Debt Dynamics and Global Imbalances: Some Conventional Views Reconsidered

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Abstract

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We use a general-equilibrium model to explain the rise in global trade and payments imbalances since the mid-1990s, and then to construct adjustment paths to a steady state. Assuming that the shocks giving rise to the imbalances do not suddenly reverse, simulated movements in the U.S. trade deficit and exchange rate are smaller and more gradual than suggested by partial-equilibrium analyses. An important factor reducing the size of the adjustments is a simulated real interest rate on U.S. external liabilities that is below both the interest rate on external assets and the U.S. real economic growth rate. In addition, the adjustment takes place over an extended period without significantly raising the share of U.S. assets in foreign portfolios, in part because depreciation of the dollar requires continued foreign accumulation of U.S. assets just to keep their portfolio share constant.

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I. INTRODUCTION

It is generally agreed that large U.S. trade deficits and corresponding trade surpluses in other countries must be reversed over time to establish a sustainable configuration of global trade and capital flows. Furthermore, standard debt dynamics indicate that the longer the imbalances persist, the larger will be the eventual adjustment, as the U.S. will need to run a higher trade surplus to service greater external debt. These considerations, combined with projections of continuing large U.S. external deficits in the near term and a relatively low responsiveness of the U.S. trade balance to exchange rate changes, have raised concerns about disruptive movements in global financial markets as the imbalances unwind.

Several partial-equilibrium analyses have been performed of the changes in trade balances and exchange rates required to reverse global imbalances.\(^2\) A common approach has been to calculate the required adjustment in the U.S. trade deficit, and then to use conventional trade price elasticities to derive the exchange rate depreciation needed to achieve such a reduction in the trade deficit. If, for instance, U.S. external debt stabilizes at 50 percent of GDP, and the difference between the long-term real interest rate and real growth rate is 2 percentage points, the U.S. would need to run a steady-state trade surplus of 1 percent of GDP. Given the actual trade deficit in 2005 of about 5½ percent of GDP, and a rule of thumb that a 10 percent depreciation in the real exchange improves the trade balance by 1 percent of GDP, a depreciation of some 65 percent in the dollar (in log terms) would be required.\(^3\) Refinements can reduce this back-of-the-envelope calculation to levels that appear more realistic, such as allowing for valuation and activity effects. Conversely, the adjustment could be even larger given projections of widening U.S. current account deficits under unchanged real exchange rates and/or more pessimistic elasticity assumptions.\(^4\)

More sanguine analyses focus on the capital account side of the imbalances, emphasizing the ability of the rest of the world to continue to accumulate U.S. financial assets in an environment of increasing financial deepening and capital mobility.\(^5\) It is difficult, however, to judge the merits of the views of these “portfolio optimists” versus those of “elasticity pessimists” in the absence of an integrated treatment of international portfolio allocation, trade balances, and savings and investment.\(^6\) The current paper fills this gap using a general-

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\(^2\) See Blanchard and others (2005), Obstfeld and Rogoff (2005), Roubini and Setzer (2004).

\(^3\) Equivalent to a depreciation of close to 50 percent in arithmetic terms.

\(^4\) See, for instance, Mann (2004).


\(^6\) Cline (2005) discusses the implications of the imbalances for the U.S. trade and capital accounts, but not in a unified model. Edwards (2005) also looks at both sides of the imbalances in a partial-equilibrium framework.
equilibrium model that endogenizes the interaction between these variables for two stylized regions—the U.S. and the rest of the world. The explicit treatment of saving and investment behavior under the assumptions of rational expectations and dynamic optimization ensures that the paths for external balances and the exchange rate are intertemporally consistent. The general-equilibrium nature of the model also ensures that the unwinding of the imbalances leads to a steady state characterized by stable stock/flow relationships.

The model is first used to assess the factors underlying the global imbalances since the mid-1990s. In particular, shocks are calibrated such that the simulation path during 1996-2005 is broadly consistent with actual developments. These shocks involve a shift in international portfolio preferences toward U.S. assets, combined with a rise in demand for imported goods both in the U.S. and the rest of the world. Adjustment paths are then constructed to a steady state based on alternative assumptions about the evolution of the shocks. The associated changes in exchange rates and trade balances are smaller and more gradual than indicated by partial-equilibrium analyses. In the baseline scenario, the long-run exchange rate depreciation is about one third of the value implied by the above rule-of-thumb calculation, while the decline in the trade deficit is about 60 percent as large, even though U.S. net foreign liabilities rise to over 90 percent of GDP; the current account deficit remains close to its recent share of GDP. These results are reasonably robust to alternative assumptions about the model’s parameters. Nevertheless, a sudden unwinding of the portfolio shift would cause an abrupt adjustment of the U.S. exchange rate—even then, however, the effects would be no larger than the adjustments observed during 1986–89.

The differences relative to the rule-of-thumb calculation are explained by three factors. The first is a real interest rate on U.S. external debt that stabilizes at less than both the interest rate on external assets and the real economic growth rate, altering the conclusions of standard debt dynamics. Even though the U.S. risk-free real interest rate is less than the real growth rate, the simulation paths are dynamically efficient, as the marginal product of capital includes a private risk premium. The second factor is a shift in relative consumption in the U.S. and abroad that reduces the exchange rate depreciation needed to narrow the U.S. trade deficit. The third is exogenous to the model itself—the U.S. experiences valuation gains on international investments that offset the effect of the trade deficit on long-run debt dynamics.

In terms of other analyses of global imbalances, this paper is closest to the approach of Blanchard, Giavazzi and Sa (2005), henceforth BGS, extending their framework to incorporate a general-equilibrium specification of saving/investment behavior, interest rates, and stock-flow dynamics. It also resembles Caballero and others (2006) in generating a decline in the real interest rate on U.S. external debt in response to shocks that give rise to external imbalances. Their framework, however, does not consider real interest rates that are less than the growth rate, nor does it aim to replicate the imbalances that have actually arisen, making it difficult to judge the implications for how they might unwind. Faruqee and others
(2005) use a general-equilibrium model to analyze the unwinding of global imbalances, but without an explicit specification of international portfolio effects, and a consumption-wealth framework that limits the analysis of the eventual distribution of international assets.

This paper is organized as follows. The next section reviews developments in global trade and capital flows, as well as evidence on real interest rates. The third discusses debt dynamics, and how the relationship between the real interest rate and the real growth rate affects the steady-state growth path. The model is described in section four, and the solution paths shown in section five for various assumptions about the model parameters and the shocks. The final section provides concluding remarks.

**II. Stylized Facts**

This section reviews trade and capital flows since the mid-1990s to characterize the rise in global imbalances, identifying factors that are unusual in terms of standard economic relationships and previous experience. The apparent shifts in structural relationships are used to identify the shocks to the simulation model. The section then turns to the historical evidence on real interest rates on U.S. assets, and in particular on external assets and liabilities, to establish stylized facts that are relevant to the model calibration.

**A. Evolution of Global Imbalances**

Figure 1 shows the behavior of the U.S. current account and trade balances since 1979. Reasons for concern about global imbalances are evident from the remarkable widening in the U.S. external deficits from about 1 percent of GDP in the mid-1990s to about 6 percent of GDP in 2005—roughly twice as large as the deficits that led to the Plaza Accord in the mid-1980s. The persistence of the deficits has also been notably greater than in the 1980s.

*Figure 1. U.S. External Balances, 1979-2005*

Source: IMF, World Economic Outlook database.
The shift in the U.S. trade position since the mid-1990s is even more remarkable looked at in real terms. Figure 2 compares movements in the U.S. real trade balance during 1995–2005 with that during 1979–89—both are indexed to zero in the initial year to show the relative movements during the two episodes. The deterioration in the trade balance was somewhat larger during 1995–2002 than during 1979–86, but the differences in the paths are even more pronounced subsequently, with the real trade deficit continuing to widen during 2003-05, whereas the deterioration during 1982–86 was largely reversed by 1989.

The decline in the trade balance has been associated with a sharp rise in the ratio of imports to GDP, as opposed to a significant decline in exports (Figure 3). Since 1995, the ratio of real imports to GDP has risen by 7 percentage points, compared with only 4 percentage points over the previous 15 years. The surge in imports is also notable compared with the smaller increase observed in the first half of the 1980s, in spite of the greater appreciation in the real exchange rate during that period (as shown below). On the export side, the ratio of U.S. exports to GDP has leveled off since 1995 after trending up over the previous decade, but has not shown a sustained decline in response to the appreciation of the dollar.

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7 The strength of imports as the dollar has appreciated appears at odds with evidence that import price passthrough is low, which would reduce the response of import volumes to exchange rate movements (Gust and Sheets (2006)).
In terms of macroeconomic factors that affect trade flows, Figure 4 compares the behavior of the U.S. real effective exchange rate (CPI-based) during 1995-2005 with 1979-89. From the initial period to the peak, the real exchange rate appreciated by about 40 percent in the first half of the 1980s compared with only 25 percent during 1995-2001. Thus, real exchange rate appreciation was larger in the mid-1980s than during the build-up of the current imbalances, while the decline in the real trade balance was smaller in the earlier period. The differences in the behavior of the real trade balance after the exchange rate starts to depreciate is even more striking—the trade deficit narrowed sharply during 1986-89, whereas it has continued to widen since 2002. It appears, then, that the behavior of the U.S. trade deficit since 1995 is at odds with real exchange rate movements, at least from the perspective of the 1980s.
Another macroeconomic factor sometimes cited as driving the U.S. trade deficit in recent years is faster growth in U.S. output than in the rest of the world (ROW). We assess the importance of relative growth rates by looking at various measures of the share of the U.S. in world GDP: nominal, using market exchange rates to convert GDPs into U.S. dollars; real, aggregating GDP at 1995 exchange rates; and real, using purchasing-power-parity (PPP) rates (Figure 5). Faster growth in the U.S. than in the ROW would be reflected in a rise in the U.S. share of world GDP. The nominal U.S share has fluctuated between 25 and 32 percent since 1980—the peaks in 1995 and then 2002 reflect by transitory dollar appreciation. Both measures of the U.S. real shares are stable until the late 1990s, after which they show modest declines. Thus there is no evidence of a sustained differential in real growth rates that would explain the widening trade deficit.

Given that trade flows are more directly influenced by spending than output, it is also useful to examine the role that consumption has played in widening the U.S. trade deficit. Relative consumption movements are indeed more consistent with trade developments—in particular, the U.S. experienced a surge in consumption starting in 1997-98, while ROW consumption weakened starting in 2002 (Figure 6). These shifts, however, only explain a modest share of the wider trade deficit. With activity elasticities of between one and two, for instance, combined with initial shares of trade in U.S. GDP of about 10 percent, the divergence in U.S. and ROW consumption would have contributed between ¾ to 1½ percent of GDP to the trade deficit.

Engel and Rogers (2006) show that the U.S. share of the GDP of G-7 countries has risen due to slow growth in Europe. Using the share in world output is more appropriate for current purposes, however, given the impact of emerging market countries on global trade and capital flows.
Figure 6. Ratio of Real Private Consumption to GDP, 1985-2005

One factor that may partially explain the anomalous relationship between the trade balance and its determinants is mismeasurement of the U.S. real exchange rate. The series in Figure 4 is calculated using relative CPIs, thus its appropriateness depends on the assumption that CPIs are measured in a comparable way across countries. There were important changes, however, in the construction of the U.S. CPI in the 1990s, in part associated with the Boskin Commission report. While no official estimate is available, these changes may have reduced growth in the U.S. CPI by close to 1 percentage point per year, or two thirds of the bias identified by the Commission prior to the early 1990s. Whether or not these changes improved the construction of the CPI, to the extent that they were not adopted by other countries, movements in the measured real exchange rate would be distorted. To illustrate the impact, we assume that U.S. CPI growth has been 1 percentage point per year lower since 1996 due to methodological changes—in their absence, the real exchange rate would follow the adjusted path shown in Figure 4. It peaks at more than 30 percent above its initial level by 2002 and then depreciates by significantly less than the measured series during 2002-05. To the extent that this more accurately reflects movements in U.S. competitiveness it would reduce—but not eliminate—the anomaly of the widening U.S. trade deficit.

Boskin and others (1996).

See Gordon (2000) and (2006). BLS (1997), (1998), and (1999) discuss changes introduced from the mid-1990s on. The correction for “formula bias” in 1995-96 is estimated to have reduced CPI growth by ¼ percentage point per year, and that for high-level substitution bias in 1999 to have further reduced growth by a similar amount. There is little evidence on adjustments for quality change—the BLS (1997) estimates that they reduced CPI growth by 1.1 percentage points in 1995, but does not indicate by how much they have changed over time. In any case, quality adjustments would have increased with the introduction of hedonic price measures for electronic goods in the late 1990s, and for other products thereafter.
The counterpart to large U.S. trade and current account deficits has been a significant increase in indebtedness to the ROW (Figure 7). From 1982 to 1995, the U.S. shifted from being a net creditor vis-à-vis the ROW to a slight debtor, with external liabilities of 4 percent of GDP at end-1995. U.S. net debt then rose to slightly over 20 percent of GDP by end-2005, equivalent to an increase of $2.2 trillion. This increase in net debt, however, is actually smaller than the $4.0 trillion cumulative U.S. current account deficit during 1996-2005—the role of valuation effects in explaining this discrepancy is discussed below.

**Figure 7. U.S. Net Foreign Assets, 1982-2005**

Along with the rise in net debt, gross U.S. external positions increased sharply starting in the mid-1990s. ROW claims on the U.S. almost doubled to 110 percent of U.S. GDP by 2005; relative to ROW GDP, they more than doubled to 43 percent (Figure 8). Foreign holdings of U.S. assets have risen in spite of a decline in their yield (as discussed below), suggesting that it reflects a shift in ROW portfolio preferences as opposed to yield incentives.

**Figure 8. U.S. External Assets and Liabilities, 1982-2005**
Financial inflows to the U.S. have been associated with strong demand for low-risk assets, including government and corporate bonds. Part of this demand has reflected a sharp rise since the mid-1990s in foreign reserve holdings, much of which is held in U.S. assets (Figure 9). In the past 10 years, the ratio of official reserves to ROW GDP has doubled, presumably reflecting increased demand for liquidity associated with volatility in capital flows. Foreign official holdings of U.S. assets rose from about 3 percent of ROW GDP in 1995 to 7 percent in 2005, accounting for about one fifth of the overall increase in foreign holdings of U.S. assets.

![Figure 9. ROW Holdings of Official Reserves, 1971-2005](image)

This review of the stylized facts suggests that global imbalances have been associated with two phenomena that began in the mid-1990s. One is a sustained rise in the U.S. trade deficit, led by higher import volumes, that goes beyond the increase implied by indicators of competitiveness and activity. The other is a sharp rise in ROW holdings of U.S. assets, accompanied by appreciation of the U.S. dollar and falling yields on these assets. As discussed in Section VI, these phenomena are reflected in the model simulations as a shift in U.S. demand for imports, and a rise in the desired share of U.S. assets in ROW portfolios.

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12 Lane and Milesi Ferretti (2006b) point to another “break” in the stylized facts in the mid-1990s—previously, U.S. current account imbalances were largely offset by changes in Europe’s current account, whereas the recent U.S. deficit is associated with surpluses outside of Europe.
B. Real Interest Rates

A property of the simulation model is a risk-free real interest rate that can be less than the real economic growth rate. The evidence is reviewed here to establish that such a relationship is not abnormal from a historical perspective. For example, real interest rates on U.S. government bonds have typically been close to or below the U.S. real growth rate; and current yields on inflation-indexed bonds are below most estimates of future real growth. On the external side, real return on U.S. external liabilities has also been close to the real growth rate, and well below that on U.S. external assets.

Jenkins (1990) reviews real interest rates over several centuries. Since 1900, the real interest rate on U.S. government bonds has averaged 1¼ percent, compared with economic growth of 3¼ percent (Table 1). Since the 1970s, the short-run relationship has been quite volatile, perhaps reflecting errors in expectations as inflation rose in the 1970s and then fell in the 1980s and 1990s. Taking the 1970-2005 period as a whole, however, the real interest rate was about ¼ percentage point below the real growth rate.13

<table>
<thead>
<tr>
<th>Growth Rate of Real GNP</th>
<th>Real Interest Rate: Long-Term Govt. Bonds</th>
<th>Real Bond Rate Minus Real Growth Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1900-09</td>
<td>-0.5</td>
<td>-4.2</td>
</tr>
<tr>
<td>1910-19</td>
<td>-5.1</td>
<td>-8.0</td>
</tr>
<tr>
<td>1920-29</td>
<td>5.9</td>
<td>2.3</td>
</tr>
<tr>
<td>1930-39</td>
<td>4.7</td>
<td>4.6</td>
</tr>
<tr>
<td>1940-49</td>
<td>-3.3</td>
<td>-7.7</td>
</tr>
<tr>
<td>1950-59</td>
<td>0.9</td>
<td>-3.0</td>
</tr>
<tr>
<td>1960-69</td>
<td>1.7</td>
<td>-2.5</td>
</tr>
<tr>
<td>1970-79</td>
<td>-0.9</td>
<td>-4.1</td>
</tr>
<tr>
<td>1980-89</td>
<td>5.9</td>
<td>2.5</td>
</tr>
<tr>
<td>1990-2005</td>
<td>3.9</td>
<td>0.9</td>
</tr>
<tr>
<td>Average:</td>
<td>3.2</td>
<td>1.3</td>
</tr>
<tr>
<td></td>
<td>-1.9</td>
<td></td>
</tr>
</tbody>
</table>


The above real interest rates are calculated using ex post inflation—ex ante real interest rates are available from inflation-indexed Treasury securities, although for a shorter period. Since 1997, the real yield on 10-year bonds peaked at 4 percent in 2000, but has remained at or below 2½ percent since mid-2002. As of end-November, the yield curve for indexed bonds

13 Catão and Mackenzie (2006) also provide evidence that current real interest rates do not appear low from a longer-term perspective. Applying a pricing model based on equity yields, they derive a long-term U.S real interest rate of 2.2 percent, close to recent yields on indexed Treasury securities.
has been virtually flat at about 2¼ percent for maturities out to 30 years. With typical estimates of longer-term growth for the U.S. economy in a range of around 3 percent, it appears that markets anticipate an extended period over which the risk-free real interest rate will be less than the real growth rate.

Of more direct relevance to this paper, however, are returns on U.S. external liabilities, of which treasury securities are only one component. Gourinchas and Rey (2005b) estimate real returns on U.S. external assets and liabilities, divided by asset class, over the period 1952Q1-2004Q1 (Table 2). The return on U.S. external assets has consistently exceeded that on liabilities, with the gap increasing to over 3 percentage points in the post Bretton Woods period. This “exorbitant privilege” reflects two effects: (i) U.S. external liabilities predominantly low-yielding asset classes, such as Treasury securities; and (ii) within asset classes, the return on U.S. liabilities is lower than on its assets. The real return on U.S. external liabilities is estimated to have averaged 3½ percent in the post Bretton Woods period—only slightly above the average U.S. real growth rate of 3 percent.

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14 Meissner and Taylor (2006) argue, in contrast, that the yield differential on U.S. external assets and liabilities has trended down since the 1960s, and thus that the exorbitant privilege is shrinking. Using BEA data starting in 1976 with assets valued at current cost, however, the yield gap shows no trend during 1977-2005.
Table 2. Real Returns on U.S. External Assets and Liabilities, 1953-2004
(average, percent per year)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Assets: Total</strong></td>
<td>5.7</td>
<td>4.0</td>
<td>6.8</td>
</tr>
<tr>
<td>Equities</td>
<td>13.7</td>
<td>10.8</td>
<td>15.5</td>
</tr>
<tr>
<td>FDI</td>
<td>9.6</td>
<td>9.4</td>
<td>9.7</td>
</tr>
<tr>
<td>Debt</td>
<td>4.3</td>
<td>4.8</td>
<td>4.1</td>
</tr>
<tr>
<td>Other</td>
<td>3.4</td>
<td>2.4</td>
<td>4.1</td>
</tr>
<tr>
<td><strong>Liabilities: Total</strong></td>
<td>3.6</td>
<td>3.8</td>
<td>3.5</td>
</tr>
<tr>
<td>Equities</td>
<td>10.3</td>
<td>11.6</td>
<td>9.4</td>
</tr>
<tr>
<td>FDI</td>
<td>9.6</td>
<td>9.7</td>
<td>9.3</td>
</tr>
<tr>
<td>Debt</td>
<td>0.5</td>
<td>0.8</td>
<td>0.3</td>
</tr>
<tr>
<td>Other</td>
<td>1.2</td>
<td>1.2</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Source: Gourinchas and Rey (2005b), Table 1.

The real returns calculated by Gourinchas and Rey consist of both measured yields on U.S. external assets, as well as valuation adjustments to existing assets. However, published BEA data on U.S. external positions at current cost only begin in 1976, while market value data start in 1982, and explicit valuation adjustments in 1989. Gourinchas and Rey construct prior estimates that are, in principle, similar to those made by the BEA, but their appropriateness is difficult to verify. Using published BEA data, measured yields on U.S. external assets have consistently exceeded those on liabilities since 1977 (Figure 11). After surging in the early 1980s, real yields have declined, with that on U.S. liabilities averaging ½ percent during 2000-05, and on U.S. assets about 2 percent. During 1977-2005, the real yield on U.S. external liabilities averaged about 1¼ percent, well below the real economic growth rate.

Figure 11. Real Yields on U.S. External Assets and Liabilities, 1977-2005

![Figure 11. Real Yields on U.S. External Assets and Liabilities, 1977-2005](image-url)
The contribution of the gap between yields on U.S. assets and liabilities to the “exorbitant privilege” can be calculated by taking the average yield on assets and liabilities and applying it to U.S. net foreign assets. The difference between this hypothetical net investment income, assuming yields were equal, and observed investment income in the current account is then attributable to the yield differential (Figure 12). In its absence, the U.S. would have recorded an investment income deficit of about 1 percent of GDP in 2005, whereas actual income was slightly positive. The yield differential has thus generated an exorbitant privilege of about 1 percent of GDP in recent years, up from less than ½ percent through most of the 1980s.

As mentioned above, total returns consist of measured yields plus valuation adjustments. A valuation adjustment for U.S. NFA on a market-value basis can be constructed starting in 1983. As the yearly adjustments are volatile, Figure 13 shows a cumulative moving average. A series is also shown starting in 1989 that excludes the impact of exchange rate changes, and thus includes only the “price” and “other” components of the BEA’s three adjustments. Valuation adjustments have generally been positive, fluctuating around 1 percent of GDP on average since 1983, although dollar appreciation in the late 1990s lowered the exchange-rate adjustment as the dollar value of U.S. assets fell. This effect was reversed during 2002-05, while large positive adjustments from the other components raised the cumulative average to about 1¼ percent of GDP by 2005. During 1996-2005, positive valuation adjustments averaged about 2 percent of GDP, explaining the large gap between the cumulated current account deficit and the change in net foreign liabilities shown in Figure 7.

15 See Tille (2005), and also Lane and Milesi-Ferretti (2006a), for discussions of the currency composition of U.S. international assets and liabilities.
The series in Figure 13 is a net adjustment—starting in 1989, it can be decomposed into valuation changes in gross U.S. external assets and liabilities. These averaged 3¾ percent of assets and 1¾ percent of liabilities respectively during 1989-2005. As a result, the real return on external assets was significantly higher than the measured yield—6¼ percent versus 2½ percent—while the real return on liabilities was 3½ percent versus a yield of 1½ percent. Thus the difference in returns on assets and liabilities was 3 times larger than the difference in yields, implying that most of the U.S. exorbitant privilege during 1989-2005 derived from valuation adjustments. Recognizing this statistical “manna from heaven” (Cline (2005)) is important in assessing the prospects for U.S. external imbalances. It is also controversial, given that the series are relatively short and the data volatile. In any case, to anticipate the model simulations, positive valuation adjustments to U.S. NFA are assumed to stabilize at 1 percent of GDP per year over the long-run horizon in the baseline scenario. The reduction in the trade deficit needed to restore debt sustainability is reduced by an equivalent amount.

This evidence on real interest rates and real growth rates is relevant to U.S. fiscal/external debt dynamics, but not to all private agents. Firstly, real growth in individual incomes is generally less than that of the economy as a whole. Secondly, private debt usually commands a significant liquidity/risk premium. For example, the yield on BAA U.S. corporate bonds has exceeded that on 10-year treasury securities by about 2 percentage points since the late 1960s (Figure 14). While only a rough measure of the private cost of capital, even this relatively small spread implies that the growth path can be dynamically efficient even if the real returns on risk-free, highly-liquid assets are less than the real growth rate.\(^{16,17}\)

16 Historically, the equity risk premium suggests a significantly larger differential (Mehra and Prescott (1985)).
III. DEBT DYNAMICS

This section discusses debt dynamics, both to introduce notation and underscore the importance of the gap between the real interest rate and the real growth rate in determining steady-state relationships. We first decompose the change in the debt-to-GDP ratio into new debt incurred in period t and the effect of GDP growth on the lagged debt ratio:

\[ \frac{D_t}{Y_t} - \frac{D_{t-1}}{Y_{t-1}} = \Delta(D/Y)_t = \Delta D_t / Y_{t-1} - \Delta Y_t / Y_{t-1} \left( \frac{D_t}{Y_t} \right) , \]

(1)

where \( D_t \) is debt at time \( t \), \( Y_t \) is GDP, and \( \Delta \) indicates the change from the previous period. Debt is taken here to represent U.S. net external liabilities.

The change in debt from period \( t-1 \) to \( t \) (\( \Delta D_t \)) equals the current account deficit (\( CAD_t \)) plus valuation effects that arise from movements in interest rates or exchange rates. Abstracting from the latter, using lower-case letters to indicate ratios to GDP, and denoting real GDP growth as \( g \) and inflation as \( \pi \), equation (1) becomes:

\[ \Delta d_t = cad_t - (g_t + \pi_t) d_{t-1} \]

(2)

where \( \Delta Y_t / Y_{t-1} = g_t + \pi_t \).

Equation (2) can be expressed in terms of the “primary” current account deficit, i.e. the current account deficit excluding net income on international investment. This will be taken

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17 See Abel and others (1989) for evidence in favor of dynamic efficiency for the U.S. economy.
to mean the goods and non-factor services balance, or the trade deficit \((td)\). The relationship between the trade and current account deficits is then (where \(i_t\) is the nominal yield on external debt):

\[
\text{\(td_t = \text{cad}_t - i_t \cdot d_{t-1}\),}
\]

and equation (2) becomes:

\[
\Delta d_t = td_t + (r_t - g_t) \cdot d_{t-1}.
\]

Along a steady-state growth path where stock-to-income ratios are constant, the left-hand sides of equations (2) and (4) are zero by definition, and the relationships between external debt and the two definitions of the external balance are:

\[
\text{\(\text{cad}^s = (\pi^s + g^s) \cdot d^s\)}
\]

\[
-td^s = (r^s - g^s) \cdot d^s.
\]

The steady-state current account deficit equals the nominal economic growth rate times the debt-to-GDP ratio, while the negative trade deficit (i.e. the trade surplus) equals the difference between the real interest rate and the growth rate times the debt-to-GDP ratio.

As shown in equation (2), the dynamics relating the current account to the debt-to-GDP ratio are stable when nominal growth is positive, as \(\partial \Delta d_t / \partial d_{t-1} < 0\) holding the current account constant. In this case, the debt-to-GDP ratio converges to \(\text{cad} / (g + \pi)\). In contrast, the dynamics of equation (4) are unstable when \(r > g\), and the debt ratio explodes for an arbitrary trade balance. In this case, the trade balance must adjust to be consistent with a given debt ratio, with the trade surplus being larger the higher is the debt ratio. When \(r < g\), however, this result is overturned—a permanent trade deficit becomes consistent with a stable debt-to-GDP ratio of \(td / (g - r)\), which is higher the larger is the trade deficit. This is important in understanding why the drawn-out unwinding of the imbalances in the model simulations does not lead to a “snowballing” of the size of the adjustment; as external debt accumulates, the equilibrium trade deficit increases, reducing the size of the adjustment from the current level.

The above abstracts from valuation effects arising, for instance, from changes in exchange rates—it also assumes the same interest rate on foreign assets and liabilities. We relax these assumptions by dividing net external debt into gross assets and liabilities. External liabilities are assumed to be denominated in the currency of the issuing country. Using \(gfa\) to indicate gross U.S. holdings of foreign assets, \(er\) to indicate the exchange rate (the foreign currency price of U.S. dollars), and \(r^a\) and \(r^l\) to indicate the yields on foreign assets and liabilities respectively, two terms are added to equation (4):

\[
\Delta d_t = td_t + (r_t^a - g_t) \cdot d_{t-1} + (r_t^a - r_t^l) \cdot \left(\frac{gfa_{t-1}}{er_{t-1}}\right) + \left(\frac{er_t}{er_{t-1}}\right) \cdot \left(\frac{gfa_{t-1}}{er_{t-1}}\right).
\]
The term in the interest differential indicates that a higher yield on U.S. external assets than on liabilities causes the U.S. debt ratio to decline over time (or rise less quickly than would otherwise be the case). The last term indicates that dollar appreciation raises net external debt by reducing the dollar value of U.S. holdings of foreign assets.\textsuperscript{18} Both effects have been important in explaining the evolution of U.S. external debt, and also play a role in the model simulations, given the positive differential between the interest rates on U.S. assets and liabilities and the depreciation of the dollar over the simulation horizon.

\section*{IV. Simulation Model}

\subsection*{A. Structure}

The simulation model consists of two regions: the U.S. and the rest of the world (ROW).\textsuperscript{19} It embodies dynamic optimization and rational expectations; a numerical simulation algorithm yields exact solutions, obviating the need for linear approximations.\textsuperscript{20} The model is expressed in real terms, abstracting from nominal rigidities, which are of limited relevance to long-run asset dynamics. Generally speaking, the structure is consistent with that of recent dynamic general-equilibrium (DGE) models, although it has some distinctive features that are important to the simulation results. These include an overlapping-generations specification of household behavior that allows the equilibrium risk-free interest rate to be below the economic growth rate. At the same time, the existence of a risk premium on private capital ensures that the equilibrium is dynamically efficient. The other distinctive feature of the model is imperfect substitutability between foreign and domestic assets—in the simulations, portfolio shifts play a central role in driving movements in the exchange rate and interest rates, and the results depend importantly on their assumed path. The model relationships are described first, followed by a discussion of parameter values.

The consumption framework is based on Blanchard (1985) and Yaari (1965), as described in the Appendix. Individuals face a constant probability of death (\(\delta\)); aggregate population growth is then determined by the birth rate less the death rate. Individuals discount future income and utility at a rate that reflects the probability of death, in addition to the risk-free real interest rate (\(r\)). The “effective” interest rate faced by households, then, is \(r + \delta\). As long

\begin{footnotesize}
\begin{enumerate}
\item \textsuperscript{18} See Lane and Milesi-Ferretti (2005) for a decomposition of movements in international investment positions based on these factors, and Tille (2005) for the effects of exchange rate movements on U.S. NFA. Gourinchas and Rey (2005a) analyze the historical importance of valuation adjustments for the U.S. Ghironi, Lee, and Rebucci (2006) examine the effects of asset price movements on external balance sheets in a theoretical model.
\item \textsuperscript{19} The aggregation of the rest of the world precludes an analysis of the distribution of the adjustment across regions. See Calvo and Talvi (2006), Faruqee and others (2005), and Lane and Milesi-Ferretti (2006b) for discussions of how different regions could be affected by an unwinding of global imbalances.
\item \textsuperscript{20} The algorithm uses the stacked-time Newton-based technique described in Julliard and others (1998).
\end{enumerate}
\end{footnotesize}
as individual income growth is less than this rate, household wealth and consumption will be bounded, and opportunities for “Ponzi games” do not exist.21

In this framework, consumption \( (C_t) \) is a fraction of total wealth, which in turn is the sum of human wealth \( (WH_t) \) and financial wealth \( (WF_t) \) expressed in units of the consumption good. Human wealth is the discounted value of expected lifetime after-tax labor income \( (YH_t - TAX_t) \). Financial wealth is the sum of the domestic capital stock \( (K_t) \), net foreign assets \( (NFA_t) \), and government bonds \( (B_t) \). The wealth relationships are then: \(^{(7)}\)

\[
WF_t \equiv K_t + NFA_t + B_t
\]

\[
W_t \equiv WH_t + WF_t / p_t^c
\]

where

\[
WH_t \equiv \sum_{t=2}^{\infty} \frac{YH_z - TAX_z}{\prod_{s=t}^{\infty} (1 + r_s + \delta + \hat{p})}
\]

\[
YH_t \equiv (1 - \mu) Y_t / p_t^c
\]

The ratio of consumption to wealth \( (\gamma_t) \) is time varying, and depends on current and future real interest rates, the rate of time preference, and the probability of death:

\[
\gamma_t^{-1} = \frac{(1 + \gamma_{t+1})}{2 - (1 + \rho + \delta)^{\alpha}(1 + r_t + \delta)^{(\alpha - 1)}}.
\]

Each region is assumed to produce a homogeneous good that is an imperfect substitute for the output of the other region. Total consumption, then, is a bundle of home goods \( (CH_t) \) and imports from the other region \( (CF_t) \). The degree of substitutability is determined by the parameter \( \theta \) in a CES utility function, where \( \alpha \) represents the share of the home good:

\[
C_t = \gamma_t W_t
\]

\[
\gamma_t^{-1} = \frac{(1 + \gamma_{t+1})}{2 - (1 + \rho + \delta)^{\alpha}(1 + r_t + \delta)^{(\alpha - 1)}}.
\]

\[
C_t = (\alpha^{1/\theta} CH_t^{(\theta - 1)/\theta} + (1 - \alpha)^{1/\theta} CF_t^{(\theta - 1)/\theta})^{\theta/(\theta - 1)}.
\]

The price of the home good (by convention the U.S.) in units of the foreign good equals the real exchange rate \( (rer_t) \), with an increase in \( rer \) implying a rise in its relative price. The

---

21 The Appendix contrasts these properties with those of infinite-horizon models, and in particular the requirement in the latter that the equilibrium real interest rate exceed the real growth rate.

22 Only a portion of government debt represents net real wealth because the associated tax burden (for currently-alive individuals) is subtracted from human wealth.

23 Note that the rate used to discount future labor income includes the population growth rate \( (\hat{p}) \), so that human wealth reflects only of the future labor income of those currently alive.

24 See, for instance, Kollman (2006).
consumer price index \((p_t^c)\), expressed relative to the price of domestic output, is determined by \(\theta\), \(\alpha\), and the real exchange rate:

\[
p_t^c = \left(\frac{1}{\alpha} \theta \left(\frac{\theta}{\alpha}\right) + (1 - \alpha) \left(\frac{\theta}{\alpha}\right)^{\theta/(\theta-1)}\right)
\]

(10)

Consumption of the imported good relative to the domestic good is also a function of the real exchange rate:

\[
\frac{C_F}{CH_t} = \left(\frac{1 - \alpha}{\alpha}\right) \theta^i
\]

(11)

Aggregate output of each region \((Y_t)\) is produced by labor and capital using a CES production function. The supply of individual labor is fixed, so labor supply is proportional to the population \((P_t)\). Output is then:

\[
Y_t = \left(\mu^{1/\kappa} P_t^{(\kappa-1)/\kappa} + (1 - \mu)^{1/\kappa} (\Gamma_t P_t)^{\kappa/(\kappa-1)}\right)
\]

(12)

where \(\Gamma_t\) is an index of labor-augmenting technological progress, \(\mu\) is the capital share, and \(\kappa\) is the elasticity of substitution between labor and capital. Capital depreciates at rate \(\lambda\), and investment \((I_t)\) consists only of the domestic good. There are no fixed costs to adjusting the capital stock, so the net marginal product of capital equals the risk-free interest plus an exogenous risk premium \((\text{risk})\), and the market value of capital equals the sum of depreciated past investment. The relations determining the optimal capital stock and its movement over time are then:

\[
K_t = \frac{\mu Y_t}{r_t + \text{risk} + \lambda}
\]

(13)

\[
K_t = (1 - \lambda) K_{t-1} + INV_{t-1}
\]

(14)

The specification of equation (13) has implications for dynamic efficiency—as the cost of capital includes a risk premium, the private borrowing cost that determines the marginal product of capital is \(r + \text{risk}\). Because of the “wedge” created by \(\text{risk}\) between the risk-free interest rate and the private borrowing cost, the marginal product of capital exceeds the economic growth rate as long as \(r + \text{risk} > g\). When this condition holds, dynamic efficiency obtains even if the risk-free interest rate is less than the economic growth rate.

Each region has a government that consumes only the domestic good; any utility derived from government consumption is assumed to be additively separable from private consumption. Primary government spending \((G_t)\) can be financed either by lump-sum taxes \((TAX_t)\) or issuance of government debt \((B_t)\) at the risk-free interest rate. The government’s intertemporal budget constraint is:

\[
B_t = (1 + r_{t-1}) B_{t-1} + G_t - TAX_t
\]

(15)
The government adjusts taxes to achieve a target for the ratio of public debt to GDP \((B/Y)^T\), with the speed of convergence of the actual to target ratio determined by \(\psi\):

\[
B_t / Y_t = \psi \left( (B/Y)^T - B_{t-1}/Y_{t-1} \right). \tag{16}
\]

Regarding international portfolio allocation, a growing literature derives optimal portfolio shares, including the “home bias” phenomenon, in a stochastic setting.\(^{25}\) The theoretical apparatus of these models is complex, and we make no attempt to endogenously characterize portfolio allocation in this paper. Instead, it is assumed that desired portfolio shares can be expressed as linear functions of the expected yield differential between home and foreign assets, as in BGS (2005). The share of domestic financial wealth allocated to holdings of foreign assets \((GFA_t)\) then equals an intercept \((\omega_t)\) plus a parameter times the expected excess return on the foreign asset \((\text{ret}_t^{ex})\). A similar relationship exists for the ROW, where its holdings of U.S. assets are negatively related to the return on ROW assets. External liabilities are denominated in units of output of the issuing region. The intercept terms are subject to shocks in the simulations, and thus can be time-varying. The excess return equals the difference between foreign and domestic interest rates \((r_t^{ROW} \text{ and } r_t \text{ respectively})\) adjusted for the expected change in the real exchange rate. The parameter \(\tau\) determines the responsiveness of portfolio shares to the excess return:

\[
GFA_t / \text{ret}_t = (\omega_t + \tau \text{ret}_t^{ex})WF_t,
\]

\[
GFA_t^{ROW} \text{ret}_t = (\omega_t^{ROW} - \tau \text{ret}_t^{ex})WF_t^{ROW}. \tag{17}
\]

\[
1 + \text{ret}_t^{ex} = \frac{1 + r_t^{ROW}}{1 + r_t} \left( \frac{\text{rer}_t}{\text{rer}_{t+1}} \right).
\]

To ensure dynamic stability, home bias in portfolio allocation is assumed, implying that the share of foreign assets in the domestic portfolio is less than one half (i.e. \(\sigma < 1/2\)).

Imports in both regions \((CF_t \text{ and } CF_t^{ROW} \text{ respectively})\) are determined by equation (11), where imports are denominated in the output of the exporting country. The exports of each region equal the imports of the other region. The U.S. trade balance \((TB_t)\) equals exports minus imports valued in units of domestic output. The trade balance of the ROW equals the negative of the U.S. trade balance times the real effective exchange rate. Thus:

\[
TB_t \equiv CF_t^{ROW} - CF_t / \text{ret}_t, 
\]

\[
TB_t^{ROW} \equiv -TB_t \text{ret}_t. \tag{18}
\]

---

25 See Kollman (2006), Engel and Matsumoto (2006), and Devereux and Sutherland (2006).
Net foreign assets of the U.S. equal its gross holdings of ROW assets divided by \( rer_t \), less ROW holdings of U.S. assets. Again, net foreign assets of the ROW equal the negative of U.S. net foreign assets converted in units of ROW output:

\[
NFA_t \equiv \frac{GFA_t}{rer_t} - GFA_{t,ROW}^{ROW} \\
NFA_{t,ROW} \equiv NFA_t \times rer_t
\]  

(19)

An additional relationship describes the change in net foreign assets from periods \( t-1 \) to \( t \):

\[
\frac{GFA_t}{rer_t} - GFA_{t,ROW}^{ROW} = TB_{t-1} + TRNS_{t-1} + (1+r_{t-1,ROW})GFA_{t-1} / rer_t - (1+r_{t-1})GFA_{t-1,ROW}^{ROW}
\]  

(20)

The variable \( TRNS \) represents the exogenous “manna from heaven” received by the U.S. in the form of valuation gains on external assets, as discussed in Section II. Taken together, the system of ten equations given by (11) plus (17) through (20) proximately determines \( C_F, C_F^{ROW}, TB_t, TB_t^{ROW}, NFA_t, NFA_t^{ROW}, GFA_t, GFA_t^{ROW}, rer_t, \) and \( rer_t^{ex} \).

**B. Parameterization**

The model is parameterized to yield an initial steady state around which the shocks that give rise to the imbalances are introduced. To generate a steady state, the behavioral parameters in the two regions are set to be equal, and trade is assumed to be balanced. The main difference between the regions, then, is their relative size—the U.S. share in world GDP is set at 30 percent, typical of its nominal share over the past two decades.

Population growth is set at 1 percent per year, similar to U.S. population growth during 1980-2005 (in the ROW it averaged about 1½ percent, but slowed toward the end of the period). The probability of death is 2 percent per year, implying an economically-active expected life of 50 years; the birth rate is set at 3 percent per year. Per capita GDP growth is 2 percent per year in both regions, slightly less than the average over the last 25 years of 2.1 percent in the U.S. and 2.5 percent in the ROW; aggregate GDP growth is then 3 percent, reflecting both productivity and population growth. To construct the current account balance, an assumption for U.S. inflation is required to convert real interest rates into their nominal equivalents (see equation (5) above)—a value of 2½ percent is assumed, in line with the expected rate implied by inflation-indexed Treasury bonds.

The capital share in gross output in both regions is set at 25 percent, while capital depreciation is assumed to be 5 percent per year. The risk premium on capital is set at 2 percentage points, based on the average difference between the BAA corporate bond rate and the U.S. treasury yield since the late 1960s. These values imply a ratio of the net capital stock to GDP of about 2.5, and a share of gross private investment in output of 20 percent. The elasticity of substitution between labor and capital, \( \kappa \), equals one half.
The baseline risk-free real interest rate is 3¼ percent, equal to the average real return on U.S. external liabilities during 1983-2005, and just above the real growth rate of 3 percent. To calibrate the model accordingly, the intertemporal elasticity of substitution in consumption is first set to an assumed value of 0.5 (see Appendix). With this value for \( \sigma \), a rate of time preference of 0.8 percent per year yields a real interest rate of 3¼ percent. The share of the home good in consumption, \( \alpha \), is set at 0.85 for the U.S., which implies an initial share of (real) imports in GDP of 9 percent, close to the value observed in 1995. Trade is balanced, such that the imports of the ROW (and thus U.S. exports) equal U.S. imports.

The elasticity of substitution between domestic and foreign goods in consumption determines the response of the trade balance to the real exchange rate. As discussed in BGS (2005), estimated price elasticities place the required exchange rate movement to achieve a 1 percent of GDP change in the U.S. trade balance at anywhere from 7 percent to 15 percent.\(^{26}\) We calibrate the model such that a 10 percent appreciation in the real exchange rate reduces the nominal trade balance by 1 percent of GDP, starting from a position of balanced trade, implying a value for \( \theta \) of 1.20. As well as lying within the usual range of estimates, this value is conveniently equal to that used in the rule-of-thumb calculation in the introduction.

U.S. net foreign liabilities were small in 1995 (4 percent of GDP), and for simplicity are calibrated to zero. Gross U.S. holdings of foreign assets were 50 percent of GDP, equivalent to 15 percent of U.S. financial assets. Gross ROW holdings of U.S. assets then equal 6 percent of ROW financial assets, given the difference in the relative size of the two regions. Assets are assumed to be denominated in the currency of the issuing region, which is important in determining currency valuation effects in the simulations. This assumption is approximately true for foreign holdings of U.S. assets, but not for U.S. holdings of foreign assets: estimates for 2005 indicate that about 35 percent of U.S. external assets were dollar-denominated (Tille (2005)). No figures are available for 1995, and the data are initialized around the assumption that this share is zero. In any event, exogenous valuation effects are added in the initial years of the simulation (1996-2005) to account for observed valuation adjustments—by 2005, simulated gross asset positions match up closely with estimates of their foreign-currency component, thus valuation effects from exchange rate movements from 2006 on are consistent with the observed currency composition of assets in 2005. The degree of asset substitutability is determined by \( \tau \), for which there is little empirical evidence. BGS use three alternative values: 10, 1, and 0.1. For the baseline scenario, we take the middle value of 1, such that a 1 percentage point change in the yield differential generates a 1 percentage point change in portfolio shares. This value yields portfolio effects that appear plausible, but scenarios are also run with the alternative values used by BGS.

\(^{26}\) See Hooper and Marquez (1995). Gust and Sheets (2006) find a similar response, observing that it is relatively insensitive to the degree of trade price passthrough.
The baseline ratio of primary government spending to GDP is set at 15 percent, while the ratio of government debt to GDP is 30 percent. The tax rate is then consistent with sustaining a constant debt-to-GDP ratio. The speed of adjustment of the actual to the target debt-to-GDP ratio is determined by $\psi$ — we assume a value of 0.1, implying a half-life of the adjustment of the debt-to-GDP ratio to its target of 6 years. Clearly these assumptions are broad brush, but the simulations described below are insensitive to a wide range of fiscal values.

V. SIMULATION RESULTS

A. Initial Equilibrium

A steady-state analogue to the dynamic model is used to derive an initial, pre-shock equilibrium for the period 1995 to 2250. The trade balance and net foreign asset positions equal zero, the real exchange rate is normalized to unity, and the two regions share the same real interest rate and growth rate (3¼ percent and 3 percent respectively). The capital-to-output ratios are both 2.5, with an associated net marginal product of capital of 5¼ percent. Note that the marginal product of capital exceeds the economic growth rate, thus the steady-state growth path is dynamically efficient notwithstanding a (risk-free) real interest rate that is less than the real growth rate. The terminal values for the forward-looking variables in 2251 are determined by the steady-state model.

B. Calibration of Shocks

It remains to determine the shocks that generate the emergence of the external imbalances. The discussion in Section II points to two factors: (i) an upward shift in international portfolio preferences for U.S. assets, and (ii) an upward shift in U.S. demand for imported goods. This is also broadly consistent with the approach in BGS (2005). In addition, we introduce a third (smaller) shock, in the form of an upward shift in ROW demand for imports from the U.S., which explains the resilience of U.S. exports since the mid-1990s in spite of the strength of the dollar. Finally, exogenous valuation gains on U.S. external assets are introduced during 1996-2005 to reflect the observed gap between the cumulative current account and the change in net foreign assets—these average 3 percent of GDP per year, more than offsetting the endogenous valuation losses generated by the model from appreciation of the dollar. After 2005, the U.S. is assumed to benefit from permanent, but smaller, valuation gains in the form of exogenous “manna from heaven” that is constant at 1 percent of GDP, consistent with the historical experience. In addition, the valuation effects from exchange rate movements are fully endogenous in the model simulations.

The sizes of the shocks were set such that the simulations broadly match the stylized facts during 1996-2005 for certain variables—in particular, the appreciation in the U.S. real

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27 The steady-state model replaces lags and leads in the dynamic model with their contemporaneous values, adjusted for steady-state growth where variables are non-stationary.
exchange rate, the deterioration in the U.S. external balance, the rise in U.S. imports relative to GDP, and the deterioration in U.S. net foreign assets. Thus these simulation results broadly line up with the actual data by construction, although the model also generates paths for other endogenous variables that are similar to actual developments. This calibration yields a shift in ROW preferences for U.S. assets of 10 percent of total assets—i.e., the intercept $\omega_t^{ROW}$ in the equation for the share of U.S. assets in ROW assets increases by 0.10. The import share in the U.S. consumption function, $1 - \alpha_t$, is assumed to rise by 0.04, while $1 - \alpha_t^{ROW}$ rises by 0.01.\footnote{As discussed in BGS (2006), instead of exogenous shifts in import shares, these changes could be modeled as income elasticities of import demand that are greater than unity (and higher in the U.S. than in the ROW).} To generate simulation paths that are consistent with the gradual movements in the exchange rate, trade flows and portfolio positions observed in the actual data, the shocks are phased in over a 12-year period.\footnote{There are two ways of phasing-in shocks to rational expectations models. One assumes that agents have perfect foresight; the other assumes that agents only incorporate shocks observed up to the current period. The main difference is that the perfect-foresight approach tends to give more “jumpiness” in the forward-looking variables. Apart from this, the model dynamics are similar.} In particular, the shock in period $t$, $shk_t$, equals $\lambda shk^* + (1 - \lambda) shk_{t-1}$, where $shk^*$ is the full size of the shock and the adjustment parameter $\lambda$ equals 0.25. The shocks stabilize over the long run in the baseline scenario.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure15.png}
\caption{Baseline Shocks to Model, 1995-2025}
\end{figure}

\section*{C. Baseline Simulation}

The baseline simulation results are shown for 1995-2025 in Figure 16. In terms of the variables for which the shocks are calibrated, the real exchange rate peaks at 32 percent above its initial value in 2002, similar to the movement in the adjusted series discussed earlier based on differences in CPI measurement between the U.S. and the ROW. The U.S. trade balance moves from zero in 1995 to a deficit of 5½ percent of GDP in 2005, while the current account deficit rises to 6 percent of GDP. U.S. net external liabilities reach 22 percent of GDP in 2005, as the exogenous valuation effects mentioned above partially offset the
current account deficit plus negative valuation effects from dollar appreciation. The other variable used to calibrate the size of the shocks is the share of imports in U.S. GDP, which rises from 9 percent in 1995 to close to 18 percent in 2005, similar to the observed increase.

The initial years of the simulation feature a sharp rise in the share of U.S. assets in foreign portfolios, from 6 percent in 1995 to 14 percent in 2005.\[^{30}\] In response to the shift toward U.S. assets, their yield falls, reaching about 2½ percent by 2010, while that on foreign assets rises to 3½ percent. Interest rate effects of this magnitudes are consistent with the (limited) empirical evidence—Dooley and others (2005) find a decline of about 1 percentage point in U.S. yields as a result of foreign accumulation of official reserves, while Warnock and Warnock (2005) estimate a similar impact due to higher-than-normal foreign investment in U.S. bonds. On this basis, the assumed degree of asset “insubstitutability” in the model appears to yield plausible interest rate effects. If anything, the effect appears small relative to the observed yield gap on U.S. external assets and liabilities over the historical period.

With the shocks replicating the main features of the rise of the imbalances, the interesting question is what the model implies in terms of how they unwind under alternative assumptions about the shocks. Under the baseline assumption that the shocks stabilize over the long run, the exchange rate depreciates by 16 percent (in logs) between 2005 and 2025, or slightly less than 1 percent per year. The trade deficit narrows to about 3 percent of GDP by 2025, while the current account deficit remains close to 6 percent of GDP.

The baseline simulation results are shown for the period extending to 2100 in Figure 17, when the variables are close to steady-state values. The real exchange rate returns to its original level, implying a decline from 2005 of 25 percent. The steady-state current account deficit is 6 percent of GDP, while the trade deficit is 1¼ percent of GDP; U.S. net foreign liabilities end up at 93 percent of GDP. The co-existence of a steady-state trade deficit with large foreign liabilities reflects the low real interest rate on the latter of 2.8 percent in the steady state—below both the real growth rate and the yield on U.S. foreign assets. While the risk-free real interest rate is below the growth rate, dynamic efficiency holds, as the marginal product of capital incorporates a private risk premium (Figure 18).

\[^{30}\] Corresponding to a rise in U.S. gross foreign liabilities to 80 percent of GDP, and in gross foreign assets to 58 percent. Actual U.S. liabilities and assets in 2005 were 110 percent and 88 percent of GDP respectively. Of the latter, about 30 percent of GDP were dollar-denominated. Netting out this dollar component, the model yields gross positions consistent with the currency mismatch between U.S. assets and liabilities.
Figure 16. Baseline Simulation Results, 1995-2025

Real Effective Exchange Rate
(Index, 1995=1)

U.S. Imports and Exports
(Percent of GDP)

U.S. External Balances
(Percent of GDP)

Net Foreign Assets
(ratio to GDP)

Expected Returns on Assets
(Percent )

U.S. & ROW Portfolio Shares
(Foreign share in total assets)

Source: Staff calculations.
Figure 17. Baseline Simulation Results, 1995-2100

Source: Staff calculations.
The effect of dollar depreciation on the trade deficit is amplified by movements in consumption in the U.S. and the ROW. In particular, U.S. consumption during 1996-2005 is boosted by declining interest rates and real income gains due to real exchange rate appreciation, while the reverse occurs in the ROW (Figure 19). These movements in relative consumption are consistent with the stylized facts described in Section II (Figure 6). As the long-run asset dynamics play out and U.S. indebtedness to the ROW rises, the consumption shifts are reversed. These activity effects reduce the change in the exchange rate needed to lower the U.S. trade deficit, helping to explain why the model results differ from the rule-of-thumb calculation.

The rise in U.S. net external debt to over 90 percent of GDP in the steady state appears large, and raises questions about the willingness of the rest of the world to accumulate U.S. assets. The increase in debt is associated with only a modest rise in the share of U.S. assets in
foreign portfolios, however—from 14 percent in 2005 to 16½ percent in the steady state, a much smaller increase than already observed since the mid-1990s. In terms of why the rise in the share of U.S. assets in foreign portfolios is not greater, the ROW is larger than the U.S., and assets are large relative to GDP in both regions. Thus an increase in U.S. assets in foreign portfolios of 2½ percentage points translates into an increase in U.S. external debt to GDP of some 20 percentage points. Beyond relative size, there are two factors endogenous to the simulation that limit the rise in the share of U.S. assets in foreign portfolios. The first is exchange-rate valuation effects: depreciation of the dollar reduces the value of U.S. assets in ROW portfolios. As a result, the ROW must accumulate U.S. assets just to keep portfolio shares constant—this valuation effect contributes 25 percentage points to the rise in U.S. external debt. Finally, as financial wealth is redistributed from the U.S. to the ROW, the financial assets of the ROW rise relative to GDP while the reverse occurs in the U.S. As U.S. financial wealth falls (in relative terms), and the yield differential in favor of foreign assets declines, so do U.S. gross holdings of foreign assets. So the rise in U.S. net foreign debt is associated, in part, with a decline in U.S. holdings of foreign assets as opposed to higher foreign holdings of U.S. assets.

D. Alternative Paths for the Shocks

The baseline scenario assumes that the shocks generating the imbalances stabilize, but do not reverse, over the long run. Here we examine the implications of alternative assumptions about how the shocks could evolve. Many scenarios are, of course, possible—the three described below are designed to illustrate a range of alternatives: at one extreme, the shocks continue to grow in the near term, leading to even larger imbalances than those currently observed; at the other, the shock to portfolio preferences suddenly reverses, reflecting “flight” from U.S. assets. An intermediate case involves an assumed “freezing” by foreign central banks of the nominal value of their U.S. dollar reserve assets.

The first alternative involves shocks that rise further over the next several years, illustrating how the magnitude and speed of the adjustment are affected when the global imbalances are larger and more drawn out. In particular, the size of the shocks was increased by 30 percent, and the speed at which the shocks are phased in was cut in half from 0.250 to 0.125, such that they are fully phased in only by 2013. The exchange rate peaks somewhat later, and similarly for the U.S. external deficits (Figure 20). Perhaps more interestingly, the eventual adjustment in the exchange rate is about the same as that in the baseline scenario, while that in the trade balance is slightly smaller, in spite of the larger shocks—the steady-state trade deficit is 1½ percent of GDP versus 1¼ percent in the baseline, even though U.S. net foreign liabilities end up at 120 percent of GDP. One implication of debt dynamics under low real interest

31 Most of this further rise in the share of U.S. assets in foreign portfolios reflects a higher return on U.S. assets adjusted for expected exchange rate movements, as the dollar stabilizes in the steady state.
Figure 20. Larger and More Drawn-Out Shocks, 1995-2100

Real Effective Exchange Rate
(Index, 1995=1)

U.S. Imports and Exports
(Percent of GDP)

U.S. External Balances
(Percent of GDP)

Net Foreign Assets
(ratio to GDP)

Expected Returns on Assets
(Percent)

U.S. & ROW Portfolio Shares
(Foreign share in total assets)

Source: Staff calculations.
rates, then, is that the unwinding of global imbalances does not necessarily require greater adjustment in exchange rates and trade balances even if the initial imbalances are larger.

The next scenario assumes a partial reversal of the shift into U.S. assets, based on a hypothetical “freezing” of the nominal level of U.S. assets held as official reserves by other countries. Specifically, it is assumed that nominal reserves (in dollars) are held constant until the ratio of reserves to ROW GDP returns to its 1995 level. Given that about one fifth of the increase in foreign holdings of U.S. assets since 1995 is accounted for by official reserves (Figure 9 in Section II), the decline in the ROW portfolio share is one fifth of the increase observed since 1995. In this scenario, the adjustments in the exchange rate and U.S. external deficits would be significantly faster than in the baseline through 2017, and level off thereafter (Figure 21). But, even with this faster adjustment, the decline in the exchange rate averages only 1½ percent per year during 2005-2017, suggesting a relatively soft landing. This assumes, of course, that the private component of the original portfolio shift is unaffected by the freezing of official reserve holdings.

A more extreme, “hard-landing” scenario was constructed where the portfolio shift reverses in a short period of time, assumed to be 2007. The decline in the dollar is more dramatic than in the previous exercises, amounting to 33 percent (in logs) on impact (Figure 22). The trade and current account deficits are immediately reversed, reflecting the absence of adjustment dynamics such as J-curve effects in the model, which is clearly unrealistic in the context of such an abrupt movement in the exchange rate. The other unrealistic aspect of the model given such an abrupt shock is the absence of nominal price stickiness. With price stickiness, monetary policy would generally “lean against the wind” of the inflationary impact of sharp exchange rate depreciation, in turn moderating the decline.32 In any case, it is interesting to note that the exchange rate depreciation in this extreme case is similar to that which occurred from 1986-89, and was absorbed without excessive turmoil in financial markets. It is also notable that the size of the exchange rate depreciation is still only about one half of that suggested by the back-of-the-envelope calculation described in the introduction.

Another simulation was run in which positive valuation adjustments to U.S. NFA disappear over the long run (specifically, falling from 1 percent of GDP to zero during 2010-25). This reduction in “manna from heaven” raises slightly the speed of adjustment of the exchange rate during 2006-25 but otherwise has little impact on the transition path. Differences relative to the baseline simulation are more apparent in the steady state, as the sustainable trade deficit is 1 percent of GDP lower without ongoing valuation adjustments. The real exchange rate is 6 percent more depreciated, while the steady-state ratio of U.S. net foreign debt to GDP rises to 105 percent. The additional adjustment in the real exchange rate, beyond that in

---

32 Price stickiness could also cause output losses through the effect of tighter monetary policy on aggregate demand that are not reflected in the simulation given the assumption of complete nominal flexibility.
Figure 21. Phase-Out of International Reserve Accumulation, 1995-2025

Real Effective Exchange Rate
(Index, 1995=1)

U.S. Imports and Exports
(Percent of GDP)

U.S. External Balances
(Percent of GDP)

Net Foreign Assets
(ratio to GDP)

Expected Returns on Assets
(Percent)

U.S. & ROW Portfolio Shares
(Foreign share in total assets)

Source: Staff calculations.
Figure 22. “Hard-Landing” Scenario, 1995-2025

Real Effective Exchange Rate
(Index, 1995=1)

U.S. Imports and Exports
(Percent of GDP)

U.S. External Balances
(Percent of GDP)

Net Foreign Assets
(ratio to GDP)

Expected Returns on Assets
(Percent)

U.S. & ROW Portfolio Shares
(Foreign share in total assets)

Source: Staff calculations.
the baseline scenario, is limited by the fact that higher net foreign debt and the loss of income from abroad reduces U.S. consumption, and thus import demand.

E. Alternative Parameter Values

Simulations were also run with different values for some key parameters to assess the sensitivity of the results to the baseline calibration. The first was to assume an initial level of the real interest rate well above the real growth rate—a gap of 2 percentage points was chosen, as it corresponds to that used in the rule-of-thumb calculation. In particular, raising the rate of time preference 2½ percent gives an initial real interest rate of 5 percent. The medium-term dynamics are affected only modestly by a higher interest rate, with both the dollar and the trade deficit reversing somewhat faster than in the baseline scenario (Figure 23). In the steady state, the trade balance moves into a small surplus, reflecting the higher carrying cost of the external debt with a real interest rate on external liabilities of about 4½ percent; the larger adjustment in the trade balance implies that the equilibrium real exchange rate is about 7 percent more depreciated than in the baseline. Nevertheless, the total depreciation is only about one half of that in the rule-of-thumb calculation, reflecting: the positive yield gap between external assets and liabilities; the assumed stream of favorable valuation effects; and the role of movements in relative consumption on the trade balance.

Another calibration issue is the degree of asset substitutability. Alternative values were considered based on the range used by BGS (2005), i.e. 10 times smaller and 10 times larger than the baseline value for \( \tau \) of unity. Assuming \( \tau \) equals 0.1 does not substantially affect the results—movements in the main variables, such as the external balances and portfolio shares, are modestly smaller than in the baseline scenario. Otherwise the dynamics are similar, so the results are not shown here. In contrast, when the degree of substitutability is much larger, the results change substantially. Indeed, no reasonable specification of the shocks generates an exchange rate movement as large as observed from 1995-2002. Even doubling the size of all shocks yields an exchange rate appreciation of less than 20 percent, while at the same time generating huge changes in portfolio shares. The difficulty in finding reasonable shocks under the assumption of high asset substitutability is not surprising, given the important role that portfolio effects play in creating the imbalances: moving toward the traditional assumption of perfect asset substitutability rules out such effects, leaving it difficult to explain the rise of the imbalances.

Given that portfolio shifts appear to have played a major role in generating the imbalances, it is useful to explore the possibility that asset substitutability is higher in the long run than in the short run, and thus that the portfolio effects observed to date will not persist. This assumption implies a gradual narrowing of the gap between the equilibrium yields on U.S. and ROW assets, in effect unwinding the effects of the portfolio shift. To assess the implications of this assumption, a time-varying degree of asset substitutability is introduced.
Figure 23. Higher Initial Real Interest Rate, 1995-2100

Real Effective Exchange Rate
(Index, 1995=1)

U.S. Imports and Exports
(Percent of GDP)

U.S. External Balances
(Percent of GDP)

Net Foreign Assets
(ratio to GDP)

Expected Returns on Assets
(Percent)

U.S. & ROW Portfolio Shares
(Foreign share in total assets)

Source: Staff calculations.
in the model in the form of a rising path for $\tau$—in particular, $\tau$ increases by 10 percent per year from 2010 on. By 2034, $\tau$ reaches 10, implying a high degree of asset substitutability, and by 2060 assets are effectively perfect substitutes. The adjustments in the exchange rate and the U.S. external balances are considerably more rapid than in the baseline scenario, being essentially complete by 2025 (Figure 24). During 2005-15, the real exchange rate depreciates by almost 2½ percent per year on average, roughly three times the rate in the baseline scenario. U.S. net foreign assets turn positive as the portfolio effects unwind and the real interest rate converges back toward its initial level of 3¼ percent.

Another scenario was run using lower trade price elasticities than in the baseline calibration. Specifically, $\theta$ was lowered from 1.2 to 1.0, such that a 15 percent change in the real exchange rate is needed to generate a 1 percent of GDP movement in the trade balance (versus 10 percent in the baseline). To obtain movements in the real exchange rate and the trade balance during 1996-2005 similar to the observed data, the size of the upward shift in U.S. import demand was raised to 0.05 from 0.04 to compensate for the lower response of trade flows to exchange rates. The main difference compared with the baseline is a larger depreciation in the exchange rate, both over the adjustment period to 2025 and in the steady state (Figure 25). The average rate of depreciation during 2005-25 is 1.1 percent per year compared with 0.8 percent in the baseline, and the steady-state exchange rate is 10 percent more depreciated. Thus the total depreciation in the exchange rate from its peak is greater than in the baseline, as expected, but the difference is less than proportional to the increase to 15:1 from 10:1 in the exchange rate change needed to achieve a given movement in the trade balance. This is because a lower path for U.S. consumption, reflecting a weaker terms of trade and thus real incomes, reinforces the impact of the weaker exchange rate.

**Figure 25. Alternative Trade Price Elasticities, 1995-2150**
Figure 24. Increasing Asset Substitutability, 1995-2100

**Real Effective Exchange Rate**
(Index, 1995=1)

**U.S. Imports and Exports**
(Percent of GDP)

**U.S. External Balances**
(Percent of GDP)

**Net Foreign Assets**
(ratio to GDP)

**Expected Returns on Assets**
(Percent)

**U.S. & ROW Portfolio Shares**
(Foreign share in total assets)

Source: Staff calculations.
Finally, a simulation was run in which the rest of the world grows more rapidly than the U.S. over an extended period, continuing trends in recent years. As the share in the world economy of rapidly-growing emerging market economies has risen, the real share of the U.S. in world output has fallen since the late 1990s, and this trend is projected to continue based on WEO projections (Figure 26). The main factor underlying the declining U.S. share is an acceleration in real per capita GDP growth in the rest of the world to about 3½ percent per year during 2001-10, above the baseline assumption of 2 percent per year. This higher growth rate is introduced as a long-lived shock (50 years) to the model, to reflect an assumed process of real income catch-up to the levels in advanced economies.

![Figure 26. Ratio of U.S. to ROW GDP, 1995-2010](image)

The results are shown in Figure 27. The obvious difference from the baseline simulation is the path of the dollar, which depreciates only slightly during 2005-15 and appreciates thereafter. The mechanism responsible for dollar appreciation is the rise in ROW relative to U.S. output, which drives down the relative price of ROW goods given the assumption that they are imperfect substitutes for U.S. goods. This result contrasts with the predictions of the Balassa-Samuelson model, which supposes a two-sector framework, with higher productivity growth occurring in the tradables sector. As tradable goods are assumed to be perfect substitutes in that framework, the ROW real exchange rate would appreciate with higher productivity growth. While the evidence suggests that real exchange rate movements are mostly attributable to imperfect substitutability between traded goods, consistent with the specification in this paper, it would nevertheless be useful to explore the results in a framework with non-traded goods.33

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Figure 27. Higher Foreign Growth, 1995-2025

Real Effective Exchange Rate
(Index, 1995=1)

U.S. Imports and Exports
(Percent of GDP)

Real Effective Exchange Rate
(Index, 1995=1)

U.S. Imports and Exports
(Percent of GDP)

U.S. External Balances
(Percent of GDP)

Net Foreign Assets
(ratio to GDP)

Expected Returns on Assets
(Percent)

U.S. & ROW Portfolio Shares
(Foreign share in total assets)

Source: Staff calculations.
Another aspect of this simulation that is perhaps surprising from a theoretical standpoint is the virtual absence of an impact on current account balances when ROW income is assumed to grow more quickly. In particular, traditional theoretical models suggest that the ROW current account balance would deteriorate as households consume more in anticipation of higher future income, and as a higher marginal productivity of capital attracts a capital inflow to the ROW. There are two reasons why this effect is not observed in this model: (i) under the relative income hypothesis, ROW households raise planned consumption growth in response to higher income growth, reducing the initial dissaving that would otherwise take place; and (ii) the assumption of imperfect asset substitutability reduces the tendency for capital to flow to the ROW—indeed, there is even a reverse effect, as demand for U.S. assets by the ROW increases as ROW portfolios expand more rapidly. This aspect of the results appears consistent with the historical record, at least for fast-growing developing countries.  

F. Fiscal Policy

Thus far there has been no discussion of the role of fiscal policies in generating the imbalances or contributing to their resolution. The reason is that, in the model used here, fiscal policy has little impact on external balances or the exchange rate. In particular, the model specification allows only one channel for deviations from Ricardian equivalence—i.e. new entrants to the population who bear some of the future burden of financing current-period fiscal deficits. With a birth rate of 3 percent per year, it would take large and long-lived changes in deficits to have a significant impact on household wealth. Otherwise, the model has no tax or spending mechanisms that generate distortions to relative prices, or liquidity constraints that would raise the sensitivity of consumption to tax changes. Taking into consideration the limited role of fiscal policy in the model, and the fact that the actual U.S. fiscal position has not changed dramatically since the mid-1990s, the model does not assign a significant role to U.S. fiscal policy as a source of global imbalances.

In addition to the absence of fiscal distortions, the assumption of imperfect asset substitutability affects the external responses to a fiscal shock compared with traditional models. For example, a fiscal stimulus has an ambiguous effect on the real exchange rate under imperfect asset substitutability, as the impact of a higher domestic real interest rate in boosting foreign demand for domestic assets is limited by the degree of asset substitutability, while increased domestic spending causes the trade balance to deteriorate. Furthermore, a fiscal stimulus that raises the ratio of government debt to GDP has unusual portfolio effects


35 Other factors, such as liquidity constraints, could increase the effect of tax cuts by raising the effective discount rate of households—Bayoumi and Sgherri (2006), for instance, estimate an excess discount rate of 15-25 percent per year.

36 Consistent with Bernanke (2005), who observes that U.S. fiscal consolidation is desirable in its own right.
under imperfect asset substitutability—as households increase their total asset holdings to accommodate more government debt, their demand for external assets rises, ceteris paribus. As a result, higher government debt can “crowd in” net foreign assets if portfolio shares are relatively insensitive to yield differentials.

The latter channel plays an important role under the baseline assumption for asset substitutability. To illustrate, a shock was performed where the target debt-to-GDP ratio was doubled to 60 percent from 30 percent (implying a cut in taxes and a rise in the fiscal deficit of 2 percent of GDP in the initial years). Under the baseline calibration, the higher fiscal deficit is associated with (small) trade surpluses, contrary to the twin-deficits view. The trade surpluses follow from the desire of households to increase their holdings of foreign assets, which causes the real exchange rate to depreciate. Figure 28 shows the results in terms of the ratio of the change in the trade balance relative to that in the fiscal balance over the first 10 years of the simulation. When the same shock is run under perfect asset substitutability, the “twin deficits” result holds, with the change in the trade balance rising from 10 to 25 percent of the change in the fiscal balance.

![Figure 28. “Twin Deficits”: Effect on Trade Balance of Change in Fiscal Balance](image)

These results indicate that it would take large changes in fiscal policy, combined with a higher degree of asset substitutability than in the model, for fiscal policy to play a significant role in explaining the global imbalances. In any case, a more promising model for assessing the role of fiscal policy would need to incorporate a richer specification of distortionary effects and deviations from Ricardian equivalence.37

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37 See, for instance, the framework of Botman and Kumar (2006).
VI. CONCLUDING REMARKS

This paper uses a general-equilibrium model to replicate the stylized facts of the emergence of global imbalances, and then to describe adjustment paths to a full stock/flow equilibrium. In this sense, it addresses gaps in partial-equilibrium analyses concerning the interaction of exchange rate movements, interest rates, saving and investment, and portfolio shares. The adjustment paths reflect smaller and more gradual movements in the exchange rate and external balances than generally implied by partial-equilibrium treatments. In addition, a significant long-run rise in the ratio of U.S. net foreign debt to GDP is associated with only a modest further increase in the share of U.S. assets in foreign portfolios, supporting the view of “portfolio optimists” who emphasize the ability of the rest of the world to continue to absorb significant U.S. external deficits. In this sense, the results are consistent with the relatively benign view of Greenspan (2004) as to how the imbalances may play out.

Of course, the results are a function of the assumed paths for the shocks, and should be considered simulations as opposed to forecasts. More dramatic outcomes are possible if behavior suddenly reverses, as shown in the hard-landing scenario. In addition, the smooth adjustment paths under the other characterizations of the shocks are a function of the forward-looking behavior in the model, which rules out phenomena such self-fulfilling market panics and herding behavior that could trigger abrupt shifts in asset preferences. Incorporating such effects, particularly in the context of nominal price rigidities and thus a role for monetary policy, would provide a richer description of possible outcomes.

A controversial aspect of the analysis is perhaps an initial U.S. real interest rate that is only modestly above the real growth rate, and then falls below it in response to the portfolio shift toward U.S. assets. We have aimed to show that these properties are consistent with both the stylized facts and dynamic optimization, and in this sense are not “abnormal.” Their implications for debt dynamics may help to explain the gap between the alarming results of some partial-equilibrium calculations and the apparently more benign view in markets of the prospects for global imbalances. The deterministic simulations generated by the model, however, are not well suited to illustrating the risks associated with unforeseen changes in the long-run economic environment. In this sense, the rather benign conditions in the baseline scenario, and the associated build up of large gross and net international asset positions, could themselves engender future vulnerabilities that would motivate risk premia that are not incorporated in the model structure.

Other directions could be taken to enhance the ability of the model to account for various features of the imbalances, including the incorporation of J-curve effects in trade, liquidity-constrained households, and distortionary fiscal policies. The realism of the initial conditions could also be enhanced by incorporating oil trade, as well as more complete modeling of the “exorbitant privilege” of the U.S. in global capital markets. Finally, it would be desirable to
provide a stronger theoretical and empirical basis for the portfolio shifts in the model, given their central role in driving the results.
APPENDIX. CONSUMPTION, WEALTH AND REAL INTEREST RATES

The simulation model generates a path for the risk-free real interest rate that is below the real economic growth rate, which is sometimes thought to be inadmissible in an optimizing model. This annex discusses conceptual aspects of interest rate determination, contrasting the properties of infinite- and finite-horizon models, to motivate this property.

A. Infinite-Horizon Model

A convenient starting point for analyzing interest-rate determination is the infinite-horizon Ramsey model with positive real growth. Consider an agent who maximizes expected utility over an infinite horizon. Utility \( U \) is derived from consumption \( c \) in period \( t \) according to the function \( U(c_t) \), discounted to the initial period by \( (1 + \rho)^{-t} \). Utility is maximized subject to the constraint that the present value of consumption cannot exceed that of “endowment” income \( endow_t \) plus the value of an asset that yields rate \( r \) per period. Endowment income grows at a constant rate \( g \). The utility maximization problem then is:

\[
\text{maximize } \sum_{t=0}^{\infty} \frac{U(c_t)}{(1 + \rho)^t} \\
\text{subject to } \sum_{t=0}^{\infty} \frac{c_t}{(1 + r)^t} \leq \sum_{t=0}^{\infty} endow_t \left( \frac{1 + g}{1 + r} \right)^t + a_0.
\]

The condition that the infinite sum on the right-hand side of the budget constraint be bounded requires that \( r > g \). This immediately establishes the result that the real interest rate must exceed the real growth rate in this infinite-horizon framework.

The Euler equation corresponding to the utility maximization problem is:

\[
U'(c_t) = \left( \frac{1 + r}{1 + \rho} \right) U'(c_{t+1}).
\]

Tying down the consumption path corresponding to equation (21) requires a specific utility function; assuming constant relative risk aversion implies:

\[
U(c) = \frac{c^{1-1/\sigma}}{1-1/\sigma} = \ln(c) \text{ if } \sigma = 1
\]

\[
U''(c) = c^{-1/\sigma} > 0
\]

\[
U''(c) = (-1/\sigma) c^{-(1+1/\sigma)} < 0 \text{ if } \sigma > 0
\]

The parameter \( \sigma \) represents the intertemporal elasticity of substitution (the inverse of the coefficient of relative risk aversion). The associated Euler equation is:
\[
\frac{c_{t+1}}{c_t} = \left( \frac{1+r}{1+\rho} \right)^\sigma
\]  

Equation (23) shows that consumption growth is related positively to the real interest rate and negatively to the rate of time preference: a higher interest rate lowers the relative price of future consumption, while a higher rate of time preference reduces its utility. A larger \( \sigma \) increases the response of consumption growth to \( r \) and \( \rho \). When \( \sigma \) is zero, in contrast, consumption is constant over time regardless of the values of \( r \) and \( \rho \).

In a steady state, this model requires that consumption growth equal growth in endowment income—otherwise, the ratio of assets to income would not be constant. The left-hand side of equation (24) can then be replaced by \( (1+g) \) to give the steady-state real interest rate (\( r^{ss} \)):

\[
1+g = \left( \frac{1+r^{ss}}{1+\rho} \right)^\sigma \\
\Rightarrow \quad 1+r^{ss} = (1+\rho)(1+g)^{1/\sigma} \\
\Rightarrow \quad r^{ss} \approx \rho + g/\sigma
\]  

Thus the real interest rate approximately equals the rate of time preference plus the real growth rate divided by \( \sigma \). When \( \sigma \) is unity, the real interest rate equals \( \rho + g \), consistent with the “modified golden rule.”\(^{38}\) For values of \( \sigma \) below unity—typical of the empirical evidence—the steady-state real interest rate is higher than this level.\(^{39}\) Taking a value of 0.5, for instance, gives a real interest rate equal to the rate of time preference plus twice the real growth rate—well above observed levels.

Other features of the infinite-horizon model make it unattractive for applied analysis. For instance, all agents must have equal rates of time preference and real growth rates for a steady state to exist, and the asset-to-income ratio is a random walk. From a policy perspective, the infinite-horizon model implies Ricardian equivalence, contrary to the evidence that spending is affected by the timing of government taxation.

\[ \text{B. Finite-Horizon Model} \]

We consider now a model in which agents have finite horizons, based on Blanchard (1985) and Yaari (1965). Individuals face a constant probability of death (\( \delta \)) in each period, giving

\(^{38}\) The modified golden rule is consistent with maximizing the sustainable level of consumption discounted at rate \( \rho \). As can be seen from equation (24), however, this outcome does not correspond to utility maximization except in the case where \( \sigma \) equals unity.

\(^{39}\) Hall (1988) found a value close to zero. More recently, Vissing-Jorgenson (2002) and Guvenen (2005) obtain higher values for households with positive assets, but still well below unity.
an expected lifetime of $\delta^{-1}$. The birth rate, $b$, then determines population growth. Individuals discount future income and utility at a rate that reflects the probability of death. The assets of those who die are assumed to be returned to a central insurance pool, and any debts are extinguished. The insurance pool charges an actuarially fair interest rate of $r + \delta$ to borrowers, and pays this rate to creditors. For households, then, the only relevant interest rate is $r + \delta$.

The probability of death appears in both the numerator and denominator of the Euler equation for the consumption path:

$$\frac{c_{t+1}}{c_t} = \left(\frac{1 + r + \delta}{1 + \rho + \delta}\right)^{\sigma}. \tag{25}$$

It also is added to the discount rate in the budget constraint. These changes have three implications. The first is that steady-state growth of individual consumption no longer needs to equal the growth rate of endowment income—population turnover generates a stable ratio of assets to income even if individuals accumulate or decumulate assets. The second is that the present value of individual endowment income is bounded when $r > g - \delta$, as opposed to $r > g$. Thus the risk-free real interest rate can be below the growth rate by a margin up to the probability of death, without creating opportunities for “Ponzi games.” The third is that the growth rate of the economy exceeds that of individual income when the population is growing, which has important implications for aggregate debt dynamics.

In this model, the real interest rate and the stock of financial assets are jointly determined. For instance, in a small open economy facing an exogenous world real interest rate, the ratio of assets to income is determined by the world interest rate; in a closed economy, the domestic real interest rate is determined jointly with the capital stock. This model has other desirable properties: Ricardian equivalence does not hold, and stable ratios of assets to income exist even when time preferences are heterogeneous.

Nevertheless, this “plain vanilla” version of the finite-horizon model still yields implausibly high real interest rates for values of $\sigma$ below unity. In particular, a high interest rate is needed in equation (25) to make the consumption path slope up sufficiently to keep pace with income growth. One way to generate a more plausible real interest rate is to assume a negative rate of time preference. Alternatively, we can appeal to the “relative income” hypothesis, which supposes that utility is derived, not from absolute consumption, but from its level relative to that of other individuals. The utility function then becomes:

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40 The idea that utility is derived from relative versus absolute consumption is not new; see Veblen (1899). Empirically, the hypothesis is consistent with evidence on subjective well-being. Diener and others (1999) conclude that there is no link between secular economic growth and well-being, although cyclical fluctuations in income may have some impact. Oswald (1997) finds that measured well-being has risen only marginally in the U.S. during the post-war period in spite of a three-fold increase in per capita income.
where $\bar{c}$ is the average level of consumption. The associated Euler equation is:

$$
\frac{c_{t+1}}{c_t} = \left(\frac{\bar{c}_{t+1}}{\bar{c}_t}\right)^{-\sigma} \left(\frac{1+r_t+\delta}{1+\rho+\delta}\right)^\sigma.
$$

(26)

The presence of growth in average consumption on the right-hand side of equation (26) generates an inherent upward slope to the consumption path, as households increase consumption over time to maintain the same relative standard of living. When average consumption is expected to rise at the rate of income growth, the Euler equation becomes:

$$
\frac{c_{t+1}}{c_t} = (1+g)^{-\sigma} \left(\frac{1+r_t+\delta}{1+\rho+\delta}\right)^\sigma.
$$

Therefore,

$$
\Rightarrow \frac{c_{t+1}}{c_t} = (1+g) \left(\frac{1+r_t+\delta}{(1+\rho+\delta)(1+g)}\right)^\sigma.
$$

The relative income hypothesis thus “builds in” consumption growth that matches income growth, even for values of $\sigma$ well below one, reducing the real interest rate that is consistent with a given real growth rate. The parameterization of the simulation model, for instance, generates a baseline real interest rate of 3¼ percent with a real growth rate of 3 percent.
REFERENCES


