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On the Properties of Various Estimators for Fiscal Reaction Functions

Oya Celasun and Joong Shik Kang

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Research Department

On the Properties of Various Estimators for Fiscal Reaction Functions

Prepared by Oya Celasun and Joong Shik Kang[1]

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Abstract

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This paper evaluates the bias of the least-squares-with-dummy-variables (LSDV) method in fiscal reaction function estimations. A growing number of studies estimate fiscal policy reaction functions—that is, relationships between the primary fiscal balance and its determinants, including public debt and the output gap. A previously unexplored methodological issue in these estimations is that lagged debt is not a strictly exogenous variable, which biases the LSDV estimator in short panels. We derive the bias analytically to understand its determinants and run Monte Carlo simulations to assess its likely size in empirical work. We find the bias to be smaller than the bias of the LSDV estimator in a comparable autoregressive dynamic panel model and show the LSDV method to outperform a number of alternatives in estimating fiscal reaction functions.

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Author(s) E-Mail Address: ocelasun@imf.org; jskang@wisc.edu

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I. INTRODUCTION

It is well known that the least-squares-with-dummy-variables (LSDV) method leads to biased estimates of dynamic panel data models when samples have a short time dimension. Although the properties of the LSDV and various alternative estimators have been the subject of a sizable body of work in the context of autoregressive distributed lag models, the case of other types of dynamic models have not been studied in detail. This is a gap in the literature, since there are a host of econometric models in which there is a dynamic relationship between the dependent variable and the explanatory variables—such as consumption and wealth, investment and the stock of capital, or the fiscal balance and public debt—where the use of panel data methods can be useful.

This paper focuses on estimating fiscal reaction functions from panel data. There has been an increased interest in characterizing fiscal policy behavior by estimating relationships between the primary (non-interest) fiscal balance and its determinants—including the level of public debt and the output gap.[2] Many studies in the literature use panel data techniques, given the typically short time dimension of data on public debt and deficits.[3] Two key questions that these studies seek to answer are whether (i) fiscal policy satisfies the intertemporal budget constraint and (ii) whether it is countercyclical. A positive estimated response of the primary balance to increases in the public debt suggests that fiscal behavior satisfies the intertemporal budget constraint, and hence the long-run solvency criterion (Bohn, 1998). Likewise, a positive response of cyclically-adjusted primary fiscal balances to the gap between actual and potential output indicates that fiscal policy is countercyclical (Galí and Perotti, 2003).

Most studies in the literature estimate fiscal reaction functions using the LSDV method. The commonly estimated fiscal reaction function specification is:

$$p_{i,t} = \alpha + \rho d_{i,t-1} + x'_{i,t}\beta + \eta_i + \varepsilon_{i,t}, \quad t = 1,...,T, \quad i = 1,...,N, \tag{1}$$

where $p_{i,t}$ is the primary balance as a share of GDP in country i and time t, $d_{i,t-1}$ is the level of outstanding public debt as a share of GDP, $x_{i,t}$ is a vector of explanatory variables, including the output gap, η_i are unobserved individual effects, and $\varepsilon_{i,t}$ is a time and country specific disturbance. The evolution of debt is given by the debt-dynamics equation:

[2] See Abiad and Baig (2005), Abiad and Ostry (2005), Celasun, Debrun and Ostry (2006), Favero (2002), Galí and Perotti (2003), IMF (2003, 2004), Bohn (1998), Mélitz (1997), and Wyplosz (2005), among others.

[3] For instance, in developing economies, where a growing interest in debt sustainability has resulted in many studies of fiscal policy behavior, the number of years for which public debt data are available is typically less than 15, preventing reliable estimations of country-specific fiscal behavior. Another reason for using panel data has been interest in common fiscal policy trends in groups of countries that have adopted joint policy frameworks, such as the European Economic and Monetary Union (EMU) countries.

$$d_{i,t-1} = f_{i,t-1} d_{i,t-2} - p_{i,t-1} + b_{i,t-1},$$

where $f_{i,t} = (1+r_{i,t})/(1+g_{i,t})$ is the ratio of the gross real interest rate to the gross real GDP growth rate and $b_{i,t} \sim N(0,\sigma_b^2)$ are shocks to public debt that are not captured in the government's primary budget.[4] Using equation (1) and iterating backward, the debt-dynamics equation yields the MA representation of $d_{i,t-1}$:

$$d_{i,t-1} = \sum\nolimits_{j=0}^{\infty} \left(\prod\nolimits_{k=0}^{j} \left(f_{i,t-1-k} - \rho \right) \right) \left(b_{i,t-1-j} - x'_{i,t-1-j}\beta - \eta_i - \varepsilon_{i,t-1-j} \right). \tag{2}$$

Since the unobserved country-specific primary-surplus effects η_i are correlated with lagged debt, $d_{i,t-1}$, the assumption of random-effects is inappropriate for estimating equation (1). Hence, equation (1) is typically estimated using the LSDV method to account for the fixed differences in primary balances across countries.[5]

A previously unexplored methodological issue in fiscal reaction estimations relates to the dependence of lagged debt, $d_{i,t-1}$, which partly represents the accumulation of past deficits, on lagged disturbances to the primary balance, $\varepsilon_{i,t-s}$, $s > 1$. Such a correlation between a regressor and the past shocks to the dependent variable—which leads to the violation of the strict exogeneity assumption—is known to bias the LSDV estimator when the time dimension of the sample is small.[6] The standard reference for this bias is the much-studied first-order autoregressive (AR(1)) dynamic panel data model:

$$p_{i,t} = \alpha + \rho p_{i,t-1} + x'_{i,t}\beta + \eta_i + \varepsilon_{i,t}, \qquad |\rho| < 1, \qquad t = 1,...,T, \quad i = 1,...,N. \tag{3}$$

In this model, the lagged dependent variable $p_{i,t-1}$ can be expressed as

$$p_{i,t-1} = \sum\nolimits_{j=0}^{\infty} \rho^j (x'_{i,t-1-j}\beta + \eta_i + \varepsilon_{i,t-1-j}). \tag{4}$$

Thus, like lagged debt in equation (1), it is correlated with past idiosyncratic disturbances, $\varepsilon_{i,t-s}$, $s > 1$. Nickell (1981) has shown this correlation to lead to a negative expected bias in

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[4] Shocks to the public debt that are not accounted for in the primary budget could arise from privatization receipts, below-the-line expenditures such as banking sector bailouts, or the effects of exchange rate movements on debt valuation.

[5] Galí and Perotti (2003) use country dummies and instrument for the output gap, which is likely to be endogenous to contemporaneous primary surplus disturbances. Celasun, Debrun, and Ostry (2005) present results from an array of estimations, including those where both debt and the output gap are instrumented.

[6] Strict exogeneity necessitates that a regressor is orthogonal to past, present, and future disturbances to the dependent variable. See Chapter 10 of Wooldridge (2001) for a discussion of this assumption.

the LSDV estimator of ρ—the coefficient on $p_{i,t-1}$, which violates the strict exogeneity assumption in equation (3)—in samples with a small time dimension.[7] Monte Carlo studies find the LSDV bias on ρ to be large when equation (3) is estimated using samples with $T < 30$ (Kiviet, 1995; Judson and Owen, 1999). The bias arising from the correlation between past primary surplus shocks and debt is potentially important in LSDV estimations of (1) for debt-sustainability assessments, since it is the coefficient on debt—which is not strictly exogenous—that determines if fiscal policy satisfies the intertemporal budget constraint.

This paper compares the LSDV bias in estimating the fiscal reaction function model (1) with the well known LSDV bias in the context of the standard AR(1) model (3). It also evaluates the expected biases of various alternative estimators that could be used to estimate fiscal reaction functions. The common feature of the fiscal reaction function and the AR(1) models is the violation of the strict exogeneity assumption (by $d_{i,t-1}$ in equation (1) and $p_{i,t-1}$ in equation (3)). The differences are in functional form: (i) whereas the lagged dependent variable $p_{i,t-1}$ in (3) depends solely on past determinants of primary surpluses by equation (4), there is a potential for exogenous shocks, $b_{i,t-s}$, or shocks to $r_{i,t-s}$ and $g_{i,t-s}$, to affect $d_{i,t-1}$ independently of past primary surplus behavior; (ii) idiosyncratic disturbances $\varepsilon_{i,t-s}$, $s > 1$, enter the MA representation of $d_{i,t-1}$ with a negative sign in equation (2), while they enter the MA representation of $p_{i,t-1}$ positively in equation (4); and (iii) in the MA representation of $d_{i,t-1}$ in equation (2), the weights in the summation are not the powers of ρ as they are in the MA representation of $p_{i,t-1}$ given in equation (4). All these factors are likely to affect the relative size and direction of biases in estimating equations (1) and (3).

The next section analytically derives the asymptotic biases of the ordinary-least-squares (OLS) and LSDV estimators of ρ in estimating fiscal reaction functions from short panels. The derivations indicate that the use of OLS and LSDV methods on equation (1) would be expected to lead to negative and positive biases, respectively, on the estimates of ρ—the opposite signs of their corresponding biases in estimating ρ in the AR(1) model (3). Using Monte Carlo simulations, Section III then analyzes the expected size of the OLS and LSDV biases in estimations of equations (1) and (3). The results indicate the expected biases are smaller in the fiscal reaction function model than in the AR(1) model. Section III also explores, using Monte Carlo simulations, how the biases in estimating equation (1) are affected by altering various aspects of the fiscal reaction function model and compares the performance of various estimators. Section IV concludes.

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[7] The use of country dummies is equivalent to running a regression on data that are expressed in deviations from country-specific means. Although this controls for the correlation between lagged debt and the fixed effects, it leads to a correlation between demeaned-debt and the demeaned-error term in short samples, exerting a bias on ρ. (See Bond, 2002, for an intuitive discussion.)

II. Biases of Ordinary-Least-Squares (OLS) and Least-Squares-with-Dummy Variables (LSDV) Estimators: Analytical Solutions

We consider the generalized form of equation (1):

$$p_{i,t} = \alpha + \rho d_{i,t-1} + \sum_{k=1}^{K} \gamma_k y_{i,k,t} + \sum_{j=1}^{J} \beta_j x_{i,j,t} + \eta_i + \varepsilon_{i,t}, \qquad i = 1,...,N, t = 1,...,T \tag{5}$$

where $p_{i,t}$ is the primary balance to GDP ratio; $d_{i,t-1}$ is the debt-GDP ratio, defined as in equation (2); $y_{i,k,t}$ and $x_{i,j,t}$ are endogenous and exogenous variables, respectively, with unknown coefficients γ_k and β_j; $\eta_i \sim N(0,\sigma_\eta^2)$ where $\sigma_\eta^2 \geq 0$ is an unobserved fixed effect; and $\varepsilon_{i,t} \sim N(0,\sigma_\varepsilon^2)$, with $\sigma_\varepsilon^2 > 0$, is a disturbance term. It is further assumed that (i) $E(\varepsilon_{i,t}\varepsilon_{j,s})$ for $i \neq j$ or $t \neq s$, (ii) $E(\eta_i\eta_j) = 0$ for $i \neq j$, (iii) $E(\eta_i\varepsilon_{j,t}) = 0 \;\; \forall i, j, t$, (iv) $E(x_{i,k,t}\eta_j) = E(y_{i,k,t}\eta_j) = 0 \;\; \forall i, j, k, t$, (v) $E(x_{i,k,t}\varepsilon_{j,s}) = 0 \;\; \forall i, j, k, t, s$, (vi) $E(y_{i,k,t}\varepsilon_{j,s}) = 0 \;\; \forall i, j, k, t, s \neq t$, (vii) $E\left(b_{i,t}\varepsilon_{j,s}\right) = E\left(b_{i,t}\eta_j\right) = 0 \;\forall i, j, t, s$. Assumption (vi) implies that $y_{i,k,t}$ could potentially be correlated with the contemporaneous idiosyncratic disturbance, $\varepsilon_{i,t}$, while (v) indicates that $x_{i,k,t}$ is strictly exogenous. Note that equation (2) indicates that $E(d_{i,t}\varepsilon_{i,s}) < 0$ for $s < t$, and assumptions (i), (iii)-(vii) indicate that $E(d_{i,t}\varepsilon_{i,s}) = 0$ for $s \geq t$. Two other key assumptions in the analytical derivations are that $f_{i,t} \;\; \forall i, t$ are deterministic constants and that $f_{i,t} > \rho \;\; \forall i, t$. The first assumption is made for simplicity, the second assumption is plausible since $f_{i,t}$ is typically close to or slightly larger than one, while most existing estimates suggest that ρ, the primary surplus response to debt accumulation, is less than 0.10. We relax the assumption that $f_{i,t}$ are deterministic constants in the Monte Carlo simulations.

It is well-known that the OLS estimator is inconsistent in the presence of individual-specific fixed effects in dynamic panel data models. In the estimation of equation (5), the presence of η_i would be expected to exert a downward bias on the OLS estimate of ρ since the individual effects η_i and $d_{i,t-1}$ are negatively correlated by equation (2). An analytical proof of the negative bias of the OLS estimator is provided in Appendix A for the case where $\gamma_k = \beta_j = 0 \;\; \forall j, k$.

The LSDV method is equivalent to applying the OLS estimator to data that is expressed in deviations from country-specific means. To analyze the asymptotic LSDV bias—where we refer to an expression of the form $\text{plim}_{N\to\infty}(\hat{\rho} - \rho)$ as a bias in the remainder of the paper—we start by defining the following matrices, where each variable is expressed in deviations from its country-specific means over the time horizon:

$$\tilde{p}_t = [p_{i,t} - p_{i.}], \quad N \times 1 \text{ vector},$$
$$\tilde{d}_{t-1} = [d_{i,t-1} - d_{i.-1}], \quad N \times 1 \text{ vector},$$
$$\tilde{Y}_t = [y_{i,k,t} - y_{i,k.}], \quad N \times K \text{ matrix},$$
$$\tilde{X}_t = [x_{i,j,t} - x_{i,j.}], \quad N \times J \text{ matrix},$$
$$\tilde{\varepsilon}_t = [\varepsilon_{i,t} - \varepsilon_{i.}], \quad N \times 1 \text{ vector},$$
$$\gamma = [\gamma_1 ... \gamma_K] \quad K \times 1 \text{ vector},$$
$$\beta = [\beta_1 ... \beta_J] \quad J \times 1 \text{ vector},$$

where for any variable $z_{i,t}$, $z_{i.} \equiv (1/T)\sum_{t=1}^{T} z_{i,t}$ and $z_{i.-1} \equiv (1/T)\sum_{t=0}^{T-1} z_{i,t}$. Using these matrices, we then rewrite equation (5) in deviation form as follows:

$$\tilde{p}_t = \rho \tilde{d}_{t-1} + \tilde{Y}_t \gamma + \tilde{X}_t \beta + \tilde{\varepsilon}_t, \quad t=1,...,T,$$

and stack these equations over time horizons:

$$\tilde{p} = \rho \tilde{d}_{-1} + \tilde{Y}\gamma + \tilde{X}\beta + \tilde{\varepsilon} = \tilde{Z}\delta + \tilde{X}\beta + \tilde{\varepsilon},$$

where $\tilde{p} = [\tilde{p}_1 \ ... \ \tilde{p}_T]'$, $\tilde{d}_{-1} = [\tilde{d}_0 \ ... \ \tilde{d}_{T-1}]'$, and $\tilde{\varepsilon} = [\tilde{\varepsilon}_1 \ ... \ \tilde{\varepsilon}_T]'$ are $NT\times 1$ vectors, $\tilde{Y} = [\tilde{Y}_1 \ ... \ \tilde{Y}_T]'$ is a $NT\times K$ matrix, $\tilde{X} = [\tilde{X}_1 \ ... \ \tilde{X}_T]'$ is a $NT\times J$ matrix, $\tilde{Z} = [\tilde{d}_{-1} \ \tilde{Y}]$ is a $NT\times(K+1)$ matrix, and $\delta = [\rho \ \gamma]'$ is a $(K+1)\times 1$ vector. Further, we define $M \equiv \mathrm{I} - \tilde{X}\left(\tilde{X}'\tilde{X}\right)^{-1}\tilde{X}'$, where I is the $J \times J$ identity matrix. The LSDV estimator for the coefficient vector δ is given as:

$$\tilde{\delta} = \delta + \left(\tilde{Z}'M\tilde{Z}\right)^{-1}\tilde{Z}'M\tilde{\varepsilon}.$$

We compute the asymptotic bias by taking probability limits as $N \to \infty$:

$$\operatorname*{plim}_{N\to\infty}\left(\tilde{\delta} - \delta\right) = \operatorname*{plim}_{N\to\infty}\left(\tilde{Z}'\tilde{Z} - \tilde{Z}'\tilde{X}\left(\tilde{X}'\tilde{X}\right)^{-1}\tilde{X}'\tilde{Z}\right)^{-1} \operatorname*{plim}_{N\to\infty}\tilde{Z}'\tilde{\varepsilon} \tag{6}$$

since $E\left(x_{i,k,t}\varepsilon_{j,s}\right) = 0 \ \ \forall i, j, k, t, s$. We now study the sign of the bias expression given in equation (6) under alternative assumptions about the model. We first consider the simplest benchmark case where there are no regressors other than $d_{i,t-1}$ in equation (5) and no independent shocks that affect public debt in equation (2), $\sigma_b^2 = 0$. Second we consider the case where there are exogenous shocks to public debt, $\sigma_b^2 > 0$. Third we consider the effect of exogenous regressors, and fourth we consider the implications of having endogenous regressors in the equation.

Case 1: $\gamma = \beta = 0$ and $\sigma_b^2 = 0$.

In this benchmark specification we assume that there are no regressors other than $d_{i,t-1}$ and there are no independent shocks that affect debt, i.e $\sigma_b^2 = 0$ in equation (2). In this simplest case, $\delta = \rho$, $\tilde{Z} = \tilde{d}_{-1}$, and $M = \mathrm{I}$. Then the policy reaction function and the debt dynamics equation can be rewritten as follows:

$$p_{i,t} = \alpha + \rho d_{i,t-1} + \eta_i + \varepsilon_{i,t}\,, \quad t = 1, 2, \ldots, T \text{ and } i = 1, 2, \ldots, N, \tag{7}$$
$$d_{i,t-1} = f_{i,t-1} d_{i,t-2} - p_{i,t-1}$$

The LSDV estimator of ρ is:

$$\hat{\rho} = \frac{\sum_{t=1}^{T}\sum_{i=1}^{N}\left(d_{i,t-1} - d_{i,-1}\right)\left(p_{i,t} - p_{i.}\right)}{\sum_{t=1}^{T}\sum_{i=1}^{N}\left(d_{i,t-1} - d_{i,-1}\right)^2} \tag{8}$$

When T is kept fixed, the asymptotic bias of this estimator is given by:

$$\underset{N\to\infty}{p\lim}(\hat{\rho} - \rho) = \frac{\sum_{t=1}^{T} \underset{N\to\infty}{p\lim} \frac{1}{N} \sum_{i=1}^{N}\left(d_{i,t-1} - d_{i,-1}\right)\left(\varepsilon_{i,t} - \varepsilon_{i.}\right)}{\sum_{t=1}^{T} \underset{N\to\infty}{p\lim} \frac{1}{N} \sum_{i=1}^{N}\left(d_{i,t-1} - d_{i,-1}\right)^2} = \frac{\sum_{t=1}^{T} E_i\left(d_{i,t-1} - d_{i,-1}\right)\left(\varepsilon_{i,t} - \varepsilon_{i.}\right)}{\sum_{t=1}^{T} E_i\left(d_{i,t-1} - d_{i,-1}\right)^2}, \tag{9}$$

where E_i represents the expectation of a random variable taken across all countries for a fixed time period. Note that in the numerator of Equation (9), $(d_{i,t-1} - d_{i,-1})$ and $(\varepsilon_{i,t} - \varepsilon_{i.})$ are not orthogonal. Given equation (2), $d_{i,t-1}$ is correlated positively with $-\varepsilon_{i,t-1}/T$ in $-\varepsilon_{i.}$, $\varepsilon_{i,t}$ is positively correlated with $-d_{i,t}/T$ in $-d_{i,-1}$, and $-\varepsilon_{i,t}/T$ in $-\varepsilon_{i.}$ is negatively correlated with the terms $-d_{i,t+s}/T$, s = 0,...,T-1-t in $-d_{i,-1}$. The magnitudes of these correlations and the expected bias decline with *T*, the time-dimension of the sample. Since the denominator of (9) is always positive by construction, the direction of the bias is determined by the sign of the numerator. As we are also interested in the size of the bias, however, we will also derive the analytical expression for the denominator. We first rewrite $d_{i,t-1}$ and $d_{i,-1}$ in MA representation. By substituting equation (7) in the debt dynamics equation and iterating, we get the MA representation for $d_{i,t}$:

$$d_{i,t} = -\left(\sum_{j=0}^{\infty} A_{i,t,j}\right)\eta_i - \left(\sum_{j=0}^{\infty} A_{i,t,j}\varepsilon_{i,t-j}\right), \tag{10}$$

where $A_{i,t,0}=1$ and $A_{i,t,k}=\prod_{l=o}^{k-1}\left(f_{i,t-l}-\rho\right)$ for $k\geq 1$ and the constant α is omitted without loss of generality. We then obtain the MA representation for $d_{i,t-1}$ and $d_{i,-1}$ as:

$$d_{i,t-1}=-\left(\sum_{j=0}^{\infty}A_{i,t-1,j}\right)\eta_i-\left(\sum_{j=0}^{\infty}A_{i,t-1,j}\varepsilon_{i,t-j-1}\right)\text{, and} \tag{11}$$

$$d_{i,-1}=\frac{1}{T}\sum_{t=0}^{T-1}d_{i,t}=\frac{1}{T}\sum_{t=1}^{T}d_{i,t-1}=A_i-\frac{1}{T}\sum_{t=1}^{T}\sum_{j=0}^{\infty}A_{i,t-1,j}\varepsilon_{i,t-j-1}\,, \tag{12}$$

where $A_i=\frac{1}{T}\sum_{t=1}^{T}A_{i,t-1}$ and $A_{i,t-1}=-\left(\sum_{j=0}^{\infty}A_{i,t-1,j}\right)\eta_i$.

In Appendix I we prove that for the benchmark policy reaction function (7), the bias of the LSDV estimator, $\underset{N\to\infty}{p\lim}(\hat{\rho}-\rho)$, is positive and can be expressed as:

$$\frac{\sum_{t=1}^{T}E_i\left(d_{i,t-1}-d_{i,-1}\right)\left(\varepsilon_{i,t}-\varepsilon_i\right)}{\sum_{t=1}^{T}E_i\left(d_{i,t-1}-d_{i,-1}\right)^2}=\frac{\frac{\sigma_\varepsilon^2}{T}\sum_{t=1}^{T}\sum_{j=0}^{T-t-1}A_{i,t+j,j}}{\sigma_\varepsilon^2\left(Q+R\right)+\sigma_\eta^2 S}>0 \tag{13}$$

where

$$Q=\sum_{t=1}^{T}\left(\sum_{j=0}^{\infty}A_{i,t-1,j}{}^2-\frac{2}{T}\left\{\sum_{j=0}^{t-1}\left(A_{i,t-1,j}\sum_{k=0}^{T-t+j}A_{i,t-1-j+k,k}\right)+\sum_{j=1}^{\infty}\left(A_{i,t-1,t+j-1}\sum_{k=1}^{T}A_{i,T-k,T-k+j}\right)\right\}\right)\,,$$

$$R=\frac{1}{T}\left\{\sum_{j=0}^{T-2}\left(\sum_{k=0}^{j}A_{i,T-1-j+k,k}\right)^2+\sum_{j=0}^{\infty}\left(\sum_{k=1}^{T}A_{i,T-k,T-k+j}\right)^2\right\}\text{, and}$$

$$S=\sum_{t=1}^{T}\left\{-\left(\sum_{j=0}^{\infty}A_{i,t-1,j}\right)+\frac{1}{T}\sum_{s=1}^{T}\sum_{j=0}^{\infty}A_{i,s-1,j}\right\}^2$$

In contrast to the LSDV bias on the coefficient of the lagged dependent variable in the AR(1) model, the LSDV bias on ρ —the coefficient of lagged debt in equation (7)—is positive and decreasing in σ_η^2 .[8]

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[8] By contrast the LSDV bias on ρ in the AR(1) model is independent of σ_η^2. In the AR(1) model, the weights on the distributed lagged terms in the MA representation of the lagged dependent variable correspond to the powers of the autoregressive parameter ρ, which leads σ_η^2 to drop from the bias expression (Nickell, 1981).

Case 2: $\gamma = \beta = 0$ and $\sigma_b^2 > 0$.
As in Case 1, $\delta = \rho$, $\tilde{Z} = \tilde{d}_{-1}$, and $M = \mathrm{I}$. However, since now $b_{i,t}$ is included in the debt accumulation equation (2), the MA representation of $d_{i,t}$ becomes:

$$d_{i,t} = -\left(\sum_{j=0}^{\infty} A_{i,t,j}\right)\eta_i - \left(\sum_{j=0}^{\infty} A_{i,t,j}\varepsilon_{i,t-j}\right) + \left(\sum_{j=0}^{\infty} A_{i,t,j}b_{i,t-j}\right).$$

Following the same steps in the proof for Case 1, we obtain the asymptotic bias as:

$$\begin{aligned} \underset{N\to\infty}{p\lim}\left(\hat{\rho}-\rho\right) &= \underset{N\to\infty}{p\lim}\left(\tilde{d}_{-1}'\tilde{d}_{-1}\right)^{-1} \underset{N\to\infty}{p\lim}\,\tilde{d}_{-1}'\tilde{\varepsilon} \\ &= \frac{\sum_{t=1}^{T} E_i\left(d_{i,t-1}-d_{i,-1}\right)\left(\varepsilon_{i,t}-\varepsilon_i\right)}{\sum_{t=1}^{T} E_i\left(d_{i,t-1}-d_{i,-1}\right)^2} = \frac{\frac{\sigma_\varepsilon^2}{T}\sum_{t=1}^{T}\sum_{j=0}^{T-t-1} A_{i,t+j,j}}{\left(\sigma_\varepsilon^2+\sigma_b^2\right)\left(Q+R\right)+\sigma_\eta^2 S}, \end{aligned} \tag{14}$$

since $E_i\left(b_{i,t}\varepsilon_{j,s}\right) = E_i\left(b_{i,t}\eta_j\right) = 0 \ \forall i, j, t, s$ by assumption (vii). Notice that the only difference from the bias in Case 1 is the addition of $\sigma_b^2\left(Q+R\right)$ to the denominator. Thus, if $Q+R>0$, the bias remains positive in Case 2 and equation (14) would then imply that the larger is σ_b^2, the larger is the denominator, therefore the smaller is the bias. Intuitively, a larger σ_b^2 implies that past primary balance shocks, $\varepsilon_{i,t-s}$, $s > 1$, account for a smaller share of the total variance of debt, $d_{i,t-1}$. A larger σ_b^2 is therefore likely to reduce the magnitude of the LSDV bias, which is increasing in the correlation between past primary balance shocks and debt. However, since it can not be shown analytically that $Q+R>0$, we study the implications of $\sigma_b^2 > 0$ in our Monte Carlo simulations.

Case 3: $\gamma = 0, \beta \neq 0$ and $\sigma_b^2 = 0$.
In this case, exogenous regressors are included in the model, so $\delta = \rho$, $\tilde{Z} = \tilde{d}_{-1}$, and $M = \mathrm{I} - \tilde{X}\left(\tilde{X}'\tilde{X}\right)^{-1}\tilde{X}'$. The bias is given as:

$$\begin{aligned} \underset{N\to\infty}{p\lim}\left(\hat{\rho}-\rho\right) &= \underset{N\to\infty}{p\lim}\left(\tilde{d}_{-1}'M\tilde{d}_{-1}\right)^{-1} \underset{N\to\infty}{p\lim}\,\tilde{d}_{-1}'\tilde{\varepsilon} \\ &= \underset{N\to\infty}{p\lim}\left(\tilde{d}_{-1}'\tilde{d}_{-1} - \tilde{d}_{-1}'\tilde{X}\left(\tilde{X}'\tilde{X}\right)^{-1}\tilde{X}'\tilde{d}_{-1}\right)^{-1} \underset{N\to\infty}{p\lim}\,\tilde{d}_{-1}'\tilde{\varepsilon}. \end{aligned}$$

The bias in this case is larger than in Case 1, as the inclusion of M <I reduces the denominator:

$$\underset{N\to\infty}{p\lim}\left(\tilde{d}_{-1}'\tilde{d}_{-1}-\tilde{d}_{-1}'\tilde{X}\left(\tilde{X}'\tilde{X}\right)^{-1}\tilde{X}'\tilde{d}_{-1}\right)<\underset{N\to\infty}{p\lim}\left(\tilde{d}_{-1}'\tilde{d}_{-1}\right),$$

since the matrix $\tilde{X}\left(\tilde{X}'\tilde{X}\right)^{-1}\tilde{X}'$ is positive semidefinite.

Case 4: $\gamma \neq 0, \beta = 0$ and $\sigma_b^2 = 0$.
In this case, endogenous regressors are included in the model. Using the formula for the inverse of block matrices, we can rewrite the bias expression in equation (6) as follows:

$$\begin{aligned}
\underset{N\to\infty}{p\lim}\left(\tilde{\delta}-\delta\right) &= \underset{N\to\infty}{p\lim}\left(\tilde{Z}'\tilde{Z}\right)^{-1}\underset{N\to\infty}{p\lim}\,\tilde{Z}'\tilde{\varepsilon} \\
&= \underset{N\to\infty}{p\lim}\begin{pmatrix}\tilde{d}_{-1}'\tilde{d}_{-1} & \tilde{d}_{-1}'\tilde{Y} \\ \tilde{Y}'\tilde{d}_{-1} & \tilde{Y}'\tilde{Y}\end{pmatrix}^{-1}\underset{N\to\infty}{p\lim}\begin{pmatrix}\tilde{d}_{-1}'\tilde{\varepsilon} \\ \tilde{Y}'\tilde{\varepsilon}\end{pmatrix} \\
&= \underset{N\to\infty}{p\lim}\begin{pmatrix}P^{-1} & -P^{-1}\tilde{d}_{-1}'\tilde{Y}\left(\tilde{Y}'\tilde{Y}\right)^{-1} \\ -\left(\tilde{Y}'\tilde{Y}\right)^{-1}\tilde{Y}'\tilde{d}_{-1}P^{-1} & \left(\tilde{Y}'\tilde{Y}\right)^{-1}(\mathrm{I}+\tilde{Y}'\tilde{d}_{-1}P^{-1}\tilde{d}_{-1}'\tilde{Y}\left(\tilde{Y}'\tilde{Y}\right)^{-1}\end{pmatrix} \\
&\quad\times\underset{N\to\infty}{p\lim}\begin{pmatrix}\tilde{d}_{-1}'\tilde{\varepsilon} \\ \tilde{Y}'\tilde{\varepsilon}\end{pmatrix},
\end{aligned}$$

where $P \equiv \tilde{d}_{-1}'\tilde{d}_{-1}-\tilde{d}_{-1}'\tilde{Y}\left(\tilde{Y}'\tilde{Y}\right)^{-1}\tilde{Y}'\tilde{d}_{-1}$. The asymptotic bias, $\underset{N\to\infty}{p\lim}\left(\hat{\rho}-\rho\right)$, can then be expressed as follows:

$$\underset{N\to\infty}{p\lim}\left(\hat{\rho}-\rho\right)=\underset{N\to\infty}{p\lim}\,P^{-1}\underset{N\to\infty}{p\lim}\,\tilde{d}_{-1}'\tilde{\varepsilon}-\underset{N\to\infty}{p\lim}\,P^{-1}\tilde{d}_{-1}'\tilde{Y}\left(\tilde{Y}'\tilde{Y}\right)^{-1}\underset{N\to\infty}{p\lim}\,\tilde{Y}'\tilde{\varepsilon}\,.$$

The first term of the above expression, $\underset{N\to\infty}{p\lim}\,P^{-1}\underset{N\to\infty}{p\lim}\,\tilde{d}_{-1}'\tilde{\varepsilon}$, is greater than the corresponding term in the bias expression for Case 1, $\underset{N\to\infty}{p\lim}(\tilde{d}_{-1}'\tilde{d}_{-1})^{-1}\underset{N\to\infty}{p\lim}\,\tilde{d}_{-1}'\tilde{\varepsilon}$, since $\tilde{Y}\left(\tilde{Y}'\tilde{Y}\right)^{-1}\tilde{Y}'$ is positive semi definite and $P<\tilde{d}_{-1}'\tilde{d}_{-1}$. The sign of the second term, $\underset{N\to\infty}{p\lim}\,P^{-1}\tilde{d}_{-1}'\tilde{Y}\left(\tilde{Y}'\tilde{Y}\right)^{-1}\tilde{Y}'\tilde{\varepsilon}$, is determined by the correlation structures between $y_{i,k,t}$, $d_{i,t-1}$ and $\varepsilon_{i,t}$. As a result, the presence of endogenous regressors has an ambiguous effect on the size and sign of the LSDV bias on ρ.

The four cases studied above suggest that the use of country-specific dummy variables in the estimation of equation (5) would be expected to exert a positive bias on the estimate of ρ when the time dimension of the panel is small. The presence of exogenous shocks to debt ($b_{i,t}$) is likely to reduce the magnitude of this bias, whereas the inclusion of exogenous

regressors in the fiscal reaction function increases the expected magnitude of the bias. It is not possible to analytically determine the effect of endogenous regressors on the direction or size of the LSDV bias. If such endogenous regressors are instrumented, however, the use of country dummies would still be expected to exert a positive bias on the estimate of ρ. In sum, the true value of ρ would be expected to lie between the downward-biased OLS estimate and the upward-biased estimate obtained using the LSDV method (with instrumentation of any potentially endogenous regressors).

In empirical estimations of the model, these complications that we have studied as different cases are likely to be present simultaneously. The next section uses Monte Carlo simulations to evaluate the size and direction of the expected bias under various complications to the model.

III. Monte Carlo Experiments

This section uses a Monte Carlo approach to assess the expected LSDV bias in estimating equation (5) under various model assumptions. Since the main point of reference for the potential LSDV bias in dynamic panel data models is the much-studied bias in the canonical AR(1) model, we first evaluate the biases on the parameter estimates of equation (5) against the benchmark of the biases in a comparable AR(1) model given in equation (3). We then examine how altering various aspects of equation (5) changes the biases of various estimators.

For the comparison of the LSDV bias in equations (3) and (5), we try to keep the models as similar as possible. In both equations, we assume that there is only one exogenous explanatory variable $x_{i,t}$, generated by the following process:

$$x_{i,t} = \lambda x_{i,t-1} + \xi_{i,t} \qquad \xi_{i,t} \sim N(0, \sigma_{\xi}^2). \tag{15}$$

Kiviet (1995), who evaluates alternative estimators for the AR(1) model and also assumes that $x_{i,t}$ is generated by equation (15), shows that the LSDV bias in estimating equation (3) depends on the signal-to-noise ratio of the relationship, defined as:

$$\begin{aligned} \sigma_s^2 &= \operatorname{var}\left(p_{i,t} - \frac{1}{1-\rho}\eta_i - \varepsilon_{i,t}\right) \\ &= \beta^2 \sigma_{\xi}^2 \left[1 + \frac{(\rho+\lambda)^2}{1+\rho\lambda}[\rho\lambda - 1] - (\rho\lambda)^2\right]^{-1} + \frac{\rho^2}{1-\rho^2}\sigma_{\varepsilon}^2. \end{aligned} \tag{16}$$

In the case of the AR(1) model, the higher is σ_s^2, the more powerful is $x_{i,t}$ in explaining $p_{i,t}$. Kiviet (1995) shows that the bias of the LSDV estimator is decreasing in the signal-to-noise ratio. We evaluate the LSDV bias in estimating equations (3) and (5) with equal signal-to-noise ratios so that the difference in expected biases can be attributed mainly to differences in the functional form of the two specifications. For equation (5) we assume that $f_{i,t} = f$ $\forall i,t$ for simplicity and we define the signal-to-noise ratio as follows:

$$\sigma_s^2 = \mathrm{var}(p_{it} - \frac{1-f}{1-f+\rho}\eta_i - \varepsilon_{it}) = \mathrm{var}\left[\rho\sum_{j=0}^{\infty}(f-\rho)^j[b_{t-1-j} - \beta x_{t-1-j} - \varepsilon_{t-1-j}] + \beta x_{it}\right]$$
$$= \rho^2\left[\frac{\sigma_b^2}{1-\theta^2} + \beta^2\sigma_\xi^2[1 - \frac{(\theta+\lambda)^2}{(1+\theta\lambda)}(1-\theta\lambda) - (\theta\lambda)^2]^{-1} + \frac{\sigma_\varepsilon^2}{1-\theta^2}\right] + \beta^2\frac{\sigma_\xi^2}{1-\lambda^2} \quad (17)$$

where $\theta = f - \rho$. As in Kiviet's definition, the signal-to-noise ratio of the relationship is the variance of the dependent variable attributable to factors other than the *cumulative* impact of the fixed-effect, η_i, and the contemporaneous idiosyncratic disturbance, ε_{it}.

In our first set of simulations, we generate data using the processes in equations (3) and (5), where (2) is used to generate data for $d_{i,t-1}$. We follow broadly the elements of Kiviet's (1995) simulation design. We allow ρ to alternate between 0.1 and 0.4, in the lower range of Kiviet's parameters, which are more realistic for fiscal reaction functions. In each case, we run simulations for two values of the signal-to-noise ratio, σ_s^2: 2 and 8. λ is set to 0.5 in all simulations. β is set to $1-\rho$ so that in equation (3) a change in ρ only affects the short run dynamic relationship between $p_{i,t}$ and $x_{i,t}$, and not the steady-state relationship. For equation (5), we set $f_{i,t} = 1.05 \;\; \forall i,t$ for simplicity. As in one of Kiviet's (1995) simulation designs we set $\sigma_\eta = \sigma_\varepsilon(1-\rho)$ for equation (3) and $\sigma_\eta = \dfrac{\sigma_\varepsilon(1-f+\rho)}{1-f}$ for equation (5), so that the idiosyncratic disturbance and the individual effect contribute equally to the variance of $p_{i,t}$ in both equations. Thus, given the choices for ρ, σ_ε^2, σ_s^2, β, and λ, and setting $\sigma_b^2 = 0$ for comparability, the only remaining parameter of the model—σ_ξ^2—is determined by equations (16) and (17). We set $N = 100$, and run 1000 simulations for different time series lengths: *T* = 5, 10, 20, and 30.[9] For each run, we estimate equations (3) and (5) using the OLS and LSDV estimators, and report the average of the estimated parameters and standard errors across the 1000 simulation runs.

The results are summarized in Table 1. They confirm that the OLS estimator is upward biased and the LSDV estimator is downward biased for ρ in the AR(1) model (3). By contrast, and these estimators lead to negatively and positively biased estimates of ρ, respectively, in the fiscal reaction function model (5).[10] While in both equations, the LSDV bias declines with the signal-to-noise ratio and with *T*, the LSDV biases in equation (5) are smaller in absolute value than they are in equation (3) for a

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[9] We set $d_{i,0} = x_{i,0} = 0$ and discard the first 51 observations of the series.

[10] Kiviet (1995) investigates samples of size *T*=3, 6. Our estimated bias magnitudes for equation (3) when *T*=5 are close to those reported in Kiviet (1995) for *T*=6.

Table 1. Estimates of Equation (3)

						OLS					
							$\hat{\rho}$			$\hat{\beta}$	
M	N	T	ρ	β	σ_s^2	Estimate	Bias	S.E.	Estimate	Bias	S.E.
1000	100	5	0.1	0.9	2	0.4236	0.3236	0.0402	0.6589	-0.2411	0.0569
1000	100	5	0.1	0.9	8	0.2842	0.1842	0.0322	0.7627	-0.1373	0.0351
1000	100	5	0.4	0.6	2	0.5945	0.1945	0.0322	0.4735	-0.1265	0.0465
1000	100	5	0.4	0.6	8	0.5106	0.1106	0.0264	0.5280	-0.0720	0.0269
1000	100	10	0.1	0.9	2	0.4211	0.3211	0.0357	0.6565	-0.2435	0.0562
1000	100	10	0.1	0.9	8	0.2802	0.1802	0.0281	0.7633	-0.1367	0.0309
1000	100	10	0.4	0.6	2	0.5925	0.1925	0.0274	0.4705	-0.1295	0.0406
1000	100	10	0.4	0.6	8	0.5068	0.1068	0.0224	0.5281	-0.0719	0.0233
1000	100	20	0.1	0.9	2	0.4218	0.3218	0.0332	0.6622	-0.2378	0.0456
1000	100	20	0.1	0.9	8	0.2808	0.1808	0.0259	0.7663	-0.1337	0.0281
1000	100	20	0.4	0.6	2	0.5935	0.1935	0.0243	0.4742	-0.1258	0.0366
1000	100	20	0.4	0.6	8	0.5078	0.1078	0.0200	0.5298	-0.0702	0.0210
1000	100	30	0.1	0.9	2	0.4204	0.3204	0.0324	0.6631	-0.2369	0.0425
1000	100	30	0.1	0.9	8	0.2790	0.1790	0.0249	0.7676	-0.1324	0.0264
1000	100	30	0.4	0.6	2	0.5927	0.1927	0.0233	0.4743	-0.1257	0.0341
1000	100	30	0.4	0.6	8	0.5067	0.1067	0.0187	0.5304	-0.0696	0.0198

						LSDV					
							$\hat{\rho}$			$\hat{\beta}$	
M	N	T	ρ	β	σ_s^2	Estimate	Bias	S.E.	Estimate	Bias	S.E.
1000	100	5	0.1	0.9	2	-0.0327	-0.1327	0.0339	0.9430	0.0430	0.0701
1000	100	5	0.1	0.9	8	0.0346	-0.0654	0.0282	0.9212	0.0212	0.0351
1000	100	5	0.4	0.6	2	0.1840	-0.2160	0.0386	0.6421	0.0421	0.0683
1000	100	5	0.4	0.6	8	0.2851	-0.1149	0.0317	0.6224	0.0224	0.0323
1000	100	10	0.1	0.9	2	0.0369	-0.0631	0.0237	0.9309	0.0309	0.0450
1000	100	10	0.1	0.9	8	0.0705	-0.0295	0.0199	0.9144	0.0144	0.0238
1000	100	10	0.4	0.6	2	0.2926	-0.1074	0.0261	0.6362	0.0362	0.0439
1000	100	10	0.4	0.6	8	0.3473	-0.0527	0.0211	0.6177	0.0177	0.0217
1000	100	20	0.1	0.9	2	0.0657	-0.0343	0.0207	0.9210	0.0210	0.0387
1000	100	20	0.1	0.9	8	0.0843	-0.0157	0.0173	0.9096	0.0096	0.0211
1000	100	20	0.4	0.6	2	0.3382	-0.0618	0.0215	0.6300	0.0300	0.0379
1000	100	20	0.4	0.6	8	0.3710	-0.0290	0.0173	0.6141	0.0141	0.0192
1000	100	30	0.1	0.9	2	0.0754	-0.0246	0.0191	0.9162	0.0162	0.0355
1000	100	30	0.1	0.9	8	0.0889	-0.0111	0.0161	0.9073	0.0073	0.0196
1000	100	30	0.4	0.6	2	0.3534	-0.0466	0.0192	0.6255	0.0255	0.0348
1000	100	30	0.4	0.6	8	0.3787	-0.0213	0.0158	0.6116	0.0116	0.0179

Table 1 (Concluded). Estimates of Equation (3)

						OLS					
							$\hat{\rho}$			$\hat{\beta}$	
M	N	T	ρ	β	σ_s^2	Estimate	Bias	S.E.	Estimate	Bias	S.E.
1000	100	5	0.1	0.9	2	0.0579	-0.0421	0.0026	0.8524	-0.0476	0.0379
1000	100	5	0.1	0.9	8	0.0894	-0.0106	0.0021	0.8880	-0.0120	0.0145
1000	100	5	0.4	0.6	2	0.0554	-0.3446	0.0031	0.3080	-0.2920	0.0206
1000	100	5	0.4	0.6	8	0.1066	-0.2934	0.0091	0.3513	-0.2487	0.0156
1000	100	10	0.1	0.9	2	0.0580	-0.0420	0.0021	0.8390	-0.0610	0.0330
1000	100	10	0.1	0.9	8	0.0891	-0.0109	0.0019	0.8841	-0.0159	0.0126
1000	100	10	0.4	0.6	2	0.0558	-0.3442	0.0025	0.2876	-0.3124	0.0165
1000	100	10	0.4	0.6	8	0.1112	-0.2888	0.0087	0.3377	-0.2623	0.0159
1000	100	20	0.1	0.9	2	0.0583	-0.0417	0.0018	0.8223	-0.0777	0.0297
1000	100	20	0.1	0.9	8	0.0891	-0.0109	0.0017	0.8795	-0.0205	0.0105
1000	100	20	0.4	0.6	2	0.0557	-0.3443	0.0021	0.2954	-0.3046	0.0153
1000	100	20	0.4	0.6	8	0.1101	-0.2899	0.0082	0.3433	-0.2567	0.0138
1000	100	30	0.1	0.9	2	0.0588	-0.0412	0.0018	0.8111	-0.0889	0.0267
1000	100	30	0.1	0.9	8	0.0894	-0.0106	0.0017	0.8769	-0.0231	0.0089
1000	100	30	0.4	0.6	2	0.0558	-0.3442	0.0018	0.2916	-0.3084	0.0138
1000	100	30	0.4	0.6	8	0.1113	-0.2887	0.0078	0.3411	-0.2589	0.0124
						LSDV					
							$\hat{\rho}$			$\hat{\beta}$	
M	N	T	ρ	β	σ_s^2	Estimate	Bias	S.E.	Estimate	Bias	S.E.
1000	100	5	0.1	0.9	2	0.2012	0.1012	0.0209	0.8967	-0.0033	0.0570
1000	100	5	0.1	0.9	8	0.1076	0.0076	0.0057	0.8998	-0.0002	0.0146
1000	100	5	0.4	0.6	2	0.5521	0.1521	0.0329	0.6213	0.0213	0.0413
1000	100	5	0.4	0.6	8	0.4141	0.0141	0.0102	0.6020	0.0020	0.0102
1000	100	10	0.1	0.9	2	0.1374	0.0374	0.0120	0.8911	-0.0089	0.0360
1000	100	10	0.1	0.9	8	0.1026	0.0026	0.0031	0.8994	-0.0006	0.0089
1000	100	10	0.4	0.6	2	0.4703	0.0703	0.0217	0.6195	0.0195	0.0272
1000	100	10	0.4	0.6	8	0.4060	0.0060	0.0064	0.6017	0.0017	0.0066
1000	100	20	0.1	0.9	2	0.1171	0.0171	0.0079	0.9002	0.0002	0.0308
1000	100	20	0.1	0.9	8	0.1012	0.0012	0.0020	0.9000	0.0000	0.0076
1000	100	20	0.4	0.6	2	0.4372	0.0372	0.0155	0.6199	0.0199	0.0242
1000	100	20	0.4	0.6	8	0.4030	0.0030	0.0046	0.6016	0.0016	0.0060
1000	100	30	0.1	0.9	2	0.1104	0.0104	0.0063	0.9044	0.0044	0.0279
1000	100	30	0.1	0.9	8	0.1007	0.0007	0.0016	0.9003	0.0003	0.0069
1000	100	30	0.4	0.6	2	0.4260	0.0260	0.0137	0.6173	0.0173	0.0224
1000	100	30	0.4	0.6	8	0.4021	0.0021	0.0042	0.6014	0.0014	0.0055

given combination of the signal-to-noise ratio and ρ. In the estimations of equation (5) from samples with $T \geq 10$, the LSDV bias is smaller than 10 percent of the true value of ρ and affects only the second decimal of the estimate. Further, the LSDV bias in equation (5) seems particularly sensitive to the signal-to-noise ratio: the ρ bias when $\sigma_s^2 = 8$ is typically less than one tenth of the bias when $\sigma_s^2 = 2$, whereas in equation (3), the ρ bias when $\sigma_s^2 = 8$ is about half of the bias when $\sigma_s^2 = 2$. So factors that increase the signal to noise ratio of the fiscal reaction function equation, such as the presence of shocks to debt, $b_{i,t}$ with $\sigma_b^2 > 0$, would be expected to further decrease the relative LSDV bias in estimating equation (5).

The next set of Monte Carlo experiments focus on the estimation of equation (5), allowing for various realistic complications to the model. We calibrate the parameters of equation (5) to data available for a panel of emerging market countries for the period 1990-2004.[11] The sample dimensions are fixed at $N = 30$ and $T = 15$, corresponding approximately to the currently available sample size for fiscal debt and deficits in emerging market economies. As time series data indicates that real interest rates typically exceed real growth rates by several percentage points in emerging market countries, f is drawn from a uniform distribution between 0.95 and 1.10. If we set $f = 1.025$, at the mid-point of that range, the debt accumulation identity suggests that $b_{i,t} = d_{i,t} + p_{i,t} - 1.025 d_{i,t-1}$, which approximately has a mean equal to zero and standard deviation σ_b equal to 11 in the full sample, and a standard deviation equal to 4 if values of $b_{i,t}$ that exceed minus or plus 10 percent of GDP are excluded. Estimating equation (15) on output gap data for the set of countries included in the same dataset, using OLS, LSDV, and the Blundell and Bond (1998) system-GMM estimator (using twice lagged levels and differences of the output gap as instruments) suggests that λ is approximately 0.50, and $\sigma_\xi = 3$. Finally, estimating equation (1) with lagged debt and the output gap as the two explanatory variables and with country dummies yields a coefficient $\beta = 0.35$ on the output gap when the output gap is instrumented, and a coefficient of $\beta = 0.20$ when the output gap is not instrumented. The standard deviation of country fixed effects is estimated to be $\sigma_\eta = 2.8$ and the standard deviation of disturbances to be $\sigma_\varepsilon = 2.5$.[12] Finally, a large number of regressions using different techniques suggest ρ to be between 0.02 and 0.06 in emerging market economies (Celasun, Debrun, and Ostry, 2006).

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[11] The dataset is described in Celasun, Debrun, and Ostry (2006).

[12] That β is estimated to be larger with instrumentation indicates that primary surplus disturbances, $\varepsilon_{i,t}$ have a negative impact on the output gap, $y_{i,t}$.

Based on these findings, we consider three scenarios, for which we consider four values of ρ, ρ = 0, 0.03, 0.06, 0.09. Other than the use of the data-consistent parameter values as described above, the scenarios allow for the following "incremental" complications relative to the version of the model studied in Table 1. In the first scenario, rather than setting $f_{i,t}$ = 1.05 $\forall i,t$ as in the simulations in Table 1, we draw $f_{i,t}$ from a uniform distribution between [0.95,1.10], but we still set the mean and variance of shocks to debt to zero, $\sigma_b = 0$, and assume that the output gap is exogenous to primary surplus shocks, $\sigma_{py} = 0$. In the second scenario, we relax the second assumption; shocks to the debt accumulation equation $b_{i,t}$ are drawn from a normal distribution with zero mean and a standard deviation of σ_b = 5 (which is close to the standard deviation of $b_{i,t}$ in the data when outliers are excluded). In the third scenario, we additionally allow the output gap to be endogenous. The following equation is used to generate the output gap series in the third scenario:

$$y_{i,t} = x_{i,t} + \sigma_{py}\varepsilon_{i,t}, \qquad \sigma_{py} \leq 0. \tag{18}$$

where $x_{i,t}$ is generated using equation (15). The parameter values used in the three scenarios are summarized in Table 2.

Table 2. Parameter Values Used in Simulations

Scenario	ρ	β	σ_ε	σ_η	f	λ	σ_ξ	σ_b	σ_{py}
I	0.0, 0.03, 0.06, 0.09	0.35	2.5	2.8	Uniform[0.95,1.1]	0.5	3	0	0
II	0.0, 0.03, 0.06, 0.09	0.35	2.5	2.8	Uniform[0.95,1.1]	0.5	3	5	0
III	0.0, 0.03, 0.06, 0.09	0.35	2.5	2.8	Uniform[0.95,1.1]	0.5	3	5	-0.01

In each simulation run for scenarios I and II, we generate the data using equations (2), (5), and (15). For scenario III, we generate the data using equations (2), (5), (15) and (18). In scenarios I and II, we estimate equation (5) using the following techniques: OLS, LSDV, Arellano and Bond's (1991) one-step GMM estimator (GMM), Blundell and Bond's (1998) one-step system GMM estimator (SGMM), and Blundell and Bond's (1998) one-step system GMM estimator using only the second and third lags of debt as instruments for $d_{i,t-1}$ (SGMMR). For scenario III, where the output gap is endogenous to contemporaneous primary balance shocks, instead of LSDV we use a two stage least squares estimator that includes country fixed effects and instruments for the output gap $y_{i,t}$ with $x_{i,t}$ (LSDVIV).

The results, given in Table 3, indicate that the true value of ρ lies between the average OLS and LSDV (LSDVIV in Scenario III) estimates in the 1000 runs. The size of LSDV bias in

estimating ρ does not necessarily increase with the value of ρ itself, as it does in the AR(1) model (Judson and Owen 1999). As a percentage of the true value of ρ, the bias is larger in scenario I than in scenarios II and III, where the bias is typically around 30 percent or less. This confirms the conjecture in Case 2 of the analytical derivations: the presence of independent shocks to debt, $b_{i,t}$, reduce the bias of the LSDV estimator. All estimators yield a very small estimate of ρ when the true value of ρ is zero, so none of the estimators would erroneously indicate debt sustainability ($\hat{\rho}$>0) when in fact the primary surplus is not responsive to debt accumulation ($\rho = 0$). The LSDV and LSDVIV estimators have the lowest bias and the lowest RMSE for ρ, while GMM is a close runner up. The SGMM estimates of ρ are severely negatively biased, as are the OLS estimates.

While the presence of endogenous—as opposed to exogenous—regressors does not significantly alter the size of the bias in estimating ρ, it vastly increases the RMSE of all estimators of β, the coefficient on the endogenous variable. The RMSE of the LSDVIV estimator of β is particularly high, most likely due to the use of a high number of instruments—including the country dummies—in the first stage.[13] So while the use of country dummies—the LSDV or LSDVIV estimations—yields the most precise estimator of ρ, it leads to the largest bias in the estimates of β, for which the GMM and SGMM estimators have the lowest RMSE.

IV. Conclusion

The Monte Carlo simulations in the paper suggest that the use of country dummies does not lead to economically large biases in the estimates of ρ, the coefficient of lagged debt in the fiscal reaction function model (1); the LSDV method yields more precise estimates of ρ compared with OLS and the Arellano and Bond (1991) and Blundell and Bond (1998) GMM methods. If other regressors in the fiscal reaction function such as the output gap are potentially endogenous to contemporaneous primary balance shocks and would need to be instrumented, however, the Arellano and Bond (1991) and Blundell and Bond (1998) GMM estimators—where the difference or level of debt is instrumented with the lagged levels or differences in debt and exogenous instruments are used for the potentially endogenous regressors—are the best-performing estimators for the coefficients of the endogenous variables. These results suggest estimating the fiscal reaction functions using the LSDV or the Arellano and Bond (1991) and Blundell and Bond (1998) GMM methods, depending on the variable and coefficient estimate of interest. Although tests of intertemporal solvency would preferably be based on LSDV methods, tests of fiscal policy countercyclicality would preferably be based on GMM methods using exogenous instruments for the output gap.

[13] See Stock and Yogo (2005) for a discussion of the weak instruments problem and related tests.

Table 3. Estimates of Parameters of Equation (5), Various Scenarios

$\hat{\rho}$					$\hat{\beta}$				
OLS (S.E.) [RMSE]	LSDV (S.E.) [RMSE]	GMM (S.E.) [RMSE]	SGMM (S.E.) [RMSE]	SGMMR (S.E.) [RMSE]	OLS (S.E.) [RMSE]	LSDV (S.E.) [RMSE]	GMM (S.E.) [RMSE]	SGMM (S.E.) [RMSE]	SGMMR (S.E.) [RMSE]
Scenario I									
$\rho = 0.00$									
-0.008	0.001	0.000	-0.007	-0.007	0.344	0.349	0.348	0.348	0.348
(0.000)	(0.002)	(0.003)	(0.000)	(0.000)	(0.056)	(0.058)	(0.072)	(0.059)	(0.060)
[0.008]	[0.003]	[0.003]	[0.007]	[0.007]	[0.056]	[0.058]	[0.072]	[0.059]	[0.060]
$\rho = 0.03$									
0.010	0.050	0.041	0.011	0.012	0.346	0.345	0.347	0.350	0.349
(0.001)	(0.011)	(0.016)	(0.001)	(0.002)	(0.057)	(0.058)	(0.072)	(0.059)	(0.060)
[0.020]	[0.023]	[0.020]	[0.019]	[0.018]	[0.057]	[0.059]	[0.072]	[0.059]	[0.060]
$\rho = 0.06$									
0.021	0.138	0.147	0.023	0.024	0.345	0.334	0.335	0.350	0.349
(0.002)	(0.030)	(0.057)	(0.003)	(0.006)	(0.057)	(0.059)	(0.071)	(0.060)	(0.061)
[0.039]	[0.084]	[0.104]	[0.037]	[0.036]	[0.057]	[0.061]	[0.072]	[0.060]	[0.061]
$\rho = 0.09$									
0.026	0.202	0.241	0.029	0.032	0.341	0.333	0.326	0.350	0.347
(0.003)	(0.040)	(0.076)	(0.007)	(0.011)	(0.056)	(0.059)	(0.069)	(0.060)	(0.061)
[0.064]	[0.119]	[0.169]	[0.062]	[0.059]	[0.057]	[0.062]	[0.073]	[0.060]	[0.061]
Scenario II									
$\rho = 0.00$									
-0.007	0.001	0.000	-0.007	-0.007	0.336	0.350	0.348	0.344	0.344
(0.000)	(0.002)	(0.003)	(0.000)	(0.001)	(0.061)	(0.058)	(0.072)	(0.060)	(0.061)
[0.007]	[0.002]	[0.003]	[0.007]	[0.007]	[0.062]	[0.058]	[0.072]	[0.060]	[0.061]
$\rho = 0.03$									
0.011	0.040	0.037	0.012	0.013	0.340	0.347	0.348	0.346	0.347
(0.001)	(0.010)	(0.015)	(0.002)	(0.002)	(0.058)	(0.059)	(0.071)	(0.059)	(0.061)
[0.019]	[0.014]	[0.017]	[0.019]	[0.017]	[0.059]	[0.059]	[0.071]	[0.059]	[0.061]
$\rho = 0.06$									
0.022	0.080	0.092	0.023	0.026	0.342	0.346	0.344	0.348	0.348
(0.002)	(0.017)	(0.036)	(0.004)	(0.006)	(0.057)	(0.059)	(0.071)	(0.059)	(0.061)
[0.038]	[0.026]	[0.048]	[0.037]	[0.034]	[0.058]	[0.059]	[0.071]	[0.059]	[0.061]
$\rho = 0.09$									
0.029	0.110	0.127	0.032	0.038	0.341	0.346	0.344	0.348	0.348
(0.003)	(0.018)	(0.039)	(0.008)	(0.012)	(0.058)	(0.059)	(0.071)	(0.060)	(0.062)
[0.061]	[0.027]	[0.054]	[0.059]	[0.054]	[0.059]	[0.059]	[0.071]	[0.060]	[0.062]

Table 3 (Concluded). Estimates of Parameters of Equation (5), Various Scenarios

$\hat{\rho}$					$\hat{\beta}$				
OLS (S.E.) [RMSE]	LSDVIV (S.E.) [RMSE]	GMM (S.E.) [RMSE]	SGMM (S.E.) [RMSE]	SGMMR (S.E.) [RMSE]	OLS (S.E.) [RMSE]	LSDVIV (S.E.) [RMSE]	GMM (S.E.) [RMSE]	SGMM (S.E.) [RMSE]	SGMMR (S.E.) [RMSE]
Scenario III									
$\rho = 0.00$									
-0.007	0.001	0.000	-0.007	-0.007	0.362	0.346	0.372	0.354	0.247
(0.000)	(0.002)	(0.003)	(0.000)	(0.001)	(0.360)	(1.768)	(0.641)	(0.598)	(1.046)
[0.008]	[0.002]	[0.003]	[0.007]	[0.007]	[0.360]	[1.767]	[0.641]	[0.598]	[1.051]
$\rho = 0.03$									
0.011	0.039	0.037	0.011	0.012	0.358	0.343	0.368	0.351	0.289
(0.001)	(0.010)	(0.016)	(0.002)	(0.002)	(0.363)	(1.780)	(0.519)	(0.507)	(0.873)
[0.019]	[0.014]	[0.017]	[0.019]	[0.018]	[0.363]	[1.779]	[0.519]	[0.507]	[0.875]
$\rho = 0.06$									
0.021	0.079	0.091	0.023	0.025	0.357	0.332	0.361	0.348	0.314
(0.002)	(0.018)	(0.036)	(0.004)	(0.006)	(0.370)	(1.796)	(0.473)	(0.482)	(0.872)
[0.039]	[0.026]	[0.048]	[0.038]	[0.036]	[0.370]	[1.796]	[0.473]	[0.482]	[0.872]
$\rho = 0.09$									
0.028	0.110	0.126	0.030	0.035	0.358	0.329	0.358	0.351	0.323
(0.003)	(0.019)	(0.040)	(0.007)	(0.012)	(0.382)	(1.797)	(0.463)	(0.491)	(0.928)
[0.062]	[0.027]	[0.053]	[0.060]	[0.056]	[0.382]	[1.796]	[0.463]	[0.490]	[0.928]

Notes: GMM and SGMM denote the Arellano and Bond (1991) and Blundell and Bond (1998) one-step difference and one-step system GMM estimators, respectively. SGMMR is the Blundell and Bond (1998) one step system GMM estimator where only the second and third lags of $d_{i,t-1}$ are used as instruments; GMM and SGMM instruments start at the second lag. The LSDVIV estimator for scenario III is a two stage least squares estimator that instruments for $y_{i,t}$ using $x_{i,t}$ and includes country dummies. The standard errors and the RMSE of the estimates are given in parentheses and brackets, respectively. All parameter values other than ρ are given in Table 2. Each Monte Carlo experiment consists of 1,000 draws.

I. Bias of OLS Estimator When $\gamma = \beta = 0$ and $\sigma_b^2 = 0$

In the case where $\gamma = \beta = 0$ and $\sigma_b^2 = 0$, the OLS estimator is given by:

$$\hat{\rho}_{OLS} = \frac{\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T} d_{i,t-1}p_{i,t}}{\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T} d_{i,t-1}^2} = \frac{\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T} d_{i,t-1}\left(\rho d_{i,t-1} + \eta_i + \varepsilon_{i,t}\right)}{\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T} d_{i,t-1}^2} = \rho + \frac{\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T} d_{i,t-1}\left(\eta_i + \varepsilon_{i,t}\right)}{\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T} d_{i,t-1}^2}$$

So, the bias of the OLS estimator is:

$$\underset{N\to\infty}{p\lim}(\hat{\rho} - \rho) = \frac{\frac{1}{T}\sum_{t=1}^{T} \underset{N\to\infty}{p\lim}\frac{1}{N}\sum_{i=1}^{N} d_{i,t-1}\left(\eta_i + \varepsilon_{i,t}\right)}{\frac{1}{T}\sum_{t=1}^{T} \underset{N\to\infty}{p\lim}\frac{1}{N}\sum_{i=1}^{N} d_{i,t-1}^2}$$

We use the MA representation for $d_{i,t-1}$, to simplify the numerator and denominator:

$$d_{i,t-1} = -\left(\sum_{j=0}^{t-2} A_{i,t-1,j}\right)\eta_i - \left(\sum_{j=0}^{t-2} A_{i,t-1,j}\varepsilon_{i,t-j-1}\right) + A_{i,t-1,t-1}d_{i,0}$$

where $A_{i,t,0} = 1$ and $A_{i,t,k} = \prod_{l=0}^{k-1}\left(f_{i,t-l} - \rho\right)$ for $k \geq 1$. Plugging this expression into the numerator yields:

$$-\left(\frac{1}{T}\sum_{t=1}^{T}\sum_{j=0}^{t-2} A_{i,t-1,j}\right)\sigma_\eta^2 + \frac{1}{T}\left(\sum_{t=1}^{T} A_{i,t-1,t-1}\right)\operatorname{cov}\left(d_{i,0},\eta_i\right)$$

Similarly, the denominator of the bias can be rewritten as follows:

$$\frac{\sigma_\eta^2}{T}\sum_{t=1}^{T}\left(\sum_{j=0}^{t-2} A_{i,t-1,j}\right)^2 + \frac{\sigma_\varepsilon^2}{T}\sum_{t=1}^{T}\left(\sum_{j=0}^{t-2} A_{i,t-1,j}^2\right) + \frac{1}{T}\sum_{t=1}^{T} A_{i,t-1,t-1}^2 \underset{N\to\infty}{p\lim}\frac{1}{N}\sum_{i=1}^{N} d_{i,0}^2$$
$$-2\frac{1}{T}\sum_{t=1}^{T}\left(A_{i,t-1,t-1}\sum_{j=0}^{t-2} A_{i,t-1,j}\right)\operatorname{cov}\left(d_{i,0},\eta_i\right)$$

Since $d_{i,0}$ are assumed to be either nonstochastic or generated by the same process as $d_{i,t}$, $\operatorname{cov}\left(d_{i,0},\eta_i\right)$ is either zero or negative. In addition, assuming that the initial values are bounded, that is $\underset{N\to\infty}{p\lim}\frac{1}{N}\sum_{i=1}^{N} d_{i,0}^2 < \infty$, implies that the OLS estimator is downward biased.

A. Bias of LSDV Estimator in Case 1 ($\gamma = \beta = 0$ and $\sigma_b^2 = 0$)

We consider the numerator of the bias term first, in particular the terms inside the summation over the time horizon. Since $E_i\left(d_{i,t-1}\varepsilon_{i,t}\right) = 0$ by construction, the denominator is decomposed into three parts as follows:

$$E_i\left(d_{i,t-1} - d_{i,-1}\right)\left(\varepsilon_{i,t} - \varepsilon_i\right) = -E_i\left(d_{i,t-1}\varepsilon_i\right) - E_i\left(d_{i,-1}\varepsilon_{i,t}\right) + E_i\left(d_{i,-1}\varepsilon_i\right) .$$

We calculate each term on the right-hand side as follows.

First term: $-E_i\left(d_{i,0}\varepsilon_i\right) = 0$ when t=1 since $d_{i,0}$ is given. When t>1, the following holds:

$$\begin{aligned}
-E_i\left(d_{i,t-1}\varepsilon_i\right) &= E_i\left(\sum_{j=0}^{\infty} A_{i,t-1,j}\varepsilon_{i,t-j-1}\right)\left(\frac{1}{T}\sum_{s=1}^{T}\varepsilon_{i,s}\right) \\
&= \frac{\sigma_\varepsilon^2}{T}\left(A_{i,t-1,t-2} + A_{i,t-1,t-3} + \ldots\ldots + A_{i,t-1,0}\right) = \frac{\sigma_\varepsilon^2}{T}\sum_{j=0}^{t-2} A_{i,t-1,j}
\end{aligned}$$

Second term:

$$\begin{aligned}
-E_i\left(d_{i,-1}\varepsilon_{i,t}\right) &= E_i\left(\frac{\varepsilon_{i,t}}{T}\sum_{s=1}^{T}\sum_{j=0}^{\infty} A_{i,s-1,j}\varepsilon_{i,s-j-1}\right) \\
&= \frac{\sigma_\varepsilon^2}{T}\left(A_{i,t,0} + A_{i,t+1,1} + \ldots\ldots + A_{i,T-1,T-t-1}\right) = \frac{\sigma_\varepsilon^2}{T}\sum_{j=0}^{T-t-1} A_{i,t+j,j}
\end{aligned}$$

Third term:

$$\begin{aligned}
E_i\left(d_{i,-1}\varepsilon_i\right) &= -E_i\left(\frac{1}{T}\sum_{s=1}^{T}\sum_{j=0}^{\infty} A_{i,s-1,j}\varepsilon_{i,s-j-1}\right)\left(\frac{1}{T}\sum_{s=1}^{T}\varepsilon_{i,s}\right) \\
&= -\frac{\sigma_\varepsilon^2}{T^2}\left(\begin{array}{r}
A_{i,1,0} + A_{i,2,1} + \ldots\ldots + A_{i,t-1,t-2} + A_{i,t,t-1} + A_{i,t+1,t} + \ldots\ldots + A_{i,T-1,T-2} \\
+A_{i,2,0} + \ldots\ldots + A_{i,t-1,t-3} + A_{i,t,t-2} + A_{i,t+1,t-1} + \ldots\ldots + A_{i,T-1,T-3} \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
+A_{i,t,0} + A_{i,t+1,1} + \ldots. + A_{i,T-1,T-t-1} \\
\ldots\ldots\ldots\ldots\ldots \\
+A_{i,T-1,0}
\end{array}\right). \\
&= -\frac{\sigma_\varepsilon^2}{T^2}\sum_{s=1}^{T-1}\sum_{j=0}^{s-1} A_{i,s,j}
\end{aligned}$$

Noting that the third term is constant for t=1,…,T, we simplify the summation of the three terms across all time horizons as follows:

$$\sum_{t=1}^{T} E_i\left(d_{i,t-1}-d_{i,-1}\right)\left(\varepsilon_{i,t}-\varepsilon_i\right)=\frac{\sigma_\varepsilon^2}{T}\sum_{t=1}^{T}\left(\sum_{j=0}^{t-2}A_{i,t-1,j}+\sum_{j=0}^{T-t-1}A_{i,t+j,j}-\frac{1}{T}\sum_{s=1}^{T-1}\sum_{j=0}^{s-1}A_{i,s,j}\right)$$

$$=\frac{\sigma_\varepsilon^2}{T}\left[\sum_{t=1}^{T}\sum_{j=0}^{t-2}A_{i,t-1,j}+\sum_{t=1}^{T}\sum_{j=0}^{T-t-1}A_{i,t+j,j}-\sum_{t=1}^{T}\frac{1}{T}\sum_{s=1}^{T-1}\sum_{j=0}^{s-1}A_{i,s,j}\right]$$

$$=\frac{\sigma_\varepsilon^2}{T}\left[\sum_{t=2}^{T}\sum_{j=0}^{t-2}A_{i,t-1,j}+\sum_{t=1}^{T}\sum_{j=0}^{T-t-1}A_{i,t+j,j}-\sum_{s=1}^{T-1}\sum_{j=0}^{s-1}A_{i,s,j}\right]$$

$$=\frac{\sigma_\varepsilon^2}{T}\left[\sum_{t=1}^{T-1}\sum_{j=0}^{t-1}A_{i,t,j}+\sum_{t=1}^{T}\sum_{j=0}^{T-t-1}A_{i,t+j,j}-\sum_{t=1}^{T-1}\sum_{j=0}^{t-1}A_{i,t,j}\right]$$

$$=\frac{\sigma_\varepsilon^2}{T}\sum_{t=1}^{T}\sum_{j=0}^{T-t-1}A_{i,t+j,j}>0,$$

where we note that $E_i\left(d_{i,t-1}\varepsilon_i\right)=0$ when t=1 in the third equality above. Thus, the numerator of the bias term is positive.

Using the MA representation of $d_{i,t-1}$ and $d_{i,-1}$, we then rewrite the terms inside the summation in the denominator of the bias:

$$E_i\left(d_{i,t-1}-d_{i,-1}\right)^2=E_i\left[\left\{A_{i,t-1}-\left(\sum_{j=0}^{\infty}A_{i,t-1,j}\varepsilon_{i,t-j-1}\right)\right\}-\left\{A_i-\frac{1}{T}\sum_{t=1}^{T}\sum_{j=0}^{\infty}A_{i,t-1,j}\varepsilon_{i,t-j-1}\right\}\right]^2$$

$$=E_i\left(A_{i,t-1}-A_i\right)^2+E_i\left(\sum_{j=0}^{\infty}A_{i,t-1,j}\varepsilon_{i,t-j-1}\right)^2$$

$$-\frac{2}{T}E_i\left\{\left(\sum_{j=0}^{\infty}A_{i,t-1,j}\varepsilon_{i,t-j-1}\right)\left(\sum_{s=1}^{T}\sum_{j=0}^{\infty}A_{i,s-1,j}\varepsilon_{i,s-j-1}\right)\right\}$$

$$+\frac{1}{T^2}E_i\left(\sum_{s=1}^{T}\sum_{j=0}^{\infty}A_{i,s-1,j}\varepsilon_{i,s-j-1}\right)^2$$

Taking similar steps similar to those for the numerator, we calculate each of the above four terms as follows:

*First term***:**

$$E_i\left(A_{i,t-1}-A_i\right)^2=E_i\left[\left\{-\left(\sum_{j=0}^{\infty}A_{i,t-1,j}\right)+\frac{1}{T}\sum_{s=1}^{T}\sum_{j=0}^{\infty}A_{i,s-1,j}\right\}\eta_i\right]^2$$

$$=\sigma_\eta^2\left\{-\left(\sum_{j=0}^{\infty}A_{i,t-1,j}\right)+\frac{1}{T}\sum_{s=1}^{T}\sum_{j=0}^{\infty}A_{i,s-1,j}\right\}^2$$

Second term:

$$E_i\left(\sum_{j=0}^{\infty}A_{i,t-1,j}\varepsilon_{i,t-j-1}\right)^2=\sigma_\varepsilon{}^2\left(A_{i,t-1,0}{}^2+A_{i,t-1,1}{}^2+\text{..........}\right)=\sigma_\varepsilon{}^2\sum_{j=0}^{\infty}A_{i,t-1,j}{}^2$$

Third term:

$$-\frac{2}{T}E_i\left\{\left(\sum_{j=0}^{\infty}A_{i,t-1,j}\varepsilon_{i,t-j-1}\right)\left(\sum_{s=1}^{T}\sum_{j=0}^{\infty}A_{i,s-1,j}\varepsilon_{i,s-j-1}\right)\right\}$$

$$=-\frac{2\sigma_\varepsilon{}^2}{T}\left[\begin{array}{l}\left\{\begin{array}{l}A_{i,t-1,0}\left(A_{i,t-1,0}+A_{i,t,1}+A_{i,t+1,2}+\text{......}+A_{i,T-1,T-t}\right)\\+A_{i,t-1,1}\left(A_{i,t-2,0}+A_{i,t-1,1}+A_{i,t,2}+\text{......}+A_{i,T-1,T-t+1}\right)\\+\text{......}+A_{i,t-1,t-1}\left(A_{i,0,0}+A_{i,1,1}+A_{i,2,2}+\text{......}+A_{i,T-1,T-1}\right)\end{array}\right\}\\+\left\{\begin{array}{l}A_{i,t-1,t}\left(A_{i,0,1}+A_{i,1,2}+A_{i,2,3}+\text{......}+A_{i,T-1,T}\right)\\+A_{i,t-1,t+1}\left(A_{i,0,2}+A_{i,1,3}+A_{i,2,4}+\text{......}+A_{i,T-1,T+1}\right)+\text{......}\end{array}\right\}\end{array}\right]$$

$$=-\frac{2\sigma_\varepsilon{}^2}{T}\left[\sum_{j=0}^{t-1}\left(A_{i,t-1,j}\sum_{k=0}^{T-t+j}A_{i,t-1-j+k,k}\right)+\sum_{j=1}^{\infty}\left(A_{i,t-1,t+j-1}\sum_{k=1}^{T}A_{i,T-k,T-k+j}\right)\right]$$

*Fourth term***:**

$$\frac{1}{T^2}E_i\left(\sum_{s=1}^{T}\sum_{j=0}^{\infty}A_{i,s-1,j}\varepsilon_{i,s-j-1}\right)^2$$

$$=\left\{\left(A_{i,T-1,0}\right)^2+\left(A_{i,T-1,1}+A_{i,T-2,0}\right)^2+\text{......}+\left(A_{i,T-1,T-2}+A_{i,T-2,T-3}+\text{......}+A_{i,1,0}\right)^2\right\}$$

$$+\left\{\left(A_{i,T-1,T-1}+A_{i,T-2,T-2}+\text{......}+A_{i,0,0}\right)^2+\left(A_{i,T-1,T}+A_{i,T-2,T-1}+\text{......}+A_{i,0,1}\right)^2+\text{......}\right\}$$

$$=\frac{\sigma_\varepsilon{}^2}{T^2}\left[\sum_{j=0}^{T-2}\left(\sum_{k=0}^{j}A_{i,T-1-j+k,k}\right)^2+\sum_{j=0}^{\infty}\left(\sum_{k=1}^{T}A_{i,T-k,T-k+j}\right)^2\right]$$

Noting that the fourth term is constant for $t = 1,\ldots, T$, we calculate the summation of the above four terms over all time horizons as follows:

$$
\begin{aligned}
&\sum_{t=1}^{T} E_i\left(d_{i,t-1}-d_{i,-1}\right)^2 \\
&\quad = \sigma_\eta^2 \sum_{t=1}^{T}\left\{-\left(\sum_{j=0}^{\infty} A_{i,t-1,j}\right)+\frac{1}{T}\sum_{s=1}^{T}\sum_{j=0}^{\infty} A_{i,s-1,j}\right\}^2 \\
&\quad + \sigma_\varepsilon{}^2 \left[\begin{array}{l}
\displaystyle\sum_{t=1}^{T}\left(\sum_{j=0}^{\infty} A_{i,t-1,j}{}^2 - \frac{2}{T}\left\{\sum_{j=0}^{t-1}\left(A_{i,t-1,j}\sum_{k=0}^{T-t+j} A_{i,t-1-j+k,k}\right)+\sum_{j=1}^{\infty}\left(A_{i,t-1,t+j-1}\sum_{k=1}^{T} A_{i,T-k,T-k+j}\right)\right\}\right) \\
\displaystyle +\frac{1}{T}\left\{\sum_{j=0}^{T-2}\left(\sum_{k=0}^{j} A_{i,T-1-j+k,k}\right)^2+\sum_{j=0}^{\infty}\left(\sum_{k=1}^{T} A_{i,T-k,T-k+j}\right)^2\right\}
\end{array}\right]
\end{aligned}
$$

Combining the numerator and denominator given above yields the expression (1.13) in the text.

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