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Banking Competition, Risk, and Regulation

Wilko Bolt and Alexander F. Tieman

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Banking Competition, Risk, and Regulation

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Abstract

In a dynamic theoretical framework, commercial banks compete for customers by setting acceptance criteria for granting loans, taking regulatory requirements into account. By easing its acceptance criteria a bank faces a trade-off between attracting more demand for loans, thus making higher per period profits, and a deterioration of the quality of its loan portfolio, thus tolerating a higher risk of failure. Our main results state that more stringent capital adequacy requirements lead banks to set stricter acceptance criteria, and that increased competition in the banking industry leads to riskier bank behavior. In an extension of our basic model, we show that it may be beneficial for a bank to hold more equity than prescribed by the regulator, even though holding equity is more expensive than attracting deposits.

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I. INTRODUCTION

While banking competition has been the subject of extensive research in the field of microeconomics, a large part of this literature has primarily focused on the liabilities' side of the bank's balance sheet. This recent strand of research has stressed the importance of competition for deposits among banks and its consequences for a bank's risk taking behavior, financial regulation, and financial stability as a whole. [2] In this article however, we take a different approach by focusing on the asset side of the bank's balance sheet. We present a dynamic framework in which commercial banks compete for loans by setting acceptance criteria. In this model, we analyze the consequences of loan competition for a bank's risk taking behavior and probability of default. The model also allows us to investigate the sensitivity of bank behavior to capital adequacy regulation.

Loosely stated, competition drives margins down and thus hurts profits. As a consequence, the bank's charter value declines, yielding stronger incentives to take on more risk, which induces higher failure rates. In our model of banking competition, a bank with looser acceptance criteria for granting loans attracts more demand. However, by loosening the acceptance criteria, the quality of the bank's loan portfolio is negatively affected, causing higher default probabilities. Through the process of screening loan applicants a bank's choice of acceptance criteria also affects the quality of its competitor's loan portfolio. In this context, banks face a 'winner's curse' in bank lending, resulting from the ability of rejected, on average riskier, loan applicants to shop around until some bank is willing to extend a loan (see e.g., Broecker (1990), Schaffer (1998), or Dell'Ariccia, Friedman, and Marquez (1999)). In our model, competition for loans induces a strategic trade-off for the bank between boosting market share in the short run and securing continuation of its operations in the long run. This article identifies the consequences of these strategic choices for bank vulnerability.

When competition undermines prudent banking, banking regulation and prudential supervision come into play. More specifically, in our study the main objective of banking regulation is to limit the probability of bank failure. We assume that the regulatory authority can only use a capital adequacy requirement (CAR) as its instrument. In setting their acceptance criteria and attracting loans in this way, banks must take the capital adequacy ratio imposed by the regulator into account. Given this imposed CAR, a bank's management seeks to maximize the discounted stream of future profits of a

[2] Matutes and Vives (2000) express the view that effective regulation should encompass deposit rate regulation and asset restrictions. Cordella and Yeyati (1998) find that lower entry costs foster competition in deposit rates and reduce banks' incentives to limit risk exposure. In a dynamic model of moral hazard, Hellmann, Murdock, and Stiglitz (2000) find that a combination of capital requirement and deposit rate controls performs best to curb risk taking behavior by banks. Also, along the same lines, see Keeley (1990). For a spatial model of competition for depositors, see Freixas and Rochet (1997). For a survey on these topics the reader is referred to Bhattacharya, Boot, and Thakor (1998).

bank. Our analysis shows that it is indeed the case that higher CARs lead to less risk taking both by tightening the acceptance criteria and by banks having larger reserves. Consequently, higher CARs lead to lower failure rates.

Capital adequacy requirements have always played an important role in banking regulation. However, the 'one-size-fits-all' capital ratio, as prescribed in the 1988 Basel Capital Accord, may induce the wrong incentives and consequently lead to riskier bank behavior. A new framework, best exemplified by the proposal for a new Basel Capital Accord, Basel II, intends to align regulatory capital requirements more closely with underlying risks, and to provide banks with a menu of options for the assessment of capital adequacy, enhancing incentives for better risk management. We implement this new type of regulation by imposing a contingent regulatory rule in our model, linking higher risk appetites with higher capital requirements. Our analysis confirms that implementing such a contingent rule is very effective in meeting the regulator's target of achieving a maximum admissible failure rate.

Further, looking at actual capital ratios one often sees that banks hold more capital than prescribed, indicating that the imposed CAR is not a binding restriction. Indeed, an extended version of our model in which banks endogenously choose their level of equity shows that a bank will choose to hold excess capital over the regulatory minimum and tighten acceptance criteria. Optimizing over equity and acceptance criteria simultaneously allows the bank to make a better cost-benefit trade-off, which results in higher charter values and lower failure rates.

The article is outlined as follows. In the next section we describe the model setup. In Section III we first examine the basic model with a uniform distribution of returns analytically in Section A, and then look at the effects of optimal regulation by a contingent rule in this framework in Section B. Section C presents further numerical analyses of the basic model with a more flexible beta distribution. In Section IV we extend the basic model to incorporate the banks with an endogenous choice of equity. Finally, Section V concludes.

II. The Basic Model

Consider competition between two banks, labeled $i = 1, 2$, which both operate for T periods, $t = 1, \dots, T$. Bank managers are risk neutral. Following Chu (1999), to abstract from agency problems, we simply assume that bank managers are the bank's shareholders. A bank's management chooses its optimal bank lending policies by stipulating its acceptance criteria for granting loans, which in turn determine the riskiness of the bank's loan portfolio. In our model, these acceptance criteria are captured by a single index parameter $\alpha_i \geq 0, i = 1, 2$. A higher value of the parameter α_i corresponds to looser acceptance criteria and is assumed to negatively affect the quality of the loan portfolio, inducing higher failure rates. However, by loosening the acceptance criteria it becomes easier for consumers to take out loans, i.e., their overall loan capacity increases, yielding an increase in the demand for loans. Formally, bank i faces the following linear demand curve

$$L_i(\alpha_i, \alpha_j) = L + l_1\alpha_i - l_2\alpha_j, \quad \text{with } l_1 > l_2 > 0, \ i, j = 1, 2, \ j \neq i. \tag{1}$$

Thus, the loans of the two banks may be regarded as (imperfect) substitutes, where the parameter l_2 reflects the degree of substitution between the loans of the two banks. By imposing demand curves which depend only on acceptance criteria, the nature of competition among the two banks in this model is completely due to competition in these criteria, and not in the lending rates. In fact, the model can be regarded as Bertrand competition with acceptance criteria as strategic variables.

All potential loans are (imperfectly) diversified and share the same expected return per unit of loanable funds. The probability that a loan is not or only partly repaid varies with α_i. To keep the analysis as transparent and tractable as possible, we follow Schaffer (1998) by assuming that the lending rate is exogenous and fixed. Although one would expect that loans are priced according to their risk exposure, there is not much empirical evidence on the relation between lending rates and borrowing risk. Risk proxies turn out to be insignificant in Berger and Udell (1995) and Blackwell and Winters (1997). Relationship banking and cross-selling provide explanations for this missing link; banks are able to bind their customers for future business, or provide cheap loans in order to attract customers for other profitable business, see e.g., Machauer and Weber (1998) and Berlin and Mester (1997).

Similar to Matutes and Vives (2000), denote by $\tilde{R}_i$ the stochastic return per unit of supplied loans by bank i. The random variable $\tilde{R}_i$ is distributed according to a distribution function F_i which is twice continuously differentiable with positive density f_i on the interval $[\underline{\gamma}_i, \overline{\gamma}_i]$, with $\underline{\gamma}_i \geq 0$, $i = 1, 2$. The density f_i and therefore $\tilde{R}_i$ depends on the acceptance criteria, i.e. $f_i = f_i(\tilde{R}_i; \alpha_i, \alpha_j)$, and $\tilde{R}_i = \tilde{R}_i(\alpha_i, \alpha_j)$, $\alpha_i, \alpha_j \in [\underline{\alpha}, \overline{\alpha}]$, $i \neq j$. First, by raising α_i, the quality of the loan portfolio held by bank i is negatively affected. This takes the form of a reduction in the expected return, labeled $R^e(\alpha_i, \alpha_j) = \mathbb{E}(\tilde{R}_i(\alpha_i, \alpha_j))$, of the loan portfolio and an increase in its variance, i.e., $dR^e(\alpha_i, \alpha_j)/d\alpha_i < 0$, $dvar(\tilde{R}(\alpha_i, \alpha_j))/d\alpha_i > 0$. Second, banks face a 'winner's curse.' In an environment where screening is not perfect and banks do not know the track record of previous rejections of loan applicants, high-risk customers that are accepted by the bank with the easiest criteria exhibit a high probability of having been have been rejected by all or some of the other banks. This 'easy' bank accumulates a substantial part of the high-risk customers. Since in the absence of this bank these applicants would remain shopping around until they are accepted, the presence of this bank improves the risks in the loan portfolio of the other banks. That is, $dR^e(\alpha_i, \alpha_j)/d\alpha_j > 0$, $dvar(\tilde{R}(\alpha_i, \alpha_j))/d\alpha_j < 0$.[3]

Our main focus is on banking competition in the loan market. To simplify, we abstract from considerations at the liabilities side of the banks' balance sheet by assuming that at the beginning of every period t, both banks have unlimited access to an inelastic supply of deposits. Deposits at bank i are labeled D_i. Deposits are short and will be repaid at the end of the period at a fixed deposit rate $r^d > 1$. There is no maturity mismatch since loans are also assumed to be of short duration. Moreover, both banks accommodate the public with all loans demanded. Apart from loans L_i, and deposits D_i, both banks need to issue equity E_i, $i = 1, 2$, in order to be able to deal with possible bad returns on their loan portfolio. Since equity bears a higher risk than deposits, risk-averse equity holders demand an equity premium of $\rho > 0$ on top of the risk free rate r^d.[4] Following Hellman, Murdock, and Stiglitz (2000) we regard the cost of capital as the dilution cost to the owners. Shareholders know, or rationally anticipate, the expected return of the bank in every period, and they will provide capital as long as the expected return on investment equals their exogenous opportunity cost $r^d + \rho$.[5] Banks thus are price takers on the market for equity.

A higher ratio of equity versus (risky) loans on a bank's balance sheet means that it has a lower probability of bankruptcy. As in the Basel Capital Accord, the banking industry regulator prescribes a minimum equity-to-loans ratio to the commercial banks by setting a capital adequacy ratio of $100k$ percent, $0 < k < 1$. In the standard version of the model we assume that no bank will raise more equity than necessary for adhering to the regulator's requirements, since equity is more costly than deposits. This assumption

[3]When banks perfectly know the history of loan applicants, e.g. via credit bnreaus with shared databases, the winner's curse argumentation no longer holds. This implies that $f_i(\cdot; \alpha_i, \alpha_j)$ does not depend on α_j, see Bolt and Tieman (2001). In this article this amounts to setting $a_2 = 0$ in Section's III and IV.

[4]In an extended version of their model, Hellman, Murdock, and Stiglitz (2000) determine the rate of return on equity endogenously as the equilibrium rate that clears the market for bank equity capital. They show that in equilibrium $\rho > 0$. Alternatively, within a general equilibrium framework, Gorton and Winton (1997) also derive that bank capital is costly.

[5]In addition, using a continuity argument we have verified that our results are not qualitatively affected when we model a simple endogenous relation between the equity premium and acceptance criteria.

results in $E_i = kL_i$, $i = 1, 2$, where the equality sign follows from the binding nature of the capital adequacy requirements. In Section IV we posit an extended model in which the amount of equity is endogenously determined by bank management.

The standard setup above leads bank i to make a (stochastic) profit or loss. Note that, because of limited liability, the maximum loss the equity holders of a bank may suffer never exceeds the amount of equity initially raised by the bank. Here, we focus on excess profits of the bank, i.e., profits which remain after repayment of deposits with interest and equity including compensation demanded by the shareholders. This profit function looks like

$$\begin{aligned}\pi_i(\alpha_i, \alpha_j) &= \max\left\{\tilde{R}_i L_i(\alpha_i, \alpha_j) - r^d D_i - (r^d + \rho)E_i\ ,\ -(r^d + \rho)E_i\right\}\\ &= \max\left\{(\tilde{R}_i - [r^d + \rho k])L_i(\alpha_i, \alpha_j),\ -(r^d + \rho)kL_i(\alpha_i, \alpha_j)\right\},\ i, j = 1, 2,\ j \neq i,\end{aligned} \tag{2}$$

where we used the binding nature of the capital adequacy requirement $E_i = kL_i$ and the initial balance sheet equality $D_i = (1 - k)L_i$. A bank which realizes nonnegative expected excess profits is able to meet (on average) the return demanded by the equity holders and will thus be able to raise equity on the capital market in the future.

At the end of each period, the regulatory authority to which both banks are subject inspects the balance sheet of both banks. If the return on its loan portfolio is not sufficient to repay all depositors, that is, whenever equity becomes negative, the regulator closes down the bank and its management is dismissed. Utility for the management after dismissal is set to 0. After bankruptcy of the bank, a new bank identical to the old one comes into operation with a new management which takes the place of the bankrupted bank. The probability of such a bankruptcy is equal to

$$\begin{aligned}\theta_i(\alpha_i, \alpha_j) = \Pr[\mathit{bankruptcy}] &= \Pr\left[\tilde{R}_i L_i(\alpha_i, \alpha_j) - r^d D_i - (r^d + \rho)E_i < -(r^d + \rho)E_i\right]\\ &= \Pr\left[\tilde{R}_i L_i(\alpha_i, \alpha_j) < r^d(1 - k)L_i(\alpha_i, \alpha_j)\right],\end{aligned} \tag{3}$$

which, since $L_i(\alpha_i, \alpha_j)$ cancels out, translates into

$$\theta_i(\alpha_i, \alpha_j) = F_i\left(r^d(1 - k)\right) = \int_{-\infty}^{r^d(1-k)} f(x; \alpha_i, \alpha_j)dx. \tag{4}$$

Combining (2) and (4) yields expected excess profits

$$\begin{aligned}\pi_i^e(\alpha_i, \alpha_j) = \mathbb{E}(\pi_i(\alpha_i, \alpha_j)) = -\theta_i(\alpha_i, \alpha_j)(r^d + \rho)kL_i(\alpha_i, \alpha_j) +\\ \int_{r^d(1-k)}^{\infty} f(x; \alpha_i, \alpha_j)(x - [r^d + \rho k])L_i(\alpha_i, \alpha_j)dx, \qquad i, j = 1, 2,\ j \neq i.\end{aligned} \tag{5}$$

Following Diamond (1989) and Hellman, Murdock, and Stiglitz (2000), we will look at symmetric Nash equilibria in the limit as $T \to \infty$. In particular, we are interested in a stationary equilibrium, where players act myopically. To derive this equilibrium, we need to make two further assumptions. First, we assume that banks can make only one-period loans which are not carried over to the next period.[6] Second, we assume that when a bank is not bankrupt at the end of period t, it continues operations into period $t+1$ with the same initial balance sheet as in period t. Implicitly, we assume that excess profits are paid out as extra dividends, and negative excess profits are compensated by shareholders supplying additional equity and thus recapitalizing their bank. This closure rule stresses that bank owners would be willing to provide new equity to cover any losses in a period, so as to hold on to their future charter value. Given the sunk losses, recapitalizing makes the shareholder better off compared to bankruptcy followed by investment in a new identical bank.[7]

In the stationary equilibrium we consider, where α_i and α_j are the same in all periods, a bank's charter value, i.e., its discounted expected excess profits, are given by

$$V_i = \lim_{T\to\infty} \sum_{t=0}^{T} (\delta_i(\alpha_i, \alpha_j))^t \, \pi_i^e(\alpha_i, \alpha_j) = \frac{\pi_i^e(\alpha_i, \alpha_j)}{1 - \delta_i(\alpha_i, \alpha_j)}, \tag{6}$$

where we denote $\delta_i(\alpha_i, \alpha_j) = [1 - \theta_i(\alpha_i, \alpha_j)]/(r^d + \rho)$. Hence, in our model the endogenous discount factor $\delta_i(\alpha_i, \alpha_j)$ consists of a multiplication of two components. First, the exogenous rate of time preference $1/(r^d + \rho)$ of equity holders, since they are the recipients of excess profits, and second, the endogenous continuation probability of the flow of profits, given by $1 - \theta_i(\alpha_i, \alpha_j)$. From the charter value V_i the bank's trade-off is clear: by setting its acceptance criteria parameter α_i it must optimally balance the positive effect on the numerator (increased demand) against the negative effect on the denominator (decrease in the endogenous discount factor).

Since we assume that banks behave myopically, they take the future strategies of their

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[6]If loans would carry over to the next period, the usual equilibrium concept to be used is Markov perfect equlibrium, which would complicate our analysis considerably. In our view the myopic stationary equilibrium makes good sense if players for some reason are not able to coordinate on a 'good' equilibrium.

[7]Bhattacharya, Plank, Strobl, and Zechner (2002) consider a model of optimal closure rules and they demonstrate how capital adequacy, bank auditing and closure regulation can be designed to mitigate risk taking behavior by banks. In Perotti and Suarez (2001) a different type of argument is pursued by analyzing a policy rule that allows solvent institutions to take over failed banks. This way, they provide banks with an incentive to act prudently. As a consequence the market structure and corresponding charter value may temporarily change. Such a nonstationarity would complicate our analysis considerably and is beyond the scope of this paper.

opponents as given. Therefore, as players do not perceive any influence on subsequent play, the continuation value of the game V_i is fixed. Hence, in the myopic stationary equilibrium, bank i's maximization problem in a period t becomes

$$\max_{\alpha_i} \ \pi_i^e(\alpha_i, \alpha_j) + \delta_i(\alpha_i, \alpha_j)V_i. \tag{7}$$

As V_i is taken as given, the first order condition for a maximum implies

$$\frac{\partial \pi_i^e(\alpha_i, \alpha_j)}{\partial \alpha_i} + \frac{\partial \delta_i(\alpha_i, \alpha_j)}{\partial \alpha_i} V_i = 0 \tag{8}$$

Using stationarity, we can insert (6) to get

$$\frac{\partial \pi_i^e(\alpha_i, \alpha_j)}{\partial \alpha_i} + \frac{\partial \delta_i(\alpha_i, \alpha_j)}{\partial \alpha_i} \frac{\pi_i^e(\alpha_i, \alpha_j)}{1 - \delta_i(\alpha_i, \alpha_j)} = 0, \tag{9}$$

which is equivalent to

$$(1 - \delta_i(\alpha_i, \alpha_j)) \frac{\partial \pi_i^e(\alpha_i, \alpha_j)}{\partial \alpha_i} + \pi_i^e(\alpha_i, \alpha_j) \frac{\partial \delta_i(\alpha_i, \alpha_j)}{\partial \alpha_i} = 0. \tag{10}$$

Equation (10) characterizes the myopic stationary equilibrium of our game.

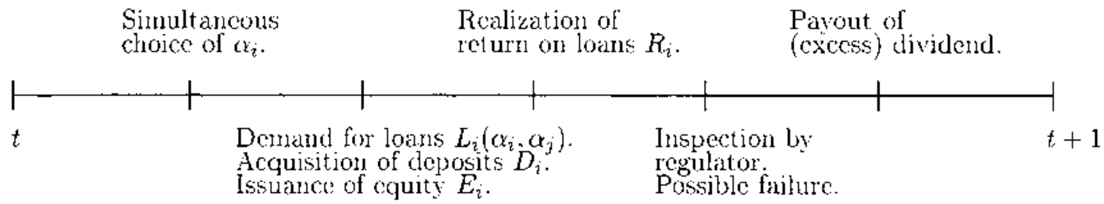
Numerically we can show that if we assume that current actions only affect current continuation probabilities and hence tomorrow's discount factors, i.e., $\delta_i(t+1) = \delta_i(\alpha_i(t), \alpha_j(t))$, this equilibrium above corresponds to the limit of the unique subgame perfect Nash equilibrium of the game with finite T, when we let $T \to \infty$.[8] In effect, our framework enables us to select one particular equilibrium of the infinitely many subgame perfect equilibria in the infinitely repeated game with an endogenous discount factor.[9]

The timing of events during a time period is illustrated in Figure 1. At the beginning of period t both banks choose their acceptance criteria α_i, $i = 1, 2$ *simultaneously*. At the

[8] Naturally, the subgame perfect equilibrium actions in a fixed number of periods close to the last period T will differ from the derived myopic stationary equilibrium actions of the game. However, by backward induction, this number of periods is fixed and does not grow with T. Hence, we can show that if $T \to \infty$, the unique SPE actions of the finite game become arbitrarily close to the myopic stationary equilibrium actions as characterized in (10) in almost all periods. Numerical results for the uniform and beta distribution are computed using the Mathematica 4.1 package, and are available upon request.

[9] Dynamic games with an infinite number of periods generally feature a continuum of equilibria. Here, we focus on the myopic stationary equilibrium of the game and ignore possible bootstrap equilibria such as e.g., trigger-strategy equilibria, or tacit collusion; see Fudenberg and Tirole (1991) for a further discussion.

Figure 1. The Timing of Events.



same time, in order to fund their loan portfolios, both banks attract the necessary deposits and raise equity. After loans are made, deposits attracted, and equity issued, the stochastic return on the loans is realized. Subsequently, at the end of period t, the regulator assesses whether the bank has positive equity. If not, the bank is declared bankrupt, and ceases operations. A new identical bank takes its place. In case of positive equity, the bank repays its depositors the amount of deposits plus interest, and equity is paid out including compensation and (possibly negative) excess dividends to the shareholders. The bank continues its operations, and the game proceeds to period $t + 1$.

III. Strategic Analysis of the Basic Model

Since the choice of the acceptance criteria parameter α_i of one bank influences the profits of the other bank, our model of bank behavior presents a strategic decision-making problem for both banks. Given α_j, bank i's manager maximizes the discounted sum of profits over α_i. This yields bank i's reaction function, $i = 1, 2$, to bank j's behavior, $j = 1, 2, j \neq i$. Formally, bank i's first order condition (10) can be rewritten as

$$\epsilon_\pi(\alpha_i, \alpha_j) = \epsilon_\delta(\alpha_i, \alpha_j), \tag{11}$$

where $\epsilon_\pi(\alpha_i, \alpha_j) = (\partial \pi_i^e / \partial \alpha_i)(\alpha_i / \pi_i^e)$ denotes the elasticity of profits with respect to α_i, and $\epsilon_\delta(\alpha_i, \alpha_j) = (\partial(1 - \delta_i)/\partial \alpha_i)(\alpha_i/(1 - \delta_i))$ the 'elasticity' of the discount factor with respect to α_i. In fact, equation (11) presents a marginal cost-benefit trade-off: the gains of taking up more risk in terms of increasing current profits are just outweighed by the future losses expressed in terms of a lower discount factor, that is, a higher failure rate. In principle, solving (11) leads to bank i's reaction function $\alpha_i^R(\alpha_j)$. In a symmetric equilibrium one must have $\alpha_i = \alpha_j$, so that $\epsilon_\pi(\alpha^*, \alpha^*) = \epsilon_\delta(\alpha^*, \alpha^*)$, implicitly defines the (symmetric) equilibrium value α^*.

A. A Uniform Distribution of Returns

For a general distribution of returns, an explicit analytical expression for α^* cannot be found. Therefore, we perform the analysis for a uniform distribution. Consider a uniform distribution for bank i with support $x \in [A - a(\alpha_i, \alpha_j), B]$, $0 \leq A < B$, and $a(\alpha_i, \alpha_j) = a_1(\alpha_i - a_2\alpha_j)$. Given the discussion in the previous section, we impose $a_1 > 0$, $0 \leq a_2 < 1$. That is, setting a higher α_i decreases the expected return and increases the variance of bank i's loan portfolio, while increasing α_j has has the opposite effect on bank i's portfolio. We assume that the magnitude of the 'own' effect, as measured by a_1, is larger than the 'cross' effect, as measured by a_1a_2; therefore, $0 \leq a_2 < 1$. The density function is given by $f(x;\alpha_i,\alpha_j) = \frac{1}{B-A+a(\alpha_i,\alpha_j)}$, when $x \in [A - a(\alpha_i, \alpha_j), B]$ and 0 elsewhere.[10] We label expected returns when banks set $\alpha_i = \alpha_j = 0$, i.e., take minimal risk, by $\bar{R} = \frac{A+B}{2}$. For this uniform distribution the charter value of bank i is equal to

$$V_i(\alpha_i,\alpha_j) = \begin{cases} L(\alpha_i,\alpha_j)\frac{C_1+C_2a_1(\alpha_i-a_2\alpha_j)}{C_3+C_4a_1(\alpha_i-a_2\alpha_j)}, & \text{when } F(r^d(1-k)) \in [0,1] \\ \\ L(\alpha_i,\alpha_j)\frac{C_5}{C_6+C_7a_1(\alpha_i-a_2\alpha_j)}, & \text{when } F(r^d(1-k)) = 0 \end{cases} \tag{12}$$

with

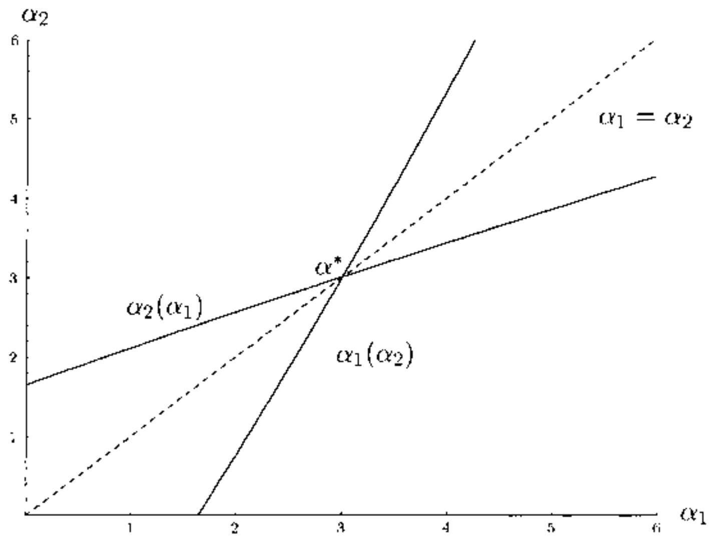
$$\begin{aligned} C_1 &= (r^d+\rho)(B^2 + ((1-k)r^d)^2 + 2Ak(r^d+\rho) - 2B(r^d+k\rho)), \\ C_2 &= 2k(r^d+\rho)^2, \\ C_3 &= 2\left((1-k)r^d + B(r^d+\rho-1) - A(r^d+\rho)\right), \\ C_4 &= 2(r^d+\rho), \\ C_5 &= (B-(1-k)r^d)(r^d+\rho)\left(B - r^d - k(r^d+2\rho)\right), \\ C_6 &= 2(B-A)(r^d+\rho-1), \\ C_7 &= 2(r^d+\rho-1). \end{aligned}$$

Solving $\alpha_j = \alpha_j^R(\alpha_i^R(\alpha_j))$ yields the symmetric equilibrium value α^* as a function of the exogenous variables of the model. The explicit analytical expression for α^* can be found in the Appendix, point 1.

Figure 2 presents the reaction curves $\alpha_i^R(\alpha_j)$, $i,j = 1,2$, $j \neq i$ of the two commercial banks for a uniform distribution of returns with support $[0.7 - a(\alpha_i,\alpha_j), 1.7]$, $a_1 = a_2 = 0.25$, so that $\bar{R} = 1.2$. We assume a deposit rate $r^d = 1.05$, an equity premium $\rho = 0.1$ and a demand function $L_i(\alpha_i,\alpha_j) = 2 + 5\alpha_i - 2\alpha_j$. The CAR is set to $k = 8\%$, the current minimum capital requirement as prescibed by the Basel Accord. The

[10] To ensure nonnegative gross returns, we must have that $\alpha^* \leq \frac{A}{a_1(1-a_2)}$ in a symmetric equilibrium.

Figure 2. The Two Reaction Curves



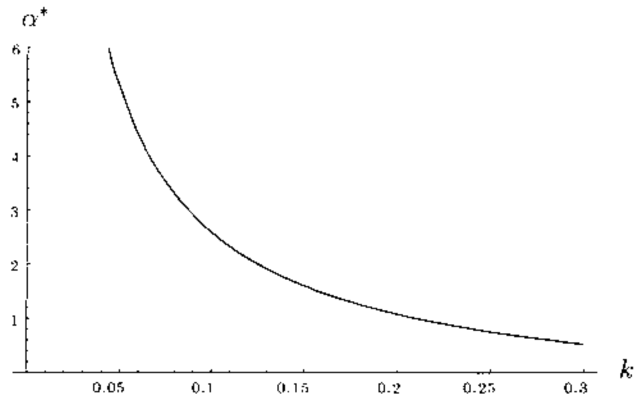
intersection of the reaction curves is a graphical representation of α^*, which calculations show to be equal to 3.0. Note that because of the symmetry of the problem the intersection point of the reaction curves lies on the line $\alpha_1 = \alpha_2$, which is also drawn in the figure.

Interest goes out to the comparative statics of α^* with respect to an increase in the capital adequacy ratio k and to increased competition, which, as seen from equation (1), is described by an increase in l_2. In Figure 3 we show the dependence of α^* on the capital adequacy ratio k for the parameter values mentioned above. The figure clearly show the general features of the comparative statics on k. For the given set of parameter values, an increase in the capital adequacy ratio leads to less risk taking on the part of the banks which translates to stricter acceptance criteria for granting loans. Therefore, an increase in the CAR leads to a decline in the total demand for loans.

Result 1. *The higher the fixed capital adequacy ratio, the less risk commercial banks take on.*

Result 1 intuitively follows from considering the two reaction curves in Figure 2. Consider two Nash equilibria at different CARs, $\alpha^*_{k_1}$ and $\alpha^*_{k_2}$, with $k_2 > k_1$. Since $\frac{\partial^2 V}{\partial \alpha_i \partial k}$ is negative at $\alpha^*_{k_1}$ (see Appendix, point 2), raising k from k_1 to k_2 yields a negative derivative $\frac{\partial V}{\partial \alpha_i}$ at $\alpha^*_{k_1}$. This means that the optimal reaction $\alpha_i(\alpha^*_{k_1})$ is lower at $k = k_2$ than at $k = k_1$, i.e., is below $\alpha^*_{k_1}$. Thus, the reaction curve of bank 1 shifts to the left, while that of bank 2 shifts downward. The intersection point of the two new reaction

Figure 3. The Dependence of α^* on k



curves therefore yields a lower equilibrium value, i.e., $\alpha^*_{k_2} < \alpha^*_{k_1}$.

Comparative statics results of α^* on l_2 are shown in Figure 4. From the figure, we see that more intense competition leads to more risk taking by the banks. This is caused by the fiercer competition for market share. Since more risky portfolios are the consequence of looser acceptance criteria for granting loans, it implies that the total demand for loans increases when competition becomes more intense.

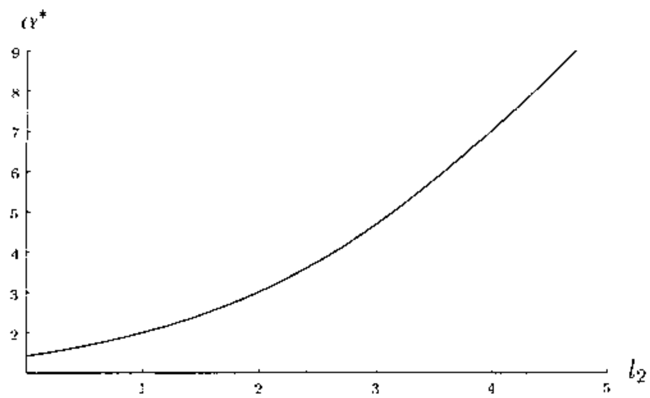
Result 2. *The more intense the banking competition, the more risk commercial banks take on.*

Seen from a technical point of view, Result 2 follows from standard Bertrand price competition considerations. Increased competition harms per-period profits and has no direct effect on the probability of bankruptcy. In order to partially compensate for the profit decrease, the banks will increase the risk in their portfolio, by setting lower acceptance criteria, thereby ceteris paribus increasing demand. Of course, through this channel of loan portfolios bearing more risks, indirectly the failure rate will increase.[11]

In contrast, e.g., Petersen and Rajan (1995) find that decreased competition may increase credit availability to firms with lower credit quality. In their model, market concentration enables the creditors to better internalize the benefits of assisting the

[11]Loose acceptance criteria in an environment with more competition is in accordance with empirical findings in the Dutch mortgage market. Indeed, increased competition in this market which is not so much in lending rates but more in acceptance criteria, has led to increased vulnerability of the commercial banks' balance sheets.

Figure 4. The Dependence of α^* on l_2



firms. Our results follow because we focus on transactional lending and ignore the basic rent extraction problem.

B. Risk-Adjusted Capital Requirements

In this section we assume that the banking regulator can apply a contingent rule as its instrument. Such a rule specifies how the CAR depends on the risk behavior of a commercial bank. The regulator set a maximum admissible failure probability $P_{max} \in [0, 1]$ for commercial banks. Formally, $k(\alpha_i, \alpha_j, P_{max})$ solves $\theta_i(\alpha_i, \alpha_j, k(\alpha_i, \alpha_j, P_{max})) \leq P_{max}$. For small values of P_{max} this inequality will be binding, which yields, for the case of a uniform distribution of returns

$$k(\alpha_i, \alpha_j, P_{max}) = 1 - \frac{1 + B - (1 - P_{max})(B - A + a_1(\alpha_i - a_2\alpha_j))}{r^d}. \tag{13}$$

Since $B > A - a_1(\alpha_i - a_2\alpha_j)$, it easily follows that $\frac{\partial k(\alpha_i, \alpha_j, P_{max})}{\partial P_{max}} < 0$. That is, when the maximum admissible probability of bankruptcy is decreased, the banks will be required to hold more equity in relation to their level of risky loans. From the expression for $k(\alpha_i, \alpha_j, P_{max})$ we have that $\frac{\partial k(\alpha_i, \alpha_j, P_{max})}{\partial \alpha_i} > 0$, confirming that indeed a regulator would like to see a bank with a more risky portfolio hold more equity.

Result 3. *The optimal risk-adjusted capital adequacy ratio is higher for a lower admissible probability of failure.*

Result 4. *The optimal risk-adjusted capital adequacy ratio is higher when a commercial bank takes on more risk.*

The regulator now communicates to the commercial banks that it will set the CARs according to $k(\alpha_i, \alpha_j, P_{max})$. The banks will thus optimize their charter value, given these risk-weighted CARs. Substituting $k(\alpha_i, \alpha_j, P_{max})$ in the charter value of the banks and subsequent computations yields new reaction curves. For tractability, we choose to fix $a_2 = 0$, implying that bank i's density function is independent from α_j (see also footnote 3). This gives an elegant linear expression for the equilibrium solution

$$\alpha^{**} = \frac{l_1 \left(A - \frac{a_1 L}{l_1} + \frac{B P_{max} - r^d}{1 - P_{max}} + \frac{\left(B - r^d\right) r^d}{(1 + P_{max}) r^d + 2\rho} \right)}{a_1 \, (2 l_1 - l_2)}. \tag{14}$$

The comparative statics for α^{**} in the case of risk-weighted regulation show that α^{**} is increasing in P_{max}, i.e., when the regulator sets a lower admissible probability of bankruptcy, the banks will react by taking less risk and vice versa (see Appendix, point 3). Thus, indeed the risk weighting of the CARs has the desired effect that the risks in the banking system will diminish. At the same time, because the banks are placing more demanding conditions on their supply of loans, the total loan market demand declines.

Result 5. *A lower admissible probability of failure leads to less risk taking by the commercial banks.*

C. A Beta Distribution of Returns

One of the main advantages of choosing a uniform distribution to model the stochastic returns on the bank's loan portfolio is its simple mathematical form and the ability to derive analytic results. However, its drawback lies in the restrictive nature of its constant density function. A more natural and flexible distribution to model the stochastic process of returns is a beta distribution, which has the appealing feature that a change in its parameters yields a shift in the probability mass, while its support remains constant. However, the cost is in losing analytical tractability and we therefore present a numerical exercise to grasp the robustness of the results presented in the previous two subsections.

Let $\tilde{R}_i = \bar{R}(1 - \tilde{X}_i)$ where $\tilde{X}_i$ follows a beta distribution on the interval $[0, 1]$ with parameters $(a(\alpha_i, \alpha_j), b)$, $b > 0$. Again we specify $a(\alpha_1, \alpha_j) = a_1(\alpha_i - a_2\alpha_j)$, $a_1 > 0$,

$0 \leq \ a_2 \ < \ 1$. Then, the support of $\tilde{R}_i$ is $[0, \bar{R}]$, and $R^e(\alpha_i, \alpha_j) = \frac{\bar{R}b}{b+a_1(\alpha_i - a_2\alpha_j)}$, which is decreasing (increasing) in α_i (α_j).[12] For a fixed parameter b, given α_j, larger values of α_i shift the probability mass towards the region of lower returns. The opposite holds for α_j. Further, no risk corresponds to $\alpha_i = \alpha_j = 0$ in which case $R^e(0,0) = \bar{R}$.

As before, a higher α_i induces on the one hand a higher probability of default but on the other hand enables bank i to attract more loans, thus enhancing its current profits. Given the choice α_j of its competitor, bank i has to weigh these two opposing forces optimally. Table 1 presents the symmetric Nash equilibria $\alpha_i^* = \alpha_j^* = \alpha^*$ for different values of the capital adequacy requirement, and shows corresponding equilibrium probabilities of default, volumes of loans and charter values. This numerical example closely follows the one presented in Section A in terms of parameter values: we put a deposit interest rate $r^d = 1.05$, average return on loans without risk $\bar{R} = 1.2$, equity premium $\rho = 0.1$, demand for loans at $L_i(\alpha_i, \alpha_j) = 2 + 5\alpha_i - 2\alpha_j$ and the beta density parameters $a_1 = a_2 = 0.25$ and $b = 5$. The table shows that a higher CAR induces lower risk taking in the sense of stricter acceptance criteria, i.e., a lower α^* in equilibrium. As a consequence the bankruptcy probabilities drop, a result which is stated in Result 6 below. These findings confirm our results of Section A and point to robustness with respect to the choice of the probability distribution of returns.

Table 1. Symmetric Nash Equilibria for Different CAR k

k	α^*	θ_i^*	L_i^*	V_i^*
0.05	1.38	0.088	6.14	2.90
0.08	1.36	0.069	6.09	2.97
0.10	1.36	0.059	6.07	2.99
0.15	1.34	0.040	6.02	3.01
0.20	1.32	0.027	5.95	2.97

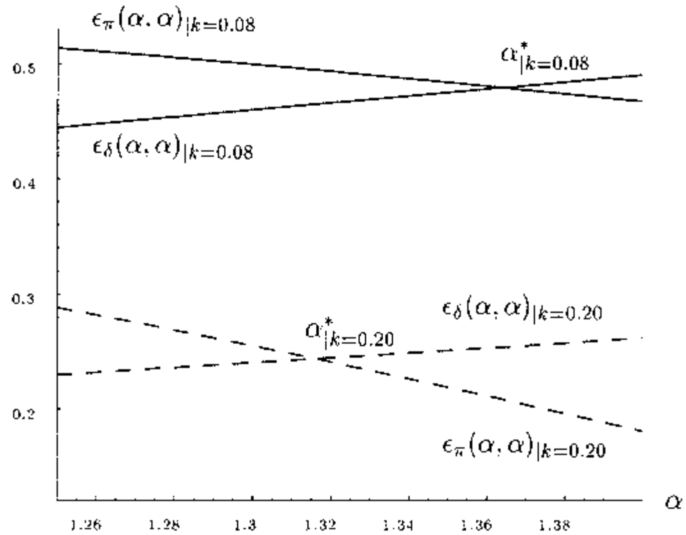
Explanatory Note: Parameter values are $a_1 = a_2 = 0.25$, $b = 5$, $\bar{R} = 1.2$, $r^d = 1.05$, $\rho = 0.1$, $L = 2$, $l_1 = 5$, $l_2 = 2$.

Note that in Table 1 the charter value of the banks first increases up to $k = 0.15$, and then decreases. Hence, for $k \leq 0.15$, this suggests that when banks could freely choose the level of their equity-to-loans ratio, they would choose a ratio higher than k, thereby rendering the capital adequacy constraint nonbinding. We extend the current model to incorporate such an unconstrained choice of equity in Section IV.

Result 6. *Higher fixed capital adequacy ratios lead to lower failure probabilities of the*

[12] The variance of the returns is increasing (decreasing) for relevant values of α_i (α_j).

Figure 5. Illustration of the Nash Equilibrium for $k = 0.08$ (solid) $k = 0.20$ (dashed)



commercial banks.

Result 6 is stated in terms of failure rates and not in terms of risk taking by banks, as in Result 1. When referring to risk taking, a further qualification is needed in the beta returns case. As illustrated in Figure 5, by varying k, the curves $\epsilon_\pi(\alpha,\alpha)$ and $\epsilon_\delta(\alpha,\alpha)$ shift simultaneously. In general, the net effect on α^* is ambiguous and depends on market structure conditions.[13] Figure 5 shows these curves for capital ratio $k = 0.08$ (solid lines) and $k = 0.20$ (dashed lines). Here, we see that, on the one hand, the 'per-period profit elasticity' line shifts to the left for the increased capital ratio, since a higher k leads to lower per-period profits, and on the other hand, the 'failure rate elasticity' line shifts to the right indicating that probability of failure decreases as k rises. Given our choice of the parameters, the net effect of a shift in k from 0.08 to 0.20 is a lower value of α^*, as also indicated by Table 1.

As in the previous section, we have also checked the sensitivity of the results to more intense competition, as measured by an increase in l_2. Table 2 shows the results for a fixed CAR of $k = 0.08$ in terms of an optimal α^*, and corresponding failure rate, volume

[13]In Bolt and Tieman (2001) we show that if the ratio $L/(l_1 - l_2)$ is small enough, that is, when the loan demand increases relatively fast in α^*, Result 1 carries over to the beta distribution case.

of loans and charter value. From the table we see that the analytic results found for a uniform distribution of returns are confirmed by the beta distribution case: More fierce competition leads to the commercial banks to take on more risk, which results in higher failure rates and lower charter values.

Result 7. *The more intense the banking competition, the more risk commercial banks take on, the higher the failure rates, and the lower the charter values.*

Table 2. Symmetric Nash Equilibria for Different Values of l_2 at $k = 0.08$

l_2	α^*	θ_i^*	L_i^*	V_i^*
1	1.05	0.052	6.22	3.66
2	1.36	0.069	6.09	2.97
3	1.91	0.102	5.82	2.01
4	2.95	0.169	4.95	0.83

Explanatory Note: Other parameter values are $a_1 = a_2 = 0.25$, $b = 5$, $\bar{R} = 1.2$, $r^d = 1.05$, $\rho = 0.1$, $L = 2$, $l_1 = 5$.

We continue by analyzing optimal regulation as in Section B. We derive $k(\alpha_i, \alpha_j, P_{max})$ such that $\theta_i(\alpha_i, \alpha_j, k(\alpha_i, \alpha_j, P_{max})) = P_{max}$ holds. In calculating an optimal strategy the banks will take this contingency rule into account. The resulting equilibrium α^{**} determines an equilibrium capital ratio $k^{**} = k(\alpha^{**}, \alpha^{**}, P_{max})$.

Table 3. Symmetric Nash Equilibria in the Presence of Risk-Adjusted Regulation

P_{max}	α^{**}	k^{**}	L_i^{**}	V_i^{**}
0.01	1.06	0.29	5.17	2.99
0.02	1.17	0.22	5.51	3.04
0.03	1.25	0.18	5.75	3.04
0.04	1.32	0.15	5.95	3.02
0.05	1.37	0.14	6.12	2.99

Explanatory Note: Parameter values are $a_1 = a_2 = 0.25$, $b = 5$, $\bar{R} = 1.2$, $r^d = 1.05$, $\rho = 0.1$, $L = 2$, $l_1 = 5$, $l_2 = 2$.

Table 3 gives the results for different values of P_{max}. It turns out that the imposed contingent rule is effective in reducing the bankruptcy probability to the imposed level. It results in higher required capital ratios and induces less risk taking. For instance, to get the probability of bankruptcy down to 3 percent the induced capital adequacy ratio rises to 18 percent, while a failure rate of 1 percent corresponds to a CAR of 29 percent in this example. Thus we observe qualitatively the same behavior as in Result 5: A lower admissible probability of failure leads to less risk taking by the commercial banks.

Optimal risk-adjusted regulation has interesting welfare implications. When comparing Table 1 with Table 3, fixing the failure rate at 4 percent, risk taking by banks is reduced when they are feeded a contingency rule, i.e., $\alpha^{**} < \alpha^{*}$. At the same time the banks' charter values increase, i.e., $V_i(\alpha^{**}, \alpha^{**}) > V_i(\alpha^{*}, \alpha^{*})$. Thus, while the utility of the regulator (which depends on the failure rate of the banks only) remains constant, the value for the banks increases in case of optimal risk adjusted regulation.[14] A complete welfare ranking is however not possible, since the consumer side of the economy is not modelled. Consumer surplus might actually decline since the supply of loans is reduced.

IV. An Extension: Endogenous Equity

In the basic model, we assumed that banks will never hold more equity than the minimum amount prescribed by the regulator, since raising equity is more costly than attracting deposits. However, in real life one observes that many banks hold more equity than the minimal amount prescribed according to the CARs. This may be partly motivated by the buffer function of bank capital. With excess capital on the balance sheet, sudden losses on a bank's loan portfolio will not immediately lead to regulatory action.[15] While equity is more expensive to a bank than deposits, in a dynamic context, holding more equity also prolongs the expected lifetime of the bank in operation, thus yielding a more extended flow of profits. Bank's management weighs the marginal benefits of taking on more risk by easing its acceptance criteria against the marginal costs.

Below, we investigate a model in which we drop the assumption $E_i = kL_i$. Such a model presents bank i with an optimization problem over both acceptance criteria and equity, i.e., over the two variables α_i and E_i, subject to the condition that $E_i/L_i \geq k_{min}$, with k_{min} now being the minimum capital requirement imposed by the regulator. It results in expressions for per period profits and the probability of bankruptcy that are different from those presented before. Using the balance sheet equality $L_i = E_i + D_i$, per period

[14] In Bolt and Tieman (2002) we characterize under which conditions optimal risk adjusted regulation leads to superior outcomes for the banks and the regulator.

[15] Peek and Rosengren (1995a, 1995b) report a direct link between regulatory enforcement actions and shrinkage of bank lending. They find that banks which have faced regulatory action quickly increased their capital ratios at the expense of their profitability. Hence, it seems a sensible busiuess strategy to create a capital buffer to avoid this type of costly regulatory action.

profits amount to

$$\begin{aligned}\pi(\alpha_i, \alpha_j, E_i) &= \max\left\{\tilde{R}_i L_i(\alpha_i, \alpha_j) - r^d D_i - (r^d + \rho)E_i\ ,\ -(r^d + \rho)E_i\right\} \\ &= \max\left\{(\tilde{R}_i - r^d)L_i - \rho E_i\ ,\ -(r^d + \rho)E_i\right\},\end{aligned} \tag{15}$$

while the probability of bankruptcy of bank i is given by

$$\theta_i(\alpha_i, \alpha_j) = Pr\left(\tilde{R}_i < r^d\left[1 - \frac{E_i}{L_i(\alpha_i, \alpha_j)}\right]\right) = F\left(r^d\left[1 - \frac{E_i}{L_i(\alpha_i, \alpha_j)}\right]\right). \tag{16}$$

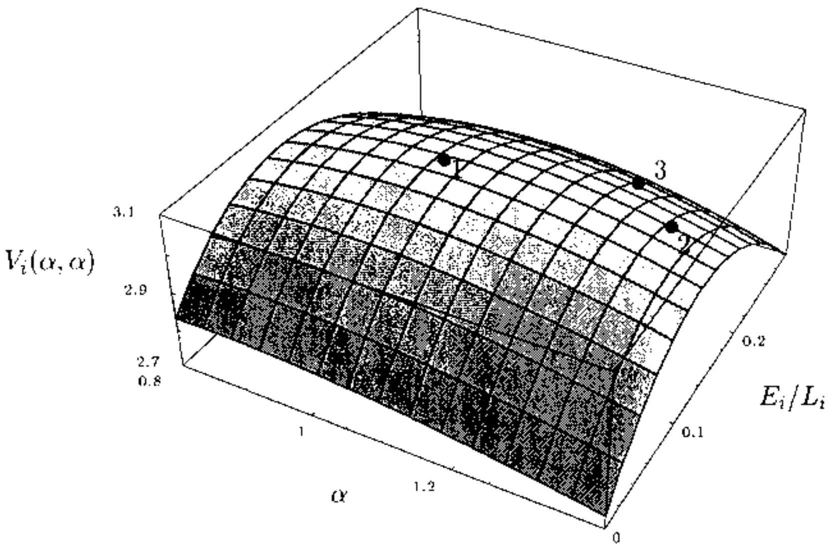
We analyze this model for the beta distribution of returns as specified in Section C, with the same parameter values as before: $a_1 = a_2 = 0.25$, $b = 5$, $\bar{R} = 1.2$, $r^d = 1.05$, $\rho = 0.1$, $L = 2$, $l_1 = 5$, $l_2 = 2$. Figure 6 plots the (symmetric) charter value for different values of $\alpha = \alpha_i = \alpha_j$ and equity-to-loans ratio E_i/L_i. One may verify that the symmetric maximum in Figure 6 is attained for $\tilde{\alpha} = 1.04$ and $\widetilde{E/L} = 13.3$ percent (with a failure rate of 3.4 percent, supply of loans 5.1, and charter value 3.07), however this maximum does not correspond to the symmetric Nash equilibrium, since players cannot credibly commit to the corresponding actions. Given that bank i chooses $\alpha_i = \tilde{\alpha}$ and $E_i/L_i = \widetilde{E/L}$, bank j is better off by deviating from these actions. Numerical optimization yields the symmetric Nash equilibrium for $\alpha^* = 1.35$ and $(E/L)^* = 13.5$ percent, which in this example is well above $k_{min} = 8$ percent, as currently prescribed in the Basel Accord. Hence, since $k_{min} \leq 13.5$ percent, capital requirements do not impinge on the equilibrium outcome. The Nash equilibrium bankruptcy probability rises to 4.5 percent at a charter value of 3.01. Indeed we conclude that banks, when unconstrained in their choice of equity, may choose to hold more equity than prescribed to them by the regulator in order to lower their probability of failure. The Nash volume of the loans is equal to 6.0, thus slightly smaller than in the case of fixed regulation at $k = 8$ percent.[16]

Result 8. *When given the choice, commercial banks may choose to hold more equity than the minimum equity level required by the regulator.*

As in the previous section, in order to better align capital requirements with underlying risks, the regulator may impose a contingent rule. In this extended model this means setting a contingency rule with respect to equity, in the sense that more risk must be countered by more equity. Such a rule is equivalent to the contingent rule on k in the

[16]We were unable to derive analytical solutions for the model with endogenous equity and a uniform distribution of returns. The numerical results for such a model are qualitatively same as the results obtained here for a beta distribution.

Figure 6. Charter Values for the Endogenous Equity Model



Explanatory Note: Dot labeled 1 corresponds to the unconstrained maximum $(\tilde{\alpha}, \widetilde{E/L}) = (1.04, 0.13)$, dot 2 to the (symmetric) Nash equilibrium $(\alpha^*, (E/L)^*) = (1.35, 0.13)$, and dot 3 to the (symmetric) 'contingency rule' Nash equilibrium $(\alpha^{**}, (E/L)^{**}) = (1.25, 0.21)$ with $P_{max} = 0.03$.

fixed equity case of Section III. Let the 'contingent equity' rule be denoted by $E(\alpha_i, \alpha_j)$, such that $\theta(\alpha_i, \alpha_j, E(\alpha_i, \alpha_j)) \leq P_{max}$. Calculations show that reducing failure rates in this way is effective. For example $P_{max} = 3$ percent -which is strictly smaller than the Nash equilibrium bankruptcy probability of 4.5 percent in the example above-, in equilibrium a bank takes on less risk and holds more equity than in the unconstrained case without a contingent rule, i.e., $\alpha^{**} = 1.25$ and $(E/L)^{**} = 21.5$ percent. The effect on the volume of the loan market is slightly contractionary, decreasing from 6.0 tot 5.8. These findings exactly correspond to the third row of Table 3 of Section III and illustrate the next result.

Result 9. *In a setting with endogenous equity, imposing an optimal risk-adjusted minimum level of equity reduces commercial banks' failure rates.*

V. Conclusions

We have modeled a framework in which commercial banks compete for loans by setting acceptance criteria. The model enables us to investigate the effects of regulation of and competition among the banks. We examine the model analytically for a uniform distribution of returns and numerically for a beta distribution. The most important conclusions to be drawn are that increased competition will lead to more risk taking by the banks. The mechanism here is that more competition leads ceteris paribus to lower per period profits, which lowers the costs of bankruptcy to the bank management and thus makes them more prone to risk seeking in order to increase demand. We therefore conclude that increased competition in the banking industry warrants increased attention of the financial sector regulatory authority. In this context, setting a higher fixed capital adequacy requirement enables the regulator to reduce the probability that commercial banks go bankrupt.

As an alternative to fixed capital adequacy requirements, a regulator can implement risk-weighted regulation, as is the tendency reflected in the new Basel proposal, Basel II, on capital adequacy. Our analysis shows that risk-weighted regulation is effective: the contingent capital adequacy rule increases in the risk taken by the bank and decreases in the maximal failure probability deemed acceptable by the regulator. We conclude that risk-adjusted capital adequacy requirements are thus a useful tool for the regulatory authority to control the increased vulnerability of the banking system as a consequence of, among other factors, increased competition.

We have extended the basic model by specifying a model in which bank management maximizes profits over both the acceptance criteria index and equity. We show that management may choose to hold more equity than required by the regulator, even though raising equity is more expensive than attracting deposits. The rationale here is that although additional equity lowers per period profits, it prolongs the expected lifetime of the bank and thus the expected length of the stream of future profits. In such a setting, risk-adjusted capital adequacy requirements are useful when the regulator wants to reduce the failure probabilities below their equilibrium values.

Appendix

1. The explicit expression for α^* in the uniform case is

$$\alpha^* = \frac{1}{4\,a_1^2\,(1-a_2)^2 k\,l_1\,\left(r^d+\rho\right)^2} \quad \times$$
$$\Bigg(\Big(a_1\,\left(r^d+\rho\right)\,\left(B^2\,l_2 + r^d\,\left(4\,k\,\left(A-(1-k)\right)\,l_1 + 2\,(1-k)\,k\,l_2 + (1-k)^2\,l_2\,r^d\right) +\right.$$
$$4\,A\,k\,l_1\,\rho - 2\,B\,\left(k\,\left(2\,l_1 - l_2\right)\,\left(r^d-1\right) + l_2\,r^d + 2\,k\,l_1\,\rho\right) - a_2 l_1\,\left(B^2 +\right.$$
$$\left.\left. r^d\left(2k\,(2A-(1-k)) + (1-k)^2\,r^d\right) + 4Ak\rho - 2B\,\left(r^d + k\left(r^d + 2\rho - 1\right)\right)\right)\right)\Big) +$$
$$\Big(a_1^2\,\left(B-(1-k)\,r^d\right)\,\left(B - 2\,k - (1-k)\,r^d\right)\,\left(r^d+\rho\right)^2 \;\times$$
$$\left(B^2\,l_2^2 - r^d\,\left(8kl_1\,\left(a_1\,L + (A-(1-k))\,l_1\right) - 8k\,(A-(1-k))\,l_1\,l_2 - 2k\,(1-k)\,l_2^2 - \right.\right.$$
$$\left.(1-k)^2\,l_2^2\,r^d\right) - 8\,k\,l_1\,\left(a_1\,L + A\,\left(l_1 - l_2\right)\right)\,\rho + 2\,B\,\left(k\,\left(2\,l_1 - l_2\right)^2\,\left(r^d - 1\right) - \right.$$
$$\left.\left. l_2^2\,r^d + 4\,k\,l_1\,\left(l_1 - l_2\right)\,\rho\right) + a_2^2 l_1\,\left(B^2 l_1 + 2Bl_1\,\left(k\left(r^d-1\right) - r^d\right) + \right.$$
$$\left.(1-k)\,l_1 r^d\left(2k + (1-k)\,r^d\right) - 8a_1 kL\left(r^d+\rho\right)\right) - 2a_2\,\left(B^2 l_2 - r^d\,(4kl_1\,(A-(1-k)) - \right.$$
$$\left.2kl_2\,(2A-(1-k)) - r^d l_2 (1-k)^2\right) - 4Ak\rho(l_1 - l_2) - 8a_1 kL\left(r^d+\rho\right) -$$
$$\left.\left. B\left(2r^d l_2 - 2k\left(l_2 + 2l_1 (r^d + \rho - 1) - l_1 (r + 2\rho)\right)\right)\right)\right)\Big)^{1/2}\Bigg).$$

2. In the uniform case, some algebraic manipulations yield

$$\frac{\partial^2 V_i}{\partial k \partial \alpha_i} = \frac{a_1(r^d+\rho)L_i(\alpha_i,\alpha_j)((B-r^d)-(B-(A-a(\alpha_i,\alpha_j))(r^d+\rho))}{((B-(A-a(\alpha_i,\alpha_j))(r^d+\rho)-(B-(1-k)r^d)^2} +$$
$$\frac{(B-(A-a(\alpha_i,\alpha_j))(r^d+\rho)-r^d(B-(1-k)r^d)}{(B-(A-a(\alpha_i,\alpha_j))(r^d+\rho)-(B-(1-k)r^d)} \times$$
$$\frac{l_1((B-(A-a(\alpha_i,\alpha_j))(r^d+\rho)-(B-(1-k)r^d)) + a_1 L(\alpha_i,\alpha_j)(r^d+\rho)}{(B-(A-a(\alpha_i,\alpha_j))(r^d+\rho)-(B-(1-k)r^d)} +$$
$$\frac{r^d l_1((B-(A-a(\alpha_i,\alpha_j))(r^d+\rho)-(B-(1-k)r^d)) + 2a_1 L(\alpha_i,\alpha_j)(r^d+\rho)}{(B-(A-a(\alpha_i,\alpha_j))(r^d+\rho)-(B-(1-k)r^d)} \times$$
$$\frac{-B^2 + r^d(2a(\alpha_i,\alpha_j)-(1-k)^2 r^d) + 2a(\alpha_i,\alpha_j)k\rho - 2Ak(r^d+\rho) - 2B(r^d+2\rho)}{((B-(A-a(\alpha_i,\alpha_j))(r^d+\rho)-(B-(1-k)r^d))^2}.$$

Under our choice of parameter values, this cross-partial derivative is negative at $\alpha_1 = \alpha^*_{k_1} = 3.0$, given $\alpha_2 \leq \alpha^*_{k_2}$.

3. From $B > r^d$ and $l_1 > l_2$, it follows that

$$\frac{\partial \alpha^{**}}{\partial P_{max}} = \frac{4l_1(B-r^d)(r^d+\rho)(P_{max}r^d+\rho)}{a_1(1-P_{max})^2(2l_1-l_2)((1+r^d)P_{max}+2\rho)^2} > 0.$$

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