On the Optimal Adherence to Money Targets in a New-Keynesian Framework: An Application to Low-Income Countries

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Abstract

Many low-income countries continue to describe their monetary policy framework in terms of targets on monetary aggregates. This contrasts with most modern discussions of monetary policy, and with most practice. We extend the new-Keynesian model to provide a role for “M” in the conduct of monetary policy, and examine the conditions under which some adherence to money targets is optimal. In the spirit of Poole (1970), this role is based on the incompleteness of information available to the central bank, a pervasive issues in these countries. Ex-ante announcements/forecasts for money growth are consistent with a Taylor rule for the relevant short-term interest rate. Ex-post, the policy maker must choose his relative adherence to interest rate and money growth targets. Drawing on the method in Svensson and Woodford (2004), we show that the optimal adherence to ex-ante targets is equivalent to a signal extraction problem where the central bank uses the money market information to update its estimate of the state of the economy. We estimate the model, using Bayesian methods, for Tanzania, Uganda (both de jure money targeters), and Ghana (a de jure inflation targeter), and compare the de facto adherence to targets with the optimal use of money market information in each country.
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There has been a revolution in the practice and analysis of monetary policy over the last 20 years. Many central banks have adopted an explicit inflation targeting strategy, using interest rates as their operational target, and with very little role for monetary aggregates in the conduct of policy. From a theoretical point of view, the new consensus on monetary economics is reflected in the New Keynesian modeling approach—the workhorse model for studying monetary policy questions—which does not assign money a special role in controlling inflation. In these models, monetary aggregates do not enter the monetary policy transmission mechanism. Considering further that the relation between money and prices (and output) is often volatile and unstable, a policy message from these models suggests disregarding monetary aggregates altogether in the analysis of monetary policy.

This approach contrasts with actual policy regimes in many low-income countries (LICs). Most countries in sub-Saharan Africa that have some degree of exchange rate flexibility continue to describe their monetary policy in terms of monthly or quarterly targets on monetary aggregates, typically narrow money (IMF (2008)). In addition, the analysis of monetary policy in most central banks in LICs, and at the IMF, remains closely related to the quantity theory MV=PQ, with money targets set so as to be consistent with a desired path for prices (P) given real output (Q) and under some assumption about the trajectory of velocity (V). Because money targets are often missed, the policy discussion focuses on assessing whether these misses reflect policy errors or unexpected changes in velocity.

One way of thinking about this anomaly is to suppose that LICs (and the IMF in this context) remain heavily influenced by thinking from before the 1990s and have been relatively slow to adopt modern frameworks. This may be true, but we think it is more fruitful to begin by studying under what conditions some attention to monetary aggregates might make sense. Given the predominance of the new-Keynesian framework in the policy and academic debate, our objective in this paper is to introduce a role for money by making the minimum necessary changes to the standard model. In particular, we maintain the assumption that money does not play a direct role in the monetary transmission mechanism. We thus extend the new-Keynesian framework to address two questions: how should central banks in low income countries set targets on money growth? More importantly, how should central banks respond to deviations from these targets?

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1See Clarida and others (1999).
2Although these models feature long run neutrality of money, this does not imply any causality running between money and prices.
3See Woodford (2008) for a detailed discussion.
Our rationalization of why money matters is based on the informational role of money. Information gaps in LICs are pervasive. Key variables like output and inflation are observed imperfectly and with substantial lags. For example, quarterly GDP is unavailable in many countries and there are often no economy-wide wage data at all. Output gaps in particular are hard to assess on the basis of standard indicators of economic activity: measures of capacity utilization are scarce, agricultural supply shocks dominate, and informal markets are pervasive. A second informational channel relates to imperfections in financial markets. Even key markets such as interbank and government bond markets are often thin and opaque. Contract enforcement may be poor. Meanwhile, a large fraction of the population does not even participate in formal financial markets. In these circumstances, a readily observed interest rate such as the interbank money market rate may bear only a loose connection to the (latent or shadow) interest rate relevant to private sector decisions.

Monetary aggregates, on the other hand, are measured accurately and with little lag. And these aggregates are systematically related to key variables such as output and the relevant interest rate through money demand. The relation is subject to money demand shocks, but with sufficient information gaps, money targets may nonetheless be worth attention.

To assess whether money has an informational role, we introduce a simple type of information incompleteness in an otherwise standard new-Keynesian model. In our information structure, the authorities periodically choose a target for both the growth rate of the monetary aggregate and the short term interest rate over the next period (one quarter). These ex-ante targets are mutually consistent: for any interest rate target, there is an associated money target, and vice versa, through the money demand function.

We assume targets are chosen so that the expected short term interest rate follows a standard forward-looking Taylor rule satisfying the Taylor principle. The choice of such a rule seems natural, since it embodies basic principles of modern monetary policy: that policy should be forward-looking, that it should let bygones be bygones, and that nominal interest rates should respond more-than-one-for-one to deviations in inflation from target. We show that such a rule has important implications for the corresponding money growth target: 1) money growth should aim toward its flexible-price value, 2) it should correct for the lagged money gap (the deviations between real money balances and the level of real money balances if prices were flexible), 3) it should accommodate deviations in current inflation from its target, and 4) its response to expected inflation and the output gap should depend on the interest and income elasticity of money demand.

While the ex-ante mapping between money and interest rates helps clarify some desirable properties of money targets, it does not yet justify any role for money. In our model, this potential role comes in the subsequent period, when the central bank observes only the outcome of the money market, i.e.,

\[4\] We discuss other possible rationales for why money might matter in section 5 below.
interest rates and monetary aggregates. Since the economy is subject to shocks, the ex ante equivalence between the money and interest rate targets breaks down. The authorities must decide whether to adhere to the interest rate target, the money aggregate target, or to give some weight to both, where the parameter \( \lambda \) is the relative weight on interest rates. The informational problem faced by the central bank is further complicated by the assumption that the interest rate that the central bank observes—and can control—is a noisy indicator of the rate relevant for policy decisions. In our view, this is a useful shortcut to model some of the shortcomings of financial markets in LICs described above, although the same relation will hold in models with limited participation in the formal financial system and shocks to the distribution of income between those that participate in financial markets and those that do not.\(^5\)  

Given this structure, we show that the optimal weight to be attached to money aggregates relative to interest rates can be thought of as the solution to a signal extraction problem faced by the central bank, similar to the problem in Svensson and Woodford (2003, 2004). The money market information helps the central bank update estimates of the output gap and expected inflation, which then affect monetary policy through their implication for the Taylor rule. The signal extraction problem is not trivial, because the money market outcome also depends on the monetary policy response, and the Kalman filter must be revised to account for this circularity.\(^6\)  

As in Poole (1970), the optimal weight turns out to depend on various characteristics of the economy, such as the noisiness of interest rates, the volatility of money demand shocks, the interest elasticity of money demand, and the volatility of real shocks. Most of the results are intuitive: money deserves more weight when money demand is not excessively volatile, the interest elasticity of money demand is low, observed interest rates are noisy, or the volatility of real shocks is high, for example.  

Ultimately, the importance of money is an empirical question. We therefore estimate the model for Ghana, Tanzania and Uganda. Uganda and Tanzania are de jure money targeters; Ghana followed a de jure inflation targeting regime since 2004, with an interest rate instrument, but authorities continue to pay attention to monetary aggregates.\(^7\) We use Bayesian methods, which help us estimate the model despite the limited time-series available for these two countries.  

Our estimation strategy has three main features. First, we impose the same priors on each country, which implies that differences in estimated parameters across countries only reflect information contained in the data. Second, we take the weight given to the interest rate target relative to the  

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\(^5\)See Portillo (2010).  

\(^6\)In the context of an optimal monetary policy problem, Svensson and Woodford (2004) refer to this circularity as the failure of the separation principle between signal extraction and (the choice of optimal) monetary policy.  

\(^7\)Ghana and Uganda are relatively well-studied cases: Alichi and others (2010) apply a New Keynesian-type model—without money—to Ghana, while Berg and others (2010) calibrate a micro-founded DSGE to Uganda to understand the impact of aid shocks and the monetary policy response during the period 2000–2004.
money target ($\lambda$) as a free parameter and assume a relatively diffuse prior with a mean of 0.5. Finally, we do not directly observe targets on interest rates and money growth but rather infer them from the behavior of actual interest rates and money aggregates and from the structure of the model.

Our results are the following. The econometric estimate of $\lambda$ across countries is consistent with each country’s de jure policy regime: Ghana’s adherence to its model-based interest rate target is almost complete ($\lambda = 0.93$), whereas Uganda adheres more strongly to money targets ($\lambda = 0.34$) and Tanzania’s estimate is somewhere between the two ($\lambda = 0.66$).

With the estimates for parameters and volatilities, we then calculate the optimal $\lambda$s for each country. We find that monetary authorities in Uganda are using money market information in a near-optimal way, as the estimated $\lambda$ is close to its optimal value (0.32). Ghana, on the other hand, and Tanzania to a lesser extent, would benefit from paying additional attention to money-market developments. In the case of Ghana, the optimal lambda is much lower (0.36), while Tanzania’s is slightly higher (0.44). These results are justified by the relatively moderate volatility of money demand and the nosiness of observed interest rates.

We also assess the costs of not optimally adhering to money targets by looking at the resulting volatilities for inflation and the output gap. Cost-push shocks are volatile in all three countries, and these shocks create a well-known tradeoff between inflation and the output gap. The optimal $\lambda$ is that which best balances this tradeoff, consistent with the preferences of the monetary authorities captured by the estimated coefficients in the policy rule. In all countries, strict adherence to money targets ($\lambda = 0$) would result in lower inflation volatility but at the cost of higher volatility in the output gap. Choosing $\lambda$ optimally yields a slightly more volatile inflation but helps achieve lower output gap volatility.

Our work is closely related to the seminal work of Poole (1970) on the choice of policy instrument, and the work of Friedman (1975) on the use of intermediate targets. Analytically, our main innovation is to embed the target choice question in a fully articulated inter-temporal model, whose information structure rationalizes the notion that adherence to operational targets depends on imperfect information. Thus, we model jointly the choice of the monetary policy committee and the decision about what to do between meetings, as Walsh (2003) describes the problem in Poole (1970).

Our work is also closely related to more recent work, starting with Svensson and Woodford (2003, 2004), on the optimal use of indicator variables in forward-looking models. In particular, Coenen, Levin and Wieland (2005) and Lippi and Neri (2007) quantify the relevance of money as an indicator variable in the Euro area. These papers simultaneously solve for optimal monetary policy and the optimal use of indicator variables—such as, but not limited to, money—in the conduct of policy. Our

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8See Friedman (1990) for a thorough discussion.
analysis is considerably simpler, in that we abstract from optimal monetary policy design and focus exclusively on the optimal use of an indicator variable. In addition, the type of incompleteness of information these authors focus on is different from ours. We believe our information structure, based on Aoki (2003), is ideally suited to analyze money-targeting issues that are common in low-income countries, such as the ex-ante choice of targets and how to interpret ex-post target misses. Finally, unlike those papers, we treat money as the only indicator variable available, which greatly enhances its potential informational role and justifies its central place in the policy framework in LICs. While our assumption is extreme, we believe it is not too removed from the informational problems faced by many low income countries. However, the simplicity of the model and the scarcity of the data argue against taking our results too literally.

The remainder of the paper is structured as follows. Section 2 presents the model. Section 3 estimates the model to quantify the potential of monetary aggregates as a source of information, while section 4 derives the optimal use of money market information. Section 5 discusses other possible rationalizations for the relevance of money on LICs. Section 6 concludes.

II. THE MODEL

The model is based on the standard new-Keynesian model of the kind described in Clarida, Gali and Gertler (1999), for a closed economy, and Monacelli (2004), for an open economy. We build on the variant in Berg and others (2006), which adds some features not explicitly micro-founded to better conform to the needs of policy-makers. The non-policy equations of the model are the following.

**Output**

Observed quarterly output is divided between potential output and the output gap:

\[ y_t = y^*_t + ygap_t, \]  

where potential output is assumed to follow a random walk with drift \( \frac{g}{4} \):

\[ y^*_t = y^*_{t-1} + \frac{g}{4} + \epsilon^*_t, \]  

and the output gap is pinned down by a hybrid IS curve:

\[ ygap_t = \beta_y E_t(ygap_{t+1}) + (1 - \beta_y)ygap_{t-1} - \beta_R (RR_t - RR^*_t) + \beta_z (z_t - z^e_{t+1}) + \beta_w (ygap^w_t - E_t(ygap^w_{t+1})). \]

**Note:** The simplicity of the information structure argues against using this model—in its current form—for developed countries or even emerging markets.
where $z_t$ is the real exchange rate, $RR_t$ is the ex-ante real interest rate $RR_t = R_t - E_t(\pi_{t+1})$ and $ygap_t^w$ is the world’s output gap.

**Real interest rates**

The ex-ante real interest rate $RR_t = R_t - E_t(\pi_{t+1})$ affects the output gap only insofar as it deviates from the natural rate of interest $RR^*_t$, which depends in part on shocks to potential output and shocks to the equilibrium real interest rate:

$$RR^*_t = RR^* + \frac{(1-\beta_R)}{\beta_R} \epsilon_t y^* + \epsilon_{RR}^*;$$

The second term in equation (4), which relates potential output to the equilibrium real interest rate, can be derived from a standard new-keynesian model.\(^{10}\)

**Inflation**

Inflation depends on expected and lagged inflation, the lagged output gap, and the rate of real exchange rate depreciation:

$$\pi_t = \alpha_{lead}E_t(\pi_{t+1}) + (1-\alpha_{lead})\pi_{t-1} + \alpha_y ygap_{t-1} + \alpha_z(z_t-z_{t-1}) + \epsilon^\pi_t$$

(4)

**Uncovered interest parity**

The open economy dimension of the model is completed by a UIP-type equation relating the expected rate of exchange rate depreciation and interest rate differentials across two countries:

$$RR_t - RR^*_t = 4(z_e^{t+1} - z_t) + \epsilon_{RR}^*$$

(5)

Regarding $z_e^{t+1}$, the expected real exchange rate for next period, we allow for deviations from rational expectations:

$$z_e^{t+1} = \theta E_t(z_{t+1}) + (1-\theta)z_{t-1}$$

(6)

**Money demand**

The model features a standard equation for real money balances in level $m_t$:

$$m_t = \chi_yy_t - \chi_RR_t + u_t,$$

(7)

\(^{10}\)In fully micro-founded models, the IS equation also holds in levels, which implies that the natural rate of interest must adjust in response to $\epsilon^y_t$, to ensure the IS equation is consistent with the path for potential output.
where \( u_t \) follows a random walk with drift:

\[
\begin{align*}
    u_t &= \mu + u_{t-1} + \epsilon^u_t \\
\end{align*}
\]

(8)

**Observed nominal interest rates**

Finally, we extend the standard model by assuming that the observed short term interest rate—the one under the direct control of the central bank—is a noisy measure of the nominal interest rate that matters for private decisions:

\[
R^N_t = R_t + \epsilon^N_t
\]

(9)

We think of this relation as resulting from a combination of limited participation in the financial system and redistribution shocks between participating and non-participating agents.

### A. Monetary policy under complete information

The discussion of monetary policy is divided in two parts. The first section describes how money targets are set and what they imply for short-term interest rates, under the assumption that the central bank has complete information about current endogenous variables and shocks. The second section analyzes how the modeling of incomplete information affects the ex-post adherence to previously announced targets.

#### 1. Money targets

We consider here the nature of the monetary policy reaction function with the monetary aggregate—the growth rate of nominal money balances—as instrument. There are several ways to set targets on money. One obvious possibility, which we explore below, is to set targets following—informally—basic principles of monetary policy. We show that the resulting reaction function is the equivalent of the familiar interest rate reaction function. The exercise serves to highlight, however, a series of practical issues in the implementation of operational targets on money.

The objective of monetary policy is to anchor inflation expectations while steering the economy toward its flexible price equilibrium, i.e., an equilibrium where the output gap is zero, the ex-ante real interest rate equals the equilibrium real interest rate, and inflation equals its target or long run value (\( \pi^* \)). To the extent that monetary aggregates are used as monetary policy instruments, targets on these variables should be consistent with such objectives. This implies that, by analogy with the

\[\text{One obvious possibility is simply to follow a constant money growth rule that does not respond to any endogenous variable.}\]
Taylor rule, monetary aggregates themselves should aim toward their flexible price counterpart while also helping implement an active monetary policy. We therefore start by defining flexible price values for real monetary variables, which are identified with a “star” (\(^*\)). Flex-price real money balances are given by:

\[
\Delta m_t^* = \chi y \Delta y_t^* - \chi R \Delta R R_t^* + \Delta u_t,
\]

which is similar to the first difference of the money demand equation in (7), with the exception that right-hand side variables have been replaced by their flex-price levels. The nominal growth rate for money at the flex-price equilibrium is the following:

\[
\Delta M_t^* = \Delta m_t^* + \pi^*,
\]

\(\Delta M_t^*\) provides the starting point for setting targets on money growth. One important implication is that the authorities should accommodate any exogenous change in money demand given by \(u_t\), which highlights one of the well known challenges of money targeting: identifying and forecasting changes in velocity.

It is not sufficient for the central bank to target \(\Delta M_t^*\), for two reasons. First, the economy was most likely not at its flex-price equilibrium at time \(t - 1\). We therefore consider correcting targets by the lagged difference between the actual value of real money balances last quarter and its flex-price counterpart:

\[
\Delta M_t^T = \Delta M_t^* - mgap_{t-1},
\]

where the money gap (\(mgap_t\)) is defined as:

\[
mgap_t = m_t - m_t^*,
\]

i.e., the amount of excess real money balances (excess liquidity) relative to its flexible price level. Identifying the lagged money gap is therefore another component of money targeting.

In addition to the above adjustment, we want to consider an active monetary policy, which implies that the money growth target must respond to inflationary pressures and or fluctuations in the output gap. An important issue is whether policy makers should attempt to contain current or expected inflationary pressures. The literature on interest rate rules emphasizes the forward-looking nature of monetary policy: authorities should let current inflation bygones (i.e., deviations of current inflation from its target) be bygones and instead respond to deviations of future inflation from target. The implication of such adage for nominal targets is that current inflation misses should be accommodated, while expected inflation misses should elicit a reduction in money growth. Otherwise, by not adjusting to current inflationary misses, monetary policy would risk being
excessively tight or loose when it is not necessary. The money target should thus be:

$$
\Delta M_t^T = \Delta M_t^* - mgap_t - 1 + (\pi_t - \pi^*) - \delta(E_t(\pi_{t+1} - \pi^*) - \gamma ygap_t,
$$

(13)

where we have assumed that the authorities target next period’s inflation.

To derive the implications of equation (13) for the relevant nominal interest rate, we replace it in equation (7) and solve for the nominal interest rate, which gives the following target for $R_t$:

$$
R_t^T = RR_t^* + \pi^* + \phi_\pi E_t(\pi_{t+1} - \pi^*) + \phi_ygap ygap_t,
$$

(14)

where:

$$
\phi_\pi = \frac{\delta}{\chi_R}, \quad \phi_ygap = \frac{(\gamma + \chi_y)}{\chi_R}.
$$

By setting money targets in the way that has just been described, the authorities are in fact ensuring that the relevant interest rate follows a forward looking Taylor rule, where the parameters are a combination of policy parameters (from (13)) and short-run money demand parameters. The interest elasticity of money demand ($\chi_R$) plays a key role. A low $\chi_R$ raises the interest rate response to inflation, in which case, a relatively mild response of money growth targets to expected inflation will satisfy the Taylor principle and provide a sufficient degree of policy hawkishness.

In the case of the output gap, the active response of money targets to that variable is compounded with the endogenous response of interest rates to the output gap implied by the money demand equation. This suggests that, if $\chi_{R1}$ is sufficiently low and $\chi_{Y1}$ is moderately high, then the authorities may want to accommodate a positive output gap, i.e., set $\gamma < 0$, when setting money targets! While such findings may appear counterintuitive, they highlight the subtleties associated with money targeting.

In sum, a money-targeting rule that satisfies certain basic principles of monetary policy is equivalent to a forward-looking Taylor rule, under complete information. While establishing such equivalence is useful and can help money-targeting countries decide on their targets, it does not justify using money as the operational target variable of monetary policy. As indicated in the previous discussion, money targeting does require more information relative to its corresponding Taylor rule representation: on money demand shocks, on the shape of the money demand equation and on lagged values of the money gap. While this additional information is perfectly available under the

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12See Clarida, Gali and Gertler (1999). Contrary to this assertion, recent papers have found that there are important advantages to not let bygones be bygones, i.e., that it is preferable to target the price level rather than inflation, especially when the probability of policy mistakes is higher (see Gorodnichenko and Shapiro (2007) and Aoki and Nikolov (2004)). We do not evaluate the implications of price-level targeting here.

13For simplicity, we abstain from introducing lagged interest rates in the Taylor Rule, which has been found to be a desirable feature of optimal interest rate rules (see Woodford (1999)).
assumption that information is complete, it becomes a first-order concern when there is considerable uncertainty about the state of the economy. As we will see, a strict targeting of money is not the optimal solution for this very reason. That does not imply however, that, money targets should not play any role in the conduct of monetary policy. We address this issue in the next section.

B. Monetary policy under incomplete information

We now introduce a relatively simple form of incompleteness of information, which is based on Aoki (2003). First, we assume that interest rate and money target announcements are made on the basis of \( t-1 \) information available to the central bank.\(^{14}\) We will to refer to the previously announced target as \( \Delta M^T_{t|t-1} \), and its formula is the following:

\[
\Delta M^T_{t|t-1} = E^C_B(t-1)(\Delta M^T_t),
\]

where \( \Delta M^T_t \) is defined as in (13), and \( E^C_B(t-1) = E(\cdot | I^C_B(t-1)) \). \( I^C_B \) refers to the set of variables available to the central bank when announcing the money growth target. In line with our previous discussion, there is an interest rate target \( R^T_{t|t-1} \) that is consistent with the money target defined in (13):

\[
R^T_{t|t-1} = E^C_B(t-1)(RR^*_{t}) + \pi^* + \phi_\pi(E^C_B(t-1)(\pi_{t+1}) - \pi^*) + \phi_{ygap}E^C_B(t-1)(ygapt)
\]

In this paper we assume that information at time \( t-1 \) is complete, i.e., the central bank is fully aware of all the shocks and values for endogenous variables at the time:

\[
E^C_B(t-1)(\cdot) = E(t-1)(\cdot) = E(\cdot | I_{t-1}),
\]

where \( I_{t-1} \) refers to the entire set of variables (both endogenous and exogenous) determined at time \( t-1 \).

Note that since \( E_{t-1}(\epsilon^R_{t}) = 0 \), the expected value—at time \( t-1 \)—for the observed short term interest rate \( R^N_{t} \) is the same as the interest rate target on \( R_t \):

\[
E_{t-1}(R^N_{t}) = R^T_{t|t-1}
\]

Ex-ante, \( \Delta M^T_{t|t-1} \) and \( R^T_{t|t-1} \) are mutually consistent targets for monetary policy. When time \( t \) arrives however, the central bank observes \( R^N_{t} \) and \( \Delta M_{t} \). It must decide whether to make the observed rate

\(^{14}\)Such assumption is consistent with the standard implementation of money targeting in African countries, where targets are announced at least one quarter in advance, often in the context of an IMF-supported program.
adhere to its interest rate target or whether instead to make the money growth rate adhere to its pre-announced target.

This tradeoff can be characterized with the following equation—which we assume the central bank is fully committed to:

$$\lambda(R_t^N - R_{t|t-1}^T) - (1 - \lambda)(\Delta M_t - \Delta M_{t|t-1}^T) = 0$$  \hspace{1cm} (18)

There are two ways of interpreting equation (18), depending on how it is rearranged. Placing the short term interest rate on the left hand side, we get the following relation:

$$R_t^N = R_{t|t-1}^T + \frac{(1 - \lambda)}{\lambda} (\Delta M_t - \Delta M_{t|t-1}^T)$$

We can think of this version of equation (18) as indicating how the short-term interest rate is revised following current developments in the money market. To the extent that money growth at time $t$ can help the central bank revise its forecast of its target variables (either the output gap, the equilibrium real interest rate, or expected inflation), then the interest rate should deviate from its previously announced target. This suggests a useful way for choosing $\lambda$: $\lambda$ should be lower when money growth can help improve the forecast of the target variables.

We can also rearrange equation (18) by putting money growth in the left hand side:

$$\Delta M_t = \Delta M_{t|t-1}^T + \frac{\lambda}{(1 - \lambda)} (R_t^N - R_{t|t-1}^T)$$

We can interpret this version of equation (18) in the following way: it describes whether the authorities decide (and by how much) to inject more or less money than was originally targeted, based solely on what they observe in the money market at time $t$, i.e., on the deviations between the actual interest rate and what was previously announced/forecasted. For example, the central bank may decide to inject more liquidity if interest rates are higher than expected, or it may also decide to stay put and keep money growth equal to its target.

Of course, both interpretations are mutually consistent. However, the first one is more useful in frameworks where the interest rate is the operational target of policy, whereas the second version is better suited for frameworks where money is the operational target instead.
More generally, equation (18) is meant to capture the problem of adjusting the operational target in-between policy meetings, i.e., between target announcements. Regardless of the operational target chosen, the information available during this period is generally limited to high frequency money market information. This is especially the case for low-income countries, where macroeconomic data are often limited, poor in quality, and not available on a timely basis. The challenge for policy makers is to make the best use of this information, i.e., to choose a value of $\lambda$. We leave the discussion on how to choose the optimal $\lambda$ for section 4.

Note that the model can be solved, simulated and estimated for any given value of $\lambda$. Indeed, for estimation purposes, it is not necessary to assume that the central bank makes the optimal use of the money market information. Our approach is to treat $\lambda$ as a free parameter and estimate it using the available data, the results of which are presented in the next section. In section 4, we then compare the estimated parameter with the optimal $\lambda$ derived by using the estimated structural parameters and variances.

III. Estimating the Model

A. Bayesian Estimation Strategy

We estimate the model to quantify the variances of the shocks and the parameters of the model, since it is the relative magnitudes of these values that will govern how relevant monetary aggregates should be in the conduct of monetary policy. We use Bayesian methods for the estimation.

For the purpose of the estimation, the state-space representation of the model’s solution will serve as the model’s state transition equation. The measurement equation links the set of observed variables $O_t$ to the variables in the model $X_t$:

$$O_t = \Lambda X_t$$

(19)

where $O_t = [\pi_t, y_t, z_t, R_t, \Delta M_t, ygap_t^w, RRe_t^w]'$. The matrix $\Lambda$ defines the relationship between the observed and the state variables.\footnote{See Chapter 9 in Walsh 2003.}

\footnote{The exchange rate is also potentially informative and available at high frequency. We return to this point in the conclusion.}

\footnote{See Schorfheide (2000) and Lubik and Schorfheide (2006) for details of the methodology and its advantages over other methods in estimating DSGE models.}

\footnote{The data set consists of quarterly observations on several macroeconomic variables over the period 2003:4-2008:4, 2002:2-2008:4, and 1998:1-2008:4 for Ghana, Tanzania and Uganda, respectively. The variables under considerations are real GDP, CPI, nominal interest rate, exchange rate (period average) and reserve money. The quarterly real GDP were constructed by interpolating annual series using the data on exports, imports and fiscal revenues (Tanzania and Uganda); and domestic credit to private sector (Ghana) following the approach described in Fernandez (1981). All data were}
Given this representation, we compute the likelihood function for the observable variables recursively using the Kalman filter, which is then combined with the prior distributions to form the posterior densities of the parameters. Because the latter cannot be directly simulated, we use Monte Carlo Markov Chain methods which approximate the generation of random variables from the posterior distribution, after finding the parameters that maximize the posterior density using an optimization routine.\(^{19}\)

We impose dogmatic priors for several parameters (Table 1). Some of these are the steady-state values of the model \((\pi^*, g^*, r_r^*)\) and some are the persistence of the exogenous processes which are defined outside the model \((\rho_{\text{ygap}w}, \rho_{\text{RR}w})\). Varying \(\theta\) (the forward-looking component in the UIP equation) does not affect the joint distribution of the vector of observable variables—it is not identified—and hence it is calibrated as well.\(^{20}\) The vector of estimated parameters is given by \(\Upsilon\):

\[
\Upsilon = [\beta_{\text{lead}}, \beta_R, \beta_w, \alpha_{\text{lead}}, \alpha_y, \alpha_z, v^*, \phi_l, \phi_{ygap}, \lambda]
\]

where all parameters are assumed to be independent \textit{a priori}.

### Table 1: Calibrated parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Ghana</th>
<th>Tanzania</th>
<th>Uganda</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\pi^*)</td>
<td>10</td>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td>(g^*)</td>
<td>12</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>(r_r^*)</td>
<td>5</td>
<td>3.5</td>
<td>3</td>
</tr>
<tr>
<td>(\theta)</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>(\rho_{\text{ygap}w})</td>
<td>0.82</td>
<td>0.82</td>
<td>0.82</td>
</tr>
<tr>
<td>(\rho_{\text{RR}w})</td>
<td>0.82</td>
<td>0.82</td>
<td>0.82</td>
</tr>
</tbody>
</table>

Tables 2-5 present the prior distributions for the parameters and the shocks. The type of distributions that are chosen take into account the range of plausible parameter values. Specifically, the beta distribution is used for the parameters with values in the range between 0 and 1; while the gamma distribution is used for parameters that are positive and might have a value greater than 1. All the variances are assumed to have an Inverted Gamma distribution, which usually represents the structure of the shocks well. We impose the same priors for all countries. Prior means are guided in

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\(^{19}\)We employ a random walk Metropolis-Hasting algorithm to approximate the moments of the posterior distribution and to calculate the harmonic mean estimator. We use Dynare.

\(^{20}\)The identification problem mentioned here arises from the high correlation between \(\alpha_z\) and \(\theta\) in the posterior distribution through equations (4) and (5). This makes impossible to identify these two parameters. Because we estimate structural parameters which are in fact a function of several “deep” parameters, the estimation analysis requires a careful investigation of the parameter space.

obtained from IMF’s International Financial Statistics. Foreign output gap and real interest rate are proxied by U.S. variables retrieved from OECD’s Analytic Database.
part by calibrated or estimated parameter values available in other studies such as Berg, Karam and Laxton (2006), Adam and others (2006), Peiris and Saxegaard (2007), and Harjes and Ricci (2008).

Table 2. Prior and posterior distributions for parameters and volatilities: Ghana

<table>
<thead>
<tr>
<th>Shocks</th>
<th>Distribution</th>
<th>Mean</th>
<th>Std. dev.</th>
<th>Mean</th>
<th>90% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_y$</td>
<td>Beta</td>
<td>0.3</td>
<td>0.15</td>
<td>0.5335</td>
<td>0.3455</td>
</tr>
<tr>
<td>$\beta_R$</td>
<td>Gamma</td>
<td>0.07</td>
<td>0.1</td>
<td>0.1954</td>
<td>0.0965</td>
</tr>
<tr>
<td>$\beta_z$</td>
<td>Gamma</td>
<td>0.10</td>
<td>0.05</td>
<td>0.0951</td>
<td>0.0264</td>
</tr>
<tr>
<td>$\beta_w$</td>
<td>Gamma</td>
<td>0.25</td>
<td>0.2</td>
<td>0.2119</td>
<td>0.0027</td>
</tr>
<tr>
<td>$\alpha_{lead}$</td>
<td>Beta</td>
<td>0.4</td>
<td>0.05</td>
<td>0.6967</td>
<td>0.6819</td>
</tr>
<tr>
<td>$\alpha_y$</td>
<td>Gamma</td>
<td>0.25</td>
<td>0.2</td>
<td>0.4077</td>
<td>0.0057</td>
</tr>
<tr>
<td>$\alpha_z$</td>
<td>Gamma</td>
<td>0.2</td>
<td>0.1</td>
<td>0.1501</td>
<td>0.0375</td>
</tr>
<tr>
<td>$\chi_y$</td>
<td>Gamma</td>
<td>0.7</td>
<td>0.2</td>
<td>0.7598</td>
<td>0.3833</td>
</tr>
<tr>
<td>$\chi_R$</td>
<td>Gamma</td>
<td>0.4</td>
<td>0.1</td>
<td>0.3773</td>
<td>0.2250</td>
</tr>
<tr>
<td>$\nu^*$</td>
<td>Gamma</td>
<td>1.00</td>
<td>0.2</td>
<td>1.0660</td>
<td>0.7601</td>
</tr>
<tr>
<td>$\phi_\pi$</td>
<td>Gamma</td>
<td>2</td>
<td>0.2</td>
<td>1.7637</td>
<td>1.3131</td>
</tr>
<tr>
<td>$\phi_{ygap}$</td>
<td>Gamma</td>
<td>0.5</td>
<td>0.2</td>
<td>0.4137</td>
<td>0.1237</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Beta</td>
<td>0.5</td>
<td>0.2</td>
<td>0.9285</td>
<td>0.8444</td>
</tr>
<tr>
<td>$\sigma_\pi$</td>
<td>Inverted Gamma</td>
<td>2</td>
<td>Inf.</td>
<td>8.0704</td>
<td>5.5556</td>
</tr>
<tr>
<td>$\sigma_y^*$</td>
<td>Inverted Gamma</td>
<td>0.5</td>
<td>Inf.</td>
<td>3.2986</td>
<td>2.3317</td>
</tr>
<tr>
<td>$\sigma_{RR^*}$</td>
<td>Inverted Gamma</td>
<td>0.5</td>
<td>Inf.</td>
<td>1.4262</td>
<td>0.1755</td>
</tr>
<tr>
<td>$\sigma_v$</td>
<td>Inverted Gamma</td>
<td>4</td>
<td>Inf.</td>
<td>5.4720</td>
<td>3.5696</td>
</tr>
<tr>
<td>$\sigma_R^*$</td>
<td>Inverted Gamma</td>
<td>4</td>
<td>Inf.</td>
<td>4.0923</td>
<td>1.1416</td>
</tr>
<tr>
<td>$\sigma_{RR}$</td>
<td>Inverted Gamma</td>
<td>5</td>
<td>Inf.</td>
<td>16.0788</td>
<td>10.4299</td>
</tr>
<tr>
<td>$\sigma_{ygapw}$</td>
<td>Inverted Gamma</td>
<td>0.25</td>
<td>Inf.</td>
<td>0.3863</td>
<td>0.2922</td>
</tr>
<tr>
<td>$\sigma_{RR^w}$</td>
<td>Inverted Gamma</td>
<td>0.3</td>
<td>Inf.</td>
<td>0.7366</td>
<td>0.5400</td>
</tr>
</tbody>
</table>

Our choice is also guided by common-held views about low-income countries, as well as country-specific information. To reflect a possible weakness of the interest-rate channel, the prior mean for $\beta_R$ is set at a relatively low level (0.07). The prior means for $\chi_y$ and $\chi_R$ are imposed in light of the empirical studies on money demand in both Ghana and Uganda (Kallon, 1992; and Andoh and Chappell, 2002, Nachega, 2001; and Kararach, 2002). Regarding priors for monetary policy parameters, we set $\phi_\pi$ and $\phi_{ygap}$ to 2 and 0.5, respectively; the prior mean of $\lambda$ is chosen to be 0.5 for all countries, with relatively large variance.

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21 See Mishra and others (2010) for a recent study on the interest rate channel—and the monetary transmission mechanism—in low-income countries.

22 See also Rother (1999) and Hamori (2008) for the estimation of money demand function in African countries.
Table 3. Prior and posterior distributions for parameters and volatilities: Uganda

<table>
<thead>
<tr>
<th>Shocks</th>
<th>Prior Distribution</th>
<th>Mean</th>
<th>Std. dev.</th>
<th>Mean</th>
<th>90% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_y$</td>
<td>Beta</td>
<td>0.3</td>
<td>0.15</td>
<td>0.7470</td>
<td>0.5836</td>
</tr>
<tr>
<td>$\beta_R$</td>
<td>Gamma</td>
<td>0.07</td>
<td>0.1</td>
<td>0.3301</td>
<td>0.1330</td>
</tr>
<tr>
<td>$\beta_z$</td>
<td>Gamma</td>
<td>0.1</td>
<td>0.05</td>
<td>0.0853</td>
<td>0.0188</td>
</tr>
<tr>
<td>$\beta_w$</td>
<td>Gamma</td>
<td>0.25</td>
<td>0.2</td>
<td>0.2136</td>
<td>0.0023</td>
</tr>
<tr>
<td>$\alpha_{lead}$</td>
<td>Beta</td>
<td>0.4</td>
<td>0.05</td>
<td>0.6707</td>
<td>0.6319</td>
</tr>
<tr>
<td>$\alpha_y$</td>
<td>Gamma</td>
<td>0.25</td>
<td>0.2</td>
<td>0.6036</td>
<td>0.1701</td>
</tr>
<tr>
<td>$\alpha_z$</td>
<td>Gamma</td>
<td>0.2</td>
<td>0.1</td>
<td>0.1344</td>
<td>0.0345</td>
</tr>
<tr>
<td>$\chi_y$</td>
<td>Gamma</td>
<td>0.7</td>
<td>0.2</td>
<td>1.3506</td>
<td>0.8762</td>
</tr>
<tr>
<td>$\chi_R$</td>
<td>Gamma</td>
<td>0.4</td>
<td>0.1</td>
<td>0.5817</td>
<td>0.4433</td>
</tr>
<tr>
<td>$\nu^*$</td>
<td>Gamma</td>
<td>0.75</td>
<td>0.2</td>
<td>0.9798</td>
<td>0.5855</td>
</tr>
<tr>
<td>$\phi_\pi$</td>
<td>Gamma</td>
<td>2</td>
<td>0.2</td>
<td>2.0030</td>
<td>1.6507</td>
</tr>
<tr>
<td>$\phi_{ygap}$</td>
<td>Gamma</td>
<td>0.5</td>
<td>0.2</td>
<td>0.5227</td>
<td>0.2032</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Beta</td>
<td>0.5</td>
<td>0.2</td>
<td>0.3377</td>
<td>0.1785</td>
</tr>
<tr>
<td>$\sigma_\pi$</td>
<td>Inverted Gamma</td>
<td>2</td>
<td>Inf.</td>
<td>6.7967</td>
<td>5.5682</td>
</tr>
<tr>
<td>$\sigma_y^*$</td>
<td>Inverted Gamma</td>
<td>0.5</td>
<td>Inf.</td>
<td>2.8802</td>
<td>2.2455</td>
</tr>
<tr>
<td>$\sigma_{RR^*}$</td>
<td>Inverted Gamma</td>
<td>0.5</td>
<td>Inf.</td>
<td>0.4795</td>
<td>0.0985</td>
</tr>
<tr>
<td>$\sigma_v$</td>
<td>Inverted Gamma</td>
<td>4</td>
<td>Inf.</td>
<td>3.3189</td>
<td>2.5337</td>
</tr>
<tr>
<td>$\sigma_{R^*}$</td>
<td>Inverted Gamma</td>
<td>4</td>
<td>Inf.</td>
<td>6.1232</td>
<td>4.6535</td>
</tr>
<tr>
<td>$\sigma_{RR}$</td>
<td>Inverted Gamma</td>
<td>5</td>
<td>Inf.</td>
<td>15.0235</td>
<td>12.3039</td>
</tr>
<tr>
<td>$\sigma_{ygap}$</td>
<td>Inverted Gamma</td>
<td>0.25</td>
<td>Inf.</td>
<td>0.3688</td>
<td>0.3065</td>
</tr>
<tr>
<td>$\sigma_{RR^*}$</td>
<td>Inverted Gamma</td>
<td>0.3</td>
<td>Inf.</td>
<td>0.6055</td>
<td>0.5042</td>
</tr>
</tbody>
</table>

B. Results

The parameter and shock estimates are given in Tables 2-5, based on 200,000 draws from the posterior density, which is sufficient to obtain convergence and relative stability in the parameter moments.

There are several results worth emphasizing. Contrary to our priors, the interest elasticity in the IS equation ($\beta_R$) is relatively high across all countries, which suggests that aggregate demand is sensitive to the (shadow) interest rate. Furthermore, forward looking behavior plays an important role in the monetary transmission mechanism, as $\beta_y$ and $\alpha_{lead}$—the parameters that describe how expectations of the future affect both the output gap and inflation—are greater than 0.5 in all three countries.
Table 4. Prior and posterior distributions for parameters and volatilities: Tanzania

<table>
<thead>
<tr>
<th>Shocks</th>
<th>Prior Distribution</th>
<th>Results from Posterior Maximization</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Distribution</td>
<td>Mean</td>
</tr>
<tr>
<td>$\beta_y$</td>
<td>Beta</td>
<td>0.3</td>
</tr>
<tr>
<td>$\beta_R$</td>
<td>Gamma</td>
<td>0.07</td>
</tr>
<tr>
<td>$\beta_z$</td>
<td>Gamma</td>
<td>0.1</td>
</tr>
<tr>
<td>$\beta_w$</td>
<td>Gamma</td>
<td>0.25</td>
</tr>
<tr>
<td>$\alpha_{lead}$</td>
<td>Beta</td>
<td>0.4</td>
</tr>
<tr>
<td>$\alpha_y$</td>
<td>Gamma</td>
<td>0.25</td>
</tr>
<tr>
<td>$\alpha_z$</td>
<td>Gamma</td>
<td>0.2</td>
</tr>
<tr>
<td>$\chi_y$</td>
<td>Gamma</td>
<td>0.7</td>
</tr>
<tr>
<td>$\chi_R$</td>
<td>Gamma</td>
<td>0.4</td>
</tr>
<tr>
<td>$\nu^*$</td>
<td>Gamma</td>
<td>0.45</td>
</tr>
<tr>
<td>$\phi_{\pi}$</td>
<td>Gamma</td>
<td>2</td>
</tr>
<tr>
<td>$\phi_{ygap}$</td>
<td>Gamma</td>
<td>0.5</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Beta</td>
<td>0.5</td>
</tr>
<tr>
<td>$\sigma_{\pi}$</td>
<td>Inverted Gamma</td>
<td>2</td>
</tr>
<tr>
<td>$\sigma_{\nu^*}$</td>
<td>Inverted Gamma</td>
<td>0.5</td>
</tr>
<tr>
<td>$\sigma_{RR^*}$</td>
<td>Inverted Gamma</td>
<td>0.5</td>
</tr>
<tr>
<td>$\sigma_v$</td>
<td>Inverted Gamma</td>
<td>4</td>
</tr>
<tr>
<td>$\sigma_R$</td>
<td>Inverted Gamma</td>
<td>4</td>
</tr>
<tr>
<td>$\sigma_{RR}$</td>
<td>Inverted Gamma</td>
<td>5</td>
</tr>
<tr>
<td>$\sigma_{ygap}^w$</td>
<td>Inverted Gamma</td>
<td>0.25</td>
</tr>
<tr>
<td>$\sigma_{RR}^w$</td>
<td>Inverted Gamma</td>
<td>0.3</td>
</tr>
</tbody>
</table>

The posterior distribution assigns a much larger probability density to the higher values of $\lambda$ for Ghana (0.99). As mentioned earlier, the country abandoned the reference to money in its policy framework and relies on the short-term interest rate as an operational target during the period we analyze. In Uganda, consistent with its current de jure framework, money targets play a bigger role in policy decisions: the estimated $\lambda$ is much lower (0.34). The estimate for $\lambda$ in Tanzania is somewhat in the middle (0.66), which suggests that Tanzania’s adherence to the money targeting framework is more flexible than Uganda’s.

Regarding volatilities, the estimated standard deviation of money demand shocks ($\sigma_{\nu}$) is larger in Tanzania and smaller in Uganda; while the estimated volatility of interest rate noise is larger in Uganda and smaller in Ghana. While it is clear that money demand shocks (interest rate noise) make it more (less) costly to adhere to money targets, determining how the relative magnitude of shocks and the value of key parameters of the model would affect the optimal target adherence necessitates a more formal analysis, which is taken up in the next section.
We now present a method for deriving the optimal value of $\lambda$. We use the representation of policy where money growth is the left-hand side variable in the policy rule, although results are exactly the same if one uses the alternative representation. Allowing for money growth to deviate from its previously announce target can be thought as adjusting the policy instrument in response to additional information on the central bank’s target variables:

$$\Delta M_t = E(\Delta M^T_{t-1} | I_t, R_t^N)$$ (21)

When measured as deviations from the previously announced target, and after applying the law of iterated expectations, equation (21) can be restated as follows:

$$\Delta M_t - \Delta M^T_{t-1} = \chi_y E(\Delta y^*_t - E_{t-1}(\Delta y^*_t) | R_t^N - R^T_{t|t-1}) - \chi_R E(\Delta RR^*_t - E_{t-1}(\Delta RR^*_t) | R_t^N - R^T_{t|t-1})$$

$$... + E(\Delta u_t - E_{t-1}(\Delta u_t) | R_t^N - R^T_{t|t-1}) + E(\pi_t - E_{t-1}(\pi_t) | R_t^N - R^T_{t|t-1})$$

$$... - \delta E(\pi_{t+1} - E_{t-1}(\pi_{t+1}) | R_t^N - R^T_{t|t-1}) - \gamma E(ygap_t - E_{t-1}(ygap_t) | R_t^N - R^T_{t|t-1})$$ (22)

In light of equation of equation (22), choosing $\lambda$ amounts to a signal extraction problem, where the central bank revises the estimates of all the variables that influence its policy decision following unexpected movements in the observed interest rate relative to its initial forecast.

At this stage it is helpful to further clarify what is meant by the optimality of $\lambda$. The money market information—observed interest rates for those countries that control the money supply—serves as an indicator variable. Therefore the choice of the optimal weight to assign to unexpected movements in this variable, when adjusting the policy instrument (money growth), is based on whether these unexpected movements help reassess the value of all the target variables that enter the monetary policy rule. Specifically, as will be shown below, the optimal value of lambda is directly related to the best linear projection of the right hand side of equation (22). We rely on the Kalman filter to provide such projection, and the optimal value of lambda is derived from the value of the Kalman gain.\textsuperscript{23} Note that the optimality is based on the information content of the observed variables and not on any measure of welfare.

This signal extraction problem is not straightforward, however. In a static model, the variables that the central bank is trying to forecast, which influence its policy decision, are themselves influenced by the policy choices. The solution of the (static) model needs to incorporate this circularity. Moreover, as explained in Svensson and Rogoff (2004), the problem increases in a dynamic setting,\textsuperscript{23}

\textsuperscript{23}See Chapter 13 in Hamilton (1995), for a detailed presentation of the Kalman filter.
where the private sector’s expectations of the future also depend on the signal-extraction problem, both present and future.\textsuperscript{24}

While our model is potentially subject to this additional complication, the type of incompleteness of information that we introduce helps avoid it. In particular, we can safely assume that the private sector’s expectations of the future are fully consistent with the version of the model where the authorities have complete information about the state of the economy. This assumption is valid because, once the uncertainty faced by the central bank at time $t$ is resolved, announcements of interest rate and money growth targets for time $t + 1$ and beyond are the rational expectations forecast for these variables. In practice, this implies that we can solve for the expected future values of all the endogenous variables in terms of current variables using the version of the model with complete information. The model can then be reduced to a system of equations involving contemporaneous and lagged values of endogenous variables only, which facilitates the signal-extraction problem considerably.

To show how lambda is derived—in a way that is consistent with the previous discussion—we start by writing the complete information version of our model in vector form:

$$\Gamma_1 E_t X_{t+1} + \Gamma_2 X_t + \Gamma_3 X_{t-1} = \Gamma \varepsilon_t,$$

where $X_t = (\Delta M_t, Y_t)$, $Y_t = (R_t, \text{ygap}_t, \pi_t, \Delta y_t, \Delta M^*_t, \Delta y^*_t, RR_t, \Delta z_t, RR^*_t, RR^*_t, \text{ygap}_t^w, m_t, m^*_t, \text{mgap}_t, R_t|_{t-1}, \Delta M_t|_{t-1}, E_t[\text{ygap}_{t+1}], E_t[\pi_{t+1}], \Delta u_t)$. The vector $\varepsilon_t$ includes all the structural shocks but excludes the interest-rate noise $\varepsilon^R_t$. Note that the benchmark version of the model—with incomplete information—has the same structure as the above system except for the policy equation, which we have ordered first.

The rational-expectations solution of the complete-information model admits the following representation:

$$X_t = \Upsilon X_{t-1} + \Upsilon \varepsilon_t,$$  \hspace{1cm} (23)

We can now use $\Upsilon X$ to solve for $E_t X_{t+1}$ in our benchmark model. Inserting $\Upsilon X$ in the block of $\Gamma_1$ that describes the non-policy part of the model, and after rearranging terms, yields the following dynamic system for the $Y$ vector:

$$Y_t = A Y_{t-1} + B \Delta M_t + B_2 \Delta M_{t-1} + C C \varepsilon_t,$$  \hspace{1cm} (24)

\textsuperscript{24}In an optimal monetary policy framework, Svensson and Woodford (2004) refer to this problem as a failure of the separation principle, i.e., that the central bank’s optimal estimate of the state of the economy is not independent of the parameters that describe the central bank’s loss function. This is a general property of models with information that is both incomplete and asymmetric, such as ours.
Taking first differences between equation (24) and a version of equation (24) with the expectations operator (evaluated at time $t - 1$) applied to both sides yields:

$$\widetilde{Y}_t = Y_t - E_{t-1}[Y_t] = B(\Delta M_t - E_{t-1}[\Delta M_t]) + C\varepsilon_t = B\Delta \tilde{M}_t + C\varepsilon_t,$$

(25)

Equation (25) describes how non-policy variables, measured as deviations from time $t - 1$ expectations of those variables, depend on the policy response, measured as deviations in money growth from its previously announced target; it shows how changes in the policy stance will affect the equilibrium value of all variables—including the short-term interest rates and those variables targeted by the central bank.

We can now introduce the observed variable ($\tilde{R}_t^N = R_t^N - R_{t|t-1}^T$) as:

$$\tilde{R}_t^N = \Gamma_R \tilde{Y}_t + \epsilon_t^{RN},$$

(26)

where $\Gamma_R$ is a row vector with zeros everywhere except in the position that corresponds to $R_t^N$’s location in the $Y$ vector. While $\tilde{R}_t^N$ is an observed variable, it is not exogenous, as it depends on $\Delta \tilde{M}_t$. For the purposes of the signal extraction, we therefore create a new variable, $\tilde{\varphi}_t$, which is a linear combination of the two variables observed in the money market. This variable depends on exogenous shocks only:

$$\tilde{\varphi}_t = \tilde{R}_t^N - \Gamma_R B \Delta \tilde{M}_t = \Theta[R_t^N \Delta \tilde{M}_t]' = \Gamma_R C\varepsilon_t + \epsilon_t^{RN} = H\varepsilon_t + \epsilon_t^{RN},$$

(27)

where $\Theta$ is a row vector of size 1*2. We can now use equation (27) as the observation equation in the Kalman filtering problem, where the objective is to estimate $\varepsilon_t$. Defining $Q$ as the covariance matrix for structural shocks ($\varepsilon_t$) and $\Omega$ as the variance of interest rate noise $\epsilon_t^{RN}$, the Kalman gain matrix is given by:

$$K = QH(H'QH + \Omega)^{-1},$$

The equation used to update the estimates $\hat{\varepsilon}_t$ of current unobserved shocks is the following:

$$\hat{\varepsilon}_t = K\tilde{\varphi}_t = K\Theta[R_t^N \Delta \tilde{M}_t]'$$

(28)

Note that, once an estimate of $\hat{\varepsilon}_t$ has been derived, we can solve for the best forecast of the vector $\tilde{Y}_t$:

$$\tilde{Y}_t = P\hat{\varepsilon}_t = (I - B\Gamma_{\Delta M})^{-1} C\hat{\varepsilon}_t,$$

where $\Gamma_{\Delta M}$ is simply the vector representation of the money growth rule from equation (11). We also solve for the choice of the policy instrument—its deviation from the previously-set target—which is given by:

$$\Delta \tilde{M}_t = P_{\Delta M}\hat{\varepsilon}_t = \Gamma_{\Delta M}P\hat{\varepsilon}_t,$$

(29)
Finally, we can use equations (25), (26) and (27) to solve for the relation between \( \Delta \tilde{M}_t \) and \( \tilde{R}_t^N \):

\[
\Delta \tilde{M}_t = \Phi \tilde{R}_t^N,
\]

where \( \Phi \) is given by:

\[
\Phi = (1 - P_{\Delta M} K \Theta(1, 2))^{-1} P_{\Delta M} K \Theta(1, 1)
\]

We can use \( \Phi \) to derive the optimal value for \( \lambda \):

\[
\lambda = \frac{\Phi}{1 + \Phi}
\]

### A. Optimal \( \lambda \): Results for Ghana, Tanzania and Uganda

Table 5 summarizes the results. For Uganda and Tanzania, the optimal \( \lambda \) is close to the estimate from the data. This would seem to imply that both central banks are extracting information from the money market in a near-optimal way, even though Tanzania would benefit from higher adherence to their money targets. In the case of Ghana however, the optimal \( \lambda \) is much lower than the empirical estimate, which suggests the authorities would also benefit from paying closer attention to monetary aggregates.\(^{25}\) Such result reflects, inter alia, the lower volatility of money demand shocks and noisy observed short term interest rates in all three countries.

<table>
<thead>
<tr>
<th>Table 5: Estimated and optimal lambda (( \lambda ))</th>
<th>Ghana</th>
<th>Tanzania</th>
<th>Uganda</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated Lambda</td>
<td>0.9285</td>
<td>0.6642</td>
<td>0.3377</td>
</tr>
<tr>
<td>Optimal Lambda</td>
<td>0.3634</td>
<td>0.4421</td>
<td>0.3246</td>
</tr>
</tbody>
</table>

### 1. Sensitivity Analysis

We also perform a sensitivity analysis to assess how the optimal value of lambda varies with changes in key volatilities and parameters of the model. For the sake of brevity, we focus on Uganda, although results are similar for Ghana and Tanzania.

Figure 1 summarizes the sensitivity analysis exercise. First, the optimal lambda increases as money-demand volatility increases. Second, lambda decreases as the interest elasticity of money demand increases, since it implies the cost of policy mistakes—by adhering too strictly to money targets—is smaller. These results are consistent with the analysis in Poole (1970).

\(^{25}\)In the case of Ghana, the results are not very robust to changing the priors, and should be interpreted with even-more-than-usual care.
Figure 1. Uganda, Sensitivity Analysis for the Optimal Lambda
The optimal lambda also declines as the volatility of cost-push shocks increases, as well as with an increase in the volatility of shocks to the natural rate of interest. Although these shocks were not included in the original Poole analysis, their implication for lambda is straightforward. In the case of cost push shocks, adherence to money targets ensures that unexpected increases in inflation lead to an increase in the nominal interest rate, thus automatically tightening policy. Shocks to the natural rate of interest are equivalent to shocks to the IS curve in the IS/LM framework used in Poole. Adherence to money targets also helps offset these shocks by ensuring policy is automatically tightened. However, the volatility of the latter shocks is very small so the impact on the optimal lambda is negligible.

Finally, the noisiness of short-term interest rates plays an important role in the value of the optimal lambda. As the observed short-term interest rates become a noisier indicator of the relevant interest rate, monetary authorities pay less attention to movements in interest rates and adhere to their previously-announced money targets. However, even when noise volatility is set to zero, Uganda’s optimal lambda does not increase beyond 0.7, which implies it is still preferable to let interest rates fluctuate somewhat relative to their target.

2. Welfare implications of changing $\lambda$

As discussed in Coenen, Levin and Wieland (2007), there are several ways of assessing the optimal use of indicator variables—which in our case amounts to assessing the implications of various $\lambda$. One option is to focus on whether the uncertainty surrounding the estimation problem faced by the policymaker is reduced following the inclusion of the indicator variable in the central bank’s information set. This is done by comparing mean square errors from the central bank forecast. Another option is to assess the policy makers’ loss function directly.

We follow an approach that is similar to the evaluation of the loss function. In our case, we have not formally specified a central bank loss function, and we have assumed instead that policy makers follow a simple Taylor Rule. We therefore simply assess the implications of lambda for inflation and output gap volatility, which are implicit measures of welfare.

Table 6 summarizes the results for four different cases: the two extremes ($\lambda = 0$ and $\lambda = 1$), the optimal $\lambda$, and the estimated $\lambda$. In all three countries, strict adherence to interest rate targets generates highly volatile movements in both inflation and the output gap. Choosing the optimal lambda does not generate the lowest inflation volatility, although it does generate the lowest volatility of the output gap, while strict adherence to money targets generates the smallest movements in inflation but at the cost of larger movements in the output.
These results can be rationalized by the presence of relatively volatile cost push shocks in all three countries. Under strict adherence to money targets, these shocks generate an automatic increase in interest rates, which help offset the impact on inflation by tightening monetary policy and contracting the output gap. This explains why inflation volatility is smaller, and output gap movements are larger, when $\lambda = 0$. However, this combination of inflation and output volatility is not in accordance with the estimated coefficients in the Taylor rule. This can be deduced from the observation that, when policymakers optimally use the money market information to improve on their initial forecast of target variables, which is the case when $\lambda$ is chosen optimally, the policy response is more muted. More generally, the policy makers’ preferences, which are implicitly contained in the estimated coefficients in the Taylor rule, call for a smaller interest rate response.

These results also suggest that strict adherence to interest rate targets is not desirable either. However, extreme cases excluded, the welfare gains in terms of inflation and output stability are relatively small. This is because of our modeling of information incompleteness: uncertainty is always resolved at the end of period $t$ so it does not have long-lasting effects. We leave a more complete treatment of information incompleteness for future work.

Table 6: Inflation and output gap volatility

<table>
<thead>
<tr>
<th></th>
<th>Ghana</th>
<th></th>
<th>Uganda</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\lambda = 0$</td>
<td>12.08</td>
<td>$\lambda = 0$</td>
<td>10.33</td>
</tr>
<tr>
<td></td>
<td>Optimal lambda ($\lambda = 0.3634$)</td>
<td>12.80</td>
<td>Optimal lambda ($\lambda = 0.3246$)</td>
<td>10.95</td>
</tr>
<tr>
<td></td>
<td>Estimated lambda ($\lambda = 0.9285$)</td>
<td>14.49</td>
<td>Estimated lambda ($\lambda = 0.3377$)</td>
<td>10.99</td>
</tr>
<tr>
<td></td>
<td>$\lambda = 1$</td>
<td>14.71</td>
<td>$\lambda = 1$</td>
<td>14.20</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Tanzania</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\lambda = 0$</td>
<td>12.02</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Optimal lambda ($\lambda = 0.4421$)</td>
<td>12.38</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Estimated lambda ($\lambda = 0.6642$)</td>
<td>13.08</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\lambda = 1$</td>
<td>14.60</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3. Discussion

Our results are suggestive and should be interpreted with caution, for several reasons. First, the money targets we have focused on are those that have been derived from the model, which may be different from the ones that are actually set by the authorities. In the case of Uganda and Tanzania, these are in principle available, but Ghana does not set quarterly targets for money growth. It may be the case that, given the method by which actual targets are set, it may be preferable to deviate substantially—ex-post—from those targets, relative to what our results suggest.

Second, we have assumed that the only source of contemporaneous information comes from the money market. This assumption may not be very realistic, as the authorities do observe the nominal exchange rate contemporaneously. They may also observe preliminary (noisy) measures of inflation and output. These additional sources of inflation are likely to weaken the relevance of monetary aggregates in the policy framework.

We have assumed that observed interest rates are a noisy measure of the relevant interest rate, and we have found this channel to be quantitatively important for the optimal adherence to money targets. However, our results still obtain qualitatively even if interest rates are perfectly observed.

V. Alternative Rationalizations for the Role of Money in Policy Frameworks in LICs

In this paper, we have chosen one particular hypothesis on why money (potentially) matters. However, there are other possible reasons, which we briefly discuss in this section.

First, LICs in many cases possess weak institutions, including for the implementation of monetary and fiscal policy. Many have a recent history of high inflation, fiscal dominance of monetary policy, and public debt crises. Central banks may thus lack credibility to engage in inflation targeting or inflation targeting ‘lite’. In these circumstances, monetary aggregates may serve as visible signs of adherence to adequate monetary policy—‘a tripwire.’ Indeed, this monetary aggregate targeting may well have helped stabilize in the 1990s in a context in which fiscal deficits had been driving high inflation.\(^{26}\)

This explanation may explain the historical use of monetary targeting in many LICs, but it is inadequate as an explanation for the many LICs that achieved several years of low inflation. First, it seems unlikely that money targets continues to play this anchoring role once inflation has stabilized. Experience suggests many cases in which LICs that have achieved a measure of stabilization observe

\(^{26}\)Ghosh and others (2005) suggests this role for monetary aggregate targeting.
large and above-target growth rates of money without adverse effects on inflation or macroeconomic stability more generally. More systematic evidence is also consistent with this assessment (IMF 2008).

Moreover, as we have shown here, rigid adherence to money targets in the interest of credibility is a potentially costly monetary policy strategy implying excessive output and inflation volatility. Thus, given the above result, it should not be surprising that in countries that have achieved some credibility, policy is executed more flexibly.

An alternative explanations for the unusual emphasis on monetary aggregates in LICs is based on the special characteristics of financial markets. With thin markets, dependence on bank finance, and poor contract enforcement, interest rates may not play the same role in influencing aggregate demand or signaling the stance of policy, while there might be a more direct connection between monetary aggregates and activity or inflation. More broadly, it may be that money’s special role relates to the importance of the banking system relative to financial markets or other features of the financial system characteristic of LICs.

We have attempted to incorporate part of this discussion in our model through our assumption that observed interest rates are a noisy indicator of the relevant interest rate for private sector decisions. However, this should be thought of as a first stage in the modeling of the financial sector in LICs, and more work is needed in this area.

VI. Conclusion

We have shown how a standard New-Keynesian model can be extended to provide a potentially important role for monetary aggregates in the conduct of monetary policy. Empirical estimates of the model suggest that money should play an important role in all the three countries we consider.

We believe our results point to interesting, qualitative, conclusions but the limitations of the model, scarcity of data, and strong identifying assumptions argue against taking them too literally.

We keep most of the model very standard. As O’Connell (2010) emphasizes, LIC-specific modifications in any of the core relationships of the model will matter for the analysis of the policy framework, including the optimal use of money market information. One notable weakness is the treatment of the exchange rate. Almost all LICs that target money also conduct a managed-floating exchange rate regime. And the exchange rate is, like money aggregates, observable at high frequency. We leave this for future work.

We also think it makes sense to proceed in careful steps in modifying standard macro models to introduce other key features of low income countries. Indeed, we see this paper as part of a broader
research program to bring modern analytic frameworks to bear on the macroeconomic policy challenges of low-income countries. Thus, Berg, Mirzoev, Portillo, and Zanna (2010) build a tractable open economy new-Keynesian model with a number of LIC-specific features to better understand how the interaction of fiscal and monetary policies affect the macroeconomic effect of aid shocks. Berg, Gottschalk, Portillo and Zanna (2010) apply a two-sector growth model that focusses on the role of public investment to analyze the medium-long-run implications of persistent aid shocks.
REFERENCES


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