Some International Issues in Commodity Taxation

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Abstract

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This paper reviews issues and evidence concerning tax-motivated, cross-border commodity transactions. A distinction is drawn between “arbitrage trades” (driven by cross-country differences in tax rates) and “tax not paid” transactions (motivated by the opportunity to pay no tax at all on transactions with international aspects). Assessment of the severity of the associated policy problems faces the difficulty that the observed extent of cross-border transactions conveys no information on the induced inefficiency that the possibility of such transactions may generate. Given the difficulty of securing coordination of national tax policies, much of the emphasis in dealing with these problems in the coming years is likely to be on administrative cooperation.

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I. INTRODUCTION

Cross-border transactions motivated by the desire to reduce taxes and other charges have a long history, and an almost entirely dishonorable (if sometimes glamorous) one. The purpose of this paper is to consider the problems posed by one class of such transactions: those intended to avoid or evade domestic commodity taxes. 2 As regional integration proceeds, and countries seek to eliminate border controls between themselves, so the severity of the potential problems grows. They have become especially evident, and a significant policy concern, in many European Union (EU) member states. They are also a continuing issue in federations that allocate some indirect tax powers to lower-level governments. Indeed problems of this kind arise, and are likely to become more intense, in many parts of the world.

Reflecting the generality of the issue, the geographical scope of this paper is broad. But particular interest naturally attaches to experience in the EU, for it is there that the difficulties of alleviating restrictions on internal trade whilst retaining substantial decentralization of tax powers (to the member states) have become most pressing. Similar problems will clearly arise elsewhere if currently less-advanced projects of regional integration take root. The paper also pays special attention—despite the title—to experience at subnational level in Canada and the United States, which have considerable experience with the corresponding issues created by indirect taxes levied at provincial, state, and local levels.

In other respects, however, the focus of the paper is narrow. Attention is concentrated on the nature and effects of transactions intended to directly undermine the destination principle. As will be noted, but little elaborated upon, commodity taxes can have important international implications even in the absence of such transactions. Moreover, the emphasis here is on understanding the nature and extent of the problems, rather than on their resolution, which is taken up only briefly at the end.

The next section sets out some key issues, and Section III considers their quantitative significance. Section IV briefly considers the distinctive problems (and whether there are any) associated with the Internet. Section V concludes.

II. PROBLEMS OF DESTINATION TAXATION

Commodities are almost everywhere taxed, at least at national level, according to the destination principle: that is, tax on a commodity is supposed to be charged at the rate specified by, and the revenue to accrue to, the country in which the commodity is consumed.

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2 The term "commodity" is used here to embrace both goods and services, the distinction between which will at some points be important.
The last major instance of origin taxation\(^3\) at the national level—in some bilateral trades amongst some CIS countries—is currently unwinding in the wake of Russia’s movement to the destination principle for CIS trade (except for energy) in July 2001. We leave aside here the question of whether pursuit of the destination principle is indeed wise,\(^4\) and focus instead on problems that arise under the destination principle, both when it is fully enforced and—nowadays the more obvious concern—when it is not.

### A. Problems to the Extent that Destination Principle is Applied...

Under the destination principle, countries that are unable to deploy tariffs have an incentive to impose relatively high destination-based taxes on their importables (and relatively low destination taxes on their exportables) so as to bring about favorable movements in their terms of trade (Friedlander and Vandendorpe, 1968)\(^5\) or to shift rents from foreign firms, effects likely to be amplified by the activities of domestic lobby groups.

Such implicit protectionism certainly appears to be evident in the EU. Wine exporting countries set relatively low rates of consumption tax on wine (and succeeded in protecting these politically by ensuring low, and in some case zero, minimum excise rates); those with no substantial wine production, conversely, tend to set relatively high taxes. Countries with national producers of relatively low quality, cheap cigarettes rely more heavily on ad valorem rather than specific taxation of cigarettes, since this amplifies the price disadvantage of higher price imports. It is hard to rationalize such features of excise tax systems in terms of differing national preference structures: to the extent that residents of beer-producing countries also have consumption patterns oriented to beer, for example, that would suggest a low price elasticity of demand for beer and so point on efficiency grounds to a relatively high tax rate, not a relatively low one.

Though most evident in the EU—where such implicit protectionism was the main indirect tax policy issue until about a decade ago—similar problems can be expected to emerge increasingly clearly elsewhere in the years ahead. As countries’ ability to deploy explicit tariffs to protectionist ends is weakened by continuing trade liberalization and, in particular, the proliferation of regional trading blocs, so the temptation to use destination-based taxes for similar purpose will strengthen.

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\(^3\) Meaning taxation in the country of production. Note though that the impact of an origin-based tax on international transactions depends critically on the extent to which such taxes levied on intermediate goods are credited against further tax levied by the country of import.

\(^4\) For an argument that it is, see Keen and Smith (1996).

\(^5\) Under origin taxation, in contrast, the same terms of trade motive points instead to relatively heavy (light) taxation of exportables (importables). In the US, for example, coal-producing states have used severance taxes as de facto export taxes on sales to other states (Kolstad and Wolak, 1983).
B. ....and Problems to the Extent that It Is Not

The most immediate indirect tax problems in many parts of the world (or at least the most visible ones) are related to the difficulties of enforcing the destination principle. These are the primary concern in the rest of this paper.

The problems arise not least, but also not only, in the EU. Movements towards regional integration, which generally involve (at least as a final objective) an easing of physical controls between participating countries, may also weaken traditional methods for enforcing the border tax adjustments required under the destination principle. Technological developments may also make commodity movements easier, a particular concern in this area—discussed separately in Section IV—being the likely impact of the continuing expansion of internet-based transactions.

The implication of these enforcement problems is that some commodities may be consumed without payment of the tax due in the consuming country. There would be some evasion of consumption taxes even in a completely closed economy, of course. The concern here is that permeable borders may give rise to a distinct class of tax-reducing activities. These activities do not simply reduce tax revenues. They generate a wasteful diversion of resources to transactions motivated solely by tax considerations. Moreover, they may also give countries an incentive to cut their tax rates in order to preserve these tax revenues, with such tax competition resulting in a mutually harmful general reduction in tax rates.

To assess these risks to revenues and of intensified tax competition, it is important to understand the nature of the border-crossing, tax-motivated transactions at issue. These can take many forms, from the occasional cross-border purchase by final consumers at one extreme to substantial commercial smuggling at the other. They can be classified in a number of ways. For some purposes, it may be important to distinguish between legal activities—with some derogations, individuals in the EU, for example, are unrestricted in their ability to purchase tax-paid goods in other member states for their own use—and downright crooked ones. But in some respects the more important distinction for tax policy purposes is one that has rarely been made in the analytical literature, between:

- Cross-border transactions which involve some payment of indirect tax outside the country of final consumption; and

- Those that involve no payment of tax.

The first kind of transaction, essentially the transshipment of commodities purchased duty-paid abroad, is driven by weighing the gain arising from the cross-country difference in tax rates against the costs of transportation and the prospective loss—in respect of purchases made for resale, which will generally be illegal—from detection and punishment. These are therefore naturally be referred to as arbitrage transactions.

The second kind of transaction is driven simply by the existence of the domestic tax, the importance of the international dimension merely being that it provides additional devices for
evading that tax. Prominent amongst these are "diversion" frauds, under which goods are claimed to be destined for export to another country, or in transit—and so relieved of tax—but in fact diverted to the domestic market. Commodities may be exported, for example, and then re-imported with the connivance of corrupt customs officials. Indeed in some cases the goods may not even leave the country; there is no smuggling as such, the apparent export trail being on paper only. These are naturally referred to (echoing industry usage) as tax-not-paid (TNP) transactions. Such frauds are an issue, it should be noted, not only in relation to excises but also in relation to VAT. Under current arrangements for the enforcement of the destination principle, exports are zero-rated: that is, VAT is not charged on output and that paid on inputs is refunded. This creates an incentive to falsely claim that goods sold in the domestic market were exported.

The distinction between arbitrage and TNP transactions has been little recognized in the theoretical public finance literature; an oversight that, the next subsection argues, is of some importance.

C. Analyzing Commodity Tax Competition

The pioneering work on strategic commodity taxation by Mintz and Tulkens (1986) showed that this is an area in which reasonably general models give few general conclusions; even the existence of equilibrium is problematic. Since then, much use has been made of simple spatial models along the lines of Kanbur and Keen (1993). Some further use will be made below, so that it is useful to recall the essentials.

The Kanbur-Keen model is of a world comprising two countries of unit length, with a border between them. The population in each is uniformly distributed, but may differ between them: the larger (upper case) country has population H, the smaller has population h. Consumers buy one unit of the good or none, and may do so either at home or by traveling to the border and paying the foreign tax. In the latter case, they incur a transport cost $s$, where $s$ denotes their distance from the border—a cost that can in a wider interpretation be taken as a metaphor for informational or other heterogeneities across consumers—so that a consumer located in the large country (where the specific tax rate is $T$) will shop across the border (paying tax $t$) if:

$$T > t + \delta s.$$  \hfill (1)

Assuming that governments seek to maximize their tax revenue, Kanbur-Keen show that there exists a unique non-cooperative equilibrium with tax rates:

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6 In some cases the tax paid could even be negative, as a refund of tax may be claimed (on grounds of exportation) more than once for the same commodities.
where $\theta = h/H \cdot 1$ denotes the relative size of the small country. An immediate implication is that the small country sets the lower tax rate in equilibrium. Intuitively, the small country has less to lose from cutting its tax rate than does the large, since its domestic tax base is smaller, and also has more to gain because the foreign tax base into which it can tap is larger. Subsequent analyses have shown that this conclusion is fairly robust, continuing to apply if the difference in size is instead modeled as a difference in the lengths of the countries (Nielsen, 2001), if the large country acts as a Stackelberg leader (indeed the extent of undercutting is then even more marked: Hvidt and Nielsen, 2001, Wang 1999), if the distribution of the world population is assumed to be continuous, or if governments maximize not revenue but welfare (assuming, importantly, that preferences are identical; Trandel, 1994). What is important to the result, however, is the underlying assumption that governments do not differ in the relative strength of their preference for public goods (hence tax revenue) and private consumption, a point explored by Haufler (1996). We return later to the question of whether the prediction of lower tax rates in smaller countries is borne out in practice.

All the models cited, it should be emphasized, consider only arbitrage transactions: tax is always paid somewhere. But given the potential and probably growing importance of TNP transactions (see below) it is important to consider the outcome of tax competition games when cross-border transactions are TNP. While there has been some modeling of TNP transactions themselves, and the ways in which companies may seek to camouflage such business (Thursby and Thursby (2000)), the implications for strategic tax-setting appear to have been ignored.

One way to get a first, rough sense of these strategic implications is by recasting the Kanbur-Keen model as a situation in which cross-border shoppers pay not the foreign tax but no tax at all; that is, each country effectively faces a foreign country that sets its tax at zero. Condition (1) becomes simply $T > \delta$, so that all those living closer to the border than $T/\delta$ will buy TPN. Each country’s revenue is then the product of $T(1-(T/\delta))$ and its population size, maximizing which leads both to set a tax of:

$$t^N = T^N = \frac{\delta}{2}.$$  \hspace{1cm} (4)

7 An important exception is Cremer and Gavhari (2000), who show the possibility for strategic use of the degree of enforcement of domestic taxes.
In sharp contrast to the noncooperative situation under arbitrage trade, country size thus play no role in shaping the equilibrium: whereas the larger country set the higher tax rate under arbitrage trade, under TNP trade large and small set the same tax. The reason is straightforward: each now faces the same problem of simply protecting its own domestic tax base.

It is natural to go further than this and compare the levels of taxation and revenue under arbitrage and TNP trade implied by (2)–(4). This is more than a little artificial, since there is no reason to suppose the ease of arbitrage buying and of taking advantage of TNP—as parameterized by δ—to be the same; these are two quite different situations. Supposing nevertheless that, as a natural benchmark, δ is the same in the two cases, the implications are clear-cut. As one would expect, the larger country unambiguously sets a lower tax rate under TNP than under arbitrage trade: the competition it faces from the availability of a zero tax rate calls for a more aggressive response than is called for by the low positive rate set by the smaller country under arbitrage trade. The small country, on the other hand, if small enough (δ<0.25), sets a higher tax rate in equilibrium under TNP than under arbitrage trade: loosely speaking, instead of competing very aggressively to attract cross-border shopping it sets a modest rate so as to raise at least some revenue from its domestic base. The implications for revenue, however, are entirely clear-cut: this is lower in both countries, small and large, under TNP than under arbitrage trade,8 tending to confirm the natural suspicion that strategic responses are likely to make the effects of such trade especially pernicious.

III. HOW SIGNIFICANT ARE THE PROBLEMS?

Such empirical work as there is in the area has focused more on the consequences of evasion/avoidance activities than on the welfare costs of implicit protection, and it is on the former that we focus here too.

A. The Revenue at Stake

The natural first question for a country to ask itself is how much revenue it loses from cross-border activities. Problems naturally often show up first in border regions. For the late 1980s, for example, Fitzgerald et al (1988) report survey evidence that consumers living in counties of the Republic of Ireland bordering Northern Ireland bought about two-thirds of their petrol and almost all their alcohol north of the border, accounting for around 10 percent of their

\[ R^N = \delta H \left( \frac{2 + \theta}{3} \right)^2 \]

whilst under TNP it is in each case the population size multiplied by (δ/2)(1−(1/2))=δ/4.

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8 This claim follows on recalling that Kanbur-Keen showed equilibrium revenue under arbitrage transactions to be:
total expenditure. The issue becomes still more serious, of course, once such effects loom large in the broader national system, which they can: Fitzgerald et al (1988) also estimate a revenue loss for the Republic from cross-border shopping equal to about 5 percent of revenues from all commodity taxes.

Turning more generally to the empirical literature on the extent and nature of cross-border transactions, a fundamental difficulty is immediately evident: many of the transactions at issue, sometimes all, are illegal. Systematic data are thus hard to find, and the account here consequently impressionistic.

The most pressing cross-border issues in commodity taxation generally concern the excises, especially on tobacco products and alcohol, since in these cases tax levels (and differentials) are particularly high relative to transport costs (and, in many cases, the expected cost of any penalties). In the early 1990s, for instance, the potential gain from smuggling a semi-trailer of cigarettes into Canada was in the order of US$350,000 (Fleenor (1998)). Table 1 reports a smattering of estimates on revenue losses from cross-border transactions in excisable goods.

Table 1. Selected Revenue Losses from Cross-Border Transactions
(as percent of revenues collected, unless indicated)

<table>
<thead>
<tr>
<th></th>
<th>Tobacco</th>
<th>Beer</th>
<th>Wine</th>
<th>Spirits</th>
<th>Petrols</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>10-14</td>
<td>n.a.</td>
<td>n.a.</td>
<td>40-55</td>
<td>1-3</td>
</tr>
<tr>
<td>Denmark</td>
<td>13</td>
<td>22</td>
<td>14</td>
<td>26</td>
<td>4</td>
</tr>
<tr>
<td>Sweden</td>
<td>25</td>
<td>18</td>
<td>27</td>
<td>44</td>
<td>1</td>
</tr>
<tr>
<td>UK</td>
<td>7.5</td>
<td>4.3</td>
<td>7.5</td>
<td>3.3</td>
<td>n.a.</td>
</tr>
<tr>
<td>US</td>
<td>13</td>
<td>$35 mn gain;</td>
<td>n.a.</td>
<td>n.a.</td>
<td>n.a.</td>
</tr>
</tbody>
</table>


Notes: 1/ Figures are for 1994/5 (a period after substantial cigarette tax cuts had begun in Canada).
2/ For tobacco and alcohols: 2000, excise revenues only.
3/ For 1996/7. Cross-border movements only, legal and other.
4/ This is the estimated proportion of cigarettes smoked in the United States, bought either out-of-state or abroad.
5/ For comparison, average state and local revenues from all alcohol taxes in the late 1990s were in the order of US$160 million.

These figures are not directly comparable across countries, corresponding to somewhat different notions of evasion/avoidance and different methods of estimation. There is also of course a good deal of self-selection, in that the countries which take most trouble to measure the effects of cross-border activities (or at least to publicize them) are likely to be these with the greatest losses. Estimates of the revenue that some countries gain from arbitrage activities...
are difficult to find. But in Estonia, for example, sales to visitors have been estimated to account for about 50 percent of all legal sales (Taal (2000)). What is clear from the miscellany of figures in Table I, in any event, is that the sums involved are in some cases very significant: certainly large enough to be a real consideration in tax design and administration. The Canadian losses from tobacco transactions remain sizeable, it is worth noting, even after the massive reductions in tobacco taxes in early 1994.

A key question, for the reasons discussed above, is the breakdown of cross-border transactions between arbitrage and TNP trades. The latter are especially difficult to measure, so that hard information on this remains sparse. At a global level, one estimate is that one-third of internationally traded cigarettes are smuggled, and that “...most smuggled cigarettes have never paid duty in any jurisdiction...” (Action on Smoking and Health, 2000). Certainly TNP trade was an important part of the well-known Canadian experience with tobacco taxes in the early 1990s. One high-profile prosecution arising from this episode was that of a US-based subsidiary of RJR Reynolds, which admitted diverting to the Canadian market, via a Native American reservation spanning the border, purported exports to Estonia and Russia. The action was brought not in Canada but in the United States, the offense being the evasion of U.S. taxes.

Firmer estimates of TNP traffic are hard to come. In the UK, there are signs that they are now extensive. Customs & Excise give no official estimate but note that their actions prevented diversion of £572 million in 1996/97; this compares to about the loss of about £1.2 billion from cross-border smuggling. Since the former figure takes no account of TNP trade not detected, it could well be that such trade is at least as important in the UK as arbitrage trade. There appears, moreover, to be a sense that TNP trade in Europe is increasing as the weaknesses of current administrative arrangements become better appreciated by large-scale smugglers. The European Commission, for example, has recently launched legal action against R.J. Reynolds and Philip Morris, alleging deliberate over-supply to Eastern Europe in the knowledge that cigarettes would be smuggled back in to the EU.9 In the context of U.S. state taxes, recent work by Thursby and Thursby (2000) suggests that commercial smuggling (which will include some arbitrage trade) accounted for about one-third of the revenue loss from inter-state transactions.

While cross-country differences in rates of general sales taxation are in many cases quite high, it is usually thought that the associated revenue losses from non-excisable goods are moderate, perhaps because most such commodities are not readily transportable. Looking at one of the largest VAT differentials between contiguous member states of the EU, Gordon and Nielsen (1997) estimated the loss of VAT revenue to Denmark from cross-border shopping into Germany to be only about 0.8 percent. This though was before the advent of the single market; and in any event is a sizeable absolute amount. And there is at least one case in which the revenue impact of cross-border shopping induced by a general sales tax

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appears to have been significant: that of the GST in Canada. Examining the impact of its introduction in 1991, Boisvert and Thirsk (1994) find that cross-border shopping resulted in the burden of the tax being substantially shifted back onto producers, while Ferris (2000) estimates a static revenue loss from cross-border shopping into the United States of around 6 percent of GST collections.

Potentially the more serious and general cross-border VAT problem is the prospect of TNP transactions resulting from fraudulent zero-rating: goods, that is, which are claimed to be exported (either to another member state or outside the union), and hence receive refund of any VAT charged on inputs, but are subsequently sold without tax being paid. This has become a significant concern in the EU. Again, no careful estimate of the revenue at stake appears to be available, but the European Commission (2001) reports that detected “carousel” frauds of this kind involved a revenue loss of around €500mn. The same issue is likely to arise in other parts of the world as continuing regional integration leads countries to seek to zero-rate intra-regional exports without the aid of border controls, and as federations seek ways to implement decentralized VATs; the treatment of inter-state exports is a major concern, for example, in the design of the state level VATs due for introduction in India in 2003.

B. From Static Revenue Losses to Welfare

The revenue losses normally reported, as in Table 1, are calculated by applying the domestic tax rate to some estimate of the quantities on which tax has been avoided. It is important to recognize, however, that this is at best a highly imperfect indicator of the extent of the full revenue and welfare losses implied by opportunities for avoidance and evasion through cross-border transaction. This is for three main reasons.\(^{10}\)

One is that the revenue losses measured in this static way may not be recoverable, in the sense that some of the consumption that the cross-border activities generated would not occur if the destination tax were fully applied. In the UK, for example, in their preferred estimates of revenue loss Customs & Excise scale back the figures for alcoholic drinks in Table 1 by 20–30 percent; cross-border purchases of tobacco, in contrast, are assumed to fully displace domestic sales. Again, however, data limitations mean that there appear to be few direct estimates of these effects. In thinking through their likely magnitude, a useful and somewhat surprising implication of the model of the consumption decision in the presence of cross-border shopping developed by Christiansen (1994) is that the extent to which cross-border purchases displace taxed domestic purchases depends not, as one might expect, on the price elasticity of demand for the affected commodities, but on the income elasticity: the displacement is complete only if the income elasticity of demand is zero, and less the greater is that elasticity.\(^{11}\) The intuition for this is that the consumer’s optimization problem in the

\(^{10}\) There are others not mentioned below. For instance, tax not paid on the income earned through these illegal activities might also, by the same token, be regarded as a revenue loss.

\(^{11}\) This follows from equation (A.2) of Appendix I.
presence of such arbitrage opportunities is akin to one in which all goods must be purchased at the domestic price but lump-sum income is received equivalent to the gain from tax arbitrage. The stronger is the income effect for the taxed good, the more the income gain from cross-border shopping tends to expand demand and so partially offset the direct impact on taxed domestic sales.\textsuperscript{12}

The extent to which the apparent loss is recoverable will also depend on the incidence of the tax. It is commonly assumed in discussions (and models) of cross-border transactions that this is wholly on the consumer. But that may well not be the case: a relatively high domestic tax rate may lead to a reduction in the domestic producer price and/or a depreciation of the exchange rate as a consequence of consumers’ ability to escape the tax by shopping abroad. By the same token, a reduction in the domestic tax rate intended to stem the flow of cross-border shopping will then tend to increase the domestic producer price, and/or lead to an exchange rate appreciation, so mitigating the expansion of domestic taxed sales that would otherwise be expected.

Second, domestic consumers presumably derive some benefit from purchasing goods abroad at a lower consumer price than they would pay at home, and those who arrange such activities derive some increase in income. These welfare gains would be reduced if such activities were to be precluded, a loss that should be weighed against any revenue gain in arriving at any overall assessment in terms of national welfare. There may also be distributional issues associated with these effects, since the gains may be concentrated amongst particular groups, such as those living near borders (a case in point being the South East of England).

The third consideration is still more fundamental. This is that the observed extent of cross-border activities reflects tax policy decisions that are themselves affected by the possibility of such activities. To take an extreme example, a country suffering a revenue loss from cross-border shopping could eliminate arbitrage trades by setting its tax rate at the same level as that in the lower tax country. But while the measured revenue loss from these activities would then be zero, revenue and welfare would clearly be lower than they would have been had the country been able to sustain whatever level of domestic tax it would have chosen if it were able to implement the destination principle. More generally, the observed outcome is the product of a potentially quite complex game of tax setting and tax competition, the intensity and effects of which are poorly proxied by the observed extent of cross-border transactions. It is the possibility of such transactions, rather than the actuality, that is the core issue.

The full welfare and revenue effects of cross-border transactions could be measured, in principle, by comparing the observed outcome with that which would emerge if the

\textsuperscript{12}Since the income elasticity for cigarettes seems likely to be lower than that for alcoholic drink, this observation is broadly consistent with the differing assumptions that Customs & Excise make on offsetting of cross-border shopping on alcoholic drinks and tobacco.
destination principle were firmly enforced. The evident difficulty with this approach is that of characterizing the latter, hypothetical equilibrium.

There is, however, an alternative approach that casts some light on the likely magnitude of the inefficiencies created by the possibility of cross-border arbitrage transactions. As noted above, the strong theoretical prediction is that tax competition will lead to tax rates being set that are inefficiently low. Thus a natural reform to consider is a uniform increase in tax rates in both countries, setting $dT = dt > 0$. This would leave the incentive for arbitrage trades unchanged, but mitigate the general downward bias in tax levels. The effects of such a coordinated reform, starting from a non-cooperative equilibrium of the tax competition game, are analyzed briefly in Appendix I. Perhaps surprisingly, it emerges that the low tax country unambiguously benefits from such a reform, while the effect on welfare in the high tax country is unclear. To see why, note that, since each county chooses its own rate optimally in the noncooperative equilibrium, a small reform affects the welfare of each only through the induced change in the other country's tax rate. For the low tax country, the only relevant effect is thus an induced increase in cross-border shopping and hence in revenue: a clear gain. For the high tax country, on the other hand, although there is a revenue gain from the reduction in cross-border shopping induced by the increase in the low tax rate, consumers now also face a price increase, implying a reduction in welfare that could in principle dominate the revenue gain. In terms of collective welfare, it is shown in Appendix I that the welfare gain from such a coordinated tax increase (equal to ten percent of the higher of the tax rates), expressed as a proportion of tax revenue collected in the high tax country, is:

$$\alpha \left( \Gamma e \left( \frac{t+T}{T-t} \right) - 1 \right) 0.1,$$

(5)

where $\alpha$ is the static measure of the revenue loss in the high tax country (as a proportion of revenue collected), $\Gamma$ is the marginal cost of public funds (assumed for simplicity to be the same in both countries), $e$ is the elasticity of cross-border trade with respect to the tax differential, and it is assumed that cross-border purchases entirely displace domestic sales.

One implication of (5) bears some emphasis. For a given overall level of taxes $t + T$, and assuming a constant elasticity of cross-border sales $e$, the welfare loss from non-cooperative tax-setting is actually greater the smaller is the tax differential between the two countries, and hence the less cross-border shopping occurs in equilibrium. This amplifies the caution expressed earlier on the un informativeness of observed trades as an indicator of the severity of the policy problem. Clearly too the welfare gain from coordination is greater the more sensitive are cross-border transactions to tax differentials, which raises an important empirical question taken up in the next section.

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13 This is clearly extreme. More generally, a sufficient condition for the welfare gain in (5) to be decreasing in $T-t$ is that the third derivative of the "transport cost" function be nonpositive: see Appendix I.
To derive some sense of the possible orders of magnitude of the welfare issues at stake, take the example of cross-border traffic in tobacco between the UK and France. In this case, from Table 1, \( a \) is 0.075. The excise rates are around £1.05 in the UK \((T)\) and 0.2p in France \((t)\). Views differ quite widely on the value of the marginal cost of public funds \( \Gamma \), varying in the US debate (where the issue has been most studied) from say 1.2 to 2. This leaves the elasticity of arbitrage transactions with respect to the tax differential, \( e \). No empirical estimates appear to be available, so that it is again natural to assume a range of values: from a low of, say, 1 (corresponding to the case, popular in theoretical work, of transport costs quadratic in distance) to a high of 2.\(^{14}\) Table 2 calculates the welfare change in equation (5) at a variety of these combinations of \( \Gamma \) and \( e \).

These figures are no more than illustrative, but taken in that spirit suggest that the gains, though not dramatic, can be noticeable when tax arbitrage is price sensitive (reflecting a strong downward pressure on tax rates). A welfare gain of 1 percent of the value of UK tobacco tax receipts, even if shared between the UK and France, would be worth having. The figures also emphasize the importance of getting some empirical sense of the sensitivity of arbitrage trade, \( e \).

### Table 2. Collective Welfare Gains from Coordination (as percent of tax revenue in high tax country)

<table>
<thead>
<tr>
<th>MCPF (( \Gamma ))</th>
<th>Elasticity of cross-border sales (( e ))</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
</tr>
</thead>
<tbody>
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C. How Intense is International Competition in Commodity Taxation?

A key concern in this area of international taxation, as in others, is the possibility that the prospect of tax-induced cross-border transactions will lead governments to set tax rates at levels lower than—or, more generally, different, from\(^{15}\)—those they would otherwise choose. What evidence is there that this has happened?

\(^{14}\) Assuming no income effects, cross-border sales at 5 percent of home sales and a tax differential equal to 2 percent of the lower tax rate, the estimated elasticity of home sales with respect to the “foreign” tax of 6 reported by Walsh and Jones (1988) would imply an elasticity \( e \) of 2.4.

\(^{15}\) As noted in Section II (A), there are important cases in which countries may choose to set destination-based taxes higher than they otherwise would, essentially in order to tax foreigners by some exercise of some market power. We leave these cases aside in this discussion.
Plenty, if one takes governments at their word. Over the last few years, UK Chancellors of the Exchequer have frequently referred to low taxes abroad as a significant constraint on their ability to raise excises on tobacco, wine, and, especially, spirits. In Sweden, the government recently chose to respond to an EU requirement to lower the tax on wine relative to that on beer by cutting the wine tax rather than by raising that on beer, rationalizing this in terms of the extensive cross-border purchasing of beer. More generally still, the adoption of minimum VAT and excise rates in the EU was intended to stem a perceived risk of downward spiraling of rates. And there have clearly been cases in which cross-border transactions have driven policy. The Danish excise cuts of the later 1980s, for instance, appear to have been a response to large-scale cross-border shopping into Germany. The most spectacular instance, however, is the 1994 cut in Canadian federal and provincial cigarette taxes in the face of substantial smuggling from the United States: cigarette taxes in Quebec, for instance, were more than halved.

It is important, nevertheless, to look for more systematic evidence—manifested in deeds rather than words—of tax competition concerns showing themselves in indirect tax policy. It could be the case, for instance, that politicians use the rhetoric of tax competition to sell policies that they wish to pursue for other reasons, perhaps to placate powerful interest groups. (In Norway, for example, cross-border shopping into Sweden was cited as a reason for a reduction in the rate of VAT on food—to the same 12 percent rate as in Sweden—that some groups had long been urging on distributional grounds). Moreover, tax competition may have effects on tax-setting that are pervasive and powerful but not of so dramatic a nature as to give rise to high-profile public statements.

There are miscellaneous pieces of evidence that are at least consistent with the kind of interdependencies in tax policy at issue. The variance of cigarette tax levels in the EU, for example, has halved over the past 20 years (although the average level has risen). A more structured approach to the issue is to estimate reaction functions in tax-setting, seeking thereby to identify any systematic dependence of each jurisdiction’s tax policy on the policies of others. There is now a growing empirical literature on this (concisely surveyed by Brueckner (2001)), though almost all focusing on tax relations within federal systems (where there arise issues of both horizontal tax competition between lower-level jurisdictions and vertical competition between federal and lower-levels) and addressed to corporate and property taxes rather than indirect taxes. In an international context, empirical work on tax reaction functions is only just beginning. Besley, Griffiths, and Klemm (2001) report preliminary results for a range of taxes within the OECD, modeling each country’s effective tax rate on a variety of bases as a function of the corresponding average effective tax rate in all other OECD economies. The preliminary results suggest that for both excises and, perhaps more surprisingly, VAT/sales taxes, there is a strong positive relationship.

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16 I owe this example to a referee.
Such findings can be interpreted in a number of ways other than competition for mobile tax bases. They might reflect yardstick competition, with citizens in each jurisdiction in part disciplining their own policy-makers on the basis of inferences drawn from tax-setting in other jurisdictions (evidence for this from the United States being given by Besley and Case, (1995)). Or they might reflect copycatting behavior that is no more than a fad. Understanding the underlying reason for the dependency is important for gauging the proper policy response (if any): the illustrative calculations of the welfare gains from coordination in the previous section, for instance, presume the initial equilibrium to be one driven by tax competition, and so would likely be inappropriate if the observed outcome reflected instead the outcome of some political game. It is in order to identify a distinct tax competition effect that Besley et al. (2001) estimate reaction functions for a variety of taxes, the hypothesis being that—as they indeed find to be the case—tax dependency should be more evident on more mobile bases.

Another approach is to ask whether the observed outcomes are consistent with other predictions of the tax competition models summarized in Section II (C) above. In this context the result that small countries should set lower tax rates in equilibrium comes quickly to mind. There seems no obvious reason, for example, why yardstick competition should lead to such an outcome. So: Is that prediction borne out in practice?

There are resonances of this result to be found. In the context of business taxation, the archetypal tax haven, for instance, is typically a country with small domestic tax base: almost all of the countries in the OECD’s list of tax havens are evidently small. Trandel (1994) also notes that there are instances of US state excises that match the prediction of lower rates in sparsely populated states close to heavily populated regions (as for example New Hampshire and Boston, Massachusetts). Luxembourg has long had the lowest cigarette taxes in the EU. But it is equally clear that there are important counter-examples: the Nordic countries, in particular, are clearly in some sense “small”, yet are also notable for high taxes—both indirect and more generally—rather than low. The natural way to explain this is by supposing there to be a higher preference for public goods in the Nordic countries. But in fact the problem may be even deeper than the existence of such counter-examples, which should otherwise average out if preferences for public expenditure are uncorrelated with size. For it is an empirical regularity that small countries do systematically have larger governments, perhaps because of economies of scale in the provision of public goods (Alesina and Wacziarg, (1998)). Whatever the reason, there appear to be systematic effects at work to counter-act the conclusion from simple tax competition models that small countries will set lower tax rates.

Moreover, size is not all that matters. Inserting a third country into the linear world of Kanbur and Keen (1993), Ohsawa (1999) shows that taxes will also tend to be systematically

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17 In principle, of course, one could apply an appropriate welfare test for any possible reform from any initial starting point (without using, for instance, the simplifying assumptions behind equation (5)). For practical purposes, however, this may require information on behavior far from the observed outcome.
lower in the intermediate country, because cutting its tax rate generates sales across two borders rather than one. This little-pursued result provides another rationale for the low excise rates of centrally-located Luxembourg, for instance, whilst also being consistent with the traditionally higher tax rates in the Nordic countries.

Another way of gauging the likely intensity of commodity tax competition is to assess how mobile cross-border transactions are to tax differentials: the more sensitive they are, the greater the potential revenue gain from setting a relatively low tax rate. Christiansen (1994) shows, for example, that an increase in the elasticity of taxable sales at home would lead a country subject to outward cross-border shopping to lower its tax rate. In this spirit, Crawford, Smith, and Tanner (1999) seek to identify any effect of the loosening of fiscal controls in the EU at the start of 1993 in increasing the elasticity of demand for excisables in the UK: they find none.

There has been relatively little empirical work, however, focused directly on the tax-sensitivity of cross-border sales—a surprising fact that presumably reflects the dearth of reliable data on the extent of such transactions. Such work as there is suggests quite strong effects, at least within those groups, perhaps quite small, that live close to the relevant borders. Fox (1986) finds that a one percentage point increase in the Tennessee state tax rate shifts 1 percent to 4 percent of county-level sales between border counties in- and out-of-state. For counties in West Virginia, Walsh and Jones (1988) find that a one percent increase in the state sales tax rate would reduce grocery sales in border counties by a little under 6 percent. Ferris (2000) finds a significant impact of consumer price differentials, and hence of tax differentials, on cross-border trips from Canada into the United States. In a somewhat different context, Goolsbee (2000) finds that, conditional on having internet access, the elasticity of online spending with respect to the tax rate of the state in which the purchaser resides is nearly 4. These are considerable effects, but their overall impact is limited by the relatively small size of the groups concerned (though in the case of the internet, of course, the size of that group is increasing rapidly).

There clearly remains much to learn about inter-jurisdictional patterns of commodity taxation. In some respects the most striking observation is the persistence of quite wide differentials. In the United States, for example, cigarette excises continue to vary from US$10 per carton in some states to US$0.25 in others (the latter tending to be tobacco producers). This is perfectly consistent of, course, with a strong effect of tax competition on the overall level of taxes: without this, perhaps the pressures to tax cigarettes indirectly through health litigation in the courts would have been less. Nevertheless, such persistent differentials suggest that the potential difficulties posed by cross-border transactions have been to some degree mitigated. Transport costs can be only part of the explanation for this, given the potential for diversion fraud and the huge profits to be earned on illicit trade. A large part of the explanation must be the success of administrative interventions. The effectiveness of these becomes especially problematic, of course, in contexts of regional integration, where control must be shifted away from physical border checks and towards more challenging audit-based methods. In this as in other areas of international taxation, a key question for the future, only now starting to be addressed in the EU, is the extent to
which enhanced international administrative cooperation can substitute for tax policy measures addressed to a likely continued increase in the ease of cross-border transactions.

IV. WHO'S AFRAID OF THE INTERNET?

No discussion of international commodity tax issues is nowadays complete without some reference to the impact of the Internet, concern being widely been expressed that this will significantly jeopardize the revenues currently raised from indirect taxation. These worries have been expressed most loudly\(^{18}\) in the United States.

In most contexts, however, these risks do not seem to be as profound as sometimes feared. It should be emphasized, in particular, that the problems faced in the United States, which have been the focus of much of the discussion in this area, arise in large part from idiosyncratic legal difficulties (described below) that simply do not and need not arise elsewhere. And even in the United States the scale of the problem should not be over-stated: one careful study, based on industry forecasts of internet usage, projects an incremental static loss of state and local sales tax revenue in 2003 from this source of a little under US$11 billion (Bruce and Fox (2000)), which compares to total revenue from these sources in 1998 of around US$190 billion.

One reason for this relatively low figure—and a reassuring feature more generally, especially in countries that tax intermediate transactions less heavily than does the U.S. states—is that the vast majority of internet transactions are business-to-business (B2B), so that one would generally not wish to tax them anyway.\(^{19}\) Under a VAT, in particular, international sales to businesses would be zero-rated. It is business-to-consumer internet transactions that are the main issue, and these remain relatively limited.

The most fundamental point, however, is that the immediate impact of the growth of transactions through the internet is quantitative rather than qualitative. In so far as the international or inter-state movement of goods to which transactions give rise involve passing through border controls—a book ordered by a customer in the UK from a firm in the United States, say—the proper destination-based tax can be levied when the good crosses the border, exactly as at present. The consequent challenge for the tax authorities is thus likely to be the administrative (and nontrivial) one of dealing with significantly greater flows of small consignments.

This cannot be done, of course, when—as between the states of the U.S., and between the members of the EU—there are no border controls on the movement of goods. Since it is

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\(^{18}\) And most memorably too, in Newman's (1995) evocative prediction that the state and local sales taxes will become "roadkill on the information superhighway."

\(^{19}\) There are exceptions to this, of course. In particular, environmental taxes should generally be levied on B2B transactions as on any other.
generally impractical to levy taxes on final consumption on the purchasers themselves—as U.S. experience with widely-evaded use taxes has long shown—the practical problem then is to ensure that the vendor charges the proper tax of the state of destination. It is here that the U.S. peculiarities arise, the problem being that a decision of the U.S. Supreme Court\(^\text{20}\) prohibits states from requiring out-of-state vendors to withhold tax on sales into their jurisdiction (this being held to be violate the interstate commerce clause of the constitution). A firm can only be required to charge a state’s sales tax when it has, in effect, a substantial physical presence there. No such difficulty arises in the EU, where firms making distance sales in excess of specified thresholds are simply required to withhold the tax of the destination state. Nor need it arise in any other region that decides to remove internal border controls. The central enforcement issue then becomes the identification of such remote sellers; and this is not intrinsically more difficult than for any other trader.

It is in relation to services and digitalized product that the greatest inherent problems arise. For these intangibles, physical border controls in any event have little role in tax collection: customs officers are little help in identifying purchases of the services of a foreign lawyer, or the downloading of a data base. While there are new challenges in relation to digitalized products, not least in physically identifying vendors, these products are not, as yet, of the first order of importance. In Sweden, for instance, it has been estimated that only around 3 percent of the VAT base falls into this category. The more immediate difficulties arise in relation to straightforward services.

Here too the effect of the internet is to intensify existing problems rather than create new ones. But the pre-existing problems in the international taxation of services were in some cases deeper, and their intensification consequently more challenging. Since services are generally intangible, the taxation of services requires careful definition of the place of supply. Under the EU's Sixth VAT Directive, for instance, services other than those on a positive list are taken to be supplied in the country of residence of the supplier; those listed are taxed by reverse charging (that is, the purchaser is required to pay the tax). The default position is thus that services acquired outside the EU are not subject to VAT. As services become more important, so this becomes more problematic as a potential source of TNP trade. The problem arises forcefully in relation to telecommunications services, a homogenous product that is easily sourced across national boundaries. In the EU, for instance, the problem arose that these could be purchased VAT-free from nonresident companies, which was extremely attractive to final consumers and exempt firms. Simplifying somewhat, the response has been to require any firm selling telecom services into an EU member state to establish a tax presence there and charge the proper tax. More generally, since it is sales of services to consumers that is in the prime concern, and since reverse charging on such transactions is essentially impracticable, the conceptually correct response is clear: to formulate rules of supply for B2C transactions in terms of the location of the final

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\(^{20}\) McLure (1999) emphasizes, however, that a constitutional amendment is not needed to overturn the effect of *Quill:* the Supreme Court made it plain that Congress could specifically authorize such requirements.
consumer. This, indeed, is the approach that the OECD (2001) has advocated for B2C sales, and which has recently been adopted by the EU, from July 2003, non-EU suppliers of digital products and radio and TV services into the EU will be required to register in some member state and to charge tax on sales to final consumers at the rate of the country in which the purchaser is located. The challenge this leaves is that of identifying traders who have no substantive presence in the destination jurisdiction. As was argued to be the case for goods, so too in relation to services this is likely to require substantially more cooperation between national tax authorities than has been the case. And in so far as the transactions at issue would otherwise escape tax altogether, the pattern of incentives gives reason to hope that there will ultimately emerge cooperation commensurate with the emerging problems.

V. CONCLUDING REMARKS

There is no doubt that cross-border transactions can on occasion reach proportions significant enough to have a marked impact on indirect tax policy. But no deep academic research is needed to know that. What is more troubling is how little is known about the importance (or otherwise) of these issues outside evidently dramatic and unmistakable episodes. For analysis in this area must deal not only with the lack of data on what are commonly illegal transactions but also, and perhaps more fundamentally, with the awkward fact that one cannot infer even from an absence of such transactions that there is no policy problem: such an outcome would arise both when commodities are completely immobile, and the issue non-existent, and, at the opposite extreme, when they are so perfectly mobile that countries spontaneously adjust their tax rates to eliminate cross-border transactions. The empirical work on international interactions in tax-setting needed to cast light on these issues is only just beginning.

Once instances of significant trading in cross-border transaction and/commodity tax competition have been identified, the question arises as to the appropriate policy response arises. There are cases in which the consequences may actually be beneficial, either because of imperfections in the political process or because of a beneficial undermining of monopoly power (as Trandel (1992) shows to be a possible consequence of evasion of the U.S. use tax). Key issues in the literature have included the case for commodity tax harmonization and the effects of imposing a minimum tax constraint, as in the EU. The former may have some general appeal in ensuring a more efficient allocation of consumption (Keen, 1987). But perhaps the more persuasive rationale, in some contexts, is as a brute force means of undermining the use of domestic taxes for implicit protection (Keen (1989; Lockwood

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21 For B2B transactions, the OECD advocates reverse charging.


23 The revenue is then to be passed on to the country of consumption by the country of location. B2B sales by non-EU suppliers of these services are to be taxed by reverse charge, again as under the OECD guidelines.
(1997); Lahiri and Raimondos-Møller (1997)), especially given the difficulty of using such legal provisions as the prohibition of implicit protection through domestic taxes under the Treaty of Rome (Article 95) to deal with any but the most egregious cases. An alternative strategy, pursued in the EU in relation to VAT and the excises, is to impose a minimum tax rate. In the Kanbur-Keen model, this has the striking and attractive feature that both countries gain, including the small country forced to raise its tax rate: the large country gains because it becomes less constrained by outward tax-shopping, while, more surprisingly, the small also gains because of the induced increase in the large country’s tax rate (which tends to increase the extent of cross-border shopping). But this attractive property—implying that all can gain from coordination without any need for compensating transfers—is not robust: it fails if the large country is a Stackelberg leader (Hvidt and Nielsen (2001), Wang (1999)) and is not generally valid when governments maximize welfare (Haufler (1996)). Another recent strand of the literature has sought to address the VAT fraud problems noted above by finding ways to implement the destination principle without zero-rating exports, either by adopting a “CVAT” that imposes a distinct creditable tax on inter-state trade (McLure (2000)), or a “PVAT” that requires proof of tax payment in the destination country before allowing zero-rating by the seller (Poddar and Hutton (2001)) or a “VIVAT” under which all intermediate transactions within a region are taxed at a common rate, whether or not they cross borders (Keen and Smith (1996 and 2000)).

But substantive measures of policy coordination measures of this kind have proved politically difficult, even in the EU, doubtless reflecting both sensitivities on the principle of national tax sovereignty issue and baser conflicts of interest. The emphasis in dealing with cross-border commodity transactions, as with other problems of international taxation, appears to be shifting to measures of administrative cooperation, where it may be easier to establish a mutuality of interest. This would certainly seem to be key in addressing the problems of tax-not-paid traffic which, it has been argued here, have received less attention in the public finance literature than they deserve. A key issue for the future is the extent to which measures of administrative cooperation between countries can substitute for deeper policy reform.
APPENDIX I. WELFARE EFFECTS OF COMMODITY TAX COORDINATION

Imagine a world comprising two countries, home (lower case) and foreign (upper case), each inhabited by single representative consumer of the type in Christiansen (1994). The home consumer has preferences $U(X,X_o) + rG$, where $X$ denotes the commodity on which interest focuses, $X_o$ is consumption of an untaxed numeraire, $r > 0$ and $G$ is the quantity of some publicly provided good. The taxed good may be purchased either at home (paying specific tax $T$) or abroad (at tax rate $t$), it being assumed throughout that in equilibrium $T > t$. Thus $X = \chi + A$, where $\chi$ denotes purchases at home and $A$ purchases abroad. Purchasing abroad incurs transport costs of $C(A)$, where $C$ is increasing and strictly convex. Maximizing utility subject to the budget constraint $Q(X-A) + qA + X_o = M + C(A)$, where $Q$ (resp. $q$) denotes the consumer price at home (abroad)—producer prices are assumed to be the same in both—and $M$ is lump sum income, cross-border purchases are $A(T-t)$, where $A = C'(A)$ (the prime indicating a derivative). Rewriting the budget constraint, it is convenient to represent preferences by the indirect utility function:

$$V(Q,M+S(T-t),G) = \max_{X,X_o} \{U(X,X_o) + rG \mid QX + X_o = M + S(T-t)\}$$

where $S(y) = yA(y) - C(A(y))$. Note, by Roy's identity, that home purchases are then of the form:

$$\chi(Q,T-t,M) = X(Q,M+S(T-t)) - A(T-t)$$

The description for the foreign country is analogous. Revenue in each country depends on the tax rates in both. In the home (high tax) country, it is $R(T,t) = TH(Q,T-t)$; abroad, it is $r(t,T) = tx(q,m) + tA(T-t)$. The policy problem of the home country is then to choose $T$ to maximize $W(T,t)D V(Q,M+S(T-t),R(T,t))$; and analogously abroad.

Consider then the welfare effects of a small coordinated increase in both tax rates, $dT = dt > 0$, starting from a noncooperative equilibrium. Since $W_T = w_t = 0$ at any such initial position, all that matters to each country is the effect of the change in the other country's tax rate. For the foreign (low tax) country, $dw = tA'dT$, which is strictly positive. For the home country, noting that $S' = A$, the welfare effect is $dW = (-A + \mu T(\partial Y/\partial(T-t)))dt$, the sign of which is uncertain. Summing the two, noting from (A.2) that $\partial Y/\partial(T-t) = -X_A + A'$ (where $Y$ denotes $M + S(T-t)$), setting $X_A = 0$ and $\gamma = \Gamma$, one finds:

$$dW + dw = (\mu(T+t)A' - A)dT$$

Rearranging this gives (5) of the text, where $\alpha \equiv A/\chi$ and $e \equiv \partial \ln(A)/\partial \ln(T-t)$. Verifying a claim made in the text, note that it is sufficient for $dW + dw$ in (A.3) to be decreasing in $T-t$ that $A''$ (which has the sign of $C''$) be nonpositive.
REFERENCES


