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# IMF Working Paper

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## A Closed Form Multivariate Linear Filter

by Francis Vitek

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I N T E R N A T I O N A L M O N E T A R Y F U N D

## **IMF Working Paper**

Monetary and Capital Markets Department

### **A Closed Form Multivariate Linear Filter**

**Prepared by Francis Vitek<sup>1</sup>**

Authorized for distribution by Ulric Eriksson von Allmen

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### **Abstract**

This paper considers the problem of jointly decomposing a set of time series variables into cyclical and trend components, subject to sets of stochastic linear restrictions among these cyclical and trend components. We derive a closed form solution to an ordinary problem featuring homogeneous penalty term difference orders and static restrictions, as well as to a generalized problem featuring heterogeneous penalty term difference orders and dynamic restrictions. We use our Generalized Multivariate Linear Filter to jointly estimate potential output, the natural rate of unemployment and the natural rate of interest, conditional on selected equilibrium conditions from a calibrated New Keynesian model.

JEL Classification Numbers: C32; E32; E52

Keywords: Closed form; Multivariate linear filter; New Keynesian model; Potential output; Natural rate of unemployment; Natural rate of interest

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Contents	Page
I. Introduction .....	<a href="#">3</a>
II. The Multivariate Filters.....	<a href="#">4</a>
A. The Ordinary Multivariate Linear Filter .....	<a href="#">4</a>
B. The Generalized Multivariate Linear Filter .....	<a href="#">5</a>
III. The New Keynesian Model .....	<a href="#">6</a>
A. Households.....	<a href="#">7</a>
B. Firms.....	<a href="#">8</a>
C. Policy.....	<a href="#">10</a>
D. Equilibrium .....	<a href="#">10</a>
IV. Estimation of Potential Output and Natural Rates.....	<a href="#">12</a>
A. Restrictions.....	<a href="#">12</a>
B. Results .....	<a href="#">14</a>
Figure 1 . Estimated Trend Components from Multivariate versus Univariate Filters .....	<a href="#">15</a>
Figure 2 . Estimated Cyclical Components from Multivariate versus Univariate Filters.....	<a href="#">16</a>
C. Robustness.....	<a href="#">16</a>
Table 1 . Sensitivity of Estimation Results to Smoothing Parameter Perturbations.....	<a href="#">17</a>
Table 2 . Sensitivity of Estimation Results to Weight Parameter Perturbations .....	<a href="#">18</a>
Table 3 . Sensitivity of Estimation Results to Structural Parameter Perturbations .....	<a href="#">18</a>
V. Conclusion .....	<a href="#">18</a>
Appendix A . Proof of Proposition 1 .....	<a href="#">18</a>
Appendix B . Proof of Proposition 2 .....	<a href="#">20</a>
References.....	<a href="#">23</a>