International Policy Coordination and Simple Monetary Policy Rules

Wolfram Berger and Helmut Wagner
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This paper studies the optimal design of monetary policy in an optimizing two-country sticky price model. We suppose that the production sequence of final consumption goods stretches across both countries and is associated with vertical trade. Prices of final consumption goods are sticky in the consumer’s currency. Pursuing an inward-looking policy, as suggested in recent work, is not optimal in this set-up. We also ask which simple, i.e. non-optimal, targeting rule best supports the welfare maximizing policy. The results hinge critically on the degree of price flexibility and the relative importance of cost-push and productivity shocks. In many cases, a strict targeting of price indices like producer or consumer price indices is dominated by rules that allow for some fluctuations in prices such as nominal income or monetary targeting.

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I. INTRODUCTION

What is the optimal design of monetary policy in open economies? This is a longstanding issue in monetary economics. From the most recent debate it is far from clear that monetary policy in open economies should have any international dimension at all. Several writers including Clarida and others (2002), Gali and Monacelli (2003), and Obstfeld and Rogoff (2002) make a strong case in favor of an inward-looking monetary policy. They argue that producer prices should be chosen as a target for welfare maximizing monetary policy. The baseline of these studies is that international integration in goods and financial markets decreases the need to take account of macroeconomic developments abroad when deciding about the optimal monetary policy stance. According to these authors there is in effect no difference in the policy problem faced by policymakers in closed and open economies. Of course, the thesis of the optimality of an inward-looking monetary policy has not remained undisputed. Some authors, for example, Kollmann (2002), Smets and Wouters (2002), and Sutherland (2004a), point out that this policy prescription is only valid for relatively simple models. More complex models imply that targeting producer prices is generally not optimal.

Hence, in fact, there is at present little or no consensus in the literature on optimal monetary policy in times of highly integrated goods and financial markets on issues such as: what monetary target should optimally be chosen in an open economy? What weight should be assigned to external factors in the optimal monetary rule? And, provided the optimal rule is infeasible to implement, which simple, that is nonoptimal, targeting rule can best support the efficient resource allocation?

Employing a stochastic general equilibrium framework of the New Keynesian type, this paper addresses these questions by accounting for an important change in the nature of international cross-country linkages brought about by globalization (Hummels and others, 2001; and Yi, 2003). The production sequence of final consumption goods increasingly stretches across many countries and is associated with vertical trade. In the light of these changes, the interdependence of countries is increasingly based on trade along vertical production chains. That is, cross-border trade involves both intermediate and final goods. If these feature of globalized markets are taken into account, it turns out that optimal monetary policy should clearly have an international dimension.

We differentiate between final consumption goods and the intermediate goods needed to produce them. A fraction of final goods producers has to set the nominal prices of consumer goods in advance of the realization of shocks, while prices of intermediate goods are taken to be perfectly flexible. We further suppose that consumption goods prices are
sticky in the consumer’s (local) currency. As in Sutherland (2004a), both productivity and cost-push shocks are considered. World aggregate welfare is maximized when monetary policy responds to both types of shocks irrespective of whether they originate at home or abroad. In contrast to the related model by Devereux and Engel (2004) we find that despite local currency pricing optimal monetary policy implies a flexible exchange rate.

Optimal monetary rules, however, involve very demanding information requirements that may prevent their practical implementation. Kollmann (2002) and Svensson (2004), among others, point out that simple, that is nonoptimal, rules seem to reflect actual central bank behaviour quite well. We therefore go on to ask which simple targeting rule comes closest in welfare terms to the optimal rule. Producer price targeting, consumer price targeting, nominal income targeting and monetary targeting are investigated. The results hinge critically on the degree of price stickiness and the relative importance of productivity and cost-push shocks.

Generally, nominal income targeting and monetary targeting seem to fare better than producer price and CPI targeting. This result only reverses if the degree of price flexibility is comparatively small and productivity shocks are much more important than cost-push shocks. In this case, a strict targeting of price indexes maximizes welfare. A policy of CPI targeting is generally (slightly) superior to producer price targeting under these circumstances. If, however, cost-push shocks are a matter of concern for the economy, nominal income targeting and monetary targeting are unambiguously preferable to any kind of price targeting. Nominal income targeting is found to always dominate monetary targeting irrespective of the parameter combination. A numerical simulation of the model, however, shows that the difference between both in welfare terms is very small. As opposed to the optimal policy rule and to the other simple rules, monetary targeting leads to a fixed exchange rate and thus prevents optimal terms of trade adjustments.

The remainder of the paper is structured as follows. The model is developed in the next section. In Section III, the welfare criterion is derived and the determination of consumption and output is discussed. In Section IV, the optimal monetary policy rule and welfare under the optimal policy rule are derived. Section V compares the welfare results of four simple targeting rules. Section VI concludes.

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2 Deviations from the law of one price both at the disaggregated level and in aggregate price indices are well established. See, for example Engel and Rogers (2001), and Parsley and Wei (2001).
II. THE MODEL

A. Basic Assumptions

The world economy consists of two equally sized countries inhabited by a continuum of households of the yeoman-farmer type. Households over the [0,1] interval live in the home country, while households in the (1,2] interval are residents of the foreign country. Analogously, goods over the [0,1] interval are produced in the home country while goods in the (1,2] interval are produced in the foreign country. In the following sections, the equations for the representative home household are presented while the equations for the representative foreign household are omitted most of the time. Generally, mirror images hold for the foreign country. Throughout the paper, an asterisk indicates a foreign variable.

Following, e.g., Devereux and Engel (2004) and Sutherland (2004a) we suppose that consumption goods are produced by two type of agents. The first type of agents, called "fixed-price agents", is required to set prices before shocks occur and monetary policy is set. The second type, called "flex-price agents", operates in markets where prices are set after the realization of shocks and the setting of monetary policy. In both countries, the share of fixed-price agents in the population is given by \( v \), so that \( 1 - v \) is the share of flex-price agents. \( v \) can therefore be interpreted as a measure of the degree of price stickiness.\(^3\) Markets for intermediate goods, however, are characterized by full price flexibility. That is, all intermediate goods producers are flex-price producers. This framework allows for a meaningful discussion of price targeting while preserving the ability of monetary policy to influence real activity. In the following, the subscript \(^1\) (\(^2\)) will indicate variables related to fixed-price (flex-price) agents.

B. Preferences and Prices

The utility of the representative home household is given by

\[
U = E \left[ \log C + \chi \log \frac{M}{P} - Ky(z) \right], \chi > 0
\]  

(1)

\( C \) denotes a consumption index defined below; \( M \) denotes the domestic end-of period money stock, \( P \) is the consumer price index (also defined below) and \( y(z) \) is the output of intermediate good \( z \). \( E \) is the rational expectation operator and \( K \) denotes a stochastic shock to the labor supply (productivity shock). The third term on the rhs expresses the disutility of work effort in terms of output.\(^4\) Both home and foreign shocks are symmetrically distributed

\(^3\) This framework can be thought of as a static version of Calvo (1983)’s staggered price setting.

\(^4\) The production technology is linear in work effort (see below).
over a finite interval around zero with \( E[\log K] = E[\log K^*] = 0 \) and 
\[ VAR[\log K] = VAR[\log K^*] = \sigma_K^2. \]

The consumption indices are defined as:
\[
C = \lambda \left( C_H \right)^\gamma \left( C_F \right)^{1-\gamma}, \quad C^* = \lambda \left( C_F^* \right)^\gamma \left( C_H^* \right)^{1-\gamma}
\]
(2)
where \( \lambda = \gamma^{-\gamma} \left( 1-\gamma \right)^{(\gamma-1)} \) and \( \frac{1}{2} \leq \gamma \leq 1 \). \( \gamma \) measures the share of home (foreign) goods in home (foreign) agents’ consumption basket. The set-up thus allows for a home bias in consumption.

Combined with the assumption of zero initial non-monetary wealth, the structure of the preferences implies that financial markets are redundant (see Cole and Obstfeld (1991) and Corsetti and Pesenti (2005)). Home and foreign consumers share their consumption risks perfectly without financial markets. The ratio of marginal utilities in consumption is equal to the ratio of aggregate prices, \( \left( C/C^* \right)^{-1} = P/(SP^*) \), where \( P \) and \( P^* \) denote the home and the foreign countries’ CPI and \( S \) is the exchange rate expressed as the price of foreign currency in home currency.

\( C_H \) and \( C_F \) are indices of home and foreign differentiated consumption goods with
\[
C_H = \mu \left( C_{H,1} \right)^{\nu} \left( C_{H,2} \right)^{1-\nu}, \quad C_F = \mu \left( C_{F,1} \right)^{\nu} \left( C_{F,2} \right)^{1-\nu}
\]
(3)
with \( \mu = \nu^{-}\nu \left( 1-\nu \right)^{(1-\nu)} \). The bundles of fixed-price and flex-price consumption goods are defined as CES aggregates over individual consumption goods with the elasticity of substitution \( \varphi \) for all bundles. \( C_{H,i}(z) \) and \( C_{F,i}(z^*) \) denote a home household’s consumption of a particular brand produced by type \( i \). \( z \) denotes a home variety, \( z \in [0,1] \), while \( z^* \in (1,2] \) denotes a foreign variety.

The price indices corresponding to the consumption goods indices are derived in the usual way. The CPI and the producer price indices are given by
\[
P = P_H^\gamma P_F^{1-\gamma}
\]
(4)
\[
P_H = P_{H,1}^{\nu} P_{H,2}^{1-\nu}, \quad P_F = P_{F,1}^{\nu} P_{F,2}^{1-\nu}.
\]
(5)

The price indices of fixed-price and flex-price goods are
$P_{H,1} = \left[ \frac{1}{U} \int_0^U P_{H,1}(z)^{1-\phi} \, dz \right]^{1-\phi}$, $P_{H,2} = \left[ \frac{1}{U} \int_U^1 P_{H,2}(z)^{1-\phi} \, dz \right]^{1-\phi}$

$P_{F,1} = \left[ \frac{1}{U} \int_1^{1+U} P_{F,1}(z^*)^{1-\phi} \, dz^* \right]^{1-\phi}$, $P_{F,2} = \left[ \frac{1}{U} \int_1^{2} P_{F,2}(z^*)^{1-\phi} \, dz^* \right]^{1-\phi}$

$P_{H,i}(z)$ and $P_{F,i}(z^*)$ are individual goods prices in the home currency. The law of one price is assumed to hold so that, e.g. $P_{F}(z^*) = P_{F}^*(z^*)S$. Purchasing power parity, however, does generally not hold because of the home bias in consumption. Only if the home bias vanishes for $\gamma = \frac{1}{2}$ purchasing power parity holds.

C. Production

We differentiate between final consumption goods and the intermediate goods needed to produce them. In both countries, producers in the final consumption goods sector and in the intermediate goods sector enjoy a degree of monopoly power. Final commodities are produced by bundling a continuum of differentiated intermediate goods. Both countries operate the same technology.

$Y_{H,i}(z) = \kappa y_H^{\alpha_i} y_i^{1-\alpha_i}$, \hspace{1cm} $i = 1, 2,$

with $\kappa = \alpha^{1-\alpha}(1-\alpha)^{-(1-\alpha)}$. $y_H$ and $y_F$ denote bundles of home and foreign intermediate goods, which are defined as CES aggregates over individual intermediate goods with an elasticity of substitution of $\omega$. The demand for home intermediates stems from home and foreign countries’ consumption goods producers, so that the aggregate equilibrium conditions for home and foreign intermediate goods read

$y = y_H + y_F = \left[ \frac{p_H}{p_I} \right]^{-1} (\alpha y_H + (1-\alpha) y_F)$, $y^* = y_F^* + y_F = \left[ \frac{p_F^*}{p_I} \right]^{-1} ((1-\alpha)Y_H + \alpha Y_F^*)$.

$p_H$ and $p_F^*$ are the price indices of the home and foreign intermediate goods bundles. $p_I$ denotes the intermediate goods price index which is given by $p_I = (p_H)^{\alpha} (Sp_F^*)^{1-\alpha}$. The law of one price holds for individual intermediate goods. Purchasing power parity in terms of intermediate goods bundles, however, does not hold owing to the home bias in the production of final goods $\alpha$.

It is further assumed that each household produces a differentiated intermediate good. The production of an intermediate goods requires labor input only. The domestic and foreign
production technologies are identical and are linear in hours of work. One unit of labor input yields one differentiated intermediate good.\textsuperscript{5}

**D. Consumption and Money Demand**

Households decide optimally about their (intratemporal) consumption allocation and their money holdings. Home household’s demands for home fixed-price and flex-price goods \((C_{H,i}(z)\text{ and } C_{H,i}^*(z))\), and for foreign fixed-price and flex-price goods \((C_{F,i}(z^*)\text{ and } C_{F,i}^*(z^*))\) are given by

\[
C_{H,i}(z) = \frac{1}{\psi} \left[ \frac{P_{H,i}(z)}{P_{H,i}} \right]^{\phi} C, \quad C_{H,i} = \psi \left[ \frac{P_{H,i}}{P_{H}} \right]^{-1} \ C \\
C_{H,i}(z) = \frac{1}{1-\psi} \left[ \frac{P_{H,i}(z)}{P_{H,i}} \right]^{\phi} C, \quad C_{H,i} = (1-\psi) \left[ \frac{P_{H,i}}{P_{H}} \right]^{-1} \ C \\
C_{F,i}(z^*) = \frac{1}{\psi} \left[ \frac{P_{F,i}(z^*)}{P_{F,i}} \right]^{\phi} C, \quad C_{F,i} = \psi \left[ \frac{P_{F,i}}{P_{F}} \right]^{-1} \ C \\
C_{F,i}(z^*) = \frac{1}{1-\psi} \left[ \frac{P_{F,i}(z^*)}{P_{F,i}} \right]^{\phi} C, \quad C_{F,i} = (1-\psi)(1-\gamma) \left[ \frac{P_{F,i}}{P_{F}} \right]^{-1} \ C.
\]

Foreign demands can be described by similar equations.

The representative home household of type \(i = 1, 2\) faces a budget constraint that is given by:

\[
M - M_0 = (1 + \tau^f) \left[ P_{H,i}(z) C_{H,i}(z) + \bar{S} \bar{P}_{H,i}(z) C_{H,i}^*(z) \right] \\
+ (1 + \tau^\text{int}) P_{H}(z) y(z) - PC - PT - p_{H} y_{H} - p_{F} y_{F}.
\]

\(M\) and \(M_0\) are money holdings at the beginning and at the end of the period. \(T\) denotes real lump-sum taxes in terms of the consumption index and \(\tau^\text{int}\) and \(\tau^f\) are production subsidies for final goods (\(f\)) and intermediate goods (\(\text{int}\)) producers. The government budget

\[
\text{\textsuperscript{5} The production technology can be explicitly written as } y(z) = K^{-h}\text{ with } h\text{ denoting the work effort of the representative household. Shocks } K > 0 \text{ are therefore negative productivity shocks reducing the quantity of goods produced with a given labour input.}
\]
constraint can be formulated as \( M - M_0 - \tau' PHY - \rho^{\text{int}} P_H Y_H = -PT \). Households’ optimal money demand is derived by maximizing their utility function subject to their budget constraint. The first order condition for the optimal money demand implies that money market equilibrium is given if \( M = \chi PC \). The money supply is set by the central bank by following a monetary rule that may depend on all shocks.

E. Optimal Price Setting

Consumption Goods Sector

The consumption goods sectors in both countries are subject to cost-push shocks. Cost-push shocks are introduced into the model as random fluctuations in the (net) mark-up over marginal costs that monopolistically competitive final goods producers set (see Sutherland (2004a)). In the consumption goods sector, the mark-up augmented by the production subsidy can be derived as \( \phi \equiv \phi \left[ \left( \phi - 1 \right) \left( 1 + \tau' \right) \right] \). Hence, fluctuations in \( \phi \) can either be caused by random changes in the degree of monopoly power (reflected in \( \phi' \)) or by random changes in the production subsidy \( \left( \tau' \right) \). \( \phi \) is assumed to be symmetrically distributed over a finite interval with \( E[\log \phi] = E[\log \phi^*] = 0 \) and \( VAR[\log \phi] = VAR[\log \phi^*] = \sigma_\phi^2 \).

As discussed in the Introduction, it is assumed that there is no pass-through of exchange rate changes into prices, i.e. the foreign (home) currency price of a home (foreign) good is completely insulated from exchange rate changes. Following Corsetti and Pesenti (2005), incomplete pass-through can be represented formally by \( P_{H,i}^*(z) = \tilde{P}_{H,i}(z) S^{\eta} \) and \( P_{F,j}^*(z^*) = \tilde{P}_{F,j}(z^*) S^{\eta} \) where we assume that \( \eta = \eta^* = 0.6 \). \( \tilde{P}_{H,i}(z) \) and \( \tilde{P}_{F,j}(z^*) \) denote the predetermined components of export prices that are set by final goods producers and that are denominated in the producers’ currency.

Fixed-price agents are required to set prices one period ahead of time, i.e. before shocks have been realized and monetary policy has been set. Since, in equilibrium, all fixed-price producers choose the same price, optimal prices can be expressed in terms of prices of consumption bundles. The optimal preset prices for home and foreign fixed-price consumption goods sold at home and abroad can be derived as

\[ P_{H,i}^*(z) = \tilde{P}_{H,i}(z) S^{\eta} \]

\[ P_{F,j}^*(z^*) = \tilde{P}_{F,j}(z^*) S^{\eta} \]

---

\(^6\) Corsetti and Pesenti (2005) allow \( \eta \) and \( \eta^* \) to vary between zero and one. We follow Devereux and Engel (2003) and focus on the case of zero pass-through for simplicity reasons.
Fixed-price agents incorporate a risk premium in prices which depends on the second moments of variables. Flex-price agents, however, need not incorporate a risk premium because they do not face any uncertainty. The first order condition for flex-price agents holds ex post. Optimal prices set by flex-price producers are given by

\[ P_{H,2} = \phi p_I, \quad P^*_{F,2} = \phi^* p^*_I \]  

\[ \bar{P}_{H,2} = \phi p_I / S, \quad \bar{P}^*_{F,2} = \phi^* p^*_I S. \]  

When prices are free to adjust instantaneously the law of one price holds even though the degree of pass-through is assumed to be zero. We have \( P^*_{H,2} = \bar{P}_{H,2} = P_{H,2} / S \).

### Intermediate Goods Sector

For simplification reasons it is assumed that there is perfect price flexibility and complete pass-through on the intermediate goods markets. Optimal price setting by intermediate goods producers yields

\[ p_{H} = \psi KM, \quad p^*_{F} = \psi^* K^* M^* \]  

where \( \psi \) denotes the net mark up given by \( \psi = \omega / (\omega - 1)(1 + \tau^{\text{int}}) \). For the remainder of the paper it is assumed that \( \tau^{\text{int}} = 1 / (\omega - 1) \) so that \( \psi = 1 \). The monopolistic distortion in the intermediate goods sector is completely offset by the production subsidy.

### III. Welfare, Output and Consumption

#### A. Methodology and Welfare Criterion

To solve the model we use second order approximations around the non-stochastic steady state. The non-stochastic steady state of the model, denoted by a bar over a variable, is

---

7 Devereux and Engel (2004) argue that prices of intermediate goods can be regarded as more flexible than prices of consumption goods. Moreover, empirical studies show that the degree of pass-through is by far higher for imported goods than for consumption goods (see Campa et al. (2005)).
characterized by $\bar{K} = \bar{K}^* = 1$, $\bar{\phi} = \bar{\phi}^* = 1$ and $\sigma_k^2 = \sigma_\phi^2 = 0$. All households then produce and consume the same quantity of goods.

Households’ utility function provides a measure of welfare that is used to study the welfare results under fixed and flexible exchange rates. As usual in the literature, the utility service of real balances is assumed small enough to be neglected.\(^8\) The aggregate welfare criterion can then be formulated as

$$\tilde{W} = E[\log C - Ky].$$

A second-order approximation of the welfare criterion is given by

$$\tilde{W} = E\left\{\hat{C} - \tilde{y} - \frac{1}{2}(\tilde{y} + \hat{K})^2\right\} + o^3$$

\(\tilde{W}\) denotes the expected deviation of welfare from its non-stochastic steady state value. Terms of order three and above are collected in \(o^3\). A hat over a variable indicates a log deviation of that variable from its deterministic steady state, formally defined as

$$\hat{X} = \log(X / \bar{X}).$$

Intuitively, welfare depends positively on the expected log deviation of consumption from its steady state value and negatively on the expected log deviation of work effort (output) from the steady state. Welfare further depends negatively on the fluctuations of the expected log deviation of the disutility of work effort (output) around the steady state.\(^9\)

When evaluating global welfare, we can make use of a simplification. Equations (8) and the consumption subindexes imply that $\hat{C} + \hat{C}^* - \tilde{y} - \tilde{y}^* = 0$. Welfare in the coordinated regime therefore depends only on the fluctuations of the expected log deviations of the disutility of work effort (output) around the steady state.

$$\tilde{W}^{CG} = \tilde{W} + \tilde{W}^* = -E\left\{\frac{1}{2}(\tilde{y} + \hat{K})^2 + \frac{1}{2}(\tilde{y}^* + \hat{K}^*)^2\right\} + o^3$$

The model is further simplified by assuming that all shocks are uncorrelated with one another.

---

\(^8\) Formally, \(\chi\) is assumed to be infinitesimally small, \(\chi \to 0\). See, e.g., Obstfeld and Rogoff (1995).

\(^9\) Although work effort enters households’ utility function linearly (see equation (1)), households’ utility decreases in the volatility of work effort in the approximated utility function (18). This effect is based on Jensen’s inequality as pointed out by Sutherland (2004b).
B. Determination of Output and Consumption

Before we delve into the welfare analysis of monetary policy the intuition of the model can be illustrated by looking at how output and consumption are determined. To do that we restrict ourselves to first-order accurate solutions. Note first that all pre-set prices only contain terms of order two and above. A first-order accurate solution for pre-set prices can therefore be written as $\hat{P}_{H,t} = \hat{P}_{H,t-1} = 0 + o^2$ where $o^2$ collects all terms of higher order than $n$. Equivalent equations hold for foreign pre-set prices.

Using the definition of overall consumption given in equation (2), realized consumption can be expressed as

\[
\begin{align*}
\hat{C} &= \nu \hat{M} - (1 - \nu) \left[ b \hat{K} + (1 - b) \hat{K}^* \right] - (1 - \nu) \left[ \gamma \hat{\phi} + (1 - \gamma) \hat{\phi}^* \right] + o^2 \\
\hat{C}^* &= \nu \hat{M}^* - (1 - \nu) \left[ b \hat{K}^* + (1 - b) \hat{K} \right] - (1 - \nu) \left[ \gamma \hat{\phi}^* + (1 - \gamma) \hat{\phi} \right] + o^2.
\end{align*}
\]

with $b = 1 - \gamma - \alpha (1 - 2\gamma)$ and where we considered that from equation (13) and its foreign counterpart it follows that $\hat{P}_{H,t} = \alpha \hat{K} + (1 - \alpha) \hat{K}^* + \hat{\phi} + \hat{M}$ and $\hat{P}_{F,t} = \alpha \hat{K}^* + (1 - \alpha) \hat{K} + \hat{\phi}^* + \hat{M}^*$. As long as a subset of producers is able to adjust prices, consumption in the home and in the foreign country are affected by home and foreign productivity shocks. The reason is that final goods production in both countries relies on home and foreign inputs. It is also apparent that monetary adjustments have no spill-over effects with respect to consumption. A change in home monetary policy only affects home consumption while a change in the foreign money supply only affects foreign consumption. Changes in the exchange rate that alter relative prices of flex-price consumption goods and thus give rise to an expenditure switching effect isolate the economies from monetary spill-overs in consumption.\(^{10}\)

Output of consumption goods, however, varies with monetary adjustments in both countries according to the structure of consumption preferences (i.e. the value of $\gamma$)

\[
\begin{align*}
\hat{Y}_u &= \nu \gamma \hat{M} + \nu (1 - \gamma) \hat{M}^* - (1 - \nu) \left[ \gamma \hat{\phi} + (1 - \gamma) \hat{\phi}^* \right] + o^2 \\
\hat{Y}_u^* &= \nu \gamma \hat{M}^* + \nu (1 - \gamma) \hat{M} - (1 - \nu) \left[ \gamma \hat{\phi}^* + (1 - \gamma) \hat{\phi} \right] + o^2.
\end{align*}
\]

A first-order approximation of output of intermediate goods in both countries can be written as

\(^{10}\) This knife-edge result is based on our assumption of a unitary elasticity of substitution between home and foreign goods.
\[
\hat{y} = vu \left[ K^* - \hat{K} \right] + v \left[ b\hat{M} + (1-b)\hat{M}^* \right] - (1-v) \left[ \hat{K} + \alpha \hat{\phi} + (1-\alpha) \hat{\phi}^* \right] + o^2
\]

\[
\hat{y}^* = vu \left[ \hat{K} - K^* \right] + v \left[ b\hat{M}^* + (1-b)\hat{M} \right] - (1-v) \left[ \hat{K}^* + \alpha \hat{\phi}^* + (1-\alpha) \hat{\phi} \right] + o^2
\]

with \( u = 2\alpha (1-\alpha) \). As long as there is a degree of price stickiness that allows monetary policy to affect real activity, a monetary adjustment anywhere in the world affects both countries. E.g., an increase in the home money supply stimulates home consumption demand (see above) that in turn causes an expansion of home and foreign intermediate goods output. Equations (21) also show that output production is shifted to the country with the most favorable productivity shock (first term on the rhs). Moreover, output and thus work effort are affected by cost-push shocks at home and abroad. Cost-push shocks affect the output of intermediate goods as long as there is a degree of price flexibility in the economies, i.e. \( \nu < 1 \). Flexprice producers react to a positive cost-push shock by raising consumption goods prices. The resulting contraction in demand for consumption goods translates into a fall in demand for intermediate goods at home and abroad and thus affects \( \hat{y} \) and \( \hat{y}^* \). If, however, prices of consumption goods are completely sticky, producers cannot pass the fluctuation in costs on to consumers and intermediate goods output remains unaffected. Instead, cost-push shocks lead to changes in final goods producers’ profit margin.

IV. OPTIMAL MONETARY POLICY AND POLICY COORDINATION

A. Optimal Monetary Policy Rule

Monetary policy is conducted by following policy rules. Generally, it is assumed that central banks are able to pre-commit irrevocably to money supply rules. Before simple targeting rules are considered in the next section the welfare implications of following optimal monetary policy rules are derived as a benchmark for the remainder of the paper. We concentrate on the cooperative solution as the best solution policymakers can achieve. It is supposed that a single world central bank chooses rules for the home and foreign money supplies that maximize world welfare given by \( \tilde{W} + \tilde{W}^* \).

The optimal monetary policy is set by following a monetary rule that may depend on the shocks \( K, K^*, \phi \) and \( \phi^* \). The home policy rule is given by \( M = M_0 \delta_k K + \delta_{K^*} K^* + \delta_\phi \phi + \delta_{\phi^*} \phi^* \). In log deviation form, the policy rule can be expressed as

\[
\hat{M} = \delta_k \hat{K} + \delta_{K^*} \hat{K}^* + \delta_\phi \hat{\phi} + \delta_{\phi^*} \hat{\phi}^*.
\]

The parameters \( \delta_k, \delta_{K^*}, \delta_\phi \) and \( \delta_{\phi^*} \) are chosen before shocks occur and prices are set.
B. Optimal Monetary Policy and Welfare

Optimal rules are chosen by optimally setting the feedback parameters $\delta_K, \delta_K^*, \delta_\phi$ and $\delta_\phi^*$ for the home economy and their counterparts $\delta_K^*, \delta_K^{**}, \delta_\phi^*$ and $\delta_\phi^{**}$ for the foreign economy. If monetary policy is internationally coordinated, the single world central bank specifies the home and foreign policy rules by choosing the following set of feedback parameters:

$$
\delta_K^C = \delta_K^{*C} = -\frac{(1-\gamma-\alpha)(1-2\alpha)}{\nu(2b-1)}, \quad \delta_K^{*C} = \delta_K^{*C} = -\frac{(\alpha-\gamma)(1-2\alpha)}{\nu(2b-1)}
$$

$$
\delta_\phi^C = \delta_\phi^{*C} = -\frac{\gamma(1-\nu)(1-2\alpha)}{\nu(2b-1)}, \quad \delta_\phi^{*C} = \delta_\phi^{*C} = \frac{(1-\gamma)(1-\nu)(1-2\alpha)}{\nu(2b-1)}.
$$

The superscript “C” indicates the cooperative case. The feedback parameters for the coordinated solution show that the optimal monetary policy for complete price flexibility ($\nu = 0$) is undefined. In this case output and consumption are exogenously determined and completely independent of monetary policy (see equations (19) – (21)). Hence, aggregate welfare (18) cannot be affected by monetary policy. If, however, price flexibility is less than perfect, monetary policy is able to influence the resource allocation. Following the optimal policy rules derived above, policymakers are able to realize the flex-price allocation in the coordinated regime. Global welfare is then given by

$$
\tilde{W}^{G,C} = 0.
$$

Equations (23) show that both policymakers (generally) react to both home and foreign productivity and home and foreign cost-push shocks. An inward-looking policy as suggested in recent work thus is not optimal. A productivity shock that lowers productivity in the intermediate goods sector leads to an increase in the marginal production costs of consumption goods in both countries irrespective of its origin. Since fixed-price producers by definition are not able to change their price in the face of shocks, monetary adjustments are necessary to bring about the required reallocation of resources.11

Cost-push shocks in one country also lead (generally) to a monetary policy reaction in both countries. As long as there is a minimum degree of price flexibility, responding to cost-push shocks is optimal because, as discussed above, cost-push shocks affect intermediate

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11 As stressed by Devereux and Engel (2004), the consideration of fixed-price and flex-price producers creates another source of inefficiency. In the face of stochastic productivity disturbances production of fixed- and flex-price producers will differ. While flex-price producers vary prices and thus ensure that their output remains on the efficiency frontier, fixed price producers are prevented from an optimal adjustment to shocks by price contracts.
goods output and thus work effort. Flex-price producers react to a positive cost-push shock by raising prices. The resulting contraction in demand for flex-price consumption goods translates into a fall in demand for intermediate goods at home and abroad and thus affects work effort in both countries. Monetary policy therefore has an incentive to react because the socially efficient level of work effort has not changed. For completely sticky consumption goods prices, however, there is no response to cost-push shocks when policies are coordinated. The reason is that in this case output of consumption goods is not affected by cost-push shocks and therefore work effort expended in the production of intermediates is not altered. In contrast to the work of Sutherland (2004a), not reacting to cost-push shocks is generally not optimal.

V. SIMPLE RULES AND WELFARE

A. Targeting Rules

Although equations (23) show that optimal monetary policy in this model can be described by rules that have a relatively simple form, analyzing the welfare effects of simple, non-optimal rules is worthwhile. That optimal monetary policy looks rather simple is due to the relatively simple structure of the model that allows for an analytical solution. Optimal rules, however, might become highly complicated and hardly feasible to compute in more complex models. Moreover, as Kollmann (2002) and Svensson (2004) point out, simple rules seem to reflect actual central bank behavior quite well. In addition, simple rules facilitate commitment because the public can easily monitor whether the policymaker adheres to that rule.

This section is therefore devoted to the welfare analysis of simple targeting rules. The key question now is: which simple policy rule best supports the world coordinated policy? Welfare associated with the targeting rules is thus evaluated at the global level, i.e. based on the global welfare criterion (18). The simple rules considered here are producer price index targeting, consumer price index targeting, nominal income targeting and monetary

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12 Remember that global welfare depends only on the variance of the disutility of work effort (see equation (18)).

targeting.\textsuperscript{14} We concentrate on symmetric policy regimes, i.e. we assume that the policymakers in both countries follow the same rule.\textsuperscript{15}

In our model, a targeting rule is taken to mean a rule that eliminates all fluctuations in the targeted variable. Hence, a rule that targets variable $X$ is modeled as a rule for the money supply that leads to $\hat{X} = 0$ ex post. This is in line with McCallum and Nelson (1999), who define a targeting rule as a commitment to set the policy instrument rate in order to realize a pre-defined target for the target variables.\textsuperscript{16} In this paper, the simple targeting rules are expressed as state-contingent rules for the money supplies at home and abroad.\textsuperscript{17}

\textbf{B. Welfare Ranking of Simple Targeting Rules}

When evaluating the welfare implications of alternative targeting rules one has to keep in mind that the criterion for global welfare, given in equation (18), only depends on second moments. Non-stochastic terms therefore need not be considered when calculating welfare. Since fixed-price agents enter a period with preset prices, optimal prices given in equations (11) and (12) are non-stochastic and therefore play no role in the welfare analysis (see Sutherland (2004a)). On these grounds, all nonstochastic terms are omitted in the formulations of the money supply rules (see the Appendix for details). To allow for a meaningful discussion of price targeting rules, the monetary rules and welfare in this section are derived for $\nu < 1$.

\textbf{Producer Price Targeting}

A policy of producer (domestic) price index targeting can formally be formulated as $\hat{P}_H = 0$ and $\hat{P}_F = 0$ in our model. Indicating such a policy with the superscript "PPT" this policy requires the policymakers to set the money supplies according to

\begin{equation}
\hat{M} = \beta \hat{X}
\end{equation}

\textsuperscript{14} Rules are only considered in their strict form, i.e. policymakers are assumed to ignore other objectives.

\textsuperscript{15} Sutherland (2004a) shows that asymmetric regimes are welfare inferior to symmetric ones in a model where both economies are mirror images of each other as in our model.

\textsuperscript{16} This definition is similar to the definition of a specific targeting rule introduced by Svensson (2002). Svensson, however, reserves the term target for variables that enter the policymaker’s objective (loss) function.

\textsuperscript{17} Sutherland (2004a) stresses these rules can be alternatively formulated as feedback rules that follow $\hat{M} = \beta \hat{X}$ and that do not require information about shocks. The targeting rule can be implemented by adopting a sufficiently high value for $\beta$.
\[ \dot{M}^{\text{PPT}} = -\alpha \dot{K} - (1 - \alpha) \dot{K}^* - \dot{\phi} \]  
\[ \dot{M}^{\ast\text{PPT}} = -\alpha \dot{K}^* - (1 - \alpha) \dot{K} - \dot{\phi}^*. \]  

Global welfare for a policy of targeting the producer price index is given by

\[ \tilde{W}_{G,\text{PPT}} = -2\nu^2 \left[ (1 - \gamma)(1 - 2\alpha)^2 \right] \sigma_k^2 - \left\{ \left[ \nu b + \alpha (1 - \nu) \right] \right\}^2 + \left[ \nu (1 - b) + (1 - \nu)(1 - \alpha) \right] \sigma_\phi^2 + o^3. \]  

**Consumer Price Targeting**

Targeting the consumer price index means that both policymakers adopt rules that ensure that \( \hat{P} = 0 \) and \( \hat{P}^* = 0 \). Again omitting non-stochastic terms, following these rules (denoted by the superscript "CPT") implies

\[ \dot{M}^{\text{CPT}} = -[1 - \gamma + \alpha (2\gamma - 1)] \dot{K} - [\gamma + \alpha (1 - 2\gamma)] \dot{K}^* - \gamma \dot{\phi} - (1 - \gamma) \dot{\phi}^* \]  
\[ \dot{M}^{\ast\text{CPT}} = -[1 - \gamma + \alpha (2\gamma - 1)] \dot{K}^* - [\gamma + \alpha (1 - 2\gamma)] \dot{K} - \gamma \dot{\phi}^* - (1 - \gamma) \dot{\phi}. \]  

Compared to producer price targeting, consumer price targeting implies a less aggressive response to home cost-push shocks but also specifies a reaction to foreign cost-push shocks. Inserting these rules into the welfare criterion yields

\[ \tilde{W}_{G,\text{CPT}} = -8\nu^2 \left[ \gamma (1 - \gamma)(2\alpha - 1)^2 \right] \sigma_k^2 - \left\{ \left[ \nu (1 - \gamma - b (1 - 2\gamma)) + \alpha (1 - \nu) \right] \right\}^2 + \left[ \nu (\gamma + b (1 - 2\gamma) - (1 - \nu)(1 - \alpha) \right] \sigma_\phi^2 + o^3. \]  

**Nominal Income Targeting**

In our model nominal income targeting (indicated by the superscript "NIT") can be captured by \( \hat{P} + \hat{Y}_H = 0 \) and \( \hat{P}^* + \hat{Y}_F^* = 0 \). The money supply rule for nominal income targeting can be written as

\[ \dot{M}^{\text{NIT}} = -\frac{(1 - \nu)(1 - \gamma)(1 - 2\alpha)}{1 - 2\nu (1 - \gamma)} (\dot{K} - \dot{K}^*) - \frac{(1 - \nu)(1 - \gamma)}{1 - 2\nu (1 - \gamma)} (\dot{\phi}^* - \dot{\phi}). \]  

---

\(^{18}\) Nominal income targeting is advocated by McCallum and Nelson (1999), among others.
For an extreme home bias in consumption \((\gamma = 1)\), i.e. if no foreign goods are consumed, nominal income targeting is equal to a policy of monetary targeting. Global welfare level yielded by such a policy is

\[
\tilde{\tilde{W}}_{GNT} = -\nu^2 \left[ 1 - 2(1-u)\Omega \left( 1 - 2\nu \right) \right] \left( 1 - \nu \right) \left( 1 - \gamma \right) \frac{\sigma_k^2}{\left[ 1 - 2\nu \left( 1 - \gamma \right) \right]^2} \sigma_k^2 + \sigma^3. \tag{30}
\]

with \(\Omega = (1-\gamma)(1-2b)(1-2\alpha)\).

Monetary Targeting

If monetary policy is governed by a target for the money supply, the money supply rules are immediately given by definition.

\[
\hat{M}^{MT} = 0, \quad \hat{M}^{*MT} = 0. \tag{31}
\]

In this case, monetary policy stays passive even if shocks hit. As opposed to the optimal rule and to the other simple rules a policy of monetary targeting in both countries implies a fixed exchange rate. Global welfare can be derived as

\[
\bar{W}_{GMT}^{\prime} = -2\nu^2 \left[ \frac{1}{2} - u (1-u) \right] \sigma_k^2 \left\{ 1 + \nu^2 \left[ 1 - \alpha (1-\alpha) \right] + \alpha (1-\alpha)(2\nu-1-\nu) \right\} \sigma_k^2 + \sigma^3. \tag{32}
\]

Such as nominal income targeting, monetary targeting implies that the policymaker is concerned with the stabilization of both price and real aggregates. Both strategies allow prices to fluctuate in exchange for less volatility in real activity. This clearly distinguishes these targeting rules from policies of price index targeting. As discussed in the next subsection, this feature will be of critical importance for the welfare implications of the targeting rules.

Welfare Comparison

The welfare results are further illustrated with the help of a numerical example. The parameter setting chosen for the numerical simulation is taken from the literature (see Chari et al. (2002) and Pappa (2004)). The home bias parameters in the consumption and intermediate goods sectors are set at \(\gamma = 0.8\) and \(\alpha = 0.65\). We examine three different cases.
In the first case, both productivity and cost-push shocks are important. Formally \( \sigma_k^2 = \sigma_{\phi}^2 = 1 \)

In the second case, we suppose that productivity shocks are much more important than cost-push shocks. Formally, this case can be captured by assuming that \( \sigma_k^2 = 1 \) and \( \sigma_{\phi}^2 = \frac{1}{3} \). In the third case, the assumption concerning the relative importance of shocks is reversed, i.e. we now suppose that \( \sigma_k^2 = \frac{1}{3} \) and \( \sigma_{\phi}^2 = 1 \).

The welfare yielded by each simple rule is evaluated relative to the welfare yielded by a policy of targeting the producer price index. Formally, welfare is measured as

\[
\frac{\tilde{W}_{G,C} - \tilde{W}_{G,CPT}}{\tilde{W}_{G,C} - \tilde{W}_{G,PPT}}
\]

for, e.g., the case of consumer price index targeting. This ratio gives the welfare gain that remains unexploited if a simple policy rule instead of the optimal coordinated policy rule is followed relative to the welfare loss associated with targeting the index of producer (domestic) prices. The numerical results can therefore be interpreted as reporting the (relative) welfare loss of the simple targeting rule under consideration relative to the welfare loss that arises under producer price targeting. Tables (1) – (3) summarize the welfare results for varying degrees of price flexibility.

### Table 1: Welfare Results of Simple Targeting Rules for \( \sigma_k^2 = \sigma_{\phi}^2 = 1 \).

<table>
<thead>
<tr>
<th>( \nu )</th>
<th>0.25</th>
<th>0.5</th>
<th>0.75</th>
<th>0.95</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPT</td>
<td>99.14%</td>
<td>98.54%</td>
<td>98.22%</td>
<td>98.17%</td>
</tr>
<tr>
<td>NIT</td>
<td>62.42%</td>
<td>49.05%</td>
<td>60.63%</td>
<td>88.12%</td>
</tr>
<tr>
<td>MT</td>
<td>63.01%</td>
<td>49.58%</td>
<td>60.79%</td>
<td>88.10%</td>
</tr>
</tbody>
</table>

If productivity and cost-push shocks are equally volatile, a policy of targeting the producer price index always generates the largest welfare loss of all rules considered. A policy of CPI targeting performs only slightly better. Both types of shocks affect intermediate goods output (work effort) and prices. Hence, stabilizing a combination of both, as in the case of nominal income or monetary targeting, is welfare superior to targeting a price index only. Unless prices are almost completely sticky \( (\nu \geq 0.95) \), a policy of nominal income

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19 As discussed in the Introduction, such a policy is advocated among others by Clarida et al. (2002), Galí and Monacelli (2003) and Obstfeld and Rogoff (2002).

20 Results for \( \nu = 0 \) are not shown because for complete flexibility of all prices output and work effort are completely exogenously determined.

21 Neither nominal income targeting nor monetary targeting as modelled here stabilize intermediate output directly. But by affecting final goods output monetary policy can impact intermediate goods output.
targeting, i.e. a policy of staying inactive, yields slightly higher welfare than a policy of monetary targeting. As opposed to a policy of nominal income targeting (and to the optimal rule), monetary targeting leads to a fixed exchange rate. Thus, the terms of trade of flex-price consumption goods cannot optimally adjust to shocks.

Things, however, look different if productivity shocks are the major source of concern.

Table 2: Welfare Results of Simple Targeting Rules for $\sigma_k^2 = 1$ and $\sigma_r^2 = \frac{1}{3}$

<table>
<thead>
<tr>
<th>$\nu$</th>
<th>0.25</th>
<th>0.5</th>
<th>0.75</th>
<th>0.95</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPT</td>
<td>99.16%</td>
<td>98.63%</td>
<td>98.44%</td>
<td>98.53%</td>
</tr>
<tr>
<td>NIT</td>
<td>74.20%</td>
<td>96.76%</td>
<td>169.09%</td>
<td>263.26%</td>
</tr>
<tr>
<td>MT</td>
<td>74.74%</td>
<td>97.16%</td>
<td>169.10%</td>
<td>263.19%</td>
</tr>
</tbody>
</table>

Even if monetary policy is predominantly concerned with productivity shocks, producer price targeting as recommended by the studies cited above does not constitute the best simple rule. Stabilizing the prices of domestically produced consumption goods is not sufficient to stabilize work effort because the demand for home intermediate goods also depends on demand for foreign consumption goods and thus on their prices. A policy of CPI targeting is generally superior in welfare terms to targeting the producer price index. CPI targeting implies that a combination of home and foreign consumption goods prices is stabilized.

As long as a considerable fraction of final goods prices can adjust instantaneously, stabilizing both prices and real aggregates as done by a policy of nominal income or monetary targeting is best. As discussed above, cost-push shocks in the final goods sector affect work effort expended in the intermediate goods sector (as long as the share of flex-price producers is larger than zero). Stabilizing nominal income (NIT) or nominal consumption (MT) therefore are appropriate means to stabilize work effort. The benefit of NIT and MT of course declines if the impact of cost-push shocks on work effort decreases for growing price rigidity.\(^22\) For $\nu \geq 0.5$ CPI targeting therefore is the best policy option.

Table 3: Welfare Results of Simple Targeting Rules for $\sigma_r^2 = 1$ and $\sigma_k^2 = \frac{1}{3}$

<table>
<thead>
<tr>
<th>$\nu$</th>
<th>0.25</th>
<th>0.5</th>
<th>0.75</th>
<th>0.95</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPT</td>
<td>99.13%</td>
<td>98.50%</td>
<td>98.14%</td>
<td>98.05%</td>
</tr>
<tr>
<td>NIT</td>
<td>58.50%</td>
<td>33.14%</td>
<td>24.41%</td>
<td>29.56%</td>
</tr>
<tr>
<td>MT</td>
<td>59.10%</td>
<td>33.71%</td>
<td>24.62%</td>
<td>29.57%</td>
</tr>
</tbody>
</table>

\(^22\) Further simulations not reported here show that the welfare losses associated with NIT and MT rise tremendously if $\sigma_r^2$ approaches zero. Then, the effect variations in $\nu$ have on the welfare results are clearly of the second order.
A policy of targeting the producer price index is clearly the least favorable targeting rule if cost-push shocks predominate. This is also stressed by Sutherland (2004a). As discussed there, policies that allow for price movements in order to stabilize real activity like monetary and nominal income targeting are superior to price targeting rules in the presence of cost-push shocks. The higher the degree of price stickiness the better approximations of the optimal rule monetary and nominal income targeting are compared to both price targeting rules. The impact of cost-push shocks on intermediate goods output, and thus on work effort, decreases in the degree of price stickiness (see equations (21)). In line with the optimal policy rule described by equations (23) nominal income targeting implies a weaker and weaker reaction to cost-push shocks if \( v \) approaches one. The response to cost-push shocks under both producer price index and consumer price index targeting, however, does not vary with the degree of price stickiness.

VI. CONCLUSIONS

A vertical chain of production and trade along this chain have been found to be a characteristic feature of globalized markets. Starting from this stylized fact, this paper examines how a multistage production process that involves more than one country affects the choice of a monetary policy target. While prices of intermediate goods are assumed to be perfectly flexible, a fraction of consumption goods prices has to be set in advance. It is assumed that there is full pass-through of exchange rate changes into intermediate goods prices, but zero pass-through into the prices of final consumption goods.

We are concerned first with the optimal monetary policy if policies are coordinated, for example, with the policy rule that maximizes global welfare. In a second step we investigate which simple, for example nonoptimal, targeting rule best supports the welfare maximizing policy. Pursuing an inward-looking policy, as suggested in recent work, is clearly not optimal in this setup. A comparison of the welfare effects of a range of simple targeting rules shows no unambiguous result. The degree of flexibility in consumption goods prices and the relative importance of cost-push and productivity shocks are keys for the welfare ranking. In many cases, a strict targeting of price indices such as producer or consumer price indices is dominated by rules that allow for some fluctuations in prices such as nominal income or monetary targeting. A strict targeting of consumer prices is only the best (simple) policy option when the degree of price flexibility is close to nil and productivity shocks are much more important than cost-push shocks.

\[ ^{23} \text{Moreover, nominal income targeting converges to monetary targeting for } v \text{ close to one.} \]
Optimal Price Setting

Fixed-price producers face two price setting problems. Prices are set separately for the home and for the foreign market. The price setting problem of a fixed-price consumption goods producer for goods sold at home can be formulated as follows:

\[
\max U = E \left[ \log C + \chi \log \frac{M}{P} - Ky(z) \right]
\]

s.t.

\[
M - M_0 = \left(1 + \tau^m\right) p_H(z) y(z) - PC - PT - p_H y_H - p_F y_F
\]

\[
+ \left(1 + \tau^f\right) \left[ P_{H,1}(z) C_{H,3}(z) + S P_{H,1}^*(z) C_{H,3}^*(z) \right]
\]

\[
y = y_H + y_H^* = \left[ \frac{p_H}{p_I} \right]^{-1} \left( \alpha Y_H + (1 - \alpha) Y_F^* \right)
\]

The first order condition with respect to \( P_{H,1}(z) \) is

\[
E \left[ (1 + \tau^m) (1 - \varphi) C_{H,3}(z) - \varphi \frac{C_{H,3}(z) p_I}{P_{H,1}(z)} \right] = 0
\]

The optimal price given in equation (11) can be obtained by rearranging equation (36) and considering that all producers set the same price so that \( P_{H,1}(z) = P_{H,3}^* \).

The price for a consumption good that is sold abroad is set by solving

\[
\max U = E \left[ \log C + \chi \log \frac{M}{P} - Ky(z) \right]
\]

s.t.

\[
M - M_0 = \left(1 + \tau^m\right) p_H(z) y(z) - PC - PT - p_H y_H - p_F y_F
\]

\[
+ \left(1 + \tau^f\right) \left[ P_{H,1}(z) C_{H,3}(z) + S P_{H,1}^*(z) C_{H,3}^*(z) \right]
\]

\[
y = y_H + y_H^* = \left[ \frac{p_H}{p_I} \right]^{-1} \left( \alpha Y_H + (1 - \alpha) Y_F^* \right)
\]

\[
P_{H,3}^*(z) = \tilde{P}_{H,3}(z) S^{-\eta^*}
\]

The first order condition with respect to \( \tilde{P}_{H,3}(z) \) is
\[
E \left[ (1 + \tau') \delta^{1-\nu} (1-\varphi)C^*_H(z) - \varphi \frac{C^*_H(z)P_t}{P_H(z)} \right] = 0. 
\] (36)

In equilibrium all producers set the same price so that \( \bar{P}_{H,1}(z) = P_{H,1} \). Rearranging and observing that it is assumed that \( \eta^* = 0 \) yields the price given in equation (12). Prices set by flex-price producers are similarly derived. But since all uncertainty is resolved when prices are set, the first-order condition holds ex post. Considering this, optimal prices given in equation (13) can be obtained.

An intermediate goods producer solves

\[
\max U = \log C + \chi \log \frac{M}{P} - Ky(z) \\
\text{s.t.} \\
M - M_0 = (1 + \tau^{\text{int}}) P_H(z) y(z) - PC - PT - p_H y_H - p_F y_F \\
+ (1 + \tau') \left[ P_{H,1}(z) C_{H,1}(z) + SP_{H,1}^*(z) C_{H,1}^*(z) \right] \\
y = y_H + y_H^* = \left[ \frac{P_H}{P_t} \right]^{1} \left( \alpha Y_H + (1-\alpha) Y_F^* \right)
\]

over \( p_H(z) \). The first condition reads

\[
\frac{1}{PC(1-\omega)(1+\tau^{\text{int}})} y(z) = -\omega \frac{Ky(z)}{p_H(z)}. 
\] (38)

Rearranging and considering that in equilibrium all producers set the same price yields the optimal price given in equation (15).

### A. Simple Policy Rules

**Producer Price Targeting**

The producer price indexes, i.e. the indexes of domestically produced goods, are given by

\[
P_H = \nu \hat{P}_{H,1} + (1-\nu) \left[ \hat{M} + \hat{\phi} + \alpha \hat{K} + (1-\alpha) \hat{K}^* \right]
\]

\[
P_F^* = \nu \hat{P}_{F,1}^* + (1-\nu) \left[ \hat{M}^* + \hat{\phi}^* + \alpha \hat{K}^* + (1-\alpha) \hat{K}^* \right].
\] (39)
As discussed in the main text, the welfare measure only depends on variances. The prices set by fixed price producers are by definition non-stochastic so that they are not relevant for welfare calculations. Bearing this in mind, (in order to simplify notation) the monetary rules under producer price targeting omit all non-stochastic terms and can be expressed as in equation (25).

**Consumer Price Targeting**

The consumer price indexes, i.e. the indexes of domestically consumed goods, are given by

\[
\hat{P} = \gamma \left\{ \nu \hat{P}_{H,1} + (1-\nu) \left[ \hat{M} + \hat{\phi} + \alpha \hat{K} + (1-\alpha) \hat{K^*} \right] \right\} + (1-\gamma) \left\{ \nu \hat{P}_{F,1} + (1-\nu) \left[ \hat{M}^* + \phi^* + \alpha \hat{K}^* + (1-\alpha) \hat{K^*} \right] \right\} - (1-\nu) \hat{S},
\]

(40)

\[
\hat{P}^* = \gamma \left\{ \nu \hat{P}_{F,1} + (1-\nu) \left[ \hat{M}^* + \phi^* + \alpha \hat{K}^* + (1-\alpha) \hat{K^*} \right] \right\} + (1-\gamma) \left\{ \nu \hat{P}_{F,1} + (1-\nu) \left[ \hat{M} + \phi + \alpha \hat{K} + (1-\alpha) \hat{K^*} \right] \right\} - (1-\nu) \hat{S},
\]

Rearranging and omitting all non-stochastic terms yields the monetary policy rules given in equation (28).

**Nominal Income Targeting**

Nominal incomes in the home and in the foreign country are given by

\[
\hat{Y}_H + \hat{P} = \gamma \hat{M} + (1-\gamma) \hat{M}^* - (1-\nu)(1-\gamma) \left[ \hat{M} + \phi + \alpha \hat{K} + (1-\alpha) \hat{K^*} \right] + (1-\nu)(1-\gamma) \left[ \hat{M}^* + \phi^* + \alpha \hat{K}^* + (1-\alpha) \hat{K^*} \right] + \nu(1-\gamma) \hat{P}_{F,1} - \nu(1-\gamma) \hat{P}_{H,1},
\]

(41)

\[
\hat{Y}_F + \hat{P}^* = \gamma \hat{M}^* + (1-\gamma) \hat{M}^* - (1-\nu)(1-\gamma) \left[ \hat{M}^* + \phi^* + \alpha \hat{K}^* + (1-\alpha) \hat{K^*} \right] + (1-\nu)(1-\gamma) \left[ \hat{M}^* + \phi^* + \alpha \hat{K}^* + (1-\alpha) \hat{K^*} \right] - 2(1-\nu)(1-\gamma) \hat{S} + \nu(1-\gamma) \hat{P}_{H,1} - \nu(1-\gamma) \hat{P}_{F,1}.
\]

After rearranging and omitting all non-stochastic terms, the monetary policy rules given in equation (29) can be derived.
REFERENCES


