I. INTRODUCTION

The purpose of this paper is to develop an index of financial inclusion that addresses the issue of weighting as well as that of perfect substitutability between dimensions. The paper uses factor analysis to identify financial inclusion dimensions and assign weights. The composite index is derived from a non-linear aggregation of intermediate dimensional indicators and is subsequently used to rank countries.

Financial inclusion has emerged as an important topic on the global agenda for sustainable long-term economic growth. A number of central banks both in emerging and developed countries have put in place various initiatives to promote financial inclusion in their countries. In addition to central bank’s initiatives, the IMF, G20, International Finance Corporation (IFC), the Alliance for Financial Inclusion (AFI), and the Consultative Group to Assist the Poor (CGAP) are assuming an increasingly active role at the international level in collecting the data and setting standards to improve financial inclusion.

This topic has also attracted a growing interest from the academic community. Burgess and Pande (2005), for example, find that the expansion of bank branches in rural India had a significant impact on alleviating poverty. Brune et al. (2011) conduct a field experiments in rural Malawi analyzing venues through which access to formal financial services improves the lives of the poor, with respect to saving products. Allen et al. (2013) explore determinants of financial development and inclusion among African countries.

While the importance of financial inclusion is well-established, a formal consensus on how it should be measured has yet to be reached. Different approaches have been proposed in the literature including the use of a variety of financial inclusion dimensions to econometric estimation. One of the first efforts at measuring financial sector outreach across countries was done by Beck et al. (2006). The authors designed new indicators of banking sector outreach for three types of banking services—deposits, loans, and payments—across three dimensions—physical access, affordability, and eligibility. This approach provides valuable information on particular aspects of financial inclusion, but combining these elements to evaluate overall progress accomplished by countries can be tricky. For example, in Beck et al. (2007), Albania ranks fourth in loan-income ratio but ranks 85th in bank branches per 100,000 adults. Such variation across dimensions makes it difficult to assess the state of financial inclusion in a country or across countries. Similarly, Honohan (2008) estimates the proportion of households having access to formal financial services for roughly 160 countries. Nevertheless, as Sarma (2012) puts it: “[the econometric estimates of this
approach] provide only a one-time measure of financial inclusion and are not useful for understanding the changes over time and across countries."²

In an attempt to overcome these shortcomings, Sarma (2008, 2010, and 2012) and Chakravarty and Pal (2010) have proposed composite indices of financial inclusion that incorporate various banking sector variables to reflect the level of accessibility, availability and usage of banking services. However, these indices assign equal weights to all variables and dimensions, which assumes that all dimensions have the same impact on financial inclusion.

The remainder of the paper is structured as follows: Section II discusses the definition of financial inclusion and its dimensions. Section III describes the variables used in the analysis. Section IV presents the methodology used to compute the index; Section V summarizes the main results of the index and the output of the index as it relates to country rankings. The final section of the paper concludes by suggesting some possible future extensions of the work and policy implications.

II. DEFINING FINANCIAL INCLUSION AND ITS DIMENSIONS

Financial inclusion can be broadly defined as an economic state where individuals and firms are not denied access to basic financial services based on motivations other than efficiency criteria. The 2014 Global Financial Development Report (World Bank, 2014) identifies four major forms of financial exclusion, which are classified into voluntary and involuntary exclusion.

Voluntary exclusion refers to the segment of the population or firms that choose not to use financial services either because they do not need those services due to the lack of promising projects or because of cultural or religious reasons. Since this type of exclusion is not a direct consequence of market failure, little can be done to address it. Of course, as pointed out in the aforementioned report, there is always room for improvement, by increasing, for example, financial literacy or encouraging the entry of specialized financial institutions that offer financial products tailored to meet cultural and religious requirements. From a macroeconomic viewpoint, this exclusion is driven by a lack of demand. Some individuals or firms may be involuntarily excluded from the financial system because they do not have sufficient income or, in the case of the credit markets, have an excessive lending risk profile. This type of involuntary exclusion is also not the result of market failure. A second category of involuntarily excluded entities consist of the segment of individuals and firms that are denied financial services as a result of government failures or market imperfections.

From a macroeconomic perspective, the main objective for building an inclusive financial system should be, in principle, the minimization of the percentage of individuals and firms in group 4 of Figure 1. In many developing economies, financial institutions are routinely faced

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3 See also Kempson and Whyley (1999a and 1999b).
with a number of barriers that lower their efficiency. For instance, because of various shortcomings in contract enforcement and a poor information environment, formal financial institutions in a number of developing economies are overcautious about extending loans to individuals or firms, especially small and medium enterprises (SMEs). Financial exclusion arising from incomplete/imperfect information may also arise in competitive markets. Stiglitz and Weiss (1981) demonstrate that, because of principal agent problems (moral hazard and adverse selection), individuals and firms in advanced economies may be excluded from the credit market even in equilibrium. Without complete information, and because beyond a certain interest rate level ($r^*$ in **Figure 2**) the rate of return of the loan may decrease, financial institutions may deny loans to additional applicants even if these applicants could afford a loan at higher interest rate ($r^{me}$ in **Figure 2**).

**Figure 2**: Credit rationing in a competitive market

Source: Adapted from Stiglitz and Weiss (1981).
Note: D = Demand; S = Supply
More recently, using a survey of low-income households conducted in Washington D.C., Los Angeles, and Chicago, Seidman et al. (2005) find that a significant number of individuals in those cities use informal non-bank services.

A stringent definition of financial inclusion should, therefore, theoretically be closely associated with the minimization of financial exclusion arising from market or government failures. However, distinguishing between the four categories of exclusion listed in Figure 1 is not straightforward. Information on each category may be obtained from user-side surveys, such as the World Bank’s Global Financial Inclusion (Global Findex) database. However, since survey-based data are costly to collect, there is no guarantee that such data can be made available to users with a reasonable frequency.

From a practical viewpoint, the concept of financial inclusion should be approached through its dimensions. There is a consensus, at least from a policy maker’s perspective, that financial inclusion encompasses three main dimensions, namely the outreach, usage, and quality of financial services. The outreach dimension refers to the (physical) ability to easily reach a point of service. According to the World Bank’s Global Findex survey, of the 2.5 billion of individuals excluded from financial systems worldwide, 20 percent cite the distance to a point of financial service as being the main reason for not having an account with a formal financial institution. The shortage of physical points of financial services affects mostly the populations who live in rural areas, but in a number of countries this is the case for individuals living in urban areas as well. The usage dimension measures the use of financial services, while the quality dimension measures the extent to which financial services address the needs of the consumers.

In light of the above discussion, we define financial inclusion in this paper as the optimal combination of its dimensions. The main challenge with this definition is that the data may not be readily available for some dimensions. The dimensions considered in this paper are those for which the data are reported to the IMF.

III. VARIABLES SELECTION

A number of variables could be theoretically relevant for inclusion in each of the three dimensions of financial inclusion. However, because the data for a number of these variables are usually not readily available, we use their proxies to measure each dimension.

The outreach dimension is usually defined using geographic or demographic penetration indicators. Proxies for these indicators are the number of automatic teller machines (ATMs)
and financial institutions’ branches rescaled by land mass (number of ATMs and branches per 1,000 km square) or adult population (number of ATMs and branches per 100,000 adults). The IMF disseminates the data on the number of ATMs and branches in terms of both land mass and adult population. The raw data for the number of ATMs and branches are collected from the financial service providers through the IMF’s Financial Access Survey (FAS) while land mass and adult population data used to rescale the raw data are extracted from the World Bank’s World Development Indicators (WDI) dataset. We use the geographic penetration indicators—ATMs and branches per land mass—as variables for the outreach dimension, because the physical distance to physical points of service tends to be an important barrier to financial inclusion.\(^7\) ATMs and branches refer to physical points of financial service offered by other depository corporations\(^8\) (ODCs) in a given country—that is, financial intermediaries (central bank excluded) that collect deposits included in broad money or issue liabilities that are close substitute of deposits and are included in broad money.

Typical indicators of the usage dimension are the percentage of adults with at least one type of regulated deposit account and the percentage of adults with at least one type of regulated loan account. Proxies to these two indicators are the number of regulated deposit accounts per 1,000 adults, number of regulated loan accounts per 1,000 adults, number of household borrowers per 1,000 adults, and the number of household depositors per 1,000 adults. We use the last two indicators as proxies of the usage dimension variables.\(^9\) The data for these variables are also disseminated by the IMF through its FAS website.\(^10\) Household depositors refer to households with at least one deposit account. Deposits include all types of deposits: transferable deposits, sight deposits, savings deposits, and fixed-term deposits. Also included are liabilities of money-market funds in the form of shares or similar evidence of deposit that are, legally or in practice, redeemable immediately or at relatively short notice. For the purpose of the present analysis, deposits that have restrictions on third-party transferability are also included in this category even though they are excluded from broad money. Household borrowers refer to households who have at least one loan account. Loans are financial assets that are created when a creditor lends funds directly to a debtor and are evidenced by non-negotiable documents. These include mortgage loans, consumer loans, hire-purchase credit, financial leases, securities repurchase agreements, etc.

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\(^7\) Data on the number of mobile banking service providers and mobile agents could also be included in the outreach dimension. However, comparable data do not exist at present.

\(^8\) The ODC sector includes commercial banks, credit unions, saving and credit cooperatives, deposit taking microfinance (MFIs), and other deposit takers (savings and loan associations, building societies, rural banks and agricultural banks, post office giro institutions, post office savings banks, savings banks, and money market funds).

\(^9\) We exclude the variables on the number of accounts because they could potentially introduce a bias in the dataset. In cases where an individual has multiple deposit or loan accounts, the use of formal financial services in a country would be overstated.

A variety of indicators are used to theoretically characterize the quality dimension. These indicators are classified in various sub-categories that include financial literacy, disclosure requirements, dispute resolution, and the cost of usage. Because the data on the quality dimension are rather scarce, this dimension is not considered in the computation of the proposed index. Table 1 below summarizes the final list of variables used to compute the index.

Table 1: List of Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of ATMs per 1,000 square kilometers</td>
<td>Sum of all ATMs multiplied by 1,000 and divided by total area of the country in square kilometers.</td>
</tr>
<tr>
<td>Number of branches of ODCs per 1,000 square kilometers</td>
<td>Sum of all branches of commercial banks, credit unions &amp; financial cooperatives, deposit-taking microfinance institutions and other deposit takers multiplied by 1,000 and divided by total area of the country in square kilometers.</td>
</tr>
<tr>
<td>Total number of resident household depositors with ODCs per 1,000 adults</td>
<td>Sum of all household depositors with commercial banks, credit unions &amp; financial cooperatives, deposit-taking microfinance institutions and other deposit takers multiplied by 1,000 then divided by the adult population.</td>
</tr>
<tr>
<td>Total number of resident household borrowers with ODCs per 1,000 adults</td>
<td>Sum of all household borrowers from commercial banks, credit unions &amp; financial cooperatives, deposit-taking microfinance institutions and other deposit takers multiplied by 1,000 then divided by the total adult population.</td>
</tr>
</tbody>
</table>

The size of the sample is relatively small for each year, as few countries are reporting the data for the four variables simultaneously. When all four variables are taken together, data are available for 23 countries in 2009, 26 countries in 2010, 28 countries for 2011, and 31 countries for 2012. However, as underlined in Section V, even with a small sample, the computed index casts interesting results with respect to financial inclusion.

IV. COMPUTATION OF THE INDEX

We derive the composite index by aggregating intermediate sub-indices pertaining to different dimensions. The multidimensional approach is generally implemented following a three-step sequence that consists of: (i) normalization of variables; (ii) determination of dimensional sub-indices; and (iii) aggregation of sub-indices. Most popular composite indices of well-being constructed by the United Nations Development Programme (UNDP) such as the Human Development Index (HDI), Human Poverty Index (HPI), and Gender-

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11 The concept of residency used in this paper is taken from the sixth edition of the Balance of Payments and International Investment Position Manual (http://www.imf.org/external/pubs/ft/bop/2007/pdf/bpm6.pdf). According to that definition, an institutional unit is said to be a resident of a given economy if it has a center of economic interest in that economy.
related Development Index (GDI) follow this basic sequence. Similarly, other indices of financial inclusion, such as those proposed by Sarma (2008 and 2012) and Chakravarty and Pal (2010), are based on this three-step sequence. We follow a five-step sequence to compute the index. First, like the UNDP’s approach, the variables are normalized so that the scale in which they are measured is irrelevant. Then, using factor analysis (FA) we introduce a statistical identification of financial inclusion dimensions in order to ascertain whether the statistical groups obtained from FA are the same as the theoretical dimensions. We show that such is the case. With the statistical dimensions matching the theoretical ones, we then use in the third step the statistical properties of the dataset to assign weights to both individual variables and sub-indices. Finally, unlike the UNDP’s indices which are computed using the simple geometric mean, the outcomes of the second and third steps allow us to choose in the fourth and fifth steps a weighted geometric average as the functional form of the aggregator for the computation of the dimension and composite indices, respectively.

A. Normalization of variables

Aggregation over variables that are expressed in different measurement units and have varying ranges requires normalization. Normalization is meant to address the lack of scale invariance. Various normalization approaches have been proposed in the literature. A comprehensive review of the different approaches may be found in Freudenberg (2003), Jacobs et al. (2004), and OECD (2008), among others. In more practical terms, however, the most common methods are the standardization, the min-max, and the distance to a reference. We use the distance to a reference method in this paper. The distance to a reference measures the relative position of a given variable with respect to its reference point. The reference point is usually a target to be reached in a given time frame or the value of the variable in a reference country. We define the reference point for each variable to be the maximum value of the variable across countries. This means that, for a given variable, the benchmark country is the group leader. The normalized variable is therefore bounded between 0 and 1 where a score of 1 is attributed to the leading country and the others countries are given percentage points away from the leader. If $x_{ic}$ is the raw value of variable $i$ for country $c$, and $M_i$ the maximum value of the variable across countries, then the normalized value $nx_{ic}$ of $x_{ic}$ is given by:

$$nx_{ic} = \frac{x_{ic}}{M_i} \tag{1}$$

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12 See UNDP (2010) for the computation of the HDI for example.
13 This method is chosen mainly because it is consistent with nonlinear aggregators that require prior transformation of raw variables using a logarithmic function.
14 The United States and Japan are often used as external benchmark countries.
The choice of the maximum value across countries for each variable is mainly motivated by the fact that countries with more inclusive financial systems tend to also have higher values for all variables considered in this paper. The World Bank’s Findex surveyed the users of financial services in 148 countries in 2011. The survey confirmed an important gap of financial inclusion performance between the advanced economies and developing countries, the former group having more inclusive financial systems than the latter.

In addition, this normalization method satisfies most of the required technical properties, including the scale invariance property which is provided by the fact that the image set of the normalizer is a sub-set of the unit interval.\textsuperscript{15} As indicated previously, it is also consistent with nonlinear aggregators that require prior transformation of raw variables using a logarithmic function.\textsuperscript{16}

**B. Statistical identification of dimensions**

The classification of variables in the relevant dimensions is needed to ensure proper allocation of the weights between dimensions. When a composite index is computed using a variety of variables, some variables that appear to be \textit{ex ante} good candidates for inclusion into a specific dimension may possess attributes of other dimensions, thereby making it difficult to assign the weights adequately. Hence, there is a need for a clear criterion to determine the relevant variables in each dimension. The index proposed in this paper is computed using four variables. From the theoretical perspective the outreach variables are clearly distinguishable from the usage variables. Hence, the goal in this section is to ensure that this theoretical taxonomy is confirmed statistically.

We use FA to group the variables into the relevant dimensions. FA posits that each observed variable of the dataset is a combination of unobserved factors. Coefficients that relate the observed variables to common factors are called factor loadings. Variables with high factor loadings have a high affinity with the latent variable. Following Berlage and Terweduwé (1988) and Nicoletti et al. (2000), we group variables that share higher affinity with a specific factor into the same dimension, that is, variables are included in the dimension for which they have the highest factor loading.\textsuperscript{17}

The basic form of an FA model is as follows:

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\textsuperscript{15} A useful discussion about the technical properties that normalizers should meet is provided in Chakravarty and Pal (2010).

\textsuperscript{16} A logarithmic transformation cannot be used with the standardization approach because countries with values below the average have negative normalized variables. Similarly, a logarithmic transformation applied to min-max normalized variables would require truncating the series by excluding countries where the minimum is attained.

\textsuperscript{17} We estimate the factors loading using the principal components analysis method and rotate the axes using the varimax technique.
Let $\hat{X}$ be the vector of our 4 observed random variables described in section III ($\mathbb{E}(\hat{X}) = \bar{\mu}$), $\bar{F}$ the vector of $m$ unobservable random variables called the common factors of $\hat{X}$, $\bar{\epsilon}$ the vector of specific factors of $\hat{X}$. Working with centered variables $\bar{Y} = \hat{X} - \bar{\mu}$ our $m$-factor model is given by equation 2 below:

$$\bar{Y} = L\bar{F} + \bar{\epsilon} \tag{2}$$

where the covariance of $\hat{X}$ is $\text{Cov}(\hat{X}) = \Sigma$, $L = (l_{ij})_{1 \leq i \leq 4}^{1 \leq j \leq m}$ is the matrix of factor loadings, and $l_{ij}$ the loading of the $i^{th}$ variable $Y_i$ on the $j^{th}$ common factor $F_j$.

We make the traditional assumptions of FA models that: $\mathbb{E}(\bar{F}) = \bar{O}_m$, $\text{Cov}(\bar{F}) = I_m$, $\text{Cov}(\bar{\epsilon}, \bar{F}) = O_{4 \times m}$, $\mathbb{E}(\bar{\epsilon}) = \bar{O}_4$, and $\text{Cov}(\bar{\epsilon}) = \Psi$.

These assumptions provide the following results that we use for the identification of financial inclusion dimensions and the derivation of the weights assigned to variables and dimensions:

$$\Sigma = LL + \Psi \tag{3}$$

$$L = \text{Cov}(\bar{Y}, \bar{F}) \tag{4}$$

$$\text{Var}(Y_i) = \sum_{j=1}^{m} l_{ij}^2 + \psi_i \tag{5}$$

where $\sum_{j=1}^{m} l_{ij}^2$ is the $i^{th}$ commonality, that is, the portion of the variance of $Y_i$ explained by the common factors and $\psi_i$ the specific variances. The contribution of the first factor to $\text{Var}(Y_i)$ is $l_{i1}^2$.

The dimension of $X_i$ is $j_0$ such that $\max_{1 \leq j \leq m}(l_{ij}) = l_{ij_0}$.

Since FA requires that the variables be correlated, we investigate associations among variables.\textsuperscript{18} The correlation structure of the dataset is assessed through multivariate tests of the covariance matrix of the data. First, we test if the covariance matrix is diagonal, and then add a spherical restriction using the Bartlett’s spherical test whose null hypothesis is that the covariance is the identity matrix.

\textsuperscript{18} From equation (3), it is indeed unlikely that variables that are not correlated would share common factors.
All these tests reject the null hypothesis. We conclude, therefore, that the dataset considered in this paper satisfies the required conditions for the use of FA.

All main criteria for selecting the optimal number of factors suggest that two factors should be considered each year. Grouping subsequently the variables according to their factor loadings we obtain the components of each dimension. As shown in Table 3 below, the delineation between the two theoretical dimensions is confirmed by FA. The variables included in each dimension are exactly those mentioned in the literature.

---

### Table 2: Multivariate tests of the covariance matrix

<table>
<thead>
<tr>
<th>Year</th>
<th>Null</th>
<th>LR chi2</th>
<th>Degree of freedom</th>
<th>Prob &gt; chi2</th>
</tr>
</thead>
<tbody>
<tr>
<td>2009</td>
<td>Covariance matrix is diagonal</td>
<td>67.92</td>
<td>6</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>Covariance matrix is spherical</td>
<td>72.24</td>
<td>9</td>
<td>0.00</td>
</tr>
<tr>
<td>2010</td>
<td>Covariance matrix is diagonal</td>
<td>84.25</td>
<td>6</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>Covariance matrix is spherical</td>
<td>91.44</td>
<td>9</td>
<td>0.00</td>
</tr>
<tr>
<td>2011</td>
<td>Covariance matrix is diagonal</td>
<td>66.33</td>
<td>6</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>Covariance matrix is spherical</td>
<td>70.93</td>
<td>9</td>
<td>0.00</td>
</tr>
<tr>
<td>2012</td>
<td>Covariance matrix is diagonal</td>
<td>83.95</td>
<td>6</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>Covariance matrix is spherical</td>
<td>87.34</td>
<td>9</td>
<td>0.00</td>
</tr>
</tbody>
</table>

---

19 These criteria are: the Kaiser criterion of dropping all factors with eigenvalues below 1, Joliffe, percentage of variance explained, and scree plot.
Table 3: Rotated factor loadings in 2012

<table>
<thead>
<tr>
<th>Variables</th>
<th>Factor 1</th>
<th>Factor 2</th>
<th>Uniqueness</th>
</tr>
</thead>
<tbody>
<tr>
<td># of resident household depositors with ODCs per 1000 adults</td>
<td>0.0772</td>
<td><strong>0.9465</strong></td>
<td>0.0982</td>
</tr>
<tr>
<td># of resident household borrowers from ODCs per 1000 adults</td>
<td>0.0449</td>
<td><strong>0.9466</strong></td>
<td>0.1019</td>
</tr>
<tr>
<td># of branches of ODCs per 1000 km square</td>
<td><strong>0.9811</strong></td>
<td>0.0418</td>
<td>0.0357</td>
</tr>
<tr>
<td># of ATMs per 1000 km square</td>
<td><strong>0.9667</strong></td>
<td>0.1683</td>
<td>0.072</td>
</tr>
</tbody>
</table>

Factor loadings in 2011

<table>
<thead>
<tr>
<th>Variables</th>
<th>Factor 1</th>
<th>Factor 2</th>
<th>Uniqueness</th>
</tr>
</thead>
<tbody>
<tr>
<td># of resident household depositors with ODCs per 1000 adults</td>
<td>0.0784</td>
<td>0.9291</td>
<td>0.1306</td>
</tr>
<tr>
<td># of resident household borrowers from ODCs per 1000 adults</td>
<td>0.0269</td>
<td>0.9329</td>
<td>0.1290</td>
</tr>
<tr>
<td># of branches of ODCs per 1000 km square</td>
<td>0.9786</td>
<td>0.0588</td>
<td>0.0389</td>
</tr>
<tr>
<td># of ATMs per 1000 km square</td>
<td>0.9649</td>
<td>0.1673</td>
<td>0.0410</td>
</tr>
</tbody>
</table>

Factor loadings in 2010

<table>
<thead>
<tr>
<th>Variables</th>
<th>Factor 1</th>
<th>Factor 2</th>
<th>Uniqueness</th>
</tr>
</thead>
<tbody>
<tr>
<td># of resident household depositors with ODCs per 1000 adults</td>
<td>-0.0101</td>
<td><strong>0.9530</strong></td>
<td>0.0917</td>
</tr>
<tr>
<td># of resident household borrowers from ODCs per 1000 adults</td>
<td>0.1117</td>
<td><strong>0.9410</strong></td>
<td>0.1020</td>
</tr>
<tr>
<td># of branches of ODCs per 1000 km square</td>
<td><strong>0.9886</strong></td>
<td>-0.0684</td>
<td>0.0180</td>
</tr>
<tr>
<td># of ATMs per 1000 km square</td>
<td><strong>0.9736</strong></td>
<td>0.1725</td>
<td>0.0224</td>
</tr>
</tbody>
</table>

Factor loadings in 2009

<table>
<thead>
<tr>
<th>Variables</th>
<th>Factor 1</th>
<th>Factor 2</th>
<th>Uniqueness</th>
</tr>
</thead>
<tbody>
<tr>
<td># of resident household depositors with ODCs per 1000 adults</td>
<td>-0.0138</td>
<td><strong>0.9361</strong></td>
<td>0.1236</td>
</tr>
<tr>
<td># of resident household borrowers from ODCs per 1000 adults</td>
<td>0.1074</td>
<td><strong>0.9217</strong></td>
<td>0.1390</td>
</tr>
<tr>
<td># of branches of ODCs per 1000 km square</td>
<td><strong>0.9879</strong></td>
<td>-0.0757</td>
<td>0.0183</td>
</tr>
<tr>
<td># of ATMs per 1000 km square</td>
<td><strong>0.9732</strong></td>
<td>0.1699</td>
<td>0.0240</td>
</tr>
</tbody>
</table>

C. Weights assignment

Assigning weights to variables and dimensions is not a straightforward task. Because of the complexity surrounding the allocation of weights, a number of papers that have attempted to calculate composite indices assign equal weights to all variables and dimensions. This is the case not only for most of the UNDP’s indices but also for the composite indices proposed by Sarma (2008) as well as Chakravarty and Pal (2010). Assigning equal weights to all variables and dimensions leads to the consideration that all individual variables contribute equally to the index. As a result, each normalized variable is implicitly considered as constituting a specific dimension.

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20 In the updated version of her 2008 paper, Sarma (2012) assigns weights to dimensions, yet the weights appear to have been derived arbitrarily.
We use the properties of our FA model to derive the weighting scheme. Since the variables are grouped into the relevant dimensions based on the way they load on the corresponding factor, it is legitimate to consider the proportion of the variance explained by the corresponding factor to the total variance to be the weight of the variable in the corresponding dimension. The corresponding variance is the squared factor loading. The derived weights are given in Table 4 and Table 5.\(^{21}\)

**Table 4: Weights assigned to variables**

<table>
<thead>
<tr>
<th>Year</th>
<th>Dimension 1</th>
<th>Dimension 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number of ODC branches per 1,000 km²</td>
<td>Number of ATMs per 1,000 km²</td>
</tr>
<tr>
<td>2009</td>
<td>51%</td>
<td>49%</td>
</tr>
<tr>
<td>2010</td>
<td>51%</td>
<td>49%</td>
</tr>
<tr>
<td>2011</td>
<td>51%</td>
<td>49%</td>
</tr>
<tr>
<td>2012</td>
<td>51%</td>
<td>49%</td>
</tr>
</tbody>
</table>

**Table 5: Weights assigned to dimensions**

<table>
<thead>
<tr>
<th>Year</th>
<th>Dimension 1</th>
<th>Dimension 2</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>2009</td>
<td>52%</td>
<td>48%</td>
<td>100%</td>
</tr>
<tr>
<td>2010</td>
<td>51%</td>
<td>49%</td>
<td>100%</td>
</tr>
<tr>
<td>2011</td>
<td>51%</td>
<td>49%</td>
<td>100%</td>
</tr>
<tr>
<td>2012</td>
<td>51%</td>
<td>49%</td>
<td>100%</td>
</tr>
</tbody>
</table>

**D. Functional form of the aggregator**

With the statistical dimension identification and a clear weighting scheme in place, we are now in a position to clarify the functional form of our aggregator. As stated before, our aggregator is the weighted geometric mean. We use it to calculate both the intermediate dimensional variables and the cross-dimension composite index. The reason for choosing the weighted geometric mean is that it addresses in a satisfactory manner the issue of perfect substitutability between variables within a dimension and/or between dimensions. This was the main drawback of the versions of the HDI prior to 2010 that used the arithmetic mean. In general, using a linear operator (as in previous versions of the HDI) implies considering the variables as perfect substitutes of each other. This is the case because the elasticity of substitution between variables or dimensions is equal to infinity. Perfect substitutability is not a relevant assumption in the particular case of financial inclusion. In fact, although some kind of compensation is possible between variables, it is not in general true that the

\(^{21}\) As the size of the sample expands, the weights are likely to further differentiate over time.
compensation would be in the same proportion.\textsuperscript{22} Thus, the use of a non-linear function is critical for addressing the issue of perfect substitutability. However, since we recognize that different combinations of variables pertaining to different dimensions may lead to the same level of financial inclusion, we also need a non-linear function for which the elasticity of substitution is not null. We must therefore avoid the extreme situations of both linear aggregator (because of perfect substitutability) and non-substitutability (arising from the use of a Leontief function, for example). The best aggregator will therefore provide an elasticity of substitution, which is a non-null real number. It is easy to see that our weighted geometric aggregator \( A \), which is given by equation (6) below, satisfies the required property.

Additionally, our aggregator preserves the scale invariance property of the variable in the sense that multiplying any component of the index by a scalar does not change the relative weight of the variable.

The explicit formula of our aggregator is:

\[
A = \exp \left( \frac{\sum_{i=1}^{N} w_i \log x_i}{\sum_{i=1}^{N} w_i} \right)
\]  

(6)

where \( w_i \) is the weight associated with variable \( i \).

For any \( x_{i0} \), the partial derivative of \( A \) with respect to \( x_{i0} \) is:

\[
\frac{\partial A}{\partial x_{i0}} = w_{i0} \exp \left( \frac{\sum_{i=1}^{N} w_i \log x_i}{B} \right) \frac{B x_{i0}}{B x_{i0}}
\]

Where \( B = \sum_{i=1}^{N} w_i \)

and the marginal rate of technical substitution between \( x_{i1} \) and \( x_{i0} \) is:

\[
MRTS_{x_{i1},x_{i0}} = \frac{w_{i1} x_{i0}}{w_{i0} x_{i1}}
\]

Therefore, the elasticity of substitution between \( x_{i0} \) and \( x_{i1} \) is \( \sigma = 1 \).\textsuperscript{23}

\textsuperscript{22} For the geographic outreach dimension, for example, it might be relevant that a country that has a good geographic branch penetration may compensate with somehow insufficient geographic ATM penetration.

\textsuperscript{23} The elasticity of substitution between \( x_{i0} \) and \( x_{i1} \) is the percent change in the ratio of the two variables to the percent change in \( MRTS_{x_{i1},x_{i0}} \).
In the case of the composite index where $x_i$ is the sub-index associated with dimension $i$ that is $I_i$, the isoquant from this aggregator ($\sigma = 1$) is shown in Figure 3 below and is located between the linear case ($\sigma = \infty$) and the Leontief aggregator ($\sigma = 0$).

**Figure 3: Isoquants from linear and non-linear aggregators**

V. RESULTS

The index is computed for the period from 2009 to 2012. Despite the limited size of the sample, some interesting lessons can be drawn from both dimensional and the composite index. In general, country rankings relative to one another remain stable over the observed periods. The change in the composition in rankings results largely from changes in the underlying sample. In some cases, however, countries rise and fall in the rankings due to changes in the magnitude their underlying variables. A more detailed summary of the results is presented in the Appendix.

A. Dimension 1: Outreach of financial services

The rankings of the first dimension indicate an increasing polarization of countries over time. For example, in 2009, high and upper middle income countries accounted for half of the top

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24 As the size of the sample expands, our results are likely to differentiate significantly from those generated from a non-weighted geometric mean.
ten. By 2012, these groups accounted for eight of the top ten. It is noteworthy that upper middle income countries consistently outperform high income countries in the sample. Mauritius and the Maldives in particular perform significantly better than all others in the sample for this dimension, ranking one and two in every year of the sample where their data are available. To illustrate, these two countries have an index of .99 and .94 in 2012 while the third ranked country in the sample, West Bank & Gaza, has an index of .35. Such rankings could indicate that geographically small, densely-populated countries fare best in terms of financial outreach.\footnote{Allen et al. (2013) argue that population density is more strongly associated with financial development and inclusion in Africa than in other developing countries. In their analysis, small, densely populated African countries such as Cape Verde, Comoros and Mauritius come on the top as countries with the highest levels of financial depth and inclusion on the continent. The authors, nevertheless, acknowledge these countries are not representative of the overall African experience.} The top of the rankings also contain countries from diverse regions of the world, regardless of the period. For example, in 2012 every region is represented in the top six countries: Mauritius, Maldives, West Bank & Gaza, Hungary, Thailand, and Dominican Republic. Figure 4 provides a snapshot of the average index values for the first dimension by income group for 2009-12.

The lowest ranked countries in the first dimension follow a similar pattern in terms of country income. In 2009, six of the lowest ten ranked countries are low or lower middle income. By 2012, the concentration increased to eight of ten. The regional diversification at the bottom of the rankings does not follow that of the top of the list. African and Middle East & Central Asian countries account for nearly all countries in the bottom ten for each year. In 2009, these regions combined to account for nine of the lowest ten ranked countries. In 2012, all countries in the bottom ten fell into one of these regions. The indices of the bottom two countries were significantly lower than that of the Republic of Congo, the country third from the bottom. In 2012, the indices for Central African Republic and Chad were 150 and 170 percent lower than the Republic of Congo.
B. Dimension 2: Use of financial services

The second dimension measures use of financial services by households by combining the variables for household depositors with ODCs and household borrowers from ODCs per 1,000 adults. Again, the data provide rankings for a four-year span (2009-12). The number of countries in the sample is consistent with the first dimension. A higher ranking in this dimension indicates that a higher proportion of the population makes use of the formal financial services for a given country relative to other countries in the sample.

In terms of income groups, the top of the rankings displays the same polarization as the first dimension. In 2009, seven of the top ten countries were in the high or upper middle income groups. In 2011 and 2012, these groups accounted for nine of the top ten. Unlike the first dimension, high income countries have a significantly higher average index than countries in the upper middle income group. The top ten countries appear to be more mixed in regards to

26 Relative changes in financial inclusion may be assessed over time provided countries report data for the same years.
geographic area and population relative to the first dimension. For example, Brunei Darussalam and Thailand consistently rank in the top three, despite their disparity in terms of size and population. Brunei in particular performs well in this dimension, with an index over 30 percent higher than Maldives in 2012. Regionally, the top ten also follows a similar pattern to that of the first dimension, with a wide range of regions represented. In 2012 for instance, countries from four regions are represented in top five: Brunei Darussalam, Estonia, Thailand, Hungary, and Georgia. The African region, however, is notably absent from the top of the list, regardless of the period. In fact, Botswana and Mauritius are the only two African countries to reach the top ten in any year of the sample. Figure 5 below provides an overview of the average index values for the second dimension by income group for 2009-12.

The lowest ranked countries again follow the trends of the first dimension, with low and lower middle income countries concentrated at the bottom. In 2011 and 2012, eight of the lowest ten countries fall into one of these groups. The absence of African countries at the top of the rankings for this dimension results in a greater concentration of countries from this region at the bottom of the list. In 2009, six of the bottom ten countries are from the African region. The concentration increases to eight of ten in 2012. The indices of the bottom three countries display the same significant decline as the first dimension, particularly for more recent periods.

Figure 5: Dimension 2: Use of Financial Services by Income Group, Year
C. Composite index

For 2009, the weights of dimension one and dimension two are .52 and .48, respectively. In each subsequent year, the difference narrows to .51 and .49. The even weighting for each dimension results in a composite index that largely follows the trends of the individual dimensions. By combining the two dimensions, the output of the composite index should be a ranking of countries in the sample from the most financially inclusive to the least. Countries at the top of the rankings should be more financially inclusive relative to countries at the bottom of the rankings.

The highest ranked countries show an increased presence of countries from the high and upper-middle income groups over time. In 2009, seven of ten fell into one of these groups, while in 2012, the concentration increased to eight of ten. As a result of the even weighting, the average index for high and upper middle income countries are nearly even over time. Regionally, the top ranked countries display nearly the same diversity as the first dimension. Unlike the first dimension, however, the top three countries in the composite index for 2011 and 2012 are in the Asia and Pacific region (Maldives, Thailand, and Brunei Darussalam). The top of the list does not show the same wide differences as the individual dimensions. For example, the index for the Maldives is 17 percent higher than Thailand in 2011 and 2012.

Figure 6 displays the results of the average composite index values by income group for 2009-12.

The countries ranked at the bottom of the list again display many of the trends of the individual dimensions. By way of illustration, six of the bottom ten in the composite index are low or lower middle income countries for 2010-12. Similarly, eight of ten are from the African region, an increase from six of ten in 2011. As was the case with the individual dimensions, the index rapidly declines toward the bottom, particularly in recent years. In 2012, the composite index for Central African Republic and Chad were 95 percent and 160 percent lower than that of the Republic of Congo, the country ranked third from the bottom. A summary of results of the index is provided in the Appendix.
VI. CONCLUDING REMARKS

In this paper we have presented a new index of financial inclusion that addresses many of the persistent criticisms of similar indices, namely the lack of an adequate weighting scheme for variables and dimensions and the inability of certain aggregators to capture imperfect substitutability between dimensions. The use of factor analysis method makes it possible to be less arbitrary in the identification of financial inclusion dimensions, thereby permitting proper weight assignment, while the weighted geometric mean is an appropriate aggregator of imperfect substitutes.

Our index is easy to compute and can be used not only to assess the state of financial inclusion in a country, region, or income group, but also, at the operational level, as a meaningful tool for checking the quality of financial inclusion data. Since the IMF collects the data used to generate the index on an ongoing (annual) basis, the results could be replicated to provide a more dynamic picture of the state of financial inclusion on a national or global level on a regular basis. The index could also become part of the regular toolkit for the IMF’s bilateral and multilateral surveillance work, as well as financial sector surveillance activities.

The index presents several possible avenues for further research. For example, the household depositors and borrowers variables could be replaced with the corresponding FAS variables
on SMEs. In addition, the household and SME indices could be combined to create an aggregated index. Another area of possible research would be expanding the coverage of the index to include other types of financial institutions, notably insurance corporations. Finally, should adequate data on the quality dimension become available, the inclusion of these data into the index as a possible third dimension could be explored.