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IMF Working Paper

Systemic Risk Modeling: How Theory Can Meet Statistics

by Raphael A. Espinoza, Miguel A. Segoviano, and Ji Yan

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European Department

Systemic Risk Modeling: How Theory Can Meet Statistics

Prepared by Raphael A. Espinoza, Miguel A. Segoviano, and Ji Yan

Authorized for distribution by Dora Iakova

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Abstract

We propose a framework to *link* empirical models of systemic risk to theoretical network/general equilibrium models used to understand the channels of transmission of systemic risk. The theoretical model allows for systemic risk due to interbank counterparty risk, common asset exposures/fire sales, and a “Minsky” cycle of optimism. The empirical model uses stock market and CDS spreads data to estimate a multivariate density of equity returns and to compute the expected equity return for each bank, conditional on a bad macro-outcome. These “cross-sectional” moments are used to re-calibrate the theoretical model and estimate the importance of the Minsky cycle of optimism in driving systemic risk.

JEL Classification Numbers: C14; G17; G21; G32

Keywords: Systemic risk, Minsky effect, CIMDO, Default

Author’s E-Mail Address: REspinoza@imf.org; MSegoviano@imf.org; Jiyan3@cityu.edu.hk

1 Introduction

The global financial crisis in 2008 showed us that due to systemic risk amplification mechanisms, relatively small initial losses in the financial system can become endogenous and be magnified to systemic dimensions with large welfare effects. Data constraints, the understanding of the intricacies of amplification mechanisms, how best to model those mechanisms, and how they might interact in complex financial systems impose significant impediments to both researchers and policymakers trying to develop frameworks that can adequately quantify systemic risk amplification.

We propose an innovative framework that allows to quantify losses that account for systemic risk endogeneity and amplification, while allowing to characterize the channels that cause such amplification. The proposed approach is to build a framework that embeds (i) an empirical (reduced-form) model to quantify systemic risk losses; and (ii) a theoretical general equilibrium model to characterize systemic risk channels, including interbank lending, common asset exposures and amplification mechanisms a la “Minsky”.¹ The empirical model is used to calibrate in a simple manner, with publicly available data, the theoretical model. As calibration can be updated with real time market-based data as the financial cycle evolves, or with information observed during crisis periods, it is possible to explore how the different systemic risk channels included in the theoretical model affect the endogenous losses that are amplified in periods of distress.

The applied literature has taken two main approaches to quantifying systemic risk. These are (i) the development of simulated models (network or agent-based (ABM); e.g. [Eisenberg and Noe \(2001\)](#); [Cifuentes, Ferrucci, and Shin \(2005\)](#); [Cont and Schaanning \(2017\)](#); [Geanakoplos et al. \(2012\)](#)) that attempt to explicitly model agents behavioral responses that underpin the interconnectedness structures that propagate losses among financial institutions (FIs); and (ii) the estimation of systemic risk metrics with empirical, reduced form, models that attempt to quantify, from market data, metrics that embed the impact of systemic risk amplification mechanisms (e.g. [Adrian and Brunnermeier \(2016\)](#); [Acharya et al. \(2017\)](#); [Segoviano and Goodhart \(2009\)](#); [Billio et al. \(2012\)](#)), without explicitly modeling behavior.

While simulated models have made important contributions to map the workings of specific systemic risk channels, these frameworks are usually difficult to calibrate, require granular data that does not exist in many countries and might produce estimations of systemic risk that while rooted in theory, might be inconsistent with empirically observed losses. These challenges limited the role of the early network models for policy analysis.²

¹We build upon the idea presented in [Alla et al. \(2018\)](#) to develop an encompassing framework aimed at integrating a diverse collection of data and of modeling frameworks with different characteristics as a way to maximize the information content of heterogeneous data sources and minimize potential model error.

²[Elsinger, Lehar, and Summer \(2013\)](#) also noted that the losses predicted by network models of interbank exposures were too small and thus had not been useful for policymaking during the 2008/09 financial crisis.

Alternatively, reduce-form models can incorporate amplification mechanisms that result of agents' actions, which get reflected on market prices. Systemic risk metrics estimated from these models are reduced-form, meaning that although in principle, they can capture the effects of agents' behavior, they do not provide information of the specific agents' behaviors nor define the channels of contagion that can lead to the materialization of systemic risk. The proposed framework in this paper aims to get the best of both approaches; i.e., to map specific channels (characterized with a theoretical model) while incorporating realistic measurements (quantified with an empirical model) of systemic risk that can be updated easily.

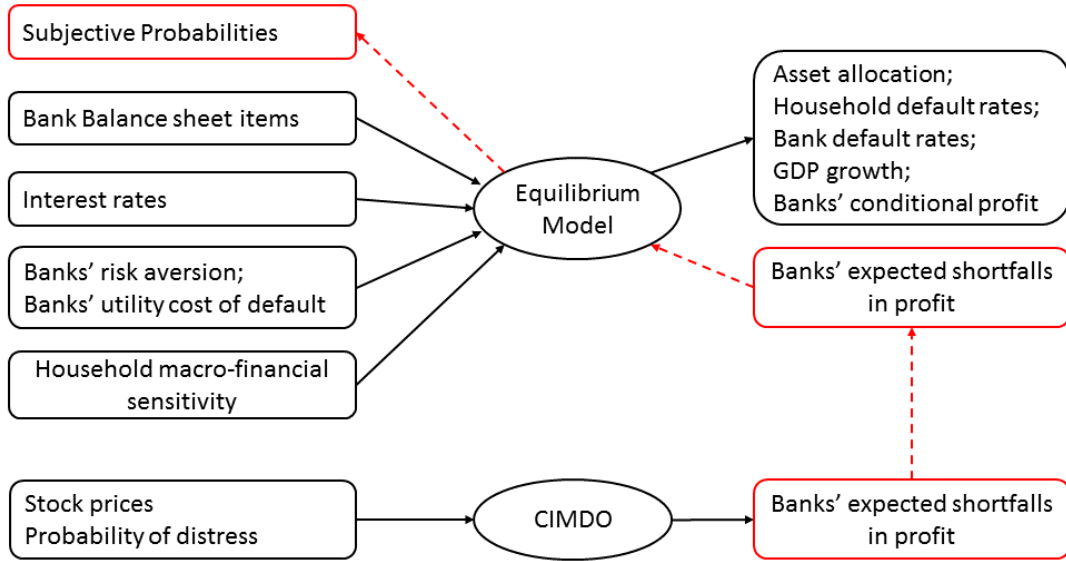
We modify and extend the equilibrium model of [Goodhart, Sunirand, and Tsomocos \(2005\)](#) to incorporate endogenous risk taking and various channels of systemic risk. In each period, banks make a strategic asset allocation choice to optimize their expected utility before the state of the macroeconomy is realized. This results in endogenous risk-taking. Systemic risk endogeneity in the model is due to four main channels: interbank contagion, common exposures, fire-sales and an amplification mechanism due to a cycle of optimism. The model allows for interbank contagion via the unsecured interbank market. Banks choose to either lend or borrow before the state of the macroeconomy is known, but the likelihood of defaulting on interbank liabilities is defined after the state of the economy is realized. The risk of default of one bank is thus transmitted throughout the interbank market to the entire banking system. The second channel of systemic risk is due to common exposures, as different banks hold assets with correlated returns due to the fact that loan losses are a function of macroeconomic outcomes. A fire sales channel is captured by the fact that distressed banks can curtail their exposure simultaneously, leading to overall lower credit in the banking system and depressed GDP. Finally, an amplification mechanism due to optimism is incorporated since the model allows bankers to depart from rational expectations. In line with [Minsky \(1977\)](#)'s argument, over a period in which the economy does well, optimism changes and bankers tend to invest more in riskier assets, potentially increasing the volatility of banking sector losses. Thus, after years of good macroeconomic outcomes, speculative investment booms can become sources of instability. The theoretical model produces estimates of economic outcomes as well as of the profits and losses in the financial system.

In parallel, we estimate the system's profit and loss distribution using an empirical approach. Our approach is to construct a multivariate density capturing the equity values for a system of banks, based on the non-parametric method proposed by [Segoviano and Goodhart \(2009\)](#).³ As with other empirical approaches, this approach, called Consistent Information Multivariate

³The CIMDO methodology is based on the minimum cross-entropy approach, where a posterior multivariate distribution the CIMDO density is recovered using an optimization procedure by which a prior density function is updated with empirical information via a set of constraints. In this implementation, the empirical estimates of the probability of distress of individual banks act as the constraints, and the derived CIMDO density is the posterior density that is the closest to the prior distribution and consistent with these constraints. This methodology and its advantages relative to other parametric multivariate densities are presented in detail in [Segoviano \(2006\)](#) and [Segoviano and Espinoza \(2017\)](#)

Density Optimization (CIMDO) can be estimated with publicly available data (without the need for highly detailed or granular supervisory information). However, the CIMDO approach offers important advantages relative to other methods in terms of implementation feasibility and estimation robustness. Instead of estimating systemic losses relying on parametric assumptions (as other risk models sometimes do), the CIMDO approach infers a distribution that is also consistent with the market perception of risk (in practice, CDS spreads);⁴ henceforth, reducing the risk of parameter misspecification.⁵ Because CIMDO parameters can be updated as empirical information changes, the method enables analysts to incorporate in a timely manner updates in systems amplification mechanisms that can experience nonlinear increases in periods of high volatility.⁶ The multivariate dimension of the framework that results of the portfolio approach adopted by CIMDO also permits to calculate the “co-movement” of profit and losses across banks, which provide several cross-sectional moments from the data (i.e. moments across banks), a useful step to calibrate the theoretical model.

Figure 1: Bridging the gap between the theory and the empirical models



⁴The CIMDO approach infers from market-based data, the unobservable interconnectedness structures (that propagate losses across FIs in a system) that are consistent with markets perceptions of risk, reflected in the observable probabilities of distress of individual FIs in the system.

⁵Using an extension of the Probability Integral Transformation (PIT) criterion advocated by [Diebold, Gunther, and Tay \(1997\)](#), the paper shows that CIMDO-inferred density forecasts perform better than parametric distributions forecasts, even when they are calibrated with the same information set.

⁶CIMDO-inferred dependence structures (characterized by copula functions) embody both linear and non-linear distress dependence among FIs and update consistently with FIs PoDs. CIMDO dependence structures are thus superior to dependence structures from parametric reduce form models that usually capture only linear dependence (correlations) or those from other reduce form models that while capturing nonlinear dependence (copula functions), these are usually assumed to remain fixed though time.

Indeed, once the profit and loss (multivariate) distribution of the banking system is estimated empirically, these estimates are plugged onto the theoretical model to reverse-engineer the estimation of the theoretical models' parameters. More specifically, the empirical estimates of the banks expected shortfalls in profits (i.e. the difference in profits between the good macro-state and the bad macro-state) are used to assess the role of the cycle of optimism, captured in the theoretical model by each banks' subjective probability that a bad macro scenario is realized (this subjective probability is allowed to differ from the 'true' probability). Figure 1 presents this framework: the components in black correspond to the standard working of a theoretical model (upper part) and the standard approach of an empirical model (bottom part). The originality of our proposal is to make a link (in red) between the two models in order to provide some model estimates of the cycle of optimism (the subjective probabilities used by each banker). Since these are consistent with the empirical quantification of the system profit and losses, we believe that our proposed framework can support policymakers to improve their understanding of the intricacies of systemic risk amplification mechanisms, although further research would be needed to fully disentangle these mechanisms.

The structure of the paper is as follows. Section 2 develops the theoretical model that we propose. Section 3 summarizes the empirical approach to computing the multivariate distribution of equity returns, and explains how it is implemented. Section 4 explains how the theoretical model is implemented on UK data (pre- and post- financial crisis, separately) and recalibrated by matching the empirical moments, and provides an analysis of the different channels of systemic risk and their interaction. Section 5 concludes the paper.

2 The model

We develop a model with heterogeneous banks and households, based on Goodhart, Sunirand, and Tsomocos (2005). In each period, the banks optimize their expected utilities by choosing the asset allocation before the state of the macroeconomy is realized, and the risk they take, i.e. the probability that they would end up defaulting on their liabilities⁷ after the state of the macroeconomy is known. Thus, default is always both a strategic choice of risk-taking as well as the outcome of bad luck. Endogenous risk-taking is a main feature of the model: banks choose the risk of default, depending on macroeconomic outcomes and the profits they make from consumer loans. Systemic risk is due to three main channels in the model. and in Figure 3.

Interbank contagion First, the model allows for direct contagion between banks via the unsecured interbank market. Banks choose to either lend or borrow before the state of the macroeconomy is known, but the likelihood of defaulting on interbank liabilities is chosen after the state of the economy is realized. The risk of default of one bank is thus transmitted throughout the interbank market to the entire banking system. For simplicity, it is assumed

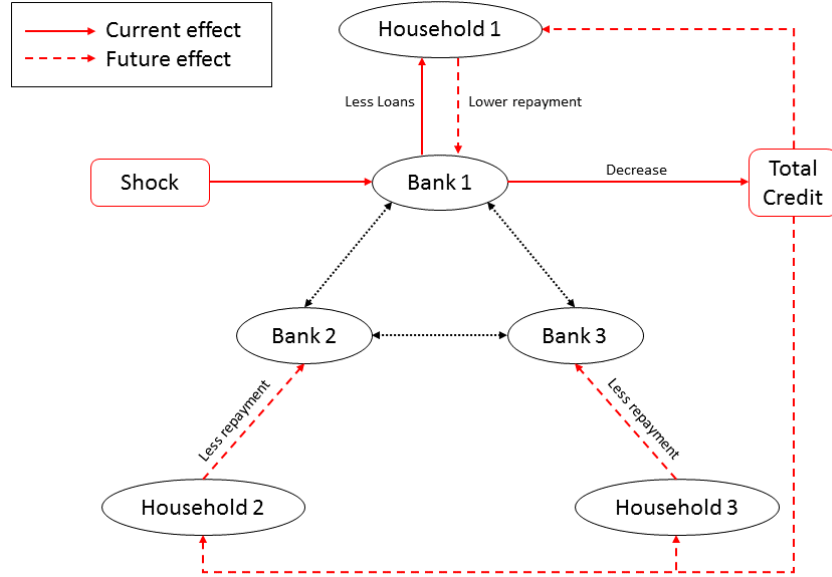
As will be made clear below, the formal choice made by each bank is the fraction of its liabilities over which it defaults. There is an equivalence between that fraction and the likelihood of default if the model was extended to include a continuum of sub-states with the bad state. The expected value of the default rate would then be equal to the likelihood of a 100 percent default rate.

that the same default rate is applied to each lender in the interbank market.⁸

Common exposure and fire sales. The second channel of systemic risk is due to common exposure. The different banks hold assets with correlated returns, since loan losses are a function of the macroeconomy. In addition, a simple “fire sales” channel is captured by the fact that distressed banks curtail their exposure, leading to lower credit, depressed GDP, and thus lower credit quality across the whole banking system. This interbank contagion and the common exposure/fire sale channels are presented schematically in Figure 2.

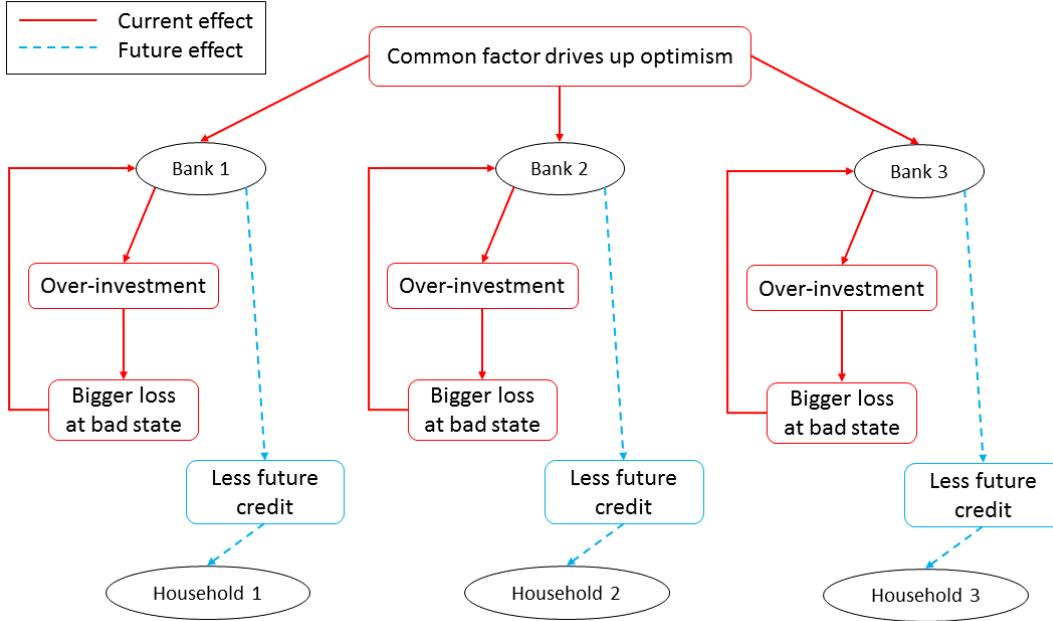
Minsky cycle. Finally, the probability of the good macro-state perceived by each bank can differ from the true probability. The model thus allows for departure from rational expectations, in line with [Minsky \(1977\)](#)’s argument that “over a period in which the economy does well, optimism changes, and agents tend to invest more in the riskier asset. The speculative investment boom is the basic instability in the economy”. We show below that when a bank becomes more optimistic, its profit and loss becomes more volatile. If there is a common trend of optimism across banks—which could happen after several years of a good economy— systemic risk is amplified. This channel is presented schematically in Figure 3.

Figure 2: Common asset exposure channel of distress dependence



⁸More detailed models of interbank exposures and default cascades are presented in [Eisenberg and Noe \(2001\)](#) and [Cifuentes, Ferrucci, and Shin \(2005\)](#).

Figure 3: Systemic Risk Amplification: Minsky channel



2.1 The economy

The economy consists of N banks, $N + 1$ households, and the central bank. The model can be thought of as a two-period model, starting at $t = 0$, and with two possible states for the macroeconomy (henceforth the “macro-state”) at time $t + 1 = 1$, the good state Up (U) and the bad state $Down$ (D). The banks are heterogeneous in the sense that they have different initial equity, risk-aversion, and attitude toward the cost of defaulting on their liabilities. Each household is assigned to borrow from a single bank. This limited participation assumption may come from history or informational constraint. The households $H^1, H^2, H^3, \dots, H^N$ borrow from bank $B^1, B^2, B^3, \dots, B^N$, respectively, and the remaining household H^{N+1} supplies deposits to all the banks. The risk-free loan rates and deposit rates are exogenous, but the repayment rates are endogenous and different across states of nature. In addition, there is an interbank market where the banks are allowed to borrow from and deposit in other banks. The Central Bank conducts open market operations (OMOs) to set the reference rate for the interbank market. Figure 4 provides a stylized presentation of the economy with its different agents and markets and the time structure of the model is shown in Figure 5.

At the beginning of period 0, the interbank, loan and deposit markets open. Banks determine how much to lend to households, and how much to deposit or borrow in the interbank market. The central bank conducts OMOs in the interbank market to fix the risk-free rate. Finally, nature decides whether a good or a bad macro-state occurs.

Figure 4: The Economy

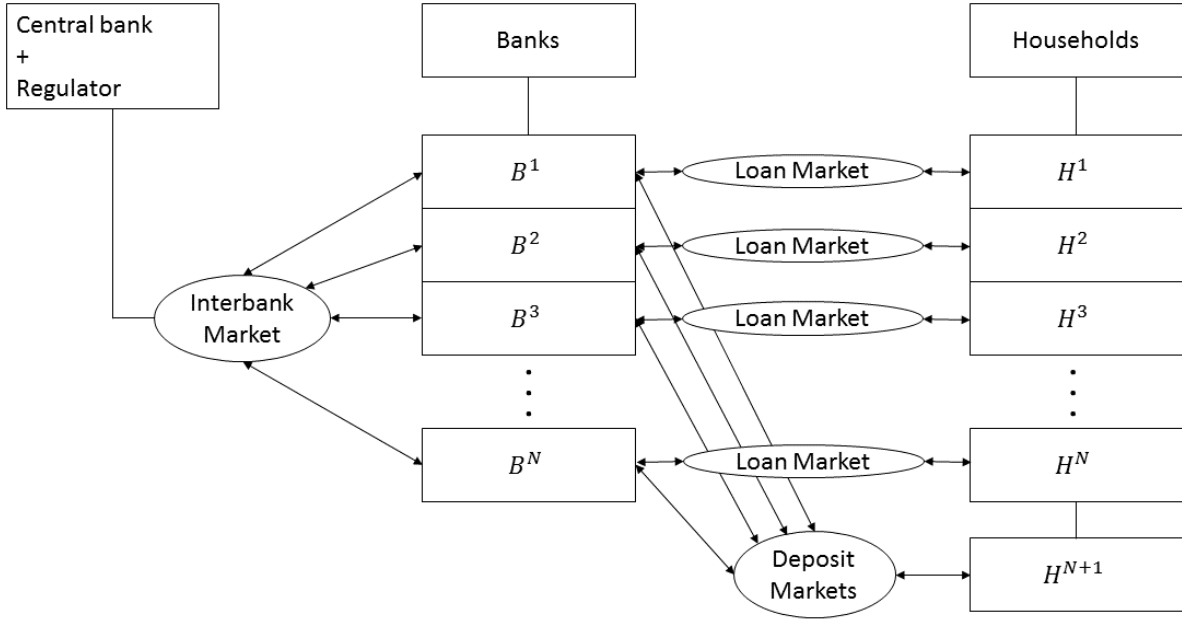


Table 1: Bank's balance sheet

Assets		Liabilities	
Loans	L	Deposits	D
Interbank loans	L^I	Interbank deposits	D^I
Market book	A	Equity	e
		Others	ϵ

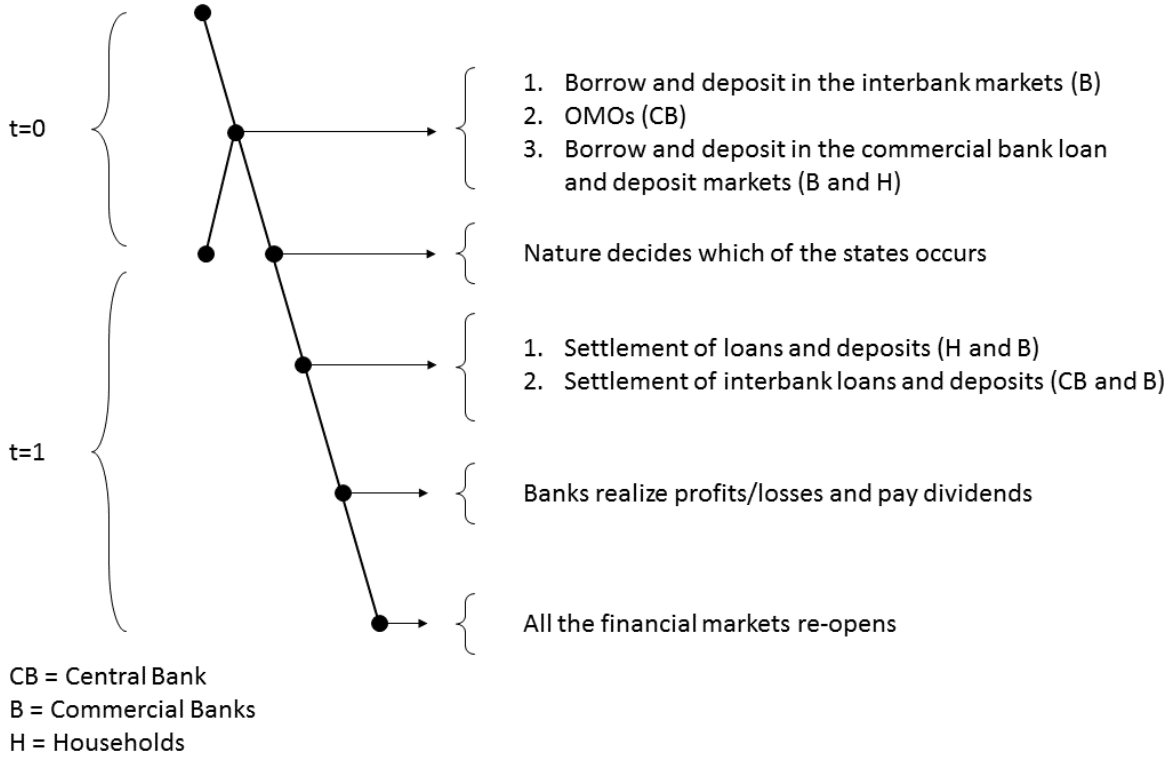
At the beginning of period 1, financial contracts are settled. In addition, the households and the banks choose the probability of default on their liabilities, a decision formalized by the repayment rate, which can be interpreted as an expected repayment rate (the bad macro-state of nature implicitly represents a set of sub-states of nature where the occurrence of default is exogenous and a probability in inverse proportion to the repayment rate). In the paper, we will use the wording “probability of default” of the bank to capture its intuition as a risk-taking decision, but the formal representation is a fraction of default on the banks’ liabilities. At the end of the period, financial markets re-open.⁹ All banks in the model are assumed to operate in a competitive environment, and thus they take all the interest rates as given. The structure of each bank’s balance sheet is shown in Table 1.

2.2 The banks’ optimization problem

The banks choose how much to leverage, by taking deposits from households and borrowing from the interbank market. They also choose how much credit to allocate to consumer loans,

⁹Goodhart, Sunirand, and Tsomocos (2006) extend the model of Goodhart, Sunirand, and Tsomocos (2005) along similar lines and prove that the infinite-period optimization problem is equivalent to a series of single-period optimization problems.

Figure 5: The Time Structure



and they choose the probability/fraction of default on their liabilities. We also assume that deposits and interbank loans have equal seniority, i.e. the default rates are the same. The timing of the model is such that the asset allocation by each bank is done before the macro-state is realized. Banks expect a higher return from investing in riskier assets (consumer loans) than in safer ones (interbank loans and market book), but consequently face greater risks. The decision on the value of the probability of default is done after the macro-state is known. A higher probability of default yields higher profits, but carries a utility cost, formalized as a default penalty.

The banks' optimization problem is given below. For the sake of simplicity, we follow [Goodhart, Sunirand, and Tsomocos \(2006\)](#) in assuming that optimization is done over a two-period horizon, i.e. at time t the banker maximizes expected utility for time $t + 1$. [Goodhart, Sunirand, and Tsomocos \(2006\)](#) also argue it is a reasonable assumption since bank managers, who can switch jobs relatively easily, tend to have short horizons. The bank b maximizes its expected utility, minus the non-pecuniary penalties that it incurs if its bank defaults on its deposit and interbank obligations. Table 2 presents the notation for all the variables in the model.

A bank $b \in B$ maximizes the following expected utility, which consists of a quadratic utility

Table 2: Variables in the model

Subscripts and Superscripts	
$t \in T \equiv \{0, 1, 2, \dots\}$	time periods
$s \in S \equiv \{U, D\}$	the set of possible states after each node
$b \in B$	the set of all the banks
Exogenous Variables	
p_s	probability of state $s \in S$
γ^b	risk-aversion coefficient in the utility function of bank $b \in B$
λ^b	bank $b \in B$'s non-pecuniary cost of default coefficient for bank $b \in B$
A_t^b	the value of risk-free asset held by bank $b \in B$ at time t
$D_t^{I,b}$	interbank deposit taken by bank $b \in B$ at time t
$L_t^{I,b}$	interbank loan issued by bank $b \in B$ at time t
e_t^b	amount of capital that bank $b \in B$ holds at time t
ϵ_t^b	the un-modelled other items in the balance sheet of bank $b \in B$ at time t
r_t^b	interest rate of the consumer loans that bank $b \in B$ extends at time t
$r_t^{D,b}$	deposit rate offered by bank $b \in B$ at time t
r_t^A	the rate of return on the safe asset at time t
ρ_t	interbank rate at time t
$\delta_{t+1,s}^b$	dividend distribution of bank $b \in B$ at time $t+1$ in state $s \in S$
$g_{b,1}, g_{b,2}, g_{b,3}$	coefficients for consumer loans repayment rate
$u_{s,1}, u_{s,2}$	coefficients for GDP level
Endogenous Variables	
U_{t+1}^b	utility of bank $b \in B$ at $t+1$
$\Pi_{t+1,s}^b$	profit/loss of bank $b \in B$ at time $t+1$ in state $s \in S$
$\Delta \Pi_{t+1,D}^b$	expected shortfall in profit of bank $b \in B$ at time $t+1$
$\Lambda_{t+1,s}^b$	default of bank $b \in B$ at time $t+1$ in state $s \in S$
L_t^b	consumer loans of bank $b \in B$ at time t
D_t^b	deposit taken by bank $b \in B$ at time t
$v_{t+1,s}^b$	repayment rate of bank $b \in B$ at time $t+1$
$v_{t+1,s}^{b,h}$	repayment rate of bank b 's consumer loans in state $s \in S$
$Y_{t+1,s}$	GDP level at time $t+1$ in state $s \in S$
M_t	central bank's supply of base money at time t
\tilde{R}_{t+1}	pooled repayment rate in the interbank market

from profits, and quadratic disutility from default,

$$\max_{\sigma_t^b = \{L_t^b, D_t^b, v_{t+1,s}^b\}} \mathbb{E}_t(U_{t+1}^b) = \sum_{s \in S} p_s \left(U^b(\Pi_{t+1,s}^b) - V^b(\Lambda_{t+1,s}^b) \right) \quad (2.1)$$

$$\text{where} \quad U^b(\Pi) = \Pi - \gamma^b \Pi^2 \quad (2.2)$$

$$V^b(\Lambda) = \lambda^b \Lambda^2 \quad (2.3)$$

where Π is the bank's profit and Λ are the unpaid obligation. In addition, the maximization problem is subject to the budget constraint at time t , $\mathcal{B}_t^b(r_t^b, r_t^{D,b})$ defined by equation (2.4). At time t , the sum of risky consumer loans, risky interbank loans, and safe assets equals the sum of deposits, interbank deposits, equity, and the other items that are not explicitly modelled,

$$L_t^b + L_t^{I,b} + A_t^b = D_t^b + D_t^{I,b} + e_t^b + \epsilon_t^b \quad (2.4)$$

The profit/loss at state s of time $t + 1$ equals the proportional repayments from the consumer loans and interbank loans, and the risk-free assets, minus the proportional repayments for the deposits and interbank deposits, as well as the initial equity and some other exogenous items,

$$\begin{aligned}\Pi_{t+1,s}^b = & v_{t+1,s}^{b,h} L_t^b (1 + r_t^b) + \tilde{R}_{t+1,s} L_t^{I,b} (1 + \rho_t) + A_t^b (1 + r_t^A) \\ & - v_{t+1,s}^b \left(D_t^b (1 + r_t^{D,b}) + D_t^{I,b} (1 + \rho_t) \right) - e_t^b - \epsilon_t^b\end{aligned}\quad (2.5)$$

and the expected shortfall in profit is defined as,

$$\Delta \Pi_{t+1,D}^b = \Pi_{t+1,U}^b - \Pi_{t+1,D}^b \quad (2.6)$$

The bank chooses the probability (or fraction) of default v_{t+1}^b after the macro-state is realized, under the constraint,

$$0 \leq v_{t+1,s}^b \leq 1 \quad (2.7)$$

Unpaid obligations for deposits and interbank deposits amount to

$$\Lambda_{t+1,s}^b = (1 - v_{t+1}^b) \left(D_t^b (1 + r_t^{D,b}) + D_t^{I,b} (1 + \rho_t) \right) \quad (2.8)$$

Bank b 's equity in period $t + 1$ is the sum of equity in the previous period, of profits, minus the dividends distributed at $t + 1$,

$$e_{t+1,s}^b = e_t^b + \Pi_{t+1,s}^b - \delta_{t+1,s}^b \quad (2.9)$$

2.3 Central bank

The decisions of the central bank are taken as exogenous. The central bank sets the interbank rate (ρ_t), and base money is then endogenous (M_t). A regulator could also affect each bank's risk-taking parameters, but the utility function of each bank, in particular the default penalty coefficients λ^b , are taken as exogenous.

2.4 Households

Each borrowing household asks for funding from the bank it is allocated to, and after the macro-state is revealed, chooses how much to default on its debt. More specifically, the i^{th} household borrows from the i^{th} bank ($i = 1, 2, 3, \dots, N$), and the $(N + 1)^{th}$ household supplies deposit to all the banks. We do not explicitly model the optimization problem of the households, but assume them to be strategic dummies on the loan and deposit market. This assumption simplifies the model, which allows us to focus on the banking sector. The repayment rates of the consumer loans is assumed to be simple function of GDP and of aggregated credit supply.

$$\ln(v_{t+1}^{b,h}) = g_{b,1} + g_{b,2} \ln(Y_{t+1,b}) + g_{b,3} \sum_{b \in B} \ln(L_t^b) \quad (2.10)$$

with exogenous coefficients $g_{b,1}, g_{b,2}, g_{b,3}$ that will be calibrated below.

2.5 GDP

GDP in each state is assumed to be a simple function of aggregate credit supply available in the previous period:

$$\ln(Y_{t+1,s}) = u_{s,1} + u_{s,2} \sum_{b \in B} \ln(L_t^b) \quad (2.11)$$

where $u_{s,1}$ and $u_{s,2}$ have to be calibrated.

2.6 Market clearing conditions

In equilibrium, the central bank clears the interbank market by setting the money supply M_t to the difference between total interbank deposits and total interbank loans:

$$\sum_{b \in B} D_t^{I,b} = M_t + \sum_{b \in B} L_t^{I,b} \quad (2.12)$$

For the sake of simplicity, the repayment rate in the interbank market is assumed to be identical across banks, and is determined by the following equation:

$$\tilde{R}_{t+1,s} = \frac{\sum_{b \in B} v_{t+1,s}^b D_t^{I,b}}{\sum_{b \in B} D_t^{I,b}} \quad (2.13)$$

2.7 Equilibrium

The equilibrium is defined as a set of asset allocations and repayment rates such that the banks optimize their expected utility under the given constraints, and the interbank market clears. The first-order conditions are given in Appendix A.

Definition 1. *The Equilibrium of the economy*

$$\mathcal{E}_t = \{(e_t^b, \epsilon_t^b, A_t^b, L_t^{I,t}, D_t^{I,t})_{b \in B}; M_t; (\lambda_t^b, \gamma_t^b)_{b \in B}; r_t^A; p; (g_{b,i})_{b \in B, i \in \{1,2,3\}}, (u_{s,j})_{s \in S, j \in \{1,2\}}\}$$

is

$$\{(\sigma_t^b)_{b \in B}; (Y_{t+1,s})_{s \in S}\}$$

iff:

1. $\sigma_t^b \in \underset{\sigma_t^b \in \mathcal{B}_t^b}{\operatorname{argmax}} \mathbb{E}_t(U_{t+1}^b) \quad b \in B$
2. Household repayments satisfy equation (2.10)
3. GDP evolves by equation (2.11)
4. Interbank market clears by equation (2.12)
5. Banks are correct in their expectation about the repayment rates on the interbank market, given by equation (2.13)

The model can be solved numerically as a system of non-linear equations. Which variables are exogenous and which variables and endogenous depends on the closure of the model, which will

be discussed below. The model's results of most interest is each banks' expected shortfall, which will be matched to its empirical counterpart, estimated using the method described in the next section.

2.8 Financial distress, systemic risk and the Minsky cycle

We conclude the description of the model by explaining how systemic risk is captured. At the bank's individual level, the fraction (or probability) of default is endogenous in the model since banks choose the (expected) repayment level on their obligations. The lending banks' first order condition w.r.t. the repayment rate is

$$1 - 2\gamma^b \Pi_{t+1,s}^b = 2\lambda^b (1 - v_{t+1,s}^b) D_t^b (1 + r_t^{D,b}), \quad b \in B^L \quad (2.14)$$

and the equation for a borrowing bank is similar. The left-hand-side is the marginal benefit of default and the right-hand-side is the non-pecuniary marginal cost of default (which can be understood as the banker's cost of bankrupting the bank). After the macro-state is realized, and repayment and profits from consumer loans are known, each bank chooses the fraction of default (v_{t+1}^b). The bank's optimal solution is such that the fraction of default is lower when profits are higher, *ceteris paribus*.

At the banking-sector level, we interpret the good macro-state in the model as a scenario where all the banks are healthy, while in the bad state at least one bank is in distress. The good state in the equilibrium model represents the scenario where no bank defaults, while the bad state represents the combination/average of the remaining $2^N - 1$ scenarios, where at least one bank is in distress. Due to the existence of direct and indirect contagion, financial distress in one bank can affect the other banks. Thus all the banks have positive probabilities (fraction) of default in the bad state. The measure of exposure to systemic risk used in the model is the conditional expected shortfall defined in equation (2.6). This measure captures, for each bank, the difference in P&L between the good macro-state and the bad macro-state.

The equilibrium model explicitly incorporates two sources of systemic risk: contagion in the interbank market and a common asset exposure channel. As illustrated by Figure ??, when a borrowing bank suffers a loss on its loan portfolio in the bad macro-state ($v_{t+1,D}^{b,h} < 1$), the banks' fraction/probability of default increases ($0 < v_{t+1,D}^b < 1$), affecting deposits and the interbank market in particular. Thus, the banks that were lending in the interbank market suffer losses. The lending banks profits ($\Pi_{t+1,D}^{b'}$) fall, as do their capital available for the next period ($e_{t+1}^{b'}$).

The model also includes a common asset exposure channel, magnified by a credit channel similar to the typical fire-sale channel (Figure 2). First, the repayment rates of loans across households are correlated (because no default occurs in the good state, whereas repayment rates are lower than 100 percent in the bad state for all banks). Second, when a bank suffers losses,

it extends less credit, which reduces GDP (since GDP depends on the sum of all bank credit) and repayment rates across the entire banking system fall (see equation (2.10)).

Finally, a Minsky cycle of optimism is possible since we allow banks' subjective probabilities that a good macro-state is realized to differ from the true probability, and the bank's beliefs affect its portfolio choice. Because there are only two macro-states of nature in the model, the bank portfolio choice is restricted to choosing how to invest between two asset classes (loans to households vs. safe assets), or equivalently, banks choose their leverage and the proportion of investment in the risky asset. We do not specify a rule for the evolution for the optimism¹⁰ but we use the model and the empirical moments to shed light on how optimism, formalized as a higher value of the probability of the good state p , might explain changes in the leverage and investment strategies of banks (higher value of m_t^b) through time.

The following proposition shows how the banks adjust their assets when subjective beliefs change.

Proposition 1. *The banks described in Section 2.2 invest more in the risky consumer loans when becoming more optimistic, i.e. $\frac{\partial L_t^b}{\partial p} > 0$. This leads to a higher volatility in the banks' profit/loss, i.e. $\frac{\partial \Pi_{t+1,U}^b}{\partial p} > 0$ and $\frac{\partial \Pi_{t+1,D}^b}{\partial p} < 0$, thus larger expected shortfalls in profit.*

The proof is given in Appendix B.

3 Empirical estimates of systemic risk

The objective of the paper is to use empirical moments to shed light on the channels of systemic risk, using the structure of the model described in the previous section. The empirical moments we use are “cross-sectional moments”, i.e. expected values for the assets of one bank, conditioning on the asset value of another bank. These moments are estimated using a multivariate density, which captures the dependence between the asset values of all the banks in the banking system.

This multivariate density is constructed using the Consistent Information Multivariate Density Optimization (CIMDO) method proposed by Segoviano (2006). Other methods could be used, but the advantage of CIMDO is that it provides good estimates of dependence “at the tail”. CIMDO is a non-parametric method and a general procedure that constructs multivariate densities from a prior distribution and a reduced set of information, in this case, information on each bank's probability of default. It ensures that the posterior distribution is the closest distribution to a prior density that is also consistent with information on the probabilities of default of each bank, which we will obtain from market data.¹¹ A short explanation of CIMDO

¹⁰Bhattacharya et al. (2015) considers a similar portfolio choice problem where belief are updated when a new state of nature is realized, following Bayes' rule.

¹¹The CIMDO method is based on the Kullback (1959) cross-entropy approach. Instead of assuming parametric probabilities to characterize the information contained in the data, the entropy approach uses the data

is provided in the next section, but the interested reader is referred to [Segoviano and Espinoza \(2017\)](#) for more details. That paper also shows that CIMDO outperforms other parametric distributions (t-distributions, mixture of normals) when focus on tail risk is essential.

3.1 Method

CIMDO infers the unknown multivariate distribution of the equity value for a system M banks, $p(l^1, \dots, l^M)$ (the posterior distribution), from the observed Probability of Distress (PoD) of each bank making up the system and from a prior multivariate distribution $q(l^1, \dots, l^M)$. The cross-entropy approach recovers the distribution that is closest to the prior distribution but also consistent with the empirically observed probabilities of default of each bank making up the banking system.

We illustrate an example of two assets below, while all results are directly applicable when $M > 2$. The two banks are characterized by their equity's logarithmic returns x and y and the optimization problem is simply defined as

$$\min_{p(\cdot, \cdot) \in S} C[p(x, y), q(x, y)] = \int \int p(x, y) \ln \left[\frac{p(x, y)}{q(x, y)} \right] dx dy \quad (3.1)$$

where $q(x, y) \in \mathbb{R}^2$ is the prior distribution and $p(x, y) \in \mathbb{R}^2$ the posterior distribution. The Kullback cross-entropy criteria $C[p(x, y), q(x, y)]$ can be thought of as the weighted average of the relative distance between p and q and is a measure of distance between the prior distribution q and the posterior distribution p .

The moment-consistency constraints incorporate information about the probabilities of distress of each bank. In line with [Merton \(1974\)](#)'s structural approach, the constraint is such that the probability of distress is equal to the probability that the value of the bank falls below a certain threshold. Imposing this constraint on the optimization problem guarantees that the posterior multivariate distribution is consistent with the probability of default observed for each bank:

$$\int \int p(x, y) \mathbb{1}_{x \geq X_d^x} dx dy = PoD_t^x \quad \text{and} \quad \int \int p(x, y) \mathbb{1}_{y \geq X_d^y} dx dy = PoD_t^y \quad (3.2)$$

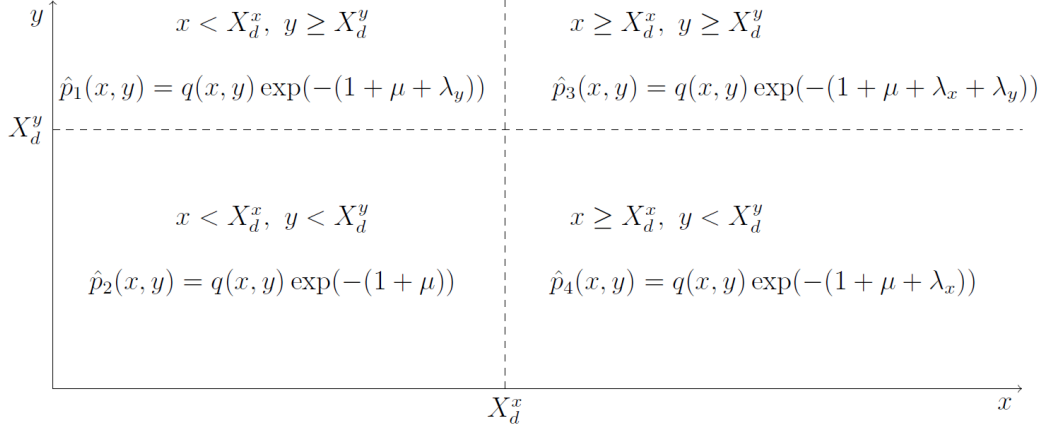
where $-x, -y$ are the equity annualized returns, and $\mathbb{1}$ is the indicator function, and the thresholds for banks x and y are X_d^x and X_d^y , respectively. In addition, the posterior density \hat{p} should sum to 1. [Segoviano and Espinoza \(2017\)](#) solve the optimization problem, and give the posterior multivariate density as

$$\hat{p}(x, y) = q(x, y) \exp \left\{ - \left[1 + \mu + (\lambda_x \mathbb{1}_{x \geq X_d^x}) + (\lambda_y \mathbb{1}_{y \geq X_d^y}) \right] \right\} \quad (3.3)$$

where the lagrange multipliers λ_x and λ_y correspond to the constraints (4.2). The lagrange multiplier μ corresponds to the constraint that the posterior distribution sums to 1. Numerically,

information to infer values for the unknown probability density.

Figure 6: CIMDO-density, adjustment factor



λ_x , λ_y , and μ are the solutions of the following system (which requires numerical integration):

$$\begin{cases} \int \int \hat{p}(x, y) \mathbb{1}_{x \geq X_d^x} dx dy = PoD_t^x \\ \int \int \hat{p}(x, y) \mathbb{1}_{y \geq X_d^y} dx dy = PoD_t^y \\ \int \int \hat{p}(x, y) dx dy = 1 \end{cases} \quad (3.4)$$

Intuitively, CIMDO adjusts the prior distribution with available information on moments using the factor $\exp\{-[1 + \mu + (\lambda_x \mathbb{1}_{x \geq X_d^x}) + (\lambda_y \mathbb{1}_{y \geq X_d^y})]\}$, which depends on the domain of the density (see Figure 6). Segoviano and Espinoza (2017) show that CIMDO tends to strengthen dependence when marginal PoDs are underestimated by the prior.

The prior density is calibrated using information on equity returns. The stock market returns' moments are used to calibrate a prior's t-distribution. The observed PoDs are crucial inputs to CIMDO. For the application to systemic risk as in this paper, 1-year ahead CDS spreads are used as proxies. The default thresholds X_d^x and X_d^y are fixed to an average through time that is consistent with a historical average of the probability of default for each bank, and with the prior distribution.¹²

3.2 Expected shortfalls in profits

The expected shortfall in profit, which is the difference between the profits/loss in the good macro-state and the P&L in the bad macro-state when banks are in distress, is our bank-specific measure of exposure to systemic risk. This measure is an outcome of the theoretical model but it can also be matched to its empirical counterpart, estimated using market data. In the good macro-state, households do not default on their loans, and thus all the banks are

¹²More precisely, for an example where the prior distribution is bivariate standard normal, the historical average of the distress threshold for each borrower is set to $X_d^x = \Phi^{-1}(\bar{\alpha}^x)$ and $X_d^y = \Phi^{-1}(\bar{\alpha}^y)$, where $\Phi(\cdot)$ is the standard normal cdf and $\bar{\alpha}^x = 1 - PoD^x$ and $\bar{\alpha}^y = 1 - PoD^y$.

healthy. In the bad macro-state, however, households partially default on their loans, damaging the banks' profitability and financial stability. The model solves for the conditional P&L at each state and thus for the expected shortfall in profits, as a function of the banks' initial endowments, utility parameters, and the monetary conditions.

The P&L can also be estimated using the empirical model. We start with an example with two banks A and B . Let us call $p(x, y)$ the joint density function of equity values' annualized returns obtained by CIMDO. The expected value of the equity of bank A given the distress of bank B is,

$$\mathbb{E}_t(EQ_{A,t+1}|B) = \frac{1}{\mathbb{P}(B)} \mathbb{E}_t(EQ_{A,t+1} \mathbb{1}_B) = \frac{1}{\mathbb{P}(B)} \int \int EQ_{A,t} e^{-x} p(x, y) \mathbb{1}_{y > X_d^y} dx dy \quad (3.5)$$

where $EQ_{A,t}$ is the equity value of bank A at time t , and event B represents the case where bank B is in distress. Given the initial value of equity, we can calculate the annualized return on equity. The change of equity value through time represents the profit/loss in each scenario.¹³

We generalize the method to a banking system with N banks. Since each bank can be either health or in distress, there are 2^N scenarios in total. S is the set of all 2^N scenarios. The scenario where no bank is in distress is denote H . Equation (3.6) gives the expected value of equity for each bank in cases where at least one bank is in distress, whereas equation (3.7) gives the expected value of equity for each bank in the case where no bank is in distress. The difference between the two values is the expected shortfall of profits ($\Delta\Pi$) due to systemic risk.

$$\begin{aligned} & \mathbb{E}_t(e_{t+1}^b | S \setminus H) \\ &= \frac{e_t^b}{1 - \mathbb{P}(H)} \int \cdots \int e^{-x^b} p(x^1, \dots, x^N) \left(1 - \mathbb{1}(x^{b'} < X_d^{b'}, \forall b' \in B)\right) dx^1 \cdots dx^N \end{aligned} \quad (3.6)$$

$$\begin{aligned} & \mathbb{E}_t(e_{t+1}^b | H) \\ &= \frac{e_t^b}{\mathbb{P}(H)} \int \cdots \int e^{-x^b} p(x^1, \dots, x^N) \mathbb{1}(x^{b'} < X_d^{b'}, \forall b' \in B) dx^1 \cdots dx^N \end{aligned} \quad (3.7)$$

$$\Delta\Pi_{t+1}^b = \mathbb{E}_t(e_{t+1}^b | H) - \mathbb{E}_t(e_{t+1}^b | S \setminus H) \quad (3.8)$$

4 Model-based assessment of the drivers of systemic risk

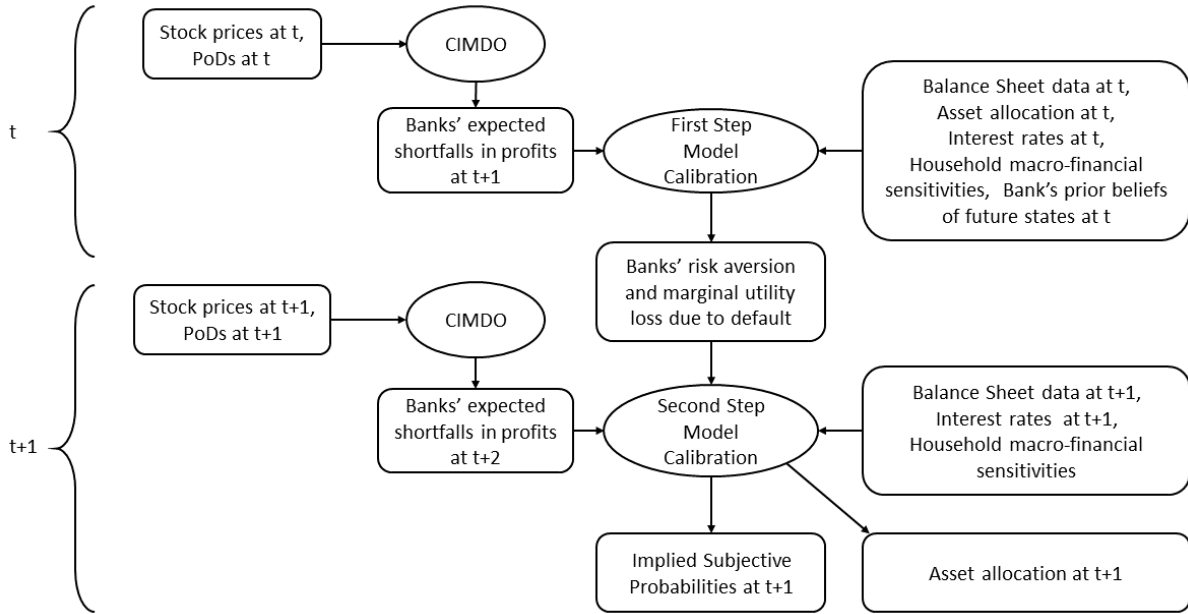
In this section, we explain how we apply the model to data on in order to assess the drivers of systemic risk in the UK banking sector. A calibration is done for two periods, before (2005-2007) and after (2014-2016) the global financial crisis. We spell out below the details of the calibration for the first period; the procedure is the same for the second period. The calibration involves two steps (see illustration in Figure 7). The first step uses data from the period June

¹³The only question that remains is how to choose the distress threshold X_d^x and X_d^y . The thresholds delimiting the region of distress for each marginal need to be fixed so that changes in PoD_t^x and in PoD_t^y affect the shape of the posterior distribution rather than the thresholds themselves. See the previous footnote for details.

2005-June 2006, and the second step uses data from June 2006 to June 2007¹⁴

In the first sub-period ($t=2005-06$), the model is calibrated using banks' balance sheet data (including the choice of leverage, and the volume of loans to households), data on interest rates, and parameter assumptions on the household macro-financial sensitivity coefficients. In addition, the expected shortfall for each bank, estimated using the approach presented in section 3 is also used for the calibration. By inverting the model, this calibration yields a calibration for the utility function of each bank, i.e. the risk aversion coefficients and the marginal disutility of default λ^b , which are not observable.

Figure 7: Model-based assessment of the drivers of systemic risk



Note: The graph shows how we assess the model parameters in a two-step calibration approach. The ovals represent the models, while the rectangles represent the variables. The directions of the arrows indicate whether a variable is input or output for the model. The first-step calibration of the equilibrium model takes the banks' expected shortfalls in profits for time $t + 1$ as inputs, and solve for the banks' risk aversion and default penalty coefficients. These two solved sets of variables are used as inputs for the second-step calibration, from which we get the implied changes in the subjective probability for the future states of each bank.

In the second sub-period ($t=2006-07$), these parameters are exogenous and set to the value obtained from the previous step. They are used, together with the model and updated balance sheet data, to predict the banks' profitability for each macro-state in 2007, and thus to compute the model-based expected shortfall for each bank. The model-based expected shortfall is compared to the empirical estimate of expected shortfall. We conjecture that the difference is

¹⁴We have quarterly balance sheet data for the UK commercial banks. Since the empirical approach uses data for one-year ahead probabilities of distress, we use balance sheet data for mid-year in order to assess the system before the crisis.

due to changes in the banks’ optimism (the Minsky cycle). By inverting the model again, this difference is set to 0 by allowing each bank’s subjective probability of a good macro-state to be different from the true probability. How each bank’s subjective probability of a good state compares to the ‘true’ probability gives us a measure of optimism. Finally, in order to check whether the procedure yields predictions consistent with data not used in the calibration, we check the leverage ratio and loan volumes predicted by the model against the actual values observed in 2007.

4.1 Data

We apply the model to the UK banking sector, which we simplify by focusing on its five largest banks (as per their asset value for 2005): Barclays plc, HSBC Bank plc, Lloyds Bank plc, the Royal Bank of Scotland plc, HBOS plc, and Standard Chartered plc.¹⁵ The first period of the model is June 2005-June 2006, while the second period is June 2006-June 2007. This period, before the financial crisis, was characterized by growing leverage and overall optimism following the years of the “Great Moderation”.

We collect data from FitchConnect and Moody’s on semi-annual balance sheets, daily stock prices, and EDFs. We set L^b to the value of total gross loans from the balance sheet data. A^b is set to the value of total securities, D^b is set to the value of total deposits, e^b is total equity, L^I is the value of loans from other banks, and D^I is the value of deposits from other banks. The data is shown in Table 5 in Appendix C (unavailable data points are linearly interpolated). We take the stock prices for the holding companies of each bank, and the expected default frequency (EDF) since 2004 as the input variables for the empirical estimation. The EDFs are used as a measure of probability of default for each bank.

4.2 Results

We first calibrate the model with data from June 2005 in order to set each bank’s utility parameters (risk aversion and marginal cost of defaulting). The model is inverted as described in Table 3. The variables are classified by column according to whether they are exogenous (RHS column) or endogenous (LHS column) according to theory. The rows show how the model is inverted: for each variable, the row shows whether the variable is solved for endogenously, is observed empirically, is estimated using market data and CIMDO, or is selected arbitrarily.

When solving the system of simultaneous equations, we calibrate the loans extended (L), deposits taken (D), safe asset holdings (A), capital level (e), and interbank loans/deposits (L^I/D^I) against the real balance sheet data in June 2005, the GDP in the good state against the nominal

¹⁵For the period 2005-2007, let bank 1, 2, 3, 4, 5 represent Barclays, HSBC, Lloyds, and RBS, respectively, whereas bank 5 represents the aggregation of the remaining two banks: HBOS and Standard Chartered. This aggregation is necessary to reduce the size of the model. We directly add the balance sheet items of the two banks to get the variables for bank 5. The stock prices and EDFs for bank 5 are aggregated by taking an weighted average of HBOS and Standard Chartered’s data by market capitalization. For the period of 2014-2016, the five banks 1, 2, 3, 4, 5 are Barclays, HSBC, Lloyds, RBS, and Standard Chartered, respectively.

Table 3: Variables for model calibration

Equilibrium Model		
	Model Endogenous Variables	Model Exogenous Variables
Endogenously Solved	$\Pi_U^b, \forall b \in B$: banks' conditional P/L in good state $\Pi_D^b, \forall b \in B$: banks' conditional profits/losses in the bad state $v^{b,h}, \forall b \in B$: repayment level of the consumer loans M : Money supply by the central bank \tilde{R} : Pooled interbank deposit repayment rate in the bad state	$\lambda^b, \forall b \in B$: the non-pecuniary default penalty coefficient $\gamma^b, \forall b \in B$: each bank's risk-aversion coefficient $e^b, \forall b \in B$: un-modelled balance sheet items $g_1^b, \forall b \in B$: the elasticity of consumer loan repayment rate with respect to GDP $u_{s,1}, s \in \{U, D\}$: coefficients for GDP
Empirically Observed	$L^b, \forall b \in B$: banks' holding of consumer loans $D^b, \forall b \in B$: banks' liability of deposits Y_U : the GDP level at the good state	$A^b, \forall b \in B$: banks' holding of safe assets $e^b, \forall b \in B$: banks' initial equity level $D^{I,b}, \forall b \in B$: banks' liability of interbank deposits $L^{I,b}, \forall b \in B$: banks' holding of interbank loans ρ : the interbank interest rate
Estimated by CIMDO	$\Delta \Pi^b, \forall b \in B$: banks' expected shortfalls in profits	
Manually Selected	$v^b, \forall b \in B$: banks' repayment rate in the bad state Y_D : the GDP level at the bad state	$r^{D,b}, \forall b \in B$: deposit interest rate $r^b, \forall b \in B$: consumer loan interest rate r^A : banks' market book return p : probability of good state g_2^b, g_3^b : coefficients for consumer loan repayment $u_{s,2}, s \in \{U, D\}$: coefficients for GDP

Notes: This table presents the variables for the equilibrium model. The values of these variables are presented in Table 6 in Appendix C. Remember that the variables which are exogenous when solving the system of simultaneous equations, do not necessarily have to be those which are exogenous in the model.

The column on the left shows the endogenous variables in the equilibrium model, while the column on the right shows the exogenous variables. This is consistent with the classification of the variables in Table 2.

When solving the simultaneous equations, the value for some exogenous variables in the model such as the default penalty coefficient ($\lambda^b, b \in B$) and the risk aversion coefficients ($\gamma^b, b \in B$) cannot be empirically observed or estimated, while some endogenous variables in the model such as the position in the consumer loans ($L^b, b \in B$) and deposit taken ($D^b, b \in B$) can be observed empirically.

Variables in the first row are endogenously computed when solving the simultaneous equations; the balance sheet items and the macroeconomic data in the second row are empirically observed; the conditional expected profits/losses in the third row are estimated by CIMDO, which captures the information about distress dependence structure; the variables in the forth row are "manually" selected.

GDP in 2006, and the interbank rate against the 12-month LIBOR at June 2005. The P&L expected shortfalls for each bank ($\Delta \Pi$) is estimated using the empirical CIMDO-distribution.¹⁶

In the model, the interest rate on interbank loans and on loans to households are equal; thus we assume the rate r^D is the same as the interbank rate ρ . The average spread between loans and deposits rates for commercial banks was 3.66% in 2005¹⁷, so we set r to be r^D plus the spread. The probability of a good state to occur (p) is selected manually to 0.94, but we run

¹⁶More specifically, CIMDO takes the stock prices and the EDFs from Jan. 2nd, 2004 to June 30th, 2005 as input variables, and generates the expected percentage change of equity value for each bank, under the condition that at least one bank is in distress in June 2006. As explained previously, CIMDO re-constructs the joint distribution of banks' equity values, which is consistent with the probability of default, and captures the information of the distress dependence structure. Thus we can calculate the conditional expected profits/losses in June 2006, using CIMDO's results and the initial value of equity for each bank in June 2005.

¹⁷FISIM data.

a robustness check afterwards. The banks' repayment rate (v) is set to be consistent with the empirically observed Expected Default Frequency (EDFs)¹⁸. The banks' market book return (r^A), the level of GDP in the bad macro-state of nature (Y_D), and the parameters driving the household's repayment rates (g_2, g_3) and the sensitivity of growth to credit (u_2) are selected manually, following the parametrization in Goodhart, Sunirand, and Tsomocos (2005). The other variables are solved endogenously by inverting the model. The values of the input variables and the simulation results are presented in Table 6 of Appendix C.

The calibration yields two parameters for each bank: a coefficient setting the bank's risk aversion ($\gamma^b, b \in B$), and a coefficient setting the marginal cost of default ($\lambda^b, b \in B$); see Table 3. These parameters are essential as they drive the banks' portfolio decision process, but they are not directly observable. Overall, the model is thus calibrated so that the bankers' utility parameters are consistent with the banks' actual portfolio choices in 2005.

4.3 Analysis of channels of contagion

We then assume that these coefficients are constant between 2005 and 2007 and use the model as well as June-2006 data to predict the banks' portfolio choices and expected P&L for June-2007. In the first exercise, we set the subjective probabilities that the good-macro state is realized to its true value, i.e. 0.94 (the Minsky cycle is added in the second exercise presented below).

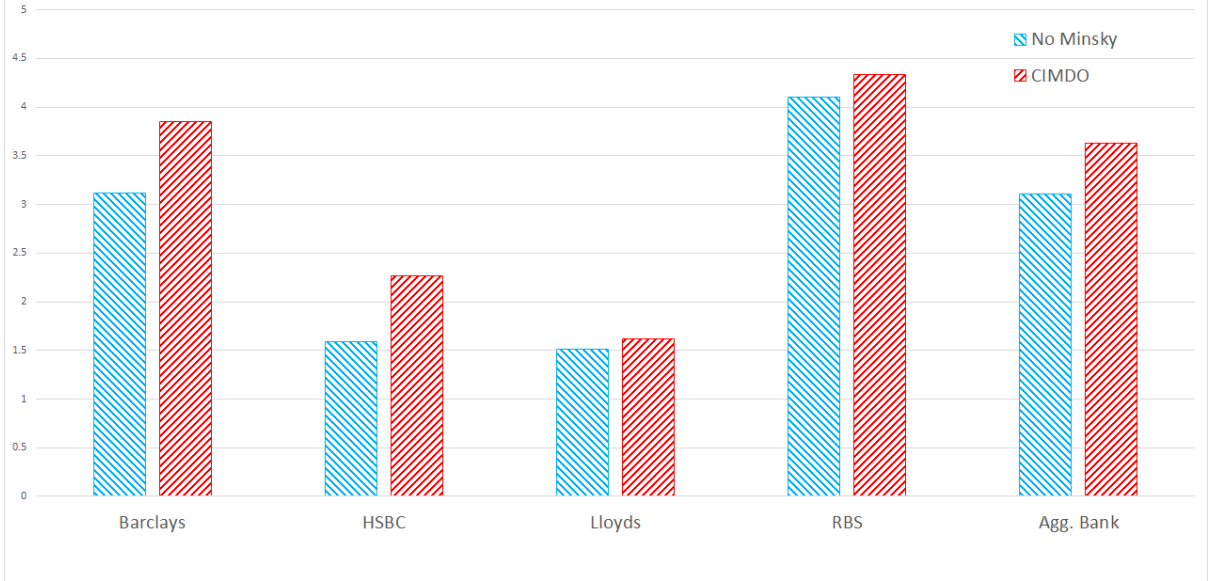
The balance sheet items (banks' equity e^b , position in the interbank market $D^{I,b}, L^{I,b}$, and other items ϵ^b) and the interbank rate (ρ) are updated to their values on June 30th, 2006. The data used is presented in Table 8 in the appendix. The model is solved in its natural way. The exogenous parameters and variables according to theory are set as exogenous. The endogenous variables according to theory are the outcome of the model. These are: the banks conditional expected P&L in the good and the bad macro-states ($\Pi_U^b, \Pi_D^b, b \in B$), the expected shortfalls in profits ($\Delta \Pi^b, b \in B$); the banks' choice to invest in the risk-free asset ($A^b, b \in B$), to issue loans to households ($L^b, b \in B$) and to take deposits ($D^b, b \in B$); the banks' fraction of default ($v^b, b \in B$); the default rate on loans to households ($v^{b,h}, b \in B$); money supply (M); the repayment rate in the interbank market in the bad state (\tilde{R}); and the level of GDP in each macro-state ($Y_s, s \in \{U, D\}$).

The numerical solution is shown in Table 7, with the same presentation as before. The model predicts that all the banks are making profits in the good macro-state, while some banks suffer from losses in the bad state. The model estimates of expected shortfall are compared to the empirical estimates (see Table 4 and Figure 8). Overall, the empirical model estimates larger expected shortfalls in profits than what theoretical model predicts. At this stage, the theoretical model only accounts for the interbank lending channel and the common asset exposure channel. We interpret the model's underestimation of expected shortfall as showing the possibility

¹⁸ v is set such that $EDF = (1 - p)(1 - v)$

that markets capture additional channels of distress dependence across banks. There are many candidate channels, for example liquidity contagion, maturity mismatch, etc., but in the next section we focus on whether the Minsky cycle could help improve the performance of the model.

Figure 8: Expected Shortfalls in Profits, June 2007



Notes: This figure compares the prediction of the theoretical model without Minsky cycle and CIMDO on the expected shortfalls in profits for each bank at June 2007. Agg. Bank represents the combination of HBOS and Standard Chartered. For each bank, the bar on the left is the theoretical model's result, while the bar on the right is CIMDO's result. CIMDO predicts larger expected shortfalls in profits in general. We know that the theoretical model incorporates the interbank lending channel and the common asset exposure channel for systemic risk transmission and amplification, while the CIMDO model should be able to capture the effects of all the channels, given the market data. Thus the difference between the two methods represents the missing channels of the theoretical model, which we attributes to the Minsky cycle.

4.4 The Minsky cycle

We now invert the model to take the empirical estimates of each bank's conditional expected shortfall as input, and thus to solve for each bank's subjective probability that a good state occurs ($p_1^b, b \in B$). Figure 1 and Table 9 summarizes the procedure. Table 10 in Appendix C presents the model results.

The model finds that $p_1^b > p_0^b, \forall b \in B$, i.e. according to the model, all the five banks in

Table 4: Banks' expected shortfalls in profits in June 2007, measured by CIMDO

$\Delta\Pi^1 = 3.8507$	$\Delta\Pi^2 = 1.3481$	$\Delta\Pi^3 = 4.8913$	$\Delta\Pi^4 = 3.0423$	$\Delta\Pi^5 = 2.8821$
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Notes: CIMDO generates the percentage change in equity value for each period. In this simulation, CIMDO predicts the change from June 2006 to June 2007.

Given the balance sheet data of equity level for each bank in June 2006, the profit at the end of the period can be computed. The difference across states represents the expected shortfall in profit.

the had become more optimistic in June 2006 than they were in June 2005. This is how the model interprets the worsening of systemic risk —i.e., the increase in expected shortfalls estimated empirically— observed through that period. The model’s prediction for banks’ (risky) loans to households ($L^b, b \in B$) is thus higher than when the Minsky cycle is turned off. Individually, the banks rationally increase their leverage and their investment in the risky asset if they believe that the probability of a good state occurring is high.

In addition, such risk-taking increases the frequency/fraction of default, which is an externality for each bank: the banks do not factor in the impact of their decision on the repayment rate in the interbank market and on future credit growth and thus the repayment rate of loans. When all the banks change their beliefs in the same direction simultaneously, their common exposure increases and the feedback loop between credit growth and loans’ repayment rates is stronger.

The model’s prediction for the leverage ratios¹⁹ is closer to actual data when the Minsky cycle is turned on. Figure 9 shows the actual leverage ratios for the main UK banks along those predicted by the equilibrium model when the Minsky cycle is turned on and off. This shows that the observed behavior in risk-taking can be represented relatively well by varying the subjective probability of the good macro-state occurring.

The results are robust to a different choice for the ‘true’ probability that the good macro-state occurs. This parameter was set to $p = 0.94$ in the calculations described above, which implies, under the binomial structure of uncertainty, that on average a crisis is expected every twenty years. We test the model with different values for $p > 0.91$, and all the results from above remain.

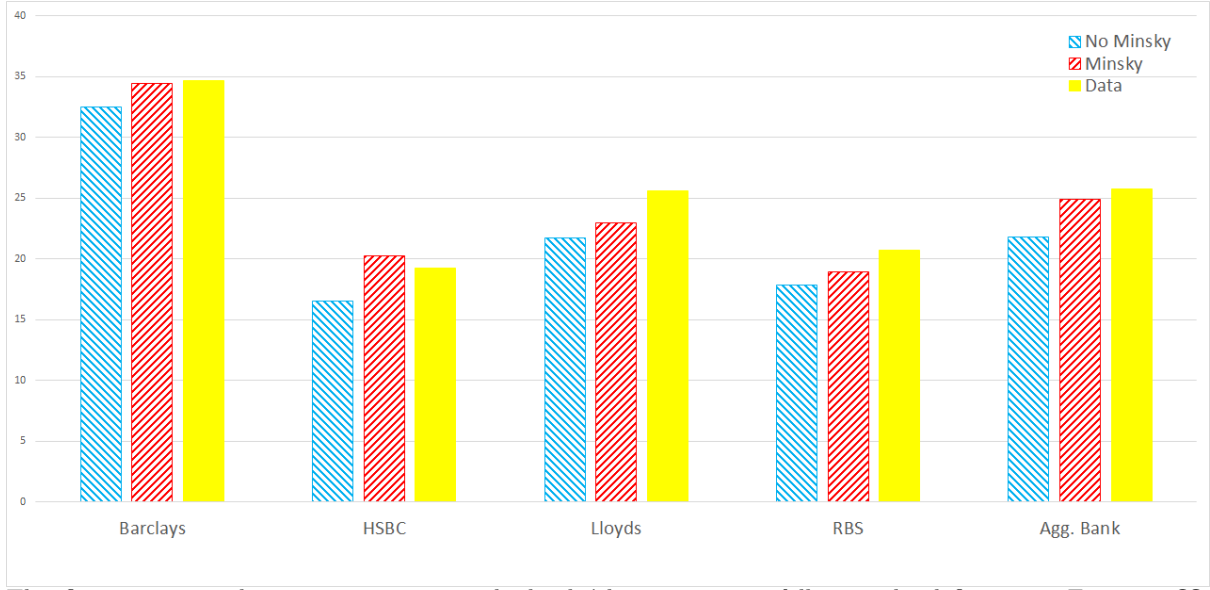
The initial choice is $p = 0.94$, which implies under the binomial structure that on average every twenty years a crisis is expected. This is a quite optimistic view. We test the model with different values for $p > 0.91$, and all the results from above remain. Remember that p is the probability of the good state in a binomial structure, thus $\frac{1}{1-p}$ represents the expected length of waiting period for a crisis to occur.

4.5 Post-Crisis Period

We re-run the simulations using data after the financial crisis (2014-2016), and the results are shown in Table 11, 12 and 13. As shown in Figure 10, the expected shortfalls in profits of each bank predicted by CIMDO are now lower than those predicted by the model that excludes the Minsky effect. To match CIMDO’s prediction, we allow the banks to update their beliefs as in the previous section. According to the results in Table 13, 3 out of the 5 banks become less optimistic, while the other 2 become more optimistic. This implies that the systemic risk

¹⁹Based on the balance sheet structure in the equilibrium model, the leverage ratio is defined as $\mathbb{L}_t^b \equiv \frac{\text{Total Asset}}{\text{Initial Equity}} = \frac{L_t^b + L^{I,b} + A_t^b}{e_t^b}$.

Figure 9: Leverage



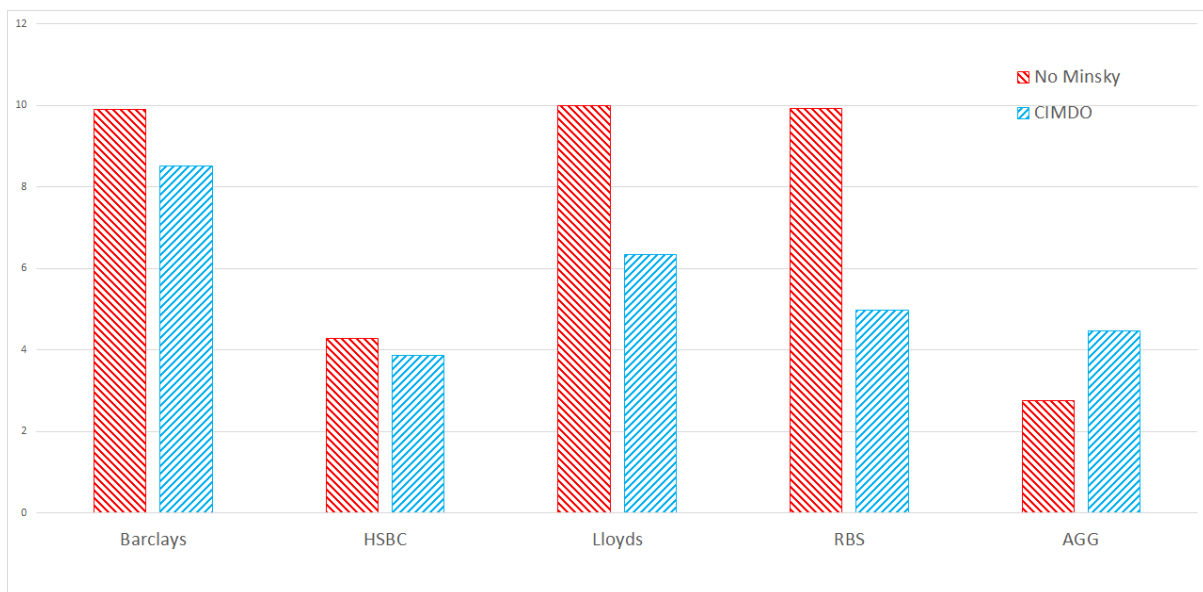
This figure presents the comparison among the banks' leverage ratios, following the definition in Equation ?? . For each bank, the bar on the left, in the middle, and on the right represents the leverage computed with data from GST model without Minsky effect, with Minsky effect, and with real data, respectively.

amplification due to the Minsky cycle of optimism is not as strong in the post-crisis period as in the pre-crisis period.

5 Conclusion

This paper has two major contributions. First, it provides an original method to combine an equilibrium model with an empirical approach to estimate systemic risk and interpret the channels driving it. Second, the paper highlights the importance of the Minsky cycle of optimism: when most of the banks become more optimistic at the same time, risk-taking increases, amplifying the traditional sources of systemic risk. We modify and extend the equilibrium model of a banking system proposed by [Goodhart, Sunirand, and Tsomocos \(2005\)](#) and show that risk in the banks' profits/losses increases when they become more optimistic, a result that we interpret as the Minsky cycle of optimism. We take the model to the data, using data from the UK banking system between 2005 to 2007 and between 2014 and 2016. The model is re-calibrated to match empirical estimates of the co-movement of banks' equity returns. Our results show that the Minsky cycle of optimism can account for a significant part of the distress dependence structure pre-crisis, although less loss in the more recent period.

Figure 10: Expected Shortfalls in Profits, Dec 2016



Appendices

A First order conditions of the equilibrium model

$$p(1 - 2\gamma^b \Pi_{t+1,U}^b) + (1 - p)(1 - 2\gamma^b \Pi_{t+1,D}^b) v_{t+1}^{b,h} = \mathcal{L}^b / (1 + r_t^b) \quad (\text{A.1})$$

$$p(1 - 2\gamma^b \Pi_{t+1,U}^b) + (1 - p) \left((1 - 2\gamma^b \Pi_{t+1,D}^b) v_{t+1}^b + 2\lambda^b (1 - v_{t+1}^b)^2 [D_{I,b}^b (1 + \rho_t) + D_t^b (1 + r_t^{D,b})] \right) = \mathcal{L}^b / (1 + r_t^{D,b}) \quad (\text{A.2})$$

$$1 - 2\gamma^b \Pi_{t+1,D}^b = 2\lambda^b (1 - v_{t+1}^b) [D_t^{I,b} (1 + \rho_t) + D_t^b (1 + r_t^{D,b})] \quad (\text{A.3})$$

where \mathcal{L}^b is the Lagrange multiplier.

B Proof of Proposition 1

Proof. We rewrite the first-order conditions and the budget constraints of the lending banks as below. To simplify the notation, remove the subscript t and superscript b .

$$p(1 - 2\gamma \Pi_U) + (1 - p)(1 - 2\gamma \Pi_D) v^h = \mathcal{L} / (1 + r) \quad (\text{B.1})$$

$$p(1 - 2\gamma \Pi_U) + (1 - p) \left((1 - 2\gamma \Pi_D) v + 2\lambda (1 - v)^2 [D(1 + r^D) + D^I(1 + \rho)] \right) = \mathcal{L} / (1 + r^D) \quad (\text{B.2})$$

$$1 - 2\gamma \Pi_D = 2\lambda (1 - v) [D(1 + r^D) + D^I(1 + \rho)] \quad (\text{B.3})$$

$$L + L^I + A = D + e + \epsilon \quad (\text{B.4})$$

$$\Pi_U = L(1 + r) + A(1 + r^A) + L^I(1 + \rho) - D(1 + r^D) - D^I(1 + \rho) - e - \epsilon \quad (\text{B.5})$$

$$\Pi_D = v^h L(1 + r) + A(1 + r^A) + \tilde{R} L^I(1 + \rho) - v [D(1 + r^D) + D^I(1 + \rho)] - e - \epsilon \quad (\text{B.6})$$

For a lending bank, L, D, v, Π_U, Π_D and the Lagrangian multiplier \mathcal{L} are endogenous, while all the other variables are exogenous. Combining (B.1) and (B.2), then plug in (B.3)

$$\begin{aligned} & p(1 - 2\gamma \Pi_U)(r - r^D) \\ &= (1 - p) \left((1 - 2\gamma \Pi_D) [v(1 + r^D) - v^h(1 + r)] + 2\lambda (1 - v)^2 [D(1 + r^D) + D^I(1 + \rho)] (1 + r^D) \right) \\ &= (1 - p)(1 - 2\gamma \Pi_D) \left(v(1 + r^D) - v^h(1 + r) + (1 - v)(1 + r^D) \right) \\ &= (1 - p)(1 - 2\gamma \Pi_D) (1 + r^D - v^h(1 + r)) \end{aligned} \quad (\text{B.7})$$

Take derivative w.r.t. p to the system, we have,

$$\begin{aligned}
(r - r^D) \left[1 - 2\gamma\Pi_U - p2\gamma\frac{\partial\Pi_U}{\partial p} \right] &= (1 + r^D - v^h(1 + r)) \left[-1 + 2\gamma\Pi_D - (1 - p)2\gamma\frac{\partial\Pi_D}{\partial p} \right] \\
- 2\gamma\frac{\partial\Pi_D}{\partial p} &= 2\lambda \left[-\frac{\partial v}{\partial p} [v(1 + r^D) - v^h(1 + r)] + (1 - v)\frac{\partial D}{\partial p}(1 + r^D) \right] \\
\frac{\partial L}{\partial p} &= \frac{\partial D}{\partial p} \\
\frac{\partial\Pi_U}{\partial p} &= (1 + r)\frac{\partial L}{\partial p} - (1 + r^D)\frac{\partial D}{\partial p} \\
\frac{\partial\Pi_D}{\partial p} &= v^h\frac{\partial L}{\partial p}(1 + r) - \frac{\partial v}{\partial p} [v(1 + r^D) - v^h(1 + r)] - v\frac{\partial D}{\partial p}(1 + r^D)
\end{aligned}$$

After some algebra, we have,

$$\frac{\partial L}{\partial p} = \frac{(1 - 2\gamma\Pi_U)(r - r^D) + (1 - 2\gamma\Pi_D)(1 + r^D - v^h(1 + r))}{2p\gamma(r - r^D)^2 + 2(1 - p)\frac{\gamma\lambda}{\gamma + \lambda}(1 + r^D - v^h(1 + r))^2} \quad (\text{B.8})$$

$$\frac{\partial\Pi_U}{\partial p} = (r - r^D)\frac{\partial L}{\partial p} \quad (\text{B.9})$$

$$\frac{\partial\Pi_D}{\partial p} = -\frac{\lambda}{\gamma + \lambda}(1 + r^D - v^h(1 + r))\frac{\partial L}{\partial p} \quad (\text{B.10})$$

The denominator of equation (B.8) is obviously positive. For the numerator, $(1 - 2\gamma\Pi_U)$ and $(1 - 2\gamma\Pi_D)$ are marginal utility, thus should be positive in equilibrium. $r - r^D$ is the spread between the consumer loan rate and the deposit rate. As long as the consumer loan does not strictly dominate the deposit, which provides an arbitrage opportunity for the banks, the interest rates should satisfy $r - r^D > 0$ and $1 + r^D - v^h(1 + r) > 0$. Thus, we can have $\frac{\partial L}{\partial p} > 0$, $\frac{\partial\Pi_U}{\partial p} > 0$, and $\frac{\partial\Pi_D}{\partial p} < 0$. Notice that the banks ignores the externalities of their choice variables on the equilibrium repayment rates of interbank loans and consumer loans. The overall effect of optimism on banks' profitability should include those as well. \square

C DATA

Table 5: UK Commercial Banks Balance Sheet data

Barclays plc						
	6/30/2005	6/30/2006	6/30/2007	12/31/2014	12/31/2015	12/31/2016
Loans to Banks	184,625	207,199	233,737	43,321	42,360	44,718
Total Gross Loans	239,951	285,497	324,517	433,222	404,138	397,404
Total Securities	339,495	391,867	485,961	811,456	600,914	582,929
Total Deposits	217,715	253,200	292,444	427,704	418,242	423,178
Deposits from Banks	209,423	235,086	270,728	59,567	48,093	48,850
Total Equity	21,785	25,539	28,721	57,982	56,419	61,946
HSBC Bank plc						
	6/30/2005	6/30/2006	6/30/2007	12/31/2014	12/31/2015	12/31/2016
Loans to Banks	35,819	41,269	55,282	25,262	23,222	21,363
Total Gross Loans	164,199	193,344	215,829	260,052	261,109	275,317
Total Securities	118,864	156,621	232,832	442,970	386,157	447,894
Total Deposits	218,256	255,116	293,661	346,507	332,830	375,252
Deposits from Banks	34,984	38,704	45,851	28,343	24,333	23,792
Total Equity	18,421	20,286	22,469	34,502	34,541	36,844
Lloyds Bank plc						
	6/30/2005	6/30/2006	6/30/2007	12/31/2014	12/31/2015	12/31/2016
Loans to Banks	31,752	36,147	33,599	24,256	24,154	5,146
Total Gross Loans	168,714	185,518	202,419	483,970	458,208	445,390
Total Securities	62,481	87,028	97,377	252,830	228,112	153,778
Total Deposits	161,996	170,739	144,866	447,067	418,326	412,998
Deposits from Banks	36,270	34,680	40,744	9,812	9,864	8,411
Total Equity	12,202	12,048	12,724	49,990	47,353	47,034
The Royal Bank of Scotland plc						
	6/30/2005	6/30/2006	6/30/2007	12/31/2014	12/31/2015	12/31/2016
Loans to Banks	61,969	72,552	87,926	23,884	18,744	17,635
Total Gross Loans	387,560	447,621	504,175	350,851	313,297	320,467
Total Securities	168,466	234,921	327,327	508,754	383,989	361,585
Total Deposits	418,199	484,596	419,015	357,649	346,962	348,969
Deposits from Banks	104,607	120,816	139,084	39,066	31,828	35,314
Total Equity	34,807	36,473	39,663	47,171	41,907	35,819
HBOS plc						
	6/30/2005	6/30/2006	6/30/2007	12/31/2014	12/31/2015	12/31/2016
Loans to Banks	16,133	17,730	10,075	49,483	32,251	48,027
Total Gross Loans	329,434	361,631	395,210	277,356	273,647	271,284
Total Securities	142,134	165,213	192,112	44,693	25,533	20,297
Total Deposits	196,240	208,137	227,117	202,936	190,046	179,317
Deposits from Banks	30,855	33,805	37,530	80,343	81,571	101,251
Total Equity	17,249	18,026	21,881	23,755	13,981	14,8355
Standard Chartered plc						
	6/30/2005	6/30/2006	6/30/2007	12/31/2014	12/31/2015	12/31/2016
Loans to Banks	11,684	9,131	10,528	53,748	43,521	59,024
Total Gross Loans	61,278	66,125	76,793	184,949	178,171	210,600
Total Securities	13,750	38,300	45,026	137,730	146,214	171,292
Total Deposits	60,647	77,790	79,926	265,371	242,339	307,522
Deposits from Banks	12,073	11,990	13,390	35,445	25,810	30,575
Total Equity	6,965	7,549	9,768	29,945	31,395	35,113

Source: FitchConnect, in Millions of GBP.

Table 6: Model Calibration, pre-crisis

Equilibrium Model					
	Model Endogenous Variables			Model Exogenous Variables	
Endogenously Solved	$\Pi_U^1 = 1.2756$	$M = 8.6231$		$\lambda^1 = 0.1364$	$g_1^1 = -0.5344$
	$\Pi_D^1 = -1.8152$	$\tilde{R} = 0.9348$		$\gamma^1 = 0.1282$	$g_1^2 = -0.3343$
	$v^{1,h} = 0.7226$			$\epsilon^1 = 10.5725$	$g_1^3 = -0.4051$
	$\Pi_U^2 = 0.8357$			$\lambda^2 = 1.5178$	$g_1^4 = -0.3845$
	$\Pi_D^2 = -0.9075$			$\gamma^2 = 0.4305$	$g_1^5 = -0.3322$
	$v^{2,h} = 0.8826$			$\epsilon^2 = 4.722$	$u_{1,U} = 3.3144$
	$\Pi_U^3 = 0.8781$			$\lambda^3 = 0.4552$	$u_{1,D} = 3.209$
	$\Pi_D^3 = -0.8629$			$\gamma^3 = 0.3277$	
	$v^{3,h} = 0.8223$			$\epsilon^3 = 5.248$	
	$\Pi_U^4 = 1.7616$			$\lambda^4 = 0.328$	
	$\Pi_D^4 = -2.6032$			$\gamma^4 = 0.1597$	
	$v^{4,h} = 0.8394$			$\epsilon^4 = 6.0382$	
	$\Pi_U^5 = 2.3990$			$\lambda^5 = 0.4292$	
	$\Pi_D^5 = -1.0903$			$\gamma^5 = 0.1648$	
	$v^{5,h} = 0.8845$			$\epsilon^5 = 20.7455$	
Estimated by CIMDO	$\Delta\Pi^1 = 3.0908$	$\Delta\Pi^4 = 4.3647$			
	$\Delta\Pi^2 = 1.7432$	$\Delta\Pi^5 = 3.4893$			
	$\Delta\Pi^3 = 1.741$				
Empirically Observed				$A^1 = 33.9495$	$A^4 = 16.8466$
				$e^1 = 2.1785$	$e^4 = 3.4807$
				$L^{I,1} = 18.4625$	$L^{I,4} = 6.1969$
	$L^1 = 23.9951$	$L^4 = 38.756$		$D^{I,1} = 20.9423$	$D^{I,4} = 10.4607$
	$D^1 = 42.7138$	$D^4 = 41.8199$		$A^2 = 11.8864$	$A^5 = 16.8466$
	$L^2 = 16.4199$	$L^5 = 39.0712$		$e^2 = 1.8421$	$e^5 = 3.4807$
	$D^2 = 21.8256$	$D^5 = 29.9815$		$L^{I,2} = 3.5819$	$L^{I,5} = 6.1969$
	$L^3 = 16.8714$	$Y_U = 137.9457$		$D^{I,2} = 3.4984$	$L^{I,5} = 10.4607$
	$D^3 = 16.1996$			$A^3 = 6.2481$	$\rho = 0.0454$
				$e^3 = 1.2202$	
				$L^{I,3} = 3.1752$	
				$D^{I,3} = 3.6270$	
Manually Selected				$r^{1,d} = 0.0454$	$g_2^1 = 0.01$
				$r^1 = 0.0820$	$g_2^2 = 0.01$
				$r^{2,d} = 0.0454$	$g_2^3 = 0.01$
	$v^1 = 0.9193$			$r^2 = 0.0820$	$g_2^4 = 0.01$
	$v^2 = 0.9778$			$r^{3,d} = 0.0454$	$g_2^5 = 0.01$
	$v^3 = 0.917$			$r^3 = 0.0820$	$g_3^1 = 0.01$
	$v^4 = 0.9489$			$r^{4,d} = 0.0454$	$g_3^2 = 0.01$
	$v^5 = 0.9558$			$r^4 = 0.0820$	$g_3^3 = 0.01$
	$Y_D = 124.1511$			$r^{5,d} = 0.0454$	$g_3^4 = 0.01$
				$r^5 = 0.0820$	$g_3^5 = 0.01$
				$r^A = 0.04$	$u_2 = 0.1$
				$p = 0.95$	

This table presents the values of variables in the model calibration, which covers the period from June 2005 to June 2006.

Bank 1, 2, 3, 4, 5 represents Barclays plc, HSBC plc, Lloyds plc, RBS plc, and the aggregated bank, respectively. The column on the left shows the endogenous variables in the equilibrium model, while the column on the right shows the exogenous variables.

Variables in the first row are endogenously computed when solving the simultaneous equations.

The expected shortfalls in the second row are estimated by CIMDO, which captures the information about distress dependence structure.

The balance sheet items and the macroeconomic data in the third row are empirically observed.

The variables in the forth row are "manually" selected. More specifically, the banks repayment rate is set consistent with EDF ($EDF = (1 - p)(1 - v)$); GDP in the bad state (Y_D) is set to be 90% of GDP observed; bank deposits in the model has the same risk as the interbank loans, thus bank deposit rates (r^d) are set equal to ρ ; the spread between r and r^d is calibrated to the average spread between loan and deposit.

Table 7: Second sub-period. Model solution, without Minsky cycle

Equilibrium Model		
	Model Endogenous Variables	Model Exogenous Variables
Endogenously Solved	$\Pi_U^b, \forall b \in B$: banks' conditional P/L in the good state $\Pi_D^b, \forall b \in B$: banks' conditional P/L in the bad state $\Delta \Pi^b, \forall b \in B$: banks' expected shortfalls in profits $v^{b,h}, \forall b \in B$: default level of the consumer loans M : Money supply by the central bank \tilde{R} : Pooled interbank deposit repayment rate in the bad state $L^b, \forall b \in B$: banks' holding of consumer loans $D^b, \forall b \in B$: banks' liability of deposits $v^b, \forall b \in B$: banks' probability of default in the bad state Y_U : the GDP level at the good state Y_D : the GDP level at the bad state	
Empirically Observed		$A^b, \forall b \in B$: banks' holding of safe assets $e^b, \forall b \in B$: banks' initial equity level $D^{I,b}, \forall b \in B^B$: borrowing banks' liability of interbank deposit $L^{I,b}, \forall b \in B^L$: lending banks' holding of interbank loan $\epsilon^b, \forall b \in B$: the other balance sheet items that are not explicitly modelled ρ : the interbank interest rate
Manually Selected		$r^{D,b}, \forall b \in B$: deposit interest rate $r^b, \forall b \in B$: consumer loan interest rate r^A : banks' market book return p : probability of good state g_2^b, g_3^b : coefficients for consumer loan repayment $u_{s,2}, s \in \{U, D\}$: coefficients for GDP
Calibrated		$\lambda^b, \forall b \in B$: the non-pecuniary default penalty coefficient for each bank $\gamma^b, \forall b \in B$: each bank's risk-aversion coefficient $g_1^b, \forall b \in B$: the elasticity of consumer loan repayment rate with respect to GDP $u_{s,1}, s \in \{U, D\}$: coefficients for GDP

This table presents the variables for the theoretical model in the second period, without Minsky effect. The main difference from Table 3 is that, the default penalty coefficient (λ^b), the risk-aversion coefficient (γ^b), the coefficient for the consumer loan repayment rate (g_1^b), and the coefficient for GDP determination ($u_{s,1}, s \in \{U, D\}$) have been calibrated in the previous step.

The values of the balance sheet items such as the banks' holding of safe assets (A^b), equity for the new period (e^b), position in the interbank market ($D^{I,b}, L^{I,b}$), and un-modelled items (ϵ^b), as well as the interbank rate (ρ) should be updated to 30th June, 2006.

Banks' holding of consumer loans (L^b), deposits taken (D^b), probability of default (v^b), the expected shortfalls in profits ($\Delta \Pi^b$), and the GDP levels (Y_s) are now endogenously solved from the simultaneous equations.

The values of the variables in this table are present in Table 8 in Appendix C.

Table 8: No Minsky effect, pre-crisis

Equilibrium Model			
	Model Endogenous Variables		Model Exogenous Variables
Endogenously Solved	$L^1 = 23.1206$ $D^1 = 43.3995$ $v^1 = 0.9233$ $\Pi_U^1 = 1.2877$ $\Pi_D^1 = -1.8322$ $\Delta\Pi^1 = 3.1199$ $v^{1,h} = 0.7169$ $L^2 = 13.7109$ $D^2 = 19.8881$ $v^2 = 0.9784$ $\Pi_U^2 = 0.8489$ $\Pi_D^2 = -0.7413$ $\Delta\Pi^2 = 1.5903$ $v^{2,h} = 0.8757$ $L^3 = 13.8571$ $D^3 = 12.3792$ $v^3 = 0.9097$ $\Pi_U^3 = 0.9538$ $\Pi_D^3 = -0.5613$ $\Delta\Pi^3 = 1.5151$ $v^{3,h} = 0.8159$	$L^4 = 34.4085$ $D^4 = 38.106$ $v^4 = 0.9499$ $\Pi_U^4 = 1.815$ $\Pi_D^4 = -2.2882$ $\Delta\Pi^4 = 4.1032$ $v^4 = 0.8329$ $L^5 = 32.7039$ $D^5 = 23.1005$ $v^5 = 0.951$ $\Pi_U^5 = 2.4375$ $\Pi_D^5 = -0.6736$ $\Delta\Pi^5 = 3.1111$ $v^5 = 0.8776$ $Y_U = 128.4768$ $Y_D = 115.6291$ $M = 9.1052$ $\tilde{R} = 0.9363$	
Empirically Observed			$A^1 = 39.1867$ $e^1 = 2.5539$ $L^{I,1} = 20.7199$ $D^{I,1} = 23.5086$ $\delta^1 = 13.5652$ $A^2 = 15.6621$ $e^2 = 2.0286$ $L^{I,2} = 4.1269$ $D^{I,2} = 3.8704$ $\delta^2 = 7.7128$ $A^3 = 8.7028$ $e^3 = 1.2048$ $L^{I,3} = 3.6147$ $D^{I,3} = 3.468$ $\delta^3 = 9.1226$ $A^4 = 23.4921$ $e^4 = 3.6473$ $L^{I,4} = 7.2552$ $D^{I,4} = 12.0816$ $\delta^4 = 11.321$ $A^5 = 20.3513$ $e^5 = 2.5575$ $L^{I,5} = 2.6861$ $D^{I,5} = 4.5795$ $\delta^5 = 25.5038$ $\rho = 0.0501$
Manually Selected			$r^{d,1} = 0.0501$ $r^1 = 0.0879$ $r^{d,2} = 0.0501$ $r^2 = 0.0879$ $r^{d,3} = 0.0501$ $r^3 = 0.0879$ $r^{d,4} = 0.0501$ $r^4 = 0.0879$ $r^{d,5} = 0.0501$ $r^5 = 0.0879$ $r^A = 0.04$ $p = 0.95$ $g_2^1 = 0.01$ $g_2^2 = 0.01$ $g_2^3 = 0.01$ $g_2^4 = 0.01$ $g_2^5 = 0.01$ $g_3^1 = 0.01$ $g_3^2 = 0.01$ $g_3^3 = 0.01$ $g_3^4 = 0.01$ $g_3^5 = 0.01$ $u_2 = 0.1$
Calibrated			$\lambda^1 = 0.1364$ $\gamma^1 = 0.1282$ $\lambda^2 = 1.5178$ $\gamma^2 = 0.4305$ $\lambda^3 = 0.4552$ $\gamma^3 = 0.3277$ $\lambda^4 = 0.328$ $\gamma^4 = 0.1597$ $\lambda^5 = 0.4292$ $\gamma^5 = 0.1648$ $g_1^1 = -0.5344$ $g_1^2 = -0.3343$ $g_1^3 = -0.4051$ $g_1^4 = -0.3845$ $g_1^5 = -0.3322$ $u_{1,U} = 3.3144$ $u_{1,D} = 3.209$

This table presents the values of variables in the model simulation without Minsky effect, which covers the period from June 2006 to June 2007.

Bank 1, 2, 3, 4, 5 represents Barclays plc, HSBC plc, Lloyds plc, RBS plc, and the aggregated bank, respectively.

Table 9: Variables for the second period with Minsky effect

Equilibrium Model		
	Model Endogenous Variables	Model Exogenous Variables
Endogenously Solved	$\Pi_U^b, \forall b \in B$: banks' conditional P/L in the good state $\Pi_D^b, \forall b \in B$: banks' conditional P/L in the bad state $v^{b,h}, \forall b \in B$: default level of the consumer loans M : Money supply by the central bank \bar{R} : Pooled interbank deposit repayment rate in the bad state $L^b, \forall b \in B$: banks' holding of consumer loans $D^b, \forall b \in B$: banks' liability of deposits $v^b, \forall b \in B$: banks' probability of default in the bad state Y_U : the GDP level at the good state Y_D : the GDP level at the bad state	$p^b, b \in B$: banks' implied subjective probability of the good state in the second period
Estimated by CIMDO	$\Delta \Pi^b, \forall b \in B$: banks' expected shortfalls in profits	
Empirically Observed		$A^b, \forall b \in B$: banks' holding of safe assets $e^b, \forall b \in B$: banks' initial equity level $D^{I,b}, \forall b \in B^B$: borrowing banks' liability of interbank deposit $L^{I,b}, \forall b \in B^L$: lending banks' holding of interbank loan $\epsilon^b, \forall b \in B$: the other balance sheet items that are not explicitly modelled ρ : the interbank interest rate
Manually Selected		$r^{D,b}, \forall b \in B$: deposit interest rate $r^b, \forall b \in B$: consumer loan interest rate r^A : banks' market book return p : prior probability of good state g_2^b, g_3^b : coefficients for consumer loan repayment $u_{s,2}, s \in \{U, D\}$: coefficients for GDP
Calibrated		$\lambda^b, \forall b \in B$: the non-pecuniary default penalty coefficient for each bank $\gamma^b, \forall b \in B$: each bank's risk-aversion coefficient $g_1^b, \forall b \in B$: the elasticity of consumer loan repayment rate with respect to GDP $u_{s,1}, s \in \{U, D\}$: coefficients for GDP

This table presents the variables for the theoretical model in the second period with Minsky effect. Compared with the case without Minsky effect (Table 7), we estimate the banks' expected shortfalls in profits by CIMDO, and solve for the banks' implied subjective probability of the good state in the second period.

The values of the variables in this table are presented in Table 10 in Appendix C.

Table 10: Minsky effect, pre-crisis

Equilibrium Model			
	Model Endogenous Variables		Model Exogenous Variables
Endogenously Solved	$L^1 = 28.0912$ $D^1 = 48.3701$ $v^1 = 0.9219$ $\Pi_U^1 = 1.4758$ $\Pi_D^1 = -2.3749$ $v^{1,h} = 0.7254$ $L^2 = 21.3795$ $D^2 = 27.5567$ $v^2 = 0.9803$ $\Pi_U^2 = 1.1391$ $\Pi_D^2 = -1.1308$ $v^{2,h} = 0.8861$ $L^3 = 15.3481$ $D^3 = 13.8702$ $v^3 = 0.9157$ $\Pi_U^3 = 1.0102$ $\Pi_D^3 = -0.6055$ $v^{3,h} = 0.8255$	$L^4 = 38.2546$ $D^4 = 41.9521$ $v^4 = 0.9527$ $\Pi_U^4 = 1.9606$ $\Pi_D^4 = -2.3781$ $v^4 = 0.8427$ $L^5 = 40.7353$ $D^5 = 31.1319$ $v^5 = 0.9598$ $\Pi_U^5 = 2.7414$ $\Pi_D^5 = -0.8922$ $v^5 = 0.888$ $Y_U = 142.9394$ $Y_D = 128.6454$ $M = 9.1052$ $\tilde{R} = 0.9377$	$p^1 = 0.9470$ $p^2 = 0.9957$ $p^3 = 0.9432$ $p^4 = 0.9431$ $p^5 = 0.9676$
Estimated by CIMDO	$\Delta\Pi^1 = 3.8507$ $\Delta\Pi^2 = 2.2700$ $\Delta\Pi^3 = 1.6157$	$\Delta\Pi^4 = 4.3387$ $\Delta\Pi^5 = 3.6336$	
Empirically Observed			$A^1 = 39.1867$ $e^1 = 2.5539$ $L^{I,1} = 20.7199$ $D^{I,1} = 23.5086$ $\delta^1 = 13.5652$ $A^2 = 15.6621$ $e^2 = 2.0286$ $L^{I,2} = 4.1269$ $D^{I,2} = 3.8704$ $\delta^2 = 7.7128$ $A^3 = 8.7028$ $e^3 = 1.2048$ $L^{I,3} = 3.6147$ $D^{I,3} = 3.468$ $\delta^3 = 9.1226$ $A^4 = 23.4921$ $e^4 = 3.6473$ $L^{I,4} = 7.2552$ $D^{I,4} = 12.0816$ $\delta^4 = 11.321$ $A^5 = 20.3513$ $e^5 = 2.5575$ $L^{I,5} = 2.6861$ $D^{I,5} = 4.5795$ $\delta^5 = 25.5038$ $\rho = 0.0501$
Manually Selected			$r^{d,1} = 0.0501$ $r^1 = 0.0879$ $r^{d,2} = 0.0501$ $r^2 = 0.0879$ $r^{d,3} = 0.0501$ $r^3 = 0.0879$ $r^{d,4} = 0.0501$ $r^4 = 0.0879$ $r^{d,5} = 0.0501$ $r^5 = 0.0879$ $r^A = 0.04$ $p = 0.95$ $g_3^1 = 0.01$ $g_2^2 = 0.01$ $g_3^3 = 0.01$ $g_2^4 = 0.01$ $g_2^5 = 0.01$ $g_3^1 = 0.01$ $g_2^2 = 0.01$ $g_3^3 = 0.01$ $g_3^4 = 0.01$ $g_3^5 = 0.01$ $u_2 = 0.1$
Calibrated			$\lambda^1 = 0.1364$ $\gamma^1 = 0.1282$ $\lambda^2 = 1.5178$ $\gamma^2 = 0.4305$ $\lambda^3 = 0.4552$ $\gamma^3 = 0.3277$ $\lambda^4 = 0.328$ $\gamma^4 = 0.1597$ $\lambda^5 = 0.4292$ $\gamma^5 = 0.1648$ $g_1^1 = -0.5344$ $g_1^2 = -0.3343$ $g_1^3 = -0.4051$ $g_1^4 = -0.3845$ $g_1^5 = -0.3322$ $u_{1,U} = 3.3144$ $u_{1,D} = 3.209$

This table presents the values of variables in the model simulation with Minsky effect, which covers the period from June 2006 to June 2007.

Bank 1, 2, 3, 4, 5 represents Barclays plc, HSBC plc, Lloyds plc, RBS plc, and the aggregated bank, respectively.

Table 11: Model Calibration, post-crisis

Equilibrium Model				
	Model Endogenous Variables		Model Exogenous Variables	
Endogenously Solved	$\Pi_U^1 = 1.9951$	$M = 0.1762$	$\lambda^1 = 0.1170$	$g_1^1 = -0.6593$
	$\Pi_D^1 = -8.0426$	$\tilde{R} = 0.899$	$\gamma^1 = 0.0291$	$g_1^2 = -0.4786$
	$v^{1,h} = 0.6476$		$\epsilon^1 = 74.2746$	$g_1^3 = -0.4759$
	$\Pi_U^2 = 1.0768$		$\lambda^2 = 0.4698$	$g_1^4 = -0.7461$
	$\Pi_D^2 = -3.2055$		$\gamma^2 = 0.1418$	$g_1^5 = -0.5328$
	$v^{2,h} = 0.7759$		$\epsilon^2 = 31.8932$	$u_{1,U} = 3.4672$
	$\Pi_U^3 = 1.7022$		$\lambda^3 = 0.8117$	$u_{1,D} = 3.3618$
	$\Pi_D^3 = -8.396$		$\gamma^3 = 0.0677$	
	$v^{3,h} = 0.778$		$\epsilon^3 = 25.4187$	
	$\Pi_U^4 = 1.4692$		$\lambda^4 = 0.1383$	
	$\Pi_D^4 = -8.734$		$\gamma^4 = 0.0196$	
	$v^{4,h} = 0.5937$		$\epsilon^4 = 43.9603$	
	$\Pi_U^5 = 0.6078$		$\lambda^5 = 0.3190$	
	$\Pi_D^5 = -2.2345$		$\gamma^5 = 0.1760$	
	$v^{5,h} = 0.7349$		$\epsilon^5 = 4.5666$	
Estimated by CIMDO	$\Delta\Pi^1 = 10.0377$	$\Delta\Pi^4 = 10.2032$		
	$\Delta\Pi^2 = 4.2823$	$\Delta\Pi^5 = 2.8422$		
	$\Delta\Pi^3 = 10.0982$			
Empirically Observed			$A^1 = 81.1456$	$A^4 = 50.8754$
			$e^1 = 5.7982$	$e^4 = 4.7171$
			$L^{I,1} = 4.3321$	$L^{I,4} = 2.3884$
	$L^1 = 43.3222$	$L^4 = 35.0851$	$D^{I,1} = 5.9567$	$D^{I,4} = 3.9066$
	$D^1 = 42.7704$	$D^4 = 35.7649$	$A^2 = 44.297$	$A^5 = 50.8754$
	$L^2 = 26.0052$	$L^5 = 18.4949$	$e^2 = 3.4502$	$e^5 = 4.7171$
	$D^2 = 34.6507$	$D^5 = 26.5371$	$L^{I,2} = 2.5262$	$L^{I,5} = 2.3884$
	$L^3 = 48.397$	$Y_U = 182.248$	$D^{I,2} = 2.8343$	$L^{I,5} = 3.9066$
	$D^3 = 44.7067$		$A^3 = 25.283$	$\rho = 0.0454$
			$e^3 = 4.9990$	
			$L^{I,3} = 2.4256$	
			$D^{I,3} = 0.9812$	
Manually Selected			$r^{1,d} = 0.0098$	$g_2^1 = 0.01$
			$r^1 = 0.0392$	$g_2^2 = 0.01$
			$r^{2,d} = 0.0098$	$g_2^3 = 0.01$
	$v^1 = 0.8726$		$r^2 = 0.0392$	$g_2^4 = 0.01$
	$v^2 = 0.9463$		$r^{3,d} = 0.0098$	$g_2^5 = 0.01$
	$v^3 = 0.9715$		$r^3 = 0.0392$	$g_3^1 = 0.01$
	$v^4 = 0.8789$		$r^{4,d} = 0.0098$	$g_3^2 = 0.01$
	$v^5 = 0.9078$		$r^4 = 0.0392$	$g_3^3 = 0.01$
	$Y_D = 164.0232$		$r^{5,d} = 0.0098$	$g_3^4 = 0.01$
			$r^5 = 0.0392$	$g_3^5 = 0.01$
			$r^A = 0.009$	$u_2 = 0.1$
			$p = 0.95$	

This table presents the values of the variables in the model calibration, which covers the period from Dec. 2014 to Dec. 2015.

Bank 1, 2, 3, 4, 5 represents Barclays plc, HSBC plc, Lloyds plc, RBS plc, and Standard Chartered, respectively. The column on the left shows the endogenous variables in the equilibrium model, while the column on the right shows the exogenous variables.

Variables in the first row are endogenously computed when solving the simultaneous equations.

The expected shortfalls in the second row are estimated by CIMDO, which captures the information about distress dependence structure.

The balance sheet items and the macroeconomic data in the third row are empirically observed.

The variables in the forth row are "manually" selected. More specifically, the banks repayment rate is set consistent with EDF ($EDF = (1 - p)(1 - v)$); GDP in the bad state (Y_D) is set to be 90% of GDP observed; bank deposits in the model has the same risk as the interbank loans, thus bank deposit rates (r^d) are set equal to ρ ; the spread between r and r^d is calibrated to the average spread between loan and deposit.

Table 12: No Minsky effect, post-crisis

Equilibrium Model			
	Model Endogenous Variables		Model Exogenous Variables
Endogenously Solved	$L^1 = 43.0063$ $D^1 = 44.4167$ $v^1 = 0.8735$ $\Pi_U^1 = 1.7749$ $\Pi_D^1 = -8.1348$ $\Delta\Pi^1 = 9.9097$ $v^{1,h} = 0.6473$ $L^2 = 26.0891$ $D^2 = 33.2612$ $v^2 = 0.9433$ $\Pi_U^2 = 1.0311$ $\Pi_D^2 = -3.2486$ $\Delta\Pi^2 = 4.2798$ $v^{2,h} = 0.7755$ $L^3 = 47.7869$ $D^3 = 43.7987$ $v^3 = 0.971$ $\Pi_U^3 = 1.6581$ $\Pi_D^3 = -8.3267$ $\Delta\Pi^3 = 9.9848$ $v^{3,h} = 0.7776$	$L^4 = 34.4472$ $D^4 = 37.8137$ $v^4 = 0.8833$ $\Pi_U^4 = 1.301$ $\Pi_D^4 = -8.6151$ $\Delta\Pi^4 = 9.9161$ $v^4 = 0.5935$ $L^5 = 18.4167$ $D^5 = 24.8334$ $v^5 = 0.9006$ $\Pi_U^5 = 0.6194$ $\Pi_D^5 = -2.1502$ $\Delta\Pi^5 = 2.7696$ $v^5 = 0.7346$ $Y_U = 181.5319$ $Y_D = 163.3787$ $M = -1.2073$ $\tilde{R} = 0.8997$	
Empirically Observed			$A^1 = 60.0914$ $e^1 = 5.6419$ $L^{I,1} = 4.236$ $D^{I,1} = 4.8093$ $\delta^1 = 52.4658$ $A^2 = 38.6157$ $e^2 = 3.4541$ $L^{I,2} = 2.3222$ $D^{I,2} = 2.4333$ $\delta^2 = 27.8784$ $A^3 = 22.8112$ $e^3 = 4.7353$ $L^{I,3} = 2.4154$ $D^{I,3} = 0.9864$ $\delta^3 = 23.4931$ $A^4 = 38.3989$ $e^4 = 4.1907$ $L^{I,4} = 1.8744$ $D^{I,4} = 3.1828$ $\delta^4 = 29.5333$ $A^5 = 14.6214$ $e^5 = 3.1395$ $L^{I,5} = 4.3521$ $D^{I,5} = 2.581$ $\delta^5 = 6.8363$ $\rho = 0.0107$
Manually Selected			$r^{d,1} = 0.0107$ $r^1 = 0.0399$ $r^{d,2} = 0.0107$ $r^2 = 0.0399$ $r^{d,3} = 0.0107$ $r^3 = 0.0399$ $r^{d,4} = 0.0107$ $r^4 = 0.0399$ $r^{d,5} = 0.0107$ $r^5 = 0.0399$ $r^A = 0.009$ $p = 0.95$ $g_2^1 = 0.01$ $g_2^2 = 0.01$ $g_2^3 = 0.01$ $g_2^4 = 0.01$ $g_2^5 = 0.01$ $g_3^1 = 0.01$ $g_3^2 = 0.01$ $g_3^3 = 0.01$ $g_3^4 = 0.01$ $g_3^5 = 0.01$ $u_2 = 0.1$
Calibrated			$\lambda^1 = 0.1170$ $\gamma^1 = 0.0291$ $\lambda^2 = 0.4698$ $\gamma^2 = 0.1418$ $\lambda^3 = 0.8117$ $\gamma^3 = 0.0677$ $\lambda^4 = 0.1383$ $\gamma^4 = 0.0196$ $\lambda^5 = 0.3190$ $\gamma^5 = 0.1760$ $g_1^1 = -0.6593$ $g_1^2 = -0.4786$ $g_3^3 = -0.4759$ $g_4^1 = -0.7461$ $g_1^5 = -0.5328$ $u_{1,U} = 3.4672$ $u_{1,D} = 3.3618$

This table presents the values of variables in the model simulation without Minsky effect, which covers the period from Dec. 2014 to Dec. 2015.

Bank 1, 2, 3, 4, 5 represents Barclays plc, HSBC plc, Lloyds plc, RBS plc, and Standard Chartered, respectively.

Table 13: Minsky effect, post-crisis

Equilibrium Model			
	Model Endogenous Variables		Model Exogenous Variables
Endogenously Solved	$L^1 = 38.5300$ $D^1 = 39.9404$ $v^1 = 0.8668$ $\Pi_U^1 = 1.6442$ $\Pi_D^1 = -7.0459$ $v^{1,h} = 0.6442$ $L^2 = 25.471$ $D^2 = 32.6431$ $v^2 = 0.9424$ $\Pi_U^2 = 1.0131$ $\Pi_D^2 = -3.2389$ $v^{2,h} = 0.7718$ $L^3 = 40.0639$ $D^3 = 36.0757$ $v^3 = 0.9679$ $\Pi_U^3 = 1.4327$ $\Pi_D^3 = -7.0443$ $v^{3,h} = 0.7739$	$L^4 = 26.9362$ $D^4 = 30.3027$ $v^4 = 0.8676$ $\Pi_U^4 = 1.0818$ $\Pi_D^4 = -6.1063$ $v^4 = 0.5906$ $L^5 = 20.7222$ $D^5 = 27.139$ $v^5 = 0.9003$ $\Pi_U^5 = 0.6867$ $\Pi_D^5 = -2.5837$ $v^5 = 0.7311$ $Y_U = 173.7495$ $Y_D = 156.3745$ $M = -1.2073$ $\tilde{R} = 0.8935$	$p^1 = 0.9479$ $p^2 = 0.9505$ $p^3 = 0.9448$ $p^4 = 0.9462$ $p^5 = 0.9558$
Estimated by CIMDO	$\Delta\Pi^1 = 8.6901$ $\Delta\Pi^2 = 4.252$ $\Delta\Pi^3 = 8.477$	$\Delta\Pi^4 = 7.1881$ $\Delta\Pi^5 = 3.2704$	
Empirically Observed			$A^1 = 60.0914$ $e^1 = 5.6419$ $L^{I,1} = 4.236$ $D^{I,1} = 4.8093$ $\delta^1 = 52.4658$ $A^2 = 38.6157$ $e^2 = 3.4541$ $L^{I,2} = 2.3222$ $D^{I,2} = 2.4333$ $\delta^2 = 27.8784$ $A^3 = 22.8112$ $e^3 = 4.7353$ $L^{I,3} = 2.4154$ $D^{I,3} = 0.9864$ $\delta^3 = 23.4931$ $A^4 = 38.3989$ $e^4 = 4.1907$ $L^{I,4} = 1.8744$ $D^{I,4} = 3.1828$ $\delta^4 = 29.5333$ $A^5 = 14.6214$ $e^5 = 3.1395$ $L^{I,5} = 4.3521$ $D^{I,5} = 2.581$ $\delta^5 = 6.8363$ $\rho = 0.0107$
Manually Selected			$r^{d,1} = 0.0107$ $r^1 = 0.0399$ $r^{d,2} = 0.0107$ $r^2 = 0.0399$ $r^{d,3} = 0.0107$ $r^3 = 0.0399$ $r^{d,4} = 0.0107$ $r^4 = 0.0399$ $r^{d,5} = 0.0107$ $r^5 = 0.0399$ $r^A = 0.009$ $p = 0.95$ $g_3^1 = 0.01$ $g_2^2 = 0.01$ $g_3^3 = 0.01$ $g_2^4 = 0.01$ $g_2^5 = 0.01$ $g_3^1 = 0.01$ $g_2^2 = 0.01$ $g_3^3 = 0.01$ $g_3^4 = 0.01$ $g_3^5 = 0.01$ $u_2 = 0.1$
Calibrated			$\lambda^1 = 0.1170$ $\gamma^1 = 0.0291$ $\lambda^2 = 0.4698$ $\gamma^2 = 0.1418$ $\lambda^3 = 0.8117$ $\gamma^3 = 0.0677$ $\lambda^4 = 0.1383$ $\gamma^4 = 0.0196$ $\lambda^5 = 0.3190$ $\gamma^5 = 0.1760$ $g_1^1 = -0.6593$ $g_1^2 = -0.4786$ $g_1^3 = -0.4759$ $g_1^4 = -0.7461$ $g_1^5 = -0.5328$ $u_{1,U} = 3.4672$ $u_{1,D} = 3.3618$

This table presents the values of variables in the model simulation with Minsky effect, which covers the period from Dec. 2014 to Dec. 2015.

Bank 1, 2, 3, 4, 5 represents Barclays plc, HSBC plc, Lloyds plc, RBS plc, and Standard Chartered, respectively.

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