Working Paper

INTERNATIONAL MONETARY FUND
Taylor Rule Under Financial Instability

Sofía Bauducco, Aleš Bulíř, and Martin Čihák
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This paper contributes to the analysis of monetary policy in the face of financial instability. In particular, we extend the standard new Keynesian dynamic stochastic general equilibrium (DSGE) model with sticky prices to include a financial system. Our simulations suggest that if financial instability affects output and inflation with a lag and if the central bank has privileged information about credit risk, monetary policy that responds instantly to increased credit risk can trade off more output and inflation instability today for a faster return to the trend than a policy that follows the simple Taylor rule with only the contemporaneous output gap and inflation. The long-run impact on consumption from the augmented policy rule appears negligible, however.

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Author’s E-Mail Address: sofia.bauducco@upf.edu; abulir@imf.org; mcihak@imf.org

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1 Sofía Bauducco (Universitat Pompeu Fabra) visited the IMF in 2007. The paper benefited from comments by Enrica Detragiache and IMF seminar participants. All remaining errors are those of the authors.
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“[A] reaction function in which the real funds rate changes by roughly equal amounts in response to deviations of inflation from a target of 2 percent and to deviations of actual from potential output describes reasonably well what this committee has done since 1986.”

Janet Yellen at the January 1995 FOMC

“[S]imple instrument rules at most explain two-thirds of the empirical variance of interest rate changes.”


I. INTRODUCTION

How should changes in financial sector soundness affect monetary policy? In this paper, we answer this question by addressing two simplifications in the standard monetary dynamic stochastic general equilibrium (DSGE) model: (i) the omission of a financial system, and (ii) the omission of forward-looking variables in the policy (Taylor) rule. In our model, the central bank has privileged information about the health of the financial system, a reasonable assumption given that many central banks are involved in financial sector supervision, and even those that are not have access to a wealth of payment system data. Our simulation results show that if the central bank responds to a deterioration in the credit risk faced by the financial system by monetary easing, using an augmented rule, such a “preemptive strike” stabilizes inflation and output better in the short run than the simple Taylor rule that is usually assumed in the DSGE model. In other words, well-informed, forward-looking discretionary policymaking, that takes into account the default rate of financial intermediaries’ lending projects and its impact on future output developments, is preferable to simple backward-looking rules.

These findings have both positive and normative implications. As regards positive implications, the findings provide model justification for the existing central bank practice of intervening against a background of financial shocks—the simple rule is an inaccurate description of central bank behavior, generally underestimating the actual policy changes. The central bank following our augmented policy rule trades off more output and inflation instability today for a faster return to the trend path tomorrow. As regards normative implications, the simulations suggest limits to the use of monetary policy instruments for such financial system stabilization, namely the size and duration of the financial shock. The central bank seems capable only of a faster reaction to the financial instability shock, keeping the nature of monetary policy unchanged and long-run consumption volatility remains practically identical under both rules. Moreover, monetary easing is unlikely to work in economies with either fixed exchange rates or a strong exchange rate channel of monetary transmission.
The paper is organized as follows. Section II reviews the recent literature on the link between monetary policy and financial sector instability. In Section III, we build a DSGE model with financial intermediaries and an augmented policy rule. In Section IV, we calibrate the model on U.S. data and present simulations from the model for a range of policy-relevant shocks. In Section V, we discuss possible extensions of the model. Section VI concludes.

II. HOW DO CENTRAL BANKS REALLY DETERMINE MONETARY POLICY UNDER FINANCIAL INSTABILITY?

In a widely quoted paper, Taylor (1993) suggested that monetary policy could be explained by a straightforward rule that links the central bank's policy rate to contemporaneous deviations of inflation and output from their target and potential levels, respectively. Since then, the “Taylor rule” has become a tool of choice for analysts, researchers, and central bank staff needing to model central bank responses to macroeconomic developments. However, it remains doubtful whether the simple, backward-looking rule—which can explain only up to two-thirds of the empirical variance of policy rate changes—is an accurate description of central banks’ behavior (e.g., Svensson, 2003).

A. The Absence of the Financial Sector from Monetary Policy Models

The financial system is conspicuously absent from the standard monetary dynamic stochastic general equilibrium (DSGE) model as well as from simpler models built for policy purposes. In fact, a typical model used by an inflation-targeting central bank does not consider banks at all and its policy rule ignores the health of the financial system. Such an omission seems puzzling, given that many central banks have financial stability as an explicit policy goal (Crockett, 1997) and that considerable time and energy are devoted by central bankers to discussing and analyzing the financial sector.²

Whether financial sector concerns should in fact influence monetary policy decisions has remained a controversial issue. On the one hand, economists such as Schwartz (1995) argued that by vigorously pursuing the goal of price stability, central banks would promote financial stability the best. Any information about the financial system is useful only to the extent that it can be used to improve the inflation forecast, while fine-tuning the policy rate to meet other objectives may do more harm than good. On the other hand, central bankers argued that financial systems are inherently fragile and central banks need to intervene when financial institutions are under stress. According to this view, restricting monetary policy to price stability considerations alone is incorrect and policymaking would be improved by incorporating financial sector concerns explicitly in the rule (Bernanke and Gertler, 1999, ²

² Financial instability is a leading indicator of the business cycle (Borio, 2006), and output costs of financial instability are estimated to be around 1 percent of GDP annually (Henry, 2004). The increasing importance that central banks attribute to financial stability can be illustrated by the rapidly growing number of financial stability reports published by central banks (see Čihák, 2006, for a review).
and Cecchetti and others, 2000). These interventions can be either proactive, in the spirit of “pricking the bubbles” or “counteracting irrational exuberances” that have not yet resulted in unfavorable financial sector developments, or reactive, adjusting the monetary stance in response to observed unfavorable financial sector developments.3

Central banks may also react to financial sector problems for reasons unconnected to monetary policy. Some central banks have a role in prudential supervision of banks or other financial institutions and may be perceived to be responsible for the soundness of the supervised institutions, including ensuring the smooth functioning of the payment and settlements systems. Owing to regulatory and supervisory agencies’ lack of independence, regulatory and monetary policy objectives may become intertwined, with the latter being subordinated to the former (Quintyn, 2007).

B. Evidence on Financial Instability and Central Banks

A host of recent empirical studies have found that central banks react to financial sector instability, but that their reaction is asymmetric, nonlinear, and reflects the nature of the underlying shock. Surprisingly, empirical results and anecdotal evidence on the link between financial instability and monetary policy have been largely ignored in theoretical papers.

Borio and Lowe (2004) estimate empirically modified Taylor rule functions for Australia, Germany, Japan, and the United States, concluding that “central banks either do not respond much to financial imbalances or, to the extent that they do, they respond asymmetrically. Policy appears to be loosened in the face of the unwinding of imbalances beyond what would be suggested by the behavior of inflation and the output gap alone, but does not seem to be tightened as imbalances build up.” In other words, they find evidence of a reactive monetary policy, but no evidence of a proactive policy. The authors measure financial instability using credit and equity-price gaps, i.e., deviations from a Hodrick-Prescott filter.

Bordo and Jeanne (2002) argue that the reactive policy of injecting liquidity ex post in the event of a credit crunch may in certain circumstances be more costly in terms of lost output than a proactive policy incorporating asset prices directly into the central bank’s objective function. They support their simple model with empirical simulations for episodes such as the Great Depression or the asset-price boom in Japan. However, their stylized model limits central banks to either “pricking the bubble” or “an after-crisis cleanup.” This seems to be an unduly narrow scope for monetary policy adjustments since a forward-looking central bank would presumably adjust its stance before a crisis erupts. Moreover, the cleanup phase is not

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3 For a discussion on the merits of proactive and reactive interventions see the general discussion of Borio and White (2003) at [http://www.kc.frb.org/PUBLICAT/SYMPOS/2003/pdf/GD32003.pdf](http://www.kc.frb.org/PUBLICAT/SYMPOS/2003/pdf/GD32003.pdf). The participants were split whether either extension is desirable, seeing both of these interventions as representing “second-best” policies as compared to the “first-best” policy of price stability.
really interesting as the scope for any change in the monetary stance during a crisis is limited (Boorman and others, 2000).

Cecchetti and Li (2005) find that policymakers react to the state of their banking system’s balance sheet, attempting to counteract or even neutralize the procyclical effect of prudential capital regulation. The resulting disintermediation has a procyclical effect and the authors conclude that "for a given level of economic activity and inflation, the optimal policy reaction dictates setting interest rates lower the more financial stress there is in the banking system when the economic activity is in the downturn." There are two problems with the Cecchetti and Li results: (i) central bank reaction is conditioned on the business cycle only; and (ii) the capital adequacy requirement is an unlikely candidate as a financial instability measure.

Bulíř and Čihák (2007) estimate a modified Taylor rule in a quarterly panel of 28 industrial and emerging market countries using a battery of seven alternative measures of financial instability and find that, irrespective of the definition used, instability has been associated with short-term interest rates below those implied by the simple rule. Moreover, the responsiveness of monetary policy to domestic financial sector vulnerability appears to be stronger in closed economies and in economies where banking supervision is inside the central bank. Quantitatively, in a country with a freely floating exchange rate, the contemporaneous impact of a one standard deviation increase in the “probability of crisis” variable is associated with interest rates that are about 0.2 percentage points below what they would be otherwise.

The literature on DSGE models has largely neglected the role of financial intermediaries in the economy. The most widely used models usually do not include a financial sector and, if they do, its role is solely to produce transaction services. Nevertheless, there have been some valuable attempts to introduce this element into the analysis. Williamson (1987) constructs a business cycle model in which financial intermediation is a determinant of business cycle fluctuations. In his model, banks exist because of asymmetric information and costly monitoring. Stochastic disturbances to the riskiness of investment projects produce equilibrium business cycles in the presence of monitoring costs: cycles are produced through costly intermediation rather than firms’ failures. There are two mechanisms through which real output fluctuates when there are stochastic disturbances. One is an intertemporal substitution mechanism, common to many real business cycle models. The other is a credit supply effect: a reduction in the amount of loans in the current period reduces the next period’s output. Though the general idea of the paper is close to ours, the model does not incorporate nominal rigidities (sticky prices) and fails to consider the role of monetary policy in smoothing fluctuations in the cycle.

Bernanke, Gertler and Gilchrist (1998) develop a new Keynesian model that includes a partial equilibrium model of the credit market in order to study how credit market frictions amplify real and nominal shocks to the economy. In the model, banks act as intermediaries
between households, which hold deposits, and entrepreneurs, which ask for loans in order to produce a homogeneous good. Their framework exhibits a “financial accelerator”, in the sense that endogenous developments in credit markets propagate and amplify shocks to the economy. As in the case of Williamson, the friction in the credit market arises due to the presence of asymmetric information and agency costs. The model does not incorporate any concept of financial instability and banks are able to diversify idiosyncratic shocks completely and, thus, the \textit{ex ante} and \textit{ex post} deposit rates are identical.

Finally, Brousseau and Detken (2001) exploit a sunspot equilibrium in a standard new Keynesian framework. In their model, the central bank may choose not to keep inflation at target in the short term in order to stabilize the financial system. They interpret the sunspot shock as financial instability because its variance is linked to a change in the policy rate. The larger the change in the policy rate, the higher the next period’s variance of the sunspot process and the simple monetary policy rule is no longer optimal. The authors provide neither an explicit modeling of the financial sector nor a clear description of the process by which financial instability affects the rest of the economy. In fact, the sunspot shock does not influence the vector of endogenous variables and the shock brings indeterminacy to the otherwise standard model. The model does not consider the use of a Taylor rule by the central bank but, instead, discusses the specification of the optimal policy with respect to the one arising from the standard model.

### III. The Model

Our model, which builds on Galí (2002), addresses two simplifications in the standard new Keynesian dynamic stochastic general equilibrium (DSGE) model: the omission of a financial system and the omission of forward-looking variables in the policy rule. We differ from a Galí-type DSGE model in two respects: (i) financial intermediaries supply external financing to some firms; and (ii) firms that are sensitive to the supply of loans and the interest rate are linked with the rest of the firms in the economy through a productivity nexus.

These innovations capture a number of well-accepted stylized facts that have been absent from the typical DSGE model used in the literature.\footnote{Needless to say, these elements have been absent also from the earlier generations of models that preceded the class of DSGE models, including those used by most central banks. See Bernanke, Gertler, and Gilchrist (1998) and Fukač and Pagan (2006) for reviews.}

- The credit channel works through a subset of firms that depend on external financing.
- Small- and medium-sized firms depend on banks for external financing much more than large firms.
- Small, start-up firms perish easily if they cannot obtain external financing.
• Large firms rely on small firms to bring about technology and productivity improvements.

• Higher lending rates make both the marginal loan and the lending portfolio of the financial system more risky.

• The central bank can observe the health of the financial system one period before the public does.

• Monetary policy actions do not result in moral-hazard behavior by banks.

The economy contains five types of agents: households; goods-producing firms that are monopolistic competitors; innovative firms that are freely competitive; financial intermediaries, which are freely competitive as well; and a central bank. While we describe them in turn below, further details of the derivations are provided in the Appendix.

A. Households

The economy is populated with a continuum of infinitely-lived and identical households that derive utility from consumption of goods and leisure, and invest their savings in a financial intermediary that pays a nominal rate \( r_t \) for one-period deposits made at time \( t-1 \).

Households consume a basket of all goods available according to

\[
c_t = \left[ \int_0^1 c_i(i) \frac{e^{\epsilon-1}}{\epsilon} \, di \right]^{\frac{\epsilon}{\epsilon-1}},
\]

where \( c \) is the contemporaneous consumption of the representative household and \( \epsilon \) is the elasticity of substitution between any two given goods indexed \( i \) and \( i' \).

The problem of the representative household can be written as

\[
\max_{c, n_t} E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, n_t),
\]

subject to its period-by-period budget constraint (in nominal terms)

\[
P_t c_t + P_t d_t = P_t w_t n_t + r_t P_{t-1} d_{t-1} + P_t \Pi_t + P_t T_t,
\]

where \( d_t \) are deposits, \( w_t \) is the wage rate, \( n_t \) is labor, \( \Pi_t \) are dividends, and \( T_t \) are lump sum taxes; all these variables are in real terms.
B. Goods Firms (“Firms That Do Not Need External Financing”)

The first segment of the corporate sector contains a continuum of infinitely-lived firms acting as monopolistic competitors. These firms produce their single perishable, differentiated good with a technology

\[ y_j(t) = a_j n_j(t), \]

where \( a_j \) is a technology shifter common to all firms. The economy-wide, competitive factor market guarantees that all firms pay the same nominal wage \( W_t = P_t w_t \) for a unit of labor employed. The key distinguishing feature of these firms is that they do not need to borrow from financial intermediaries and finance themselves through retained earnings.

Following Calvo (1983), we assume that these firms adjust their prices infrequently and that the opportunity to adjust prices follows an exogenous Poisson process. Each period there is a constant probability \( 1 - \theta \) that the firm will be able to adjust its price, independently of past history. Inability to adjust prices every period is the source of nominal rigidities that make inflation distortionary.

C. Innovative Firms (“Firms That Need External Financing”)

The non-financial corporate sector contains also firms that have to borrow from financial intermediaries to develop a project. These firms only live for two periods and operate under perfect competition. The key distinguishing feature of these firms is that, unlike the goods firms, they are dependent on external financing. In period \( t \) such a firm invests in a project in order to obtain a return in period \( t+1 \). The technology is such that

\[ \chi_j(f) = (1 + \beta) s_j(f), \]

where \( s_j(f) \) is the initial investment made by firm \( j \) in the project and \( \chi_j(f) \) is the firm-specific return of the project.

Some of these firms survive and some do not. For simplicity, we assume that a constant fraction \( \gamma \) of these firms born in period \( t \) survive with probability one in period \( t+1 \). Moreover, to capture the trade-off between risk and return, we assume that firms that survive with probability one, i.e., “risk-free” firms, are the least profitable firms. The remaining firms may die at the beginning of period \( t+1 \) with probability \( \delta_{t+1} \), where \( \delta_{t+1} \) is stochastic. A firm that does not survive obtains a zero return for its project. Finally, \( \delta_{t+1} \) is realized only at the beginning of period \( t+1 \), after firms have applied for and received loans.

The returns of innovative firms are nonstochastic, firm-specific, and distributed according to a log-normal distribution with parameters \( \mu \) and \( \sigma \). We index firms according to their
returns in the interval $[0, 1]$, denoting with 0 the firm for which $\chi(j) = \chi_{\min}$ and with 1 the firm for which $\chi(j) = \chi_{\max}$. We establish a one-to-one correspondence between the firm’s index and the lognormal cumulative distribution function. That is, firm $j$ is such that a proportion $j$ of firms obtain returns which are lower than $\chi(j)$. Then $\gamma$ is both the proportion of firms that survive with probability one next period and the most profitable non-risky firm. Figure 1 shows a stylized distribution of returns with parameters $\mu = 0.05$, $\sigma = 0.1$, and $\gamma = 0.2$. Firms that obtain returns higher than $\chi(\gamma) = 0.9664$ are risky, in the sense that they may die next period with probability $\delta_{t+1}$. On the contrary, firms with returns lower than $\chi(\gamma) = 0.9664$ survive with probability one. In Section IV we will show results under different assumptions about $\gamma$.

### D. Financial Intermediaries

The economy also contains a continuum of financial intermediaries that act as go-betweens for households and innovative firms. As there is free entry in the financial sector, banks obtain zero profits in equilibrium. These institutions receive deposits from households in period $t$ and lend to innovative firms, charging for them a lending rate $z_t$. At time $t+1$ firms that have survived repay their loans and intermediaries pay to households the return $r_{t+1}$. For simplicity, we assume zero recovery on loans to failed firms. We will assume that intermediaries are able to monitor whether a firm exists or not without a cost, but cannot distinguish between firms and thus charge the same loan rate $z_t$ to all firms. However, a firm $j$ has an incentive to ask for a loan only if its expected project return is higher than the lending rate. In other words, there is no moral hazard problem in the model.

$$\chi(j) > z_t.$$  \hfill (2)

We assume that $z_t < \chi_{\max} \quad \forall t$. Therefore, the marginal firm $\omega_t$ to ask for a loan will be such that

$$\chi(\omega_t) = z_t.$$  

In Figure 1 the marginal-firm cutoff point $\omega_t$ can be interpreted as the proportion of innovative firms that have returns lower than $z_t$ and, therefore, do not find it profitable to apply for a loan. Thus, due to the lack of funds, these firms will not be producing in period $t+1$. Conversely, the proportion of firms asking for loans will be $1 - \omega_t$.

Given the technology of innovative firms, a firm for which equation (2) holds has an infinite demand for loans. Since banks cannot distinguish among firms, they will divide their loanable resources (household deposits) into equal parts and provide the same amount of
loans to each firm that asks for one: \( l_t = \frac{d_t}{1 - \omega_t} \). This simple assumption has a profound implication: the riskiness of the whole loan portfolio is increasing in the lending rate. At higher rates fewer firms apply for a loan and the lending portfolio becomes more concentrated in the high-return/high-risk segment and thus more risky overall.

Intermediaries’ opportunity cost of a loan is a central bank bill that pays a nominal interest rate equal to \( i_t \) (policy rate) and that is for all practical purposes equivalent to short-term treasury bills. Then it has to be the case that the return on lending to firms is equal to the return on investing in the central bank paper and the loan rate will be determined as the rate such that the expected returns from loans are equal to the interest rate \( i_t \).

\[
i_t d_t = E_t(\tilde{l}_{t+1}),
\]

where \( \tilde{l}_{t+1} \) are loans that are actually repaid in period \( t+1 \). Notice that, since banks do not know the probability of firm survival at the moment they lend to them, they compute the loan rate based on their expectations of the shock to \( \delta_{t+1} \). The \textit{ex post} deposit rate will be thus:

\[
r_{t+1} = z_t \tilde{l}_{t+1} / d_t.
\]

In other words, if a smaller proportion of loans are repaid in \( t+1 \), the (ex-post) deposit rate becomes smaller, which means an increase in the spread between the deposit rate on one hand and the policy rate and the lending rate on the other hand.

**E. Technology**

The economy-wide total technology \( a_t \) consists of two components. One component is exogenous and stochastic and follows an autoregressive process

\[
\tilde{a}_t^a = \rho^a \tilde{a}_{t-1}^a + \epsilon_t^a
\]

where \( \epsilon_t^a \) is an independent and identically-distributed (i.i.d.) shock and \( \tilde{a}_t^a \) denotes log-deviations of \( a_t^a \) from steady state.

---

\(^5\) We assume that there is perfect competition and free entry in the intermediaries market, and that intermediaries are risk neutral. Therefore, no bank would charge a higher rate for loans since this would imply that some other intermediary could charge a smaller rate and capture all the demand for loans. Similarly, no bank would be willing to charge a smaller rate than \( i_t \) because it could buy the central bank paper that yields a higher return. Since banks are risk neutral, they will charge a rate for loans such that their expected returns from loans equal their opportunity cost \( i_t \). Provided that there is free entry in the market, profits will be equal to zero.
The other, additional components of technology are the projects developed by the innovative firms that asked for loans in period \( t-1 \) and survived in period \( t \). The production function for this type of technology is

\[
a_t^i = \left[ \int_{a_{ij},i}^{\tau} \left( s_i(j)\delta_t^+(j) \right)^{\frac{\tau-1}{\tau}} dj \right]^{\frac{\tau}{\tau-1}},
\]

where \( \delta_t^+(j) = 1 \) if the firm survived in period \( t \) and zero otherwise, and \( \tau \) is the elasticity of substitution between any two projects \( j \) and \( j' \). Substituting (1) into the last expression and rearranging, we obtain

\[
a_t^i = \left[ \int_{a_{ij},i}^{\tau} \left( \chi(j)\delta_t^+(j) \right)^{\frac{\tau-1}{\tau}} dj \right]^{\frac{\tau}{\tau-1}} \frac{d_{t-1}}{1-\omega_{t-1}},
\]

Finally, total technology combines both the exogenous and endogenous components according to a Cobb-Douglas function

\[
a_t = a_t^i a_t^{1-\alpha},
\]

where \( \alpha \) is the contribution of technology generated by innovative firms, \( a_t^i \), to total technology. Thus, without innovative firms, productivity growth would be limited to the exogenous component only.

F. The Central Bank

The central bank seeks to stabilize the economy, which means that it responds to the productivity and survival shocks that hit the two types of firms (\( a_t^i \) and \( \delta_t \), respectively). We examine two central bank response functions.

First, we look at a central bank that employs the traditional Taylor rule to set its instrument, a policy rate:

\[
\tilde{i}_t = \phi_\pi \tilde{\pi}_t + \phi_x x_t
\]

(4)

where \( i_t \) is the policy rate, \( \tilde{\pi}_t \) is inflation in period \( t \) and \( x_t \) is the output gap, defined as the difference between actual output and natural output (that is, output in the flexible price allocation). In order to guarantee a unique equilibrium, the rule needs to be such that \( \phi_\pi > 1 \).
Second, we look at a central bank that continually monitors financial intermediaries and their counterparts to infer the state of the economy and the impact of financial institutions’ health on the real economy. This information is likely to be collected through prudential supervision of financial intermediaries (if the central bank has prudential powers), or through the central bank’s role in the payment system. This information is confidential, i.e., exclusive to the central bank. While individual financial institutions would have detailed information about their clients, possibly better than that available to the central bank, the central bank is likely to be the only one to have such information for the financial system as a whole.

Empirical studies suggest that if central banks have recent supervisory information from on-site visits, they can achieve better predictions of financial stability than is possible based on publicly available data. If the central bank has this information, it can use it to improve the stabilization outcomes. In particular, if the chances of firm survival are good, the usual Taylor rule would apply. However, if the chances are not as good and the central bank has private information on $\delta_t$ at the beginning of period $t$, it may decide to incorporate this information in the rule. This means that the central bank’s policy response function would look as follows

$$
\hat{i}_t = \begin{cases} 
\phi_x \hat{\pi}_t + \phi_z x_t & \text{if } (\delta_{t+1} - E_t \delta_{t+1}) < 0 \\
\phi_x \hat{\pi}_t + \phi_z x_t + (\phi_\delta + \nu^\delta)(\delta_{t+1} - E_t \delta_{t+1}) & \text{otherwise}
\end{cases}
$$

(5)

where $\phi_x < 0$ and $\nu^\delta$ is either a positive or negative shock to the sensitivity of the rule to deviations of $\delta$, capturing both the reporting lags in financial stability reports and the policymaker’s nonlinear and asymmetric response to deviations of $\delta$. Note that, if equation (5) did not include the shock $\nu^\delta$, knowing $\phi_x$, $\phi_z$ and $\phi_\delta$ agents would be able to infer $\delta_t$ from the interest rate set by the central bank and the existence of signal extraction would defy the policymaker’s private information. One could also think that the central bank cannot perfectly foresee $\delta_t$ and only receives more information to compute the conditional expectation than the rest of the agents in the economy. Nevertheless, under this specification the qualitative implications of our setup will remain unchanged.

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6 For example, Berger, Davies, and Flannery (2000) find, using U.S. data, that if supervisory inspections were recent, assessments based on those inspections tended to be more accurate than equity and bond market indicators in predicting future changes in the performance of large bank holding companies. The relative predictive power of supervisory data is even higher in economies with lower public availability of data on financial soundness of banks. For instance, Bongini, Laeven, and Majnoni (2002) arrive to a similar, but stronger result for banks in East Asia during the 1996–98 crisis.
The timing of events will be as follows: at the beginning of every period, after $\delta_1$ and $a^*_i$ are realized, total technology $a_i$ is observed. Households make their decisions on consumption, saving, and labor allocations, forming their expectations based on the information they possess at the time. The central bank sets the policy rate according to the simple rule (4) in the benchmark scenario; while it employs the augmented rule (5) in the alternative scenarios, using also different values of this information in the rule ($\phi_\delta$). Therefore, in (5) the central bank uses information that the rest of agents do not possess and we want to study whether this informational advantage is relevant for the determination of the policy rate or not.

In what follows, we will calibrate the model and observe the effects of two shocks on the economy: (i) a pure technological shock; and (ii) a shock to the probability of survival of firms. We will study two different scenarios: first, the benchmark, in which the Taylor rule is defined as in (4), with the parameter values $\phi_\pi=1.5$ and $\phi_s=0.5$ used in Taylor (1993) and many of the subsequent papers, and second, a scenario in which the policy rule is as in (5) with $\phi_\delta=-0.5$. This parameterization of $\phi_\delta$ is somewhat arbitrary, but the results do not differ qualitatively if we use, for example $\phi_\delta=-0.25$ or $\phi_\delta=-0.75$. What matters from a qualitative standpoint is that $\phi_\delta < 0$.

IV. MODEL SIMULATIONS

We calibrate the model and observe the effects of two shocks: a pure technological shock and a shock to the probability of survival of firms. The time period is one quarter.

A. Parameterization

We calibrate the model using parameter values from Bernanke, Gertler and Gilchrist (1998) and Galí (2001), both of which refer to the U.S. economy. We use the following utility function for households:

$$U(c_t, n_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{n_t^{1+\phi}}{1+\phi}$$

with $\sigma = \phi = 1$. The probability of adjusting prices in a given period $1-\theta$ is set equal to 0.25, which implies average price duration of one year, a value in line with survey evidence. The discount rate $\beta$ is set to be 0.99. The serial correlation for the technology process, $\rho^a$, is assumed to be 0.95.

The process for the probability of small-firm survival is

$$\delta_t = \delta + \rho^\delta \delta_{t-1} + \epsilon_t^\delta$$
where $\varepsilon_i^\delta$ is an i.i.d. process.

Given the average duration of post-war U.S. recessions of 11 months, we choose $\rho^\delta$ to be 0.25 and the annual rate of business failure, $\delta$, is equal to 0.03, approximating the data. The share of firms that survive for sure, $\gamma$, is chosen to be 0.2, which implies a quarterly steady-state spread between loan and deposit rates of 0.76 percent and an annual loan rate of 7.28 percent.

The variances of the technological shock and probability-of-survival shocks, $\sigma^\tau$ and $\sigma^\delta$, are assumed to be 0.01 and 0.0025, respectively. The last two parameters, the participation of $a_i$ in the creation of technology and the elasticity of substitution between any two projects $j$ and $j'$, are set equal $\alpha = 0.05$ and $\tau = 4/3$, respectively. That is, we are assigning a comparatively small role of $a_i$ in the creation of total technology, but—based on U.S. data—we expect sizeable effects from shocks to the probability of default.

To assess robustness of our results, we have calculated our simulations with a range of different distributions of returns to innovative firms. The general results do not vary substantially. For exposition purposes, we choose a lognormal distribution with parameters $\mu = 0.05$ and $\sigma = 0.1$.

**B. Simulation Results**

**An exogenous technology shock**

First, we simulate an exogenous technology shock, that is, a positive shock in $a_i$ equal to its one standard deviation ($\varepsilon^\tau$). Figure 2 shows the trajectory of the shock, Figure 3 shows the response of output, output gap, inflation and labor, Figure 4 shows the evolution of the main interest rates (policy rate, deposit rate, loan rate, and effective (ex post) deposit rate) and finally Figure 5 shows the evolution of loan application ($1 - \omega$) and endogenous and total technology.

Both in Figure 3 and Figure 4 we compare the results from our model (with both specifications of the Taylor rule given in equations (4) and (5)) with what is obtained in a standard model without the innovative sector. This last case has been widely studied in the literature and will serve as a benchmark for comparison purposes. In Figure 5, we do not show results for the benchmark model since the variables graphed do not appear in such a framework.

A positive exogenous technology shock in an economy with sticky prices, imperfect competition, and no innovative sector generates lower employment, higher output, and opening of the output gap, as depicted by the alternating dashed green line in Figure 3. While all firms experience a decrease in their marginal costs, not all can adjust their prices in this
period (Gali, 2001). Thus, the consequent changes in the aggregate price level and demand will be proportionately less than the initial increase in productivity.

In the above economy, the responses of the variables to an exogenous technology shock are both different from the simple benchmark model and more pronounced. Despite the differences explained below, from period 2 on the qualitative response of all variables is identical in all three scenarios. The response of labor and the output gap in the first period is positive, becoming negative only from the second period onwards. This result follows from the initial monetary loosening based on the central bank’s private information. The path of the variables is not as smooth as before, and for some variables such as labor and output gap the response is not even monotonic. This result follows from the impact of lending conditions on the evolution of endogenous technology.

Increased volatility results from the presence of endogenous technology and bank lending. Recall that:

\[
a'_i = \left[ \int_{\omega_{t-1}}^{1} \left( \frac{e \sigma^*_t(j)}{1 - \omega_t} \right)^{\tau-1} \omega_t \frac{d_{t-1}}{1 - \omega_{t-1}} \right]^{\tau-1} d_{t-1} \frac{1}{1 - \omega_{t-1}}
\]

From this expression, we can observe that the creation of endogenous technology in period \( t \) depends on the proportion of firms applying for a loan \( \omega_{t-1} \) and deposits \( d_{t-1} \). The higher the level of deposits, the higher the amount of the loan received by each firm and, consequently, the higher the contribution of technology generated by innovative firms, \( a'_i \). The effect of \( \omega_{t-1} \) on \( a'_i \) is a priori indeterminate: while a lower \( \omega_{t-1} \) implies that more firms are obtaining loans and thus the term in brackets increases, the total amount of deposits has to be divided over a higher number of borrowers (\( \frac{d_{t-1}}{1 - \omega_{t-1}} \) decreases).

More specifically, the decrease in the policy rate due to the negative response of inflation causes loan applications to decrease on impact, whereas deposits increase because of the better expectations on future consumption (see (A14) in the appendix). The full impact on endogenous technology is positive; nevertheless, this effect only takes place in period 2. This translates into a higher expectation of output (and consequently, consumption) for period 2, which causes period 1 consumption to increase more than what is accounted by the period 1 increase in exogenous technology. Thus, labor increases, causing the output gap to increase as well (A6). From period 2 onward the higher level of endogenous technology is added to the original effect of the exogenous technology shock, which explains the amplified response of all variables to the shock.
As the technology shock dies out, loan applications, $\omega_t$, increase and deposits, $d_{t-1}$, decrease until they reach their steady state values. Nevertheless, the path of $i_t$ is not monotonic: while the overall trend is downward sloping, in some periods it increases with respect to its value in the previous period. This is so because the process for $i_t$ has no memory (there is no transmission from $i_{t-1}$ to $i_t$ as innovative firms live for two periods only) and the effect of $\omega_{t-1}$ on $i_t$ is indeterminate a priori.

Nevertheless, the economy with the innovative sector behaves much in the same way under both rules proposed. This is due to the fact that a technology shock does not generate financial instability: $\delta_t - E_\delta_t = 0 \quad \forall \ t$ and thus the reaction of the central bank to the shock will be identical under all scenarios. For the case in which $\alpha = 0$, our model nests the benchmark model without the innovative sector. Deviating slightly from the benchmark scenario, that is, setting $\alpha = 0.05$, the model yields visible departures from the standard results. There are some differences between the dashed red and solid blue lines in Figures 3 to 5, due to a small difference in the numerical solution of the model under the two specifications. These differences are, nonetheless, of a very small magnitude and do not alter the thrust of the results.

**A shock to the default probability**

Second, we consider a negative shock to the probability of survival ($\delta_t$) of one standard deviation ($\delta\varepsilon$) (Figure 6). Unlike the long-lasting technology shock, this shock is assumed to be much less persistent than the previous shock, dying out in approximately 4 quarters. The economy is described in Figures 7 to 9.

Since the two specifications of the Taylor rule that we consider generate very different dynamics for the variables of interest, we will describe in the first place their evolution when the central bank sets the policy rate according to the traditional Taylor rule (4). The benchmark model does not have an innovative sector and thus no simulations for this shock can be generated.

The negative default shock in period $t$ results in a decline in output in period 1 (Figure 7). The shock translates into fewer firms surviving the next period and, consequently, less generation of endogenous technology in period 1 (Figure 9). As a result, output in period 1 decreases. Following the same logic as in the previous case, inflation increases and aggregate demand decreases less than the fall in the natural level of output (thus, the output gap is negative). Again, the presence of the innovative shock alters significantly the responses in the first period from the ones in period 2 onwards. In period 1, the negative performance of current and future output impacts negatively on deposits. In addition to this, the higher default rate causes $\omega_t$ to increase. These two elements depress further the creation of endogenous technology in period 2, causing labor (and, consequently, the output gap) to
increase even above its steady state level. From period 2 onwards the dynamics of the variables are similar to what is obtained in the benchmark model for a negative technology shock.

We observe that when the central bank follows a simple Taylor rule, it fails to react in period 1 to $\delta_1$, even though it possesses this information (Figure 8). The central bank reacts only from period 2 onwards, when the shock has already affected the economy, widening the output gap and increasing inflation. At this point, naturally, the rule dictates that the central bank increase the policy rate so as to induce a decline in aggregate demand and thus soften the effects of the shock.

But what would happen if the central bank adjusted the policy before the shock affected total technology according to the augmented rule (5)? Given that the central bank knows $\delta_1$ at the beginning of period 1, foreseeing the negative effects on the economy caused by a higher default rate, it can decrease the policy rate in period 1, in turn lowering the loan rate (Figure 8). The change in the expected deposit rate mimics the change in the policy rate, thus stimulating aggregate demand. Moreover, the decrease in the policy rate implies a lower $\omega_1$ and it generates positive expectations over future activity and stimulates deposits. These two elements have a positive effect over endogenous technology in period 2, more than offsetting the negative effect of $\delta_1$. The deposit and loan rates follow the evolution of the policy rate by decreasing in period 1 and increasing gradually in subsequent periods (Figure 8).

Monetary policy is useful in reacting to financial stability shocks only to the extent private agents’ signal extraction is limited. Specifically, only the central bank has information on $\delta_{t+1}$ at the beginning of period $t$. Households make their decisions only with information on $a_t$ and $\delta_t$, which seems a reasonable reflection of reporting lags.

Our simulations suggest that a central bank responding to financial sector instability is able to trade off marginally higher beginning-of-the-period inflation for more stable output and inflation. Under the traditional Taylor rule, the cost of ignoring information about $\delta$ is more pronounced, resulting in longer-lasting output and higher inflation.

**C. Welfare Calculations**

Can the visually observed result of more stable output under the default probability shock and augmented Taylor rule be extended to measurable long-term welfare gains? It turns out that using the Lucas (1987) measure of welfare—comparing the variance of consumption under alternative assumptions—does not yield substantial differences between the two rules. To this end, we calculated the welfare differential as $\frac{1}{2}$ times the risk aversion coefficient ($\sigma$) times the difference in the variance of the CES consumption that was simulated for a time
span of 300 periods (exposing technology and default probability to a simultaneous shock) and repeated 100 times.

The estimates of standard deviations of consumption are practically identical for both rules, and these results are robust with respect to the weight of financial instability in the policy rule \( \phi \). For example, assuming the benchmark value, \( \phi = -0.5 \), the standard deviations of consumption under the traditional and augmented rules are virtually identical at 0.05915 and 0.05947, respectively. For a less aggressive rule, \( \phi = -0.1 \), the standard deviations are 0.05899 and 0.05892, respectively. Neither of these differences is substantial, meaning that the impact of the augmented rule clearly does not extend into more stable consumption.

These results capture the fact that the economy stabilizes faster under the augmented Taylor rule, but the output and consumption paths are more volatile initially than under the traditional rule. Indeed, in Figure 7, we observe much larger initial, two-period departures from the trend in both output and output gap simulations under the augmented rule than under the traditional rule. In other words, the central bank trades off more instability today for a faster return to the trend path tomorrow. Introduction of the financial sector and shocks thereto in the DSGE model does not change the nature of monetary policy; it only accelerates policy reaction to the signs of financial instability.

V. PRACTICAL ISSUES AND POSSIBLE EXTENSIONS

We see several practical limitations for embracing financial instability as a regular part of a policy rule, namely the magnitude and nature of the initial shock, fiscal repercussions, and exchange rate stability. Finally, we discuss the assumption of central banks’ privileged information and some practical issues and suggestions for further research.

Practical limitations

In the existing model, central bank response to financial instability is linear (as is the simple Taylor rule). In practice, the response is likely to be highly nonlinear: in their reports on financial stability, central banks claim to focus much more on shocks with systemic implications than on smaller shocks with no or limited systemic implications (see, for example, the survey in Čihák, 2006). The long-term fiscal effects of neglected financial instability can be dire. By providing additional liquidity, as opposed to closing down weak financial institutions, the central bank may delay the necessary adjustment and increase the fiscal cost of the eventual cleanup operation. Also, if the financial system is faced with a credibility crisis, a lower policy rate is unlikely to calm the depositors. Monetary easing cannot be a substitute for prudential regulation of the financial system.

The model can be enhanced by introducing other sectors, such as the external sector. Introducing the external sector requires distinguishing residents and nonresidents, and it introduces a role for the exchange rate regime. The scope for monetary easing is more limited
in a monetary regime with an exchange rate anchor or with a strong exchange rate transmission channel of monetary policy. Even small monetary easing may cause a run on the currency and force unwinding of external positions. One can expect much less enthusiasm for monetary easing in the face of financial instability in countries with significant capital flows.

The model as presented here focuses only on monetary policy reasons for reactions to financial instability. The central bank in the model responds to financial sector instability not because financial sector developments would have a direct place in its utility function, but simply because responding this way improves developments in future inflation and output. However, there may be other reasons for real central banks to react to instability that are not modeled here. In particular, many central banks have a role in prudential supervision of banks (and sometimes also other financial institutions), and may be perceived responsible for the soundness of the supervised institutions, including ensuring the smooth functioning of the payment and settlement systems, thus putting an even higher premium on financial sector stability. This may create an additional incentive for central banks to lower interest rates to alleviate financial instability. Such central banks also have at their disposal more direct measures of a regulatory or supervisory nature (e.g., they can impose limits on various financial ratios, or demand corrective action from banks’ management). However, those tend to take a longer time and may be more difficult to implement than changes in interest rates.

**Privileged information**

The simulations suggest that if a central bank has privileged information about the soundness of the financial sector, the information can be used for monetary policy purposes to stabilize output in the face of financial instability shocks. The assumption that central banks have access to privileged information is realistic: central banks usually play an important role in their country’s payment and settlement systems, and many are also prudential supervisors of banks (and in some cases other financial institutions). While individual financial institutions would usually have relatively better information than anybody else (including the central bank) about their own financial health and the financial health of their clients, they would normally not have such information about the other financial institutions in the system, and therefore about the system as a whole. In contrast, the central bank’s unique role in the payment system (and often also in prudential supervision) allows it to have privileged information about financial health on the systemic level.

An interesting question not directly addressed in our model is the central bank’s policy choice regarding the extent to which this privileged information should be revealed to the public. In our model, the public can only extract parts of the central bank’s privileged information from the policy rates. However, a central bank can of course decide to forgo a part of its information superiority, and become more transparent in its assessment of the financial sector, thereby allowing better signal extraction. Indeed, a number of central banks have tried in recent years to increase the amount of information they share about their
assessment of financial sector soundness through financial stability reports and other avenues. Some of these central banks have argued that the purpose of these publications is to influence market participants’ decisions and ultimately increase financial stability (e.g., Čihák, 2006).

The willingness and ability of central banks to communicate privileged information to the public may be limited, for several reasons. Some of these are purely legal: banking confidentiality laws in most countries completely prohibit important types of information (e.g., on performance of individual loans) from being distributed to the public. Others are policy-related: sharing information may in some cases trigger the very crisis the central bank is trying to prevent. Whatever the reasons for limiting the amount of information made public, it needs to be stressed that, reflecting these limitations, there are substantial gaps between what some central banks publish in their financial stability reports and what can be considered “good practice” (Čihák, 2006). The central banks’ choice regarding the extent of public access to financial sector information is an interesting issue that could be investigated more in further research.

VI. CONCLUSIONS

Financial instability deserves to be taken seriously because of its macroeconomic costs, but the literature on monetary policy response functions has largely ignored its impact on the behavior of the central bank, basing the policy rule only on the contemporaneous output gap and inflation. To this end, we (i) enrich the standard new Keynesian model with a financial system and firms that require external financing, and (ii) introduce a forward-looking element into the Taylor rule. Under the augmented policy rule the central bank monitors the financial system, responding to deterioration in the financial system balance sheet with instant monetary loosening. Our paper is the first one to model the central bank response to financial instability in a general equilibrium context.

The model fits the stylized facts of modern central banking particularly well. Namely, we know that central banks spend much of their resources on monitoring the economy and financial system, collecting private information that would allow them to respond to forthcoming financial instability shocks well before these shocks are transmitted into headline inflation, output, or other macroeconomic aggregates. The underlying financial shock and its transmission mechanism are integrated into the model, rather than being ad hoc as in the earlier literature.

We find that a policy rule, whereby a central bank lowers its policy rate in response to financial sector instability, yields different short-term outcomes in terms of output and inflation than the traditional Taylor rule. Our model illustrates that as long as the financial instability shock is short-lived and of reasonable magnitude, a forward-looking central bank can prop up the banking system with monetary easing, limiting the short-term fall in the level of output and consumption as compared to the traditional Taylor rule. The central bank
following the augmented rule trades off more output and inflation instability today for a faster return to the trend path tomorrow. The nature of monetary policy remains unchanged under this rule, and although the reaction of the central bank to financial instability is much faster than under the Taylor rule, the long-run impact on consumption appears negligible.
REFERENCES


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APPENDIX

To derive the equations to solve the model numerically we follow Monacelli (2004).

Households:

Households consume a basket of goods according to

$$c_t = \left[ \int_0^1 c_i(i)^{\frac{1}{\varepsilon}} \, di \right]^{\frac{\varepsilon}{1-\varepsilon}}$$

By solving

$$\max c_t = \left[ \int_0^1 c_i(i)^{\frac{1}{\varepsilon}} \, di \right]^{\frac{\varepsilon}{1-\varepsilon}}$$

s.t. $$X_t - \int_0^1 P_t(i)c_i(i) \, di = 0$$

where $$X_t$$ is total nominal expenditure, $$P_t(i)$$ is the price of good $$i$$.

We obtain

$$c_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\frac{1}{\varepsilon}} c_t$$

(A1)

where $$P_t = \left[ \int_0^1 P_t^{1-\varepsilon}(i) \, di \right]^{\frac{1}{1-\varepsilon}}$$ is the aggregate price index. Equation (A1) is the isoelastic demand of good $$i$$.

The optimization problem of households stated in the text is:

$$\max_{c_t} E_0 \sum_{t=0}^\infty \beta^t \left( c_t^{1-\sigma} - n_t^{1+\varphi} \right)$$

s.t.

$$P_t c_t + P_t d_t = P_t w t n_t + r_t P_{t-1} d_{t-1} + P_t \Pi_t + P_t T$$

The first order conditions for this problem can be summarized as

$$\frac{n_t^{\varphi}}{c_t^{\sigma}} = w_t$$
\[ \rho \tilde{n}_t + \sigma \tilde{c}_t = \tilde{w}_t \]
\[ -\sigma \tilde{c}_t = E_r (\tilde{r}_{t+1} - \tilde{\pi}_{t+1} - \sigma \tilde{c}_{t+1}) \]

where variables in hat denote log-deviations from steady state.

**Goods Firms:**

Given our assumption of sticky prices, the evolution of the aggregate price index can be written as

\[ P_t = \left[ \theta P_{t-1}^{1-\epsilon} + (1-\theta)P_t^{\text{new}} \right]^{-\frac{1}{\epsilon}} \] (A3)

A firm \( j \) that has to decide on the price of its product in period \( t \) has to take into account the fact that it will be able to reset its price in the future with probability \( 1-\theta \). Then the problem of such firm in period \( t \) can be written as

\[
\max E \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} Y_{t+k} (i) (P_t^{\text{new}} (i) - MC_{t+k})
\]

s.t. \( Y_{t+k} (i) = \left( \frac{P_t^{\text{new}} (i)}{P_{t+k}} \right)^{-\epsilon} Y_{t+k} \)

where \( Y_{t+k} (i) \) is the nominal production of good \( I \) in period \( t+k \) and, given that in our model \( C_i (i) = Y_t (i) \) \( \forall i \), the constraint is given by the demand equation (A1). We define

\[ \Lambda_{t,t+k} = \beta \left( \frac{C_{t+k}}{C_t} \right)^{-\sigma} \]

as the household marginal intertemporal rate of substitution. All variables are in nominal terms.

Rearranging the FIRST ORDER CONDITIONS of the problem and using equation (A3) we obtain

\[
P_t^{\text{new}} = \frac{E \sum_{k=0}^{\infty} \Lambda_{t,t+k} MC_{t+k} Y_{t+k} (i)}{\epsilon - 1} \frac{E \sum_{k=0}^{\infty} \Lambda_{t,t+k} Y_{t+k} (i)}{E \sum_{k=0}^{\infty} \Lambda_{t,t+k} Y_{t+k} (i)}
\]
Log-linearizing this expression we get

\[
\tilde{\pi}_t = \beta E_t \tilde{\pi}_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta} m\tilde{c}_t
\]  \quad (A4)

where \( \tilde{\pi}_t = \tilde{P}_t - \tilde{P}_{t-1} \) is inflation.

The aggregate production function is

\[
y_t = a_t n_t
\]  \quad (A5)

In log-linear terms

\[
\tilde{y}_t = \tilde{a}_t + \tilde{n}_t
\]  \quad (A6)

Cost-minimization implies the following efficiency condition for the choice of labor

\[
w_t = mc_t a_t \nu
\]  \quad (A7)

where \( \nu \) is a constant subsidy to employment that exactly offsets the distortion associated with monopolistic competition, such that in steady state the economy achieves the efficient allocation. Combining this last equation with the first FIRST ORDER CONDITIONS of households:

\[
\frac{n_t^\sigma}{c_t^{-\sigma}} = mc_t a_t \nu
\]  \quad (A8)

Replacing (A6) in (A8) and writing this equation in log-linear terms

\[
m\tilde{c}_t = (\sigma + \varphi)\tilde{y}_t - (1 + \varphi)\tilde{a}_t
\]  \quad (A9)

**Flexible-price allocation:**

Under flexible prices, the problem of firm \( i \) is

\[
\max \quad \frac{P_i(i)}{P_t} y_t(i) - \frac{W_t}{P_t} n_t(i) - T^f
\]

\[
s.t. \quad y_t(i) = a_t n_t(i)
\]

\[
y_t(i) = \left( \frac{P_i(i)}{P_t} \right)^{-\varv} y_t
\]

where \( T^f \) is a constant lump-sum tax to goods firms. From here we get
\[ P_t(i) = \frac{\epsilon}{\epsilon - 1} MC_t \]

From this equation we can conclude that under flexible prices the real marginal cost will be constant and given by \( \frac{\epsilon - 1}{\epsilon} \). Then in equation (A9), \( m\hat{c}_i \) will be 0, so

\[ 0 = (\sigma + \varphi)\hat{y}_i^* - (1 + \varphi)\hat{a}_i \]  \hspace{1cm} (A10)

\[ \hat{y}_i^* = \frac{1 + \varphi}{\sigma + \varphi} \hat{a}_i \]

This is the natural level of output (expressed as log-deviations from steady state). The star refers to the variables in the flexible price allocation.

**Forward-looking Phillips Curve:**

Subtracting (A10) from (A9) we obtain

\[ m\hat{c}_i = (\sigma + \varphi)(\hat{y}_i - \hat{y}_i^*) = (\sigma + \varphi)x_t \]  \hspace{1cm} (A11)

where \( x_t \) is the output gap defined as the difference between output and its natural level.

Substituting equation (A11) in (A4) we obtain the forward-looking Phillips Curve:

\[ \hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa x_t \]  \hspace{1cm} (A12)

where \( \kappa = \frac{(1 - \theta)(1 - \beta \theta)(\sigma + \varphi)}{\theta} \).

**Computation of \( \omega_t \) and \( z_t \):**

First remember that \( \chi(\omega_t) = z_t \). Next we have to distinguish two cases according to whether the marginal firm \( \omega_t \) is risky (i.e., \( \omega_t > \gamma \)) or not (\( \omega_t < \gamma \)).

- \( \omega_t > \gamma \)

Recall equation (3) in the text

\[ i_t d_t = E_t(z_t, \bar{t}_{t+1}) \]

\[ i_t d_t = E_t((1 - \delta_{t+1})(1 - \omega_t)l_t)z_t \]
\[ i_t = \chi(\omega_t) E_t(1 - \delta_{t+1}) \]

Then \( \omega_t \) can be computed as the cdf of a lognormal distribution with parameters \( \mu \) and \( \sigma \) for \( \frac{i_t}{E_t(1 - \delta_{t+1})} \).

- \( \omega_t < \gamma \)

Again, taking into account that

\[ i_t d_t = E_t(z_i l_{t+1}) \]

\[ i_t d_t = E_t \left( \frac{\gamma - \omega_t}{1 - \delta_{t+1}} l_t + \frac{1 - \gamma}{1 - \delta_{t+1}} (1 - \delta_{t+1}) l_t \right) z_t \]

In this case \( \omega_t \) needs to be computed by a numerical root-finding algorithm.

Let \( z^A_t \) be the loan rate that is charged when \( \omega_t < \gamma \) and \( z^B_t \) the one charged when \( \omega_t > \gamma \).

Then for a given \( i_t \):

\[ E_t(1 - \delta_{t+1})(\gamma - \omega_t) < (\gamma - \omega_t) \]

\[ \Rightarrow E_t(1 - \delta_{t+1})(1 - \omega_t) < (\gamma - \omega_t) + (1 - \gamma)E_t(1 - \delta_{t+1}) \]

\[ \Rightarrow E_t(1 - \delta_{t+1}) < \frac{(\gamma - \omega_t) + (1 - \gamma)E_t(1 - \delta_{t+1})}{1 - \omega_t} \]

So it follows that \( z^A_t < z^B_t \).

**Steady State:**

First notice that in steady state \( a^i = a^s = 1 \Rightarrow a = 1^7 \). Then \( \gamma = c = n \).

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7 In order to have \( a^i \) we normalize it so that \( a^i = \frac{1}{A^i} \left[ \int_{s_i(j) \delta^*(j)}^{\tau-1} d j \right]^{\tau-1} \), where

\[ A^i = \int_{s_i(j) \delta^*(j)}^{\tau-1} d j \]

and \( \omega \) and \( \delta^* \) are steady state values.
From equation (A2) we can see that in steady state

\[ R = \frac{1}{\beta} \]

From the first condition of the household optimization problem, we can conclude that

\[ y^{\sigma+\sigma} = w. \]  
It can be shown that in order to attain the efficient allocation, \( \nu = \frac{\varepsilon}{\varepsilon - 1} \). Also,

\[ mc = \frac{\varepsilon - 1}{\varepsilon}. \]  
From equation (A7) we find then that \( w = 1 \Rightarrow y = c = n = 1 \).

Finally, we assume \( P = 1 \), so the budget constraint of households is

\[ c + d = w^*n + R^*d + \Pi - T \]

First notice that in steady state

\[ \Pi = y - \frac{1}{\nu} w^*n - T^f \]

\[ \Pi = 1 - \frac{1}{\nu} - T^f \]

We will assume that \( T^f \) is such that \( \Pi = 0 \Rightarrow T^f = \frac{1}{\varepsilon} \).

Given that in steady state the government has a balanced budget, we know that

\[ \nu = T + T^f = \frac{\varepsilon}{\varepsilon - 1} \Rightarrow T = \frac{\varepsilon(\varepsilon - 1) + 1}{\varepsilon(\varepsilon - 1)}. \]

We will assume that, unlike profits \( \Pi \), lump-sum taxes to households are constant every period. Then substituting in the period-by-period budget constraint of households, we find that deposits in steady state are

\[ d = \frac{1}{1 - R} \frac{\varepsilon(1 - \varepsilon) - 1}{\varepsilon(\varepsilon - 1)} > 0 \quad \text{because} \ (1 - R) < 0, \ \varepsilon > 1 \]

**Computation of deposits:**

Combining the period-by-period budget constraint of households and the Euler Equation (A2) we can write

\[ c_t + \beta E_t \frac{c_{t+1}^{\sigma}}{c_t^{\sigma}} \frac{P_t}{P_{t+1}} r_t d_t - w_t n_t - \Pi_t + T_t = r_t d_t \frac{P_{t+1}}{P_t} \]  
(A13)
Iterating forward and using the transversality condition \( \lim_{j \to \infty} \beta^j \frac{c_{t+j}^{-\sigma}}{c_{t+j-1}^{-\sigma}} \frac{P_{t+j-1}}{P_{t+j}} r_{t+j} d_{t+j-1} = 0 \) we obtain:

\[
r_t \frac{P_{t-1}}{P_t} d_{t-1} = E_t \sum_{j=0}^\infty \beta^j \frac{c_{t+j}^{-\sigma}}{c_t^{-\sigma}} \left( c_{t+j} - w_{t+j} n_{t+j} - \Pi_{t+j} + T_{t+j} \right)
\]

(A14)

Log-linearizing equation (A13) we obtain

\[
\beta E_t \left( -\sigma C^{-\sigma} RD \hat{c}_{t+1} + C^{-\sigma} RD \hat{p}_t - C^{-\sigma} RD \hat{p}_{t+1} + C^{-\sigma} RD \hat{c}_{t+1} + C^{-\sigma} RD \hat{d}_t \right) \\
- \sigma C^{-\sigma} (C - WN - \Pi + T) \hat{c}_t + C^{-\sigma} \left( C \hat{c}_t - WN (\hat{w}_t + \hat{n}_t) - \Pi \hat{I}_t + T \hat{t}_t \right) = \\
- \sigma C^{-\sigma} RD \hat{c}_t + C^{-\sigma} RD \hat{p}_{t+1} - C^{-\sigma} RD \hat{p}_t + C^{-\sigma} RD \hat{c}_t + C^{-\sigma} RD \hat{d}_{t-1}
\]

As we have shown before, in steady state we have that \( C = N = W = 1 \), and \( \Pi = 0 \). Since taxes are constant, \( \hat{t}_t = 0 \). Finally, given our parameterization, \( \sigma = 1 \). The latter expression becomes

\[
\beta E_t \left( - RD \hat{c}_{t+1} + RD (\hat{r}_{t+1} + \hat{d}_t - \hat{\pi}_{t+1}) \right) + (1 - T) \hat{c}_t - (\hat{w}_t + \hat{n}_t) = -RD \hat{c}_t + RD (\hat{r}_t + \hat{d}_{t-1} - \hat{\pi}_t)
\]

This can be re-written as

\[
V(s_t) = (1 - T) \hat{c}_t - (\hat{w}_t + \hat{n}_t) + \beta E_t V(s_{t+1})
\]

where \( V(s_t) = -RD \hat{c}_t + RD (\hat{r}_t + \hat{d}_{t-1} - \hat{\pi}_t) \). Imposing the transversality condition, it is easy to see that this is a contraction. Then we can solve for \( V(s_t) \) by value function iteration. Now we need to extract \( \hat{d}_t \) from this expression. Log-linearizing expression (A2) and imposing the conditions of steady state as before we obtain

\[
\hat{c}_t + D \hat{d}_t = \hat{w}_t + \hat{n}_t + RD (\hat{r}_t + \hat{d}_{t-1} - \hat{\pi}_t)
\]

Adding and subtracting \( RD \hat{c}_t \) from the previous expression we obtain \( \hat{d}_t \)

\[
\hat{d}_t = \frac{1}{D} (\hat{w}_t + \hat{n}_t + (RD - 1) \hat{c}_t + V(s_t))
\]

Equations (A6), (A7) in log-linear terms, (A11) and (A12) are, together with the two first order conditions for households, the equations corresponding to the innovative firms and financial intermediaries stated in the text and the expression for deposits, the fundamental equations of the model.
Figure 1: Distribution of Returns of Innovative Firms

Cumulative Distribution of Lognormal with $\mu = 0.05$ and $\sigma = 0.1$
Figure 2. Shock to Exogenous Technology
(One standard deviation of the technology shock, $\varepsilon^a$)
Figure 3. Shock to Exogenous Technology: Response of Output, Output Gap, Inflation, and Labor

(One standard deviation of the technology shock, $\varepsilon^a$)
Figure 4. Shock to Exogenous Technology: Response of Policy Rate, Real Interest Rate, Loan Rate, and Effective Deposit Rate
(One standard deviation of the technology shock, $\varepsilon^a$)
Figure 5. Shock to Exogenous Technology: Response of $\omega_t$, Endogenous Technology and Total Technology
(One standard deviation of the technology shock, $\epsilon^a$)

![Graphs showing the response of $\omega_t$, endogenous technology, and total technology over different periods.](image)
Figure 6. Shock to Probability of Default
(One standard deviation of the shock to the default probability, $\varepsilon^\sigma$)
Figure 7. Shock to Probability of Default: Response of Output, Output Gap, Inflation, and Labor
(One standard deviation of the shock to the default probability, $\varepsilon^\sigma$)
Figure 8. Shock to Probability of Default: Response of Policy Rate, Real Interest Rate, Loan Rate, and Effective Deposit Rate
(One standard deviation of the shock to the default probability, $\varepsilon^\delta$)
Figure 9. Shock to Probability of Default: Response of $\omega_t$, Endogenous Technology and Total Technology

(One standard deviation of the technology shock, $\varepsilon^a$)