Monetary Policy and Exchange Rate Dynamics in a Behavioral Open Economy Model

Marcin Kolasa, Sahil Ravgotra, and Pawel Zabczyk

WP/22/112

IMF Working Papers describe research in progress by the author(s) and are published to elicit comments and to encourage debate. The views expressed in IMF Working Papers are those of the author(s) and do not necessarily represent the views of the IMF, its Executive Board, or IMF management.
IMF Working Paper
Monetary and Capital Markets Department

Monetary Policy and Exchange Rate Dynamics in a Behavioral Open Economy Model
Marcin Kolasa, Sahil Ravgotra, and Pawel Zabczyk*

Authorized for distribution by Christopher Erceg
June 2022

IMF Working Papers describe research in progress by the author(s) and are published to elicit comments and to encourage debate. The views expressed in IMF Working Papers are those of the author(s) and do not necessarily represent the views of the IMF, its Executive Board, or IMF management.

ABSTRACT: We develop an extension of the open economy New Keynesian model in which agents are boundedly rational à la Gabaix (2020). Our setup nests rational expectations (RE) as a special case and it can successfully mitigate many “puzzling” aspects of the relationship between exchange rates and interest rates. Since the model implies an uncovered interest rate parity (UIP) condition featuring behavioral expectations, our results are also consistent with recent empirical evidence showing that several UIP puzzles vanish when actual exchange rate expectations are used (instead of realizations implicitly coupled with the RE assumption). We find that cognitive discounting dampens the effects of current monetary shocks and lowers the efficacy of forward guidance (FG), but its relative importance in mitigating the so-called FG puzzle is decreasing in openness. Finally, we show that accounting for myopia exacerbates the small open economy unit-root problem, makes positive monetary spillovers more likely, and increases the persistence of net foreign assets and the real exchange rate.

JEL Classification Numbers: F41, E70, E52, E58, G40

Keywords: Monetary Policy; Exchange Rates; Bounded Rationality

Author’s E-Mail Address: MKolas@IMF.org, S.Ravgotra@Surrey.ac.uk, PZabczyk@IMF.org

* We have benefitted from comments by our discussant Lahcen Bounader and those by Tobias Adrian, Stephane Dupraz, Chris Erceg, Xavier Gabaix, Herve Le Bihan, Jesper Linde, Julien Matheron, Ruperto Mujica and Felipe Zanna. We are also grateful to participants in seminars at the IMF and University of Surrey, as well as those in the Edinburgh MMF Conference and MCM Policy Forum. Ravgotra thanks the Monetary Policy Modeling Unit for hospitality during his 2021 summer internship. The views expressed here are those of the authors and do not necessarily represent the views of the IMF, its Executive Board, or IMF management.
1. Introduction

The role of expectations in determining the effectiveness of central bank actions has recently, and yet again, taken center stage. Faced with forces persistently pushing the natural interest rate down, and having to deal with large crises such as the Great Recession or COVID-19, monetary policy makers have had to increasingly rely on forward guidance to mitigate the adverse effects of reaching the effective lower bound (ELB) on interest rates. By treating agents as forward-looking, the workhorse New Keynesian (NK) policy framework provided clear mechanisms that gave such policies traction. Indeed, if anything, the problem was that of an “embarrassment of riches”, as they appeared counterfactually powerful, a feature which soon became known as the “forward guidance puzzle” (FGP) (see also Carlstrom, Fuerst, and Paustian 2015; Giannoni, Patterson, and Negro 2015; McKay, Nakamura, and Steinsson 2016, for excellent overviews).

Researchers have since taken several distinct routes in trying to address the FGP. The one of most consequence for our paper was to replace the assumption of full rationality, which has been shown to play a key role, with behavioral alternatives. Woodford (2019), for example, developed an NK framework in which cognitive constraints translated into a finite planning horizon, beyond which agents were “backward looking”. Gabaix (2020) proposed instead a “cognitive discounting” alternative, which introduced partial myopia towards future atypical events. Notably, and as recently demonstrated by Gust, Herbst, and Lopez-Salido (2021), such models can have benefits beyond mitigating the FGP as they also provide a better fit to the data than their “fully rational” New Keynesian ancestors.

Despite these advances, little is known about the implications of such behavioral assumptions in richer theoretical environments. Arguably, this is because rational expectations are a powerful simplifying device and consistently venturing into the “wilderness” beyond them significantly increases model complexity. For example, and as shown by Gabaix (2020), allowing for time-varying government bond supply in a behavioral NK model makes agents’ intertemporal choices directly depend on the current fiscal balance, breaking Ricardian equivalence in the process. As such, our

1 Other approaches relied on moving to a perpetual youth structure (Giannoni, Patterson, and Negro 2015), introducing heterogeneous agents operating in incomplete asset markets (McKay, Nakamura, and Steinsson 2016), introducing imperfect monetary policy credibility (Campbell, Ferroni, Fisher, and Melosi 2019; Haberis, Harrison, and Waldron 2019) as well as various departures from “common knowledge” (Carlstrom, Fuerst, and Paustian 2015; Kiley 2016; Angeletos and Lian 2018).

2 Both of these introduced similar forms of discounting into the (linearized) IS and Phillips curves, in line with Angeletos and Lian (2018), where aggregate myopia originates in uncertainty about other agents’ beliefs.
goal in this paper is to advance the associated research agenda by developing an open economy extension of Gabaix’s (2020) behavioral model, and by characterizing the implications of cognitive discounting in such an environment.

Our point of departure is the well-established, new open economy macroeconomics (NOEM) paradigm. We cast the analysis in a, now standard, incomplete asset market version of that environment to ensure that the economy’s net foreign asset position becomes a relevant endogenous state variable. We then show that the behavioral extensions – our model nests rational expectations as a special case – significantly modify several key equilibrium conditions. For example, one immediate implication is that cognitive discounting exacerbates the well-known unit root problem in small open economy models, requiring stronger remedial mechanisms such as endogenous risk premia.

More fundamentally, however, the behavioral extension we study goes a long way towards resolving several important anomalies governing the relationship of exchange rates and interest rates. These include the forward premium puzzle (Fama, 1984), the predictability reversal puzzle (Bacchetta and van Wincoop, 2010), the Engel puzzle (Engel, 2016), as well as the forward guidance exchange rate puzzle (Galí, 2020).

At the same time, and in contrast to some recently advocated “rational” solutions to those uncovered interest rate parity (UIP) puzzles, our model is also consistent with the empirical evidence presented in Kalemli-Ozcan and Varela (2021). The paper demonstrates, in particular, that the first three puzzles vanish when actual exchange rate expectations are used instead of realizations coupled with the rational expectations assumption (see also Canadian and De Leo, 2021). The reason behind our behavioral model’s success is that the uncovered interest rate parity (UIP) condition holds exactly (up to a small stationarizing premium) when formulated in terms of agents’ subjective expectations, but not when reformulated in terms of what it would be rational to expect. Expressed alternatively, testing various implications of UIP on our model simulated data under the assumption of rational expectations would lead to rejections of that joint hypothesis. Crucially, however, not because UIP fails to hold, but rather because agents in our model are not rational, and so ex post exchange rate realizations provide a biased read of their ex ante expectations.

4 The problem arises on account of asset market incompleteness and is discussed more fully in Schmitt-Grohé and Uribe (2003).
5 To mitigate the UIP-related puzzles, Bacchetta and van Wincoop (2021) propose a model of delayed portfolio adjustment, while Valchev (2020) and Itskhoki and Mukhin (2021) rely instead on financial frictions.
We also find that the underlying behavioral assumptions weaken the efficacy of future policy announcements and of ‘low for longer’ type policies. Therefore, and broadly in line with closed economy results, cognitive discounting also resolves the FGP in an open economy context. Similarly – and to the extent that current monetary shocks are persistent or have persistent effects on policy rates on account of interest rate smoothing – their influence on the economy is also dampened. There are, however, notable differences between closed and open economy behavioral models. The key one is that, as we show analytically, while the sensitivity of domestic prices in the fully rational case increases approximately linearly as a function of the forward guidance horizon, this is not the case for the exchange rate. This implies that the FGP is considerably less dramatic in an open economy to begin with, and hence myopia translates into relatively less dampening of future interest rate changes, at least in an environment of high exchange rate pass-through to import prices advocated by the dominant currency paradigm literature (Gopinath, Boz, Casas, Diez, Gourinchas, and Plagborg-Moller 2020).

Finally, we show that behavioral discounting has interesting implications for business cycle properties of open economy variables and for the size of international monetary policy spillovers. According to our model, the more heavily the future is discounted by agents, the more persistent net foreign assets and the real exchange rate become, an outcome that follows from agents underestimating the persistence of interest rate changes. We also demonstrate that monetary easing in one economy is more likely to be expansionary for its trading partners if these are populated by behavioral agents. This is because cognitive discounting weakens the expenditure switching channel associated with the reaction of the exchange rate.

Our paper is related to the literature developing structural models with bounded rationality and learning. One of the early contributions is Brock and Hommes (1997), who integrated heterogeneous expectations and a partial equilibrium cob-web model into the NK framework. Evans and Honkapohja (2001) and Bullard and Mitra (2002) introduced learning where agents forecast only immediate future variables. An alternative form of learning based on infinite horizons was promoted by Preston (2005). Branch and McGough (2009) and De Grauwe (2011) proposed models where some agents are rational while others either learn adaptively or follow simple rules-of-thumb. More recently, and as an alternative to the cognitive discounting setup that we are closest to, Bordalo, Gennaioli, and Shleifer (2018) formalized the concept of diagnostic expectations and demonstrated
how they can lead to financial cycles, while Bianchi, Ilut, and Saijo (2021) showed how this concept can be introduced into fully-fledged DSGE models. In an open economy setup, Llosa and Tuesta (2008) and Zanna (2009) analyze the equilibrium determinacy properties of monetary and exchange rate rules under adaptive expectations. More recently, Du, Eusepi, and Preston (2021) look at bounded rationality in a microfounded open economy, their key finding being that learning goes a considerable way towards better accounting for exchange rate dynamics.

The remainder of this paper is structured as follows. Section 2 presents the theoretical setup with two-countries and boundedly rational agents. In Section 3 we use a linearized small open economy version of the model to present how behavioral discounting changes the key equilibrium relationships, also covering the parameter values used in numerical experiments. Section 4 discusses how myopia of agents impinges on model stationarity and equilibrium determinacy. Section 5 shows how allowing for behavioral discounting helps resolve some open economy puzzles. In Section 6, we present analytically the implications of myopia for the transmission of “surprise” as well as anticipated changes in the real interest rates. Section 7 evaluates the dynamic effects of monetary policy, including both conventional and “low for longer” policies. In section 8 we present the impact of discounting on international monetary policy spillovers. Section 9 concludes.

2. Theoretical Setup

We develop a two-country NOEM model with myopic agents. We refer to one of the economies as Home and the other as Foreign. Both are populated by a continuum of households and monopolistically competitive firms. We normalize the world population to unity and use \( \zeta \in (0, 1) \) to indicate the share of Home agents, with the mass of Foreign agents equal to \( 1 - \zeta \). The two economies are linked by trade in goods and cross-border borrowing, and have separate monetary authorities. Since both countries are isomorphic, in the rest of this section we focus only on problems faced by Home agents.

2.1. Households. The household sector is populated by a large number of infinitely-lived dynasties. At any time \( t \), household \( h \) maximizes a discounted stream of period utility flows that depends on consumption \( C^h_t \) and labor supply \( N^h_t \)

\[
U^h_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[ \frac{(C^h_T)^{1-\sigma}}{1 - \sigma} - \frac{(N^h_T)^{1+\varphi}}{1 + \varphi} \right],
\] (1)
where $0 < \beta < 1$ is the subjective discount factor, $\sigma > 0$ is the inverse of the elasticity of intertemporal substitution, $\varphi > 0$ is the inverse of the Frisch elasticity of labor supply, and $\hat{E}_t$ indicates the expected value operator under the subjective expectations of households that we shall specify subsequently. The consumption basket is made of goods produced domestically $C^h_{H,t}$ and imports $C^h_{F,t}$, aggregated according to

$$C_t^h = \left[(1-\alpha)^{\frac{1}{\eta}} \left(C^h_{H,t} \right)^{\frac{\alpha-1}{\eta}} + \alpha \left(C^h_{F,t} \right)^{\frac{\alpha-1}{\eta}} \right]^{\frac{1}{\eta-1}}, \quad (2)$$

where $0 < \alpha < 1$ controls the degree of openness and $\eta > 0$ is the trade elasticity.

Households have access to one-period bonds denominated in Home currency $B^h_t$ and in Foreign currency $B^*_{t,h}$, of which only the latter is internationally traded, and which pay nominal interest rate $i_t$ and $i_{t}^{*}$, respectively. Labor is remunerated at the real rate $W_t$, and each dynasty also receives an aliquot share in real firm profits $D_t$. The real budget constraint can hence be written as

$$C^h_t + \frac{B^h_t}{1+i_t} + \frac{Q_t}{\Phi_t} \frac{B^*_{t,h}}{1+i_{t}^{*}} = \frac{B^h_{t-1}}{\Pi_t} + \frac{Q_t}{\Pi_t} \frac{B^*_{t-1}}{\Pi_{t}^{*}} + W_t N^h_t + D_t, \quad (3)$$

where $P_t$ and $P_t^*$ are the prices of the Home and Foreign consumption baskets, $\Pi_t = P_t/P_{t-1}$ and $\Pi_t^* = P_t^*/P_{t-1}^*$ are the associated gross inflation rates, $Q_t = \varepsilon_t P_t^*/P_t$ is the real exchange rate, with $\varepsilon_t$ denoting the units of domestic currency per unit of foreign currency, and $\Phi_t = \Phi(B^*_t)$ is a risk premium that depends on the Home country’s per capita net foreign asset (NFA) position.

### 2.2. Firms

Final goods sold domestically $Y_{H,t}$ and for exports $Y^*_{H,t}$ are made of intermediate inputs indexed by $f$ and aggregated according to the following Dixit-Stiglitz technology

$$Y_{H,t} = \left[ \int_0^1 \left( Y^f_{H,t} \right)^{\frac{1}{\mu}} df \right]^{\mu}, \quad \text{and} \quad Y^*_{H,t} = \left[ \int_0^1 \left( Y^{*f}_{H,t} \right)^{\frac{1}{\mu}} df \right]^{\mu}, \quad (4)$$

where $\mu > 1$.

Intermediate inputs are produced by monopolistically competitive firms that operate a linear production function in labor

$$Y^f_{H,t} + Y^{*f}_{H,t} = N^f_t. \quad (5)$$

Firms set the same prices for domestic and export sales, quoting them in domestic currency (producer currency pricing) so that $P^f_{H,t} = \varepsilon_t P^{*f}_{H,t}$ at every time $t$. They are subject to a Calvo-style friction. More specifically, each period only a fraction $0 < \theta < 1$ of firms is allowed to reoptimize
their prices. The problem of intermediate goods producers is then to maximize

$$V_t^f = \hat{E}_t \sum_{T=t}^{\infty} \theta^{T-t} \Lambda_{t,T} \left[ P_{H,t}^f (Y_{H,T}^f + Y_{H,T}^*) - W_T N_T^f \right],$$

subject to production technology \[5\] and demand constraints implied by aggregation \[4\]. Since firms are owned by households, they discount future profits using \(\Lambda_{t,T} \equiv \beta^{T-t} u_1 (C_T, N_T)\), which is the stochastic discount factor consistent with household preferences, and they use the same subjective expectations as in equation \[1\].

2.3. Myopia. While solving their problems, households and firms form subjective expectations denoted by the operator \(\hat{E}_t\). We deviate from rational expectations by assuming that agents are myopic and cannot correctly anticipate the evolution of variables that are beyond their control. More specifically, we follow Gabaix (2020) and assume that, when agents anticipate the future, they shrink their expectations toward some benchmark, which is assumed to be the economy’s steady state.

Formally, for any variable \(X_t\) that agents take as given during optimization, the perceived equilibrium law of motion is

$$X_{t+1} - X = m G^X (X_t - X, \epsilon_{t+1}),$$

where \(X_t\) is a vector of aggregate state variables, \(\epsilon_t\) is a vector of mean-zero innovations to stochastic processes driving economic fluctuations, \(G^X\) is the equilibrium aggregate policy function for variable \(X_t\), and where variables without time subscripts indicate steady state values, with \(0 \leq m \leq 1\) denoting a cognitive discounting parameter. The standard case of rational expectations can be readily obtained for \(m = 1\), while lower values of this parameter make agents myopic in that they expect future macroeconomic conditions to revert back to the steady state faster.

It is important to stress that agents misperceive laws of motion of variables beyond their individual control. More specifically, and as in Woodford (2013), households and firms do correctly perceive the constraints defining their problems, and hence their decisions are optimal, conditional on their subjective beliefs about the future evolution of variables that they take as given.

2.4. Monetary Authority. Unless indicated otherwise, the monetary authority follows a standard Taylor-like feedback rule

$$i_t = \rho i_{t-1} + (1 - \rho) \left[ i + \phi_\pi (\Pi_t - \Pi) + \phi_y \log(Y_t/Y) \right] + \nu_t,$$
where \(0 \leq \rho < 1\) controls the degree of interest rate smoothing, \(\phi_\pi\) and \(\phi_y\) determine the reaction to deviations of inflation \(\Pi_t \equiv P_t/P_{t-1}\) and output \(Y_t \equiv Y_{H,t} + Y_{H,t}^*\) from their steady state levels, and \(\nu_t\) denotes a monetary policy shock.

2.5. **General Equilibrium.** In equilibrium, all households make identical choices so that individual allocations are equal to aggregate per capita quantities, implying \(C_t^h = C_t, N_t^h = N_t, B_t^{*,h} = B_t^*, B_t^h = B_t = 0\), where the last equality follows from the fact that bonds denominated in Home currency can only be traded by Home households.

Labor supplied by households must be equal to labor demand, leading to the following condition

\[
N_t = \int_0^1 N_t^f \, df, \quad (9)
\]

while goods market clearing requires

\[
Y_{H,t} = C_{H,t}, \quad \text{and} \quad Y_{H,t}^* = \frac{1 - \zeta}{\zeta} C_{H,t}^*. \quad (10)
\]

3. **Linearized Model for a Small Open Economy**

3.1. **Linear Approximation to Behavioral Discounting.** For tractability, we consider a linearized version of the model defined in the previous section. As shown by Gabaix (2020), this simplifying assumption allows us to approximate behavioral \(k\)-period ahead expectations of any variable \(X_t\) that agents take as given during optimization as

\[
\hat{E}_t \{X_{t+k} - X\} = m^k E_t \{X_{t+k} - X\}, \quad (11)
\]

where \(E_t\) is the rational expectations operator. As this formula illustrates, agents are globally patient with respect to steady-state variables, but myopic with respect to deviations from the steady state, especially if those occur in the distant future.

One issue worth highlighting is that we cannot proceed without precisely defining the set of variables that agents form their expectations about. For example, it matters whether households think about the real exchange rate \(Q_t\) or its rate of depreciation \(Q_t/Q_{t-1}\) when making projections about the future. To see that, define \(\dot{Q}_t = Q_t - Q\), where \(Q\) is the steady state of \(Q_t\). We could then have
one of two alternatives

{\hat{E}_t \left\{ Q_{t+1} \right\}} = m{\hat{E}_t \left\{ \hat{Q}_{t+1} \right\}}

(12)

\hat{E}_t \left\{ \Delta \hat{Q}_{t+1} + \hat{Q}_t \right\} = m{\hat{E}_t \left\{ \hat{Q}_{t+1} \right\}} + \left( 1 - m \right) \hat{Q}_t.

While the choice is arguably somewhat arbitrary, we find it natural to assume that agents make projections about levels of variables that are constant in the steady state (top equality), as opposed to their rates of change (bottom equality).

3.2. Linearized Equilibrium Conditions. When linearizing the model, we focus on the small open economy case, which obtains as the limit when \( \zeta \to 0 \). We also assume zero steady state inflation (\( \Pi = 1 \)) and a zero net foreign asset position (\( B^* = 0 \)), which also implies \( C = Y \). We define the following transformations: \( \hat{i}_t \equiv \log(1+i_t) - \log(\beta - 1) \), \( \hat{\pi}_t \equiv \log(\Pi_t) \), \( \hat{B}^*_t \equiv (B^*_t Q_t - B^*)/Y \), with corresponding expressions for their Foreign analogs. All other ‘hat’ variables are defined as percent deviations from steady state, i.e. \( \hat{X}_t \equiv (X_t - X)/X \). Below we present and discuss the linearized equilibrium conditions. As deriving some of them is not straightforward, we outline the key associated steps in Appendix A.

Solving the household problem results in the following modified IS curve

\[ \hat{C}_t = m{\hat{E}_t \left\{ \hat{C}_{t+1} \right\}} - \frac{1}{\sigma} \left( \hat{i}_t - m{\hat{E}_t \hat{\pi}_{t+1}} \right) + \left( 1 - m \right) \frac{1 - \beta}{1 + \frac{\sigma}{\mu\phi}} \hat{B}^*_t. \]

(13)

The standard New Keynesian relationship can be immediately recovered by setting \( m = 1 \). If \( m < 1 \), expectations about future consumption and inflation are discounted, similarly to the closed economy New Keynesian model considered by Gabaix (2020). Notably, however, the move to an open economy setup is associated with an extra term in the behavioral IS curve, which now additionally depends on the country’s net foreign asset position. This term crops up because Equation 13 is derived using subjectively optimal consumption plans. More specifically, agents do not apply discounting to their individual choices, i.e. \( \hat{E}_t \hat{C}^{h}_{t+k} \neq m^k \hat{E}_t \hat{C}^{h}_{t+k} \) for \( k \geq 1 \), but do it only when forming expectations about variables beyond their control. As a result, following

---

6 In his baseline formulation, Gabaix (2020) assumes that agents correctly perceive the ex ante real interest rate, which means that expected inflation in Equation 13 is not discounted. This can be seen as yet another manifestation of a certain degree of arbitrariness when defining the set of variables that agents take as given while solving their optimization problem (discussed in the preceding section). Crucially, however, none of our main findings hinge on whether we follow Gabaix in assuming no misperception of the current real interest rate, or if we instead allow for some money illusion, as implicit in Equation 13.
a positive income shock that households want to smooth over by increasing their foreign bond holdings (net foreign assets in aggregate), the equilibrium response of consumption will be stronger than suggested by the expected path of future real interest rates (see also Appendix A.3 for more details).

Relatedly, optimal bond holdings of myopic households can be shown to imply an uncovered interest rate parity (UIP) condition

\[ \hat{i}_t - m \mathbb{E}_t \{ \hat{\pi}_{t+1} \} = \hat{i}_t^* - m \mathbb{E}_t \{ \hat{\pi}_{t+1}^* - \hat{Q}_{t+1} \} - \hat{Q}_t - \phi \hat{B}_t^*, \]  

where \( \phi = \Phi'(0) \). Again, a standard risk premium-augmented UIP condition obtains for \( m = 1 \).

Optimal price setting by myopic firms leads to the following Phillips curve for domestic prices

\[ \hat{\pi}_{H,t} = m \beta \mathbb{E}_t \{ \hat{\pi}_{H,t+1} \} + \frac{(1 - \theta)(1 - \beta \theta)}{\theta} \hat{M}C_t, \]  

which collapses to the canonical New Keynesian Phillips curve for \( m = 1 \). Note that, in line with Benchimol and Bounader (2019) and the principle stated in Section 3.1, we deviate from Gabaix (2020) while deriving equation 15 (see also Appendix A.4 for details). However, this deviation does not have a material impact on any of our key results.

The remaining equilibrium conditions are not affected by discounting. In particular, and as in Gali and Monacelli (2005), the real marginal cost, deflated by the producer price index is

\[ \hat{M}C_t = \sigma \hat{C}_t + \varphi \hat{Y}_t + \frac{\alpha}{1 - \alpha} \hat{Q}_t, \]  

the consumer price inflation is given by

\[ \hat{\pi}_t = \hat{\pi}_{H,t} + \frac{\alpha}{1 - \alpha} \left( \hat{Q}_t - \hat{Q}_{t-1} \right), \]  

and the aggregate goods market clearing condition can be written as

\[ \hat{Y}_t = (1 - \alpha) \hat{C}_t + \alpha \hat{Y}_t^* + \eta \frac{\alpha (2 - \alpha)}{1 - \alpha} \hat{Q}_t. \]  

Aggregating the budget constraints of all Home agents leads to the following law of motion for net foreign assets

\[ \hat{B}_t^* = \beta^{-1} \left( \hat{B}_{t-1}^* + \hat{Y}_t - \hat{C}_t \right). \]
Finally, the linearized version of the monetary policy rule is

\[ \hat{i}_t = \rho \hat{i}_{t-1} + (1 - \rho)(\phi_\pi \hat{\pi}_t + \phi_y \hat{Y}_t) + \nu_t. \]  

(20)

Equations 13 – 20 jointly define the equilibrium evolution of \( \hat{Y}_t, \hat{C}_t, \hat{\pi}_t, \hat{\pi}_H,t, \hat{MC}_t, \hat{Q}_t, \hat{i}_t, \hat{B}_t^* \), for given monetary policy shocks \( \nu_t \) and for given laws of motion of foreign variables \( \hat{Y}_t^*, \hat{\pi}_t^*, \hat{i}_t^* \), which are exogenous from the Home country perspective (on account of the small open economy assumption).

For given foreign monetary shocks \( \nu_t^* \), the equilibrium evolution of the three foreign variables is in turn described by the following conditions

\[ \hat{Y}_t^* = m \mathbb{E}_{t+1} \hat{Y}_{t+1}^* - \frac{1}{\sigma} \left( \hat{i}_t^* - m \mathbb{E}_t \hat{\pi}_{t+1}^* \right), \]  

(21)

\[ \hat{\pi}_t^* = m \beta \mathbb{E}_t \{ \hat{\pi}_{t+1}^* \} + \frac{(1 - \theta)(1 - \beta \theta)}{\theta} (\sigma + \varphi) \hat{Y}_t^*, \]  

(22)

\[ \hat{i}_t^* = \rho \hat{i}_{t-1}^* + (1 - \rho)(\phi_\pi \hat{\pi}_t^* + \phi_y \hat{Y}_t^*) + \nu_t^*, \]  

(23)

which simply follow from the same principles as for the Home economy, but additionally account for the fact that the Foreign economy can be treated as closed.

3.3. Parameter Values Used in Numerical Simulations. To investigate the implications of behavioral discounting in an open economy, we shall combine stylized experiments allowing us to gain analytical insights with numerical simulations designed to shed light on more general or involved questions. For the latter, we will need to assign numerical values to model parameters. To that end, we borrow from the NOEM literature, closely following the parametrization used in Gali and Monacelli (2005).

Table 1 summarizes our calibration choices for the small open economy version of our model, which we obtain by making the Home country size \( \zeta \) a very low number. We also make the economy fairly open by setting the import share parameter \( \alpha \) equal to 0.4. The discount factor \( \beta \) is calibrated at 0.99, which implies a steady state interest rate of 4 per cent per annum, consistent with the values observed in many emerging economies. The elasticity of substitution between domestically produced goods and imports \( \eta \), as well as that describing household intertemporal preferences \( \sigma \), are both calibrated at 1. The inverse of the Frisch elasticity of labor supply \( \varphi \) is set to 3 and the steady state markup in the goods market \( \mu \) is calibrated at 1.2. Compared to Gali and Monacelli (2005),
Table 1. Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.4</td>
<td>Openness</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.99</td>
<td>Discount Factor</td>
</tr>
<tr>
<td>$\eta$</td>
<td>1</td>
<td>Elasticity of Substitution Between Home and Foreign Goods</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>3</td>
<td>Inverse Frisch Elasticity of Labor Supply</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1</td>
<td>Inverse Intertemporal Elasticity of Substitution</td>
</tr>
<tr>
<td>$\mu$</td>
<td>1.2</td>
<td>Gross Product Markup</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.85</td>
<td>Calvo Probability of No Price Adjustment</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.9</td>
<td>Interest Rate Smoothing Parameters</td>
</tr>
<tr>
<td>$\phi_\pi$</td>
<td>1.5</td>
<td>Inflation Feedback Taylor Rule</td>
</tr>
<tr>
<td>$\phi_y$</td>
<td>0.125</td>
<td>Output Feedback Taylor Rule</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.01</td>
<td>Intermediation Costs Debt Sensitivity</td>
</tr>
</tbody>
</table>

we choose a higher value of the Calvo probability $\theta$ (0.85) to make our model consistent with more recent empirical evidence on the slope of the Phillips curve. As is standard in the DSGE literature, we also allow for interest rate smoothing in the monetary policy rule by setting $\rho$ to 0.9, and we assume standard feedback coefficients to inflation ($\phi_\pi = 1.5$) and the output gap ($\phi_y = 0.125$). As discussed in the next section, we set the slope of the risk premium in the UIP condition $\phi$ to a relatively small value of 0.01, which is sufficient to induce stationarity for empirically relevant degrees of myopia.

Since the goal of our paper is to investigate the impact of myopia, we take an agnostic stance on the size of cognitive discounting $m$ and show outcomes for its values ranging from 0.5 to 1. For reference, we note that Gabaix (2020) uses the value of 0.85, which implies that the consumer pays just over half as much attention to an innovation that is to come a year into the future relative to the attention that she pays to an innovation today. However, some papers suggest that even lower values of $m$ cannot be ruled out. For example, empirical estimates of the Euler equation by Fuhrer and Rudebusch (2004) are consistent with $m$ as low as 0.65. Gust, Herbst, and Lopez-Salido (2021) estimate a closed economy New Keynesian model with a finite planning horizon as in Woodford (2019), and arrive at discounting in the IS curve close to 0.5. Ilabaca, Meggiorini, and Milani (2020) allow for a different degree of behavioral discounting by households and firms, obtaining respectively 0.71 and 0.41.
4. Stationarity and Determinacy

It is well understood that in small open economy models with rational agents the assumption of incomplete asset markets engenders stationarity issues (see also Schmitt-Grohe and Uribe, 2003). Among several methods to induce stationarity considered in the NOEM literature, a debt-elastic risk premium on foreign bond holdings is by far the most popular (Senhadji, 2003), and in our setup it can be introduced by setting $\phi > 0$. It is also well known that standard New Keynesian models can generate sunspot equilibria when the interest rate does not respond sufficiently strongly to endogenous variables. For example, if the policy rate in our model only reacts to inflation ($\phi_y = 0$), determinacy requires it to respond more than one-for-one to deviations of inflation from target ($\phi_\pi > 1$, Taylor principle). The corresponding cannonical rational expectations case is illustrated in the bottom right panel of Figure 1, which demonstrates, in particular, that any positive risk premium ensures stationarity, while satisfying the Taylor principle guarantees uniqueness.

As the remaining panels reveal, things change dramatically when agents are myopic. Two effects are at play. First of all, behavioral discounting exacerbates the stationarity problem. To see why, it is instructive to inspect Equation 13, which clarifies that, except for the final term, the consumption path becomes explosive whenever $m < 1$. While the endogenous evolution of net foreign assets associated with the final term mitigates this effect somewhat, the associated feedback turns out to be too weak to induce stationarity. As a result, and for standard parameter values adopted in our calibration, the case for an additional stationarizing mechanism (such as the debt elastic premium, $\phi > 0$) becomes stronger when agents form behavioral rather than rational expectations. In addition, and unlike in the rational case, we cannot use an arbitrarily small positive $\phi$ to achieve stationarity. Intuitively, this is because myopic agents are less sensitive to future values of the risk premium, and so the responses of their consumption to temporary income shocks are too small to prevent boundless accumulation or decumulation of assets, unless $\phi$ is sufficiently large. This effect is stronger for higher degrees of myopia and can be observed in Figure 1 by noting that the explosive area tends to expand when $m$ becomes smaller.

---

7 The intuition is that when $m = 1$, and in the absence of a risk premium ($\phi = 0$), the Home real interest rate becomes tied to the (exogenous from the small economy’s perspective) foreign real interest rate via the UIP condition 14. The IS curve 13 then implies that consumption inherits a unit root.

8 Clearly, the size of this area also depends on other model parameters, such as the discount factor $\beta$. This is because, for given income and consumption paths, the rate of asset accumulation is governed by the real interest rate, the steady state value of which is $\beta^{-1}$ (see Equation 19). Therefore, in general, the lower the discount factor, the higher the $\phi$ needed to induce stationarity.
Figure 1. Stationarity and Determinacy Regions

**Note:** This figure shows the type of equilibrium in the linearized version of the model for different values of behavioral discounting $m$, UIP risk premium parameter $\phi_y$, and monetary policy feedback to inflation $\phi_\pi$. All other parameters are as in our baseline calibration described in Table 1 except that we set $\phi_y = 0$.

The second consideration – first documented by Gabaix (2020) in a closed economy setup – is that behavioral discounting shrinks the indeterminacy region, so that a weaker response of the policy rate to inflation may suffice to eliminate sunspot equilibria. This effect can be clearly seen in Figure 1, where the area corresponding to indeterminacy decreases in line with the cognitive discounting parameter $m$.

Moreover, Figure 1 also reveals how the two considerations discussed above interact in a non-trivial way. In particular, for moderate degrees of discounting – such as those in its counterdiagonal panels – stability may not require a debt-elastic risk premium, but can alternatively be achieved by sufficiently deviating from the Taylor principle. As is well known from the New Keynesian literature, in that case one eigenvalue moves inside the unit circle, thus ensuring model stability by offsetting the impact of the unstable root associated with the Euler equation. Notably, however,
this alternative way of ensuring determinacy only works for moderate degrees of discounting, with the upper left panel of Figure 1 highlighting a case in which stability can only be achieved by setting the risk premium parameter $\phi$ sufficiently high. For this reason, and also for ease of comparison with extant New Keynesian contributions, in the remainder we ensure stationarity by setting $\phi = 0.01$. This proves sufficient for all the values of $m$ that we consider, while being sufficiently small to avoid unduly affecting the short-term dynamics of our model.

5. Exchange Rate Dynamics

As discussed in Section 3, our model implies a standard uncovered interest rate parity condition, except for the fact that the expectation operators appearing in it are behavioral rather than rational. Once these behavioral terms are reexpressed using their rational equivalents, we arrive at the “behavioral” UIP condition (Equation 14), which is repeated here for convenience

$$\hat{i}_t - mE_t \{\hat{\pi}_{t+1}\} = \hat{i}_t^* - mE_t \{\hat{\pi}_{t+1}^* - \hat{Q}_{t+1}\} - \hat{Q}_t,$$

and where we define $\hat{i}_t^* \equiv \hat{i}_t^* - \phi \hat{B}_t^*$ as the “premium adjusted” foreign nominal interest rate. These observations have several strong and testable implications. First, we would expect the standard UIP condition to hold when actual expectations are used instead of their rational equivalents, particularly if actual expectations were, in fact, formed in a behavioral fashion. The former appears in line with a growing body of evidence. For example, Kalemli-Ozcan and Varela (2021) show that there are no overshooting and predictability reversal puzzles – for any currency – when using actual exchange rate expectations to calculate the UIP premium. Our model therefore appears broadly consistent with these new empirical findings.

While the results above are reassuring, it is also true that there are many possible ways of deviating from rational expectations and it does not necessarily follow that the behavioral discounting route proposed by Gabaix (2020) performs well in an open economy context. In what follows we therefore

---

9 We define the “premium adjusted” nominal interest rate for analytical convenience as it allows us to write the UIP condition in its canonical form. Note that, since we calibrate $\phi$ to be small, $\hat{i}_t^*$ is very close to $\hat{i}_t$. We have verified that all results presented further in this section, and which use the “premium adjusted” foreign rate, are very similar to those obtained without such an adjustment.

10 Starting with the influential work of Fama (1984), a large literature used realized exchange rates to document violations of the UIP condition, showing, in particular, that high interest rate currencies do not sufficiently depreciate, implying – on average – excess investment returns from carry trade strategies.
analyze the forward premium puzzle of [Fama (1984)] as well as the predictability reversal puzzle of [Bacchetta and van Wincoop (2010) and Engel (2016)] through the lens of our model. Crucially, we demonstrate that our behavioral framework can match the patterns underlying those two sets of puzzles, and we also explain the economic mechanism behind its empirical success.

5.1. **Forward Premium.** Under uncovered interest rate parity, the domestic currency is expected to depreciate if the home interest rate exceeds the foreign. As alluded to above, however, [Fama (1984)] famously showed that this simple prediction is at odds with the data, where high interest currencies tend to offer higher returns even when an exchange rate depreciation is fully factored in. We now investigate whether our behavioral model can help account for this feature of the data.

We first observe that behavioral agents correctly perceive the current level of the real exchange rate and only discount its future level. Accordingly, the UIP condition (14) can be rewritten as

$$\hat{i}_t - i^*_t = m \mathbb{E}_t \{\Delta \hat{\epsilon}_{t+1}\} - (1 - m) \hat{Q}_t.$$  \hspace{1cm} (25)

Broadly, this equation points to two opposing forces affecting the evolution of the exchange rate. On the one hand, the fact that the expected future depreciation is discounted tends to generate lower returns on high-interest currencies, thus deepening the Fama puzzle. On the other hand, the real interest rate adjustment increases ex post returns owing to the negative correlation between interest rates and the real exchange rate.$^{11}$

To move beyond such qualitative statements and to compare the relative contributions of both terms, we first rewrite the behavioral UIP condition as

$$\mathbb{E}_t \{\Delta \hat{\epsilon}_{t+1}\} = \frac{1}{m} \left( \hat{i}_t - i^*_t \right) + \left( \frac{1}{m} - 1 \right) \hat{Q}_t.$$ \hspace{1cm} (26)

This formulation is designed to resemble ‘Fama (1984) regressions’ typically used to document the forward premium puzzle, i.e.,

$$\Delta \hat{\epsilon}_{t+1} = a_0 + a_1 \left( \hat{i}_t - i^*_t \right) + \epsilon_t,$$  \hspace{1cm} (27)

where empirical estimates suggest values of the slope coefficient close to zero, or even negative, while standard UIP counterfactually implies $a_1 = 1$. Population regression techniques combined

$^{11}$It is straightforward to verify that under a specification in which agents discount changes in variables the last term would vanish, exacerbating rather than addressing the forward premium puzzle. Similar issues would also arise when discounting nominal / non-stationary variables.
with the omitted variable bias formula allow us to express the value of the $a_1$ coefficient as

$$
\mathbb{E}a_1 = \frac{1}{m} + \left( \frac{1}{m} - 1 \right) \frac{\mathbb{E} \{ \hat{Q}_t (\hat{i}_t - \hat{i}_t^*) \}}{\mathbb{E} \{ (\hat{i}_t - \hat{i}_t^*)^2 \}},
$$

$$
= \frac{1}{m} + \left( \frac{1}{m} - 1 \right) \text{Corr} \left\{ \hat{Q}_t, \hat{i}_t - \hat{i}_t^* \right\} \frac{\text{Std} \{ \hat{Q}_t \}}{\text{Std} \{ \hat{i}_t - \hat{i}_t^* \}}
$$

(28)

where, to fix attention, and as is standard in the literature, we subsequently focus on the case in which monetary policy shocks are key drivers of real exchange rate and interest rate dynamics.

We first observe that for $m = 1$ the second term vanishes and so $\mathbb{E}a_1 = 1$, replicating the original forward premium puzzle. We also see that as $m$ becomes lower than unity, the first term pushes the model-implied regression coefficient above one, exacerbating the puzzle. Offsetting that, the second term is negative for $m < 1$ as when home monetary shocks are dominant, the real exchange rate is almost perfectly negatively correlated with the interest rate differential (or the home interest rate in a small open economy setup).

Whether discounting helps address the Fama puzzle thus crucially depends on the variability of the real exchange rate relative to that of the interest rate. Since the former clearly exceeds the latter in floating exchange rate regimes, and also in our model, there are reasons to expect improvements in fit. In Table 2 we confirm that conjecture by plugging in exact values of the standard deviations into Equation (28) and computing the slope coefficient for different combinations of the interest rate smoothing parameter ($\rho$) and discounting ($m$).\(^\text{12}\)

| TABLE 2. Fama Regression Coefficients |
|-----------|---------|---------|---------|---------|
| PARAMETER | $m = 0.50$ | $m = 0.75$ | $m = 0.90$ | $m = 1.00$ |
| $\rho = 0.95$ | -0.07 | -0.04 | 0.37 | 1.00 |
| $\rho = 0.90$ | 0.17 | 0.36 | 0.67 | 1.00 |
| $\rho = 0.75$ | 0.51 | 0.69 | 0.86 | 1.00 |
| $\rho = 0.50$ | 0.75 | 0.86 | 0.94 | 1.00 |

Overall, we find that large, but empirically plausible\(^\text{13}\) values of discounting coupled with high interest rate smoothing are capable of generating negative values of the Fama coefficient, in line with

\(^{12}\) We have also corroborated these numbers using Monte Carlo techniques, running actual Fama regressions on long samples of simulated data.

\(^{13}\) As noted previously, Gust, Herbst, and Lopez-Salido (2021) estimate cognitive discounting to be around 0.5.
empirical results based on realized returns. Intuitively, the reason why such parameter combinations
perform well is because higher values of smoothing decrease the volatility of interest rates relative
to the real exchange rate. This, in turn, occurs because real exchange rates depend on the whole
future path of interest rates, and the fact that these become positively autocorrelated under greater
smoothing tends to amplify the variance of the sum (i.e., makes it exceed the sum of individual
variances by more).

5.2. **Predictability Sign Reversal.** We now turn to Engel-style regressions and discuss our
model’s ability to match the empirical patterns documented in [Bacchetta and van Wincoop (2010)]
and confirmed in real form by [Engel (2016)]. To that effect, we focus on regressions specified as

\[ r_{t+1}^x \approx \hat{i}_t - \hat{i}_t^* - \Delta \hat{e}_{t+1} = b_{s,0} + b_{s,1} \left( \hat{i}_{t-s} - \hat{i}_{t-s}^* \right) + \epsilon_t, \quad (29) \]

where \( s = 0, 1, \ldots \). As first shown by [Bacchetta and van Wincoop (2010)], the coefficient \( b_{s,1} \) turns
from positive to negative in the data for some \( s \). [Engel (2016)] additionally argues that the data
satisfy an even stronger requirement, namely that

\[ \text{Cov} \left\{ \mathbb{E}_t \sum_{s=0}^{\infty} r_{t+s+1}^x, \hat{i}_t - \hat{i}_t^* \right\} < 0, \]

which can be shown to be equivalent to \( \sum_{s=0}^{\infty} b_{s,1} < 0 \). An immediate implication of this condition (Engel
condition henceforth) is that the level of the high-yielding country’s exchange rate is stronger than
implied by UIP, or, expressed alternatively, that exchange rate volatility exceeds what could be
predicted based on the simple uncovered interest rate parity condition.

To relate our open economy behavioral model to these stylized facts, Figure 2 provides an overview
of its key implications. Specifically, the left panels show our model-implied \( b_{s,1} \) coefficients as
a function of \( s \) (x-axis) for values of interest rate smoothing ranging from \( \rho = 0.95 \) in the top
panel to \( \rho = 0.5 \) in the bottom one. To highlight the role of discounting, each chart covers four
different \( m \) values, varying from \( m = 1 \) (purple line, no discounting / rational expectations) to
\( m = 0.5 \) (light-blue line, discounting in line with [Gust, Herbst, and Lopez-Salido (2021)]). Given
the equivalence between \( \text{Cov} \left\{ \mathbb{E}_t \sum_{s=0}^{\infty} r_{t+s+1}^x, \hat{i}_t - \hat{i}_t^* \right\} < 0 \) and infinite sums of \( b_{s,1} \) appearing in
the Engel condition, the right-hand panels additionally plot the cumulative sums \( \sum_{s=0}^{T} b_{s,1} \) for
values of \( T \) ranging from 0 to 500 (at which point there are no discernible differences between the
truncated and infinite sums).

---

\[ ^{14} \text{Note that for } s = 0 \text{ the above is equivalent to a Fama-type regression (Equation 27), in which } b_{0,1} = 1 - a_1. \]
Figure 2 confirms two important findings. First, and as shown in the left panels, cognitive discounting is capable of generating sign reversals in Engel regression coefficients. We see, in particular, that $b_{s,1}$ eventually becomes negative for all values of the discounting parameter $m$ except for the rational expectations (RE) case of $m = 1$. It thus appears that the RE model’s inability to match the sign reversal findings of Bacchetta and van Wincoop (2010) is a knife-edge result specific to “full rationality”; even if, admittedly, the sign-flips we document are not large. Second, and as
made clear by the right panels, our model can account for Engel’s excess volatility puzzle provided that there is sufficient cognitive discounting and that interest rate smoothing is not excessive.

To provide intuition for these findings we start by expressing excess returns as

\[ E_t r_{t+1}^x \equiv \hat{r}_t - \hat{r}_t^* - E_t \{ \Delta \hat{\varepsilon}_{t+1} \} = (m - 1) E_t \left\{ \hat{Q}_{t+1} + \hat{\pi}_{t+1} - \hat{\pi}_{t+1}^* \right\}. \] (30)

We first note that, as discussed at length in Section 7.1, the real exchange rate depreciates persistently in response to domestic policy easing. This initially leads to an accumulation of net foreign assets on account of greater price competitiveness. Over time, however, the real exchange rate ends up appreciating relative to its steady state, which facilitates net foreign asset decumulation.

An implication of Equation 30 is that, in the presence of discounting, the risk premium depends negatively on the real exchange rate. Accordingly, the evolution of the premium will be the mirror image of what we described above, i.e., an increase in the interest rate differential will lead to a persistent increase followed by a fall below zero. Which is precisely the sign reversal documented in Figure 2.

The intuition behind the second stylized fact is closely related to the argument of Bacchetta and van Wincoop (2021) who show that – in their gradual portfolio adjustment model – the Engel condition is violated whenever interest rate inertia is high. In both cases, higher interest rate autocorrelation amplifies the initial response of excess returns, which also inherit some of the underlying persistence, the confluence of which implies that they are not offset by future reversals.

6. MONETARY POLICY TRANSMISSION - INSPECTING THE MECHANISM

We now investigate the impact of discounting on monetary transmission, highlighting, in particular, the role played by openness. Our goal is to provide tractable, analytical foundations by working with Home and Foreign real interest rates defined, respectively, as \( \hat{r}_t \equiv \hat{r}_t - m E_t \hat{\pi}_{t+1} \) and \( \hat{r}_t^* \equiv \hat{r}_t^* - m E_t \hat{\pi}_{t+1}^* \). Except for slight differences in these definitions, our approach closely follows that of Gabaix (2020), i.e., we shall obtain valuable insights into how monetary policy and forward guidance propagate by studying the impact of real interest rate changes in selected periods, while holding their values fixed at all other horizons.

\[ \text{To reiterate: in contrast to Gabaix (2020), we do not assume away the impact of biases in inflationary perceptions on the real interest rate.} \]
6.1. **Real Exchange Rate.** Our point of departure is the discussion of how monetary policy affects the real exchange rate (RER). Because our focus is on the effects of domestic monetary policy actions in a small open economy, we treat all foreign variables as fixed. Accordingly, the RER reaction provides an efficient summary of key differences between the closed and open economy setups.

First, by iterating forward on the UIP condition\[14\] we arrive at

\[
\hat{Q}_t = -E_t \sum_{T=t}^{\infty} m^{T-t} \left( \hat{r}_T - \hat{r}_T^* + \phi \hat{B}_T^* \right),
\]

where responses of the real exchange rate to real interest rate changes at different horizons are also depicted in the left panel of Figure \[3\][16]. Both the formula and the figure make it immediately apparent that cognitive discounting dampens the effects of future real interest rate changes on the current exchange rate, making them less relevant the longer the horizon. In so doing, the move from rational expectations to behavioral discounting thus appears to immediately address the exchange rate forward guidance puzzle of \cite{Galí2020}.

**Figure 3. Effects of Forward Guidance: Real Exchange Rate**

Note: This figure shows the normalized initial response of the real exchange rate as a function of forward guidance horizon. The left panel depicts the case where the UIP premium $\phi = 0.01$ and the right panel corresponds to the case of no UIP premium ($\phi = 0$). In both the cases, the purple lines indicate no discounting, i.e., $m = 1$, while the blue, red and yellow lines correspond to values of $m$ equal to 0.5, 0.75 and 0.9 respectively.

It is worth noting, and may seem puzzling, that the RER response depicted in Figure 3 is declining in FG horizon at a rate faster than $m$, and even absent discounting (e.g., the purple line corresponding

\[16\] We normalize by dividing through the response to an unanticipated shock at horizon zero.
to $m = 1$ shows decay even though one could, in principle, expect a flat line). \[17\] This apparent discrepancy – between the rate of decay as a function of forward guidance horizon and $m$ – arises because of the endogenous response of net foreign assets. The strength of that response varies depending on FG horizon and it feeds back to the UIP premium. The right panel of Figure 3 corroborates this conjecture by applying Equation [31] while keeping $\phi$ equal to 0, and it shows, in particular, that in that case the purple line remains flat.

To clarify why NFA responds differently at different FG horizons, even for $m = 1$, it proves useful to iterate forward on the consumption Euler condition (Equation [13]) to obtain the following relationship

$$\hat{C}_t = -\frac{1}{\sigma} \mathbb{E}_t \sum_{T=t}^{\infty} m^{T-t} \hat{r}_T + (1 - m) \frac{1 - \beta}{1 + \frac{\nu}{\mu \phi}} \mathbb{E}_t \sum_{T=t}^{\infty} m^{T-t} \hat{B}_T. \quad (32)$$

Absent discounting, and because we fix $\hat{r}_t = 0$ at all horizons except for the forward guidance horizon $H$, the expression simplifies to

$$\hat{C}_t = \begin{cases} \forall t \leq H : & -\frac{1}{\sigma} \hat{r}_H, \\ \forall t > H : & 0. \end{cases}$$

We thus see that the initial response of consumption does not depend on $H$, and that consumption remains elevated at that higher level for the entire duration of forward guidance. For example, an anticipated 1pp decrease in the real interest rate ten periods ahead (holding real rates in all other periods unchanged) implies that consumption will be $\frac{1}{\sigma}$ percent above the steady state level for exactly eleven periods. As we shall now discuss, a similar relationship can be derived for output (see Equation [34]), with the crucial difference that, for realistic calibrations, the interest rate elasticity exceeds $\frac{1}{\sigma}$. This means that the longer the horizon of an anticipated decrease in the real interest rate, the longer the period over which the economy generates trade surpluses, which eventually translate into larger NFA accumulation.

6.2. Output. Combining the consumption Euler Equation [13] with the resource constraint [18] and the UIP condition [14] and using the small open economy assumption to eliminate foreign variables,
allows us to derive an IS curve for output[^8]

\[
\hat{Y}_t = mE_t \hat{Y}_{t+1} - \left( \frac{1 - \alpha}{\sigma} + \eta \frac{\alpha(2 - \alpha)}{1 - \alpha} \right) \hat{r}_t - \left[ \eta \frac{\alpha(2 - \alpha)}{1 - \alpha} - \frac{1 - \beta}{1 + \frac{\sigma}{\mu \phi}} \right] \hat{B}_t^*. \tag{33}
\]

Iterating forward on this equation yields

\[
\hat{Y}_t = - \left( \frac{1 - \alpha}{\sigma} + \eta \frac{\alpha(2 - \alpha)}{1 - \alpha} \right) E_t \sum_{T=t}^{\infty} m^{T-t} \hat{r}_T
\]

\[
- \left[ \eta \frac{\alpha(2 - \alpha)}{1 - \alpha} - \frac{1 - \beta}{1 + \frac{\sigma}{\mu \phi}} \right] E_t \sum_{T=t}^{\infty} m^{T-t} \hat{B}_T^*. \tag{34}
\]

where the closed economy case readily obtains by setting \( \alpha = 0 \) and imposing \( \hat{B}_t^* = 0 \) for all \( t \).

Initially focusing on the first line allows us to derive some key predictions. First we observe that, relative to a closed economy, the same change in the real interest path will have a stronger direct effect on output, unless trade elasticity \( \eta \) is very low. More specifically, the formal criterion can be stated as

\[
\eta < \frac{1 - \alpha}{2 - \alpha} \sigma^{-1} \leq \frac{1}{2} \sigma^{-1},
\]

which is satisfied for typical parameterizations used in the open economy macroeconomic literature, including papers allowing for low trade elasticity[^9]. In addition, and in line with the closed economy case, discounting dampens the effects of real interest rate changes occurring further into the future, helping mitigate the output part of the FGP.

The effect of behavioral discounting on the initial response of output to current and future real interest rate changes of a given size is depicted in the first row of Figure 4. As done for the real exchange rate, we normalize by the contemporaneous output response, with the chart clearly showing that lower \( m \) translates into a faster decay of FG efficiency (as a function of its horizon), both in the open and closed economy cases.

The differences between the open and closed economy cases can be traced back to the endogenous net foreign asset response, i.e., the second line of Equation 34 (which is absent in the closed economy case, in which \( B_t^* \equiv 0 \)). Following a pattern similar to the one described before, a longer FG horizon leads to greater accumulation of net foreign assets, which tends to depress output. The latter follows from the fact that the term in the square brackets is positive, at least for values of \( \phi \)

[^8]: See also Appendix B for details of the derivation.
[^9]: See Bodenstein (2010) for a discussion of the consequences of low trade elasticity.
Figure 4. Effects of Forward Guidance: Open vs Closed Economy

Note: This figure shows the normalized initial response of output and inflation as a function of forward guidance horizon. The left panel represents the open economy with UIP premium of 0.01 and the right panel represents the closed economy. In both the cases, the purple lines correspond to no discounting, i.e., $m = 1$, while the blue, red and yellow lines indicate values of $m$ equal to 0.5, 0.75 and 0.9 respectively.

6.3. Inflation. We first focus on the domestic component of inflation, which is characterized in Equation 15 repeated here for ease of reference

$$
\hat{\pi}_{H,t} = \beta m \mathbb{E}_t \{ \hat{\pi}_{H,t+1} \} + \left( \frac{1 - \theta}{\theta} \right) \left( \sigma \hat{C}_t + \varphi \hat{Y}_t + \frac{\alpha}{1 - \alpha} \hat{Q}_t \right),
$$

necessary to induce stationarity. Overall, and in line with Figure 4, the rate of decay is faster in an open economy, which actually also holds absent discounting. Crucially, however, when $m = 1$ FG remains very efficient over very long horizons as the purple line does not go down to zero, but rather asymptotes at around 0.6, consistent with studies confirming the presence of the FGP also in rational expectations open economy models.
and which essentially highlights that marginal costs depend on consumption, output and the real exchange rate. Building on the preceding analysis, we know that – relative to the closed economy case – a change in the real interest rates (current or future) translates into unchanged (if \( m = 1 \)) or greater responses of consumption, greater responses of output (for typical values of trade and intertemporal elasticities), with real exchange rate depreciations additionally boosting marginal cost. It then follows, as an immediate corollary of Equation (17), that higher responsiveness of domestic inflation to real interest rate changes also translates into greater responsiveness of CPI inflation.

The second row of Figure 4 highlights that the FGP for inflation is much less pronounced in an open economy, even without discounting. The reasons why may not appear immediately obvious, particularly as domestic inflation is the discounted sum of marginal costs, and all three marginal cost components depend on the future path of real interest rates. After some algebra, and invoking the small open economy assumption and the associated exogeneity of foreign rates, we can characterize the relationship between inflation and domestic real rates as follows:

\[
\hat{\pi}_t \approx -\frac{\kappa A}{\beta(1-m)} \mathbb{E}_t \sum_{T=t}^{\infty} \left[ m^{T-t+1} - (\beta m)^{T-t+1} \right] \hat{r}_T - \frac{\alpha}{1-\alpha} \mathbb{E}_t \sum_{T=t}^{\infty} m^{T-t} \hat{r}_T - \frac{\alpha}{1-\alpha} \hat{Q}_{t-1},
\]

(35)

where we have defined

\[
A \equiv \phi \left( \frac{1-\alpha}{\sigma} + \eta \frac{\alpha(2-\alpha)}{1-\alpha} \right) + \frac{1}{1-\alpha},
\]

and where the approximation comes from omitting several terms loading on NFA, which are quantitatively very small, and which would otherwise obscure the non-monotonic relationship between forward guidance horizon \( T - t \) and inflation.

Focusing on the limiting case of Equation (35) as \( \beta \to 1 \), we arrive at

\[
\hat{\pi}_t \approx -\kappa A \mathbb{E}_t \sum_{T=t}^{\infty} (T-t+1) m^{T-t} \hat{r}_T - \frac{\alpha}{1-\alpha} \mathbb{E}_t \sum_{T=t}^{\infty} m^{T-t} \hat{r}_T - \frac{\alpha}{1-\alpha} \hat{Q}_{t-1},
\]

(36)

where the first term captures effects due to marginal cost, while the other one is related to the direct effects of exchange rates on prices of imported goods. Notably, the key implications of Equation (36) are perfectly consistent with the patterns documented in Figure 4. In particular, and because of the first component (the only one present in the closed economy case), which is a product of exponential

---

20 See Appendix B for full details of the derivation.
decay and linear growth, the relationship between the effect on inflation and FG horizon is linear for no discounting \( m = 1 \), non-monotonic for small discounting (increasing and then decreasing), and decreasing for strong discounting.\(^{21}\) In an open economy the picture is made more complicated by the second component, which decays exponentially, and the relative weight on which increases in openness \( \alpha \). As a consequence, the forward guidance puzzle is less pronounced in an open economy implying that the mitigating effect of discounting on the FGP is also relatively smaller compared to the closed economy case.

7. **Dynamic Effects of Monetary Policy**

7.1. **Conventional Monetary Shocks.** Having developed intuition on how cognitive discounting affects monetary transmission, we now move on to more standard policy experiments. We first study the dynamic responses to monetary policy (MP) shocks, defined conventionally as innovations in the policy rule describing the evolution of the nominal rate.\(^{20}\) Broadly, since our empirically motivated Taylor rule features interest rate smoothing, the outcomes shall end up being a combination of standard monetary policy and forward guidance shocks studied in the preceding sections. Given these tight links, in what follows we augment numerical simulations using the analytical results just described.

Figure 5 presents impulse responses to 100bp (annualized) worth of conventional monetary easing, contrasting the closed economy version of our model (row 1) with its fully-fledged, open economy counterpart (rows 2 and 3). We immediately see that discounting fairly efficiently dampens the effects of current monetary policy shocks. Intuitively, this occurs because under interest rate smoothing the policy rate will be lower for some time into the future, which behavioral agents “cognitively discount”, i.e., they expect a less low path than the one that ends up materializing. Consistently with analytical results from the previous section, discounting matters more for domestic inflation than for the exchange rate, and hence also for CPI. As output depends on the CPI-deflated real interest rate, its response is also less sensitive to discounting in the open economy case.

Another notable open economy finding is that discounting tends to make the accumulation of net foreign assets more persistent. Consistently with the line of reasoning provided above, agents underestimate the persistence of future interest rates and so end up being surprised by their actual

\(^{21}\) This explains the hump shape also documented, though not accounted for, in Gabaix (2020).
7.2. Low for Longer Policies. We can now focus on a variant of policies often dubbed as “low for longer” (LFL). The idea, due to Reifschneider and Williams (2000), is that if the policy rate cannot be lowered by the desired amount, it may be cut by less, but kept at that level for a prolonged period of time. These policies have been typically considered effective according to model-based analyses (Kiley and Roberts, 2017), which is perhaps unsurprising as the commitment to keeping rates low means that LFL policies inherit some of the (counterfactual) potency of forward guidance.
Figure 6. Low for Longer Policy

(A) Open Economy

(B) Closed Economy

Note: In this figure, we show the impulse response functions to two and a half years of a “Low for Longer” policy for different values of $m$. In all the rows, the purple lines correspond to no discounting, i.e., $m = 1$, while the blue, red and yellow lines indicate values of $m$ equal to 0.5, 0.75 and 0.9 respectively.

The experiments we consider in this section implicitly assume that the central bank desires to provide stimulus to the economy by unexpectedly deviating from the Taylor rule by 100bp (annualized), as in Figure 5. Since the proximity of the effective lower bound makes that impossible, it therefore lowers the policy rate by 10bp, but keeps it at that level for 10 quarters, with Figure 6 presenting the outcomes. Comparing these to Figure 5 for the case of no discounting, confirms that LFL is very effective in both closed and open economy cases, which implicitly reflects the underlying FGP.

In line with our discussion in Section 6, which highlighted that the forward guidance puzzle was more pronounced in a closed economy setting, we find that “low for longer” policies are particularly
efficient in that case as well. However, even for moderate levels of discounting, e.g., $m = 0.9$, these results can flip sign, with LFL becoming less efficient than the corresponding “conventional” stimulus. Finally, but also in line with our discussion in Section 6, Figure 6 highlights that the potency of “low for longer” policies is relatively more affected by “cognitive discounting” in the closed economy case.

8. INTERNATIONAL MONETARY POLICY SPILLOVERS

As we shall show in this Section, our behavioral open economy model also features interesting implications for international monetary spillovers. To analyze these, we first combine the consumption Euler equation (13) with the resource constraint (18) and iterate on the outcome to arrive at

$$
\hat{Y}_t = \alpha \hat{Y}^*_t + \eta \frac{\alpha(2-\alpha)}{1-\alpha} \hat{Q}_t + \frac{1-\alpha}{\sigma} \sum_{T=t}^{\infty} m^{T-t} \eta T + (1-m)(1-\alpha) \frac{1-\beta}{1+\delta} \sum_{T=t}^{\infty} m^{T-t} \hat{B}_T^r. \tag{37}
$$

The first two terms on the right hand side represent two traditional channels of international spillovers. The first one captures the positive effect of an increase in foreign output, and hence demand for Home economy’s exports. The second is associated with expenditure switching effects caused by the endogenous reaction of the real exchange rate. As the exchange rate appreciates when foreign interest rates go down, this channel acts in the opposite direction to the foreign demand channel, potentially more than offsetting the positive effects of an increase in foreign output.\(^\text{22}\) The third term describes the effects of an endogenous response of the Home real interest rate, highlighting that any meaningful evaluation of international spillovers must condition on the monetary policy reaction of the recipient country.\(^\text{23}\) Finally, the last term shows up only under myopia, and will typically make the response of Home output to Foreign monetary easing smaller, as the net foreign asset position deteriorates due to an exchange rate appreciation.

To provide more insight on how behavioral discounting affects the size of spillovers, and in line with the preceding observation on their conditionality, we further assume that the Home monetary

\(^{22}\) Notably, for given reactions of foreign output and the real exchange rate, the relative importance of these two spillover channels does not depend on discounting.

\(^{23}\) Recall that, and as explained in Section 7.1, discounting moderates the Home interest rate channel.
Figure 7. Monetary Spillovers

Note: This figure shows the impulse response functions to a 25bp (100 bp annualized) negative foreign monetary policy shock. The first row shows the case where $m^* = m$ while the second row corresponds to $m^* = 1$. In both cases, the purple lines indicate no discounting, i.e., $m = 1$, while the blue, red and yellow lines correspond to values of $m$ equal to 0.5, 0.75 and 0.9 respectively.

authority always keeps the real interest rate constant, so that the third term in Equation 37 disappears and the expenditure switching channel represented by the reaction of the real exchange rate becomes exogenous to the Home economy. Iterating the foreign IS curve forward, substituting in the outcome for foreign output, and then using the UIP condition to substitute for the real exchange rate, yields

$$\hat{Y}_t \approx \left( -\frac{\alpha}{\sigma} + \eta \frac{(2 - \alpha)}{1 - \alpha} \right) E_t \sum_{T=t}^{\infty} m^{T-t} \hat{y}_T^*, \quad (38)$$

where the approximation comes from omitting terms depending on the net foreign asset position, which are quantitatively small and hence immaterial for the results.

Note that, unless the trade elasticity is very low (the formal condition being again $\eta < \frac{1-\alpha}{\sigma} \leq \frac{1}{2} \sigma^{-1}$, see Section 6.2), the coefficient on the real interest rate path is positive and hence, assuming constant real interest rates at Home at all times, output spillovers from current or expected future monetary easing abroad are negative.
How is their magnitude affected by discounting? Similarly to the case of domestic effects discussed in Section 6, the impact of future real interest rate changes on other countries’ output declines with agent myopia, so that forward guidance becomes less powerful also in the context of international spillovers. Furthermore, to the extent that foreign monetary policy easing generates persistent effects on the foreign real rate, as is typically the case because of interest rate smoothing, the international transmission of current policy easing in the Foreign economy is also weaker. This effect is illustrated in the first row of Figure 7, which also demonstrates that the role of both spillover channels decreases in the degree of myopia, resulting in a lower response in Home economy’s output, conditional on it keeping its own real interest rate constant.

Finally, it is also worth noting that spillovers can be even less negative if agents are less myopic in the Foreign economy compared to Home agents. In the more general case of \( m^* \neq m \), Equation 38 becomes

\[
\hat{Y}_t \approx -\frac{\alpha}{\sigma} \mathbb{E}_t \sum_{T=t}^{\infty} m^{*T-t} \hat{r}^{*T} + \eta \frac{(2-\alpha)}{1-\alpha} \mathbb{E}_t \sum_{T=t}^{\infty} m^{T-t} \hat{r}^{*T},
\]

where, again, the first term represents the foreign demand channel while the second one corresponds to the expenditure switching channel. Clearly, \( m^* > m \) increases the importance of the former, which acts towards positive cross-country comovement in output conditional on foreign monetary shocks. We illustrate this effect in the second row of Figure 7 by assuming \( m^* = 1 \), i.e. we consider the extreme case of fully rational foreign agents. As explained, myopia of Home agents makes the effects of foreign monetary policy easing less negative when the Home real interest rate is kept constant, and can even turn them positive for a sufficiently high degree of discounting.

9. Conclusions

In this paper, we have extended the standard open economy New Keynesian framework with incomplete financial markets by adding behavioral agents. We have shown that the resulting model significantly improves upon its version with fully rational agents along many dimensions. First, it helps resolve several anomalies related to the uncovered interest rate parity condition in a way that is consistent with recent empirical evidence reassessing those puzzles using survey-based measures of expectations. Second, accounting for myopia decreases the efficacy of policies that rely on announcements of future actions, like “low for longer”, thus mitigating the forward guidance
puzzle. Third, by decreasing the relative strength of the exchange rate channel, the behavioral open economy model can better account for international output comovement.

While incorporating behavioral aspects in a consistent way is not costless, and doing so can quickly become quite involved in more complex environments, we believe that the price is worth paying as the benefits in the form of better empirical fit and more reasonable implications are significant. As our analysis suggests, this is true both when working with closed and open economy models, but, arguably, particularly so for the latter as they are plagued with many anomalies which cognitive discounting largely cures.
REFERENCES


Gust, C., E. Herbst, and J. D. Lopez-Salido (2021): “Short-term Planning, Monetary Policy, and Macroeconomic Persistence,” CEPR Discussion Papers 16141, CEPR.


Kiley, M. T., and J. M. Roberts (2017): “Monetary Policy in a Low Interest Rate World,” Brookings Papers on Economic Activity, 48(1 (Spring), 317–396. 7.2


APPENDICES

APPENDIX A. KEY DERIVATIONS

In this Appendix we present the key steps necessary to derive the linearized equilibrium conditions of a small open economy version of our model. Unless indicated otherwise, we use the variable transformations defined in Section 3.

A.1. Household Budget Constraint and Optimality Conditions. Linearizing the budget constraint (3) yields

\[ \hat{B}^s_{t-h} + \hat{B}^h_t = \beta^{-1} \left( \hat{B}^s_{t-1-h} + \hat{B}^h_{t-1} + \mu^{-1}(\hat{W}_t + \hat{N}^h_t) + \hat{D}_t - \hat{C}^h_t \right), \]

where \( \hat{D}_t \equiv (D_t - D)/Y \), and where we used the assumption of zero steady state assets (\( B^* = B = 0 \)), as well as the result that the steady state labor share is the inverse of (gross) product markup \( \mu \).

Given the household’s utility function (1) and budget constraint (3), the optimization problem gives the following linearized Euler equations associated with Home and Foreign bond holdings

\[ \hat{C}^h_t = \hat{E}_t \hat{C}^h_{t+1} - \frac{1}{\sigma} \hat{E}_t \left\{ \dot{i}_t - \dot{\pi}_{t+1} \right\}, \]

\[ \hat{C}^h_t = \hat{E}_t \hat{C}^h_{t+1} - \frac{1}{\sigma} \hat{E}_t \left\{ \dot{i}_t^* - \dot{\pi}^*_{t+1} + \hat{Q}_{t+1} - \hat{Q}_t - \phi \hat{B}^*_t \right\}, \]

where \( \phi = \Phi'(0) \), and the intratemporal labor supply condition is

\[ \hat{W}_t = \sigma \hat{C}^h_t + \varphi \hat{N}^h_t. \]

Combining equations A.2 and A.3 yields

\[ \hat{E}_t \left\{ \dot{i}_t - \dot{\pi}_{t+1} \right\} = \hat{E}_t \left\{ \dot{i}_t^* - \dot{\pi}^*_{t+1} + \hat{Q}_{t+1} - \hat{Q}_t - \phi \hat{B}^*_t \right\}. \]

Since this equation features expectations in aggregate variables that are beyond the control of an individual agent, and which are expressed as deviations from their respective steady state values, we can use the behavioral discounting formula \( \Pi_k \) for \( k = 0, 1 \) to write

\[ \dot{i}_t - m \hat{E}_t \left\{ \dot{\pi}_{t+1} \right\} = \dot{i}_t^* - m \hat{E}_t \left\{ \dot{\pi}^*_{t+1} - \hat{Q}_{t+1} \right\} - \dot{Q}_t - \phi \hat{B}^*_t, \]

which is the UIP condition (14) in the main text.

A.2. Deriving the Individual Consumption Function. Let us iterate the linearized budget constraint forward and use the standard transversality condition to write

\[ \hat{B}^s_{t-1-h} + \hat{B}^h_{t-1} = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left( \hat{C}^h_T - \mu^{-1}(\hat{W}_T + \hat{N}^h_T) + \hat{D}_T \right). \]

35
Note that by multiplying the Euler equation (A.2) by $\beta$ and iterating forward we obtain

$$\hat{C}_t^h = (1 - \beta)\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{C}_T^h - \frac{\beta}{\sigma} \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left( \hat{i}_T - \hat{\pi}_{T+1} \right). \tag{A.8}$$

Combining the two and rearranging yields

$$\hat{C}_t^h = (1 - \beta) \left( \hat{B}_{t-1}^h + \hat{B}_t^h \right)$$

$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[ (1 - \beta) \left( \frac{\varphi + 1}{\mu \varphi} \hat{W}_T - \frac{\sigma}{\mu \varphi} \hat{C}_T^h + \hat{D}_T \right) - \frac{\beta}{\sigma} \left( \hat{i}_T - \hat{\pi}_{T+1} \right) \right]. \tag{A.9}$$

We can now use the equilibrium condition (A.4) to eliminate individual labor supply

$$\hat{C}_t^h = (1 - \beta) \left( \hat{B}_{t-1}^h + \hat{B}_t^h \right)$$

$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[ (1 - \beta) \left( \frac{\varphi + 1}{\mu \varphi} \hat{W}_T - \frac{\sigma}{\mu \varphi} \hat{C}_T^h + \hat{D}_T \right) - \frac{\beta}{\sigma} \left( \hat{i}_T - \hat{\pi}_{T+1} \right) \right], \tag{A.10}$$

and again exploit Equation (A.8) to finally obtain

$$\left( 1 + \frac{\sigma}{\mu \varphi} \right) \hat{C}_t^h = (1 - \beta) \left( \hat{B}_{t-1}^h + \hat{B}_t^h \right)$$

$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[ (1 - \beta) \left( \frac{\varphi + 1}{\mu \varphi} \hat{W}_T - \frac{\sigma}{\mu \varphi} \hat{C}_T^h + \hat{D}_T \right) - \frac{\beta}{\sigma} \left( \hat{i}_T - \hat{\pi}_{T+1} \right) \right]. \tag{A.11}$$

The equation above is the individual consumption function that incorporates labor supply choice.

A.3. Deriving the IS Curve. Since Equation (A.11) features expectations only about aggregate variables, we can apply to it the behavioral discounting formula (11) for $k = 0, 1, 2, ...$

$$\left( 1 + \frac{\sigma}{\mu \varphi} \right) \hat{C}_t^h = (1 - \beta) \left( \hat{B}_{t-1}^h + \hat{B}_t^h \right)$$

$$+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[ (1 - \beta) \left( \frac{\varphi + 1}{\mu \varphi} \hat{W}_T + \hat{D}_T \right) - \frac{\beta}{\sigma} \left( 1 + \frac{\sigma}{\mu \varphi} \right) \left( \hat{i}_T - m\hat{\pi}_{T+1} \right) \right], \tag{A.12}$$

so that it now uses the rational expectations operator rather than the subjective one. Since we no longer need to make a distinction between macroeconomic aggregates and individual choices, we can drop indexing consumption and assets by $h$ and use the Home bond market clearing condition $B_t = 0$. After some algebra, we can write Equation (A.12) recursively

$$\left( 1 + \frac{\sigma}{\mu \varphi} \right) \hat{C}_t = (1 - \beta) \left( \hat{B}_{t-1} + \hat{B}_t - m\beta \hat{B}_t^* - m\beta \hat{B}_t \right) + (1 - \beta) \left( \frac{\varphi + 1}{\mu \varphi} \hat{W}_t + \hat{D}_t \right)$$

$$- \frac{\beta}{\sigma} \left( 1 + \frac{\sigma}{\mu \varphi} \right) \left( \hat{i}_t - m\hat{E}_t \hat{\pi}_{t+1} \right) + m\beta \left( 1 + \frac{\sigma}{\mu \varphi} \right) \hat{E}_{t+1} \hat{C}_{t+1}. \tag{A.13}$$
Now we can use the budget constraint \( A.1 \) and the Home currency bond market clearing condition \( B_t = 0 \) to obtain

\[
\beta + \sigma \frac{\mu}{\varphi} \hat{C}_t = (1 - \beta)(1 - m)\beta \hat{B}^*_t + \frac{1 - \beta}{\mu} \left( \frac{1}{\varphi} \hat{W}_t - \hat{N}_t \right)
- \frac{\beta}{\sigma} \left( 1 + \frac{\sigma}{\mu \varphi} \right) (\hat{i}_t - m \hat{E}_t \hat{\pi}_{t+1}) + m \beta \left( 1 + \frac{\sigma}{\mu \varphi} \right) \hat{E}_{t+1} \hat{C}_{t+1}.
\]

(A.14)

Finally, using the optimal labor supply condition \( A.4 \) results in

\[
\hat{C}_t = m \hat{E}_{t+1} \hat{C}_{t+1} - \frac{1}{\sigma} \left( \frac{\hat{i}_t - m \hat{E}_t \hat{\pi}_{t+1}}{\hat{i}_t} \right) + (1 - m) \frac{1 - \beta}{1 + \frac{\sigma}{\mu \varphi}} \hat{B}^*_t,
\]

(A.15)

which is the aggregate IS curve (13) in the main text.

A.4. Deriving the Phillips Curve. Aggregation of intermediate inputs into final goods according to Dixit-Stiglitz formulas \( 4 \) yields the following isoelastic demand conditions

\[
Y_{fH,t} + Y_{fH,t}^* = \left( \frac{P_{fH,t}}{P_{H,t}} \right)^{\frac{1}{1-\mu}} \left[ Y_{H,t} + Y_{H,t}^* \right],
\]

(A.16)

where the aggregate price indices are

\[
P_{H,t} = \left[ \int_0^1 \left( \frac{P_{fH,t}}{P_{H,t}} \right)^{\frac{1}{1-\mu}} df \right]^{1-\mu}, \quad \text{and} \quad P_{H,t}^* = \left[ \int_0^1 \left( \frac{P_{fH,t}^*}{P_{H,t}} \right)^{\frac{1}{1-\mu}} df \right]^{1-\mu},
\]

(A.17)

and where we used the law of one price \( P_{fH,t} = \varepsilon_t P_{fH,t}^* \), which also implies \( P_{H,t} = \varepsilon_t P_{H,t}^* \).

Using the demand conditions \( A.16 \) and production technology \( 5 \) allows us to rewrite the firm problem consistent with maximization of \( 6 \) as

\[
\max_{P_{fH,t}} \hat{E}_t \sum_{T=t}^{\infty} \theta^{T-t} \Lambda_{t,T} \left( \frac{P_{fH,t}}{P_{H,t}} \right)^{\frac{1}{1-\mu}} \left[ Y_{H,T} + Y_{H,T}^* \right].
\]

(A.18)

The first order condition is

\[
\hat{E}_t \sum_{T=t}^{\infty} \theta^{T-t} \Lambda_{t,T} \left[ P_{fH,t} - \mu P_T M C_T \right] \left( \frac{P_{fH,t}}{P_{H,T}} \right)^{\frac{1}{1-\mu}} \left[ Y_{H,T} + Y_{H,T}^* \right] = 0,
\]

(A.19)

where \( M C_t \equiv W_t P_t / P_{H,t} \) is real marginal cost deflated by the producer price index.

As in a textbook closed economy case (see e.g. [Galí 2015]), linearizing around the zero inflation steady state yields

\[
\hat{P}_{H,t}^* = (1 - \beta \theta) \sum_{T=t}^{\infty} (\beta \theta)^{T-t} \hat{E}_t \left\{ \hat{\pi}_{H,t+1} + ... + \hat{\pi}_{H,T} + \hat{M} C_T \right\},
\]

(A.20)

where \( \hat{P}_{H,t}^* \equiv \log(P_{H,t}^*/P_{H,t}) \), \( M C_t \equiv \log(MC_t/MC) \), \( \hat{\pi}_{H,t} \equiv \log(P_{H,t}/P_{H,t-1}) \) and where we used the result that all reoptimizing firms choose the same price to drop the \( f \) superscript. Since the subjective expectation operator now concerns only variables beyond individual firm control and all
of them are expressed as deviations from steady state, we can apply the discounting formula (11) to obtain
\[ \hat{P}_{H,t}^\infty = (1 - \beta \theta) \sum_{T=t}^{\infty} (\beta \theta)^{T-t} \mathbb{E}_t \left\{ m\hat{\pi}_{H,t+1} + ... + m^{T-t}\hat{\pi}_{H,T} + m^{T-t}\hat{MC}_T \right\}. \] (A.21)

Note that this step differs from Gabaix (2020), who discounts all terms in the curly bracket of equation A.20 by \( m^{T-t} \). By doing so he implicitly applies myopia to nominal rather than real marginal cost, even though the former is not constant in the steady state (see also Benchimol and Bounader (2019) for a discussion).

After some algebra, this can be written recursively as
\[ \hat{P}_{H,t}^\infty - \beta \theta m \mathbb{E}_t \hat{P}_{H,t+1}^\infty = (1 - \beta \theta) \hat{MC}_t + \beta \theta m \mathbb{E}_t \{ \hat{\pi}_{H,t+1} \}. \] (A.22)

Note that the price index definition A.17 implies
\[ \hat{\pi}_{H,t} = (1 - \theta)(\hat{P}_{H,t}^\infty + \hat{\pi}_{H,t}^\infty) = \frac{1 - \theta}{\theta} \hat{P}_{H,t}^\infty. \] (A.23)

Combining it with Equation A.22 and rearranging yields
\[ \hat{\pi}_{H,t} = m \beta \mathbb{E}_t \{ \hat{\pi}_{H,t+1} \} + \frac{(1 - \beta \theta)(1 - \theta)}{\theta} \hat{MC}_t, \] (A.24)

which is Equation (15) in the main text.

A.5. Deriving the Marginal Cost Equation. The optimal composition of the consumption basket (2) implies the following formula for the aggregate price index \( P_t \) (see e.g. Gali and Monacelli, 2005)
\[ P_t = \left[ (1 - \alpha)(P_{H,t})^{1-\eta} + \alpha (P_{F,t})^{1-\eta} \right]^{\frac{1}{1-\eta}} \] (A.25)

which implies
\[ \hat{P}_{H,t} = -\frac{\alpha}{1 - \alpha} \hat{P}_{F,t} = -\frac{\alpha}{1 - \alpha} \hat{Q}_t, \] (A.26)

where \( \hat{P}_{F,t} = \log(P_{F,t}/P_t) \) and where the last equality follows from the definition of the real exchange rate \( Q_t = \varepsilon_t P_t^* \) and the small open economy version of the law of one price \( P_{F,t} = \varepsilon_t P_t^* \).

By the definition of marginal cost deflated by producer prices we have
\[ \hat{MC}_t = \hat{W}_t - \hat{P}_{H,t}, \] (A.27)

where \( \hat{P}_{H,t} = \log(P_{H,t}/P_t) \). Using Equation A.26 to substitute in for \( \hat{P}_{H,t} \) and the labor supply condition A.4 to eliminate \( \hat{W}_t \) yields
\[ \hat{MC}_t = \sigma \hat{C}_t + \varphi \hat{Y}_t + \frac{\alpha}{1 - \alpha} \hat{Q}_t, \] (A.28)

which is equation 16 in the main text.
Appendix B. Additional Derivations

B.1. Deriving Equation 33

Eliminating consumption from Equation 13 using the resource constraint results in

\[
\dot{Y}_t = m\mathbb{E}_t\dot{Y}_{t+1} + \alpha \left( \dot{Y}^*_t - m\mathbb{E}_t\dot{Y}^*_{t+1} \right) + \eta \frac{\alpha(2 - \alpha)}{1 - \alpha} \left( \dot{Q}_t - m\mathbb{E}_t\dot{Q}_{t+1} \right)
- \frac{1 - \alpha}{\sigma} \left( \dot{i}_t - m\mathbb{E}_t\dot{\pi}_{t+1} \right) + (1 - m)(1 - \alpha) \frac{1 - \beta}{1 + \frac{\sigma}{\mu \varphi}} \dot{B}^*_t,
\]

and using the UIP condition then yields

\[
\dot{Y}_t = m\mathbb{E}_t\dot{Y}_{t+1} + \alpha \left( \dot{Y}^*_t - m\mathbb{E}_t\dot{Y}^*_{t+1} \right) + \eta \frac{\alpha(2 - \alpha)}{1 - \alpha} \left( \dot{\pi}^*_t - \phi \dot{B}^*_t - m\mathbb{E}_t \{ \dot{\pi}^*_{t+1} \} - \dot{i}_t + m\mathbb{E}_t \{ \dot{\pi}_{t+1} \} \right)
- \frac{1 - \alpha}{\sigma} \left( \dot{i}_t - m\mathbb{E}_t\dot{\pi}_{t+1} \right) + (1 - m)(1 - \alpha) \frac{1 - \beta}{1 + \frac{\sigma}{\mu \varphi}} \dot{B}^*_t.
\]

When considering the effects of Home monetary policy, we can drop foreign variables as they are exogenous on account of the small open economy assumption. By rearranging and using the definition of the ex ante real interest rate \( \hat{r}_t \equiv \dot{i}_t - m\mathbb{E}_t\dot{\pi}_{t+1} \) we then arrive at

\[
\dot{Y}_t = m\mathbb{E}_t\dot{Y}_{t+1} - \left( \frac{1 - \alpha}{\sigma} + \eta \frac{\alpha(2 - \alpha)}{1 - \alpha} \right) \dot{r}_t - \left[ \eta \frac{\alpha(2 - \alpha)}{1 - \alpha} \phi - (1 - m)(1 - \alpha) \frac{1 - \beta}{1 + \frac{\sigma}{\mu \varphi}} \right] \dot{B}^*_t,
\]

which is Equation 33 in the main text.

B.2. Deriving Equations 35 and 36

By combining equations and iterating forward on the outcome, we obtain

\[
\hat{\pi}_{H,t} = \kappa \mathbb{E}_t \sum_{T=t}^{\infty} (\beta m)^{T-t} \left( \sigma \hat{C}_T + \varphi \hat{Y}_T + \frac{\alpha}{1 - \alpha} \dot{Q}_T \right),
\]

Note that each of the three variables defining real marginal cost (last bracket above) can be expressed as a function of the current and expected future real interest rates, see in particular equations 32, 34 and 31. Ignoring the terms associated with the net foreign asset position (as they are small) and consistently dropping foreign variables (on account of the the small open economy assumption) allows us to write

\[
\sigma \hat{C}_t + \varphi \hat{Y}_t + \frac{\alpha}{1 - \alpha} \dot{Q}_t \approx -A\mathbb{E}_t \sum_{T=t}^{\infty} m^{T-t} \hat{r}_T,
\]

where \( A \equiv \varphi \left( \frac{1 - \alpha}{\sigma} + \eta \frac{\alpha(2 - \alpha)}{1 - \alpha} \right) + \frac{1}{1 - \alpha} \). Plugging this into Equation yields

\[
\hat{\pi}_{H,t} \approx -\kappa A\mathbb{E}_t [\hat{r}_t + m(1 + \beta) \hat{r}_{t+1} + \ldots + m^n(1 + \beta + \ldots + \beta^n) \hat{r}_{t+n} + \ldots]
= -\kappa A\mathbb{E}_t \sum_{T=t}^{\infty} m^{T-t+1} - (\beta m)^{T-t+1} \frac{m(1 - \beta)}{m(1 - \beta)} \hat{r}_T.
\]
Recall that CPI inflation is given by Equation 17. Exploiting relationships B.6 and 31, and again ignoring terms related to net foreign assets, yields

$$
\hat{\pi}_t \approx -\frac{\kappa A}{m(1-\beta)} \mathbb{E}_t \sum_{T=t}^{\infty} [m^{T-t+1} - (\beta m)^{T-t+1}] \hat{r}_T - \frac{\alpha}{1-\alpha} \mathbb{E}_t \sum_{T=t}^{\infty} m^{T-t} \hat{r}_T - \frac{\alpha}{1-\alpha} \hat{\theta}_{t-1},
$$

(B.7)

which is Equation 35 in the main text.

In the limit $\beta \to 1$ we also have $M \to m$, and Equation B.6 becomes

$$
\hat{\pi}_{H,t} = -\kappa A \mathbb{E}_t [\hat{r}_t + 2m\hat{r}_{t+1} + (+n+1)m^n\hat{r}_{t+n} + \ldots],
$$

(B.8)

which plugged into the definition of CPI 17 results in

$$
\hat{\pi}_t = -\kappa A \mathbb{E}_t \sum_{T=t}^{\infty} (T-t+1) m^{T-t} \hat{r}_T - \frac{\alpha}{1-\alpha} \mathbb{E}_t \sum_{T=t}^{\infty} m^{T-t} \hat{r}_T - \frac{\alpha}{1-\alpha} \hat{\theta}_{t-1},
$$

(B.9)

which is Equation 36 in the text.

Finally, the relative weight of the penultimate component in the formula above is

$$
\frac{\alpha}{1-\alpha} \left( \frac{1}{\sigma} + \eta \frac{2\alpha - \alpha}{1-\alpha} \right) = \frac{1}{\varphi(\eta - \sigma^{-1})(2-\alpha) + \left( \frac{2}{\sigma} + 1 \right) \alpha^{-1}},
$$

and so it is clearly increasing in the economy’s openness $\alpha$.

**B.3. Deriving Equation 37 and 38.** Let us rearrange the output IS curve B.1 as follows

$$
\hat{Y}_t - \alpha \hat{Y}_t^* - \eta \frac{\alpha(2-\alpha)}{1-\alpha} \hat{Q}_t = m \mathbb{E}_t \left\{ \hat{Y}_{t+1} + \alpha \hat{Y}_{t+1}^* + \eta \frac{\alpha(2-\alpha)}{1-\alpha} \hat{Q}_{t+1} \right\}
$$

$$
- \frac{1-\alpha}{\sigma} \mathbb{E}_t \sum_{T=t}^{\infty} m^{T-t} \hat{r}_T + (1-m)(1-\alpha) \frac{1-\beta}{1+\frac{\beta}{1-\alpha}} \hat{B}_t^*,
$$

(B.10)

Iterating this forward yields

$$
\hat{Y}_t = \alpha \hat{Y}_t^* + \eta \frac{\alpha(2-\alpha)}{1-\alpha} \hat{Q}_t - \frac{1-\alpha}{\sigma} \mathbb{E}_t \sum_{T=t}^{\infty} m^{T-t} \hat{r}_T + (1-m)(1-\alpha) \mathbb{E}_t \sum_{T=t}^{\infty} m^{T-t} \hat{B}_t^*,
$$

(B.11)

which is Equation 37 in the main text.

To derive Equation 38 we iterate forward on the foreign IS curve 21 and use the outcome to substitute for $\hat{Y}_t^*$ above, exploiting Equation 31 to substitute for $\hat{Q}_t$. After omitting the terms associated with net foreign assets, assuming constant real interest rate in the Home economy $\hat{r}_t = 0$, and rearranging we arrive at

$$
\hat{Y}_t \approx \left( \frac{-\alpha}{\sigma} + \eta \frac{\alpha(2-\alpha)}{1-\alpha} \right) \mathbb{E}_t \sum_{T=t}^{\infty} m^{T-t} \hat{r}_T^*,
$$

(B.12)

which is Equation 38 in the main text.
Monetary Policy and Exchange Rate Dynamics in a Behavioral Open Economy Model

Working Paper No. WP/2022/112