{\partial^2 \tilde{I}_c}{\partial Z_c^2} + \dfrac{\partial^2 E_c}{\partial \tilde{I}_c^2}\left(\dfrac{\partial \tilde{I}_c}{\partial Z_c}\right)^2 + 2\dfrac{\partial^2 E_c}{\partial \tilde{I}_c \partial Z_c}\dfrac{\partial \tilde{I}_c}{\partial Z_c}\right]p_f Q \end{array}\right\}$$

The first-order condition is

$$[x_{f,c}]: \quad F(1+\lambda)Q + E_c Q - \frac{1}{\psi}\frac{(1-s_c)w_c^i x_{f,c}^{\frac{1-\psi}{\psi}}}{(\bar{\chi}_c Z_c)^{\frac{1}{\psi}}\tilde{I}_c^{\eta}p_f^{\frac{1-\psi}{\psi}}} \le 0.$$

If the optimal solution is interior, the arrival rate of an innovation is given by

$$x_{f,c} = p_f\bar{\chi}_c^{\frac{1}{1-\psi}}\left\{\psi[F(1+\lambda)+E_c]\frac{Q}{w^i}\frac{\alpha_c^{\frac{\theta}{1-\theta}}}{1-s_c}\tilde{I}_c^{\eta-\frac{(1-\beta)\theta}{1-\theta}}\right\}^{\frac{\psi}{1-\psi}}Z_c^{\frac{1}{1-\psi}}.$$

Plugging $x_{f,c}$ into the HJB equation and collecting terms with and without $\sum_{q_j\in\mathbf{q}_f} q_j$ gives

$$F(D, L_0) = \frac{\bar{\pi}L_0}{r+D}, \tag{G.2}$$

where the same argument made in the proof of proposition 1 applies to show that $F$ is constant

over time. In addition, using that $\dot{Q}/Q = g$,

$$
\begin{aligned}
(r-g)E_c - \frac{\partial E_c}{\partial A}\frac{\partial A}{\partial t} &= (1-\psi)\left\{\bar{\chi}_c[F(1+\lambda)+E_c]\right\}^{\frac{1}{1-\psi}} \left[\psi \frac{Q}{w^i}\frac{\alpha_c^{\frac{\theta}{1-\theta}}}{1-s_c}\tilde{I}_c^{\frac{\eta(1-\theta)-(1-\beta)\theta}{1-\theta}}\right]^{\frac{\psi}{1-\psi}} Z_c^{\frac{1}{1-\psi}} \\
&\quad -(1-s_c)\kappa\frac{w^i}{Q}\left(\frac{\tilde{I}_c^{1-\beta}}{\alpha_c}\right)^{\frac{\theta}{1-\theta}} + \phi\left(\frac{\sigma^2}{4\phi} - \ln(Z_c)\right) Z_c \left[\frac{\partial E_c}{\partial Z_c} + \frac{\partial E_c}{\partial \tilde{I}_c}\frac{\partial \tilde{I}_c}{\partial Z_c}\right] \\
&\quad + \frac{(\sigma Z_c)^2}{2}\left[\frac{\partial^2 E_c}{\partial Z_c^2} + \frac{\partial E_c}{\partial \tilde{I}_c}\frac{\partial^2 \tilde{I}_c}{\partial Z_c^2} + \frac{\partial^2 E_c}{\partial \tilde{I}_c^2}\left(\frac{\partial \tilde{I}_c}{\partial Z_c}\right)^2 + 2\frac{\partial^2 E_c}{\partial \tilde{I}_c \partial Z_c}\frac{\partial \tilde{I}_c}{\partial Z_c}\right],
\end{aligned} \tag{G.3}
$$

which implicitly defines $E_c = E_c(\tilde{I}_c, Z_c, w^i/Q, D, L_0)$.

If the optimal solution is a corner, then both the number of inventors hired and the fixed cost are zero. In this case it is straightforward to show that $F$, as defined in equation (G.2), and $E_c = 0$ solve the firm's HJB. Summarizing both cases, we can conclude that

$$V_c(\mathbf{q}_f, \tilde{I}_c, Z_c, A) = F(D, L_0)\sum_{q_j \in \mathbf{q}_f} q_j + \max\left\{0, E_c(\tilde{I}_c, Z_c, w^i/Q, D, L_0)Q\right\}.$$

**Entrant's Problem.** The entrant's problem is solved in exactly the same way. Repeating the argument made in the proof of proposition 1 shows that

$$V_c^e(\tilde{I}_c, Z_c, A) = \max\left\{0, E_c(\tilde{I}_c, Z_c, w^i/Q, D, L_0)Q\right\}.$$

□

I now impose free entry into all cities, which drives the value of entry to zero. Setting $E_c = 0$ in equation (G.3) determines the population of inventors in each city. Proposition G.2 shows how the economy is spatially distributed in this model.

**Proposition G.2.** *Imposing (1) free entry, (2) labor market clearing for both inventors and production workers, and (3) assuming a large number of cities $C \to \infty$ (so that the Law of Large Number applies and the average of city-specific shocks converges to its mean), the population of inventors in each city is given by*

$$I_c = I \times \frac{\left(\frac{\bar{\chi}_c}{1-s_c}\right)^{\frac{1-\theta}{\Theta}} \alpha_c^{\frac{\theta}{\Theta}}}{\sum_{c=1}^{C}\left(\frac{\bar{\chi}_c}{1-s_c}\right)^{\frac{1-\theta}{\Theta}} \alpha_c^{\frac{\theta}{\Theta}}} \times \frac{Z_c^{\frac{1-\theta}{\Theta}}}{e^{\frac{1-\theta}{\Theta}\left(\frac{1-\theta}{\Theta}-1\right)\frac{\sigma^2}{4\phi}}} \tag{G.4}$$

*where $\Theta = (1-\beta)\theta - \psi\eta(1-\theta)$. Moreover, the arrival rate of an innovation for a firm $f$ located in city $c$ is*

$$x_{f,c} = p_f\left(\kappa\frac{\psi}{1-\psi}\right)^{\psi}\bar{\chi}_c\tilde{I}_c^{\psi\eta}Z_c \tag{G.5}$$

*and the number of inventors hired by each firm in city $c$ is $i_{f,c} = \frac{\psi}{1-\psi}\kappa p_f$. Now let $N_c$ be the number of firms investing in R&D and located in city $c$, and $J_c$ be the total number of products produced by those*

*firms. Then,*

$$N_c + J_c = \left(\frac{1-\psi}{\kappa}\right) I_c \tag{G.6}$$

*Finally, it can also be shown that the population of production workers in city* $0$ *is proportional to* $L$ *(i.e.,* $L_0$ *does not vary over time), and that* $w^i$ *is not affected by the city-specific productivity shocks*

$$\frac{w^i}{Q} \propto \frac{\bar{\pi} L_0}{r+D} \left\{ \frac{1}{C} \sum_{c=1}^{C} \left( \frac{\bar{\chi}_c}{1-s_c} \right)^{\frac{1-\theta}{\Theta}} \alpha_c^{\frac{\theta}{\Theta}} \right\}^{\frac{\Theta}{1-\theta}}. \tag{G.7}$$

Again, those results are stated without proof since they are exactly analogous to the ones in proposition 2. The only difference in this case is when finding the number of active firms in each city in the equilibrium. This is done by requiring that all inventors living in the city are employed by some firm. Let $\mathcal{F}_c$ be the set of firms $f$ located in city $c$. Then

$$I_c = \int_{\mathcal{F}_c} (i_{f,c} + \kappa p_f) df,$$

where we integrate over the set of firms $f$ located in city $c$. Using the expression for $i_{f,c}$ and recalling that $p_f = 1 + |\mathbf{q}_f|$, this expression becomes

$$I_c = \frac{\kappa}{1-\psi} \int_{\mathcal{F}_c} (1 + |\mathbf{q}_f|) df.$$

Let $N_c$ be the number of firms located in city $c$ and $J_c$ the total number products that are produced by those firms. Then

$$I_c = \left(\frac{\kappa}{1-\psi}\right)(N_c + J_c),$$

which gives equation (G.6) after rearranging.

Finally, corollary G.1 determines the aggregate rate of creative destruction in this economy.

**Corollary G.1.** *The aggregate rate of creative destruction in this economy is*

$$D \propto \frac{1}{C} \sum_{c=1}^{C} \bar{\chi}_c \tilde{I}_c^{1+\psi\eta}, \tag{G.8}$$

*Proof.* The aggregate rate of creative destruction in this economy is given by

$$\begin{aligned} D &= \sum_{c=1}^{C} \int_{\mathcal{F}_c} x_{f,c} df \\ &= \left(\kappa \frac{\psi}{1-\psi}\right)^{\psi} \sum_{c=1}^{C} \bar{\chi}_c \tilde{I}_c^{\psi\eta} Z_c \int_{\mathcal{F}_c} p_f df \\ &= \left(\kappa \frac{\psi}{1-\psi}\right)^{\psi} \sum_{c=1}^{C} \bar{\chi}_c \tilde{I}_c^{\psi\eta} Z_c (N_c + J_c) \\ &= \psi^{\psi} \left(\frac{1-\psi}{\kappa}\right)^{1-\psi} \frac{1}{C} \sum_{c=1}^{C} \bar{\chi}_c \tilde{I}_c^{1+\psi\eta} Z_c. \end{aligned}$$

Note that this is exactly the same expression found in the proof of corollary 1. Since there is no change in how the local shock is defined and the population of inventors is allocated in the same way (proposition G.2), it follows that the rate of creative of creative destruction will also have the same structure as the one shown in corollary 1. □

## G.2 Estimation

The main challenge with this extended version of the model is estimating its parameters. In particular, equation (G.6) relates the number of firms, products and the population of inventors in each city. Because $J_c$ is included in that expression, I can no longer normalize the measure of products in the economy to 1 without affecting the scale of $N_c$ and $I_c$.

To determine the relative scale of $N_c$ and $J_c$, one could solve for the firm size distribution in each city. Let $\mu_c(q,t)$ be the measures of firms with $q$ products located city $c$ in period $t$. Since firms gain products at rate $x_{f,c}$ and loose them at rate $D$, it follows that

$$\frac{\partial \mu_c(q,t)}{\partial t} = x_{f(q-1),c}\mu_c(q-1,t) + (q+1)D\mu_c(q+1,t) - x_{f(q),c}\mu_c(q,t) - qD\mu_c(q,t)$$

where $x_{f(q),c}$ is the arrival rate of innovation for a firm with $q$ products. The first and second terms in the RHS of the equality account for the inflow of firms into size $q$: firms with $q-1$ products who gain a new product line and firms with $q+1$ products who loose one; the third and fourth terms account for the outflow: firms with $q$ product lines who gain one extra product and those who loose one product. Note from this differential equation that the measure of firms who own $q$ products in city $c$ is not stationary, as it depends on the realization of the local sock through the arrival rate $x_{f,c}$. As a result, the average number of products per firm in each city is also going to vary over time, making the relationship between $N_c$ and $J_c$ hard to pin down.

The simplified model in the main text will also have a non-stationary firm size distribution in each city. However, that model does not require the average number of products per firm to be determined before it is taken to the data (since the number of products does not affect the number of inventors hired). In addition, both the simplified and extended models make the same predictions about how R&D subsidies affect the distribution of the population and the aggregate rate of growth of the economy. For those reasons, the model introduced in section 2 is chosen for performing the counterfactual policy exercises in this paper.

PUBLICATIONS

**Agglomeration, Innovation, and Spatial Reallocation: the Aggregate Effects of R&D Tax Credits**
Working Paper No. **WP/2022/131**