WP/19/175

# IMF Working Paper

## The Impact of Remittances on Economic Activity: The Importance of Sectoral Linkages

By Jemma Dridi, Tunc Gursoy, Hector Perez-Saiz and Mounir Bari



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African Department

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**Abstract**

We propose a simple macroeconomic model with input-output sectoral linkages based on Acemoglu et al. (2016) to quantify how changes in aggregate demand due to additional income from household's remittances propagates through the network of input-output linkages in Sub-Saharan African countries. We first propose two network centrality measures to assess the role of some sectors as key input providers in the economy. Then, we use these measures to quantify the effect of sectoral linkages on sectoral and total output following an increase in remittances inflows. Our empirical results suggest that the effects of remittances on recipient economies increase with the degree of linkages across sectors, which is especially prominent in the case of the financial intermediation sector. Our paper contributes to the emerging macroeconomic literature on the propagation of shocks across sectors and the implications for the whole economy.

JEL Classification Numbers: F24, C67, O41, D85

Keywords: Remittances, Sub-Saharan Africa, networks, centrality

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# Contents



# List of Figures

## List of Tables

# 1 Introduction

Remittances inflows have increased significantly in recent years and have become the main financial external inflow in some developing countries, surpassing other inflows that traditionally play an important role in these countries, such as official development assistance and foreign direct investment. The World Bank estimates that remittances now make up about a third of total financial inflows in developing countries. Like other regions, Africa saw large increases in the last decade. According to World Bank Migration and Remittances database, remittances currently represent for some African countries a significant share, up to 22 percent, of their 2017 Gross Domestic Product (GDP). The magnitude of the economic impact of remittances on the receiving countries depends on how this money is spent by the recipient households. If these flows increase consumption in sectors that have strong sectoral linkages with other economic sectors, the positive effect of remittances may propagate to these sectors and have an amplified aggregate effect on the entire economy.

In this paper, we extend a framework from Acemoglu *et al.* (2016) to explain how additional income from remittances inflows affect household consumption, and how these changes in aggregate demand are amplified and propagated through the economy. Survey data from the World Bank shows that the use of remittances varies across countries, as these inflows can be used for food consumption, household construction or education, among other uses. Various consumption uses have significant implications for economic fluctuations because production sectors are interlinked in varying degrees. Changes in demand in one sector due to remittances can significantly affect other sectors connected to it through input-output linkages (Leontief matrix). Therefore, a relatively small increase in production in one sector can be amplified in the economy due to these linkages. For instance, remittances inflows may increase consumption in certain sectors such as food, retail, or education, which may have strong dependence on other sectors such as agriculture, manufacturing, or financial intermediation. As a result, a relatively small initial change in consumption may quickly propagate to the rest of the economy.

The basis of our analysis is country-level data with the input-output matrix structure of the economy, as proposed by Wassily Leontief (Leontief 1974). We use country-specific data for 35 African countries pertaining to remittances inflows, the pattern of consumption, and the input-output linkages for each economy. Using data for Sub-Saharan African (SSA) countries, and a proper calibration of the rest of the parameters of the model, we are able to quantify the effects of remittances inflows across economic sectors in SSA.

The results from our calibrated model show that input-output sectoral linkages are important to explain the magnitude of the effect of remittances on output across sectors. We first assess the importance of input-output sectoral linkages in the economy by constructing two centrality measures based on the Leontief input-output matrix, the weighted outdegree, and the Katz Bonacich centrality score. A careful analysis of these centrality measures shows that financial intermediation, and also other sectors, such as petroleum and minerals, or retail/wholesale trade, are, on average, more important for the economy than other sectors. In other words, these sectors have a key role as input providers for the rest of the economy (they are more *central* in the SSA economies).

Subsequently, using our calibrated model, we study the relationship between the intensity of linkages across sectors, and output growth across sectors due to remittances inflows. We show the robust positive relationship between sectoral linkages and real growth across sectors and countries using the two proposed network centrality

measures. Our methodology allows to quantify the effect of remittance inflows on sectoral output in sectors that are more *central* in the economy because their output is broadly used as inputs by the rest of the economic sectors. Using our calibrated model, we also quantify the effect on total output of the whole economy, and show that total output increases in economies with relatively more interlinked sectors. Finally, we simulate the effect of a 5 percent (over GDP) remittances inflow on the sector output growth, and find that the financial intermediation sector, and other sectors such as retail or wholesale, have the largest growth as percent of GDP due to these remittances inflows. Our empirical results suggest that the positive effects of remittances inflows could be amplified in economies that have a more developed structure, which may have strong economic interlinkages across sectors that lead to a greater propagation of these inflows.

These results suggest that a better understanding of input-output sectoral linkages is necessary to properly capture the full impact of remittances inflows. Remittances may expand domestic production of consumption goods and intermediate products necessary to support the increase in consumption. Furthermore, when remittances are spent within sectors which have strong linkages with the rest of the economy, the sectors that do not benefit directly from remittances expenditure may still experience a growth in demand for their output. This expansion of output should foster employment creation and stimulate investment, and these benefits may be larger as the economy is diversified, and its production structure integrated. Also, to protect employment and foster growth, policymakers should devise stimulus policies targeting sectors that exhibit high vulnerabilities to sharp declines in remittances inflows, or other types of similar demand-side shocks.

Our paper contributes to the emerging literature that uses detailed network models to understand the propagation of shocks across sectors and the implications for the macroeconomy (Gabaix, 2011; Carvalho, 2010; Acemoglu *et al.*, 2012; or Acemoglu *et al.*, 2017). In this literature, network models are used to understand how a shock to a single firm (or sector) may have a more sustained effect on the economy if it impacts the firm's output, or other firms or sectors that are connected to it through a network of input-output linkages. We follow Acemoglu *et al.* (2016) but focus only on changes that affect the demand-side. When there are changes to the demand in a sector, it propagates upstream to the sectors that produce inputs for that sector. For instance, an increase in demand for processed food tends to increase demand in the agriculture sector. In turn, the agriculture sector uses other inputs from other sectors. Therefore, there are cumulative effects working upstream.

The remainder of the paper is organized as follows: Section 2 discusses the importance of remittances for economic growth. Section 3 describes the network model. Section 4 describes the data sources. Empirical results are shown in Section 5. Section 6 concludes.

# 2 Importance of remittances

## 2.1 Literature review

This section reviews previous empirical studies, which mostly focus on the direct economic impact of remittances. The growing importance of remittances flows has given rise to a large literature that analyzes the economic impact of these flows. Several theoretical and empirical studies analyze the impact of remittances

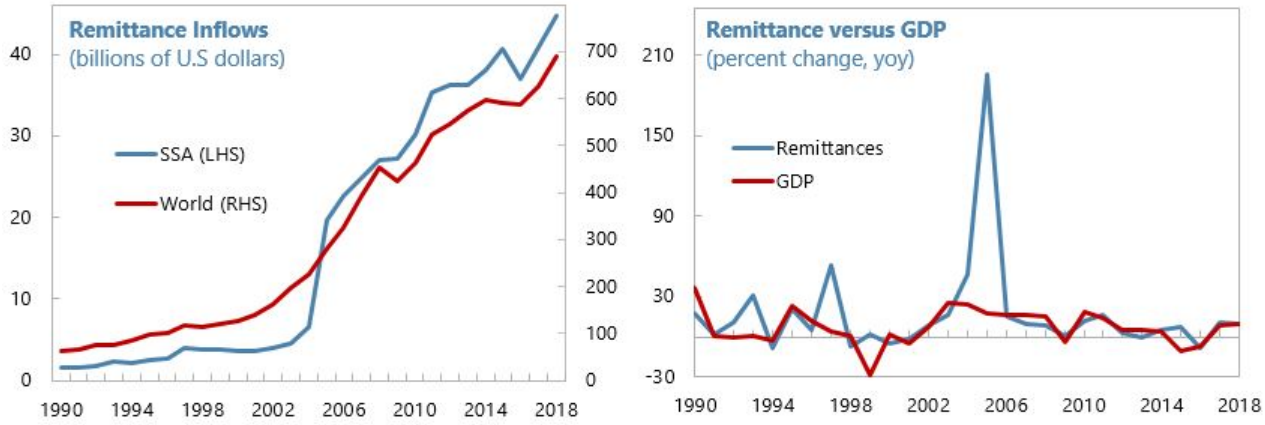
on macroeconomic variables, such as consumption, investment and growth in recipient countries, yet the results of these studies remain largely inconclusive. The existing literature is also very diverse on the spillover effects of the increase in remittances, such as the impact on poverty reduction (Ratha, 2013), financial deepening (Giuliano and Ruiz-Arranz, 2009), increases in migration (Taylor, 1999), and institutional development (Catrinescu *et al.*, 2009). There is empirical evidence that remittances contribute to economic growth, through their positive impact on consumption, savings, and investment. Remittances can also have negative impact on growth in recipient countries by reducing incentives to work, and therefore reducing labor supply or labor force participation. This may cause an appreciation of the real exchange rate in recipient economies and generating a resource reallocation from the tradeable to the non-tradeable sector, or by adversely affecting long-run growth through the Dutch disease.

Several studies found that there is a positive relationship between remittances and economic growth. On a panel of 15 countries in the Middle East and North Africa from 1980 to 2009, Mim and Ali (2012) find a positive influence of remittances on consumption, investment and economic growth. Channeled towards the accumulation of human capital, remittances act effectively on economic growth in these countries. Also, econometric specifications based on endogenous growth models find conclusive results. For example, Cooray (2012) finds a positive impact of remittances on economic growth through education and financial sector development for the economies of South Asia in 1970–2008. Rao and Hassan (2012) show that the increase in transfers has a direct positive effect on economic activity, and also an indirect positive effect through investment, the depreciation of the real exchange rate, and the development of the financial sector. Using data for 36 African countries for 1980–2004, Fayissa and Nsiah (2010) find that a 10 percent increase in remittances would result in a 0.4 percent increase in the growth rate of GDP per capita. In a sample of 34 economies in SSA for 1980–2004, Baldé (2011) suggests that transfers can have an indirect effect on economic growth through savings and investment. Singh *et al.* (2010) reveal that transfers are counter-cyclical and act as shock stabilizers in a sample of 36 countries in SSA in 1990–2008. More recently, Nsiah and Fayissa (2013) found a positive relationship between economic growth and remittances, using a panel of 64 different countries in Africa, Asia, and Latin America-Caribbean for 1987–2007.

Remittances inflows may finance investment in human capital, smooth consumption and have multiplier effects through increased household expenditures (Gupta *et al.*, 2009). Remittances can also increase investments by alleviating credit constraints in developing countries, and thereby positively affect economic growth. It has been argued that the effect of remittances through this channel would be greater for countries with a relatively underdeveloped financial system. Remittances could enhance investment by reducing the volatility of consumption, contributing to a more stable macroeconomic environment conducive to investment activities (Singh *et al.*, 2010). Barajas *et al.* (2009) pointed out that the more integrated an economy is with the world financial markets, and the more developed the domestic financial system is, the less likely that remittances flows will stimulate investment by relaxing credit constraints.

Remittances can dampen income volatility and pressures on inflation in receiving countries (Chami *et al.*, 2009). Thus, they have counter-cyclical behavior because they act as an insurance to respond to macroeconomic shocks that have emerged in the remitter's home country. Also, Ratha (2011) states that remittances act as a macroeconomic agent to muzzle the adverse effects of financial crises and thus tend to act countercyclically, while most other flows are procyclical (they decline or even come to a cease during financial crises). Hence, remittances ensure a stable consumption and output against changes in price volatility.

Figure 1: Remittances Inflows and Growth in SSA and the World, 1990-2018




Sources: World Bank Migration and Remittance database; IMF WEO database; and authors' calculations.

Conversely, remittances may have negative effects on economic growth by reducing labor supply and participation. They increase the recipients' wealth and can undermine their incentives to work, which, in turn, slows economic growth. Rodriguez and Tiongson (2001) show that Filipino households with temporary overseas migrants tend to reduce their labor participation and hours worked. Chami *et al.* (2003) show that remittances may have a negative effect on economic growth due to the presence of asymmetric information and moral hazard. Airola (2007) observes a negative elasticity between remittances and labor supply in Mexico. Analyses by Cox-Edwards and Rodríguez-Oreggia (2009) and Amuedo-Dorantes and Pozo (2006), also based on Mexican data, observe a negative relationship between remittances and labor supply only in narrow segments of the population. Using a large cross-country database, Chami *et al.* (2018) show that remittances reduce labor force participation and increase informality of the labor market.

Other studies have revealed that rising levels of remittances could be harmful to the long-run growth of recipient economies through an appreciation of the real exchange rate. These flows can appreciate the real exchange rate in recipient economies and therefore generate a resource allocation from the tradeable to the non-tradeable sector, i.e. the Dutch disease phenomenon (Amuedo-Dorantes and Pozo, 2004; Acosta *et al.*, 2009; Chami *et al.*, 2010b). This will in turn hurt economic growth.

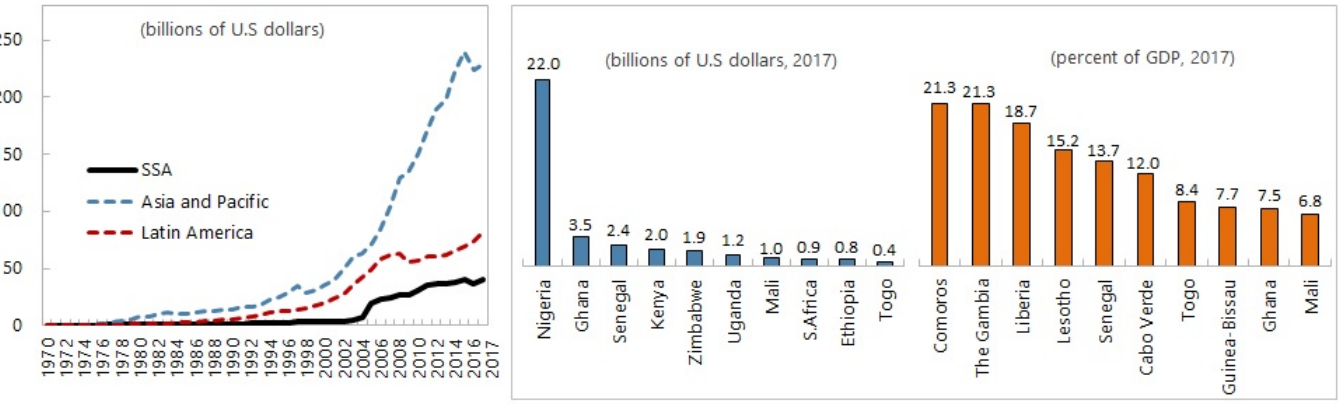
## 2.2 Recent trends in remittances

Remittances inflows in SSA have increased substantially in past decades and they have reached $34 billion in 2015, despite a deceleration during the global financial crisis (see Chami *et al.*, 2010a), and a recent decrease of 6.1 percent in 2016.[1] From 2005, growth of remittances inflows in SSA has exceeded the world average (Figure 1), but in recent years, growth has tempered down, like in the rest of the world.[2]

[1] In addition, future developments in financial technologies such as cryptocurrencies should reduce the costs of sending remittances, thereby attracting more remittances flows.

[2] The sudden increase in the growth of remittances in Sub-Saharan Africa in 2005 is owed to a dramatic increase in measured remittance inflows to Nigeria, which was about three-quarters of total remittances received by SSA as a whole. According to Mohapatra and Ratha (2011), rather than actual increases in remittances, this jump possibly captures improved data collection and measurement of said receipts.

Figure 2: Remittances in SSA




Sources: World Development Indicators, and authors' calculations.

Slow economic growth in remittance-sending countries, coupled with a decline in commodity prices, particularly oil prices, and a diversion of remittances to informal channels due to exchange rate regimes, were likely the main factors behind the marked slowdown in remittances inflows in recent years. Nigeria remains the largest remittances recipient (in dollars) in SSA. Gambia, Liberia, Comoros and Lesotho are some of the largest remittances recipients in relative terms, with inflows close to 20 percent of GDP. Figure 2 provides more details on the level of remittances across countries in SSA.

## 2.3 Remittances and economic growth

The way in which remittances are used by households has important implications for economic growth. Recipient households often channel these funds towards human capital investments, especially education, health and food, which affects long-term economic growth and thus reduce poverty (Adams Jr, 2004; Docquier *et al.*, 2012). Mohapatra and Ratha (2011) argues that the distribution of the spending behavior of remittances receipts should be considered carefully when trying to assess the impact of remittances on growth because not all of the receipts are spent on GDP-bolstering activities. Chami *et al.* (2018) show that remittances inflows are associated with a shift in the sectoral employment structure, with employment flowing from agriculture into service-oriented sectors. Table 1, from Mohapatra and Ratha (2011), shows that remittances inflows have varied uses in several African countries. Their study concludes that when trying to determine the impact of remittances inflows on growth, not only the amount of remittances matter but also on what they are spent.

To our knowledge, most empirical studies seem to have paid little attention to the multiplier effect of remittances expenditures at the sectoral level to assess a fuller impact of remittances on output. The positive relationship between growth of remittances and real GDP is observable in the data (see Figure 3), in line with a number of existing empirical studies, yet there is still ample merit in understanding and measuring the network effects that these flows generate in the economy. Table 1 shows that uses of remittances vary significantly across countries. These flows may propagate differently through sectors and may have a different economic impact depending on how these flows are spent. For instance, Stahl and Habib (1989) used a computable general equilibrium (CGE) model for Bangladesh and showed that even if only small proportions

Table 1: Uses of Remittances Receipts in Selected African Countries, by Source

**Use of Remittances by Recipient Households in Selected African Countries, by Source** [1/]

*% of total remittances*

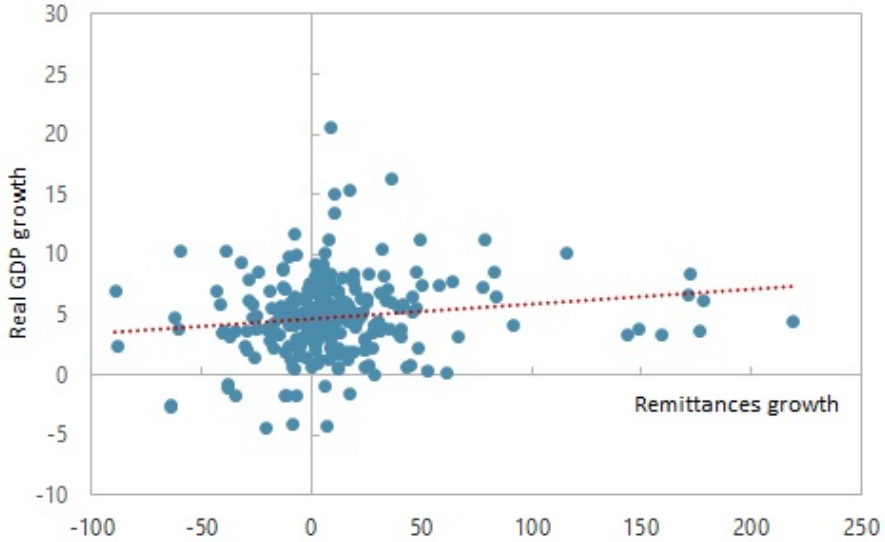
| | Burkina Faso | | | Kenya | | | Nigeria | | | Senegal | | | Uganda | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ***Use*** | Outside Africa | Within Africa | Domestic | Outside Africa | Within Africa | Domestic | Outside Africa | Within Africa | Domestic | Outside Africa | Within Africa | Domestic | Outside Africa | Within Africa | Domestic |
| New-house construction | 25.7 | 10.1 | 2.6 | 11.2 | 27.5 | 1.3 | 5.8 | 0 | 0.1 | 7 | 0.7 | 0 | 2.5 | 1.6 | 0.4 |
| Food | 23.5 | 34.9 | 48.7 | 12.8 | 14.5 | 29.7 | 10.1 | 20.1 | 1 | 52.6 | 72.6 | 81.9 | 7.6 | 9.7 | 12.4 |
| Education | 12.4 | 5.9 | 9.4 | 9.6 | 22.9 | 20.5 | 22.1 | 19.6 | 4.5 | 3.6 | 2.3 | 4.6 | 12.7 | 14.5 | 20.2 |
| Health | 11.3 | 10.1 | 12.5 | 7.3 | 5.8 | 7 | 5.1 | 12 | 10.6 | 10.7 | 7.3 | 2.9 | 6.3 | 14.5 | 24.8 |
| Business | 10.4 | 2.6 | 2.4 | 3.9 | 8.4 | 13 | 21.7 | 20.1 | 11.1 | 1.3 | 5.7 | 0.2 | 7.6 | 9.7 | 2.1 |
| clothing | 5 | 0.7 | 0.7 | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. |
| Marriage/funeral | 2.1 | 3.9 | 3.1 | 0.9 | 1.7 | 2 | 0.4 | 1 | 0.7 | 2.9 | 2.4 | 1.1 | 7.6 | 6.5 | 1.7 |
| Rent (house, land) | 1.4 | 0.6 | 1.7 | 5.7 | 0.4 | 7.4 | 4.4 | 4.9 | 0.8 | 1 | 0 | 2.2 | 5.1 | 8.1 | 4.5 |
| House rebuilding | 0.3 | 1 | 1.2 | 5.3 | 3.1 | 1.3 | 4.7 | 3.2 | 7 | 4.2 | 0.7 | 0.1 | 6.3 | 3.2 | 2.1 |
| Cars of trucks | 0.1 | 0 | 0.1 | 1.3 | 1 | 0.4 | 0 | 0 | 0.5 | 0.2 | 0 | 0 | 2.5 | 0 | 0 |
| Land purchase | 0 | 1.4 | 0.1 | 8.4 | 7 | 1.3 | 24.8 | 16.6 | 18.2 | 3 | 0 | 0 | 3.8 | 4.8 | 2.1 |
| Farm improvement[2/] | 0 | 3.9 | 1.1 | 2.3 | 0.4 | 4.4 | .. | .. | .. | .. | .. | .. | .. | .. | .. |
| Investment | .. | .. | .. | 24.2 | 0.6 | 4.7 | .. | .. | .. | .. | .. | .. | .. | .. | .. |
| Other | 7.7 | 24.9 | 16.3 | 7.2 | 6.6 | 5.9 | 0.8 | 2.6 | 3.5 | 13.5 | 8.3 | 6.9 | 38 | 27.4 | 29.8 |

Source: Mohapatra & Ratha, Migrant Remittances in Africa: An Overview (2011).

[1/] Mohapatra and Ratha's calculations based on household surveys conducted in Burkina Faso, Kenya, Nigeria, Senegal and Uganda in 2009, as part of Ghana Living Standards Survey in 2005-06.

[2/] Includes agricultural equipment.

Figure 3: Remittances and GDP Growth in SSA, 2010-18




Sources: World Development Indicators; and authors' calculations.

of remittances go to direct investment, while the majority goes to consumption, remittances could still be developmental because they tended to be spent within those sectors which had relatively strong linkages with the rest of the economy. Thus, many sectors not directly benefiting from remittances expenditures would nonetheless experience an increase in demand for their output inducing investment and fostering employment. If remittances are spent on sectors that have strong forward and backward linkages with other sectors, the overall impact on output would be even higher. Our research contributes to a better understanding of the role of these linkages, which has been generally neglected in previous studies, and will help to show a more complete representation of the effects of remittances on the economy.

# 3 Model for remittances' impact

The model presented extends the framework from Acemoglu *et al.* (2016),[3] which is used to understand how demand-side and supply-side shocks are amplified and propagated through the economy. We extend the model to consider remittances inflows as money windfalls that affect aggregate demand in the economy with upstream propagation to other sectors. We assume that these remittances inflows may be specially directed to the consumption of certain types of goods, depending on the observed preferences in each country.

## 3.1 Definitions

We consider a static perfectly competitive economy with $n$ industries. In every industry $i$, we assume that to produce a good ($y_i$), it is necesary labor ($l_i$) and intermediate goods produced by industry $j$ ($x_{ij}$). Industry $i$'s production function can then be written as,

$$y_i = l_i^{\alpha_i^l} \prod_{j=1}^{n} x_{ij}^{a_{ij}}. \tag{1}$$

Since we assume that the production function of each industry exhibits a constant returns to scale technology, the following equation is satisfied:

$$\alpha_i^l + \sum_{j=1}^{n} a_{ij} = 1. \tag{2}$$

A representative household consumes goods ($c_i$) from the $n$ industries and supplies labor ($l$), with constant elasticity of substitution (CES) utility function equal to;

$$u(c_1, c_2, ..., c_n, l) = \gamma(l) \prod_{i=1}^{n} c_i^{\beta_i}, \tag{3}$$

[3]The mentioned framework by Acemoglu et al. is based on Long and Plosser (1983).

where $\sum_{i=1}^{n} \beta_i = 1$. Consumer's income is derived from labor ($l$) and supplemented by remittances from abroad ($R$). Therefore, the budget constraint, i.e., the maximum the consumer can afford to consume given prevailing prices ($p_i$), is defined as

$$\sum_{i=1}^{n} p_i c_i = wl + R. \tag{4}$$

The number of goods produced in industry $i$ are either consumed or used as intermediate goods (as inputs) to produce in other industries (including in industry $i$). Therefore, market clearing condition for each industry $i$ is equal to

$$y_i = c_i + \sum_{j=1}^{n} x_{ji}. \tag{5}$$

We define the input ratio, $a_{ij}$, as the value of goods produced by industry $j$, and used by industry $i$ ($x_{ij}$), over the total nominal output of good $i$ produced by industry $i$:

$$\frac{p_j x_{ij}}{p_i y_i} = a_{ij}. \tag{6}$$

Accordingly, the term $a_{ij}$ can be directly obtained from the input-output table of each country. The matrix $A$ (Leontief matrix) can be written as;

$$A = \begin{pmatrix} a_{11} & a_{12} & & & & \\ a_{21} & a_{22} & & & & \\ & & \cdot & & & \\ & & & \cdot & & \\ & & & & \cdot & \\ & & & & & a_{nn} \end{pmatrix}. \tag{7}$$

We also define the matrix $\hat{A}$. The elements of this matrix are defined as $\hat{a}_{ji} = \frac{a_{ji} p_j y_j}{p_i y_i}$ (or equivalently, $\hat{a}_{ij} = \frac{a_{ij} p_i y_i}{p_j y_j} = \frac{x_{ij}}{y_j} = \frac{p_j x_{ij}}{p_j y_j}$). $\hat{a}_{ij}$ represents the share of industry $j$'s nominal output sold to industry $i$ (used as input in industry $i$). Using the matrix $\hat{A}$ we can define the Leontief inverse matrix, $\hat{H}$, as follows:

$$\hat{H} \equiv (I - \hat{A}^T)^{-1}, \tag{8}$$

and denote its typical entry by $h_{ij}$. $I$ is the identity matrix.

In order to obtain a closed form solution[4] for the main theoretical result of the paper, we assume that $\gamma(l) = (1-l)^{\lambda}$. This allows us to obtain a closed-form solution for the supply of labor by households, which greatly simplifies the analytical work.

Consistent with the literature on remittances which shows that remittances inflows can be used for specific consumption goods, we assume that the preference parameter may depend on the level of remittances, $\beta_i(R)$. This is a relatively simple way of modeling this effect.[5] In practice, this effect can be due to several reasons. For instance, uncertainty of these income flows may lead consumers to use them in a specific way, or in a way specifically intended by remitters.

Next, we present our main theoretical result, which shows how increases in remittances inflows propagate across sectors and induce an important degree of indirect expansion in output. A detailed proof of the result is shown in the appendix:

**Proposition 1.** *The impact of an increase in remittances on the output of sectors is equal to*

$$dlny=(I-\hat{A}^{T})^{-1}d\widetilde{R}, \tag{9}$$

*where $d\widetilde{R}$ is the following vector:*

$$d\widetilde{R}=\begin{pmatrix} \frac{\beta_1+(1+R)\beta_1'}{(1+\lambda)p_1y_1}dR \\ \frac{\beta_2+(1+R)\beta_2'}{(1+\lambda)p_2y_2}dR \\ \dots \\ \frac{\beta_n+(1+R)\beta_n'}{(1+\lambda)p_ny_n}dR \end{pmatrix}. \tag{10}$$

Note that $d\ln y = dy/y$ is a vector of growth of output by sector. Note also that the elements of the vector $d\widetilde{R}$ depend on the effect of remittances on every specific demand parameter, $\beta_i'(R)$.[6] Equation (9) can be expressed in a more intuitive way by using the elements of Leontief matrix in 8 as follows,

$$dlny_i = h_{ii}d\widetilde{R} + \sum_{j\neq i} h_{ij}d\widetilde{R}, \tag{11}$$

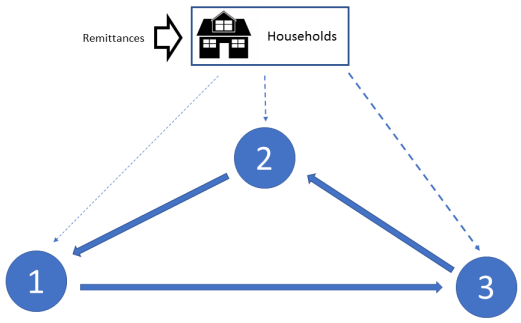
where to simplify the exposition, we assume that all the elements of vector in equation (10) are identical. In Eq. (11), the first term is the direct "own-effect" of remittances on industry $i$, and the second term is the "network-effect" of remittances on industry $i$. In the next section we provide a more intuitive explanation of the theoretical result.

[4] Our goal is to obtain a simple analytical equation that relates output growth, remittances inflows, and the network structure of the economy.

[5] This is a very simple way of modelling non-homothetic preferences, i.e., preferences that imply a demand structure such that the relative ratio of products consumed depend on the level of income.

[6] Note that to simplify the empirical analysis presented in the next section, we do not consider this effect, so $\beta_i' = 0$.

Figure 4: Simple Example of Changes in Demand and Sectoral Linkages (Three Sectors)




## 3.2 Intuition of the result

Figure 4 provides an example with a simple economy with three sectors (example adapted from Acemoglu *et al.* 2016). Sector 1 is the sole user of sector 2's output to produce good 1, sector 2 is the sole user of sector 3 to produce good 2, and sector 3 is the sole user of sector 1. When there is a change in demand in a sector, it propagates upstream to the sectors that produce inputs for that sector. For instance, an increase in demand for sector 1 leads an increase of demand for inputs used to produce good 1 (sector 2). In turn, an increase of demand for sector 2, leads to an increase of demand in sector 3, and so on. Therefore, there are cumulative effects working upstream.

Note also the crucial assumption of constant returns to scale in the production function, which implies that prices are constant. Therefore, changes in demand do not affect equilibrium prices. If this proves not to be the case, there could be downstream propagation (like the effect of supply-side shocks), which means that downstream producers would be affected by more expensive goods of upstream producers so they could substitute the more expensive sectors for less expensive ones.

We also present a simple analytical example to provide more intuition about the main result of the paper. If we assume a simple two-sector economy, the Leontief matrix $A$ is

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}, \tag{12}$$

and the Leontief inverse matrix $H$ is equal to

$$H \equiv (I - A^T)^{-1} = \begin{pmatrix} \frac{1-a_{22}}{(1-a_{11})(1-a_{22})-a_{21}a_{12}} & \frac{a_{21}}{(1-a_{11})(1-a_{22})-a_{21}a_{12}} \\ \frac{a_{12}}{(1-a_{11})(1-a_{22})-a_{21}a_{12}} & \frac{1-a_{11}}{(1-a_{11})(1-a_{22})-a_{21}a_{12}} \end{pmatrix}. \tag{13}$$

Following Proposition 1, we have that the growth of real production in sector 1 can be expressed as follows:

Figure 5: Sectors Included in the Eora MRIO Database

| Eora Database | | | |
|---|---|---|---|
| **Sectors** | | **Final Demand Components** | **Primary Inputs** |
| Agriculture | Construction | Household final consumption | Compensation of employees |
| Fishing | Maintenance and Repair | Non-profit institutions serving households | Taxes on production |
| Mining and Quarrying | Wholesale Trade | Government final consumption | Subsidies on production |
| Food & Beverages | Retail Trade | Gross fixed capital formation | Net operating surplus |
| Textiles and Wearing Apparel | Hotels and Restraurants | Changes in inventories | Net mixed income |
| Wood and Paper | Transport | Acquisitions less disposals of valuables | Consumption of fixed capital |
| Petroleum, Chemical and Non-Metallic Mineral Products | Post and Telecommunications | | |
| Metal Products | Finacial Intermediation and Business Activities | | |
| Electrical and Machinery | Public Administration | | |
| Transport Equipment | Education, Health and Other Services | | |
| Other Manufacturing | Private Households | | |
| Recycling | Others | | |
| Electricity, Gas and Water | Re-export & Re-import | | |

Source: Eora database.

$$d\ln y_1 = \frac{1-a_{22}}{(1-a_{11})(1-a_{22})-a_{21}a_{12}} d\widetilde{R}_1 + \frac{a_{21}}{(1-a_{11})(1-a_{22})-a_{21}a_{12}} d\widetilde{R}_2. \tag{14}$$

In this equation, the first term is the sector's own effect, and the second term is the network effect. The terms $d\widetilde{R}_1$ and $d\widetilde{R}_2$ are related to the preferences of households to spend the additional income from remittances. Note that in a extreme case of an economy without linkages across sectors, $a_{21} = 0$ and the network effect is zero. Therefore, intuitively the larger $a_{21}$, the larger isthe network effect.

# 4 Data

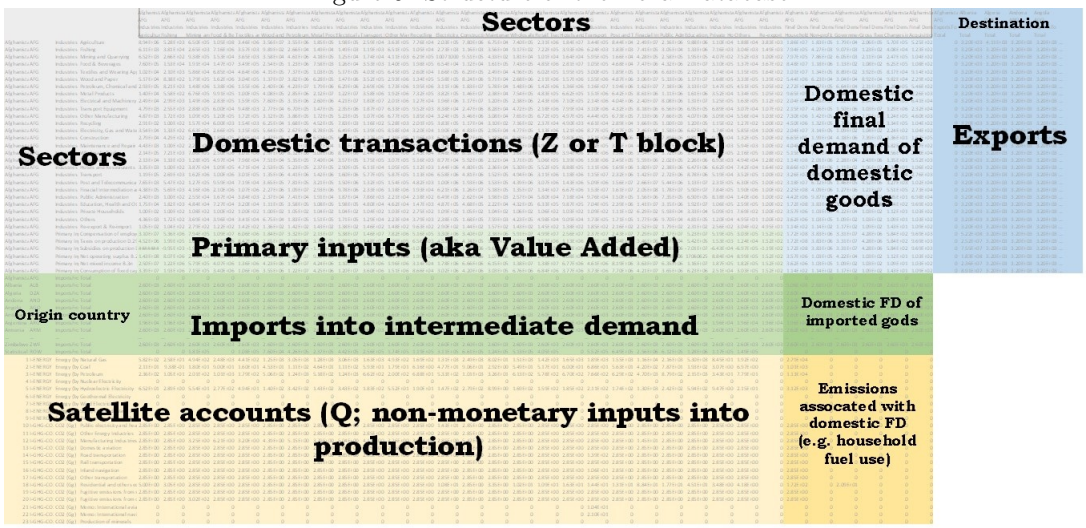
## 4.1 Eora Database

In order to analyze the impact of Remittances in SSA, this paper uses the Multi-Regional Input-Output (MRIO) database supplied by Eora. The Eora MRIO database has been widely used to conduct numerous economic studies, such as the economic effect of migration across countries, or the impact of carbon emissions on international trade, just to cite a few. Leontief considered the MRIO database as the "information system for the world economy" (Leontief 1974; Leontief 1986)

Eora's high-resolution MRIO tables track thousands of goods and commodities flowing through billions of trade and transformation steps to reach end users (Lenzen *et al.* 2013) in 187 countries. The highly detailed database can be used to quantify how a shock may propagate through supply chains to affect a particular country or sector. Raw data are primarily drawn from (i) the UN System of National Accounts, (ii) UN COMTRADE, (iii) Eurostat, (iv) IDE/JETRO, and (v) national agencies. The national accounts main aggregates comprise 126,152 data points over 38 years, expressed in current US dollars.

The Eora MRIO database provides a continuous 20-year time series of input-output tables using pro-rating, concordances matrices, and interpolation (Lenzen *et al.* 2013). The database comprises a total of over 15,000 industries for all the countries considered, and hence offers great details. Timeliness is also a unique feature of the database, as data are continually updated, accounting for revised statistics once published by data providers (Ratha, 2011). In our model we use a homogeneous set of 26 sectors as shown in Figure 5. The continuity of the time series enables a robust identification of key trends and a better understanding of the linkages across

Figure 6: Structure of the Eora Database




Source: Eora MRIO database.

sectors within individual countries. Figure 6 shows the general structure of the MRIO database.[7]

The detailed Eora MRIO database allows to map the network structure of the economy for each SSA country. Figure 7 is constructed using the Leontief input-output matrix (matrix $A$ described in the previous section), taking averages across countries for period 2011-2015. In order to improve the information content of the network structure presented, only sectoral linkages exceeding a certain threshold (of 5 percent) are shown. Figure 7 shows that financial intermediation is one of the central sectors in the economy, as it is a relevant input provider for most sectors in the economy. We show further analysis of the network structure of SSA economies in the empirical results section.

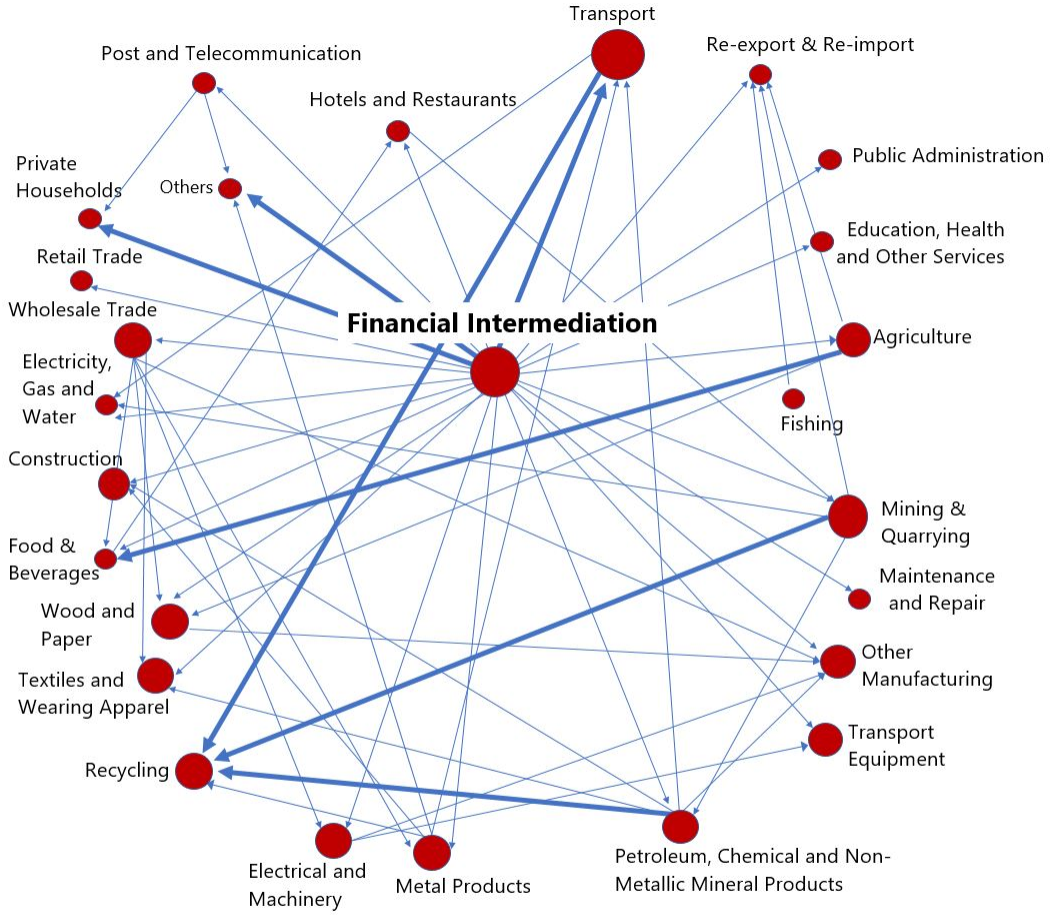
## 4.2 Remittances data

There is a concensus that the quality of remittances data is not usually good because the nature of these flows makes their measurement challenging. Remittances inlows are heterogeneous with numerous small transactions conducted by individuals through a wide variety of channels: formal channels, such as electronic wire, or through informal channels, such as cash or goods carried across borders. Therefore, the large number of remittances transactions and the multitude of channels pose challenges to the compilation of comprehensive statistics (World Bank, 2016).

Data on private remittances are derived from the World Bank Development Indicators (WDI) database. Personal remittances comprise personal transfers and compensation of employees. Personal transfers include all current transfers between resident and nonresident individuals. Compensation of employees includes income

[7]The Eora MRIO database is well known and a number of IMF publications have used it to conduct various analyses. For example, Aslam *et al.* (2017b) used a global input-output table for three countries to analyze global value chains. Similarly, Allard *et al.* (2016) used the MRIO database to assess the extent and strength of SSA's integration into global supply chains, while Aslam *et al.* (2017a) used the database to quantify the role of weak economic growth and changes in its decomposition in accounting for the slowdown in trade. Also, Cerdeiro (2016) used the detailed international trade flows from Eora to estimate the effect of the Trans-Pacific Partnership (TPP) on Latin America and the Caribbean.

Figure 7: Network Structure of Intersectoral Flows in SSA, 2011-15




Sources: Eora database; and authors' calculations.

Notes:

Only sectoral linkages exceeding a threshold of 5 percent are shown using sample averages. Bolded arrows reflect sectoral linkages of a value exceeding 20 percent and the size of red circles indicate the relative interaction volume of that sector with the rest.

of border, seasonal, and other short-term workers who are employed in an economy but are not residents, and of residents employed by nonresident entities. Therefore, private remittances are the sum of two items, which are defined in the sixth edition of the IMF's Balance of Payments Manual.

# 5 Empirical results

In this section we present the main results of the paper using SSA data for period 2011-2015. We focus on this limited period to ensure that our results are not influenced by the various economic events that have occurred in SSA in the last decades, such as financial crises and political events. The presentation of our results is divided in three subsections. We first propose two node centrality measures used to determine the importance of sectoral linkages in the economy. Then, we empirically analyze how these centrality measures vary across SSA countries and sectors. Finally, using our calibrated model, we analyze the effect of remittances inflows on the growth of economic sectors and total output across SSA countries, and how growth is related with the importance of sectoral linkages across economies.

## 5.1 Centrality measures of the sectoral network of the economy

We assess the importance of input-output sectoral linkages in the economy by constructing two node centrality measures based on the Leontief input-output matrix $A$ defined in Eq. 7. These centrality measures are proposed in Acemoglu *et al.* (2012) and in Carvalho (2014) and are intended to measure the importance of each economic sector as an upstream provider of inputs to the rest of the economic sectors.

**Measure 1 (Weighted outdegree):**

The first proposed node centrality measure, the weighted outdegree measure, is generated as an aggregate measure of the upstream importance of every sector. For a given sector $j$, the weighted outdegree measure, $d^j_{out}$, is defined as the sum of the elements of $A$ in which sector $j$ appears as an input-supplying sector,

$$d^j_{out} = \sum_{i=1}^{n} a_{ij}, \tag{15}$$

where $a_{ij}$ is the input ratio as the value of goods produced by industry $j$ and used by industry $i$, over nominal output for good $i$ produced by industry $i$, and $n$ is the total number of sectors ($n = 26$).

As explained in Carvalho (2014), this measure is equal to 0 if a sector does not supply inputs to any other sectors, and increases as the sector becomes more important as input provider to other sectors. Therefore, the greater this measure is, the higher the upstream importance of the sector $j$ as an input provider for the rest of the sectors.

**Measure 2 (Katz-Bonacich centrality score):**

The second node centrality measure complements and extends the weighted outdegree indicator. Some sectors can be key for the economy, even if they are not relevant upstream suppliers. For instance, a sector may have an average importance when considering the outdegree measure, but it may have an aggregate large impact in the rest of the economy because the immediate downstream sectors may be relevant input suppliers to other sectors, which may also be relevant input suppliers to other sectors (and so on). Therefore, the importance of a sector as input supplier may be high even if the weighted outdegree measure is low.

Therefore, the Katz-Bonacich centrality score is proposed to consider the propagation effect across sectors. To derive the Katz-Bonacich centrality score $c_j$ for a sector $j$, $c_j$, we assume that the measure is defined by some constant level $\eta$, equal across all sectors, plus a term that is proportional to the weighted sum of the centrality weights of its downstream sectors:

$$c_j = \lambda \sum_{i=1}^{n} a_{ij} c_i + \eta,$$

where we use the elements of the Leontief matrix $A$ to weight the terms, and $\lambda$ is a constant ($\lambda = 0.5$). This is a recursive definition that can be expressed in matrix form as follows

$$c = \eta(I - \lambda A')^{-1} 1,$$

where $c$ is the vector of Bonacich centrality scores, and 1 is a vector of ones.

The two centrality measures considered, weighted outdegree and Bonacich centrality score, use elements from matrix $A$. Perhaps not surprisingly, the two measures are correlated although they have different ranges of values, as it can be shown in Figure 8. There are other measures that have been proposed in the literature (see for instance, Blöchl *et al.*, 2011), but we drop them from our empirical analysis because we do not find robust results.[8]

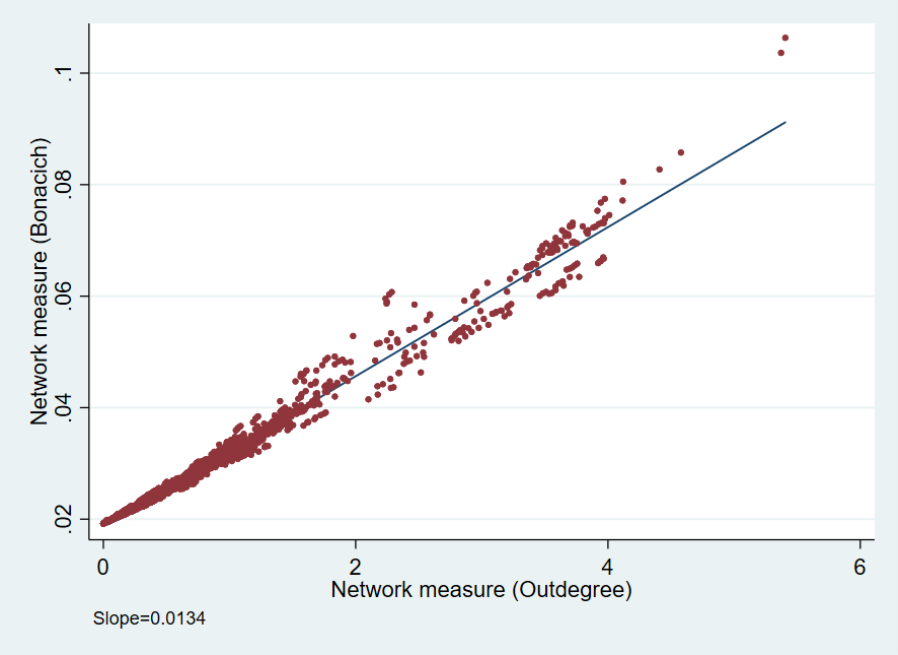
## 5.2 Importance of sectoral linkages in the economy

In this section we discuss in detail the two proposed centrality measures across countries and sectors. Figure 9 shows the scatter plot and the median value of the Bonacich measure across SSA countries for every sector. Primary sectors are shown in the left hand-side of the graph, whereas secondary sectors are in the center of the graph, and tertiary sectors are in the right hand-side of the graph. The graph shows that some sectors such as Financial intermediation, Petroleum and Minerals, or Wholesale trade are, on average, more important in the economy than the rest of the sectors. The importance of some of these sectors is consistent with the patterns observed in Figure 7. The literature on networks has shown similar results when analyzing sectoral networks in other world regions. Blöchl *et al.* (2011), and McNerney *et al.* (2013), provide a cross-country comparative perspective on the sectoral network structure for OECD countries. Blöchl *et al.* (2011) find a very high sectoral importance of the financial intermediation and wholesale trade sectors. Carvalho (2014) also shows the high importance of financial intermediation and wholesale trade in the United States. To our knowledge, our paper

[8] We have experimented with alternative measures, including betweenness, page-rank, hubs and authorities centrality measures, but we could not find a conclusive relationship between these measures and the effect of remittances on growth.

Figure 8: Correlation of the Two Network Measures Considered

Note: We show the correlation between the weighted outdegree and Bonacich centrality measures. An observation in this figure is a sector in a given country and year (period 2011-15). Source: Authors' calculations.




is the first paper that studies in detail the sectoral network structure of developing economies.

Figure 10 shows a similar pattern to Figure 9, for the case of the weighted outdegree. We find similar results (high importance for the wholesale trade and financial intermediation sectors), although we observe smaller differences across sectors, and also a smaller dispersion.

Figure 11 shows the scatter plot and the median value of the variable $\widetilde{R}$, related to the preferences of the households to use the additional income from remittances. Therefore, this variable is related to the demand-side effect of remittances. We observe greater dispersion across countries, and a relatively high median value in the food, textile or retail sectors (among others). Interestingly, we do not generally find a high value in sectors that are highly linked with other sectors in the economy (as shown in the previous figures).

## 5.3 Effect of remittances on sectoral output growth across economic sectors

We use Proposition 1 to estimate the effect of remittances on output growth for every sector. Equation 9 shows the rate of growth of output of an economic sector $i$, $dlny_i$, when there is an increase of the level of remittances equal to $dR$. Several parameters need to be calibrated in Equation 9 to derive the empirical results. The data provided by Eora can be used to obtain the nominal value of production per sector, $p_i y_i$. Also, by assuming a CES utility function, the preference parameter $\beta_i$ is calibrated and set equal to the observed ratio of consumption of good $i$ over total consumption, $\frac{p_i c_i}{\sum_{j=1}^{n} p_j c_j}$. We also assume that $\beta' = 0$ and that $\lambda = 0.8$ to

Figure 9: Sectoral Linkages across Sectors (Bonacich Centrality)

Note: We show the scatter plot of the Bonacich centrality measure by sector (median line added). An observation in this figure is a sector in a given country and year (period 2011-15). Source: Authors' calculations.

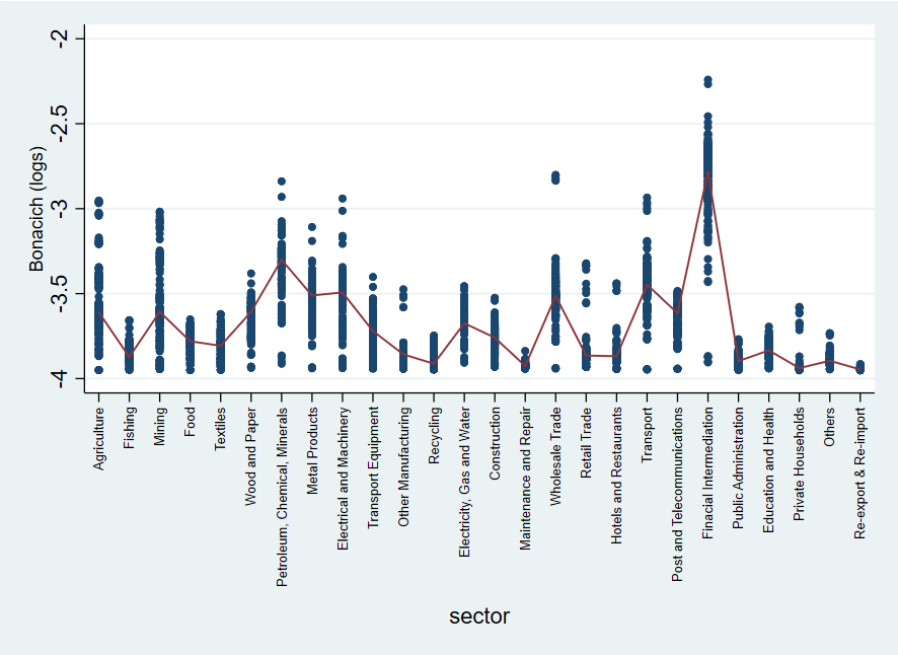



Figure 10: Sectoral Linkages across Sectors (Weighted Outdegree)

Note: We show the scatter plot of the weighted outdegree measure by sector (median line added). An observation in this figure is a sector in a given country and year (period 2011-15). Source: Authors' calculations.

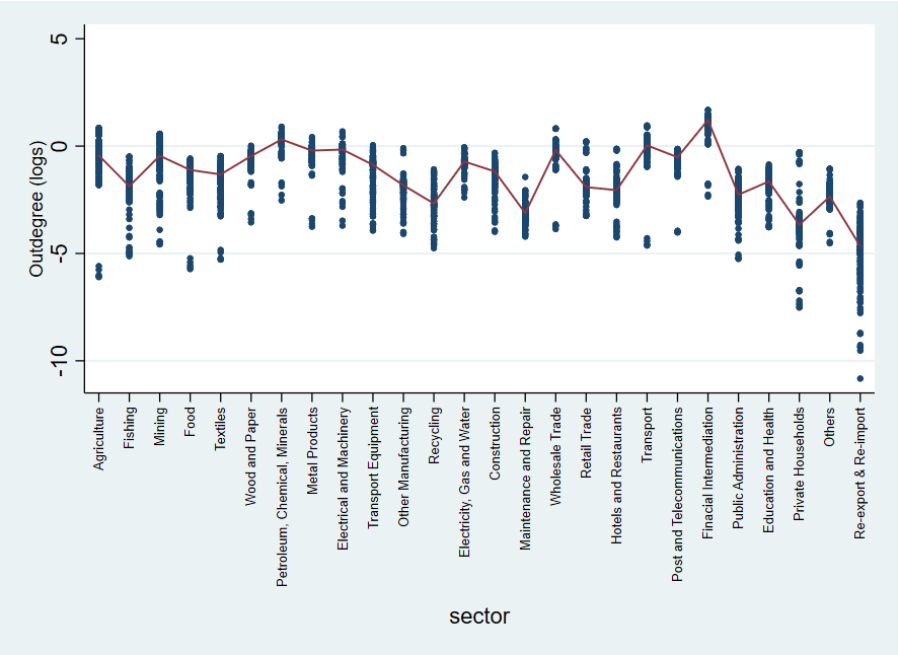
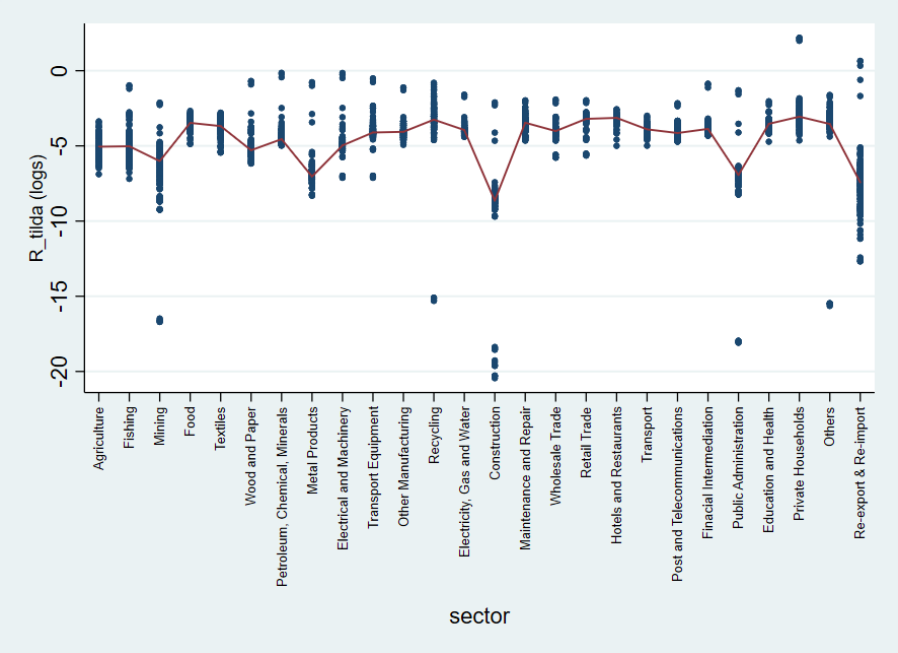


Figure 11: $\widetilde{R}$ Variable across Sectors

Note: We show the scatter plot of the $\widetilde{R}$ variable by sector (median line added). An observation in this figure is a sector in a given country and year (period 2011-15). Source: Authors' calculations.




calculate labor supply.

We consider two cases. In a first case, using the calibrated model we simulate the effect of the observed flow of remittances in the economy for period 2011-2015. In other words, we assume that, for each country, $dR$ is equal to the average observed level of remittances for period 2011-2015 and we use Proposition 1 to simulate the effect on output growth for every sector in every country. The results are shown in Table 2, which displays the rate of growth of output for some relevant sectors for each country included in our database. We have selected a subset of primary, secondary and tertiary sectors. The results show large differences across sectors. For instance, Madagascar received 415 millions of USD in the analysis period as remittances inflows, which is equivalent of 4.2 percent of its GDP. This flow of remittances had the largest effect on the financial intermediation sector (output equal to 1.71 percent), but the effect was much smaller on the agriculture sector (output growth equal to 0.05 percent). Even if the agricultural sector is important in many SSA countries, other sectors may be more relevant in terms of sectoral linkages, which may affect sectoral output growth. Also, The Gambia, Liberia and Lesotho have the largest inflow of remittances, relative to their GDP, and the effect on the food or wood sectors was also relatively large. At the other extreme, Angola, DRC and Namibia have the lowest inflow of remittances as a percentage of their GDP.

In the second case, we study the effect of a flow of remittances that represents 5 percent of the overall GDP in each economy. This allows us to compare the effect of remittances inflows on sectors across countries, so countries receive the same inflow of remittances, in relative terms. Table 3 shows the results of this exercise. Compared to the first case, the observed differences across countries are smaller for a given sector, but there

Table 2: Effect of Remittances Inflows on the Economic Sectors (Output), by Country, Period 2011-15

| | Growth as % of GDP | | | | | | | | | | | Remitt | Remitt |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Country | Agricult. | Food | Wood | Petrol | Equip | Restau | Telecom | Financial | Retail | Wholesale | Educat | (M USD) | (% of GDP) |
| Angola | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 11.00 | 0.01% |
| Burundi | 0.34 | 0.17 | 0.05 | 0.12 | 0.04 | 0.14 | 0.08 | 0.61 | 0.17 | 0.12 | 0.26 | 48.00 | 2.15% |
| Benin | 0.15 | 0.22 | 0.09 | 0.23 | 0.08 | 0.21 | 0.19 | 1.41 | 0.27 | 0.23 | 0.50 | 228.60 | 3.39% |
| Burkina-Fasso | 0.24 | 0.25 | 0.09 | 0.27 | 0.11 | 0.23 | 0.14 | 1.09 | 0.26 | 0.27 | 0.38 | 305.40 | 3.66% |
| Botswana | 0.01 | 0.01 | 0.00 | 0.01 | 0.00 | 0.01 | 0.01 | 0.07 | 0.02 | 0.02 | 0.03 | 30.00 | 0.22% |
| Cote Ivoire | 0.02 | 0.06 | 0.01 | 0.09 | 0.04 | 0.08 | 0.07 | 0.51 | 0.12 | 0.08 | 0.20 | 373.00 | 1.46% |
| Cameroon | 0.02 | 0.05 | 0.01 | 0.06 | 0.02 | 0.05 | 0.05 | 0.36 | 0.07 | 0.05 | 0.13 | 231.80 | 0.86% |
| DRC | 0.01 | 0.01 | 0.00 | 0.01 | 0.00 | 0.01 | 0.01 | 0.06 | 0.01 | 0.01 | 0.02 | 37.40 | 0.12% |
| Cape Verde | 0.88 | 0.85 | 0.37 | 0.83 | 0.28 | 0.80 | 0.84 | 4.43 | 0.96 | 0.85 | 1.58 | 185.80 | 11.43% |
| Ethiopia | 0.16 | 0.31 | 0.68 | 0.90 | 0.70 | 0.52 | 0.58 | 1.02 | 0.68 | 0.63 | 0.63 | 601.80 | 8.72% |
| Ghana | 0.05 | 0.21 | 0.04 | 0.19 | 0.08 | 0.39 | 0.21 | 1.45 | 0.41 | 0.46 | 0.62 | 2052.40 | 5.00% |
| Guinea | 0.05 | 0.07 | 0.02 | 0.06 | 0.02 | 0.09 | 0.07 | 0.44 | 0.12 | 0.09 | 0.20 | 82.00 | 1.45% |
| Gambia | 1.87 | 1.39 | 0.67 | 1.34 | 0.49 | 1.22 | 1.20 | 5.93 | 1.50 | 1.46 | 2.26 | 158.40 | 16.12% |
| Kenya | 0.78 | 0.48 | 0.06 | 0.19 | 0.00 | 0.23 | 0.16 | 0.35 | 0.24 | 0.11 | 0.19 | 1290.20 | 3.28% |
| Liberia | 0.52 | 2.39 | 1.30 | 2.43 | 0.52 | 1.90 | 1.94 | 13.93 | 2.92 | 2.39 | 4.55 | 481.20 | 34.51% |
| Lesotho | 1.42 | 1.68 | 0.74 | 1.52 | 0.60 | 1.37 | 0.93 | 6.65 | 1.62 | 1.42 | 2.79 | 482.60 | 21.36% |
| Madagascar | 0.05 | 0.22 | 0.10 | 0.26 | 0.13 | 0.25 | 0.25 | 1.71 | 0.32 | 0.26 | 0.63 | 415.20 | 4.19% |
| Mali | 0.40 | 0.62 | 0.26 | 0.66 | 0.22 | 0.60 | 0.51 | 4.01 | 0.75 | 0.63 | 1.36 | 844.00 | 9.29% |
| Mozambique | 0.03 | 0.06 | 0.02 | 0.05 | 0.02 | 0.06 | 0.06 | 0.31 | 0.09 | 0.06 | 0.14 | 161.40 | 1.13% |
| Mauritius | 0.20 | 0.35 | 0.05 | 0.11 | 0.01 | 0.30 | 0.12 | 0.52 | 0.03 | 0.15 | 0.13 | 249.00 | 2.62% |
| Malawi | 0.01 | 0.03 | 0.02 | 0.04 | 0.02 | 0.03 | 0.03 | 0.23 | 0.05 | 0.04 | 0.09 | 31.80 | 0.57% |
| Namibia | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.01 | 0.03 | 0.01 | 0.01 | 0.01 | 11.60 | 0.10% |
| Niger | 0.19 | 0.12 | 0.03 | 0.06 | 0.03 | 0.12 | 0.09 | 0.55 | 0.16 | 0.13 | 0.26 | 143.60 | 2.11% |
| Nigeria | 0.35 | 0.28 | 0.08 | 0.19 | 0.10 | 0.31 | 0.23 | 2.18 | 0.45 | 0.31 | 0.66 | 20769.20 | 7.01% |
| Rwanda | 0.17 | 0.18 | 0.07 | 0.17 | 0.07 | 0.16 | 0.14 | 1.04 | 0.20 | 0.18 | 0.38 | 153.60 | 2.72% |
| Senegal | 0.21 | 0.38 | 0.17 | 0.41 | 0.17 | 0.49 | 0.36 | 2.67 | 0.66 | 0.50 | 0.96 | 1673.20 | 8.01% |
| Sierra Leone | 0.39 | 0.12 | 0.03 | 0.08 | 0.03 | 0.42 | 0.12 | 0.64 | 0.17 | 0.87 | 0.35 | 63.60 | 2.18% |
| Sao Tome | 0.41 | 0.39 | 0.24 | 0.48 | 0.16 | 0.37 | 0.39 | 1.85 | 0.50 | 0.46 | 0.76 | 17.40 | 5.36% |
| Swaziland | 0.04 | 0.06 | 0.04 | 0.12 | 0.04 | 0.07 | 0.06 | 0.51 | 0.08 | 0.07 | 0.16 | 28.40 | 1.13% |
| Seychelles | 0.04 | 0.06 | 0.03 | 0.09 | 0.03 | 0.13 | 0.08 | 0.47 | 0.15 | 0.12 | 0.19 | 17.80 | 1.46% |
| Togo | 0.45 | 0.72 | 0.38 | 0.90 | 0.32 | 0.71 | 0.68 | 4.96 | 0.85 | 0.79 | 1.60 | 355.40 | 10.49% |
| Tanzania | 0.14 | 0.32 | 0.23 | 0.66 | 0.22 | 0.31 | 0.35 | 2.94 | 0.41 | 0.43 | 0.79 | 392.00 | 5.03% |
| Uganda | 0.12 | 0.27 | 0.13 | 0.34 | 0.12 | 0.27 | 0.24 | 1.87 | 0.35 | 0.30 | 0.69 | 921.20 | 4.55% |
| South Africa | 0.02 | 0.03 | 0.01 | 0.03 | 0.01 | 0.01 | 0.02 | 0.07 | 0.04 | 0.02 | 0.02 | 990.40 | 0.28% |
| Zambia | 0.01 | 0.02 | 0.01 | 0.02 | 0.01 | 0.02 | 0.01 | 0.12 | 0.02 | 0.02 | 0.04 | 55.60 | 0.28% |

Source: Authors' calculations.

remain relatively large differences in some cases, which can be attributed to differences in sectoral linkages, and consumer preferences across countries. The sectors with the largest growth are financial intermediation, education and retail, whereas the sectors with lowest growth are wood (and paper) and transport equipment.

### 5.4 Importance of linkages in explaining sectoral growth

In this section we show the relationship between real sectoral growth due to remittances inflows, and the importance of input-output sectoral linkages in each sector. We consider a remittances inflow equivalent to 5 percent of GDP in all countries and estimate the sectoral growth as a function of the importance of sectoral linkages, using the two network measures that we have previously proposed.

Although sectoral growth is obtained from Equation 9, which depends on matrix $A$, and the same matrix is used to construct the network measures, the relationship between the two variables plotted in each graph does not need to be perfectly correlated (or even correlated) for several possible reasons. First, there is no perfect relationship between matrix $A$ and the defined network measures. Second, there are other parameters that vary across countries, such as the preference parameter $\beta_i$. The comovement of all these variables could potentially reduce or eliminate any possible correlation between the variables of interest. Our goal is to quantify this relationship, which gives an estimate of the importance of sectoral linkages in explaining sectoral growth.

We first display the importance of the demand size effect in Figure 12. We show the relationship between sectoral growth and the value of variable $\widetilde{R}$, and we find a clear positive relationship between both variables. A one percent increase in the variable $\widetilde{R}$ increases sectoral growth by 0.45 percent.

In Figures 13 and 14 we display the sectoral growth and the value of the network measure (both expressed in logs) in a country and sector, over the period 2011-15. The two figures show a clear positive and statistically significant relationship between network measures and sectoral growth across sectors, countries and years. A one percent increase in weighted outdegree sectoral linkages increase sectoral growth by an estimated 0.19 percent. The effect of the Bonacich measure is substantially larger (0.65 percent).

We analyze in greater detail the effect of linkages on output growth by using regression analysis that controls for sector, country, and year fixed effects. Table 4 shows that when the sectoral linkages (measured with the weighted outdegree measure) increase by 1 percent, the sectoral growth is about 0.16 percent higher when sector fixed effects are not considered. When sector, country, and year fixed effects are considered, the impact on sectoral growth is much larger and close to 0.6 percent. This may suggest that fixed effects that positively affect sectoral growth may be negatively correlated with the network measures used (sectors that grow more due exogenous reasons are less interconnected with other sectors). This could indicate that by disregarding sectoral fixed effects, we are underestimating the effect of sectoral linkages. The parameter of interest is significant in all cases at the 1 percent level.

Table 5 shows that the Bonacich network measure has a much stronger effect on sectoral growth than the weighted outdegree measure. When the sectoral linkages increase by 1 percent, the sectoral growth is about 0.5 percent higher when not considering sector fixed effects. After controlling for sector, country, and year fixed effects, a 1 percent increase in this network measure increases sectoral growth by about 1.6 percent,

Table 3: Effect of a 5 Percent Remittances Inflows on the Economic Sectors (Output) by Country, Period 2011-15

| | Growth in % of GDP | | | | | | | | | | | Remitt | Remitt |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Country | Agricult. | Food | Wood | Petrol | Equip | Restau | Telecom | Financial | Retail | Wholesale | Educat | (M USD) | (% of GDP) |
| Angola | 0.18 | 0.36 | 0.13 | 0.31 | 0.12 | 0.32 | 0.27 | 2.02 | 0.43 | 0.34 | 0.77 | 5752.76 | 5.00% |
| Burundi | 0.80 | 0.39 | 0.12 | 0.28 | 0.10 | 0.32 | 0.19 | 1.43 | 0.39 | 0.27 | 0.61 | 111.88 | 5.00% |
| Benin | 0.22 | 0.32 | 0.13 | 0.34 | 0.12 | 0.31 | 0.28 | 2.08 | 0.39 | 0.34 | 0.73 | 336.86 | 5.00% |
| Burkina-Fasso | 0.33 | 0.35 | 0.13 | 0.37 | 0.14 | 0.31 | 0.19 | 1.49 | 0.36 | 0.37 | 0.52 | 417.12 | 5.00% |
| Botswana | 0.16 | 0.26 | 0.07 | 0.26 | 0.06 | 0.34 | 0.19 | 1.58 | 0.42 | 0.34 | 0.57 | 681.35 | 5.00% |
| Cote Ivoire | 0.06 | 0.21 | 0.05 | 0.29 | 0.15 | 0.28 | 0.24 | 1.73 | 0.40 | 0.29 | 0.67 | 1273.58 | 5.00% |
| Cameroon | 0.11 | 0.30 | 0.07 | 0.32 | 0.12 | 0.31 | 0.26 | 2.07 | 0.39 | 0.32 | 0.76 | 1345.86 | 5.00% |
| DRC | 0.20 | 0.37 | 0.18 | 0.47 | 0.16 | 0.34 | 0.32 | 2.41 | 0.43 | 0.40 | 0.83 | 1505.20 | 5.00% |
| Cape Verde | 0.39 | 0.37 | 0.16 | 0.37 | 0.12 | 0.35 | 0.37 | 1.94 | 0.42 | 0.37 | 0.69 | 81.31 | 5.00% |
| Ethiopia | 0.09 | 0.18 | 0.39 | 0.51 | 0.40 | 0.30 | 0.33 | 0.58 | 0.39 | 0.36 | 0.36 | 345.01 | 5.00% |
| Ghana | 0.05 | 0.21 | 0.04 | 0.19 | 0.08 | 0.39 | 0.21 | 1.45 | 0.41 | 0.46 | 0.62 | 2053.84 | 5.00% |
| Guinea | 0.18 | 0.26 | 0.08 | 0.20 | 0.08 | 0.31 | 0.23 | 1.53 | 0.40 | 0.30 | 0.70 | 283.28 | 5.00% |
| Gambia | 0.58 | 0.43 | 0.21 | 0.42 | 0.15 | 0.38 | 0.37 | 1.84 | 0.47 | 0.45 | 0.70 | 49.12 | 5.00% |
| Kenya | 1.19 | 0.72 | 0.09 | 0.29 | 0.00 | 0.34 | 0.24 | 0.53 | 0.36 | 0.16 | 0.29 | 1965.11 | 5.00% |
| Liberia | 0.08 | 0.35 | 0.19 | 0.35 | 0.08 | 0.27 | 0.28 | 2.02 | 0.42 | 0.35 | 0.66 | 69.73 | 5.00% |
| Lesotho | 0.33 | 0.39 | 0.17 | 0.35 | 0.14 | 0.32 | 0.22 | 1.56 | 0.38 | 0.33 | 0.65 | 112.95 | 5.00% |
| Madagascar | 0.06 | 0.26 | 0.11 | 0.31 | 0.15 | 0.30 | 0.30 | 2.03 | 0.38 | 0.32 | 0.75 | 495.34 | 5.00% |
| Mali | 0.22 | 0.33 | 0.14 | 0.36 | 0.12 | 0.32 | 0.28 | 2.16 | 0.40 | 0.34 | 0.73 | 454.22 | 5.00% |
| Mozambique | 0.15 | 0.25 | 0.08 | 0.21 | 0.08 | 0.29 | 0.29 | 1.39 | 0.39 | 0.28 | 0.63 | 717.00 | 5.00% |
| Mauritius | 0.38 | 0.66 | 0.09 | 0.21 | 0.02 | 0.56 | 0.23 | 1.00 | 0.06 | 0.29 | 0.24 | 475.39 | 5.00% |
| Malawi | 0.08 | 0.28 | 0.14 | 0.37 | 0.14 | 0.30 | 0.27 | 2.04 | 0.40 | 0.35 | 0.74 | 277.49 | 5.00% |
| Namibia | 0.21 | 0.20 | 0.10 | 0.22 | 0.09 | 0.33 | 0.26 | 1.56 | 0.39 | 0.38 | 0.64 | 570.55 | 5.00% |
| Niger | 0.46 | 0.27 | 0.07 | 0.15 | 0.07 | 0.30 | 0.21 | 1.30 | 0.39 | 0.30 | 0.62 | 340.46 | 5.00% |
| Nigeria | 0.25 | 0.20 | 0.05 | 0.13 | 0.07 | 0.22 | 0.17 | 1.56 | 0.32 | 0.22 | 0.47 | 14816.51 | 5.00% |
| Rwanda | 0.31 | 0.33 | 0.13 | 0.32 | 0.13 | 0.30 | 0.26 | 1.92 | 0.38 | 0.33 | 0.70 | 282.69 | 5.00% |
| Senegal | 0.13 | 0.24 | 0.10 | 0.26 | 0.11 | 0.31 | 0.22 | 1.66 | 0.41 | 0.31 | 0.60 | 1043.96 | 5.00% |
| Sierra Leone | 0.90 | 0.28 | 0.08 | 0.19 | 0.07 | 0.95 | 0.29 | 1.47 | 0.40 | 1.98 | 0.81 | 145.55 | 5.00% |
| Sao Tome | 0.38 | 0.37 | 0.22 | 0.45 | 0.15 | 0.34 | 0.36 | 1.73 | 0.46 | 0.43 | 0.71 | 16.23 | 5.00% |
| Swaziland | 0.20 | 0.29 | 0.17 | 0.53 | 0.19 | 0.31 | 0.29 | 2.26 | 0.37 | 0.32 | 0.71 | 126.08 | 5.00% |
| Seychelles | 0.13 | 0.20 | 0.11 | 0.30 | 0.09 | 0.45 | 0.29 | 1.61 | 0.50 | 0.40 | 0.64 | 60.99 | 5.00% |
| Togo | 0.21 | 0.34 | 0.18 | 0.43 | 0.15 | 0.34 | 0.33 | 2.36 | 0.41 | 0.38 | 0.76 | 169.48 | 5.00% |
| Tanzania | 0.13 | 0.32 | 0.23 | 0.65 | 0.22 | 0.31 | 0.35 | 2.92 | 0.41 | 0.43 | 0.78 | 389.64 | 5.00% |
| Uganda | 0.13 | 0.29 | 0.14 | 0.37 | 0.13 | 0.30 | 0.27 | 2.06 | 0.38 | 0.33 | 0.76 | 1012.01 | 5.00% |
| South Africa | 0.27 | 0.59 | 0.18 | 0.47 | 0.21 | 0.12 | 0.33 | 1.32 | 0.63 | 0.28 | 0.30 | 17737.71 | 5.00% |
| Zambia | 0.15 | 0.31 | 0.12 | 0.32 | 0.12 | 0.31 | 0.26 | 2.09 | 0.41 | 0.32 | 0.76 | 1001.68 | 5.00% |
| AVERAGE | 0.28 | 0.33 | 0.13 | 0.33 | 0.12 | 0.34 | 0.27 | 1.73 | 0.40 | 0.38 | 0.64 | | |

Source: Authors' calculations.

Figure 12: Changes in $\widetilde{R}$ and Sectoral Output Growth (Weighted Outdegree)

Note: We show the relationship between sectoral output growth and the $\widetilde{R}$ variable. An observation in this figure is a sector in a given country-year for period 2011-15. Source: Authors' calculations.

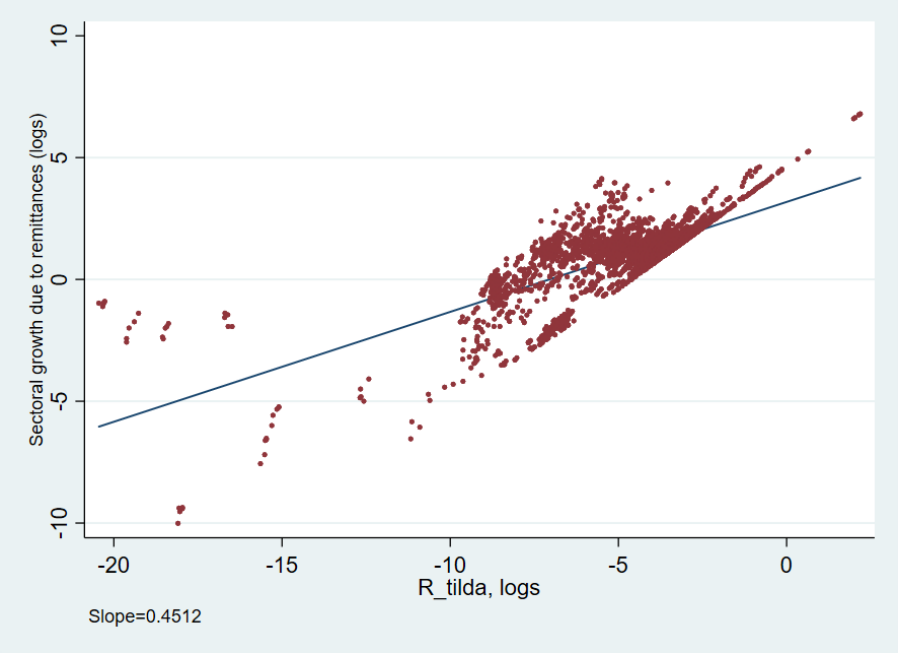



Figure 13: Sectoral Linkages and Sectoral Output Growth (Weighted Outdegree)

Note: We show the relationship between sectoral output growth and the weighted outdegree measure. An observation in this figure is a sector in a given country-year for period 2011-15. Source: Authors' calculations.

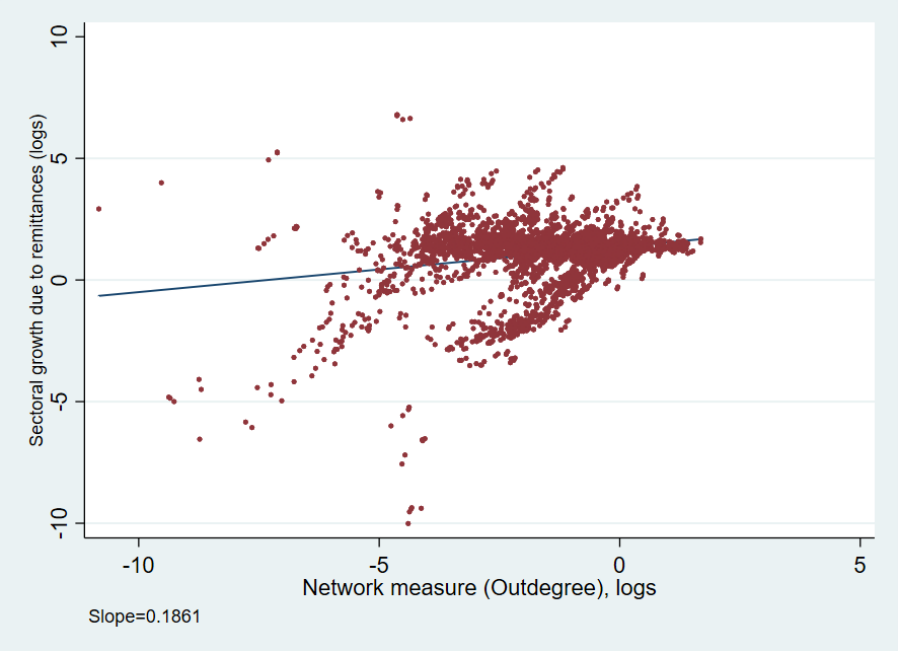


Figure 14: Sectoral Linkages and Sectoral Output Growth (Bonacich Centrality)

Note: We show the relationship between sectoral output growth and the Bonacich centrality measure. An observation in this figure is a sector in a given country-year for period 2011-15. Source: Authors' calculations.

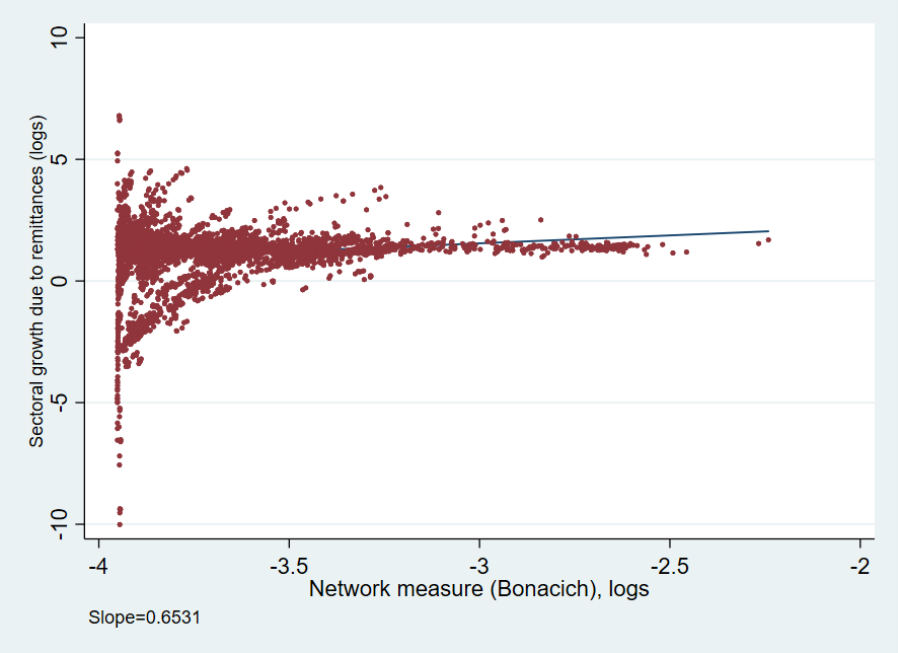



Table 4: Relationship between Sectoral Linkages and Sectoral Output Growth (Weighted Outdegree)

| | (1) Sectoral growth | (2) Sectoral growth | (3) Sectoral growth | (4) Sectoral growth | (5) Sectoral growth |
|---|---|---|---|---|---|
| Weighted outdegree (log) | 0.186*** | 0.218*** | 0.387*** | 0.653*** | 0.642*** |
| | (0.0204) | (0.0194) | (0.0400) | (0.0413) | (0.0408) |
| Observations | 4,550 | 4,550 | 4,550 | 4,550 | 4,550 |
| R-squared | 0.053 | 0.112 | 0.489 | 0.590 | 0.601 |
| Country fixed effects | NO | YES | NO | YES | YES |
| Sector fixed effects | NO | NO | YES | YES | YES |
| Year fixed effects | NO | NO | NO | NO | YES |

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Note: OLS regression. One observation is a country-sector-year in period 2011-2015. Variables expressed in logs. Robust standard errors.

Table 5: Relationship between Sectoral Linkages and Sectoral Output Growth (Bonacich Centrality)

| | (1) Sectoral growth | (2) Sectoral growth | (3) Sectoral growth | (4) Sectoral growth | (5) Sectoral growth |
|---|---|---|---|---|---|
| Bonacich (log) | 0.653*** | 0.713*** | 1.170*** | 1.620*** | 1.568*** |
| | (0.0549) | (0.0521) | (0.162) | (0.151) | (0.149) |
| Observations | 4,550 | 4,550 | 4,550 | 4,550 | 4,550 |
| R-squared | 0.021 | 0.070 | 0.439 | 0.493 | 0.506 |
| Country fixed effects | NO | YES | NO | YES | YES |
| Sector fixed effects | NO | NO | YES | YES | YES |
| Year fixed effects | NO | NO | NO | NO | YES |

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Note: OLS regression. One observation is a country-sector-year in period 2011-2015. Variables expressed in logs. Robust standard errors.

where in the weighted-outdegree method the same increase is about 0.6 percent. Moreover, in all cases, the estimated parameter are statistically significant at the 1 percent level.

### 5.5 Effect of sectoral linkages on total output

We now study the effect of sectoral linkages on growth of total output in the economy due to a 5 percent (of GDP) remittances inflows. We calculate the median value of the sectoral link measure across sectors in each country and compare it with the growth in total output in each economy. Since we have found a positive effect of sectoral linkages on the growth of individual sectors, we also find a positive and intuitive relationship between the median value of sectoral linkages, and growth across countries. Figures 15 and 16 show a clear positive effect, which is consistent with the previous findings. This suggests that more developped economies that have sectors that are more interlinked with the rest of the economy, are also more likely to generate greater total economic output as a result of remittances inflows.

## 6 Conclusion

In this paper we propose and calibrate a simple macroeconomic model with input-output sectoral linkages to analyze how additional income from remittances increases household consumption, and this propagates through the network of input-output sectoral linkages in the economy. Our results are based on individual country-level remittances data, and detailed Leontief input-output matrices for SSA countries. We use the calibrated model to estimate sector-level output growth due to an increase of remittances inflows.

We show that sectoral linkages are important to explain economic growth across sectors due to an increase in remittances inflows. Our results contribute to the literature that studies the effects of remittances inflows on economic growth, and also to the emerging literature that uses detailed network models to understand the

propagation of shocks. More broadly, our results suggest that a better understanding of input-output sectoral linkages is key to properly capture the full impact of remittances inflows on the recipient economy. Our results indicate that even when utilized for non-investment purposes, remittances may expand domestic production of consumption and intermediate goods necessary to support the increase in consumption. Furthermore, when remittances are spent within sectors that have strong linkages with the rest of the economy, the sectors that do not directly benefit from remittances inflows may still experience output growth. The overall expansion of output will create employment opportunities and stimulate demand for investment goods. Hence, the external stimulus provided by remittances inflows would be more beneficial to a country, the more its economy is diversified, and its production structure integrated. This underscores the importance of diversifying the SSA economies. Also, to foster employment and growth, policymakers should devise stimulus policies targeting sectors that exhibit high vulnerabilities to sharp declines in remittances inflows, including those due to worsening economic conditions in sender countries.

Our results seem to indicate the potential for further research in this area, especially for developing countries that have not progressed enough in the "quality ladder", and where sectoral sophistication and interlinkages are limited. Our research contributes to the literature on remittances and economic development by considering the importance of input-output sectoral linkages, and highlights the potential for future research work in this area.

Figure 15: Sectoral Linkages and Total Output Growth (Weighted Outdegree)

Note: We show the relationship between total output growth and the weighted outdegree measure. An observation in this figure is a country-year in period 2011-15. Source: Authors' calculations.

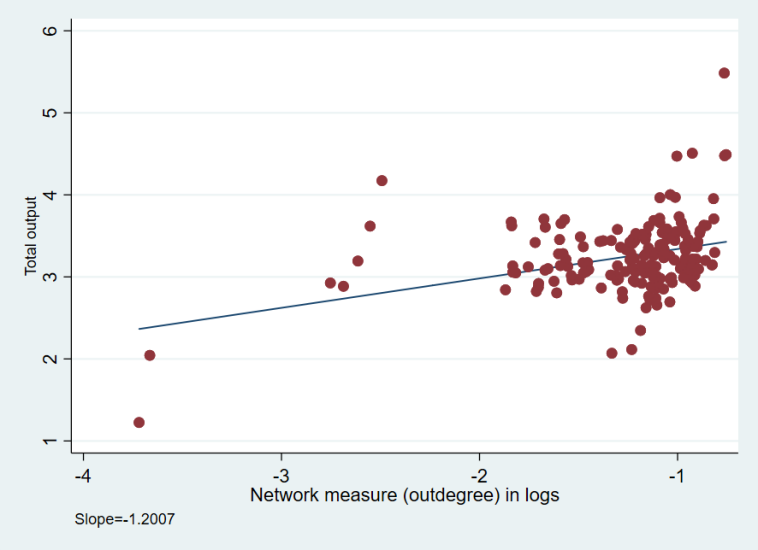
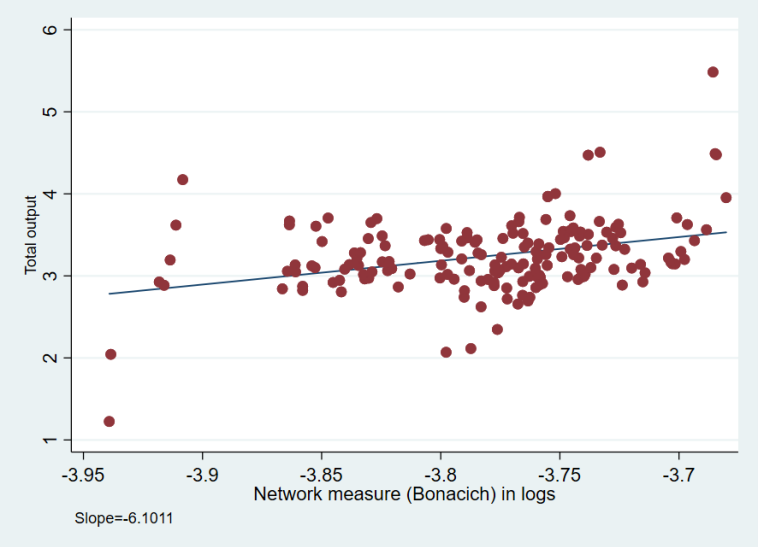



Figure 16: Sectoral Linkages and Total Output (Bonacich Centrality)

Note: We show the relationship between total output growth and the Bonacich centrality measure. An observation in this figure is a country-year for period 2011-15. Source: Authors' calculations.



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# A Proof of Proposition 1

We assume perfect competition in every industry. Profit maximization problem for producer of good $i$ is

$$\max_{x_{i1},...,x_{in},l_i} p_i y_i - \sum_{j=1}^{n} p_j x_{ij} - w l_i, \tag{16}$$

subject to

$$y_i = l_i^{\alpha_i^l} \prod_{j=1}^{n} x_{ij}^{a_{ij}}. \tag{17}$$

First order conditions for producer $i$ are

$$p_i \frac{a_{ij}}{x_{ij}} l_i^{\alpha_i^l} \prod_{j=1}^{n} x_{ij}^{a_{ij}} - p_j = 0 \quad \text{for} \quad j = 1, ..., n, \tag{18}$$

$$p_i \frac{\alpha_i^l}{l_i} l_i^{\alpha_i^l} \prod_{j=1}^{n} x_{ij}^{a_{ij}} - w = 0. \tag{19}$$

These first order conditions imply

$$p_i a_{ij} y_i = p_j x_{ij} \Leftrightarrow a_{ij} = \frac{p_j x_{ij}}{p_i y_i}, \tag{20}$$

$$\alpha_i^l = \frac{w l_i}{p_i y_i}. \tag{21}$$

Also, consumers maximize utility subject to the budget constraint, which gives the following first order conditions:

$$\gamma(l) \frac{\beta_i}{c_i} \prod_{i=1}^{n} c_i^{\beta_i} = \mu p_i, \tag{22}$$

$$\gamma'(l) \prod_{i=1}^{n} c_i^{\beta_i} = -\mu w. \tag{23}$$

These first order conditions imply

$$\frac{p_i c_i}{\beta_i} = \frac{p_j c_j}{\beta_j}. \tag{24}$$

We can arrange these first order conditions in Eq. 24 as follows:

$$\sum_{j=1}^{n} \beta_j \frac{p_i c_i}{\beta_i} = \sum_{j=1}^{n} p_j c_j \Leftrightarrow \frac{p_i c_i}{\beta_i} \sum_{j=1}^{n} \beta_j = \sum_{j=1}^{n} p_j c_j$$

$$\Leftrightarrow \frac{p_i c_i}{\beta_i} = \sum_{j=1}^{n} p_j c_j = wl + R \Leftrightarrow \frac{p_i c_i}{\beta_i} = wl + R \tag{25}$$

where the last expression is obtained from the budget constraint.

We can express labor supply from first order conditions as

$$\frac{\gamma(l)}{\gamma'(l)} \frac{\beta_i}{c_i} = -\frac{p_i}{w} \Leftrightarrow \frac{\gamma(l)}{\gamma'(l)} \beta_i = -\frac{p_i c_i}{w}, \tag{26}$$

$$\sum_{i=1}^{n} \frac{\gamma(l)}{\gamma'(l)} \beta_i = -\sum_{i=1}^{n} \frac{p_i c_i}{w} \Leftrightarrow \frac{\gamma(l)}{\gamma'(l)} \sum_{i=1}^{n} \beta_i = -\frac{1}{w} \sum_{i=1}^{n} p_i c_i \tag{27}$$

Which can be expressed as

$$\frac{\gamma(l)}{\gamma'(l)} = -\frac{wl + R}{w}. \tag{28}$$

If we assume that $\gamma(l) = (1-l)^{\lambda}$then we can solve for the labor supply:

$$\frac{(1-l)^{\lambda}}{-\lambda(1-l)^{\lambda-1}} = -\frac{wl + R}{w} \Leftrightarrow \frac{-1}{\lambda}(1-l) = -\frac{wl + R}{w} \Leftrightarrow l(1+\lambda) - 1 = -\lambda R, \tag{29}$$

$$l = \frac{1 - \lambda R}{1 + \lambda}. \tag{30}$$

Therefore, from Eq. 25 and the fact that $w = 1$ (labor is the numeraire), we have

$$p_i c_i = \beta_i (l + R),$$

$$pc_i = \beta_i \left( \frac{1 - \lambda R}{1 + \lambda} + R \right) = \beta_i \frac{1 + R}{1 + \lambda}. \tag{31}$$

From here, we can differentiate the expression. Let's assume consumer preferences depend on $R$ so we have$\beta_i(R)$.To express the differential term

$$d(p_i c_i) = \frac{(\beta_i + (1+R)\beta_i')dR}{1 + \lambda}. \tag{32}$$

By defining $d\hat{R}_i \equiv \frac{\beta_i + (1+R)\beta_i'}{1+\lambda} dR$ we can re-arrange the previous expression:

$$d(p_i c_i) = d\hat{R}_i. \tag{33}$$

From the market clearing condition in Eq. 5 and Eq. 32, we have

$$dy_i = dc_i + \sum_{j=1}^{n} dx_{ji} \tag{34}$$

Therefore

$$\frac{d(p_i y_i)}{p_i y_i} = \frac{d\hat{R}_i}{p_i y_i} + \sum_{j=1}^{n} a_{ji} \frac{d(p_j y_j)}{p_i y_i} \Leftrightarrow \tag{35}$$

$$\frac{d(p_i y_i)}{p_i y_i} = \frac{d\hat{R}_i}{p_i y_i} + \sum_{j=1}^{n} \hat{a}_{ji} \frac{d(p_j y_j)}{p_j y_j}. \tag{36}$$

where $\hat{a}_{ji} = \frac{a_{ji} p_j y_j}{p_i y_i}$ (or equivalently, $\hat{a}_{ij} = \frac{a_{ij} p_i y_i}{p_j y_j}$)

Since we have a CRS production function, prices are constant, and therefore

$$\frac{d(p_i y_i)}{p_i y_i} = \frac{dy_i}{y_i} = d\ln y_i. \tag{37}$$

Therefore, we can express Eq. (36) as

$$d\ln y_i = \frac{d\hat{R}_i}{p_i y_i} + \sum_{j=1}^{n} \hat{a}_{ji} d\ln y_j, \tag{38}$$

$$d\ln y = \hat{A}^T d\ln y + d\widetilde{R} \Leftrightarrow \tag{39}$$

$$d\ln y = (I - \hat{A}^T)^{-1} d\widetilde{R}, \tag{40}$$

where $d\widetilde{R}$ is the following vector:

$$d\widetilde{R} = \begin{pmatrix} \frac{\beta_1 + (1+R)\beta_1'}{(1+\lambda) p_1 y_1} dR \\ \frac{\beta_2 + (1+R)\beta_2'}{(1+\lambda) p_1 y_2} dR \\ \cdots \\ \frac{\beta_n + (1+R)\beta_n'}{(1+\lambda) p_n y_n} dR \end{pmatrix}. \tag{41}$$

Alternatively, we can express the growth expression in Eq. (40) in absolute terms. From Eq. (36), we have

$$d(p_i y_i) = p_i y_i \frac{d\hat{R}_i}{p_i y_i} + \sum_{j=1}^{n} \hat{a}_{ji} \frac{p_i y_i}{p_j y_j} d(p_j y_j) \tag{42}$$

$$\Leftrightarrow d(p_i y_i) = d\hat{R}_i + \sum_{j=1}^{n} \hat{a}_{ji} \frac{p_i y_i}{p_j y_j} d(p_j y_j) \tag{43}$$

Since, $\hat{a}_{ji} = \frac{a_{ji} p_j y_j}{p_i y_i}$ , this can be simplified to

$$d(p_i y_i) = d\hat{R}_i + \sum_{j=1}^{n} a_{ji} d(p_j y_j) \tag{44}$$

which can be expressed in matrix form as follows

$$d(py) = (I - A^T)^{-1} d\hat{R} \tag{45}$$

Note that since we have CRS in the production function, prices are constant and therefore output growth in nominal o real terms is the same.