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Deficit Limits, Budget Rules, and Fiscal Policy

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Deficit Limits, Budget Rules, and Fiscal Policy

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Abstract

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published to elicit comments and to further debate.

The paper presents a simple model for discussing the effects of deficit limits and budget rules
on fiscal policy. I find that limits on deficit-output ratios provide incentives to implement
procyclical policies when the economy is in intermediate states, and countercyclical policies
only in very “good” and very “bad” economic times. As a result, fiscal “reaction functions”
are not monotonically related to the state of the economy. Deficit limits are found to exert
discipline only provided the limit is tight and the expected sanction large, albeit at a
relatively large welfare cost. Moreover, when fiscal choices are made under a veil of
ignorance about the output gap, an increase in volatility is likely to raise the level of the
budget deficit. Finally, concerning the design of fiscal frameworks, when excessive deficits
arise from a political bias, deficit limits should be symmetric and not state-contingent.

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I. INTRODUCTION

This paper presents a simple framework for discussing the incentive effects of deficit limits and budget rules on the conduct of fiscal policy.

The literature on fiscal frameworks has expanded rapidly in the past few years, possibly because of the introduction of the Stability and Growth Pact (SGP) in the Euro area, the adoption of the golden rule in the United Kingdom, and the implementation of a variety of fiscal rules in many countries.

In a nutshell, all these frameworks aim at getting rid of the “dirty water” of (politically motivated) excessive budget deficits without sacrificing the “baby” of fiscal stabilization. But how should that be done? Many observers have expressed criticisms of the existing frameworks, particularly the SGP, for fostering procyclical policies, lacking incentives for discipline in good times, and lacking a credible enforcement mechanism, particularly for large countries (Buti, Brunila, and Franco, 2002). Somewhat independently from this debate, researchers have tried to assess the stabilization properties of fiscal policy, trying to gauge whether policy has been pro or countercyclical in different regions (e.g., Fatás, and others, 2003; Kaminski, Reinhart, and Vegh, 2004). While there is a consensus that fiscal policy behaves differently in bad and good times, it is fair to say that, so far, this literature has proved rather inconclusive: results change considerably depending on the countries and periods analyzed. Finally, recent contributions have stressed that fiscal policy choices are made under substantial uncertainty as to the state of the economy, which makes the task of stabilization quite daunting (Balassone, Ley, and Kumar, 2005).

This paper clarifies and brings together these issues, evaluates possible reform proposals, and compares the welfare levels associated with different budget rules. The analysis focuses on two main issues. The first is the disciplining effects of deficit limits. I ask the following questions: under what conditions are deficit limits effective in providing fiscal discipline? Are they preferable to simple-minded balanced-budget rules? The second issue is stabilization policy. Here the relevant questions are: what do budget rules imply for the policymakers’ incentives to implement stabilization policies? Should budget rules be contingent on the state of the economy?

The analysis is set in a static framework where the focus is on budget deficits and surpluses. Issues relating to debt accumulation and sustainability are not explicitly addressed. This is not because they are deemed less important, on the contrary, but for two reasons: (i) analytical simplicity; and (ii) existing fiscal frameworks have been set largely in terms of (different concepts of) deficits (e.g., the SGP, the golden rule, etc.). Even when debt criteria are used, they are hardly ever made operational (e.g., the SGP). This probably reflects the

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2 I have tried to keep technicalities to a minimum, or to relegate them into Appendix I. For a dynamic stochastic model of optimal taxation with debt and in the presence of spending barriers, see Manasse, 1996.
view that since the level of the public debt reflects past deficits, governments cannot be made fully responsible for decisions taken by their predecessors.

The paper does not attempt to provide a justification for the existence of such rules in terms, say, of underlying political economy factors or optimal contracts in principal-agent models: here I take the rules as given and simply work out their implications. The model can also be interpreted as describing how financial markets discipline governments pursuing excessive deficits: rating agencies and investors sanction the authorities by raising borrowing costs whenever the deficit ratio exceeds a “critical threshold.” The model describes an economy where it is optimal, from a welfare perspective, to have transitory deficits and surpluses for stabilization purposes. Policymakers, however, pursue excessive deficits due to the presence of a political (electoral) distortion. The remedy of ceilings on deficit-output ratios (or market sanctions) makes policymakers face the following trade-off: they either forego the economic benefits of stabilization and the political benefits of excessive deficits by abiding to the constraint, or they violate the limit and face possible sanctions.

I reach the following main conclusions. First, limits on deficit-output ratios, or market sanctions for excessive deficits, provide incentives to implement procyclical policies when the economy is in intermediate states: here, in order to avoid the sanction, the government optimally keeps the deficit-output ratio just below the required limit, so that a contraction of output is met by a budget tightening. Countercyclical policies are implemented only in very “good” and very “bad” times: in the former, the constraint is not binding, while in the latter it is optimally violated, since abiding is too costly in a recession. As a consequence, the fiscal “reaction functions” are not monotone in the state of the economy, and attempts to estimate these relationships are bound to produce inconsistent results unless the break-points are precisely identified. Second, when fiscal choices are made under a veil of ignorance about the state of the economy, an increase in volatility raises the optimal deficit, provided political distortion is sufficiently high and the probability of a sanction sufficiently low. Intuitively, in a volatile environment the government has an incentive to run a larger deficit, in the hope that a favorable outcome will take care of the limit. Third, a numerical example suggests that deficit limits exert disciplining effects only when the limit is tight and the expected sanction high, albeit at a relatively large welfare cost. Fourth, concerning the design of fiscal frameworks, when the excessive deficits arise from a political bias, fiscal limit should be symmetric and not state-contingent.

The paper’s outline is the following: Section II presents the model and compares first and second best fiscal policy rules. Section III analyzes deficit limits for the cases where output gaps are/are not observed prior to the policy choice. A simple numerical example is also presented. Section IV concludes by summarizing the policy and empirical implications of the analysis.
II. A SIMPLE MODEL OF FISCAL POLICY

This section presents a stylized model whose purpose is to study the incentives of budget rules for fiscal policy. I start with the benchmark case where the government chooses the budget in the absence of explicit rules: this sets the stage for the discussion of alternative regimes. For simplicity, the policymaker has one instrument, the level of the budget deficit. I assume that there is a continuum of identical consumers, whose indirect utility function, $W$, depends on the (logarithm of the) ratio of fiscal deficit to trend output, $d$, and on the (logarithmic) deviation of output from trend, $e$, the “output gap.” Thus, $W=W(d,e)$. This function displays some intuitive properties:

(i) $W_d(d^*,0) = 0$;

(ii) $W_d(d^*,e) > 0$ for $e < 0$; and

(iii) $W_{de}(d,e) < 0$; iv) $W_{dd}(d,e) < 0$.

The first and the last properties imply that when output is at potential, $e=0$, there exists an optimal deficit-output ratio $d^*$ that maximizes welfare. The second property requires that in recession, $e<0$, raising the deficit above $d^*$ yields a positive marginal utility. The third states that the marginal utility of a deficit falls when the economy improves. This condition, we will see, implies the optimality of stabilization policy. The optimality of stabilization policies can be rationalized in several ways. For example, in Bordignon, Manasse, and Tabellini (2001), households derive utility from leisure and from the consumption of a private and a public good. In bad times, the tax base shrinks, and this has two implications: on the one hand tax distortions rise; on the other, tax revenues and the supply of the public good fall, so that the marginal utility of the public good rises. As a result, it is optimal to cut tax rates and spend more in bad times. The opposite applies in good times. Other justifications for stabilization policy range from tax smoothing considerations when shocks are transitory (Barro, 1974), to imperfect capital markets and borrowing constraints. To economize in partial derivatives, I follow Bottazzi and Manasse (2005), and take a simple functional form that satisfies the requirements above, as described in the following equation (1):

$$W(d,e) = - (d-d^*)^2/2 - e(d-d^*) + c(e) = - d^2/2 - ed + c(e)$$

where, in order to save notation, I have redefined $d$ as the (logarithmic) deviation of the deficit ratio from the optimal value, $d^*$. Note that when output is below potential ($e<0$) running a higher (than optimal) deficit, $d>0$, raises welfare, and vice versa when the economy is hit by a good shock ($e>0$). The term $c(e)$ captures the consumption effects of good and bad states. This term can be justified if liquidity constraints prevent consumption

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3 Thus, $d=ln(D/Y)$, $e=ln(Y/Y^p)$, and $D$, $Y$, $Y^p$ denote the deficit, actual and trend (potential) output, respectively.

4 I follow the convention of indicating a partial derivative with a subscript, e.g., $W_x$ is the partial derivative of $W$ with respect to $x$. 

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smoothing and insurance. I assume that $c(e)$ is increasing and concave in $e$, so that consumption and welfare depend positively on the output gap $e$ and negatively on its volatility. For analytical convenience, I consider the functional form $c(e) = e - e^2$.\(^5\) The policymaker’s objective function does not exactly coincide with that of consumers’, due to the presence of a political distortion. It reads:

$$W^p(d,e) = W(d,e) + \omega d \quad (2)$$

The new term represents a *deficit bias*, $\omega > 0$: even when output is at potential ($e=0$) the policymaker benefits from running a higher-than optimal deficit for political (for example, electoral) reasons. One simple interpretation is that policymakers maximize a weighted average of consumer’s welfare and political contributions (see, Grossman and Helpman, 1994) or “electoral support.” Here electoral support is assumed to be proportional to spending on (or tax breaks for) selected groups (or lobbies). Alternatively, in the context of a monetary union, this additional term can be thought of as a negative international externality of deficit spending, which is not internalized by the national government.\(^6\) Note that this political distortion applies independently of the cycle $e$. Finally, I assume that the “output gap” $e$ is uniformly distributed between $[-a,a]$, $a>0$, with probability density function $f(e) = 1/2a$, so that the mean is zero and the variance $\sigma^2 = a^2/3$. In this simple formulation, it is useful to think of $a$ as a mean-preserving spread. Next I assume that output gaps are observable, and are realized before fiscal decisions are made.

### A. First and Second Best Policies

In this section, I briefly characterize the model’s first best, and discuss the second best outcomes obtained when political distortions are present. I review the consequence of “balanced” budget (noncontingent) rules and discuss the features of an optimal rule that implements the first best.

**First best policy**

It is immediate to show from equation (1) that the optimal state-contingent deficit policy that maximizes consumers’ welfare is

\(^5\) The term $c(e)$ enters separately from the choice variable, $d$, so it does not affect the analysis (but only the welfare calculations). The chosen functional form can be interpreted as a linearization of an increasing and concave function, or, in expected terms, as a constant absolute risk-aversion (CARA) utility function.

\(^6\) For example, Feldstein (2005) argues that a monetary union creates a strong deficit bias because a deficit in a single nation does not cause the rise in the nation’s interest rate nor a depreciation of the domestic currency. Under this interpretation, $W^p$ represents the national government’s objective function, and $W$ denotes the welfare of the representative union-wide agent.
\[ d = D^F(e) = -e \]  
with \( E_e(D^F(e)) = 0; \ Var(D^F(e)) = \sigma^2 \)  

where \( E_e(.) \) denotes the expectation taken with respect to \( e \). From equation (3), the optimal fiscal reaction function raises the deficit in recession and lowers it in booms, depending on the size of the output gap, \( e \). Note, however, that the deficit is at the optimal level, \( d=0 \), on average. Substituting equation (3) into (1), and taking expectations, gives the first best level of welfare:

\[ E_e W(D^F(e), e) = -\sigma^2/2 \]  

Thus, the stabilization policy rule alleviates the negative welfare costs of output volatility by lowering (halving) the negative effects of uncertainty (compare (4) and (1)).

**Second best (political distortions) policy**

For electoral reasons, policymakers derive a higher marginal utility from deficit spending than consumers. When policymakers maximize (2) instead of (1), the optimal policy becomes

\[ d = D^P(e; \omega) = \omega - e \]  
with \( E_e(D^P(e; \omega)) = \omega; \ Var(D^P(e; \omega)) = \sigma^2 \)  

As before, the policymaker optimally raises or cuts the deficit whenever output falls below or rises above potential. However, due to the political distortion of \( \omega > 0 \), the deficit is now on average higher than optimal. Substituting (5) into (1), and taking expectations, gives expected welfare:

\[ E_e W(D^P(e; \omega), e) = -(\omega^2 + \sigma^2)/2 \]  

Welfare is lower than in the first best because of the incentive to run an excessive deficit.

**Fixed ("balanced") budget rule**

Suppose a country’s constitution mandates a fixed ("balanced") budget \( d=0 \) in all states of the economy. From equation (1), expected welfare simply becomes

\[ E_e W(0, e) = -\sigma^2 \]  

Comparing this with expression (6), we see that the balanced budget rule removes the costs of the political bias, \( \omega \), but forgoes the benefits of stabilization (the standard trade-off in Barro-Gordon types of models). Thus, this rule is preferable to the discretionary policy only when output uncertainty is low compared to the political bias (\( \sigma < \omega \)). Empirically, one should observe this type of rules in countries with low output volatility and/or high political distortions.
The optimal rule

By comparing the utility function of consumers to that of the policymaker, it is worth noting that a (Walsh-type of) rule that rewards or penalizes the policymaker, depending on whether he chooses an excessive surplus or a deficit, can implement the first best. When subject to a rule that offers a reward (or sanction) \( w \) that is proportional to the excess surplus, \( w(d) = -\omega d \), the policymaker utility becomes identical to that of the representative consumer:  

\[ W'(d,e) = W(d,e) + \omega d + w(d) = W \]  

Hence, in this case the policymaker will choose the first best policy. The rationale here is that, since the marginal incentive of the politician to run an excessive deficit (\( \omega \)) is independent of the state of the economy, the fiscal constraint should not be state contingent and should apply symmetrically over states, penalizing deficits and rewarding surpluses. In practice, this can be done by letting the fiscal authorities accumulate “credits” when running surpluses (in good times) to be spent in order to run deficits (in bad times). Since bad and good time average out, this scheme is viable by definition.

In summary, in the first best world the policymaker stabilizes the economy, and the budget is at the optimal level (“in balance”) on average. When political distortions are introduced, the stabilization nature of policy is preserved, but the policymaker now runs an excessive deficit over the cycle. Strict balanced budget (noncontingent) rules alleviate the political distortion but sacrifice stabilization: they are preferable to the politically distorted outcome only insofar the output volatility is low with respect to the deficit bias. Finally, a simple rule that implements the first best must be symmetric and non state-contingent.

III. DEFICIT LIMITS

My interest here is to describe the policy incentives provided by some common “fiscal frameworks,” such as ceilings on deficit-output ratio. This section makes an important point: deficit-output limits typically induce procyclical policies in intermediate states, and countercyclical policies in very “good” and “bad” states. Interestingly, limits are optimally violated in recessions, even if they exert discipline on deficit spending. I consider the following framework: a constraint on the deficit \( \Phi(d,e) \leq 0 \) is imposed, so that whenever the constraint is violated, a penalty \( \varphi(d,e) \) is levied on the policymaker, with a known probability. By assumption, both the constraint and the penalty rule depend on \( d \) and (possibly) on the state of the economy \( e \). For example, the constraint that the (excess) budget deficit should not exceed a limit of \( X \) percent of GDP can be written, in logarithmic terms, as  

\[ \Phi(d,e) = d - e - x \leq 0, \text{ where } x = \ln(X). \]  

---

7 Note that each of the \( N \) consumers’ utility becomes \( W' = W - w(d)/N \approx W \) for \( N \) large.

8 Recall that \( d = \ln(D/Y^p) \) and \( e = \ln(Y/Y^p) \), so that \( d-e \) represents the logarithm of the deficit-output ratio, \( d-e = \ln(D/Y) \). Hence \( D/Y \leq X \) implies, taking logs of both sides, \( d-e \leq x \).
The penalty is enforced by an external regulator and ultimately paid by consumers. I assume that a sanction can be commuted only with a given probability, $0 < p < 1$. This probability can be thought of in two ways. Under the first interpretation, $p$ is determined by the monitoring technology available to the regulator. In other words, the policymaker may fudge the balance sheets, show a lower deficit than the actual one, and get away with it with probability $1 - p$. Alternatively, $p$ can be taken to represent, literally, the probability that the rule will be enforced. In this respect, $p$ represents an (inverse) measure of the policymaker clout and bargaining power with respect to the regulator, or a measure of the political independence of the regulator. One simple and interesting rule is the following:

A penalty $\phi(d,e)$ is imposed (with probability $p$) when the (log) deficit ratio, $d-e$, exceeds the threshold $x$, $\Phi(d,e) = d-e-x > 0$; the penalty, $\phi(d,e)$, is proportional to the difference between the (log) deficit-output ratio and the threshold, i.e., $\phi(d,e) = \phi(d-e-x)$, with $\phi > 0$ (9) denoting the constant of proportionality; no penalty applies when the fiscal constraint is not binding, $\Phi(d,e) \leq 0$.

This type of sanction is very common. Think, for example, of speed limits for cars where sanctions are proportional to the excess of the actual speed over the maximum-limit. Also, this penalty closely resembles the SGP. This framework can also be interpreted as describing the discipline device provided by financial markets: when the deficit ratio exceeds a “critical threshold,” rating agencies and investors sanction the government by raising borrowing costs proportionately. A simple way to summarize this framework is to note that, in expected terms, the penalty can be written as the max $[p\phi(d-e-x), 0]$.

Stabilization and discipline incentives

**Stabilization**

I now study the implications of this rule for optimal fiscal policy. When subject to the budget limit, the government maximizes a constrained welfare function, given by

$$W^c = -d^2/2 - ed + c(e) + \omega d - \max[p\phi(d-x-e), 0]$$

(10)

The following proposition is easily established:

**Proposition 1.** When the government is subject to the budget limit described in (9), the optimal policy is:

$$d = D^c(e; \omega) = \begin{cases} \omega - e & \text{if } e \geq e \equiv (\omega - x)/2 \\
 x + e & \text{if } e > e \geq e \equiv e - p\phi/2, \\
 \omega - e - p\phi & \text{if } e < e \end{cases}$$

(11)

Proof: a simple geometrical proof is provided in the appendix.
The optimal (constrained) reaction function is depicted in Figure 1. The positively sloped line \( d=x+e \) represents the fiscal limit on the deficit-output ratio: below this line the limit does not bind, while above it does, and the penalty is imposed (with probability \( p \)). The downward sloping parallel lines represent the fiscal reaction functions in good and bad states, described in equation (11). When the economy is in a boom, in \( e > \bar{e} \), the (excess) deficit associated with the optimal (unconstrained) policy, \( d = \omega - e \), is below the ceiling, so that the limit does not bind and the government does not have to abide with the ceiling: it freely pursues its strategy of stabilization policy cum deficit bias. However, when the economy worsens and the output gap falls below \( \bar{e} \), the fiscal limit becomes binding, and the government chooses the highest possible deficit that avoids the sanction. In the range \([\bar{e}, e]\) it is therefore optimal to keep the deficit just below the limit, \( d = e + x \), and to move along the positively sloped segment.

Figure 1. The Deficit Reaction Function

The reason is that here the expected cost of the penalty exceeds the stabilization and electoral benefits of deficit spending, thus the authorities choose to be restrained by the ceiling. Note that when the output gap rises in this range, marginally relaxing the constraint, the government raises the deficit proportionally: the optimal (constrained) policy becomes procyclical. Finally, when the economy falls into recession, \( e < \bar{e} \), the government does not want to abide to the excessive deficit limit, since the cost of forgoing stabilization (plus the electoral benefits of the deficits) exceeds the expected penalty. Here the deficit moves up along the negatively sloped (constrained) fiscal reaction function \( d = \omega - e - p\varphi \). In this range, policy is again countercyclical. Interestingly, even if the deficit limit is being violated, it still provides fiscal discipline, since the government cuts the deficit in order to reduce the

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\(^9\) This requires that \( e = \frac{-e - p\varphi}{2} > -a \), that is, the penalty must not be too high, so that point \( e \) is above the lower support of the distribution of shocks, \(-a\). In Figure 1, we also assume that \( \omega < x \), and \( \bar{e} < a \).
expected sanction (in the absence of the constraint, the government would be moving on the higher parallel reaction function). In summary, the deficit limit acts like a peculiar “tax” on excessive deficit: highly distortionary in intermediate states, “lump-sum” in bad states, and ineffective (zero tax rate) in good states.

**Discipline**

It is natural to ask how does the rule perform on average, in terms of restricting the policymaker inclination for excessive deficit spending. In order to calculate the average disciplining effect of the limit, we need to find the average excess deficit implied by (11). After a little algebra, the average deficit turns out to be

$$E_c D^c(e; \omega) = E_c D^c(\cdot \mid -a \leq e < \bar{e}) + E_c D^c(\cdot \mid \bar{e} < e \leq e) + E_c D^c(\cdot \mid e < a) = \omega - \left( \frac{p\varphi(e + a) + (\omega - x)(\bar{e} - e) - (\bar{e}^2 - e^2)}{2a} \right)$$

The interpretation of equation (12) is straightforward. The term in parenthesis measures the disciplining effect of the limit. From Figure 1 we know that the limit effectively constrains fiscal choices in two ranges. First, discipline is exerted in bad states, in the range measuring \( e - (-a) = e + a \), where the excess deficit is cut down by \( p\varphi \). This effect is given by the first term in the parenthesis. Second, in the intermediate states in the range \( \bar{e} - e \), the policymaker sticks closely to the limit. The contribution to discipline of this effect is measured by the second term in the bracket. Intuitively, the political bias is mitigated only in so far the limit is sufficiently tight, \( x < \omega \). The relative importance of the two effects is described by the last term, which equals \( (\bar{e} - e)(e + \bar{e}) \). Discipline is lessened if the range \( (\bar{e} - e) \) rises, since there the deficit is kept on the positive line \( d = x + e \), which is above the reaction function \( d = \omega - e - p\varphi \). Clearly, when the policymaker expects no sanction \( (p\varphi = 0) \), all terms in the parenthesis vanish, and the government’s fiscal rule and average deficit revert to the politically distorted outcome (see equation (5)).

**Deficit limits under uncertainty**

In this section, I briefly discuss how deficit limits work when uncertainty makes stabilization unfeasible. It is often argued that fiscal decisions are generally made under a veil of ignorance as to the state of the economy: output gaps are measured with delay and are subject to substantial revisions (see Balassone et al., 2005). When the deficit is chosen prior to the realization of the shock, the government’s objective function becomes

$$E_c W^c = -d^2/2 + \omega d - \sigma^2 - E_c \varphi(d, e)$$

where the expected penalty is given by the following expression:
\[ E_e \varphi(d,e) = \int_{-a}^{d-x} \frac{p \varphi(d-e-x)}{2a} \, de = \frac{p \varphi}{4a} (d-x+a)^2 \]  
(14)

Intuitively, the expected sanction is inversely proportional to the volatility parameter, \(a\), in the denominator of (14), and directly proportional to the (squared) maximum penalty. This occurs in the “worst case scenario,” when \(e=-a\) is realized and the limit is exceeded by \(d-x-(a)\). By substituting equation (14) into (13), and maximizing with respect to \(d\), one finds the optimal deficit:

\[ d = \lambda(a) \omega - (1-\lambda(a))(a-x) \]  
(15)

where the weight \(0<\lambda<1\) is defined as \(\lambda(a)= \frac{2a}{(2a+p\varphi)<1}\), and \(\lambda_o(a)>0\). The optimal deficit is a linear combination of the political bias, \(\omega\), and of the lowest possible “effective” threshold \((x-a)\). When uncertainty \(a\) rises, there are two opposite effects on the expected penalty and, as a consequence, on choice of the deficit. On one hand, the penalty become less likely, \(\lambda_o(a)>0\), since it is incurred only above the threshold: hence, the government has an incentive to run a larger deficit, in the hope that a favorable outcome will take care of the limit. On the other hand, as volatility rises, the maximum sanction rises: this reduces the optimal deficit (see the \((a-x)\) term in (15)). It is easy to see that in the case where the sanction is lump-sum, \(\varphi(d,e)= k >0\), this second effect disappears. Thus, the following Proposition 2 is easily established.

**Proposition 2.** (i) When the government chooses the deficit prior to learning the realization of the output gap, and the penalty rule is given by equation (9), an increase in volatility raises the optimal deficit, provided the political distortion is sufficiently strong, the limit sufficiently tight, and the probability of the sanction sufficiently low, i.e., \(\partial d/\partial a > 0\) provided \( \omega-x>p\varphi/2\). (ii) When the sanction is lump-sum, an increase in volatility always raises the optimal deficit, i.e., \(\varphi(d,e)= k >0\) implies \(\partial d/\partial a >0\).

**Numerical example**

How effective are deficit limits for providing discipline, and how serious are the consequences of the induced procyclicality? It is instructive to compare the average excess deficits and expected losses (the negative of expected welfare) that apply under different fiscal frameworks, when the output gap is observed prior to the choice of \(d\). Table 1 shows the baseline calculations, which assume that the deficit ceiling is 3 percent, that the (excess) deficit to trend-output ratio due to political bias is 4 percent, that the probability of a sanction is 1/2, the sanction coefficient \(\varphi=1\), and the maximum output gap is 50 percent.\(^{10}\) By construction, the first best and the balanced budget rule achieve a zero excess deficit on average. The latter, however, is associated with a lower expected welfare (higher expected loss) since it forgoes stabilization. The deficit limit framework performs better than the

\(^{10}\) Thus, in the baseline calculations, \(x=ln(3), \omega=ln(4), a=ln(1.5), \varphi=1, p=1/2\).
unconstrained equilibrium with political distortions, and achieves almost a 1 percentage point reduction in the deficit output ratio in the baseline calculation. Table 2 shows that the relationship between the size of the sanction and the excess deficit is not monotone: first, as expected, the excess deficit declines as the sanction (or its probability) rises. However, when the sanction is raised further, discipline falls. This is due to the fact that for very high penalties, the government is never willing to take the risk of violating the limit, so that, when the output is low, it always keeps the deficit close to the ceiling (point e in Figure 1 is eventually pushed to the left of -a). Stricter limits, x (Table 3), improve welfare and exert a noticeable disciplining effect. In particular, budget limits seem to exert a considerable discipline when large political distortions are present (Table 4). This table compares the unconstrained outcome (Political Distortion) with the Deficit Limit outcome, for increasing values of the parameter \( \omega \). As political distortions rise, the limit becomes tighter and restrains the excess deficit effectively, but this comes at the price of increasing procyclicality and welfare losses. Finally, Table 5 compares the performance of the various frameworks for different values of the output gap volatility parameter \( a \). As volatility rises, deficit limits tend to become marginally less effective, while they imply increasing welfare losses that soon exceed the unconstrained politically distorted outcome.

### Table 1. Baseline Calculation

\( (\varphi=1, p=1/2, \omega=\ln(4), x=\ln(3), a=\ln(1.5)) \)

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<tbody>
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<td>Expected loss</td>
<td>0.84</td>
<td>0.99</td>
<td>0.05</td>
<td>0.03</td>
</tr>
<tr>
<td>Average deficit</td>
<td>3.08</td>
<td>4.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Source: Author’s calculations.

### Table 2. Change in Penalty

<table>
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<tr>
<th>Penalty</th>
<th>Expected Loss</th>
<th>Average Excess Deficit</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.68</td>
<td>4.00</td>
</tr>
<tr>
<td>0.50</td>
<td>0.78</td>
<td>3.44</td>
</tr>
<tr>
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<td><strong>0.84</strong></td>
<td><strong>3.08</strong></td>
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<tr>
<td>1.50</td>
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</tr>
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<td>0.88</td>
<td>2.78</td>
</tr>
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<td>2.90</td>
</tr>
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</tr>
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<td>0.81</td>
<td>3.54</td>
</tr>
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</tr>
<tr>
<td>5.00</td>
<td>0.56</td>
<td>5.05</td>
</tr>
</tbody>
</table>

Source: Author’s calculations. The bold entries represent the baseline.
Table 3. Change in Ceiling

<table>
<thead>
<tr>
<th>Deficit Limit</th>
<th>Ceiling</th>
<th>Expected Loss</th>
<th>Average Excess Deficit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.00</td>
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<td>0.75</td>
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</tr>
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<td><strong>0.84</strong></td>
<td><strong>3.08</strong></td>
</tr>
<tr>
<td></td>
<td>3.5</td>
<td>0.91</td>
<td>3.23</td>
</tr>
<tr>
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<td>4.0</td>
<td>0.98</td>
<td>3.36</td>
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</tr>
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<td>6.5</td>
<td>0.73</td>
<td>3.91</td>
</tr>
<tr>
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<td><strong>7.0</strong></td>
<td><strong>0.54</strong></td>
<td><strong>4.00</strong></td>
</tr>
</tbody>
</table>

Source: Author’s calculations. The bold entries represent the baseline.

Table 4. Change in Political Bias

<table>
<thead>
<tr>
<th>Deficit Limit</th>
<th>Political Bias</th>
<th>Expected Loss</th>
<th>Average Excess Deficit</th>
<th>Political Distortion</th>
<th>Expected Loss</th>
<th>Average Excess Deficit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<tr>
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<td>0.65</td>
<td>2.81</td>
<td>0.81</td>
<td><strong>3.50</strong></td>
<td><strong>3.50</strong></td>
</tr>
<tr>
<td></td>
<td><strong>4.00</strong></td>
<td><strong>0.84</strong></td>
<td><strong>3.08</strong></td>
<td><strong>0.99</strong></td>
<td><strong>4.00</strong></td>
<td><strong>4.00</strong></td>
</tr>
<tr>
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<td>1.16</td>
<td>4.50</td>
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<tr>
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<td>1.32</td>
<td>5.00</td>
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<tr>
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<td>3.84</td>
<td>1.48</td>
<td>5.50</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6.00</td>
<td>1.53</td>
<td>4.08</td>
<td>1.63</td>
<td>6.00</td>
<td></td>
</tr>
<tr>
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<td>4.31</td>
<td>1.78</td>
<td>6.50</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7.00</td>
<td>1.83</td>
<td>4.53</td>
<td>1.92</td>
<td>7.00</td>
<td></td>
</tr>
<tr>
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<td>1.98</td>
<td>4.76</td>
<td>2.06</td>
<td>7.50</td>
<td></td>
</tr>
</tbody>
</table>

Source: Author’s calculations. The bold entries represent the baseline.
IV. CONCLUSIONS

This paper has presented a simple framework for discussing the incentive effects of budget rules and deficit limits. When output gaps are observable, so that stabilization policy is feasible (arguably the case of most industrialized economies), “balanced” budget (noncontingent) rules alleviate the political distortion but sacrifice stabilization: they are preferable to the politically distorted outcome only insofar the output volatility is low with respect to the deficit bias. A simple fiscal rule that implements the first best must be symmetric and non state-contingent, that is, it must penalize deficits and reward surpluses. By contrast, limits on deficit-output ratios typically induce procyclical policies in intermediate states and countercyclical policies in very good and very bad states. These limits are optimally violated in recessions, even if they exert discipline on deficit spending. A numerical example suggests that deficit ceilings achieve fiscal discipline, provided limits are tight and expected sanctions are high, albeit at a large welfare cost. The analysis implies that reform proposals that aim at improving the stabilization properties of such frameworks by relaxing the constraint or by lowering (expected) sanctions are likely to encourage excessive deficits and reduce welfare. A case in point is the recent proposals of the European Council of Economic Ministers (2005): the stated objective of enhancing flexibility is effectively

Table 5. Change in Volatility

<table>
<thead>
<tr>
<th>Volatility</th>
<th>Deficit Limit</th>
<th>Political Distortion</th>
<th>Balanced Budget</th>
<th>First Best</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ave. exc. Deficit</td>
<td>Ave. exc. Deficit</td>
<td>Ave. exc. Deficit</td>
<td>Ave. exc. Deficit</td>
</tr>
<tr>
<td>1.50</td>
<td>0.84</td>
<td>3.08</td>
<td>0.99</td>
<td>4.00</td>
</tr>
<tr>
<td>1.60</td>
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<td>1.00</td>
<td>4.00</td>
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<tr>
<td>1.70</td>
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<tr>
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</tr>
<tr>
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<tr>
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<td>1.09</td>
<td>4.00</td>
</tr>
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<td>3.10</td>
<td>1.10</td>
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</tr>
<tr>
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<td>1.56</td>
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</tr>
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<td>2.70</td>
<td>1.60</td>
<td>3.10</td>
<td>1.13</td>
<td>4.00</td>
</tr>
</tbody>
</table>

Source: Author’s calculations. The bold entries represent the baseline.
achieved by introducing multiple and ill-defined escape clauses; that is, by weakening the punishment arm (the “excessive deficit procedure”). Moreover, in countries where output volatility is extremely large, so that stabilization policy is hardly feasible (arguably in many emerging markets), deficit limits are unlikely to exert discipline, since governments have an incentive to “bet on resurrection”: they may run excessive deficits and cross their fingers in the hope that good output realizations will take care of the limits. Finally, the model has an interesting empirical implication for the estimation of fiscal policy reaction functions. When a government pursuing excessive deficits faces a sanction, either rules-based or market determined, its fiscal reaction function is no longer monotone in the state of the economy. Hence, unless the break-points are precisely identified, the attempts to estimate these policy reaction functions are bound to be inconclusive.
PROOF OF PROPOSITION 1

The simplest way to prove the Proposition is by geometry. Figure 2a. plots the unconstrained welfare function $W(d,e)$, for a given shock $e$, the constraint function, $\max(.)$, a broken line, and the constrained welfare function $W^c(.)$, a dotted curve, which is simply the difference between the two functions. Figure 2a. represents the case where the point, when the constraint has a kink, $x+e$, falls at the right of (or coincides with) the value that maximizes $W$, $\omega-e$. In this case, that is when $e \geq \bar{e} = (\omega-x)/2$, the value $d = \omega-e$ also maximizes the constrained welfare function. Thus, for $e \geq \bar{e}$, $D^c(e) = \omega-e$

Figure 2a. Welfare and Penalty Function: $e \geq \bar{e}$

Figure 2b. shows the case where the point, when the constraint has a kink, $x+e$, falls to the left of point $\omega-e$, that is $e < \bar{e}$. At the same time, the slope of the constraint, $p\phi$, is lower than the slope of the welfare function at the point $x+e$, $W_d(x+e, e) = (\omega-e) - (x+e) > 0$, that is $e < \bar{e} \equiv \bar{e} - p\phi/2$. Therefore, the constrained welfare function keeps rising beyond point $x+e$, and achieves an interior optimum at $d = \omega-e - p\phi = \text{argmax} \ [W - p\phi(d-x-e)]$. Hence, for $e < \bar{e}$, the optimal policy is $D^c(e) = \omega-e-p\phi.$
Finally, Figure 2c. portrays the case where the kink, $x+e$, falls to the left of point $\omega-e$, that is $e<\bar{e}$, but the slope of the constraint, $p\phi$, is larger (or equal) than the slope of the welfare function at the point $x+e$, that is $e \leq \bar{e}$. In this case the constrained welfare function has a maximum at point $d=x+e$. Thus, for $e \leq e < \bar{e}$, the optimal policy is $D^c(e)=x+e$, which completes the proof.
REFERENCES


