Trade Policy with Heterogeneous Traders: Do Quotas Get a Bum Rap?

Kala Krishna and Ling Hui Tan
IMF Working Paper

IMF Institute

Trade Policy with Heterogeneous Traders: Do Quotas Get a Bum Rap?

Prepared by Kala Krishna and Ling Hui Tan1

Authorized for distribution by Jorge Roldos

April 2007

Abstract

This Working Paper should not be reported as representing the views of the IMF.

The views expressed in this Working Paper are those of the author(s) and do not necessarily represent those of the IMF or IMF policy. Working Papers describe research in progress by the author(s) and are published to elicit comments and to further debate.

This paper considers the effects of trade policy—tariffs and quotas—when importing is done by competitive traders who are identical ex ante but differ ex post. We show that the standard equivalence results no longer hold and the conventional ranking of tariffs and quotas is turned on its head: quotas are not as bad for welfare as previously believed, while tariffs may restrict trade by more than originally intended. Furthermore, the allocation of property rights (quota licenses) has real effects beyond the distribution of rents; this, in turn, has implications for the effects of corruption on welfare.

JEL Classification Numbers: F13, L10

Keywords: trade policy, tariffs, quotas, firm heterogeneity, entry

Author’s E-Mail Address: kmkrishn@princeton.edu; ltan@imf.org

1 Princeton University/Pennsylvania State University/NBER and IMF Institute, respectively. This paper was initiated while Kala Krishna was a visiting scholar at the IMF Institute. We thank Andres Rodriguez Clare and seminar participants at the IMF Institute and Queens University for their helpful comments. Kala Krishna gratefully acknowledges support from the IMF Institute and Princeton University.
Contents

I. Introduction ................................................................. 3
II. The Model ................................................................. 5
III. Equilibrium Under Free Trade ........................................... 6
IV. Restricted Trade: Quotas .................................................. 8
    A. Welfare Under a Quota ............................................... 11
    B. Allocation of Quota Rent .......................................... 11
V. Restricted Trade: Specific Tariffs ......................................... 14
    A. Welfare Under a Specific Tariff ................................... 16
    B. Specific Tariffs, Quotas, and Equivalence ......................... 16
VI. Restricted Trade: Ad Valorem Tariffs .................................... 20
VII. Conclusion .................................................................. 22
Appendix .............................................................................. 24
References ........................................................................... 29

Figures
1. Competitive Equilibrium .................................................. 6
2. The Free Entry Condition ................................................ 7
3. Import Equivalent Policies .............................................. 8
4. Comparing Rents: Quotas, Specific and Ad-valorem Tariffs ....... 9
5. Quotas Versus Specific Tariffs .......................................... 17
I. Introduction

This paper analyzes the impact of conventional trade policy instruments such as tariffs and quotas in a simple competitive model of supply with heterogeneous traders facing fixed costs and capacity constraints. Traders in our model are import distributors or agents who bring products and services from another country into their home country for distribution and resale.

The role of traders—middlemen who buy and sell a product but neither produce nor consume it themselves—has been largely ignored in the international trade literature. Yet in practice, traders play a key role in international commerce because potential buyers (home consumers) and sellers (producers from the rest of the world) usually cannot meet directly and conduct trades. In other words, traders are necessary to facilitate trade. In return, they make their money by buying low and selling at what the market will bear. A recent Washington Post article (Timberg, 2006) described the activities of an African importer thus: “He strapped $5,000 to his waist and flew off to Asia in search of a new kind of fortune, built not on making goods but trading them... He met with Chinese businessmen in Singapore and quickly spent the $5,000 he brought and $15,000 more he had kept in reserve in Nigeria. In return, he got four used Honda cars and one and a half shipping containers full of car parts. [He] calculates that he can sell the cars and auto parts for $34,000, a profit of 70 percent on his investment.” But trading is neither costless nor riskless—as with any business venture, trading involves setup costs and financial risks.\(^2\) These considerations play an important part in the behavior of traders.

In our model, imports are supplied by such profit-maximizing competitive traders. There is free entry into (and exit from) the import industry: anyone can become a trader by paying a fixed cost which allows him to buy a good from the world market at a random price and to sell it in the integrated domestic market at the market-clearing price. In practice, such fixed costs typically involve, at the minimum, the cost of registering and setting up an import business. The traders face capacity constraints, so no one trader can dominate the market and there is perfect competition. This setup reflects reasonably well trade in various types manufactured products where import and export are mostly carried out by small businesses; it does not apply to other goods such as primary commodities where an integrated world market exists, and highly specialized products with a limited number of buyers and sellers. The model is used to understand what happens when traders get only a part of what their efforts obtain. This could be because there are trade policies in place like tariffs or quotas, or because corruption transfers rents and/or raises costs, both fixed and variable. In this manner, our work contributes to the literature on tariffs (both specific and ad valorem) versus quotas in international trade as well as that on corruption and licensing.\(^3\)

Our model can be seen as a competitive analogue of the monopolistically competitive heterogeneous firm setting common in the international trade/industrial organization literature. In those models, it is relatively complicated to obtain insights into the effects of trade policy; see for example, Melitz and Ottaviano (2005), and Baldwin and Forslid (2006). Furthermore, those models focus primarily on export behavior. In contrast, we focus on importers. By looking at heterogeneous traders in the absence of complications caused by product differentiation and monopolistic competition, we are able to obtain clean results and shed some light on the pure effect of cost heterogeneity and fixed costs on the impact of trade policy in the long term (when

\(^2\)Examples of risks include unexpected fluctuations in prices and delivery times.

\(^3\)For a survey of some of the older literature in this area, see Krishna (1990).
entry/exit can occur) versus the short term (when entry/exit cannot occur).

Why is it important to model the role of traders explicitly? We do so not simply to inject a dose of realism into the analysis but because the size of the import industry matters for the amount of trade that takes place and the consequent level of social welfare. And the size of the import industry, in turn, is affected by the costs and risks involved in importing. This is where our model differs from the standard partial equilibrium analysis of trade policy under perfect competition: by explicitly introducing entry costs and an element of uncertainty for all potential traders—factors that are crucial in determining the entry decisions of traders and ultimately, the outcome of trade policies—we show that neglecting the role of traders can lead one astray in evaluating the effects of various trade restrictions. Thus, the fundamental contribution of this paper lies in its implications for trade policy, which differ quite substantially from the norm.

There is an enormous literature on the ranking of “equivalent” policies in trade. The classic paper in a competitive setting is McCulloch (1973) which argues that equivalence needs to be carefully defined: import equivalent policies may be ranked one way while domestic consumption equivalent ones may be ranked in the opposite manner. In another classic paper, Bhagwati (1965) shows that market structure matters: while tariffs and quotas may be equivalent under competition, they are not under monopoly. Since then, this work has been extended to a variety of settings including oligopoly, uncertainty, and so on.4 However, there has been no analysis of how such policies affect the size of the import industry (via entry) in a competitive setting, and what this means for welfare. We show that when heterogeneous traders and entry costs are introduced into the picture, quotas differ substantially from tariffs via their entry effects. The standard equivalence results no longer hold and the conventional ranking of tariffs and quotas can be turned on its head: quotas are not as bad for welfare as previously believed, while tariffs may restrict trade by more than originally intended.

Our model also provides a new insight into the effect of corruption. The conventional wisdom is that corruption cannot be good for social welfare: to the extent that it results in a pure transfer of rents from one set of agents to another, it is at best neutral to aggregate welfare (though of course, not to the distribution of welfare), but to the extent that anticipated corruption discourages investment, it can be harmful to welfare. In contrast, we show that there are circumstances in which corruption can actually improve social welfare. When there is an import quota in place, the way in which the quota licenses are distributed matters a great deal. We show that entry is (constrained) optimal if importers have to pay for the quota licenses but not if they are given the licenses for free according to some rationing rule, therefore welfare is higher whenever the quota rent is captured by some domestic agent(s) other than the traders themselves.5 This suggests that society as a whole will be better off if the quota licenses were “sold” by corrupt customs officials through bribery than if the quota licenses were given away to traders by a benevolent government. The way in which quotas are allocated thus affects aggregate welfare, and not just its allocation across agents.

The paper proceeds as follows. Section II sets up the model. Section III characterizes the free trade equilibrium with endogenous entry of traders, and shows that this equilibrium is socially optimal. Section IV looks at what happens when trade is restricted by a quota set below the free

---

4 More recently, Jørgensen and Schröder (2005) compare ad valorem and specific tariffs in terms of welfare in monopolistically competitive models using the standard CES utility function introduced by Spence-Dixit-Stiglitz.

5 Whether the quota rents are captured by the government or by corrupt domestic agents does not matter for aggregate welfare, only for its distribution across domestic agents.
trade level of imports. For a given quota, entry will adjust (down) to the socially optimal level but of course, the smaller the quota, the lower the welfare. We also consider the effects of selling the quota rights (by the government or through bribery) versus giving them to the traders, and show that the former welfare dominates the latter: the allocation of property rights affects market outcomes, and corruption can actually have beneficial welfare effects. Sections V and VI consider, respectively, a specific tariff and an ad valorem tariff that generate the same initial level of imports as the quota. A specific tariff results in greater exit of traders and lower welfare than does a quota. An initially import equivalent ad valorem tariff reduces entry and welfare even more than does an initially import equivalent specific tariff. Section VII concludes.

II. The Model

Assume there is an integrated domestic market for a particular good where a single price, $P$, prevails. Demand in this market is given by $Q^D(P)$. For simplicity, assume the good is not produced at home. Domestic consumers cannot directly access the world supply of this good. Instead, they are served by traders who import the good from the world market and sell it domestically. This world market is not integrated in the sense that there is no single price that prevails: rather, there is a distribution of prices denoted by $F(c)$ which this small country takes as given.

Suppose there is a continuum of these traders with mass $N$. Every trader has a fixed entry cost, $f_e$, that he has to incur to enter the market. Once this is paid, the trader gets a draw from the known distribution of prices $F(c)$. In other words, paying the entry cost allows a trader to access the world market at a random price and to sell in the domestic market at the domestic market-clearing price. There are limits to the scale at which the traders can operate as each trader has limited resources at his disposal. For simplicity, assume that each trader has the capacity to import one unit.

The setup is as follows. First, the trader decides whether to enter the market or not. If he enters, he is matched with a seller in the world market and gets a draw of $c$ from $F(c)$. Depending on the draw, the trader decides whether to buy the good or not, keeping in mind that he receives no direct utility from the good, only the profit from selling the good domestically at the market clearing price. If the domestic price is $P$, only those traders who draw a cost of $c \leq P$ will choose to import the good. Thus, supply from a unit mass of traders is $F(P)$. As usual, the mass of traders in equilibrium is determined so that their expected profit at the time of entry is zero. In what follows, we work with a static setting.

---

6 We can also interpret $Q^D(P)$ as domestic excess demand as long as domestic entry is fixed, for the positive part of the analysis.

7 There are several ways to think about this. We can just assume a price distribution in the world market without specifying where it comes from (which is what we do here). Or we can assume that the traders do not know their productivity (or costs) prior to entry. After entry, each trader draws a productivity, and hence a cost—he is then able to import one unit at the world price plus his costs. High productivity traders would thus be willing to supply at a lower price. Finally, we can think of $c$ as inversely related to quality, which is variable and cannot be contracted upon in the world market. So there could be a single price in the world market per “effective” unit, but profit in the domestic market is variable due to differences in quality.

8 Capacity constraints of some sort are necessary for internal consistency of a model with cost heterogeneity and perfect competition.

9 It is easy to convert this into its dynamic analogue in steady state (à la Melitz (2003)) by assuming a constant
III. Equilibrium Under Free Trade

Under free trade, if a mass of $N$ traders enters the market, supply will be equal to $NF(P)$, and the free trade market clearing price, $P^F(N)$, will be determined by the intersection of demand and supply:

$$Q^D(P^F(N)) = NF(P^F(N)).$$

Note that $P^F(N)$ is decreasing in $N$: the price of the import falls as more traders enter the market. The free trade situation is illustrated in Figure 1, which shows a downward sloping demand curve $P^D(Q)$ and an upward sloping supply curve $P^S(Q,N)$ drawn for a mass of $N$ traders.

Consider a trader deciding whether to enter the market or not. If he enters and draws a cost below the market clearing price, he will earn quasi rent equal to the difference between the two, i.e., $P^F(N) - c$; if he draws a cost greater than or equal to $P^F(N)$, he will not import and his quasi rent will be zero. Hence, his expected profit or quasi rent is:

$$r^F(N) = \int_{0}^{P^F(N)} (P^F(N) - c) f(c)\, dc = \int_{0}^{P^F(N)} F(c)\, dc$$

(where the second equality follows from integration by parts). Total quasi rent earned in the economy, $Nr^F(N)$, is equal to the area between the supply curve and the equilibrium price, that is, the area $OAB$ in Figure 1. Since $P^F(N)$ is decreasing in $N$, $r^F(N)$ must also be decreasing in $N$.

Entry will occur until each trader’s expected quasi rent equals the fixed cost of entry. Hence, the exogenous death rate for all firms, and setting the mass of entrants in each time period to exactly compensate for these deaths. We choose not to do more than note this in the interests of simplicity.
Figure 2. The Free Entry Condition

The equilibrium level of entry under free trade (which we will denote by \( N^F \)) is determined by:

\[
r^F (N^F) = \int_0^{P^F(N^F)} F(c) \, dc = f_e.
\]

as depicted in Figure 2. The equilibrium level of imports is \( Q^F = Q^D \left( P^F \left( N^F \right) \right) \).

**Assumption 1** \( f_e < \int_0^P F(c) \, dc \) where \( P \) is the price at which demand becomes zero.

If Assumption 1 holds, there is a unique free entry equilibrium with \( N^F > 0 \). An increase in the entry cost \( f_e \) will reduce the equilibrium mass of traders \( N^F \).

Let us now turn to the welfare implications of our model. We know that in the case of homogeneous goods and market power (Mankiw and Whinston, 1986) there is too much entry due to the “business stealing effect”: firms do not internalize the fact that their entry dissipates the profits of other firms and as a result, more of them enter the market than is socially optimal. On the other hand, in the case of differentiated products and monopolistic competition (Dixit and Stiglitz, 1977) there may be too much entry or too little entry relative to the social optimum. Is entry optimal in the case of competitive heterogeneous traders?

For any \( N \), welfare is the sum of consumer surplus and producer surplus (quasi rent) less entry costs:

\[
W^F(N) = \int_{P^F(N)}^{\bar{P}} Q^D(P) \, dP + N \int_0^{P^F(N)} F(c) \, dc - N f_e.
\]

\(^{10}\)In Figure 2, since \( r^F (N) \) is decreasing in \( N \), the intersection of \( r^F (N) \) and \( f_e \) will occur at \( N^F > 0 \) only if \( f_e \) lies below the vertical intercept of \( r^F (N) \).
Proposition 1  The free trade equilibrium results in a level of entry that is socially optimal, i.e., 
\( dW^F (N) / dN = 0 \) at \( N = N^F \).

Proof. In the Appendix. ■

IV. Restricted Trade: Quotas

Suppose that we are at the free trade equilibrium with \( N^F \) traders in the market when a binding 
quota of \( Q \) is imposed, where \( Q \) is less than the free trade level of imports. Traders now have to 
purchase a quota license from the government in order to sell the imported good in the domestic 
market. What happens to entry?

The equilibrium is easy to depict in a picture, as is done in Figure 3. The price in the domestic 
market (the demand price) is such that \( Q^D (P) = Q \); call this \( P^D (Q) \). For any given mass of 
traders, \( N \), the value of a quota license, \( L (N, Q) \), is equal to the difference between the demand 
price \( P^D (Q) \), and the supply price \( P^S (N, Q) \), where the latter is given by setting supply equal 
to the quota level:

\[
NF (P^S (N, Q)) = Q. 
\] (5)

Note that \( P^S (N, Q) \) is decreasing in \( N \) and increasing in \( Q \). The license price, \( L (N, Q) \), equals 
the distance \( DF \) in Figure 3.

The quasi rent function facing each trader is now:
as long as the quota is binding. Like $P^S (N, \bar{Q})$, $r^Q (N, \bar{Q})$ is decreasing in $N$ and increasing in $\bar{Q}$. The area $OCD$ in Figure 3 corresponds to the quasi rent earned by all traders who enter, so $r^Q (N, \bar{Q})$ is equal to $(1/N)$ of the area $OCD$.

How does $r^Q (N, \bar{Q})$ compare with $r^F (N)$? Define $N_0 (\bar{Q})$ to be the level of $N$ where the supply curve intersects demand at $P^D (\bar{Q})$. This inverse supply curve is depicted in Figure 3. Since $\bar{Q}$ is less than the free trade level of imports, $N_0 (\bar{Q})$ must be below $N^F$. Once $N$ falls to $N_0 (\bar{Q})$ and below, the quota will no longer be binding. Therefore, for a given $\bar{Q}$, $r^Q (N, \bar{Q})$ will be identical to $r^F (N)$ when $N \leq N_0 (\bar{Q})$. When $N > N_0 (\bar{Q})$, $r^Q (N, \bar{Q}) < r^F (N)$: this follows from Equations (2) and (6) since $P^S (N^F, \bar{Q}) < P^F (N^F)$. This is depicted in Figure 4.

Equilibrium entry under the quota, which we will denote by $N^Q (\bar{Q})$, is determined by the intersection of $r^Q (N, \bar{Q})$ and $f_e$: 

$$r^Q (N, \bar{Q}) = \int_{0}^{P^S (N, \bar{Q})} F(c) \, dc$$

(6)
\[ P^S(N^Q(Q), Q) \int_0 F(c) dc = f_e. \]  

(7)

Note that \( N^Q(Q) \) is increasing in \( Q \), as a reduction in the quota level decreases \( N_0(Q) \) and shifts the curve \( r^Q(N, Q) \) downwards in Figure 4.

**Lemma 1** The equilibrium supply price is invariant with respect to \( Q \). In other words, \( P^S(N^Q(Q), Q) = P^F(N^F) \).

**Proof.** This follows from comparing the free entry conditions under free trade and under the quota (Equations (3) and (7)). Since the expected quasi rent per trader must equal \( f_e \) in equilibrium, \( P^S(N^Q(Q), Q) \) must be equal to \( P^F(N^F) \) and be independent of \( Q \).

**Proposition 2** If we start at the free trade equilibrium and impose a binding quota, there will be a net exit of traders over time and the new equilibrium will be characterized by a lower level of entry compared with free trade. Average cost will first fall and then rise (average productivity will first rise and then fall) as traders first change their import and then their entry decisions in response to the quota. Once entry adjusts, in the new equilibrium, average cost (productivity) will return to its free trade level. The price of a quota license will fall over time as the number of traders shrinks, but it will remain positive in the new equilibrium.

**Proof.** We can think of the free trade equilibrium as equivalent to a quota set at the free trade level of imports with a mass of \( N^F \) traders. Thus the impact effect of imposing a binding quota (i.e., reducing the quota from \( Q^F \) to \( Q \)) will be to make it unprofitable for incumbent traders who have cost draws between \( P^S(N^F, Q) \) and \( P^F(N^F) \) to import. The supply price drops to \( P^S(N^F, Q) \). This will lower average cost; in other words, maintaining our analogy with the monopolistically competitive heterogeneous firm setting, it will raise average productivity. The license price is \( L(N^F, Q) \).

Over time, there will be a net exit of traders from the market.\(^{11}\) In the new equilibrium, the mass of traders, \( N^Q(Q) \), is smaller than that under free trade: since \( r^Q(N, Q) \) lies below \( r^F(N) \) when \( N > N_0(Q) \), it must intersect \( f_e \) at a point between \( N_0(Q) \) and \( N^F \). As \( N \) adjusts down from \( N^F \) to its new equilibrium \( N^Q(Q) \), the supply price (i.e., the cost of the marginal trader) will rise and average cost will also rise (or average productivity will fall). As the supply price rises, the license price falls.

In the new equilibrium, the supply price returns to \( P^F(N^F) \), so average cost and productivity return to their free trade levels: this follows from Lemma 1. Hence, entry will always adjust to completely offset the reduction in the supply price caused by the binding quota. The equilibrium license price, \( L(N^Q(Q), Q) \), will therefore be lower than \( L(N^F, Q) \) but will still be positive.

Thus, our model provides some stark but interesting and potentially testable predictions for the path of cost/productivity in an industry in response to a quantitative restriction. In Section IV.B, we show that this depends on the way in which the quota rights are distributed.

\(^{11}\)In Figure 3, the area \( OAB \) represents free trade profits while the area \( OCD \) represents total quasi rent under the quota \( Q \). At \( N = N^F \), it is clear that the quasi rent of \( OCD \) is not enough to cover entry costs—recall that \((1/N^F) \) of the area \( OAB \) is equal to \( f_e \), and \( OAB > OCD \). Thus, traders will exit.
A. Welfare Under a Quota

Let us assume that the quota rights are sold and that these revenues go to the government. For any $N$ and $\bar{Q}$, welfare is the sum of consumer surplus, producer surplus, and license revenue, less entry costs:

$$ W^Q(N, \bar{Q}) = \int_{P^D(\bar{Q})}^{\bar{P}} Q^D(P) dP + N \int_0^{P^S(N, \bar{Q})} F(c) dc + \left[P^D(\bar{Q}) - P^S(N, \bar{Q})\right] \bar{Q} - N f_e. \quad (8) $$

**Lemma 2** The level of entry is socially optimal, given the quota level.

**Proof.** In the Appendix. ■

Note that in equilibrium, total quasi rent exactly equals total entry costs so welfare comprises only consumer surplus and license revenue:

$$ W^Q(N^Q(\bar{Q}), \bar{Q}) = \int_{P^D(\bar{Q})}^{\bar{P}} Q^D(P) dP + \left[P^D(\bar{Q}) - P^S(N^Q(\bar{Q}), \bar{Q})\right] \bar{Q}. \quad (9) $$

**Lemma 3** An increase in the quota always raises welfare.

**Proof.** In the Appendix. ■

B. Allocation of Quota Rent

We have assumed so far that quota licenses are sold, with the revenues accruing to the government. But this is often not the case; it is quite common for quota licenses to be awarded to some or all importers based on certain criteria such as past import performance. In this section, we show that the allocation of quota rent has an impact on the entry decision of traders. This means that the details of quota implementation—who receives the licenses and under what conditions—affect not just the distribution of the rents, but the equilibrium size of the import industry.

To illustrate, let us consider the scenario where quota licenses are not sold but rather awarded to the traders free of charge. To avoid confusion, we will refer to such an arrangement as “free” quota to distinguish it from the previous case where traders had to pay for the quota licenses.

For concreteness, assume that quota licenses are tradeable. When a trader chooses to enter, he pays the fixed cost, and upon doing so, he is allocated a license. This allocation is certain if the mass of traders that enter does not exceed the quota, but occurs with a probability $Q/N$ if $N > \bar{Q}$. Traders need not import to get a license, they just need to enter (e.g., obtain a business
registration number). Since license markets work without friction, we can assume that every trader that enters sells his license for \( L(N, \bar{Q}) \) and then buys it back if his cost realization is below the supply price \( P^{S}(N, \bar{Q}) \).

Let \( r^{q}(N, \bar{Q}) \) denote the expected quasi rent per trader under the “free” quota.\(^{12}\) If \( N \leq N_{0}(\bar{Q}) \), then \( L(N, \bar{Q}) = 0 \), \( P^{S}(N, \bar{Q}) = P^{F}(N) \), and \( r^{q}(N, \bar{Q}) = r^{F}(N) \)—in other words, the quota is not binding. But if \( N > N_{0}(\bar{Q}) \), then \( L(N, \bar{Q}) > 0 \) and \( P^{S}(N, \bar{Q}) < P^{F}(N) \). Furthermore, \( \bar{Q}/N < \bar{Q}/(NF(P^{S}(N, \bar{Q}))) = 1 \), thus, entrants are not assured of a license. As a result, they obtain \((\bar{Q}/N) L(N, \bar{Q})\) in expected terms from selling any licenses they are given and \( P^{S}(N, \bar{Q}) \)

\[
\int_{0}^{P(c)} F(c) \, dc \text{ from their productive (importing) activities. More compactly, expected earnings of an entrant are:}
\]

\[
r^{q}(N, \bar{Q}) = \min \left[ 1, \frac{\bar{Q}}{N} \right] L(N, \bar{Q}) + \int_{0}^{P^{S}(N, \bar{Q})} F(c) \, dc.
\]

(10)

Clearly, \( r^{q}(N, \bar{Q}) > r^{Q}(N, \bar{Q}) \) for any \( N \) and \( \bar{Q} \), as long as the value of the quota license is positive (i.e., when \( N > N_{0}(\bar{Q}) \)—in other words, when \( N > N_{0}(\bar{Q}) \), entrants must make more in expected terms at any given \( N \) and \( \bar{Q} \) than when they have to buy licenses.

**Lemma 4** For \( N > N_{0}(\bar{Q}) \), \( r^{q}(N, \bar{Q}) \) is decreasing in \( N \), and decreasing in \( \bar{Q} \) if the quota is close to the free trade level of imports, but increasing in \( \bar{Q} \) if the quota is relatively restrictive.

**Proof.** In the Appendix. \( \blacksquare \)

Equilibrium entry under the “free” quota, which we will denote by \( N^{q}(\bar{Q}) \), is determined by the intersection of \( r^{q}(N, \bar{Q}) \) and \( f_{e} \).\(^{13}\)

**Proposition 3** Giving the quota licenses to traders results in more entry and lower average cost (higher average productivity) compared to selling the quota licenses to traders. If the quota is not too restrictive, giving the licenses to traders can even raise entry above the free trade level. However, entry is sub-optimally high given the quota level and equilibrium welfare is lower when quota licenses are given away than when they are sold.

**Proof.** If we start at the free trade equilibrium and impose a binding “free” quota, the impact effect will be to lower the supply price to \( P^{S}(N^{F}, \bar{Q}) \), as in the previous case; this will lower average cost (or raise average productivity). The value of a license is \( L(N^{F}, \bar{Q}) \) but unlike the previous case, this amount is part of the trader’s profit.

Over time, there could be a net exit or net entry of traders in the market: if the quota is restrictive, expected quasi rent per trader falls on impact (when \( N = N^{F} \) and there will be net exit, but if the quota is not very restrictive, expected quasi rent per trader rises on impact and there will be net entry (from Lemma 4). If there is net exit, the supply price will rise (average

\(^{12}\) We use the lower case \( q \) as the superscript to differentiate the quasi rent function from \( r^{Q}(N, \bar{Q}) \), used previously. 

\(^{13}\) Note that \( N^{q}(\bar{Q}) \) is decreasing in \( \bar{Q} \) if the quota is close to the free trade level of imports, but it is increasing in \( \bar{Q} \) if the quota is relatively restrictive.
productivity will fall) as \( N \) falls; if there is net entry, the supply price will fall further (average productivity will rise further) as \( N \) increases.

In the new equilibrium, the mass of traders will be \( N^q (\bar{Q}) \), where \( N^q (\bar{Q}) \) is greater than \( N^Q (\bar{Q}) \) and possibly greater than \( N^F \). In Figure 4, the quasi rent curves, \( r^Q (N, \bar{Q}) \), \( r^F (N) \), and \( r^q (N, \bar{Q}) \) coincide at \( N_0 (\bar{Q}) \); beyond that point, \( r^q (N, \bar{Q}) > r^Q (N, \bar{Q}) \). It follows, therefore, that there is greater entry when licenses are given to traders than when they are sold, i.e., \( N^q (\bar{Q}) > N^Q (\bar{Q}) \).

In fact, it is possible for equilibrium entry under the “free” quota to exceed \( N^F \), the equilibrium entry level under free trade. When the quota is close to the free trade level of imports, \( r^q (N, \bar{Q}) \) is decreasing in \( \bar{Q} \), so \( r^q (N, \bar{Q}) > r^F (N) \), as depicted in Figure 4. As a result, more traders will be encouraged to enter the market. Thus, when the quota is not too restrictive and the quota rights are given to the traders, the equilibrium level of entry would be higher than under free trade, i.e., \( N^q (\bar{Q}) > N^F \).

Since entry is greater when the quota rents are given to traders, the “free” quota must remain binding in equilibrium, and the equilibrium supply price must be lower compared to the previous case when the quota licenses were sold: \( P^S (N^q (\bar{Q}), \bar{Q}) < P^S (N^Q (\bar{Q}), \bar{Q}) = P^F (N^F) \). Therefore, average cost is also lower (average productivity is higher) than in the previous case.

Turning to welfare, note that \( \text{W}^q (\bar{Q}, N) \), like \( \text{W}^Q (\bar{Q}, N) \), is the sum of consumer surplus, producer surplus, and license rents, less entry costs, for any \( N \) and \( \bar{Q} \). We know from Lemma 2 that given \( \bar{Q} \), entry is optimal when the quota licenses are sold. Since \( N^q (\bar{Q}) > N^Q (\bar{Q}) \), it follows that given \( \bar{Q} \), there is excessive entry under the “free” quota. In the free entry equilibrium, when all quasi rents are competed away, welfare is made up solely of consumer surplus:

\[
\text{W}^q (\bar{Q}, N^q (\bar{Q})) = \int_{P^D(\bar{Q})} \bar{Q}^D(P)dP. \tag{11}
\]

It is clear from Equations (9) and (11) that welfare is lower with the “free” quota. The reason is that license rents are not frittered away in excessive entry when traders have to pay for their quota licenses, but are so frittered away when they do not. □

Proposition 3 demonstrates that the common practice by many governments of devising rationing rules to distribute quota licenses leads to inferior welfare outcomes compared to simply selling the licenses outright.\(^{15}\)

Proposition 3 has a further interesting implication: welfare is higher when quota rents are not appropriated by the traders but accrue to some other domestic agent(s). If quota rents are captured by the traders (under a “free” quota), there will be too much entry for a given quota

\(^{14}\)In Figure 3, under free trade, total quasi rent is equal to the area of the producer surplus triangle \( OAB \) when \( N = N^F \). When a “free” quota is imposed at \( N = N^F \), total quasi rent is equal to the area of the producer surplus triangle plus license revenue, i.e., area \( OGF\bar{D} \). If the quota is not too restrictive (i.e., if the quota is close to the free trade level of imports), \( OGF\bar{D} \) must be larger than \( OAB \) because the increase in quasi rent from the license revenue (rectangle \( DF\bar{G}C \)) must exceed the decrease (triangle \( DF\bar{B} \)): when the quota is close to the free trade level of imports, the area of triangle \( DF\bar{B} \) approaches zero because its height \( (HB) \) and base \( (DF) \) approach zero, but the area of rectangle \( DF\bar{G}C \) does not approach zero as fast since \( FG \) does not approach zero, though \( DF \) does.

\(^{15}\)Note that this is different from the question of allowing quota licenses to be traded. Even if trade is allowed, entry will be distorted by not selling the quota to begin with.
and consequently lower welfare, but if quota rents are captured by some other domestic agent, then entry is optimal, given the quota, and welfare is higher. In the previous case, we assumed that it was the government that sold the quota licenses and thereby captured the quota rent, but it could just as well be corrupt customs officials that “sell” the quota licenses through bribery. Whether the quota rents are captured by the government or by corrupt officials does not matter for aggregate welfare, only for its distribution across domestic agents; as long as the quota rents go to domestic agents other than the traders, aggregate welfare will be higher than under a “free” quota. This implies that society will actually be better off if the quota licenses are “sold” via bribery and corruption than if they were given away to the traders by a benevolent government. Thus, we have uncovered another channel through which corruption can have real effects, rather than simply distributional effects; in this example, the real effects are actually welfare enhancing.

We will now compare the equilibrium effects of “equivalent” trade policies, that is, trade policies that generate the same level of imports as the quota, \( Q \), with the free trade mass of traders, \( N^F \). This focuses attention on import equivalence abstracting from entry, which is a likely starting point for policy makers who typically lack the information necessary to predict the induced entry effects of different trade policies.\(^{16}\)

V. Restricted Trade: Specific Tariffs

Let us start again from the free trade equilibrium with a mass of \( N^F \) traders, and consider now a specific tariff that will reduce imports to the level of the quota, \( \bar{Q} \), as depicted in Figure 3. This is the sort of scenario that a policy maker is likely to consider in choosing between a quota and a tariff to restrict imports by a certain amount. The specific tariff is thus initially import equivalent to the quota, and equal to the quota license price at \( N = N^F \):

\[
z(N^F, \bar{Q}) = L(N^F, \bar{Q}). \tag{12}
\]

For notational simplicity, let us denote \( z(N^F, \bar{Q}) \) by \( \bar{z} \). In Figure 3, this is equivalent to shifting the supply curve up by \( L(N^F, \bar{Q}) \), or the distance \( DF \). Pre-tariff supply is \( NF(P) \) and post-tariff supply is \( NF(P - \bar{z}) \). The post-tariff supply curve has a vertical intercept at \( E \) where the distance \( OE \) equals \( DF \). The intersection of demand and post-tariff supply determines the quantity imported and the price paid by the consumer, \( P^D(N, \bar{z}) \):

\[
NF \left( P^D(N, \bar{z}) - \bar{z} \right) = Q^D \left( P^D(N, \bar{z}) \right). \tag{13}
\]

Note that \( P^D(N, \bar{z}) \) is decreasing in \( N \) and increasing in \( \bar{z} \). The marginal trader receives the supply price, \( PS(N, \bar{z}) = P^D(N, \bar{z}) - \bar{z} \) where \( PS(N, \bar{z}) \) is decreasing in both arguments.\(^{17}\)

Each trader now expects to receive quasi rent of:

\(^{16}\)Our results generalize to policies that are equivalent given any number of traders in the market, not necessarily the free trade number of traders. This is shown in Section V.B.

\(^{17}\)This can be seen by writing Equation (13) as: \( NF(P^S(N, \bar{z})) = Q^D(P^S(N, \bar{z}) + \bar{z}) \) and partially differentiating it with respect to \( N \) and \( \bar{z} \). For any given \( N \), an increase in \( \bar{z} \) drives a bigger wedge between demand and supply, so the price of the marginal supplier must fall.
Since \( P^S (N, \tilde{z}) \) is decreasing in both arguments, \( r^z (N, \tilde{z}) \) is also decreasing in both arguments.

**Lemma 5** \( r^z (N, \tilde{z}) \) lies below \( r^Q (N, Q) \) for \( N < N^F \) and above \( r^Q (N, Q) \) for \( N > N^F \).

**Proof.** At \( N = N^F \), the specific tariff is equivalent to the quota so \( \tilde{z} = L (N^F, Q) \), and \( P^S (N^F, \tilde{z}) = P^S (N^F, Q) \), therefore \( r^z (N^F, \tilde{z}) = r^Q (N^F, Q) \). As \( N \) falls below \( N^F \), the quota license price falls but the specific tariff remains unchanged, so \( \tilde{z} > L (N, Q) \); hence, \( P^S (N, \tilde{z}) < P^S (N, Q) \), and \( r^z (N, \tilde{z}) < r^Q (N, Q) \) for \( N < N^F \). By an analogous argument it can be shown that for \( N > N^F \), \( r^z (N, \tilde{z}) > r^Q (N, Q) \).

Equilibrium entry under the specific tariff, which we will denote by \( N^z (\tilde{z}) \), is determined by the intersection of \( r^z (N, \tilde{z}) \) and \( f_c \). Note that \( N^z (\tilde{z}) \) is decreasing in \( \tilde{z} \). Furthermore, it follows from the same argument as Lemma 1 that the equilibrium supply price is the same as the free trade price, that is: \( P^S (N^z (\tilde{z}), \tilde{z}) = P^F (N^F) \).

**Proposition 4** If we start at the free trade equilibrium and impose a specific tariff that is initially import equivalent to the quota, there will be a net exit of traders over time and the new equilibrium will be characterized by a lower level of entry compared with the quota. Average cost will first fall and then rise (average productivity will first rise and then fall) as traders change their import and entry decisions in response to the tariff. Imports will fall over time as the number of traders shrinks. In the new equilibrium, average cost (productivity) will return to its free trade level, and imports will be lower than the quota level.

**Proof.** If we start at the free trade equilibrium and impose a specific tariff \( \tilde{z} \), the impact effect will be to make it unprofitable for incumbent traders who have cost draws between \( P^S (N^F, \tilde{z}) \) and \( P^F (N^F) \) to import. As in the case of the quota when traders have to pay for the licenses, the supply price drops to \( P^S (N^F, \tilde{z}) \), which is equal to \( P^S (N^F, Q) \). Hence average cost falls (average productivity rises).

Over time, there will be a net exit of traders from the market. Since quasi rent under the specific tariff is equal to that under the quota at \( N = N^F \) (from Lemma 5), using the same logic as before (when the quota licenses were sold), it follows that quasi rent per trader, \( r^z (N^F, \tilde{z}) \) is less than the entry cost. Hence, there will be a net exit of traders. As \( N \) falls, the supply price rises: in Figure 3, both the pre-tariff and post-tariff supply curves will steepen but remain parallel to each other and anchored at \( O \) and \( E \) respectively.

In the new equilibrium, the mass of traders is smaller than that under the quota: Lemma 5 implies that the intersection of \( f_c \) with \( r^z (N^F, \tilde{z}) \), which determines the equilibrium mass of traders, \( N^z (\tilde{z}) \), must occur below \( N^Q (Q) \) as depicted in Figure 4. Hence, \( N^z (\tilde{z}) < N^Q (Q) < N^q (Q) \) and final imports under the initially import equivalent tariff will be less than the quota. In the new equilibrium, the supply price returns to \( P^F (N^F) \), so average cost and productivity return to their free trade levels: entry will always adjust to completely offset the reduction in the supply price caused by the tariff. ■
A. Welfare Under a Specific Tariff

For any $N$ and $\tilde{z}$, welfare is the sum of consumer surplus, producer surplus, and tariff revenue, less entry costs:

$$W^z (N, \tilde{z}) = \int_{P^D(N, \tilde{z})}^{\bar{P}} Q^D(P) dP + N \int_{0}^{P^S(N, \tilde{z})} F(c) dc + \tilde{z} Q^D(P^D(N, \tilde{z})) - N f_e.$$  \hspace{1cm} (15)

**Lemma 6** Welfare is decreasing in $\tilde{z}$ for a given $N$, i.e., $\partial W^z (N, \tilde{z}) / \partial \tilde{z} < 0$. Welfare is increasing in $N$ for a given $\tilde{z}$, i.e., $\partial W^z (N, \tilde{z}) / \partial N > 0$, for $N \leq N^z (\tilde{z})$; in other words, the level of entry is sub-optimally low, given the specific tariff.

**Proof.** In the Appendix. ■

In equilibrium (when $N = N^z (\tilde{z})$), all quasi rents are competed away so welfare consists of consumer surplus plus tariff revenue only:

$$W^z (N^z (\tilde{z}), \tilde{z}) = \int_{P^D(N^z (\tilde{z}), \tilde{z})}^{\bar{P}} Q^D(P) dP + \tilde{z} Q^D(P^D(N^z (\tilde{z}), \tilde{z}))$$  \hspace{1cm} (16)

**Lemma 7** A reduction in the specific tariff always raises welfare.

**Proof.** In the Appendix. ■

B. Specific Tariffs, Quotas, and Equivalence

So far we have used the free trade equilibrium with $N^F$ traders as our starting point for analyzing the effects of “equivalent” policies. But in practice, the starting point for policy analysis need not be the free trade equilibrium: if there are trade policies already in place, the number of traders in the market could be greater or less than the free trade number. To see how the different notions of equivalence affect our results, it is useful to look at Figure 5. This has the tariff $z$ on the vertical axis and the mass of traders $N$ on the horizontal axis. The line $FF$ depicts combinations of $z$ and $N$ that are consistent with the free trade level of imports, $Q^F$:

$$Q^D(P^D(N, z)) = Q^F.$$  \hspace{1cm} (17)

Since $P^D(N, z)$ is decreasing in $N$ and increasing in $z$, the $FF$ line has to be upward sloping as depicted: an increase in $z$ (for a given $N$) raises the price paid by consumers and reduces imports while an increase in $N$ (for a given $z$) lowers the price paid by consumers and increases imports,
Figure 5. Quotas Versus Specific Tariffs
so an increase in \( z \) must be accompanied by an increase in \( N \) in order to keep imports constant along \( FF \). By definition, \( FF \) intersects the horizontal axis at \( N^F \), since \( P^D (N^F, 0) = P^F (N^F) \).

Similarly, the upward sloping line \( QQ \) depicts combinations of \( z \) and \( N \) that are consistent with the quota, \( Q \):

\[
Q^D (P^D (N, z)) = Q.
\]

(18)

As the quota is set below the free trade level of imports, the \( QQ \) line must lie above and to the left of \( FF \). Moreover, its intersection with the horizontal axis occurs at \( N_0 (\bar{Q}) \), the level of entry in the absence of tariffs that makes the quota just binding.

Likewise, the upward sloping line \( ZZ \) depicts combinations of \( z \) and \( N \) that are consistent with the free entry equilibrium output level under the specific tariff, \( \bar{z} \):

\[
Q^D (P^D (N, z)) = Q^\bar{z} (\bar{z})
\]

(19)

where \( Q^\bar{z} (\bar{z}) = Q^D (P^D (N^\bar{z} (\bar{z}), \bar{z})) \). Since we know that \( Q^\bar{z} (\bar{z}) \) is less than \( \bar{Q} \), the \( ZZ \) line must lie above and to the left of \( QQ \).

The fourth component of the diagram is the free entry line, \( EE \). The line \( EE \) depicts combinations of \( z \) and \( N \) such that the quasi rent per trader exactly offsets the cost of entry:

\[
r^z (N, z) = f_e.
\]

(20)

As noted earlier, \( r^z (N, z) \) is decreasing in both arguments. Hence the \( EE \) line must slope downwards as depicted in Figure 5. Also, the \( EE \) line must cut the horizontal axis at \( N = N^F \) since that point represents the free trade equilibrium.

The intersection of the \( QQ \) line and the \( EE \) line gives the free entry equilibrium with a quota of \( \bar{Q} \) (when the quota licenses are sold). This occurs at point \( B \) on the line \( EE \), where \( z \) and \( N \) are such that the free entry condition is met and imports are equal to \( \bar{Q} \). At point \( B \), \( z \) is exactly equal to the license price \( L (N^Q (\bar{Q}), \bar{Q}) \) due to tariff quota equivalence. Since the \( EE \) line is downward sloping, \( N_0 (\bar{Q}) < N^Q (\bar{Q}) < N^F \) as was shown earlier.

**Proposition 5** Equilibrium entry under the quota (when licenses are sold) is higher than under a specific tariff that is initially import equivalent to the quota at \( N > N^Q (\bar{Q}) \). If equivalence is defined at \( N < N^Q (\bar{Q}) \), then the opposite holds.

**Proof.** We can use Figure 5 to depict the import equivalent specific tariff at any initial \( N \). For example, suppose we start at \( N = N^F \). From \( N^F \) on the horizontal axis, go up to the \( QQ \) curve to point \( A \): the vertical distance gives the level of \( z \) needed to get imports of \( \bar{Q} \). This is what we had denoted by \( \bar{z} \) in the previous subsection. Set the specific tariff at this level and find the number of traders that enter using the \( EE \) line: point \( C \) on the \( EE \) line will give the equilibrium entry, \( N^z (\bar{z}) \), with this tariff. It is clear from Figure 5 that \( N^z (\bar{z}) < N^Q (\bar{Q}) < N^F \).\(^{18}\)

\(^{18}\)Since \( N^q (\bar{Q}) > N^Q (\bar{Q}) \), we can say that entry under the specific tariff is lower than entry under the (initially) import-equivalent quota, no matter how the licenses are distributed.
To generalize, suppose that we start at some arbitrary $N = \tilde{N}$. Following the same procedure as above, we can determine the specific tariff equivalent to the quota $\tilde{Q}$ by going up to the $QQ$ line in Figure 5: the vertical distance gives $z(\tilde{N}, \tilde{Q})$. The free entry level of traders at this tariff—call it $\tilde{N}^z$—is obtained by going across to the $EE$ line at $z = z(\tilde{N}, \tilde{Q})$. If $\tilde{N} > N^Q(\tilde{Q})$, then $\tilde{N}^z < N^Q(\tilde{Q})$ and equilibrium welfare under the tariff is higher than when equivalence is defined at $N = N^F$, but lower than $W^Q(N^Q(\tilde{Q}), \tilde{Q})$.\footnote{Equilibrium welfare under the quota remains at $W^Q(N^Q(\tilde{Q}), \tilde{Q})$ as before.} If $\tilde{N} < N^Q(\tilde{Q})$, then $\tilde{N}^z > N^Q(\tilde{Q})$ and equilibrium welfare under the tariff is higher than $W^Q(N^Q(\tilde{Q}), \tilde{Q})$.\footnote{If $\tilde{N} = N^Q(\tilde{Q})$, then $\tilde{N}^z = N^Q(\tilde{Q})$, i.e. if we start start at $N^Q(\tilde{Q})$ and impose an import equivalent specific tariff, there will be no change in $N$. If $\tilde{N} = N_0(\tilde{Q})$, then $\tilde{N}^z = N^F$: if we start start at $N_0(\tilde{Q})$, the import equivalent specific tariff will be zero. If $\tilde{N} < N_0(\tilde{Q})$, then the import equivalent specific tariff becomes a subsidy, and $\tilde{N}^z > N^F$.} Basically, the smaller is $\tilde{N}$, the smaller will be the import equivalent specific tariff and the closer the equilibrium level of entry will be to the free trade equilibrium, $N^F$, so that there is less of a distortion under the specific tariff than under the quota when the licenses are sold. \hfill \blacksquare

This result argues for caution when tariffying quotas as a means to liberalize trade. A common strategy to convert quotas to tariffs is for the government to auction the quota rights and use the realized license prices as guides to setting tariffs. Proposition 5 shows that in computing the tariff equivalent of a quota, one has to make sure that $z$ is exactly equal to $L(N^Q(\tilde{Q}), \tilde{Q})$: a tariff equivalent calculated at any other level of $N$ will not be equivalent and hence may not have the desired result. In addition, the allocation of quota licenses needs to be considered in defining the “equivalent” specific tariff. Recall that both entry and the implicit license price are higher when the quota licenses are given to the traders than when they are sold. In Figure 5, the “free” quota equilibrium is on the $QQ$ line, northeast of point $B$ (and possibly northeast of point $A$). It is easy to see that tariffying the quota at $N^q(\tilde{Q})$ rather than at $N^Q(\tilde{Q})$ would lead to a net exit of traders.

**Proposition 6.** Equilibrium welfare under the quota (when licenses are sold) is higher than under a specific tariff that is initially import equivalent to the quota at $N > N^Q(\tilde{Q})$. If equivalence is defined at $N < N^Q(\tilde{Q})$, then the opposite holds.

**Proof.** Since welfare is maximized with respect to $N$ for a given $\tilde{Q}$ at $N = N^Q(\tilde{Q})$ (Lemma 2), we know that the iso-welfare contour corresponding to $W^Q(N^Q(\tilde{Q}), \tilde{Q})$ must be tangent to the $QQ$ line at point $B$. This is depicted in Figure 5. Additionally, we know that $W^Q(N^Q(\tilde{Q}), \tilde{Q}) < W^F(N^F)$ (since welfare is increasing in $\tilde{Q}$, from Lemma 3). Therefore, the iso-welfare contour corresponding to $W^Q(N^Q(\tilde{Q}), \tilde{Q})$ must lie outside that corresponding to free trade, where welfare is maximized at point $N^F$ on the horizontal axis.

If we start at $N^Q(\tilde{Q})$ and impose an import equivalent specific tariff, there will be no change in $N$, and welfare will stay at $W^Q(N^Q(\tilde{Q}), \tilde{Q})$. But if equivalence is defined at $N = N^F$, then equilibrium welfare with the specific tariff (at point $C$) is equal to $W^z(N^z(\tilde{z}), \tilde{z})$. Note that $\tilde{z}$ (represented by the height of point $C$) is greater than the equilibrium license price under the quota, $L(N^Q(\tilde{Q}), \tilde{Q})$ (represented by the height of point $B$). Since $N^z(\tilde{z})$ is less than $N^Q(\tilde{Q})$, it follows that the quota that is import-equivalent to $\tilde{z}$ at $N = N^z(\tilde{z})$ has to be smaller than $\tilde{Q}$. From Lemma 3, we know that welfare is lower the smaller the quota. Therefore:

\[ W^z(N^z(\tilde{z}), \tilde{z}) < W^Q(N^Q(\tilde{Q}), \tilde{Q}) < W^F(N^F). \]
By an analogous argument, it can be shown that equilibrium welfare under the specific tariff is higher than under the equivalent quota (when licenses are sold) when equivalence is defined at \( N < N^Q (\bar{Q}) \). \(^{21}\)

How does equilibrium welfare under the “free” quota compare with welfare under the specific tariff? In general we cannot say, since in Figure 5, the iso-welfare contour that goes through the “free” quota equilibrium which lies to the right of \( B \) along \( QQ \), can be above or below \( W^z (N^z (\bar{z}), \bar{z}) \).

VI. Restricted Trade: Ad Valorem Tariffs

Now let us once again return to the free trade equilibrium and impose an ad valorem tariff that is import equivalent to the quota \( Q \). The tariff is set at \( t(N^F, \bar{Q}) \) so that imports are equal to \( \bar{Q} \) with \( N = N^F \), hence:

\[
1 + t(N^F, \bar{Q}) = \frac{P^{D}(\bar{Q})}{P^{S}(N^F, Q)}.
\]

For notational simplicity, denote \( t(N^F, \bar{Q}) \) by \( \bar{t} \).

Since we assume a single price for the good in the domestic market, and the ad valorem tariff is levied on the domestic market price, all traders will pay the same tariff amount in dollar terms (as with the specific tariff earlier). Thus, the ad valorem tariff equals the import equivalent specific tariff at \( N = N^F \) in nominal terms. However, as the ad valorem rate is fixed, the total payment depends on the supply price in the market, which of course, depends on the mass of traders entering the market—as more traders enter, the supply price drops and as a result, the dollar amount of the tariff also drops. Thus, an ad valorem tariff is like a specific tariff that declines with \( N \). More formally, for any \( N \), given \( \bar{t} \), pre-tariff supply is \( NF (P) \) and post-tariff supply is \( NF (P/ (1 + \bar{t})) \). Hence, the equilibrium demand price is given by \( P^{D}(N, \bar{t}) \) which solves:

\[
NF \left( \frac{P^{D}(N, \bar{t})}{1 + \bar{t}} \right) = Q^{D} \left( P^{D}(N, \bar{t}) \right).
\]

Note that \( P^{D}(N, \bar{t}) \) is decreasing in \( N \) and increasing in \( \bar{t} \). The supply price of the marginal trader is \( P^{S}(N, \bar{t}) = P^{D}(N, \bar{t}) / (1 + \bar{t}) \); \( P^{S}(N, \bar{t}) \) is decreasing in both arguments.\(^{22}\) Each trader pays a tariff amount of \( \bar{t}P^{S}(N, \bar{t}) \). Denote this by \( T(N, \bar{t}) \). As \( N \) rises, \( P^{S}(N, \bar{t}) \) falls, so \( T(N, \bar{t}) \) falls. In this way, the ad valorem tariff is like a specific tariff that falls as \( N \) rises.

Quasi rent per trader under the ad valorem tariff is equal to:

\[
r^t (N, \bar{t}) = \frac{P^{S}(N, \bar{t})}{0} \ F (c) \ dc.
\]

\(^{21}\)But free trade welfare is still the highest.

\(^{22}\)This can be seen by writing Equation (22) as: \( NF (P^{S}(N, \bar{t})) = Q^{D} ((1 + \bar{t}) P^{S}(N, \bar{t})) \) and partially differentiating it with respect to \( N \) and \( \bar{t} \).
Since $P^S(N, \tilde{t})$ is decreasing in both arguments, so is $r^t(N, \tilde{t})$.

**Lemma 8** $r^t(N, \tilde{t})$ lies below $r^z(N, \tilde{z})$ for $N < N^F$ and above $r^z(N, \tilde{z})$ for $N > N^F$.

**Proof.** At $N = N^F$, the ad valorem tariff is equivalent, in nominal terms, to the specific tariff, i.e., $T(N^F, \tilde{t}) = \tilde{z}$. Therefore, $P^S(N^F, \tilde{t}) = P^S(N^F, \tilde{z})$ and as a result, $r^t(N^F, \tilde{t}) = r^z(N^F, \tilde{z})$. When $N$ increases beyond $N^F$, $T(N, \tilde{t})$ falls but $\tilde{z}$ remains fixed, so $T(N, \tilde{t}) < \tilde{z}$, hence $P^S(N, \tilde{t}) > P^S(N, \tilde{z})$ and $r^t(N^F, \tilde{t}) > r^z(N^F, \tilde{z})$. Similarly, when $N < N^F$, $T(N, \tilde{t}) > \tilde{z}$ so $P^S(N, \tilde{t}) < P^S(N, \tilde{z})$ and $r^t(N, \tilde{t}) < r^z(N, \tilde{z})$. ■

The quasi rent function under the ad valorem tariff, $r^t(N, \tilde{t})$, is depicted in Figure 4 by the dashed curve. Equilibrium entry under the ad valorem tariff, which we will denote by $N^t(\tilde{t})$, is determined by the intersection of $r^t(N, \tilde{t})$ and $f_e$. Note that $N^t(\tilde{t})$ is decreasing in $\tilde{t}$. Furthermore, it follows from the same argument as Lemma 1 that the equilibrium supply price is the same as the free trade price: $P^S(N^t(\tilde{t}), \tilde{t}) = P^F(N^F)$.

**Proposition 7** If we start at the free trade equilibrium and impose an ad valorem tariff that is initially import equivalent to the quota $\tilde{Q}$ and the specific tariff $\tilde{z}$, there will be a net exit of traders over time and the new equilibrium will be characterized by a lower level of entry compared with the specific tariff. Average cost will first fall and then rise (average productivity will first rise and then fall) as traders change their import and entry decisions in response to the tariff. Imports will fall over time as the number of traders shrinks. In the new equilibrium, average cost (productivity) will return to its free trade level, but the nominal tariff will be higher than $\tilde{z}$ and imports will be less than what they were under the specific tariff.

**Proof.** If we start at the free trade equilibrium and impose an ad valorem tariff $\tilde{t}$, the impact effect will be to make it unprofitable for incumbent traders who have cost draws between $P^S(N^F, \tilde{t})$ and $P^F(N^F)$ to import. As in the case of the specific tariff, the supply price drops to $P^S(N^F, \tilde{t})$, which is equal to $P^S(N^F, \tilde{z})$. Hence average cost falls (average productivity rises).

Over time, there will be a net exit of traders from the market, exactly with the specific tariff. However, as $N$ falls, the supply price rises and the nominal tariff, $T(N, \tilde{t})$, rises.

In the new equilibrium, the mass of traders is smaller than that under the specific tariff: Lemma 8 implies that the intersection of $f_e$ with $r^t(N, \tilde{t})$, which determines the equilibrium mass of traders, $N^t(\tilde{t})$, must occur below $N^z(\tilde{z})$ as depicted in Figure 4. Hence, $N^t(\tilde{t}) < N^z(\tilde{z}) < N^Q(\tilde{Q}) < N^q(\tilde{Q})$. As in the case of the specific tariff, entry could even drop to zero if $\tilde{t}$ is high enough (reflecting a very restrictive quota). In the new equilibrium, the supply price returns to $P^F(N^F)$, so average cost and productivity return to their free trade levels. However, the nominal tariff is greater than $\tilde{z}$ so the demand price (which is the supply price plus the nominal tariff) is higher than it was under the specific tariff, and consequently, equilibrium imports are lower than they were under the specific tariff. ■

For any $N$ and $\tilde{t}$, welfare is the sum of consumer surplus, producer surplus, and tariff revenue, less
entry costs:
\[
W^t (N, \bar{t}) = \int_{P^D (N, \bar{t})}^\bar{P} QD (P) dP + N \int_0^{P^S (N, \bar{t})} F (c) dc + T (N, \bar{t}) QD (P^D (N, \bar{t})) - N f_e. \tag{24}
\]

Lemma 9 \( \partial W^t (N, \bar{t}) / \partial t > 0 \) at \( N = N^t (\bar{t}) \). In other words, the ad valorem tariff results in a level of entry that is sub-optimally low, given the tariff rate.

Proof. In the Appendix. \( \blacksquare \)

In equilibrium (when \( N = N^t (\bar{t}) \)), all quasi rents are competed away so welfare consists of consumer surplus plus tariff revenue only:
\[
W^t (N^t (\bar{t}), \bar{t}) = \int_{P^D (N^t (\bar{t}), \bar{t})}^\bar{P} QD (P) dP + T (N^t (\bar{t}), \bar{t}) QD (P^D (N^t (\bar{t}), \bar{t})) \tag{25}
\]

Proposition 8 Equilibrium welfare under the ad valorem tariff is lower than under a specific tariff that is initially import equivalent at \( N = N^F \).

Proof. From Lemma 6, welfare under a specific tariff is decreasing in \( \bar{z} \) and increasing in \( N \) at \( N \leq N^z (\bar{z}) \). Since the equilibrium ad valorem tariff has the same effect as a specific tariff that is larger than \( \bar{z} \) with a smaller mass of traders \( (N < N^z (\bar{z})) \), it follows that free entry welfare under the ad valorem tariff must be smaller than free entry welfare under the specific tariff. Hence:
\[
W^t (N^t (\bar{t}), \bar{t}) < W^z (N^z (\bar{z}), \bar{z}) \tag{26}
\]

This can be seen in Figure 5 where points to the northwest of point \( C \) (representing combinations of \( z \) and \( N \) where \( z > \bar{z} \) and \( N < N^z (\bar{z}) \)) lie outside the iso-welfare contour corresponding to the equilibrium under the specific tariff, \( \bar{z} \). \( \blacksquare \)

VII. Conclusion

The effects of trade policy can be very sensitive to the choice of instruments once we allow for the role of traders. This is true even in the absence of imperfect competition and product differentiation. When imports are facilitated by competitive traders who are identical ex ante but heterogeneous ex post, the entry and welfare effects of tariffs and quotas differ considerably: specific tariffs tend to reduce entry and welfare more than import equivalent quotas, and ad valorem tariffs reduce entry and welfare even further. The intuition behind our results is simple. Although tariffs and quotas are equivalent given entry, their effects on entry are profoundly different. Trade restrictions by and large discourage entry and thereby reduce the supply of imports. But quotas tend to discourage entry less than tariffs do. The reason is that as traders leave the market in response to the quota, import supply shrinks—this reduces the value of a
quota license and hence the restrictiveness of the quota. By contrast, the exit of traders does not change the restrictiveness of a given specific tariff.

Our results fly in the face of the conventional preference for tariffs over quotas. However, the comparison is more nuanced when we consider how the quota is implemented. In most cases, quota licenses are not sold by the government—auction quotas are relatively rare—but given to the traders free of charge on the basis of certain criteria, such as historical import performance, or the level of investment, or even first-come-first-served. Welfare under such a “free” quota may well be lower than welfare under an initially import equivalent tariff, even though entry is greater. In fact, the most common criticism of quotas is their potential for encouraging rent-seeking behavior. “Free” quota allocation schemes give traders the incentive to expend real resources on things like opening an office in the capital city in order to be close to the license administrator—such expenditures provide no utility to either party and thus represent pure waste. If these expenditures are large, then welfare under a “free” quota will be even lower than our model suggests, and more likely to fall below that associated with an initially import equivalent tariff. In sum, it is the method of quota allocation that matters—an auction quota is preferable to an (initially) import equivalent tariff, but a “free” quota, which tends to promote wasteful rent-seeking behavior, need not be preferable to a tariff.

A widely promoted tenet of trade policy reform is to replace quotas with “equivalent” tariffs which can then be lowered in a transparent manner. (See, for example, Thomas, Nash, and associates (1991) and WTO (2005).) But finding the correct “equivalent” tariff (whether specific or ad valorem) for a given quota depends crucially on the details of the quota arrangement in place. If the quota licenses are given rather than sold to traders, as is often the case, then a large number of traders will enter the market and the value of a quota license will be high in equilibrium. Tarifying such a “free” quota will result in an excessively high tariff that will shrink the import market excessively and could lead to a reduction in welfare. The results are worse if the conversion is to an ad valorem tariff instead of a specific tariff.23

Our model also provides an alternative insight into the implications of corruption. After all, trade policy in the form of a tariff or quota is only one way in which traders can be “penalized” by giving up part of what their efforts obtain. Corruption in the domestic economy could transfer quasi rents and/or raise traders’ costs in a very similar manner. In our model, welfare under a quota is actually higher, and entry lower, when quota rents are not appropriated by the traders but by some other domestic agent, including, possibly, corrupt government agents (assuming, of course, that the welfare of this group is also counted as part of social welfare). A quota lowers social welfare relative to free trade due to the output distortion it introduces. When the quota licenses are sold, entry will adjust to its welfare-maximizing level under the quota but when quota licenses are given to the traders, there is an entry distortion on top of the output distortion. For this reason, corruption that transfers license rents from the traders to other domestic agents may improve social welfare. On the other hand, if we interpret corruption as allowing those with power to extract a fixed fee from all traders, then corruption is equivalent to a specific tax, with similar effects.

23 Moschini (1991) discusses other pitfalls in tariffication.
Appendix

Proof of Proposition 1.

\[
\frac{dW^F(N)}{dN} \bigg|_{N=N^F} = -Q^D(P^F(N))P^{F^I}(N) + \int_0^{P^F(N)} F(c) \, dc + NF(P^F(N))P^{F^I}(N) - f_e \\
= [NF(P^F(N)) - Q^D(P^F(N))]P^{F^I}(N) + \left[ \int_0^{P^F(N)} F(c) \, dc - f_e \right] \\
= 0.
\]

The first equality above comes differentiating; the second from rearranging terms; and the third from (i) market clearing—demand equals supply at price \(P^F(N^F)\) so that the first square bracketed term is zero, and (ii) the free entry condition—at \(N = N^F\), the expected level of profits exactly covers fixed cost so that the second square bracketed term is zero.

Proof of Lemma 2. Differentiating Equation (8) with respect to \(N\) for a given \(\bar{Q}\):

\[
\frac{\partial W^Q(N,\bar{Q})}{\partial N} \bigg|_{N=N^Q(\bar{Q})} = \frac{P^s(N,\bar{Q})}{\partial N} \int_0^{P^s(N,\bar{Q})} F(c) \, dc + NF(P^s(N,\bar{Q})) \frac{\partial P^s(N,\bar{Q})}{\partial N} - Q^P \frac{\partial P^s(N,\bar{Q})}{\partial N} - f_e \\
= \left[ P^s(N,\bar{Q}) \int_0^{P^s(N,\bar{Q})} F(c) \, dc - f_e \right] \\
= 0.
\]

The first equality comes from differentiation; the second from rearranging terms; and the third from (i) market clearing—supply at \(P^s(N^Q(\bar{Q}),\bar{Q})\) equals the quota level, so that the first square-bracketed term is zero, and (ii) the free entry condition—at \(N = N^Q(\bar{Q})\), the expected level of profits exactly covers fixed cost so that the second square bracketed term is zero.
Proof of Lemma 3. Differentiating Equation (9) with respect to $\bar{Q}$:

\[
\frac{dW^Q (N^Q(\bar{Q}), \bar{Q})}{dQ} = -Q^D (P^D(\bar{Q})) P^D(\bar{Q})
\]

\[
+ [P^D(\bar{Q}) - P^S (N^Q(\bar{Q}), \bar{Q})] 
\]

\[
+ P^D(\bar{Q}) \bar{Q} - \bar{Q} dP^S (N^Q(\bar{Q}), \bar{Q}) 
\]

\[
= [P^D(\bar{Q}) - P^S (N^Q(\bar{Q}), \bar{Q})] 
\]

\[
+ P^D(\bar{Q}) [\bar{Q} - Q^D(P^D(\bar{Q}))] 
\]

\[
- \bar{Q} dP^S (N^Q(\bar{Q}), \bar{Q}) 
\]

\[
= P^D(\bar{Q}) - P^S (N^Q(\bar{Q}), \bar{Q}) > 0
\]

where the first equality comes from total differentiation; the second from rearranging terms; and the third from (i) market clearing—demand at $P^D(\bar{Q})$ equals the quota level, so that the second square-bracketed term is zero, and (ii) Lemma 1. The inequality follows from the result that the license price is positive in equilibrium (Proposition 2). ■

Proof of Lemma 4. The first result in the lemma follows from differentiating Equation (10) with respect to $N$ for a given $\bar{Q}$:

\[
\frac{\partial r^d (N, \bar{Q})}{\partial N} = F (P^S (N, \bar{Q})) \frac{\partial P^S (N, \bar{Q})}{\partial N} - \left( \frac{\bar{Q}}{N} \right) \frac{\partial P^S (N, \bar{Q})}{\partial N} 
\]

\[
- \left[ P^D(\bar{Q}) - P^S (N, \bar{Q}) \right] \left( \frac{\bar{Q}}{N^2} \right) 
\]

\[
= \frac{\partial P^S (N, \bar{Q})}{\partial N} \left[ F (P^S (N, \bar{Q})) - \frac{\bar{Q}}{N} \right] 
\]

\[
- \left[ P^D(\bar{Q}) - P^S (N, \bar{Q}) \right] \left( \frac{\bar{Q}}{N^2} \right) 
\]

\[
= - \left[ P^D(\bar{Q}) - P^S (N, \bar{Q}) \right] \left( \frac{\bar{Q}}{N^2} \right) 
\]

\[
< 0
\]

where the first equality comes from total differentiation; the second from rearranging terms; and the third from market clearing—supply at $P^S (N, \bar{Q})$ equals the quota level, so that the first square-bracketed term is zero.

The second result in the lemma follows from differentiating Equation (10) with respect to $\bar{Q}$ for a
given $N$:

$$\frac{\partial r^g (N, \tilde{Q})}{\partial \tilde{Q}} = F \left( P^S (N, \tilde{Q}) \right) \frac{\partial P^S (N, \tilde{Q})}{\partial \tilde{Q}}$$

$$+ \left( \frac{\tilde{Q}}{N} \right) \left[ P^{D^t} (\tilde{Q}) - \frac{\partial P^S (N, \tilde{Q})}{\partial \tilde{Q}} \right]$$

$$+ \left[ P^D (\tilde{Q}) - P^S (N, \tilde{Q}) \right] \left( \frac{1}{N} \right)$$

$$= \frac{\partial P^S (N, \tilde{Q})}{\partial \tilde{Q}} \left[ F \left( P^S (N, \tilde{Q}) \right) - \frac{\tilde{Q}}{N} \right]$$

$$+ \left[ P^D (\tilde{Q}) - P^S (N, \tilde{Q}) \right] \left( \frac{1}{N} \right) + \left( \frac{\tilde{Q}}{N} \right) P^{D^t} (\tilde{Q})$$

$$= \left[ P^D (\tilde{Q}) - P^S (N, \tilde{Q}) \right] \left( \frac{1}{N} \right) + \left( \frac{\tilde{Q}}{N} \right) P^{D^t} (\tilde{Q})$$

where the first equality comes from total differentiation; the second from rearranging terms; and the third from market clearing—supply at $P^S (N, \tilde{Q})$ equals the quota level, so that the first square-bracketed term is zero. We know that $P^{D^t} (\tilde{Q}) < 0$ and $\left[ P^D (\tilde{Q}) - P^S (N, \tilde{Q}) \right] \geq 0$. When $\tilde{Q}$ is at (close to) the free trade level of imports, the license price will be (close to) zero, and $\frac{\partial r^g (N, \tilde{Q})}{\partial \tilde{Q}} < 0$.

**Proof of Lemma 6.** The first result in the lemma follows from differentiating Equation (15) with respect to $\tilde{z}$ for a given $N$:

$$\frac{\partial W^z(N, \tilde{z})}{\partial \tilde{z}} = -Q^D \left( P^D (N, \tilde{z}) \right) \frac{\partial P^D (N, \tilde{z})}{\partial \tilde{z}} + NF \left( P^S (N, \tilde{z}) \right) \frac{\partial P^S (N, \tilde{z})}{\partial \tilde{z}}$$

$$+ \tilde{z} Q^{D^t} \left( P^D (N, \tilde{z}) \right) \frac{\partial P^D (N, \tilde{z})}{\partial \tilde{z}} + Q^D \left( P^D (N, \tilde{z}) \right)$$

$$= \left( \frac{\partial P^D (N, \tilde{z})}{\partial \tilde{z}} - 1 \right) \left[ NF \left( P^S (N, \tilde{z}) \right) - Q^D \left( P^D (N, \tilde{z}) \right) \right]$$

$$+ \tilde{z} Q^{D^t} \left( P^D (N, \tilde{z}) \right) \frac{\partial P^D (N, \tilde{z})}{\partial \tilde{z}}$$

$$= \tilde{z} Q^{D^t} \left( P^D (N, \tilde{z}) \right) \frac{\partial P^D (N, \tilde{z})}{\partial \tilde{z}} < 0$$

where the first equality comes from differentiation; the second from rearranging terms, using the fact that $P^S (N, \tilde{z}) = P^D (N, \tilde{z}) - \tilde{z}$ so $\partial P^S (N, \tilde{z}) / \partial \tilde{z} = \left( \partial P^D (N, \tilde{z}) / \partial \tilde{z} \right) - 1$; and the third from market clearing (Equation (13)) so that the square-bracketed term is zero. The inequality follows from $Q^{D^t} \left( P^D (N, \tilde{z}) \right) < 0$ and $\partial P^D (N, \tilde{z}) / \partial \tilde{z} > 0$.

The second result in the lemma follows from differentiating Equation (15) with respect to $N$ for a
given $\tilde{z}$:

$$\frac{\partial W^z(N, \tilde{z})}{\partial N} = -Q^D(P^D(N, \tilde{z})) \frac{\partial P^D(N, \tilde{z})}{\partial N} + \int_0^{P^S(N, \tilde{z})} F(c) dc$$

$$+ NF(P^S(N, \tilde{z})) \frac{\partial P^S(N, \tilde{z})}{\partial N} + \tilde{z}Q^{D_P}(P^D(N, \tilde{z})) \frac{\partial P^D(N, \tilde{z})}{\partial N} - f_e$$

$$= \frac{\partial P^D(N, \tilde{z})}{\partial N} [NF(P^S(N, \tilde{z})) - Q^D(P^D(N, \tilde{z}))]
+ \left[ \int_0^{P^S(N, \tilde{z})} F(c) dc - f_e \right]$$

$$+ \tilde{z}Q^{D_P}(P^D(N, \tilde{z})) \frac{\partial P^D(N, \tilde{z})}{\partial N}$$

$$= \left[ \int_0^{P^S(N, \tilde{z})} F(c) dc - f_e \right] + \tilde{z}Q^{D_P}(P^D(N, \tilde{z})) \frac{\partial P^D(N, \tilde{z})}{\partial N}.$$
where the first equality comes from total differentiation and the second from the fact that
\( P^D(N^z(\tilde{z}), \tilde{z}) = P^S(N^z(\tilde{z}), \tilde{z}) \) and \( dP^S(N^z(\tilde{z}), \tilde{z})/d\tilde{z} = 0 \), so \( dP^D(N^z(\tilde{z}), \tilde{z})/d\tilde{z} = 1 \). The
inequality follows from \( Q^{D^t}(P^D(N^z(\tilde{z}), \tilde{z})) < 0 \).

**Proof of Lemma 9.** Differentiating Equation (24) with respect to \( N \) for a given \( \tilde{t} \):

\[
\frac{\partial W^t(N, \tilde{t})}{\partial N} \bigg|_{N = N^t(\tilde{t})} = -Q^D(P^D(N, \tilde{t})) \frac{\partial P^D(N, \tilde{t})}{\partial N} + \int_0^{P^S(N, \tilde{t})} F(c) dc \\
+ N F(P^S(N, \tilde{t})) \frac{\partial P^S(N, \tilde{t})}{\partial N} \\
+ \tilde{t} Q^D(P^D(N, \tilde{t})) \frac{\partial P^S(N, \tilde{t})}{\partial N} \\
+ \tilde{t} P^S(N, \tilde{t}) Q^{D^t}(P^D(N, \tilde{t})) \frac{\partial P^D(N, \tilde{t})}{\partial N} - f_e \\
= \frac{\partial P^S(N, \tilde{t})}{\partial N} \left[ N F(P^S(N, \tilde{t})) - Q^D(P^D(N, \tilde{t})) \right] \\
+ \left[ \int_0^{P^S(N, \tilde{t})} F(c) dc - f_e \right] \\
+ \tilde{t} P^S(N, \tilde{t}) Q^{D^t}(P^D(N, \tilde{t})) \frac{\partial P^D(N, \tilde{t})}{\partial N} \\
> 0
\]

where the first equality comes from total differentiation, using the fact that \( T(N, \tilde{t}) = \tilde{t} P^S(N, \tilde{t}) \); the second from rearranging terms, using the fact that \( P^D(N, \tilde{t}) = (1 + \tilde{t}) P^S(N, \tilde{t}) \); and the third from (i) market clearing (Equation (22) so that the first square-bracketed term is zero, and (ii) the free entry condition (Equation (23) so that the second square-bracketed term is zero. The inequality follows from \( Q^{D^t}(P^D(N, \tilde{t})) < 0 \) and \( \partial P^D(N, \tilde{t})/\partial N < 0 \).
References


