

I. Introduction

Following the receipt of debt relief poor countries face a classic time consistency problem: they can either constrain their absorption and keep the debt-to-GDP ratios at the post-relief level or start borrowing again, possibly in excess of prudential levels. We argue that the recurrent availability of debt-relief schemes, like the Heavily Indebted Poor Countries (HIPC) Initiative and the Multilateral Debt Relief Initiative (MDRI), provide incentives for the latter option.¹ The prospect of future debt relief motivates indebted countries to contract more debt, increase consumption, and lower investment. In doing so, these countries are driven by the past behavior of donors who have granted debt relief to countries whose debts have exceeded some arbitrary levels. While donor surveillance of poor-country economic programs prevents some of the excessive debt accumulation trajectories, it is unlikely to eliminate the dilemma completely.

After the oil and commodity price shocks of the 1970s and 1980s, most low-income countries closed their external financing gaps through borrowing and their debt-to-GDP ratios quickly increased to the point where they could not service these loans. Arrears to external lenders became widespread, and bilateral official lenders started to offer increasingly generous refinancing schemes in the context of the Paris Club.² These offers were, however, piecemeal and debt continued to increase until the mid 1990s. By the early 1990s the international community started to call for a coordinated effort between bilateral and multilateral creditors to grant debt relief to those countries that were committed to pursue sustainable macroeconomic policies under the IMF-supported adjustment programs. The underlying idea was simple. First, countries with a track record of responsible macroeconomic policies will have their slate wiped clean off debts that were by now clearly unserviceable. Second, looking forward, leaders of the newly debt-free countries would spurn the excessive borrowing of their predecessors and increase investment to encourage growth and reduce poverty.

The 1996 HIPC initiative succeeded in bringing the average debt-to-GDP ratio for countries at the completion point to 26 percent, significantly below the 1996 peak of 128 percent, thus fulfilling the first objective (Figure 1). Regarding the second objective of debt sustainability, will HIPCs remain “debt free” or will they be tempted to accumulate debt again? On the one hand, to limit moral hazard, the HIPC initiative contained a sunset clause, making the initiative a one-off event and sending a signal that HIPC eligibility would not be unlimited (International Monetary Fund, 2004). On the other hand, the sunset clause was extended four times as progress under the initiative was slower than anticipated and HIPC eligibility was gradually extended. In early 2010 the international financial institutions did not foresee any systemic debt difficulties in low-income countries, however, these statements could be hardly construed as a firm pre-commitment of no future debt relief.³

¹The HIPC initiative is a comprehensive approach to debt reduction for poor countries with unmanageable debt burdens. The MDRI provides relief to selected low-income countries to help them reach the Millennium Development Goals. For a description of the initiatives see <http://www.imf.org/external/np/exr/facts/hipc.htm>.

²The Paris Club is an informal group of nineteen official creditors devoted to assist debtor nations to sort out debt payment problems. A detailed chronology of the relief mechanisms is available at the Paris Club Annual Reports 2007, 2008 and 2009.

³See, for example, International Monetary Fund (2010).

To ascertain the consequences of the lack of pre-commitment we ask the following question: how different would be the behavior of low-income countries with and without debt relief? We formulate a dynamic stochastic general equilibrium model that explores the incentive effects of debt relief. Specifically, we ask whether the possibility of debt relief motivates poor countries to take on additional debt. In this environment we examine the dynamic implications of relief expectations on consumption, investment, and the debt-to-GDP ratio given donor's debt-relief policy. To this end, we build a parsimonious characterization of debt-relief schemes where donors' debt-relief policy is reduced to a probability rule that encompasses the criteria traditionally used by donors and international financial institutions: the debt-to-GDP ratio and adverse macroeconomic conditions, this is, negative productivity shocks. We show that this simple formulation for debt relief fits the data well. A note of caution is in order; it is beyond the scope of the paper to propose an optimal mechanism to allocate debt-relief and we leave the formulation of optimal debt-relief rules open for future research. Furthermore, in our approach, we abstract from some important issues: first, political economy effects associated with strategic behavior by borrowers and lenders; second, learning-by-doing effects resulting from past actions; and third, commitment technologies that could allow the lender to pre-commit to specific relief mechanisms (e.g. commit not to grant debt relief in the future). In particular, this last extension of our framework could further enrich the study of the determinants of debt relief from a time-consistency problem approach.

The small open economy model is calibrated to match the data for Uganda, the first HIPC-eligible country to reach the enhanced HIPC initiative completion point in 2000. The model features a minimum consumption requirement and an endogenous debt-relief policy rule. The former feature puts a floor under aggregate consumption; in particular, a country may decide to acquire additional debt to secure the subsistence minimum. The latter feature is meant to capture the relationship between low-income country debt decisions and donor relief policy. Moreover, to simplify the model, we assume that all debt is external, a reasonable simplification as domestic debt markets have been underdeveloped in HIPC.⁴

In the model debt decisions depend on the state of the world and a stochastic interest rate driven by the probability of debt relief. Although households do not know whether debt relief is going to be granted or not, they may formulate expectations thereof. On the one hand, debt relief is likely to be granted to a country with either unsustainable debt, or one that is clearly in balance of payment difficulties, or both. On the other hand, poor countries do not mechanically collect debt relief as donors may decide not to grant it.

To quantify the effect of HIPC's expectations of future debt relief on consumption, investment, and debt decisions, we contrast two scenarios. In the benchmark, debt-relief scenario, countries estimate the likelihood of obtaining debt relief based on the state of the world (realization of productivity shocks) and their current debt-to-GDP ratio. In the second scenario no debt relief is offered, irrespective of the state of the world. We find that debt relief motivates low-income countries to borrow more, consume more, and invest less. Our quantitative results cast doubt on the idea that a debt write-off would promote rapid capital accumulation and growth, while simultaneously keeping debt at a sustainable

⁴Christensen (2004) found that sub-Saharan domestic debt markets are generally small, highly short-term, and have a narrow investor base. During 1980-2000 the average domestic debt-to-GDP ratio was 7.6% in HIPC countries and only 1.6% of GDP in Uganda, or 1/30 of its external debt.

level. In our set-up poor countries face incentives to borrow unsustainably to finance consumption and wait for another relief episode rather than to finance future consumption through savings and investment alone.

We also estimate the potential welfare gains from eliminating the consumption volatility generated by debt relief. We perform two policy experiments, tweaking the original relief mechanism to respond only to either productivity shocks or to large debt. We find that a simplified debt-relief rule that compensates low-income countries for negative productivity shocks, while ignoring the debt-to-GDP ratio, would provide much better incentives for capital accumulation at the same cost to the donors. The policy implication is that the debt-relief mechanism needs to limit the incentive effect from donors' history of past write-offs that tend to reinforce expectations of future debt-relief initiatives.

Our findings build on several past contributions. Easterly (2002) argued that past debt relief benefited mostly HIPC countries with bad domestic policies rather than those with good policies. Regressing the 1980-1997 average of selected policy and macroeconomic indicators on the log of initial income, and using a dummy variable for HIPC, Easterly showed that these indicators were the worst in the HIPC. He concluded that bad policies have not only resulted in high debt accumulation, they have also neutralized past debt-relief efforts. Such behavior appears consistent with HIPC policy makers expecting still better debt-relief terms before finally committing to sustainable policies. We show that excessive borrowing is not necessarily a sign of a bad government: it may also arise with benevolent governments.

Chauvin and Kraay (2007) showed that developing countries with large debts vis-à-vis multilateral creditors are more likely to receive debt relief and that such relief does not respond to fluctuations in GDP growth. Employing data on the cross-country allocation of debt relief for 62 low-income countries, the authors find that debt relief is mainly driven by country characteristics and suggest that, unless debt relief changes those characteristics, debtors and creditors may face repeated cycles of debt relief. Our paper formalizes the notion that a low-income country may experience repeated debt-relief episodes that have only a temporary impact on the level of debt due to the incentive effect that debt-relief availability creates.

Arslanalp and Henry (2006) compared the conditions in middle-income, mostly Latin American countries that benefited from debt restructuring using the so-called Brady bonds in the 1990s and HIPC. They conclude that the HIPC initiative would fail to stimulate HIPC economic growth since their main problem is not debt overhang but a lack of strong institutions. We show that even with good institutions the current debt initiatives could lead countries back to a sequence of large debt, low investment, and increased consumption.

Koeda (2006) explored the low-income countries' incentive problem as the optimal response to lending rules, under which concessional loans are granted to countries with income below a cut-off value and commercial loans to others. The dual lending standard and low growth ensure future aid flows (soft loans). We extend Koeda's framework through modeling debt forgiveness as write-offs of existing debt, whereby donor concessional lending determines the value the interest rate can take on.

The remainder of the paper is organized as follows. After reviewing the evolution of debt

in Uganda, we outline the model and describe its calibration. In the next sections we present the simulations for the scenarios with and without debt relief, and for alternative formulations of the debt relief mechanism. The final section concludes.

II. The Debt Problem and the HIPC Initiatives

The HIPC group is composed of forty countries, most of them in sub-Saharan Africa. A poor country is considered to be heavily indebted if its annual gross national income (GNI) per capita is below the International Development Association's (IDA) eligibility threshold for concessional loans (US\$1,095 in 2009) and after traditional debt-relief mechanisms, such as loan rescheduling at a new market rate and improved repayment profiles have been applied, either the debt-to-exports ratio is still above 150 percent or the debt-to-fiscal-revenue ratio is larger than 250 percent for very open economies.

The origin of the sub-Saharan African countries' debt problem has been attributed to the short-lived increase in primary commodity prices after the 1973 oil price shock (Green, 1989). The national authorities responded to the positive terms of trade shock by expanding their infrastructure spending, partly financed by foreign borrowing. When export prices declined government expenditures were sustained by additional borrowing under the assumption that these prices would recover quickly. They did not and the debt service burden continued to increase. The second oil price shock in 1979 stoked inflation in developed economies and the resulting monetary tightening led to sharply increasing real interest rates. As the most indebted countries began to default in Latin America, risk premiums increased across the world, and the cost of commercial borrowing became prohibitive. Among the HIPCs interest payments doubled during 1980-1987. Following the debt crisis of the early 1980s, virtually all new borrowing by HIPCs was from the so-called official creditors, either industrial-country governments or international financial institutions, such as the World Bank, African Development Bank, or Inter-American Development Bank.⁵

As of 1987, 80 percent of sub-Saharan external debt was owed to official creditors and international institutions, who began to realize that these debts were not sustainable. The creditors responded with increasingly concessional facilities: the Paris Club gradually raised the write-off amount in its rescheduling from 33 percent in 1989 to 67 percent in 1994, and the IMF offered a lending facility repayable in ten years, with a grace period of five years, and an interest rate of $\frac{1}{2}$ percent. Overall debt burden continued to increase, however, as the bulk of HIPC' debt was due to multilateral lenders who could neither reschedule nor cancel them at that time. In 1996 the IMF and the World Bank launched the HIPC initiative, enhancing it in 1999, granting write-offs of multilateral debt for the first time.

Eligibility for the HIPC initiative required adoption and successful implementation of an IMF-supported adjustment program and a poverty-reducing strategy. At the onset of the

⁵For instance, according to the World Development Indicators database, between 1970 and 1987 World Bank lending to sub-Saharan Africa grew by 20 percent annually (in US\$ terms). This behavior has been highlighted by a strand of the literature as lenders' complicity in low-income countries debt accrual (Easterly, 2002).

program the country's eligible debt was either written off up to 67 percent in net present value terms or rescheduled over 23 to 40 years with a long grace period. After successfully implementing the pro-growth and poverty-reducing policies, HIPC's were eligible for cancellation of 90 percent of non-official debt and rescheduling of the remainder. On completion of the whole process the Paris Club reduced the stock of eligible debt, often beyond the 90 percent cut-off point, and the IMF, World Bank, African Development Bank, and Inter-American Bank cancelled all debt owed to them, in accordance with the Multilateral Debt Relief Initiative. As of September 2010, 30 countries benefited from these initiatives.

A. The Ugandan Experience with Debt Relief

Uganda exemplifies the travails of debt relief: during 1982-2006 the country received some sort of debt relief on seven occasions, that is, on average every $3\frac{1}{2}$ years. In 1998, Uganda became the first country to receive debt relief under the HIPC initiative, just one year after having its reforms endorsed by the IMF and World Bank. The 2005-06 and 2006-07 debt write-offs under the MDRI initiative were equivalent to further US\$3.6 billion. As a result, Uganda's total debt outstanding declined from its 1992 peak of 102 percent of GDP to about 12 percent of GDP in 2007. The debt ratio varies on the account of GDP measurement issues and exchange rate volatility, however, after smoothing the U.S. dollar GDP by the Hodrick-Prescott filter we still observe a decline from a peak of about 75 percent of smoothed GDP to 12 percent of GDP or from debt per capita of about US\$ 200 (in constant 2005 dollars) to less than US\$ 50 (Figure 2, upper panel). It is interesting to note that the 2010 World Economic Outlook projections anticipate Ugandan debt increasing to 18 percent of GDP by 2011 or almost doubling in per capita terms.

How did Uganda become "highly indebted" in the first place? While in the early 1970s the external debt was below 10 percent of GDP, it increased ten times in less than 20 years. The answer is consistent with the general HIPC story—in the late 1970s and early 1980s Uganda began to borrow more, above and beyond its debt service capacity. The annual average of external gross borrowing—measured by disbursements in 2005 constant US dollars—quadrupled from US\$ 100 million in the 1970s to about US\$ 400 million in the period until 2001 (Figure 2, bottom panel). Needless to say, the volatility of commodity prices amplified the US\$ GDP decline in the early 1990s and contributed to the spike in the debt-to-GDP ratio.

III. The Model Economy

In this section we present a dynamic stochastic general equilibrium model of a small open economy with a minimum consumption requirement and an endogenous debt-relief probability that builds on Schmitt-Grohe and Uribe (2003) portfolio adjustment costs model. The minimum consumption level prevents the country from borrowing excessively, while the availability of debt-relief provides it with the incentive to borrow more and invest less than otherwise.

The economy is modeled in a world with perfect information (agents' decisions are all observable), perfect competition (agents take the market structure as given), and perfectly enforceable contracts (the relief mechanism is available every period and the low-income country honors its debts when relief is not granted). We abstract from asymmetric information, strategic behavior, and enforcement issues in order to disentangle the effect of debt-relief availability from other possible explanations for the HIPC debt-accumulation problem and show that even in a frictionless environment this problem may arise.

Moreover, to further simplify the analysis, donors lending policy is reduced to the supply of concessional loans, available every period and treated as one-year, risk-free bonds that perfectly meet the country's borrowing needs.

A. Environment

The economy consists of a representative profit maximizing firm and an infinitely lived utility maximizing stand-in consumer. The firm rents capital, K_t , and labor, L_t , from the household⁶ and pays a competitive interest rate, r_t , and wage, w_t . The consumer has access to two saving mechanisms: capital accumulation and a one-year, risk-free bond. Debt payments are stochastically determined each year through a debt-relief lottery: they are either written-off or have to be repaid in full.

B. Technology

The productive sector of the economy is represented by a firm with a constant returns to scale technology:

$$Y_t = Z_t K_t^\alpha L_t^{1-\alpha} \quad (1)$$

with stochastically determined productivity, $Z_t = \bar{Z}e^{z_t}$, where \bar{Z} is the economy's average productivity level and z_t is a random productivity shock, which follows an AR(1) process, $z_t = \rho z_{t-1} + \varepsilon_t$, $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$ and $\rho < 1$. The capital share of income is denoted by $\alpha \in (0, 1)$.

The firm solves the standard profit maximization problem by choosing sequences of labor $\{L_t\}_{t=0}^\infty$ and capital $\{K_t\}_{t=0}^\infty$ so as to

$$\text{Max}_{\{K_t, L_t\}_{t=0}^\infty} \sum_{t=0}^\infty (Z_t K_t^\alpha L_t^{1-\alpha} - w_t L_t - r_t K_t) ; K_t, L_t \geq 0. \quad (2)$$

C. Preferences

The representative consumer has time-separable preferences represented by a period CRRA utility function with a survival level of consumption, $c_{\min} \geq 0$, time preferences,

⁶We use the terms "consumer" and "household" interchangeably throughout this section.

$\beta \in (0, 1)$, and a risk aversion coefficient, $\sigma > 0$. The consumer's expected utility is represented by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(c_t - c_{\min})^{1-\sigma}}{1-\sigma} \right]. \quad (3)$$

The above utility formulation has been previously employed to model differences in saving rates across households (Chatterjee (1994), Chatterjee and Ruvikumar (1999), Alvarez-Pelaez and Diaz (2005), and Obiols-Holms and Urrutia (2005)), to capture different intertemporal elasticities of substitution among poor and rich countries (Atkinson and Ogaki (1996, 1997)), or to study the distributional implications of economic growth (Ogaki, Ostry, and Reinhart (1996)). We introduce the survival level of consumption to capture the feature of low saving rates in poor countries. The populations of such countries are often left—after satisfying the subsistence requirements—with little or no resources for saving and are therefore prone to overborrowing to finance investment. In a stochastic environment a consumption minimum works in two directions. On the one hand, it motivates the household to borrow in order to attain the survival consumption level, i.e., the standard intertemporal smoothing role of finance (Rosenzweig and Wolpin (1993)). On the other hand, it constrains its borrowing as the household is aware that the joint occurrence of a negative productivity shock and a no-relief outcome of the lottery could push its consumption below the survival level.

At time t , the household divides its labor income, $w_t l_t$, capital income, $r_t k_t$, and bond-holding proceeds, $(1 + i_t)d_t$, among consumption, c_t , investment, x_t , and the next period portfolio allocation, d_{t+1} . In order to characterize the low-income country debt-accumulation problem, $d_{t+1} < 0$, denotes a one-period loan taken by the consumer in period t and carried over to period $t + 1$ at the interest rate i_{t+1} .

The household accumulates capital by adding investment to the current capital stock, free of depreciation, $\delta \in (0, 1)$, after covering capital adjustment costs to transform today's capital into tomorrow's capital.

$$k_{t+1} = (1 - \delta)k_t + x_t - \Psi(k_t, k_{t+1}). \quad (4)$$

Capital adjustment costs are used to ensure that the volatility of investment in the model corresponds to that observed in the data and are represented by a convex function $\Psi(k_t, k_{t+1})$ as in Mendoza (1991),

$$\Psi(k_t, k_{t+1}) = \frac{\psi}{2}(k_{t+1} - k_t)^2 \quad (5)$$

where $\psi > 0$ is a constant determining the size of the adjustment cost.

We further assume that the household faces portfolio adjustment costs of holding debt in quantities different from the steady state level of debt \bar{d} . Such costs capture transaction costs and also ensure stationarity in small open economy models with incomplete markets (Schmitt-Grohe and Uribe, 2003). In our model, the presence of these costs ensures gradual borrowing adjustments in response to productivity and debt-relief shocks, generating smooth equilibrium debt dynamics.

The debt adjustment cost function $\Xi(d_{t+1})$ is assumed to be convex as in Schmitt-Grohe and Uribe (2003), where $\kappa > 0$ is a constant determining the size of the bond holding cost:

$$\Xi(d_{t+1}) = \frac{\kappa}{2}(d_{t+1} - \bar{d})^2. \quad (6)$$

The sequential budget constraint is then:

$$c_t + x_t + d_{t+1} = w_t l_t + r_t k_t + (1 + i_t)d_t - \Xi(d_{t+1}). \quad (7)$$

D. The Debt-Relief Mechanism

Debt relief is modeled as a lottery with two possible outcomes. At time t , the household observes the exogenously determined interest rate i_t , that can take on two possible values, either -1, indicating full debt relief, or a concessional international rate, $i^* \geq 0$. Hence, with probability $\phi_t \in [0, 1]$ the household gets all its debt forgiven and does not have to make any repayment, $(1 + i_t) = 0$. With probability $(1 - \phi_t)$ the household has to repay all debt in full, $(1 + i_t) = (1 + i^*)$. To this end, debt payments are stochastically determined by the following rule:

$$i_t = \begin{cases} -1 & \text{with probability } \phi_t \\ i^* & \text{with probability } 1 - \phi_t. \end{cases}$$

Donors' debt-relief policy is represented by a probability rule, $\Phi\left(z_t, \frac{d_t}{Y_t}\right)$, that depends on the exogenous productivity shock, z_t , and the country's existing debt-to-GDP ratio, $\frac{d_t}{Y_t}$:

$$\Phi\left(z_t, \frac{d_t}{Y_t}\right) = \frac{1}{1 + e^{-(\phi_1 e^{z_t} + \phi_2 \frac{d_t}{Y_t})}} \quad (8)$$

where $\phi_1, \phi_2 \leq 0$ are constants determining the impact of productivity shocks and the country's indebtedness ratio in the relief probability function. To characterize HIPC-like initiatives with eligibility criteria favoring poor and highly indebted nations, the debt-relief rule is an increasing function of negative productivity shocks, $\Phi_z < 0$, and of the debt-to-GDP ratio, $\Phi_{\frac{d}{Y}} < 0$.

Each period, the household forms rational expectations on the outcome of the lottery. On the one hand, the larger the current debt-to-GDP ratio, the larger the probability of obtaining debt relief. On the other hand, the household may fail to attain the minimum consumption level and perish if after being hit by a negative productivity shock it were required to pay back its debts. Figure 3 illustrates the debt-relief mechanism.

E. The Household Problem

At time t , the household takes expectations over the shock, z_t , and interest rates, i_t , and chooses sequences of consumption, c_t , investment, x_t , and next period debt holdings, d_{t+1} , to maximize its lifetime utility. The labor endowment is normalized to 1 and is inelastically supplied.

For given prices, optimal decision rules for c_t , x_t , and d_{t+1} , solve the household problem:

$$\begin{aligned} & \text{Max}_{\{c_t, x_t, d_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(c_t - c_{\min})^{1-\sigma}}{1-\sigma} \right] \\ & \text{subject to} \\ c_t + d_{t+1} + x_t & \leq w_t + r_t k_t + (1 + i_t) d_t - \frac{\kappa}{2} (d_{t+1} - \bar{d})^2 \\ k_{t+1} & = (1 - \delta) k_t + x_t - \frac{\psi}{2} (k_{t+1} - k_t)^2 \\ c_t & \geq c_{\min} \\ d_{t+1} & \geq -M \\ & k_0, d_0 \text{ given} \end{aligned} \tag{9}$$

where the lower bound $d_{t+1} \geq -M$ prevents the consumer from running a Ponzi scheme.

F. Rational Expectations Equilibrium and its Recursive Representation

Given the international interest rate, i^* , a *competitive equilibrium* is a sequence of prices $\{w_t, r_t\}_{t=0}^{\infty}$, and allocations $\{c_t, k_{t+1}, d_{t+1}, K_t, L_t\}_{t=0}^{\infty}$ such that: *i*) $\{c_t, k_{t+1}, d_{t+1}\}_{t=0}^{\infty}$ solves the household's problem (9), *ii*) $\{K_t, L_t\}_{t=0}^{\infty}$ solves the firm's problem (2), and for all t , *iii*) $K_t = k_t$, and *iv*) $L_t = 1$.

More conveniently, the sequential household and firm problems can be summarized in a recursive problem solved by a benevolent social planner:

$$\begin{aligned} V(k, d, z, i) & = \text{Max}_{\{c, k', d'\}} \left\{ \frac{(c - c_{\min})^{1-\sigma}}{1-\sigma} \right. \\ & \quad + \beta \sum_{z'} \pi(z' | z) \left\{ \phi(k', d', z') V(k', d', z', -1) \right. \\ & \quad \left. \left. + (1 - \phi(k', d', z')) V(k', d', z', i^*) \right\} \right\} \\ & \text{subject to} \\ c + k' + d' & \leq \bar{Z} e^z k^\alpha + (1 - \delta) k - \frac{\psi}{2} (k' - k)^2 + (1 + i) d - \frac{\kappa}{2} (d' - \bar{d})^2 \\ \phi(k', d', z') & = \Phi \left(z', \frac{d'}{\bar{Z} e^{z'} k'^\alpha} \right) \\ d' & \geq -M \\ c & \geq c_{\min}. \end{aligned} \tag{10}$$

IV. Simulation Results

The model is solved by value function iteration over a discretized state space and we perform two simulations. In the first one we explore household choices when debt relief is available, while in the second one there is no lottery and no debt relief. Calibration parameters are obtained from previous research or selected from data such that the model with debt relief replicates empirical regularities of the Ugandan economy.

A. Calibration

In this section, we describe the selection criteria for parameter values taken from the existing literature and estimation procedures for parameters exclusive to the model (Table 1). The economy's total factor productivity is computed as the residual after accounting for productive factors, $Z_t = Y_t/K_t^\alpha L_t^{1-\alpha}$. We calculate Z_t using data from the World Economic Outlook (WEO) database on output and investment, measured as gross fixed capital formation and changes in inventories, and World Development Indicators (WDI) data on labor employment, measured as working age population.

The capital share of income is set to $\alpha = 0.3$, a standard value in the business cycle literature. The average productivity level is normalized to $\bar{Z} = 1$. The exogenous productivity shock, z_t , is assumed to follow an AR(1) process and it is estimated using the Solow residual. The realizations for z_t and the corresponding transition matrix are estimated using the Touchen's method with a grid of 9 equidistant points. Following Kehoe and Ruhl (2003) we generate a capital stock series using investment data for 1970-2006 and the capital accumulation process (4). The depreciation rate and the initial capital stock are determined jointly: for a given value for δ , the initial capital stock is selected such that the capital-output ratio in 1970 is the same as its average value during 1970-2006. Specifically, setting $\delta = 0.0489$ ensures that the 2006 capital consumption-to-output ratio, $\delta \frac{K_t}{Y_t}$, is 0.072, as observed in the Ugandan data.

The preference parameters, the discount factor, β , and the risk aversion coefficient, σ , are set in line with the estimations of the consumption minimum as in Chatterjee and Ravikumar (1999), where such minimum is estimated to be about 58 percent of average consumption. This estimate is consistent with a risk aversion parameter of 0.964 and a discount factor of 0.95.

The real interest rate, $i^* = 5.88$ percent, matches the 1983-2006 average of the Commercial Interest Reference Rate (CIRR) published by the OECD, a standard reference for concessional rates.⁷ The country does not pay the much higher market rate as the debt-relief lottery determines whether the loan carries the concessional rate or is written off.⁸ Donor lending is concessional both in the sense that the actual interest rate

⁷The CIRRs are minimum interest rates applied to official financing support for export credits and are used as reference for calculating the concessionality level of aid. We employ CIRRs with a maturity of 8.5 years as a representative rate for concessional loans received by Uganda. For the CIRR calculations see http://www.oecd.org/LongAbstract/0,3425,en_2649_34171_2428234_1_1_1_1,00.html.

⁸Uribe and Yue (2006) estimated that in 1994-2001 emerging countries faced market rates composed of the average U.S. interest rate of 4 percent and an average country premium of 7 percent. In practice, most HIPC's IMF-supported programs contained a condition of no commercial borrowing (at market rates).

is lower than commercial rates and that it allows for debt write-off.

Regarding the intrinsic model parameters in the debt probability rule (ϕ_1, ϕ_2) , we choose their values so as to fit the data. We run a restricted logit regression using the years at which Uganda received debt relief from the Paris Club during the 1982-2006 period and using debt ratios and productivity as the explanatory variables in the spirit of Chauvin and Kraay (2007). Restrictions imposed on the regression guarantee that: i) zero probability of write-off is assigned to the null debt ratio, $\Phi(z_t, 0) = 0, \forall z_t$; and ii) the steady state debt ratio matches the 1982-2006 average for Uganda. Finally, a logistic function is transformed to ensure that for each $\frac{d_t}{Y_t} \in (-\infty, 0]$ there were corresponding debt relief probability values, $\phi_t \in [0, 1)$.

The capital adjustment cost parameter ψ in equation (5) is chosen to match the relative volatility of investment and GDP. The portfolio adjustment cost parameter κ in equation (6) is calibrated to match the service fee charged by multilateral development banks on total debt outstanding and disbursed balances of concessional loans.⁹

B. The Debt-Relief Scenario

In period t , the planner observes both the productivity shock, z_t , and the period interest rate, i_t , and then decides consumption, capital, and debt to solve the optimization problem described in equation (10).

The debt-to-GDP ratio increases/decreases when the model economy is hit by a negative/positive productivity shock and the correlation between the debt-to-GDP ratio and TFP shocks is strong at 0.95 (See the full and dashed lines in Figure 4). The simulation also confirms a strong correlation between adverse productivity shocks and relief episodes. In the model economy debt relief is almost always granted when the country is hit by any of the three most negative productivity shocks and in about 1/2 half of all instances when hit with the smallest negative shock (Figure 5). In contrast, relief is not granted when the model economy is hit with positive shocks. As the shocks are observable by lenders, this result can be interpreted as donors requesting a full repayment when “times are good.”

The simulated economy replicates the Ugandan stylized facts well. Uganda received debt relief on seven occasions during 1982-2006, that is, every 3.6 years the country benefited from either a rescheduling, extension, or write-off of debt owed to the Paris Club members, implying a debt-relief probability of about 28 percent. On average, the model yields a debt-relief probability of 29.6 percent and a debt-to-GDP ratio of 52.3 percent, close to the 51.6 percent in the data (Table 2). The model also matches other relevant variables: the ratio of the standard deviation of investment to that of GDP in the data and in the model are 2.74, and 2.02, respectively; the data and model portfolio adjustment cost of average debt holdings are 0.75, and 0.76 percent, respectively; and the minimum-to-average consumption ratio of 57.6 percent in the model is close to the 58 percent ratio reported in the literature.

⁹See World Bank (2001), section on IDA eligibility, terms and graduation policies (table 3, annex II, p. 24).

C. The Scenario Without Debt Relief

In the counterfactual simulation debt relief is not available. The planner observes the realization of the shock, and then decides consumption, capital, and debt to solve problem (10) without considering debt relief. Parameter values used in the simulation are identical to those previously used in the debt-relief case (Table 1), the only difference being that we set the relief probability value, ϕ_t , equal to zero.

We compare debt, investment, and consumption as shares of GDP between the two scenarios (Table 3 and Figure 6). In the absence of debt relief, the average debt-to-GDP ratio declines to 24.3 percent, less than one half of the ratio in the former scenario, consumption-to-GDP is halved, while investment increases to 24.1 percent of GDP, as compared to 15.4 percent of GDP. As a result, GDP is on average 22 percent larger than with debt relief. These outcomes imply that, with debt relief expectations, investing in physical capital is less attractive for the country since a larger GDP reduces the chance to obtain debt relief. Consumption smoothing is provided, at least partly, directly through debt relief. In contrast, when debt relief is absent, a larger capital stock serves to smooth consumption.¹⁰

To further illustrate the distortion in resource allocation that stochastic debt relief introduces we also compare (net) investment and consumption as shares of disposable income (DI) defined as GDP plus net debt inflows, $(1 + i_t)d_t - d_{t+1}$, free of capital and debt adjustment costs:

$$DI_t = Y_t - \Psi(k_t, k_{t+1}) + (1 + i_t)d_t - d_{t+1} - \Xi(d_{t+1}) \quad (11)$$

We find that the average investment-to-DI ratios are 13 percent and 33 percent with and without debt relief, respectively. The average consumption-to-DI ratios are 87 percent and 67 percent, respectively. When debt relief is available, the country consumes a larger fraction of its disposable income, and consequently invests a lesser proportion of it.

D. Welfare Implications

Although higher in mean and levels, the consumption stream with debt relief is significantly more volatile. We evaluate the impact of economic instability generated by the debt-relief lottery on consumer welfare using the Lucas' (1987) definition of compensating variation. The cost of consumption instability is equivalent to the percentage increase in consumption, uniform across all dates and values of the shocks, needed to leave the consumer indifferent between a perfectly smooth consumption stream and a consumption path with variability about its trend. Such cost is estimated as $\frac{1}{2}$ times the risk aversion coefficient times the consumption variability. To this end, we use the standard deviation of the linearly detrended log of the optimal consumption sequence obtained in the relief and no-relief scenarios.¹¹

¹⁰This finding is in line with the literature on household accumulation of wealth. Hubbard et al. (1995) showed that asset-based means testing welfare programs can have distortionary effects on savings behavior by discouraging households with low expected lifetime income to accumulate their own precautionary wealth.

¹¹A similar analysis was performed by Arellano et al., (2009) when assessing potential welfare costs associated with the increased consumption volatility that aid flows might induce. The welfare benefit of

Eliminating aggregate consumption variability in the debt-relief scenario is equivalent to a sizable increase of 0.85 percent in average consumption, while the same computation in the no-relief scenario yields a negligible increase of 0.01 percent. Such enormous difference is fully attributable to debt-relief shocks, as the contribution of productivity shocks is nugatory. While debt-relief events magnify consumption variability and increase welfare losses, the elimination of the uncertainty of debt-relief outcomes implies a potential welfare gain.¹²

V. Policy Experiments

In this section we present the macroeconomic implications of two alternative debt-relief mechanisms. Recall that in the benchmark model debt relief is driven by two events. First, the economy is hit by an exogenous negative shock and debt relief offsets the shock's impact. Second, the debt-to-GDP ratio increases either because the country borrowed too much or GDP declined owing to capital decumulation, TFP shocks, or both. We now consider the outcomes of debt-relief mechanisms activated either by changes in the debt-to-GDP ratio only or by productivity shocks only. It turns out that a relief policy responding to a country's debt-to-GDP ratio generates most of the macroeconomic differences between the debt-relief and the no-relief scenarios observed in Figure 6, while a simplified debt-relief rule that compensates for output losses from negative productivity shocks only is sufficient to motivate the household to invest optimally, i.e., as in the no-relief scenario.

A. The Debt-Relief Mechanism Responding to the Debt-to-GDP Ratio Only

By linking debt relief to the debt-to-GDP ratio only, creditors ignore the first-round effects of "bad luck." While the economy is still hit by productivity shocks, the debt relief lottery acknowledges only the eventual adverse effect of the higher debt-to-GDP ratio. Parameter values remain as in Table 1, with the exception of the constant multiplying the productivity shock parameter in the debt-relief probability function, ϕ_1 , which is set to zero.

We find that productivity shocks explain only a minor part of debt-relief expectations and that these expectations are driven chiefly by the debt-to-GDP ratio. The average debt-to-GDP ratio in the modified relief mechanism is only slightly lower than in the benchmark case: 49 percent and 52 percent, respectively (compare the first and second set of columns in Figure 6). In terms of resource allocation we find similar outcomes to those obtained under the benchmark mechanism: on average 17 percent of disposable income is

reducing aid volatility to zero was estimated to be about 0.4 percent of consumption, a rather large estimate when compared to the standard cost of business cycles for industrialized economies (one tenth of a percentage point for the United States for the post-war period according to Lucas' calculations).

¹²This result is analogous to Pallage, Robe and Bérubé (2006) finding that a welfare cost in the order of three-fourths of aggregate consumption volatility is associated with procyclical aid flows to developing countries. This cost could be potentially eliminated if aid were used as an insurance mechanism to smooth aggregate consumption.

invested and 83 percent is consumed. These results fit the behavior of a planner controlling the “borrowing” part of the debt-relief lottery and a donor who is under domestic political pressure to cancel “odious” debt. These results indicate that 87 percent of additional debt in the relief scenario is due to the country’s own borrowing decisions and only 13 percent is due to exogenous shocks.¹³

B. The Debt-Relief Mechanism Responding to Productivity Shocks Only

In this section we solve the model under the assumption that creditors only consider the realization of the productivity shock when granting debt relief, i.e. debt relief serves as an insurance mechanism. We confirm that it is the inclusion of the debt-to-GDP ratio in the debt relief rule that drives debt and consumption up relative to the no-relief scenario in Figure 6. Making debt relief respond only to truly exogenous productivity shocks (bad luck), the equilibrium dynamics of debt, consumption, and investment appear broadly identical to the no-relief scenario.

To illustrate the effect of this modified debt-relief mechanism on investment, we perform the following comparison: controlling for both the initial conditions $-(k, d, z)$ - and for the amount of debt relief granted $-(1 + i^*)d$ -, the planner chooses an investment level that is on average 90 percent higher than that chosen under the benchmark model. This additional capital is on average equivalent to 140 percent of the amount of relief granted, making debt relief more effective in encouraging investment.

VI. Conclusions

We argue that the recurrent availability of debt relief creates a quantitatively important incentive problem. Donor debt-relief policy has reinforced HIPC expectations of future debt relief as opposed to relying on domestic saving. Therefore, when debt relief is available high debt does not necessarily signal a "bad" government since it arises also with benevolent governments. We examine the effect of HIPC-like expectations of future debt relief on consumption, investment, and debt decisions by comparing simulation results from two scenarios in a dynamic stochastic general equilibrium model of a small open economy, calibrated using Ugandan data for the 1982-2006 period. In the debt-relief scenario the country estimates the likelihood of obtaining write-offs based on the realization of exogenous productivity shocks and its current debt-to-GDP ratio. In the counterfactual, no-relief scenario, consumption, investment and, debt decisions are made independently of the debt-relief lottery.

We find that the debt-to-GDP ratio is higher and the investment-to-GDP ratio is lower when debt relief is available. As a result, GDP is on average lower by more than 20 percent when the country expects a debt write-off as compared to a situation when it does not. These results are at odds with the commonly held perception that debt-relief initiatives encourage capital accumulation and a decline of debt. Welfare analysis suggests

¹³ $(52.3-48.6)/(52.3-24.3)=0.13$

that the uncertainty associated to debt relief amplifies the volatility of consumption and that there is a potential welfare gain of 0.84 percent of average consumption if such volatility is eliminated. Policy experiments with variations to the original debt relief mechanism suggest that most of the debt and investment distortions can be eliminated by providing debt relief to countries hit by negative productivity shocks only. Such modification of the debt relief mechanism is both welfare enhancing and—from the point of view of the donor community—more cost effective in spurring investment.

Finally, we argue that high debt-to-GDP ratios may re-emerge in the medium term. Unless donors credibly pre-commit to never grant debt relief in the future, the currently designed debt-relief mechanisms would distort low-income countries' decisions by encouraging them to carry larger debt, consume more, and invest less than what they would have chosen in the absence of debt relief. Our simulations suggest that it has been primarily the endogenous borrowing choice—domestic policies—that have driven the accumulation of large debts in the past four decades, while the contribution of adverse productivity shocks—"bad luck"—was negligible.