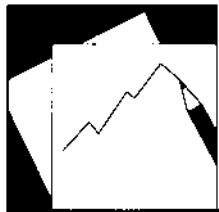


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Euro-Dollar Real Exchange Rate Dynamics in an Estimated Two-Country Model: What Is Important and What Is Not

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IMF Working Paper

Middle East and Central Asia Department

Euro-Dollar Real Exchange Rate Dynamics in an Estimated Two-Country Model: What Is Important and What Is Not

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Abstract

This Working Paper should not be reported as representing the views of the IMF.

The views expressed in this Working Paper are those of the author(s) and do not necessarily represent those of the IMF or IMF policy. Working Papers describe research in progress by the author(s) and are published to elicit comments and to further debate.

We use a Bayesian approach to estimate a standard two-country New Open Economy Macroeconomics model using data for the United States and the euro area, and we perform model comparisons to study the importance of departing from the law of one price and complete markets assumptions. Our results can be summarized as follows. First, we find that the baseline model does a good job in explaining real exchange rate volatility but at the cost of overestimating volatility in output and consumption. Second, the introduction of incomplete markets allows the model to better match the volatilities of all real variables. Third, introducing sticky prices in Local Currency Pricing improves the fit of the baseline model but does not improve the fit as much as introducing incomplete markets. Finally, we show that monetary shocks have played a minor role in explaining the behavior of the real exchange rate, while both demand and technology shocks have been important.

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I. INTRODUCTION

Most puzzles in international macroeconomics are related to real exchange rate dynamics. Fluctuations in real exchange rates can be very large and persistent, when compared to other real variables. In addition, there is clear evidence of lack of consumption risk-sharing across countries, which is at odds with the assumption of complete markets. In order to replicate these features of the data, the New Open Economy Macroeconomics (NOEM) literature has incorporated either nominal rigidities, alternative structures of assets markets, or both.

The real exchange rate (q_t) between two currencies is defined as the ratio of the two countries' price levels expressed in a common currency.² When all the components of the price level, namely domestically produced and imported goods, are sticky, it can be possible to explain some empirical features, like the high correlation between nominal and real exchange rates, and real exchange rate volatility. In the literature, pricing of imported goods are assumed to be governed either by Producer Currency Pricing (PCP), where the law of one price holds and there is perfect pass-through; or Local Currency Pricing (LCP), where the pass-through is zero in the short run.

Under complete markets, the real exchange rate should be equal to the ratio of the marginal utility of consumption across countries, because it reflects the relative price of foreign goods in terms of domestic goods. For example, assuming separable preferences and log utility, the following relationship should hold as an equilibrium condition: $q = c_t / c_t^*$, where c_t and c_t^* are the levels of domestic and foreign consumption. This relationship, which implies a correlation of one between the real exchange rate and the ratio of consumption levels in two countries, does not hold for many bilateral relationships in general. For the bilateral euro-U.S. dollar exchange rate in particular, the correlation between these variables (HP-filtered) is -0.17. Hence, models that incorporate complete markets are bound to perform poorly, even when they allow for other nominal or real rigidities. One possibility to get around this problem is to assume that agents do not have access to complete markets to ensure their wealth against idiosyncratic and country-specific shocks. Another possibility is to introduce preference shocks that affect the marginal utility of consumption, as in Stockman and Tesar (1995).

A recent paper by Chari, Kehoe, and McGrattan (2002—hereafter CKM) attempts to explain the volatility and persistence of the real exchange rate by constructing a model with sticky prices and LCP. Their main finding is that monetary shocks and complete markets, along with a high degree of risk aversion and price stickiness of one year, are enough to account for real exchange rate volatility and, to a lesser extent, for its persistence. However, their model found it difficult to account for the observed negative correlation between real exchange rates and relative consumption across countries, a fact that they labeled the *consumption-real*

² In log-linear terms, the real exchange rate is defined as $q_t = s_t + p_t^* - p_t$, where s_t is the (logarithm of the) nominal exchange in units of domestic currency per unit of foreign currency, p_t^* is the (log of the) foreign price level, and p_t is the (log of the) domestic price level.

exchange rate anomaly. In addition, CKM showed that some conventional ways of modeling of asset market incompleteness and habit persistence do not eliminate the anomaly.³

We use a Bayesian approach to estimate and compare two-country NOEM models using different assumptions of imports goods pricing and asset markets structures, thereby testing some of the key implications of CKM. Unlike them, we find that monetary policy shocks have a minor role in explaining real exchange rate volatility, and that both demand and technology shocks have had some importance. Using the Bayes factor to compare between competing alternatives, we find that what is crucial to explaining real exchange rate dynamics and the exchange rate-consumption anomaly is the introduction of incomplete markets with stationary net foreign asset positions. Somewhat surprisingly, we find that in a complete markets set up, the introduction of LCP improves the fit of the model. However, when incomplete markets are allowed for, LCP actually lowers the overall fit, overestimating real exchange rate volatility and implying a lower correlation between the real exchange rate and the ratio of relative consumptions than in the data.

We contribute to the existing literature on estimation of NOEM models. First, we focus on the relationship between relative consumption and the real exchange rate by introducing data on consumption for the United States and the euro area. Second, although our model is quite rich in shocks (we need nine shocks because we try to explain nine variables), we have left aside uncovered interest-rate parity (UIP)-type shocks, which tend to explain a large fraction of real exchange rate variability. We do so because under complete markets these shocks, at least conceptually, should not be included and also because we want to study more carefully the role of “traditional” shocks (technology, demand, monetary, and so on) in explaining real exchange rate fluctuations.⁴ Third, we believe this is the first paper to evaluate the merits of the incomplete markets assumption with stationary net foreign assets in a two-country NOEM model. Last, but not least, we perform an in-sample forecast exercise and find that the preferred model does a good job in forecasting compared to the other NOEM models, but still lags behind the performance of a vector autorregression (VAR) model.

The literature on estimating NOEM models in the spirit of CKM and Galí and Monacelli (2005) has grown rapidly, with the adoption of the Bayesian methodology to an open economy setting.⁵ For example, Lubik and Schorfheide (2006) estimate small open

³ Alternative ways to explain this anomaly typically include models with traded and nontraded goods. Selaive and Tuesta (2003a) and Benigno and Thoenissen (2005) have shown that this anomaly can be successfully addressed by models with incomplete markets and nontraded goods, with the traditional Balassa-Samuelson effect and sector-specific productivity shocks. Similarly, Ghironi and Melitz (2005) rely on aggregate productivity shocks and also find that the Balassa-Samuelson effect help to explain the consumption real exchange rate anomaly. Finally, Corsetti, Dedola, and Leduc (2004) have shown that distribution services can help to account for the real exchange rate-consumption correlation by lowering import demand elasticity.

⁴ Our benchmark model, unlike the International Real Business Cycle (IRBC) literature, always includes nominal rigidities, because we want to evaluate the relative importance of monetary shocks in explaining real exchange rate fluctuations.

⁵ Examples of closed economy applications of this methodology are Rabanal and Rubio-Ramírez (2005), and Galí and Rabanal (2004) for the United States, and Smets and Wouters (2003) for the euro area.

economy models with data for Australia, Canada, New Zealand, and the United Kingdom, to examine whether the monetary policy rules of those countries have targeted the nominal exchange rate. Justiniano and Preston (2004) estimate and compare small open economy models analyzing the consequences of introducing imperfect pass-through. Adolfson et al. (2005) estimate a medium-scale (15 variable) small open economy model for the euro area, while Lubik and Schorfheide (2005) and Batini et al. (2005) estimate a small-scale two-country model using U.S. and euro-area data.

The rest of the paper is organized as follows. In the next section, we outline the baseline model and describe the LCP and the incomplete market extensions. In Section III, we explain the data and, in Section IV, the econometric strategy. The estimation results are reported in Section V. First, we present the parameter estimates of the baseline model. Then, we analyze the parameter estimates of all the extensions along with the second moments implied by each model. We select our preferred model based on the comparison of Bayes factors, and analyze its dynamics by studying the impulse response functions. Finally, we evaluate the importance of shocks through variance decompositions. We also compare the forecasting performance of all the Dynamic Stochastic General Equilibrium (DSGE) models with respect to VAR models. Section VI concludes.

II. THE MODEL

In this section, we present the stochastic two-country NOEM model that we will use to analyze real exchange rate dynamics.⁶ We first outline a baseline model with complete markets and where the law of one price holds, in the spirit of Clarida, Galí, and Gertler (2002); Benigno and Benigno (2003); and Galí and Monacelli (2005). We add features that are known to improve the model's empirical properties, namely: home bias and habit formation in consumption, and staggered price setting with backward looking indexation. In the next section, we introduce two extensions that we are interested in comparing: incomplete markets and sticky prices of imported goods in local currency.

The model assumes that there are two countries, home and foreign, of equal size. Each country produces a continuum of intermediate goods, indexed by $h \in [0,1]$ in the home country and $f \in [0,1]$ in the foreign country. Preferences over these goods are of the Dixit-Stiglitz type, implying that producers operate under monopolistic competition, and all goods are internationally tradable. Table 1 lists all the variables of the model. The model contains nine shocks: a world technology shock that has a unit root, and country-specific stationary technology, monetary, demand and preference shocks. All stationary shocks are AR(1), except for the monetary shocks that are *iid*.

A. Households

In each country there is a continuum of infinitely lived households in the unit interval, who obtain utility from consuming the final good and disutility from supplying hours of labor. It

⁶ This type of model has been the *workhorse* of the NOEM literature after Obstfeld and Rogoff (1995).

Table 1. Variables in the Home and Foreign Countries

	Home		Foreign	
	Quantity	Price	Quantity	Price
Consumption goods				
Aggregate	C_t	P_t	C_t^*	P_t^*
Imports	$C_{F,t}$	$P_{F,t}$	$C_{H,t}^*$	$P_{H,t}^*$
Domestically produced	$C_{H,t}$	$P_{H,t}$	$C_{F,t}^*$	$P_{F,t}^*$
Intermediate goods				
Imports	$c_t(f)$	$p_t(f)$	$c_t^*(h)$	$p_t^*(h)$
Domestically produced	$c_t(h)$	$p_t(h)$	$c_t^*(f)$	$p_t^*(f)$
Production				
Aggregate (GDP)	$Y_{H,t}$	$P_{H,t}$	$Y_{F,t}^*$	$P_{F,t}^*$
Intermediate goods	$y_t(h)$	$p_t(h)$	$y_t^*(f)$	$p_t^*(f)$
	Home		Foreign	
Labor markets				
Hours worked	N_t		N_t^*	
Real wage	ω_t		ω_t^*	
Firms' labor demand	$N_t(h)$		$N_t^*(f)$	
Terms of trade	T_t		T_t^*	
Interest rates	R_t		R_t^*	
Bonds	B_t		B_t^*	
Real exchange rate			Q_t	
Nominal exchange rate			S_t	
Shocks				
World technology			A_t	
Country technology	X_t		X_t^*	
Preference	G_t		G_t^*	
Monetary	z_t		z_t^*	
Demand	η_t		η_t^*	

is assumed that consumers have access to complete markets at the country level and at the world level, which implies that consumers' wealth is insured against country-specific and world shocks, and hence all consumers face the same consumption-savings decision.⁷

⁷ Baxter and Crucini (1993) have used the same assumption in an IRBC model in order to explain the saving-investment correlation.

In the home country, households' lifetime utility function is:

$$E_0 \sum_{t=0}^{\infty} \beta^t G_t [\log(C_t - bC_{t-1}) - \frac{N_t^{1+\gamma}}{1+\gamma}]. \quad (1)$$

E_0 denotes the rational expectations operator using information up to time $t=0$. $\beta \in [0,1]$ is the discount factor. The utility function displays *external* habit formation. $b \in [0,1]$ denotes the importance of the habit stock, which is last period's aggregate consumption. $\gamma > 0$ is inverse elasticity of labor supply with respect to the real wage.

Table 2 contains additional variable definitions and functional forms. C_t denotes the consumption of the final good, which is a CES aggregate of consumption bundles of home and foreign goods. The parameter $1 - \delta$ is the fraction of home-produced goods in the consumer basket, and denotes the degree of home bias in consumption. Its analogous in the foreign country is $1 - \delta^*$. The elasticity of substitution between domestically produced and imported goods in both countries is θ , while the elasticity of substitution between types of intermediate goods is $\varepsilon > 1$.

In our baseline case, we assume that the law of one price holds for each intermediate good. This implies that $P_{H,t} = S_t P_{H,t}^*$, and $P_{F,t} = S_t P_{F,t}^*$. Note, however, that purchasing power parity (a constant real exchange rate) does not necessarily hold because of the presence of home bias in preferences. The home-bias assumption allows to generate real exchange rate dynamics in a model, like this one, with only tradable goods. From previous definitions, we can express the real exchange rate as a function of the terms of trade:

$$Q_t = \frac{S_t P_t^*}{P_t} = \left[\frac{\delta^* + (1 - \delta^*) T_t^{1-\theta}}{(1 - \delta) + \delta T_t^{1-\theta}} \right]^{\frac{1}{1-\theta}} \quad (2)$$

B. Asset Market Structure, Budget Constraint, and the Real Exchange Rate

We model complete markets by assuming that households have access to a complete set of state contingent nominal claims which are traded domestically and internationally. We represent the asset structure by assuming a complete set of contingent one-period nominal bonds denominated in home currency.⁸ Hence, households in the home country maximize their utility (1) subject to the following budget constraint:

$$C_t = \omega_t N_t + \frac{E_t \{ \xi_{t,t+1} B_{t+1} \} - B_t}{P_t} + \int_0^1 \Pi_t(h) dh, \quad (3)$$

⁸ Given these assumptions, it is not necessary to characterize the current account dynamics in order to determine the equilibrium allocations, and the currency denomination of the bonds is irrelevant.

Table 2: Definitions and Functional Forms

Consumption	$C_t \equiv \left[(1-\delta)^{\frac{1}{\theta}} (C_{H,t})^{\frac{\theta-1}{\theta}} + \delta^{\frac{1}{\theta}} (C_{F,t})^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}$ $C_t^* \equiv \left[(\delta^*)^{\frac{1}{\theta}} (C_{H,t}^*)^{\frac{\theta-1}{\theta}} + (1-\delta^*)^{\frac{1}{\theta}} (C_{F,t}^*)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}$
Consumption components	$C_{H,t} \equiv \left\{ \int_0^1 [c_t(h)]^{\frac{\varepsilon-1}{\varepsilon}} dh \right\}^{\frac{\varepsilon}{\varepsilon-1}}, \quad C_{H,t}^* \equiv \left\{ \int_0^1 [c_t^*(h)]^{\frac{\varepsilon-1}{\varepsilon}} dh \right\}^{\frac{\varepsilon}{\varepsilon-1}}$
	$C_{F,t} \equiv \left\{ \int_0^1 [c_t(f)]^{\frac{\varepsilon-1}{\varepsilon}} df \right\}^{\frac{\varepsilon}{\varepsilon-1}}, \quad C_{F,t}^* \equiv \left\{ \int_0^1 [c_t^*(f)]^{\frac{\varepsilon-1}{\varepsilon}} df \right\}^{\frac{\varepsilon}{\varepsilon-1}}$
Consumer price indices	$P_t \equiv \left[(1-\delta)(P_{H,t})^{1-\theta} + \delta(P_{F,t})^{1-\theta} \right]^{\frac{1}{1-\theta}},$ $P_t^* \equiv \left[\delta^*(P_{H,t}^*)^{1-\theta} + (1-\delta^*)(P_{F,t}^*)^{1-\theta} \right]^{\frac{1}{1-\theta}}.$
Price sub indices	$P_{H,t} \equiv \left\{ \int_0^1 [p_t(h)]^{1-\varepsilon} dh \right\}^{\frac{1}{1-\varepsilon}}, \quad P_{H,t}^* \equiv \left\{ \int_0^1 [p_t^*(h)]^{1-\varepsilon} dh \right\}^{\frac{1}{1-\varepsilon}}$
	$P_{F,t} \equiv \left\{ \int_0^1 [p_t(f)]^{1-\varepsilon} df \right\}^{\frac{1}{1-\varepsilon}}, \quad P_{F,t}^* \equiv \left\{ \int_0^1 [p_t^*(f)]^{1-\varepsilon} df \right\}^{\frac{1}{1-\varepsilon}}$
Terms of trade	$T_t = P_{F,t} / P_{H,t}, T_t^* = P_{H,t}^* / P_{F,t}^*$
Net exports	$NX_t = P_{H,t} Y_{H,t} / P_t - C_t$
Real exchange rate	$Q_t = \frac{S_t P_t^*}{P_t}$
Production functions	$y_t(h) = A_t X_t N_t(h), y_t^*(f) = A_t X_t^* N_t^*(f)$
World technology shocks	$\log(A_t) = \Gamma + \log(A_{t-1}) + \varepsilon_t^a$
Country technology shocks	$\log(X_t) = \rho_x \log(X_{t-1}) + \varepsilon_t^x, \log(X_t^*) = \rho_x^* \log(X_{t-1}^*) + \varepsilon_t^{x^*}$
Preference shocks	$\log(G_t) = \rho_g \log(G_{t-1}) + \varepsilon_t^g, \log(G_t^*) = \rho_g^* \log(G_{t-1}^*) + \varepsilon_t^{g^*}$

where B_{t+1} denotes nominal state-contingent payoffs of the portfolio purchased in domestic currency at t , and $\xi_{t,t+1}$ is the stochastic discount factor.⁹ The real wage is deflated by the country's consumer price index (CPI). The last term of the right hand side of equation (3) denotes the profits from the monopolistically competitive intermediate goods firms, which are ultimately owned by households in each country.

Combining optimality conditions of consumption in both countries under complete markets, we arrive at the following expression for the real exchange rate, that equals the ratio of marginal utilities of the two countries:

$$Q_t = \nu \frac{(C_t - bC_{t-1})}{(C_t^* - b^*C_{t-1}^*)} \frac{G_t^*}{G_t}, \quad (4)$$

where ν is a constant that depends on initial conditions (see CKM, and Galí and Monacelli, 2005). The risk-sharing condition (4) differs from the one in CKM because of the presence of both preference shocks and habit persistence.

C. Intermediate Goods Producers and Price-Setting

In each country, there is a continuum of intermediate goods producers, each producing a type of good that is an imperfect substitute of the others. As shown in Table 2, the production function is linear in the labor input, and has two technology shocks. The first one is a world technology shock, that affects the two countries the same way: it has a unit root, as in Galí and Rabanal (2004) and Ireland (2004), and it implies that real variables in both countries grow at a rate Γ . In addition, there is a country-specific technology shock that evolves as an AR(1) process.

Firms face a modified Calvo (1983)-type restriction when setting their prices. When they receive the Calvo-type signal, which arrives with probability $1 - \alpha$ in the home country, firms reoptimize their price. When they do not receive that signal, a fraction τ of intermediate goods producers index their price to last period's inflation rate, and a fraction $1 - \tau$ indexes their price to the steady-state inflation rate. This assumption is needed to incorporate trend inflation, as in Yun (1996). The equivalent parameters in the foreign country are $1 - \alpha^*$ and τ^* .

Cost minimization by firms implies that the real marginal cost of production is $\omega_t / (A_t X_t)$. Since the real marginal cost depends only on aggregate variables, it is the same for all firms

⁹ $\xi_{t,t+1}$ is a price of one unit of nominal consumption at time $t+1$, expressed in units of nominal consumption at t , contingent on the state at $t+1$ being s_{t+1} , given any state s_t in t . The complete market assumption implies that there exists a unique discount factor with the property that the price in period t of the portfolio with random value B_{t+1} is $E_t \{ \xi_{t,t+1} B_{t+1} \}$.

in each country. The overall demand for an intermediate good produced in h comes from optimal choices by consumers at home and abroad:

$$D_t(h) = c_t(h) + c_t^*(h) = \left(\frac{p_t(h)}{P_{H,t}} \right)^{-\varepsilon} \left(\frac{P_{H,t}}{P_t} \right)^{-\theta} \left[(1-\delta)C_t + \delta^* C_t^* Q_t^\theta \right]$$

Hence, whenever intermediate-goods producers are allowed to reset their price, they maximize the following profit function, which discounts future profits by the probability of not being able to reset prices optimally every period:

$$\text{Max}_{p_t(h)} E_t \sum_{k=0}^{\infty} \alpha^k \xi_{t,t+k} \left\{ \left[\frac{p_{t,t+k}(h)}{P_{t+k}} - \frac{\omega_{t+k}}{A_{t+k} X_{t+k}} MC_{t+k} \right] D_{t,t+k}(h) \right\}. \quad (5)$$

where $p_{t,t+k}(h)$ is the price prevailing at $t+k$ assuming that the firm last reoptimized at time t , and whose evolution will depend on whether the firm indexes its price to last period's inflation rate or to the steady-state rate of inflation, $D_{t,t+k}(h)$ the demand associated to that price, and $\xi_{t,t+k}$ is the k periods ahead stochastic discount factor.

The evolution of the aggregate consumption bundle price produced in the home country is:

$$P_{H,t}^{1-\varepsilon} = (1-\alpha)(\hat{P}_{H,t})^{1-\varepsilon} + \alpha \left[P_{H,t-1} \left(\frac{P_{H,t-1}}{P_{H,t-2}} \right)^\tau \bar{\Pi}_H^{1-\tau} \right]^{1-\varepsilon}. \quad (6)$$

where $\hat{P}_{H,t}$ is the optimal price set by firms in a symmetric equilibrium.

D. Closing the Model

In order to close the model, we impose market clearing conditions for all home and foreign intermediate goods. For each individual good, market clearing requires $y_t(h) = c_t(h) + c_t^*(h)$ for all $h \in [0,1]$. Defining aggregate real GDP as $Y_{H,t} = \left[\int_0^1 p_t(h) y_t(h) dh \right] / P_{H,t}$, the following market clearing condition holds at the home-country level:

$$Y_{H,t} = \left[(1-\delta)C_t + \delta^* C_t^* Q_t^\theta \right] \left(\frac{P_{H,t}}{P_t} \right)^{-\theta} + \eta_t \quad (7)$$

The analogous expressions for the foreign country are, $y_t^*(f) = c_t(f) + c_t^*(f)$, for all $f \in [0,1]$ and for aggregate foreign real GDP:

$$Y_{F,t}^* = \left[\delta C_t + (1-\delta^*) C_t^* Q_t^\theta \right] \left(\frac{P_{H,t}}{P_t} \right)^{-\theta} + \eta_t^* \quad (8)$$

We also introduce an exogenous demand shock for each country (η_t, η_t^*) that can be interpreted as government purchases, and/or trade with third countries. The model is closed by assuming that each country follows a monetary policy rule of the Taylor-type. We present the rules for each country in Section II.F. below.

E. Symmetric Equilibrium

Since we have assumed a world-wide technology shock that grows at a rate Γ , output, consumption, real wages, and the level of exogenous demand in the two economies grow at that same rate. In order to render these variables stationary, we divide them by the level of world technology A_t . Hours, inflation, interest rates, the real exchange rate, and the terms of trade are stationary.

F. Dynamics

We obtain the model's dynamics by taking a linear approximation to the steady state values at zero inflation. We impose a symmetric home bias, such that $\delta = \delta^*$. We denote by lower case variables percent deviations from steady state values. Moreover, variables with a tilde have been normalized by the level of technology to render them stationary. For instance, $\tilde{c}_t = (\tilde{C}_t - \tilde{C}) / \tilde{C}$, where $\tilde{C}_t = C_t / A_t$. The relationship between the transformed variables in the model (normalized by the level of technology) and the first-differenced variables is as follows:

$$\tilde{c}_t = \tilde{c}_{t-1} + \Delta c_t - \varepsilon_t^a, \tilde{y}_t = \tilde{y}_{t-1} + \Delta y_t - \varepsilon_t^a, \tilde{c}_t^* = \tilde{c}_{t-1}^* + \Delta c_t^* - \varepsilon_t^a, \text{ and } \tilde{y}_t^* = \tilde{y}_{t-1}^* + \Delta y_t^* - \varepsilon_t^a.$$

where Δ denotes the first difference operator. These relationships are used in the estimation strategy, since we include first-differenced real variables in the set of observable variables.

In this subsection, we focus the discussion on the equations that influence the behavior of the real exchange rate, and that will be affected by the introduction of imperfect pass-through and incomplete markets. Table 3 presents the rest of the model's equations, which are fairly standard given our assumptions. The only exception are the Taylor rules, which modify the original formulation by reacting to output growth instead of the output gap, incorporating interest rate smoothing, and an *iid* monetary shock.

The risk sharing condition delivers the following relationship between consumption in the two countries, the preference shocks, and the real exchange rate:

$$q_t = \left[\frac{(1+\Gamma)\tilde{c}_t - b\tilde{c}_{t-1}}{1+\Gamma-b} \right] - \left[\frac{(1+\Gamma)\tilde{c}_t^* - b^*\tilde{c}_{t-1}^*}{1+\Gamma-b^*} \right] - (g_t - g_t^*) + \left(\frac{1+\Gamma}{1+\Gamma-b} - \frac{1+\Gamma}{1+\Gamma-b^*} \right) \varepsilon_t^a. \quad (9)$$

As in CKM, the real exchange rate depends on the ratio of marginal utilities of consumption, which in our case include the habit stock in each country, and the preference shocks. Note

Table 3: Linearized Equations

Euler equations	$b\Delta c_t = -(1+\Gamma-b)(r_t - E_t\Delta p_{t+1}) + (1+\Gamma)E_t\Delta c_{t+1} + (1+\Gamma-b)(1-\rho_g)g_t,$ $b^*\Delta c_t^* = -(1+\Gamma-b^*)(r_t^* - E_t\Delta p_{t+1}^*) + (1+\Gamma)E_t\Delta c_{t+1}^* + (1+\Gamma-b^*)(1-\rho_g^*)g_t^*$
Labor supply	$\tilde{\omega}_t = m_t + \left[\frac{(1+\Gamma)\tilde{c}_t - b\tilde{c}_{t-1} + b\varepsilon_t^a}{1+\Gamma-b} \right], \tilde{\omega}_t^* = m_t^* + \left[\frac{(1+\Gamma)\tilde{c}_t^* - b^*\tilde{c}_{t-1}^* + b^*\varepsilon_t^{a*}}{1+\Gamma-b^*} \right]$
Goods market clearing	$\tilde{y}_{H,t} = \theta \left[\frac{2\delta(1-\delta)}{1-2\delta} \right] q_t + (1-\delta)\tilde{c}_t + \delta\tilde{c}_t^* + \eta_t, \tilde{y}_{F,t}^* = -\theta \left[\frac{2\delta(1-\delta)}{1-2\delta} \right] q_t + \delta\tilde{c}_t + (1-\delta)\tilde{c}_t^* + \eta_t^*.$
Production functions	$\tilde{y}_{H,t} = x_t + n_t, \tilde{y}_{F,t}^* = x_t^* + n_t^*$
Taylor rules	$r_t = \rho_r r_{t-1} + (1-\rho_r)\gamma_p\Delta p_{H,t} + (1-\rho_r)\gamma_y\Delta y_{H,t} + z_t$ $r_t^* = \rho_r^* r_{t-1}^* + (1-\rho_r^*)\gamma_p^*\Delta p_{F,t}^* + (1-\rho_r^*)\gamma_y^*\Delta y_{F,t}^* + z_t^*$
Terms of trade	$\Delta t_t = \Delta s_t + \Delta p_{F,t}^* - \Delta p_{H,t}$

that the innovation to world growth enters as long as the effect on the ratio of marginal utilities is different in the two countries, due to differences in the habit formation parameters.

Inflation dynamics for domestically produced goods in each country are given by:

$$\Delta p_{H,t} = \gamma_b \Delta p_{H,t-1} + \gamma_f E_t \Delta p_{H,t+1} + \kappa [\tilde{\omega}_t - x_t + \delta t_t], \quad (10)$$

$$\Delta p_{F,t}^* = \gamma_b^* \Delta p_{F,t-1}^* + \gamma_f^* E_t \Delta p_{F,t+1}^* + \kappa^* [\tilde{\omega}_t^* - x_t^* - \delta^* t_t^*]. \quad (11)$$

where for the home country, the backward and forward looking components are $\gamma_b \equiv \tau/(1+\beta\tau)$, $\gamma_f \equiv \beta/(1+\beta\tau)$, and the slope is given by $\kappa \equiv (1-\alpha\beta)(1-\alpha)/[(1+\beta\tau)\alpha]$.

Similar expressions with asterisks deliver the coefficients γ_b^* , γ_f^* , and κ^* . Domestic inflation is determined by unit labor costs (the real wage), productivity shocks, and the terms of trade. This last variable appears because wages are deflated by the CPI: an increase in imports prices will cause real wages to drop, and households will demand higher wages. As a result, domestic inflation will also increase.

When the law of one price holds, the real exchange rate and the terms of trade are linked as follows: $q_t = (1-2\delta)t_t$. The symmetric home bias assumption implies a positive comovement between the real exchange rate and the terms of trade which is consistent with the data. Thus, in this model, the real exchange rate inherits the properties of the terms of trade. With no home bias ($\delta=1/2$), the real exchange rate is constant and purchasing power

parity holds. The degree of home bias is crucial to account for the volatility of the real exchange rate: the larger the degree of home bias (smaller δ), the larger the volatility of the real exchange rate.¹⁰

Finally, the CPI inflation rates are a combination of domestic inflation and imported goods. Since prices are set in the producer currency, and the law of one price holds, the nominal exchange rate has a direct inflationary impact on CPI inflation:

$$\Delta p_t = (1 - \delta)\Delta p_{H,t} + \delta\Delta p_{F,t}^* + \delta\Delta s_t \quad (12)$$

and

$$\Delta p_t^* = \delta^* \Delta p_{H,t}^* - \delta^* \Delta s_t + (1 - \delta^*)\Delta p_{F,t}^*. \quad (13)$$

III. EXTENSIONS TO THE BASELINE MODEL

A. Incomplete Markets with Stationary Net Foreign Assets

In this section, we introduce the incomplete markets assumption. We assume that home-country households are able to trade in two nominal riskless bonds denominated in domestic and foreign currency, respectively. These bonds are issued by home-country residents in the domestic and foreign currency to finance their consumption. Home-country households face a cost of undertaking positions in the foreign bonds market.¹¹ For simplicity, we further assume that foreign residents can only allocate their wealth in bonds denominated in foreign currency. In each country, firms are still assumed to be completely owned by domestic residents, and profits are distributed equally across households.

The real budget constraint of home-country households is now given by:

$$C_t + \frac{B_t}{P_t R_t} + \frac{S_t B_t^*}{P_t R_t^* \phi\left(\frac{S_t B_t^*}{P_t}\right)} = \omega_t N_t + \frac{B_{t-1}}{P_t} + \frac{S_t B_{t-1}^*}{P_t} + \int_0^1 \Pi_t(h) dh, \quad (3')$$

where the $\phi(\cdot)$ function depends on the real holdings of the foreign assets in the entire economy, and therefore is taken as given by individual households.¹²

¹⁰ In a model with nontradable goods, this proportionality is broken down so that the real exchange rate will depend upon the relative price of tradable to nontradable goods across countries.

¹¹ This cost is needed to obtain stationarity in the net foreign asset position. See Schmitt-Grohe and Uribe (2001) and Kollman (2002) for applications in small open economies, and Benigno (2001) and Selaive and Tuesta (2003a) for applications in two-country models. Heathcote and Perri (2002) have used the same transaction cost in a two-country IRBC model.

¹² In order to achieve stationarity, $\phi(\cdot)$ has to be differentiable and decreasing in a neighborhood of zero. We further assume that $\phi(\cdot)$ equals zero when $B_t^* = 0$.

We further assume that the initial level of wealth is the same across households in each country. This assumption combined with the fact that households within a country equally share the profits of intermediate goods producers, implies that within a country all households face the same budget constraint. In their consumption decisions, they will choose the same path of consumption.

Dynamics

Under incomplete markets, the net foreign asset (NFA) position for the home country consists of the holding of foreign bonds (since domestic bonds are in net supply in the symmetric equilibrium). By definition, the NFA position of the foreign country equals the stock of bonds outstanding with the home country. The risk sharing condition holds in expected first difference terms and depends on the NFA position and preference shocks:

$$E_t q_{t+1} - q_t = \left[\frac{(1+\Gamma)E_t \Delta c_{t+1} - b \Delta c_t}{1+\Gamma-b} \right] - \left[\frac{(1+\Gamma)E_t \Delta c_{t+1}^* - b^* \Delta c_t^*}{1+\Gamma-b^*} \right] + (1-\rho_g)g_t - (1-\rho_g^*)g_t^* + \chi b_t^* \quad (9')$$

where $\chi = -\phi'(0)Y_H$ and $b_t^* = \left(\frac{S_t B_t^*}{Y_H P_t} \right)$, which substitutes equation (9) in section II.F.

The net foreign asset position becomes a state variable—its evolution depends on the stock of previous debt and on the trade deficit (or surplus):¹³

$$\beta b_t^* = b_{t-1}^* + \delta \left[\frac{2\theta(1-\delta)-1}{1-2\delta} \right] q_t - \delta(\tilde{c}_t - \tilde{c}_t^*) \quad (14)$$

Note that the effect of the real exchange rate on the NFA critically depends on the size of the elasticity of substitution: with a low elasticity, a real depreciation will imply that volumes increase less than prices decline, and hence the value of net exports declines after a real devaluation.

B. LCP by Intermediate Goods Producers

We assume price stickiness in each country's import prices in terms of local currency. Each firm chooses a price for the domestic market and a price for the foreign market under the same conditions of the modified Calvo lottery with indexation described above. This assumption generates deviations from the law of one price at the border, and nominal

¹³ This expression comes from defining the law of motion of the NFA position as $\beta b_t^* = b_{t-1}^* + nx_t$, where $nx_t = NX_t / Y_H$, and we make use of the expression of net exports in Table 2, the goods markets clearing condition in Table 3, and the consumer's optimizing conditions.

exchange rate movements generate ex-post deviations from the law of one price.¹⁴ Importantly, under the assumption of LCP, even without home bias, it is possible to generate real exchange rate fluctuations.

The overall demand (from domestic and foreign households) for an intermediate good produced in h , is given by:

$$c_t(h) = (1 - \delta) \left(\frac{p_t(h)}{P_{H,t}} \right)^{-\varepsilon} \left(\frac{P_{H,t}}{P_t} \right)^{-\theta} C_t \quad \text{and} \quad c_t^*(h) = \delta \left(\frac{p_t^*(h)}{P_{H,t}^*} \right)^{-\varepsilon} \left(\frac{P_{H,t}^*}{P_t^*} \right)^{-\theta} C_t^*.$$

Hence, whenever domestic intermediate-goods producers are allowed to reset their prices in the home and the foreign country, they maximize the following profit function:

$$\text{Max}_{p_t(h), p_t^*(h)} E_t \sum_{k=0}^{\infty} \alpha^k \xi_{t,t+k} \left\{ \frac{[p_{t,t+k}(h) - \omega_t / (A_t X_t)] c_{t,t+k}(h) + [p_{t,t+k}^*(h) S_{t+k} - \omega_t^* / (A_t X_t^*)] c_{t,t+k}^*(h)}{P_{t+k}} \right\}.$$

where $p_{t,t+k}(h)$ and $p_{t,t+k}^*(h)$ are prices of the home good set at home and abroad prevailing at $t+k$ assuming that the firm last reoptimized at time t , and whose evolution will depend on whether the firm indexes to last period's inflation rate (a fraction τ of firms) or to the steady-state rate of inflation (a fraction $1 - \tau$ of firms) when it is not allowed to reoptimize. $c_{t,t+k}(h)$ and $c_{t,t+k}^*(h)$ are the associated demands for good h in each country.

To obtain the log-linear dynamics, we first need to redefine the terms of trade:

$$t_t \equiv p_{F,t} - p_{H,t}, \text{ and } t_t^* \equiv p_{H,t}^* - p_{F,t}^*.$$

These ratios represent the relative price of imported goods in terms of the domestically produced goods expressed in local currency, for each country.¹⁵

Dynamics

The following new equations arise with respect to the baseline (PCP) case. The inflation equations for home-produced goods are:

$$\Delta p_{H,t} = \gamma_b \Delta p_{H,t-1} + \gamma_f E_t \Delta p_{H,t+1} + \kappa [\tilde{\omega}_t - x_t + \delta \tilde{x}_t], \quad (10')$$

¹⁴ Monacelli (2005) assumes that retail importers are subject to sticky prices, rather than the exporting firms in the country of origin. In his model, the law of one price holds at the border, but the pass-through is slow.

¹⁵ Note that if the law of one price holds, $t_t = -t_t^*$, but now it is no longer the case.

$$\Delta p_{H,t}^* = \gamma_b \Delta p_{H,t-1}^* + \gamma_f E_t \Delta p_{H,t+1}^* + \kappa [\tilde{\omega}_t - x_t - (1-\delta)t_t^* - q_t], \quad (10b')$$

$$\Delta p_{F,t}^* = \gamma_b \Delta p_{F,t-1}^* + \gamma_f E_t \Delta p_{F,t+1}^* + \kappa [\tilde{\omega}_t^* - x_t^* + \delta t_t^*], \quad (11')$$

$$\Delta p_{F,t} = \gamma_b \Delta p_{F,t-1} + \gamma_f E_t \Delta p_{F,t+1} + \kappa [\tilde{\omega}_t^* - x_t^* - (1-\delta)t_t + q_t], \quad (11b')$$

Similarly to the baseline case, real wages are deflated by the CPI which causes the terms of trade for each country, as well as the real exchange rate, to matter in the determination of unit labor costs and of domestic inflation.

The CPI inflation rates under LCP do not include the nominal exchange rate as a direct determinant of imported goods inflation, because the pass-through is low and import prices are sticky in domestic currency:

$$\Delta p_t = (1-\delta)\Delta p_{H,t} + \delta\Delta p_{F,t} \quad (12')$$

and

$$\Delta p_t^* = \delta\Delta p_{H,t}^* + (1-\delta)\Delta p_{F,t}^* \quad (13')$$

which substitute equations (10)–(13) of the baseline model. In addition, the market-clearing conditions in Table 3 become:

$$\tilde{y}_{H,t} = (1-\delta)\theta\delta(t_t - t_t^*) + (1-\delta)\tilde{c}_t + \delta\tilde{c}_t^* + \eta_t, \text{ and}$$

$$\tilde{y}_{F,t}^* = -(1-\delta)\theta\delta(t_t - t_t^*) + \delta\tilde{c}_t + (1-\delta)\tilde{c}_t^* + \eta_t^*.$$

C. Incomplete Markets and Sticky Prices in LCP

Under incomplete markets and LCP, the equations of the model are given by those in Section II.F, Table 3, and modified by those in Section III.B. The additional change is that while the behavior of the real exchange rate is the same as under incomplete markets (Equation 9' in Section III.A), the NFA position dynamics are given by:

$$\beta b_t^* = b_{t-1}^* + \delta(1-\delta)(\theta-1)(t_t - t_t^*) + \delta q_t - \delta(\tilde{c}_t - \tilde{c}_t^*) \quad (14')$$

which substitutes (14) in Section III.A.

IV. ESTIMATION AND MODEL COMPARISON

A. Data

We use data for the United States and euro area to estimate the model. For the United States, we use the following series (mnemonics as they appear in the Haver USECON database): quarterly real GDP (GDPH), the GDP deflator (DGDPI), real consumption (CH), and the 3-month T-bill interest rate (FTB3). Since we want to express real variables in per capita terms, we divide real GDP and consumption by total population of 16 years and over (LN16).

Data for the euro area as a whole comes from the Fagan, Henry, and Maestre (2001) dataset. (This dataset is a synthetic dataset constructed by the Econometric Modeling Unit at the European Central Bank, and should not be viewed as an “official” series.) We extract from that database real consumption (PCR), real GDP (YER), the GDP deflator (YED), and short-term interest rates. The euro zone population series is taken from Eurostat. Since it consists of annual data, we transform it to quarterly frequency by using linear interpolation.

The convention we adopt is that the home country is the euro area, and the foreign country is the United States. The real exchange rate consists of the nominal exchange rate in euros per U.S. dollar, converted to the real exchange rate index by multiplying it by the U.S. CPI and dividing it by the euro area CPI. The “synthetic” euro/U.S. dollar exchange rate prior to the launch of the euro in 1999 also comes from Eurostat; the U.S. CPI comes from the Haver USECON database (PCU) and the euro area CPI comes from the Fagan, Henry, and Maestre database (HICP).

Our sample period goes from 1973:1 to 2003:4, at quarterly frequency, which is when the euro area dataset ends. To compute per capita output and consumption growth rates and inflation, we take natural logs and first differences of per capita output and consumption, and the GDP deflator, respectively. We divide the short-term interest rate by four to obtain its quarterly equivalent. We also take natural logs and first differences of the euro/dollar real exchange rate.

Table 4 presents some relevant statistics. Interestingly, the raw data show that per capita output growth rates in the United States and the euro area are not that different (0.48 percent versus 0.47 percent), while per capita consumption and output in the euro area grow at the same rate (0.47 percent). Consumption growth in the United States displays a higher sample mean growth rate (0.53 percent) than in the euro area, which is not surprising given recent trends. Interestingly, growth rates in the euro area are less volatile than in the United States. The real exchange rate displays a small appreciating trend mean during the sample period and is much more volatile than any other series.

Real exchange rate volatility also stands out in the HP-filtered series: the bilateral real exchange rate has a standard deviation of 7.83 percent, while output and consumption in the United States have a standard deviation of 1.58 percent and 1.28 percent, respectively. Output and consumption in the euro area are less volatile, with a standard deviation of about 1 percent. Interest rates and inflation rates display high persistence, and so do all real variables when they are HP-filtered. Interestingly, only consumption in the euro area displays

Table 4: Properties of the Data for the United States and the Euro Area
(1973:1–2003:4)

Raw Data, Quarterly Growth Rates					
	Consumption Euro	Output Euro	Consumption USA	Output USA	Real Exch. Rate
Mean	0.47	0.47	0.53	0.48	-0.14
Std. dev.	0.57	0.58	0.67	0.85	4.59
Raw Data, Quarterly Rates					
	Interest Rate Euro	Inflation Euro	Interest Rate USA	Inflation USA	
Mean	2.08	1.44	1.59	1.00	
Std. dev.	0.83	0.93	0.73	0.67	
First autocorr.	0.96	0.89	0.94	0.90	
HP-Filtered Data					
	Consumption Euro	Output Euro	Consumption USA	Output USA	Real Exch. Rate
Std. dev.	0.91	1.01	1.28	1.58	7.83
Corr. with RER	-0.26	-0.06	-0.02	-0.08	1.00
First autocorr.	0.84	0.86	0.87	0.87	0.83
	Consumption Euro, USA	Output Euro, USA	Relative Cons., RER	Relative Outputs, RER	
Other Correlations	0.33	0.47	-0.17	0.04	

Source: Haver Analytics, Eurostat, and Fagan, Henry, and Maestre (2001).

Note: Relative variables are the ratio between the euro area variable and its U.S. counterpart.

a nonzero correlation of -0.26 with the real exchange rate. The correlation of output in the euro area, and output and consumption in the United States with the real exchange rate is essentially zero.

Finally, it is worth noting that the correlation between consumptions is smaller than between outputs (0.33 versus 0.47), although the size of the two correlations are smaller than those obtained using shorter sample periods, as in Backus, Kehoe, and Kydland (1992).¹⁶ The correlation of relative output with the real exchange rate is fairly small, while the correlation between the real exchange rate and relative consumptions across countries is negative (-0.17), which is at odds with efficient risk-sharing.¹⁷

¹⁶ Heathcote and Perri (2004) show that in recent years the U.S. economy has become less correlated with the rest of the world.

¹⁷ All the facts related to the U.S. economy are very similar to the ones presented in CKM.

B. Bayesian Estimation of the Model's Parameters

According to Bayes' rule, the posterior distribution of the parameters of any given model is proportional to the product of the prior distribution of the parameters and the likelihood function of the data. An appealing feature of the Bayesian approach is that additional information about the model's parameters (i.e., micro-data evidence, features of the first moments of the data) can be introduced via the prior distribution.

To implement the Bayesian estimation method, we need to numerically evaluate the prior and the likelihood function. The likelihood function is evaluated using the state-space representation of the law of motion of the model, and the Kalman filter. We then use the Metropolis-Hastings algorithm to obtain random draws from the posterior distribution, from which we obtain the relevant moments of the posterior distribution of the parameters.

Let ψ denote the vector of parameters that describe preferences, technology, the monetary policy rules, and the shocks in the two countries of the model. The vector of observable variables consists of $z_t = \{\Delta y_t, \Delta c_t, r_t, \Delta p_{H,t}, \Delta y_t^*, \Delta c_t^*, r_t^*, \Delta p_{F,t}^*, \Delta q_t\}$. The assumption of a world technology shock with a unit root makes the real variables stationary in the model in first differences. Hence, we use consumption and output growth per country, which are stationary in the data and in the model. We first-difference the real exchange rate, while inflation and the nominal interest rate in each country enter in levels.¹⁸ We express all variables as deviations from their sample mean. We denote by $L(\{z_t\}_{t=1}^T | \psi)$ the likelihood function of $\{z_t\}_{t=1}^T$.

Priors

Table 5 shows the prior distributions for the model's parameters, which we denote by $\Pi(\psi)$. For the estimation, we decide to fix only two parameters. The first one is the steady-state growth rate of the economy. Based on the evidence presented in Section IV.A, we set $\Gamma=0.5$ percent, which implies that the world growth rate of per capita variables is about 2 percent per year. In order to match a real interest rate in the steady state of about 4 percent per year, we set the discount factor to $\beta=0.995$. For reasonable parameterizations of these two variables, the parameter estimates do not change significantly. For the other parameters, gamma distributions are used as priors when nonnegativity constraints are necessary, and uniform priors when we are mainly interested in estimating fractions or probabilities. Normal distributions are used when more informative priors seem to be necessary.

Unlike other two-country model studies (e.g., Lubik and Schorfheide, 2005; and CKM), we do not impose the constraint that parameter values be the same in the two countries. However, we do use the same prior distributions for parameters across countries. We use normal distributions for the coefficients of habit formation and inverse elasticity of labor supply with respect to the real wage, centered at values commonly used in the literature (0.7

¹⁸ In this way, we avoid having to choose a particular detrending method (linear, quadratic, or HP-filter).

Table 5: Prior Distributions of the Model's Parameters

Parameter		Distribution	Mean	Std. Dev.
Habit formation	b, b^*	Normal	0.70	0.05
Labor supply	γ, γ^*	Normal	1.00	0.25
Average duration between optimal price changes	$(1 - \alpha)^{-1}, (1 - \alpha^*)^{-1}$	Gamma	3.00	1.42
Indexation	τ, τ^*	Uniform(0,1)	0.50	0.29
Fraction of imported goods	δ	Normal	0.20	0.03
Elasticity of substitution between home and foreign goods	θ	Normal	1.50	0.25
Elasticity of the real exchange rate to the NFA position	χ	Gamma	0.02	0.014
Taylor rule: inflation	γ_p, γ_p^*	Normal	1.50	0.25
Taylor rule: output growth	γ_y, γ_y^*	Normal	1.00	0.20
Taylor rule: smoothing	ρ_r, ρ_r^*	Uniform(0,1)	0.50	0.29
AR coefficients of shocks	$\rho_x, \rho_x^*, \rho_g, \rho_g^*, \rho_\eta, \rho_\eta^*$	Uniform(0,0.96)	0.48	0.28
Std. dev. technology shocks	$\sigma_x, \sigma_x^*, \sigma_a$	Gamma	0.007	0.003
Std. dev. preference shocks	σ_g, σ_g^*	Gamma	0.010	0.005
Std. dev. monetary shocks	σ_z, σ_z^*	Gamma	0.004	0.002
Std. dev. demand shocks	$\sigma_\eta, \sigma_\eta^*$	Gamma	0.010	0.005

and 1, respectively). We truncate the habit formation parameter to be between 0 and 1. We assume that the average duration of price contracts has a prior mean of 3 in the two countries, following empirical evidence reported in Taylor (1999). In this case, a gamma distribution is used.¹⁹ The prior on the fraction of price setters that follow a backward looking indexation rule is less informative and takes the form of a uniform distribution between zero and one.

The priors over the parameters that incorporate the open economy features of the model are as follows: (i) the parameter δ , which captures the implied home bias, has a prior distribution with mean 0.2 and standard deviation 0.03, implying a smaller home-bias than suggested by Heathcote and Perri (2002) and CKM; (ii) the elasticity of substitution between home and foreign goods (θ) is a source of controversy, so we center it at a value of 1.5 as suggested by CKM, but with a large enough standard deviation to accommodate other feasible parameters,

¹⁹ To keep the probability of the Calvo lottery between 0 and 1, the prior distribution is specified as average duration between optimal price changes minus one: $D=1/(1-\alpha)-1$. The shape of the prior is not much different than assuming a beta distribution for α .

even those below one;²⁰ and (iii) the parameter χ , that measures the elasticity of the risk premium with respect to the net foreign asset position, is assumed to have a gamma distribution with mean of 0.02 and a standard deviation of 0.014, following the evidence in Selaive and Tuesta (2003a and 2003b).

For the coefficients of the interest rate rule, we center the coefficients to the values suggested by Rabanal (2004), who estimates rules with output growth for the United States. Hence, γ_p has a prior mean of 1.5, and γ_y has a prior mean of 1. The same values are used for the monetary policy rule in the euro area, and we use uniform priors for the autoregressive processes between zero and one. We also truncate the prior distributions of the Taylor rule coefficients such that the models deliver a unique and stable solution.

We use uniform priors on the autoregressive coefficients of the six AR(1) shocks. We truncate the upper bound of the distribution to 0.96, because we want to examine how far the models can go in replicating persistence. We choose gamma distributions for the priors on the standard deviations of the shocks, to avoid negative values. The prior means are chosen to match previous studies. For instance, the prior mean for the standard deviation of all technology shocks is set to 0.007, close to the values suggested by Backus, Kehoe, and Kydland (1992), while the prior mean of the standard deviation of the monetary shocks comes from estimating the monetary policy rules using OLS.

Drawing from the Posterior and Model Comparison

We implement the Metropolis-Hastings algorithm to draw from the posterior. The results are based on 250,000 draws from the posterior distribution.²¹ The definition of the marginal likelihood for each model is as follows:

$$L(\{z_t\}_{t=1}^T) = \int_{\psi \in \Psi} L(\{z_t\}_{t=1}^T | \psi) \Pi(\psi) d\psi \quad (17)$$

The marginal likelihood averages all possible likelihoods across the parameter space, using the prior as a weight. Multiple integration is required to compute the marginal likelihood, making the exact calculation impossible. We approximate it by using the modified harmonic mean estimator.²²

²⁰ Trade studies typically find values for the elasticity of import demand with respect to price (relative to the overall domestic consumption basket) in the neighborhood of five to six, see Trefler and Lai (1999). Most of the NOEM models consider values of 1 for this elasticity, which implies Cobb-Douglas-type preferences in aggregate consumption.

²¹ See the appendix for some details on the estimation. Lubik and Schorfheide (2005, 2006) also provide useful details on the estimation procedure.

²² See Fernández-Villaverde and Rubio-Ramírez (2004) for computational details.

Then, for two different models (A and B), the posterior odds ratio is

$$\frac{P(A | \{z_t\}_{t=1}^T)}{P(B | \{z_t\}_{t=1}^T)} = \frac{\Pr(A)L(\{z_t\}_{t=1}^T | \text{model} = A)}{\Pr(B)L(\{z_t\}_{t=1}^T | \text{model} = B)}.$$

If there are $m \in M$ competing models, and one does not have strong views on which model is the best one (i.e., $\Pr(A)=\Pr(B)=1/M$) the posterior odds ratio equals the Bayes factor (i.e., the ratio of marginal likelihoods).

V. RESULTS

We report the results of our estimation in five stages. First, we present the posterior estimates obtained for a closed economy vis-à-vis the four specifications considered for the open economy model. Second, we perform a model comparison by evaluating the marginal likelihood for each model. Third, we compute the standard deviations and correlations of each model at the mode posterior values. Fourth, we discuss the dynamics of our preferred model by analyzing the importance of the structural shocks for real exchange rate fluctuations. And finally, we look at the one-step ahead in-sample forecast performance of all models, and compare their performance to VARs.

A. Posterior Distributions for the Parameters

Table 6 presents relevant moments of the posterior parameters of all the models. In order to have a benchmark for the open economy estimates, we first provide the results from estimating each country as a closed economy. Column I reports the mean and standard deviation of the posterior distributions of the parameters for the euro area and the United States, when both are estimated as a closed economies. We assume that within each country agents only consume home produced goods ($\delta=\theta=0$), and are not allowed to trade bonds internationally. In addition, the real exchange rate is dropped from the set of observed variables. Overall, the estimates in Column I are in line with those obtained in the literature, (e.g., Galí and Rabanal (2005), Rabanal and Rubio-Ramírez (2005) and Lubik and Schorfheide (2005)), hence we do not discuss them any further.

Our benchmark open economy model assumes complete markets and PCP. The parameter estimates are displayed in Column II. The results differ in important ways with respect to the closed economy case. First, the proportion of firms that index their prices to the lagged inflation rate increases to almost one in the euro area, while inflation remains almost purely forward looking in the United States. The average duration of price contracts decreases for the euro area to 4.77 quarters and increases significantly for the United States to 14.74 quarters, which is a fairly large number.²³ The habit persistence parameters increase

²³ This result is a consequence of having assumed a production function that is linear in the labor input. If we had assumed, as Galí, Gertler, and López-Salido (2001) that the production function is concave in labor, we would obtain smaller average price durations. Introducing firm-specific capital or real demand rigidities, as in Altig, et. al. (2005) or Eichenbaum and Fischer (2004), would also have lowered average price duration.

Table 6: Posterior Distributions

	I. Two Closed Economies		II. Complete Markets, PCP		III. Complete Markets, LCP		IV. Incomplete Markets, PCP		V. Incomplete Markets, LCP	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
b	0.57	0.04	0.78	0.02	0.72	0.03	0.62	0.03	0.64	0.03
b^*	0.61	0.04	0.69	0.04	0.68	0.04	0.55	0.04	0.54	0.03
γ	1.09	0.23	1.25	0.21	1.47	0.20	1.18	0.20	0.79	0.23
γ^*	1.00	0.21	0.86	0.25	1.02	0.23	1.16	0.20	1.03	0.23
$(1-\alpha)^{-1}$	5.94	0.89	4.77	0.46	6.29	0.62	4.28	0.39	4.12	0.47
$(1-\alpha^*)^{-1}$	7.09	0.92	14.74	1.68	12.66	1.36	5.74	0.63	4.95	0.63
τ	0.06	0.07	0.93	0.06	0.41	0.17	0.94	0.06	0.84	0.13
τ^*	0.09	0.08	0.04	0.04	0.06	0.05	0.09	0.07	0.09	0.08
δ	-	-	0.13	0.02	0.23	0.02	0.12	0.01	0.06	0.01
θ	-	-	0.04	0.03	0.01	0.01	0.44	0.00	0.91	0.01
χ	-	-	-	-	-	-	0.007	0.004	0.013	0.008
ρ_r	0.87	0.02	0.88	0.01	0.87	0.01	0.89	0.01	0.88	0.01
γ_y	1.08	0.16	0.07	0.06	0.16	0.10	1.04	0.16	0.98	0.17
γ_p	1.59	0.13	2.24	0.15	2.03	0.13	1.90	0.15	1.71	0.16
ρ_r^*	0.82	0.02	0.85	0.02	0.85	0.01	0.81	0.02	0.82	0.02
γ_y^*	0.91	0.15	1.24	0.13	1.34	0.14	1.16	0.14	1.01	0.15
γ_p^*	1.81	0.17	1.67	0.13	1.71	0.13	1.85	0.13	1.86	0.15
ρ_x	0.80	0.27	0.63	0.04	0.69	0.04	0.17	0.08	0.39	0.07
ρ_x^*	0.93	0.02	0.96	0.002	0.96	0.002	0.92	0.02	0.96	0.002
ρ_g	0.91	0.02	0.96	0.001	0.96	0.001	0.93	0.02	0.93	0.02
ρ_g^*	0.82	0.05	0.84	0.03	0.89	0.03	0.87	0.03	0.88	0.03
ρ_η	0.93	0.03	0.89	0.04	0.86	0.04	0.96	0.00	0.93	0.02
ρ_η^*	0.93	0.03	0.94	0.02	0.93	0.04	0.95	0.01	0.94	0.01
σ_x (in %)	1.93	0.45	4.45	0.52	4.27	0.48	4.00	0.55	4.23	0.54
σ_x^* (in %)	1.91	0.29	4.68	0.43	3.32	0.33	0.92	0.20	0.62	0.09
σ_g (in %)	2.32	0.29	5.72	0.40	4.78	0.31	3.64	0.45	3.41	0.49
σ_g^* (in %)	2.16	0.24	2.73	0.27	3.07	0.30	2.13	0.26	2.17	0.27
σ_z (in %)	0.20	0.02	0.23	0.02	0.22	0.02	0.20	0.01	0.19	0.01
σ_z^* (in %)	0.23	0.02	0.22	0.02	0.22	0.02	0.25	0.02	0.23	0.02
σ_a (in %)	0.70	0.02	2.58	0.21	2.03	0.17	1.39	0.12	1.44	0.14
σ_η (in %)	0.46	0.03	0.43	0.03	0.44	0.03	0.67	0.05	0.50	0.03
σ_η^* (in %)	0.69	0.05	0.67	0.04	0.66	0.04	0.89	0.06	0.70	0.05
Log-Marginal			3981.6		4033.2		4106.8		4070.9	

both in the euro area (to 0.78) and the United States (to 0.69). Estimates of the Taylor rule for the United States are similar to the ones obtained from the closed economy specification. However, there are significant changes in the estimates for the euro area: the estimated coefficient on inflation rises from 1.59 to 2.24 and the one on output decreases from 1.08 to 0.07. The degree of interest rate smoothing in each country presents minor changes with respect to the closed economy estimations.

The persistence and volatility of all shocks increases significantly in the open economy case, as the model tries to match the behavior of the real exchange rate. The standard deviation of shocks (except for monetary policy and demand shocks) is two or three times larger than in the closed economy case. In addition, the autocorrelation of technology shocks in the United States and of preference shocks in the euro area increases to 0.96, which is the upper bound allowed in the estimation. This outcome is largely the result of trying to match a highly volatile real exchange rate with less volatile output and consumption series.²⁴

We now turn to analyze the parameters that are critical in NOEM models to shape real exchange rate dynamics: the implied degree of home bias, captured by $1-\delta$, the intratemporal elasticity of substitution between goods across countries, θ , and the real exchange-rate elasticity with respect to the stock of foreign debt, χ , that arises from the incomplete markets assumption. In our benchmark NOEM model we find that the implied degree of home bias towards home goods is 0.87 which is below 0.984, the value used by CKM (2002) and Heathcote and Perri (2002). The baseline two-country model delivers a very small estimate for the elasticity θ . This result may follow from the market clearing conditions, because output and consumption are much less volatile than the real exchange rate. Another feature of the data to expect an estimated low elasticity is that the real exchange rate displays close-to-zero correlations with consumption and output in each country.

The following example sheds further light on this result. Assume for simplicity that the utility function does not exhibit habit persistence and there are no preference or world technology shocks. The risk-sharing condition under complete markets can be expressed as: $q_t = c_t - c_t^*$. This risk-sharing condition, combined with the market clearing conditions in both countries (see Table 3) and with the relationship between the real exchange rate and the terms of trade, $q_t = (1-2\delta)t_t$, delivers the following condition between relative outputs and the real exchange rate:

$$y_{H,t} - y_{F,t}^* = \left[\frac{4\delta(1-\delta)(\theta-1)-1}{(1-2\delta)^2} \right] q_t \quad (16)$$

²⁴ In our estimation we assume that shocks are orthogonal. However, the world aggregate shock indirectly adds some form of spillovers. Baxter and Crucini (1995) highlight the importance of the structure of shocks for international asset market structures in IRBC models. In particular, they find that if shocks are stationary and with substantial spillovers both complete and incomplete markets perform similarly. But if shocks are very persistent without spillovers, adding incomplete markets changes significantly the prediction of IRBC models.

Equation (16) illustrates the relationship between the volatility of relative outputs and the volatility of the real exchange rate, and highlights the need for a low value of the intratemporal elasticity of substitution between home and foreign goods, θ , in order to match the data. For a given volatility of the real exchange rate, the volatility of relative outputs is increasing in θ : a low value of θ will help in fitting the data better. Our prior distribution was centered at a value of 1.5, so the data clearly provide evidence that the value is much smaller.²⁵ Under PCP, we would observe a strong substitution in demand towards home goods following a devaluation. A value of θ close to zero is neglecting this expenditure-switching effect.

When we relax the PCP assumption allowing for deviations from the law of one price by using LCP, the results only change marginally (Column III). The estimated parameter θ remains close to zero. Two differences in the results are worth mentioning. First, the implied degree of home-bias in preferences drops from 0.87 to 0.77 in the euro area, and second, the proportion of firms that index their prices to the inflation rate drops from 0.93 to 0.41. Given the unreasonably low values obtained for θ , our estimation does not provide support for the complete asset market structure.

The LCP assumption adds endogenous volatility and persistence to the real exchange rate dynamics, so a smaller degree of home bias is needed in order to match the data. The real exchange rate under LCP can be decomposed as

$$q_t = \text{lop}_t - (1 - \delta)t_t^* - \delta r_t \quad (17)$$

$$\text{lop}_t - \text{lop}_{t-1} = \Delta s_t + \Delta p_{H,t}^* - \Delta p_{H,t} \quad (18)$$

where lop_t denotes deviations from the law of one price which arise from the LCP assumption.²⁶ When the law of one price holds, then $\text{lop}_t = 0$. Given the larger endogenous volatility of the real exchange rate and the low estimated values for θ , a lower estimated home bias parameter (i.e., higher δ) is needed.

Columns IV and V present the estimates of the model under incomplete markets with both PCP and LCP, respectively. There are some important differences with respect to the models with complete markets. First, the estimates of θ increase significantly, with point estimates of 0.45 and 0.91 under PCP and LCP, respectively. The intuition for this result can be seen from the law of motion of the NFA position (equation (14) or (14')). A real depreciation has to lead to a positive income effect to avoid having explosive NFA dynamics. For that to happen, θ has to be in the neighborhood of $1/2$ under PCP and 1 under LCP. This implicit

²⁵ Batini et al. (2005) allow for different elasticities of substitution of home and foreign goods for the euro area and the United States, and find that this elasticity is above one in both countries under complete markets.

²⁶ That is, lop_t denotes deviations from the following variable $LOP_t = S_t P_{H,t}^* / P_{H,t}$ under LCP from its PCP counterpart value of one.

restriction pushes the value of the elasticity up, although in both cases it seems to stay around the lowest possible value that delivers stable dynamics.

Given the low estimated values of the elasticity of substitution between home and foreign goods in complete market models (which are at odds with both macro and micro empirical evidence), our results give support for an asset market structure with incomplete markets. We conclude from this that the degree of financial integration (or lack thereof) is central for understanding real exchange rate dynamics.

The last two columns of Table 6 also report an estimate for χ . We find values of 0.007 and 0.013 under PCP and LCP, respectively. These values are larger than the ones found by Bergin (2004) for the G-7 countries (0.0038) and smaller than those obtained by Lane and Milesi-Ferretti (2001) from a panel of OECD countries (0.0254). Selaive and Tuesta (2003a and 2003b), estimate a risk-sharing condition similar to ours and obtain parameters that range between 0.004 and 0.071 for a sample of OECD countries. From the above results, it seems that the data give support to an incomplete asset market structure with a stationary net foreign asset position.

It is worth noting that the volatility of the shocks affecting the U.S. economy becomes much smaller under incomplete markets. Even for the productivity shock, the estimated standard deviations of the shocks are half the size of those under a closed economy set up. In the case of the euro area, the estimated volatility of the shocks is smaller, although the reduction is not as important as in the U.S. case. Finally, the estimates of the Taylor rule for the euro area become closer to what was obtained under a closed economy. Overall, then, it seems that the assumption of incomplete markets helps improve the internal dynamics of the model by requiring smaller shocks.

B. Model Comparison

The last row of Table 6 shows the marginal likelihood of the four open economy models.²⁷ The table shows that models with incomplete markets yield a higher marginal likelihood than those with complete markets (either under PCP or LCP). It also shows that the marginal likelihood is highest in the incomplete markets model with PCP. The model that ranks second is the incomplete markets model with LCP.

The differences are quantitatively very important. The (log) differences imply “decisive” evidence for the model with highest log marginal likelihood, using the Bayesian model comparison language (Kass and Raftery, 1995). For instance, the difference between the log marginals of the models in Columns IV and V is about 36. This means that we would need a prior that favors the second model over the first by a factor of 3.9×10^{15} in order to accept it after observing the data. Since this is a large number, we conclude that the incomplete market

²⁷ The marginal likelihood of the closed economies is not computed because of the different observed variables used in the estimation.

model with PCP (Column IV) outperforms the incomplete market model with LCP (Column V), which in turn outperforms the two models with complete markets.

C. Second Moments

One implication of the results just discussed is that the model with more features does not rank first in terms of the Bayes factor comparison. To understand why this is the case, let us analyze some evidence based on second moments. In all the models, the evaluation is done at the mode of the posterior distribution. Table 7 presents some second moments implied by our estimations and those in the actual data.²⁸

Table 7: Selected Second Moments in the Data and in the Models

	Euro Area				United States				
Std. Dev. (in percent)	Δc	Δy	r	Δp	Δc^*	Δy^*	r^*	Δp^*	Δq
Data	0.57	0.58	0.83	0.93	0.67	0.85	0.73	0.67	4.59
Closed Economy	0.73	0.81	0.59	0.74	0.85	1.00	0.57	0.56	-
Complete, PCP	1.30	1.27	1.43	1.31	1.24	1.27	0.63	0.79	4.96
Complete, LCP	1.30	1.18	1.10	0.90	1.17	1.16	0.77	0.72	5.32
Incomplete, PCP	1.04	1.07	0.83	1.11	0.83	0.98	0.61	0.62	4.90
Incomplete, LCP	1.03	1.11	0.82	1.22	0.86	0.97	0.60	0.63	6.83
Autocorrelations	c	y	r	Δp	c^*	y^*	r^*	Δp^*	q
Data	0.84	0.86	0.96	0.89	0.87	0.87	0.94	0.90	0.83
Closed Economy	0.87	0.85	0.91	0.73	0.88	0.85	0.89	0.77	-
Complete, PCP	0.92	0.91	0.95	0.82	0.91	0.89	0.89	0.85	0.77
Complete, LCP	0.90	0.89	0.93	0.71	0.91	0.88	0.91	0.84	0.71
Incomplete, PCP	0.87	0.85	0.94	0.79	0.88	0.83	0.87	0.72	0.72
Incomplete, LCP	0.87	0.86	0.93	0.78	0.87	0.83	0.88	0.72	0.70
Other Correlations	c, c^*	c, y	y, y^*	c^*, y^*	$c - c^*, q$	$y - y^*, q$			
Data	0.33	0.81	0.47	0.85	-0.17	0.04			
Complete, PCP	0.32	0.97	0.48	0.91	0.03	0.09			
Complete, LCP	0.21	0.93	0.60	0.87	0.20	0.13			
Incomplete, PCP	0.53	0.86	0.60	0.76	-0.37	0.09			
Incomplete, LCP	0.53	0.90	0.44	0.78	0.04	0.26			

Note: All model-based standard deviations and autocorrelations of nominal variables are computed by simulating the model at the posterior mode. Autocorrelations and cross-correlations of real variables come from simulating the model 1000 times with 124 periods at the posterior mode and applying the HP filter.

²⁸ All model-based standard deviations and autocorrelations of nominal variables are based on simulating the model at the posterior mode, to be consistent with the observable counterpart. Autocorrelations and cross-correlations of real variables come from simulating the model 1000 times with 124 periods at the posterior mode and applying the HP filter.

We find that the baseline model does a good job in explaining real exchange rate volatility and persistence, but at the cost of excessively high volatility and persistence in output and consumption in both countries, and of interest rates and inflation in the euro area. Allowing for deviations from the law of one price, gives a slightly better fit to the real variables, although it delivers less volatility and persistence in the real exchange rate. The introduction of incomplete markets allows the model to match the volatilities of all real variables best. While the model fits all U.S. variables and the nominal interest rate and inflation in the euro area fairly well, it still over-predicts output and consumption volatility in the euro area. Adding sticky prices in LCP to the incomplete markets model results in an over prediction of the real exchange rate volatility (the standard deviation rises from 4.90 percent to 6.83 percent, while in the data it is 4.59 percent), and a mild worsening of other features of the data. This is why this model, which is the one selected by CKM, does not rank best in terms of the Bayes factor.

The bottom panel of Table 7 shows cross-correlations of HP-filtered data of real variables that are typically studied in international business cycle analysis. In terms of consumption and output correlations across countries both the PCP and LCP complete market models perform quite well, but they are not so useful in explaining the correlation between the real exchange rate and relative consumptions across countries. In the data, this correlation is negative (-0.17), while we obtain positive values under the PCP and LCP models (0.03 and 0.20 respectively). CKM (2002) refer to this discrepancy between the models and the data as *the consumption real exchange rate anomaly*. CKM find this anomaly even when introducing incomplete markets. However, our proposed extension of the model allowing for incomplete markets, allows us to replicate this key feature of the data. In particular, we obtain a negative correlation between the relative consumption across countries and the real exchange rate (-0.37). Once again, extending the incomplete markets model allowing for deviations from the law of one price gives us a correlation close to zero, worsening the fit with respect to the incomplete market and PCP model.

The discussion below may help understand why the addition of incomplete markets allows us to replicate the lack of risk-sharing reported in Table 4. As argued by CKM, in a model with complete markets, the correlation between the ratio of relative consumptions and the real exchange rate is one. The introduction of preference and world technology shocks will tend to disturb this perfect correlation. Under incomplete markets and the law of one price, the NFA position would be:

$$\beta b_t^* = b_{t-1}^* + \delta \left[\frac{2\theta(1-\delta)-1}{1-2\delta} \right] q_t - \delta(\tilde{c}_t - \tilde{c}_t^*)$$

At the other extreme, under financial autarky, the NFA position would be zero at all times, and the equilibrium condition would become:

$$(\tilde{c}_t - \tilde{c}_t^*) = \left[\frac{2\theta(1-\delta)-1}{1-2\delta} \right] q_t$$

The above condition has to hold regardless of other nominal or real frictions present in the economy. It can therefore be seen that the correlation can have either sign depending on

combinations of parameters of θ and δ . In the particular case where $\theta = 1$, we would be mimicking the complete markets case and $q_t = c_t - c_t^*$ for all t . Low values of θ will generate negative correlations between relative consumptions and the real exchange rate.

The incomplete markets model is, in effect, an intermediate case between financial autarky and a complete markets setup. Therefore, it allows risk sharing to be split across countries, and induces a negative correlation between relative consumptions and the real exchange rate, for sufficiently low values of θ . The existence of preference shocks will also contribute to explain the lack of risk sharing.

Table 7 shows that each model matches a particular moment of the data better than the others. The advantage of the Bayesian approach over other methods of model comparison is that this method is likelihood-based; namely, all the implications of the model for fitting the data are contained in the likelihood function. The good news is that the model that ranks highest using the marginal likelihood criterion seems to be the one that fits most features of the data best.

D. Shocks and Real Exchange Rate Dynamics

In this subsection, we investigate the importance of the different shocks for explaining real exchange dynamics. We perform this exercise only for our “preferred” model: the incomplete markets model with PCP. The contribution of each shock to the standard deviation of the observable variables in the model are reported in Table 8.²⁹ The results show that the demand shock explains most of real exchange rate variance (49.2 percent), while the country-specific technology shocks come second, explaining 35.5 percent of the variance of the real exchange rate.³⁰ The estimated contributions of monetary and preference shocks are small (9.0 percent and 5.8 percent, respectively).³¹

Only the world technology shock and the preference shocks are able to generate a highly persistent real exchange rate response; however, these shocks do not help explain real exchange rate variability. Monetary policy shocks on their own cannot account for real exchange rate persistence: the first autocorrelation of the real exchange rate is 0.49 under monetary shocks. Contrary to CKM, our results suggest that it is very difficult that monetary shocks and sticky prices with LCP are sufficient to account for the observed real exchange rate volatility. Monetary shocks do have some importance in explaining domestic inflation

²⁹ Lubik and Schorfheide (2005) add error terms to either the UIP or the PPP equations to assess the degree of model misspecification in explaining real exchange rate dynamics.

³⁰ Note, however, that this is an upper bound for the importance of technology shocks. Since the model does not have capital in the production function or sticky wages, then price markup, wage markup, and temporary technology shocks are lumped together in one composite shock and cannot be separately identified.

³¹ Faust and Rogers (2003) find that monetary shocks explain a small share of the volatility in the nominal exchange rate. Clarida and Gali (1994) find that demand shocks explain most of the variance in the real exchange rate fluctuations.

Table 8: Contributions of the Shocks to Selected Second Moments in the Preferred Model

	Euro Area				United States				
Percent variance	Δc	Δy	r	Δp	Δc^*	Δy^*	r^*	Δp^*	Δq
Monetary shocks	10.1	14.7	2.9	14.4	10.5	14.4	5.3	14.9	9.0
Country tech. shocks	12.7	16.2	2.9	24.7	2.8	12.9	2.2	11.4	35.5
World tech. shocks	31.3	33.6	0.9	15.8	48.5	31.8	0.1	33.6	0.6
Preference shocks	41.8	19.3	90.9	44.7	31.8	16.8	79.2	37.4	5.8
Demand shocks	4.1	16.2	2.5	0.4	6.4	24.2	13.1	2.6	49.2
Autocorrelations	c	y	r	Δp	c^*	y^*	r^*	Δp^*	q
Data	0.84	0.86	0.96	0.89	0.87	0.87	0.94	0.90	0.83
Monetary shocks	0.77	0.71	0.45	0.87	0.77	0.70	0.44	0.72	0.49
Country tech. shocks	0.89	0.83	0.82	0.53	0.91	0.84	0.88	0.67	0.73
World tech. shocks	0.93	0.93	0.92	0.85	0.92	0.93	0.83	0.71	0.86
Preference shocks	0.78	0.78	0.97	0.89	0.81	0.76	0.92	0.74	0.85
Demand shocks	0.88	0.62	0.87	0.94	0.87	0.63	0.80	0.97	0.69
Other correlations	c, c^*	c, y	c, c^*	c^*, y^*	$c - c^*, q$	$y - y^*, q$			
Data	0.33	0.81	0.47	0.85	-0.17	0.04			
Monetary shocks	0.33	0.97	0.01	0.96	0.79	0.94			
Country tech. shocks	0.72	0.75	-0.03	0.61	-0.24	0.90			
World tech. shocks	0.99	1.00	0.99	1.00	0.42	0.86			
Preference shocks	-0.30	0.93	0.38	0.82	-0.80	-0.40			
Demand shocks	0.25	-0.10	0.35	-0.35	-0.74	-0.97			

Note: All model-based standard deviations and autocorrelations of nominal variables are computed by simulating the model at the posterior mode. Autocorrelations and cross-correlations of real variables come from simulating the model 1000 times with 124 periods at the posterior mode and applying the HP filter.

rates (14.4 percent and 14.9 percent for the euro area and the United States, respectively). Models with either monetary or world technology shocks as the only driving force yield positive correlations between relative consumptions and the RER of 0.79 and 0.42, respectively. In summary, in order to simultaneously account for real exchange rate dynamics and the *consumption-real exchange rate anomaly*, it is necessary to have a model with technology, preference, and demand shocks.

To better understand the dynamics of the real exchange rate, we now analyze the posterior impulse response functions. Given the importance of technology and demand shocks to explain real exchange rate volatility, we focus the analysis on those two shocks.³² Figures 1 and 2 plot the responses of: (i) consumption and output in both countries; (ii) relative consumption; (iii) the real exchange rate; and (iv) the net foreign asset position to each of these shocks.

³² We use the mode of the posterior distribution of the model's parameters to compute the impulse responses. The impulse responses to monetary, preference, and world technology shocks are available upon request.

Figure 1: Impulse Responses, U.S. Technology Shock

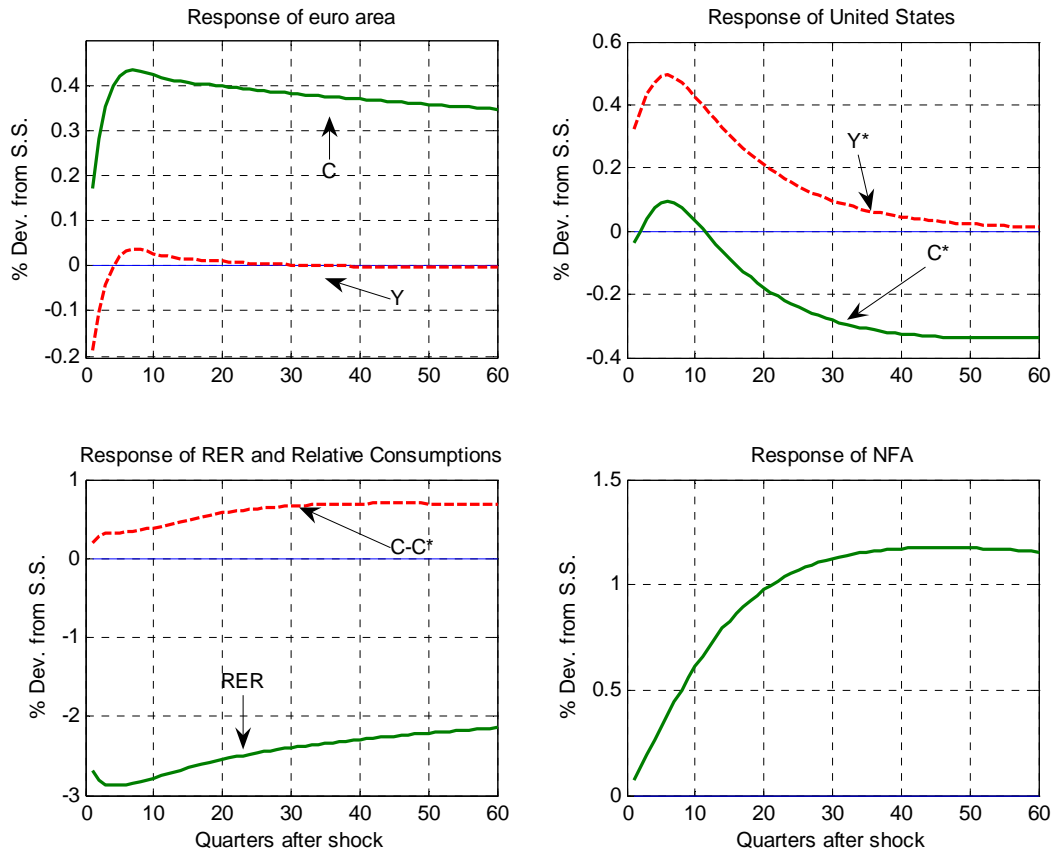


Figure 1 displays the effects of one standard deviation of a U.S. transitory technology shock. A U.S. technology shock expands output in the United States, and reduces inflation. U.S. interest rates fall through the Taylor rule, and U.S. consumption increases. Because of the decline in interest rates, the dollar depreciates (the euro appreciates), which causes consumption to expand in the euro area, but it reduces euro area output. The real exchange rate appreciation of the euro (i.e., real depreciation of the dollar) is consistent with a decrease in U.S. domestic prices (worsening in the terms of trade) due to an improvement in productivity. Because of this, we observe that the euro area net foreign asset position improves, and then returns to its the steady state value very slowly. Euro area technology shocks imply a similar pattern, but with the opposite sign and lower persistence.³³

³³ This figure is available upon request.

Figure 2: Impulse Responses, U.S. Demand Shock

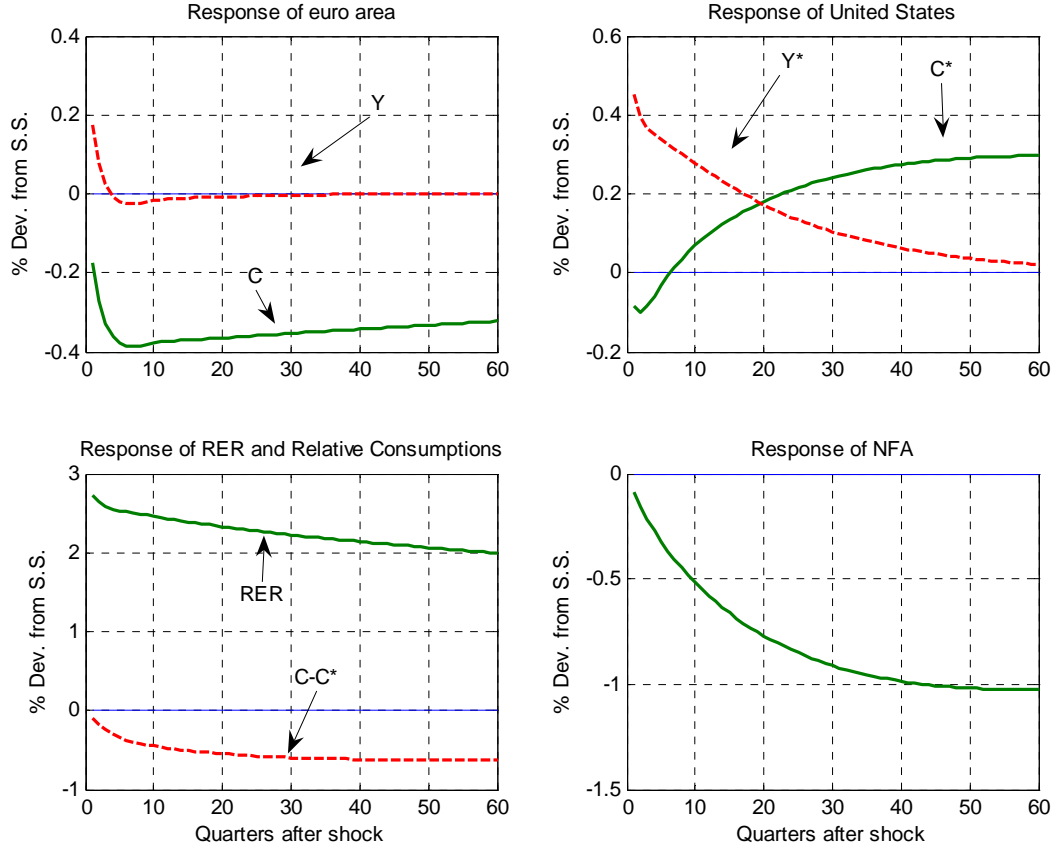


Figure 2 presents the impulse response to a demand shock in the United States. A positive demand shock in the United States generates an increase in output and inflation. U.S. interest rates increase, and therefore consumption declines in the U.S. Consumption also declines in the euro area because when the United States increases interest rates, the euro depreciates. This depreciation of the euro gives a short-lived boost to output. Consequently, to restore the balance in the United States, we observe a persistent NFA decumulation in the euro area due to wealth effects. It takes several periods for consumption in the United States to recover, and for the euro area NFA to slowly increase to its steady-state level. Remarkably, the impulse responses generated by the demand shocks will imply in both cases a negative comovement between the real exchange rate and relative consumptions, as it is observed in the data.

E. Forecasts

Table 9 presents the mean squared errors (MSE) for one-step ahead in-sample forecasts of all models, and compares them to VAR models at various lags. We also estimated VARs with the same nine observable variables as the DSGE models, with up to six lags. For each model m , denote the one-step ahead forecast as $E[z_t^j | \Omega_{t-1}, m]$, where z_t^j denotes each of the nine variables contained in the vector of observed variables z_t , and Ω_{t-1} denotes the information

Table 9: Mean Squared Errors of One Period Ahead Forecasts

(In percent)

	Euro Area				United States				RER
	Cons.	Output	Int.Rate	Inflation	Cons.	Output	Int.Rate	Inflation	
Closed economy	0.46	0.53	0.09	0.21	0.38	0.73	0.11	0.14	-
Complete, PCP	0.32	0.51	0.18	0.37	0.35	0.70	0.11	0.08	4.55
Complete, LCP	0.32	0.52	0.13	0.35	0.38	0.73	0.11	0.08	4.90
Incomplete, PCP	0.43	0.57	0.10	0.43	0.41	0.71	0.11	0.18	4.52
Incomplete, LCP	0.43	0.59	0.11	0.31	0.41	0.71	0.12	0.17	4.76
VAR(1)	0.52	0.51	0.15	0.34	0.61	0.71	0.20	0.27	4.27
VAR(2)	0.49	0.47	0.13	0.30	0.55	0.66	0.19	0.24	4.04
VAR(4)	0.44	0.41	0.12	0.28	0.46	0.53	0.16	0.21	3.73
VAR(6)	0.36	0.36	0.10	0.23	0.39	0.47	0.13	0.16	3.29

set up to period $t-1$. Then, for each model, the MSE of one period-ahead forecasts for each variable is $MSE_m^j = \sum_t \{z_t^j - E[z_t^j | \Omega_{t-1}, m]\}^2$.

All versions of the NOEM model perform quite poorly when trying to forecast the real exchange rate. The preferred model with incomplete markets and PCP is the only one of the four that beats a simple random walk with drift: the “preferred model” has a MSE of 4.56 percent, while the random walk with drift has a MSE of 4.59 percent. Interestingly, a VAR with just one lag does not perform better than the DSGE models, and it is only after including 4 lags in the VAR that the MSE for forecasting the real exchange rate drops significantly.

While none of the DSGE models does a particularly good job in forecasting real exchange rates, they are slightly better in forecasting other macro variables. In terms of forecasting consumption and interest rates in both areas, and inflation in the United States, some versions of the NOEM model outperform the VAR that includes six lags. The forecasting performance of the five DSGE models is quite similar, with no single model standing out as having the best forecasting performance for all variables.³⁴

³⁴ Using net foreign asset position data would enrich the real exchange rate dynamics, as shown by Lane and Milesi-Ferretti (2001). We also estimated the incomplete market models by using the quarterly U.S. net foreign asset position as an observed variable. The estimated parameters were broadly similar and in-sample forecast performance did not improve. Results are available upon request.

VI. CONCLUDING REMARKS

This paper estimates a two-country NOEM model for the euro area and the United States that aims to replicate observed real exchange rate volatility and persistence. We used a Bayesian approach to estimate the models' parameters and compare a baseline two-country model with models containing two main extensions: incomplete markets and sticky prices in imported goods with LCP.

Our results suggest that the complete markets assumption ends up attributing a very small role to international trade. In particular, we obtain a very low estimated parameter for the elasticity of substitution between home and foreign goods. Our near-zero estimate implies that the expenditure-switching effect of a real devaluation as a transmission mechanism is negligible. By contrast, the Bayesian estimation gives empirical support to the incomplete markets assumption. We find that the baseline model with complete markets and law of one price performs well in explaining real exchange rate dynamics, but at the cost of implying volatilities too large for other real variables, especially in the euro area. We find that the addition of incomplete markets, where the law of one price holds, produces the best fit. In particular, this model is able to simultaneously account for real exchange rate volatility and persistence along with the negative correlation between the real exchange rate and relative consumptions. Interestingly, a model with both incomplete markets and sticky import prices in local currency does not perform as well, but still outperforms models with complete markets. We show that both demand and technology shocks have played a major role in explaining the behavior of the real exchange rate, while monetary shocks have not.

There are some interesting avenues for future research, some of which we are exploring in ongoing work. We believe that the failure of the LCP assumption could be due to the fact that we are not explicitly using import price series. To explore the implications of these types of models for aggregate data, some other form of deviations from the law of one price are worth exploring. A promising line of research consists of incorporating distribution services in two-sector (tradable and nontradable goods), two-country models. Campa and Goldberg (2004) provide evidence that deviations from the law of one price at the border due to the presence of distribution services helps explain a lower exchange rate pass-through at the consumer level than at the producer level. Analyzing the out-of-sample forecasting performance of competing NOEM models, along the lines of the exercise performed by Del Negro et al. (2004) in the closed economy, and exploiting the information content of available estimates of the net foreign asset position (Lane and Milesi-Ferretti, 2001) would help clarify the role these models can have for policy formulation and analysis.

APPENDIX: THE METROPOLIS-HASTINGS ALGORITHM

To obtain a random draw of size N from the posterior distribution, a random walk Markov Chain using the Metropolis-Hastings algorithm is generated. The algorithm is implemented as follows:

1. Start with an initial value (ψ^0) . From that value, evaluate the product $L(\{x_t\}_{t=1}^T | \psi^0) \Pi(\psi^0)$.
2. For each i :

$$\begin{cases} \psi^i = \psi^{i-1} & \text{with probability } 1-R \\ \psi^i = \psi^{i,*} & \text{with probability } R \end{cases}$$

where $\psi^{i,*} = \psi^{i-1} + v^i$, v^i follows a multivariate Normal distribution, and

$$R = \min\left\{1, \frac{L(\{x_t\}_{t=1}^T | \psi^{i,*}) \Pi(\psi^{i,*})}{L(\{x_t\}_{t=1}^T | \psi^{i-1}) \Pi(\psi^{i-1})}\right\}.$$

The idea for this algorithm is that, regardless of the starting value, more draws will be accepted from the regions of the parameter space where the posterior density is high. At the same time, areas of the posterior support with low density (the tails of the distribution) are less represented, but will eventually be visited. The variance-covariance matrix of v^i is proportional to the inverse Hessian of the posterior mode and the constant of proportionality is specified such that the random draw has some desirable time series properties.

In all cases, the acceptance rates were between 20 and 30 percent, and the autocorrelation functions of the parameters decay fairly fast. We used two methods to simulate the posterior that delivered the same result (with very small numerical differences). First, we simulated the posterior 250,000 times taking as initial value the prior mean, and updating the Hessian of the posterior every 25,000 draws. We discarded all the values from that chain, and from the last value, generated a second chain of size 250,000, updating the Hessian each 25,000 draws. The second method involved finding the posterior mode using standard optimization algorithms to be used as initial value. Then, we generated a chain of 250,000 draws, updating the Hessian every 25,000 draws.

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