

# Working Paper

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### Search Unemployment with Advance Notice

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#### Abstract

This paper proposes and solves a search model in which job separation requires mandatory notice. When jobs are subject to idiosyncratic uncertainty, firms would issue advance notice even with good business conditions. We show that such precautionary policy is not pursued if it entails sufficiently high productivity losses. If workers can search on the job, an increase in advance notice increases job to job movements, reduces unemployment flows, and has ambiguous effects on unemployment. Results are consistent with the fact that North American and European labor markets, despite their differences in job security provisions, experience similar turnover rates and dissimilar unemployment flows.

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## SUMMARY

This paper shows that advance notice is an important determinant of the relationship between unemployment duration, job turnover, and on the job search. Empirically, unemployment inflows and outflows are much larger in North America than in Europe, and marked differences in unemployment flows appear consistent with the view that high job security provisions in Europe reduce unemployment turnover and increase the average duration of unemployment. However, existing evidence on gross job flows shows that job turnover (the sum of job creation and destruction) in highly regulated European markets is as high as turnover in North American markets. This paper argues that the existence of advance notice in European labor markets, along with other institutions that transform the job separation into a time-consuming process, can potentially be consistent with both sets of observations.

The paper introduces advance notice in a search-unemployment environment, and assumes that employer-initiated job separation can take place only when workers are given an institutionally determined advance notice. When jobs are subject to idiosyncratic uncertainty, firms would issue advance notice even when business conditions are favorable. The paper shows that such precautionary policy is not pursued only if it entails sufficiently high productivity losses. The length of the advance notice affects inflows and outflows into unemployment, and has ambiguous effects on equilibrium unemployment. In addition, advance notice affects labor market flows in two opposite ways. On the one hand, advance notice reduces firms' incentive to create jobs, reduces inflows into unemployment, and increases the average duration of unemployment. On the other hand, advance notice increases the stock of employed job seekers, and increases the number of job to job movements. As a result, turnover in highly regulated markets can be as high as turnover in unregulated markets, while the corresponding unemployment flows are much smaller.

## I. INTRODUCTION

The remarkable differences in net employment growth between continental Europe and the United States is at the center of the recent debate on labor market flexibility. In the policy debate, labor markets in continental Europe are often described as sclerotic and rigid markets, and high job security provisions in Europe are often held responsible for low rates of job creation and high and rates of unemployment. In academic journals, conversely, the effects of firing costs on equilibrium unemployment is overall ambiguous.<sup>1</sup> Nevertheless, the extensive costs-of-adjustment literature predicts that firing costs should unambiguously reduce turnover (Nickell, 1986). Empirically, inflow and outflow rates into unemployment appear much larger in North American than in Europe (OECD 1990,1995), and the correlation between low firing costs and very large unemployment flows in the United States and Canada appear consistent with the predictions embedded in most costs-of-adjustment models. However, recently published OECD (1994a) statistics show that, on average, job turnover in European economies is as high as job turnover in North America.

Bertola and Rogerson (1997) argue that relative wage compression is conducive to higher employer-initiated job turnover. Thus, wage-setting institutions and job-security provisions differ across the atlantic in ways that are consistent with rough uniformity of job turnover statistics. Even though the paper by Bertola and Rogerson (1997) can potentially rationalize the uniformity of turnover rates between Europe and the United States, the model they propose can not fully account for the remarkable differences in unemployment flows. Intuitively, uniformity of turnover rates and marked differences in unemployment flows imply large job to job movements in European markets. Not surprisingly, a close look at the existing evidence suggests that European labor markets experience larger job to job movements than North American markets.

In most of the existing literature, job security provisions are modeled as a simple fixed cost to be incurred when separation takes place (Bentolila and Bertola, 1990; Bertola and Rogerson, 1997). In reality, however, job security provisions are much more complex than a simple fixed firing cost, and they often transform job separation into a costly and time consuming process. This paper argues that studying the time consuming property of job separation can cast new light on the relationship between unemployment duration, job flows and on-the-job search. We show that the existence of *time to fire* in the job-worker separation process is consistent with the fact that North American and European countries experience similar turnover rates, and remarkable different unemployment flows. The argument goes as follows. Jobs are subject to idiosyncratic shocks and workers employed in low productivity jobs search for high-productivity (high-wage) jobs. In unregulated markets adverse shocks are conducive to employer

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<sup>1</sup>Bentolila and Bertola, 1990 and Nickell, 1997 find that firing costs have ambiguous effect on unemployment, whereas Lazear (1990) and Hopennaym and Rogerson (1993) find a negative employment effect.

initiated job separation, and laid off workers often experience short unemployment spells. Conversely, in highly regulated markets, firing in low productivity jobs can not take place immediately, but only after the worker is given an institutionally determined advance notice. The existence of advance notice creates an incentive to search on the job and, depending on the notice's length, workers may match to high productivity jobs without experiencing unemployment spells. The relationship between turnover and advance notice is affected by two opposing forces. On the one hand, an increase in the notice period reduces firms incentive to create job. This tends to decrease turnover. On the other hand, an increase in the notice period increases the number of job-searching employees, causing an increase in job to job movements. This tends to increase turnover. As a result, the turnover in regulated markets can be as high as the turnover in unregulated market, while the corresponding unemployment flows much smaller.

We introduce mandatory notice in a search-unemployment environment, and we assume that employer-initiated job separation can take place only when workers are given an institutionally determined advance notice. If issuing notice were completely costless, firms would continuously give advance notice to workers, just to ask them to ignore such notices if business conditions remain favorable. In reality, however, when firing notices are given, job-worker pairs are likely to suffer productivity losses. The worker is likely to reduce its commitment on the job, and to invest time and resources in looking for other jobs. We show that only if these productivity costs are high enough, firms will not give notice warnings to reduce the expected firing costs. In addition, we show that the length of the advance notice affects inflows and outflows into unemployment, and has ambiguous effects on equilibrium unemployment. Finally, advance notice increases jobs to jobs movements, reduces inflows into unemployment and increases the average duration of unemployment.

More specifically, we propose and solve a search unemployment model in the spirit of Pissarides (1990,1994), Mortensen and Pissarides (1994) and Garibaldi (1998a). Similarly to Mortensen and Pissarides (1994), we extend the traditional matching model by assuming heterogeneity in the value of the labor product, and similarly to Pissarides (1994) and Mortensen (1994), we let workers search on-the-job. Finally, in the spirit of Mortensen and Millard (1997) and Garibaldi (1998a), we model job security provisions in a search-unemployment environment.<sup>2</sup>

The paper proceeds as follows. Section II briefly reviews the existing evidence on

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<sup>2</sup>Millard and Mortensen (1997) study the effect of traditional fixed firing costs rules on aggregate flows, whereas Garibaldi (1998a) analyses the effect of firing restrictions on the dynamics of job creation and destruction. In the U.S. data compiled by Davis and Haltiwanger (1990,1992) job destruction appears much more volatile than job creation, and job reallocation, the sum of job creation and destruction, moves counter-cyclically. In continental Europe, conversely, job creation and destruction moves in a opposite but symmetric way. Garibaldi (1998a) shows that firing permissions reduce the volatility of job destruction, and cause job reallocation to be uncorrelated with net employment changes.

job flows and unemployment flows across countries. Section III discusses advance notice in European countries, and other job security provisions which transform firing into a time consuming process. Section IV introduces concepts and notation, while section V presents and solves the model, and considers the firms' incentives to issue notice when business conditions are still good, and derives a set of conditions that induce firms to issue notice only when conditions are unfavorable. Section VI presents a simulation of the model, and shows that the results of the paper are broadly consistent with the cross country evidence presented in Section II. Section VIII summarizes and concludes the paper.

## II. A BRIEF LOOK AT THE EVIDENCE

Unemployment inflows and outflows are usually defined as movements between unemployment and other labor force status (employment and out of the labor force), in a given country and in a given period of time. In general, direct estimates of unemployment flows are very difficult, if not impossible to come by. However, from labor force surveys, it is possible to obtain proxies of these flows, and the OECD regularly publishes estimates of unemployment inflows and outflows. Table 1 reports flows data for a selected number of European and North American labor markets. In survey data, unemployment inflows are proxied by the number of people who have been unemployed for less than a month, and they are usually expressed as a percentage of the people in the labor force that are not unemployed. Outflows rates are obtained as the residual between the estimated inflow rates and changes in unemployment, and they are expressed as percentage of the average unemployment stock. From Table 1 is clear that the estimated flows are substantially larger in the North American labor markets. Inflows and outflows in Canada and the U.S. are five times larger than the corresponding flows in Germany, France, Italy, Spain and the United Kingdom. Table 1 reports also aggregate statistics on unemployment and the incidence of long-term-unemployment, the latter being defined as the percentage of people with unemployment duration longer than one year. Clearly, Canada and the U.S. have a much lower proportion of long-term-unemployment than European countries. The difference between the incidence of long-term-unemployment in North American and European labor market is one of the most striking and widely documented difference between the two labor markets.

From the statistics on Table 1, we should expect that any measure of turnover in the labor market would provide a picture similar to the one reported in Table 1. Empirically, it is possible to distinguish between job turnover and worker turnover, and in the rest of the section we look at turnover data for (roughly) the same countries for which we have data on unemployment flows.

Empirically, job creation (destruction) is defined as the sum of all positive (negative) employment changes at the level of the establishment, in a given period



Table 1. Unemployment Flows Across Selected Industrial Countries

	<i>Inflows</i> <sup>1</sup>		<i>Outflows</i> <sup>2</sup>		<i>U</i> <sup>3</sup>		<i>LTU</i> <sup>4</sup>	
	1985	1993	1985	1993	1985	1993	1985	1993
Canada	2.47	2.4	29.4	25.6	7.5	11.2	10.3	11.4
France	0.32	0.34	3.7	13.9	9.4	11.6	46.8	34.2
Germany <sup>5</sup>	0.25	0.57	6.1	9	5.6	6.2	47.9	40.3
Italy	0.14	0.41	1.8	9.5	10.9	10.2	65.8	57.7
Spain	0.35	0.56	1.7	1.8	16.9	22.4	56.7	50.1
United Kingdom	0.51	0.67	6.1	9.3	7.2	10.3	48.6	42.5
United States	2.45	2.06	41.4	37.7	5.2	6.7	9.5	11.7
<sup>1</sup> Inflows into unemployed, proxied by unemployed people for less than a month <sup>2</sup> Outflows from unemployed, residual between inflows and change in unemployment stock <sup>3</sup> OECD Standardized Unemployment Rate <sup>4</sup> Incidence of Long Term Unemployment (more than a year) <sup>5</sup> Data refers only to West Germany. Source: OECD Employment Outlook (1990,1995)								

(usually the year), and in a given industry. If the industry is representative of the entire economy, we have a measure of aggregate job creation. Usually, the absolute number of job created (destroyed) is normalized by the employment size, and the resulting ratios are the rates of job creation (destruction). The sum of job creation and job destruction is the turnover rate, whereas their difference is the traditional measure of net employment change.

International comparison of turnover rates suffer from comparability problems. The size of the sample, the sectors included in the analysis, the average size of the establishment, and the unit of analysis (firms versus establishments) are all factors that affect the quality of the turnover statistics.<sup>3</sup> The OECD (1994a) has recently tried to standardize data as much as possible, and with the limits deriving from comparability problems, Table 2 presents aggregate statistics on job creation and destruction. As was already pointed out by Garibaldi et al. (1997), and Bertola and Rogerson (1997), it is remarkable to observe that job turnover in Italy is as high as job turnover in the United States. The aggregate statistics in Table 2 suggest that, on average, job turnover in continental Europe is comparable to job turnover in North American labor markets. Finally, job turnover in the United Kingdom appears extremely low, and, as it was pointed out by Contini et al. (1995), United Kingdom seems to be an obvious outlier. The clear distinction between Europe and North America, remarkable from Table 1, seems to be lost in Table 2.

<sup>3</sup>See Contini et al. (1995) for a thorough discussion on comparability issues

Table 2. Job Turnover Across Selected Industrial Countries

	JT	JC	JD	NET	Time Period
	(1)	(2)	(3)	(4)	
Canada	26.4	14.5	11.9	2.6	1983-91
France	27.1	13.9	13.2	0.6	1984-92
Germany	16.5	9	7.5	1.5	1982-90
Italy	23.4	12.3	11.1	1.3	1984-92
United Kingdom	15.3	8.7	6.6	2.1	1985-91
United States	23.4	13	10.4	2.6	1984-91
Numbers are percentage of total employment.					
<sup>1</sup> JT is Job Turnover					
<sup>2</sup> JC is Job Creation					
<sup>3</sup> JD is Job Destruction					
<sup>4</sup> NET is Net Employment Change					
Source: OECD Employment Outlook (1994)					

For most of the countries reported in Table 2 there exist also data on worker turnover, but the comparability problems across data set is really difficult (See Contini et al. 1995). Worker turnover is defined as the sum of hiring and separations, normalized for an aggregate measure of employment. In what follows we shall indicate with  $E$ ,  $U$  and  $N$  the labor force states of, respectively, employment, unemployment and non-participation, and with appropriate combinations of these letters the origin and destiny of any labor market flows, so that  $UE$ ,  $NE$  and  $EE$  indicate hirings and  $EU$ ,  $EN$  and  $EE$  indicate separations. The flow  $EE$ , which appears in both hirings and firings, corresponds to job to job movements.<sup>4</sup> In general, worker turnover is higher than job turnover, as different workers tends to rotate on the same jobs. The OECD does not publish international statistics on worker turnover, but a recent paper by Burda and Wyplosz (1994) reports turnover statistics for five industrialized countries. Similar to the comparison of job turnover statistics, international comparison of worker turnover does not suggest a clear difference between Europe and North America. In particular, worker turnover in France appears larger than worker turnover in the U.S. Similarly to the statistics in Table 2, worker turnover in the U.K. is the lowest among the selected countries.

How can we reconcile the picture given in Tables 2 and 3, with the picture given in Table 1? If unemployment inflows and outflows across the atlantic are so different, why the same difference does not hold for worker and job turnover? In the rest of this

<sup>4</sup>Formally, worker turnover reads

$$WT = \frac{S + H}{E} = \frac{UE + NE + EU + EN + 2EE}{E}, \quad (1)$$

where  $H$  and  $S$  are total hirings and total separations.

Table 3. Worker Turnover Across Selected Industrial Countries

	H (1)	S (1)	WT (1)	Year
France	28.9	30.7	59.6	1987
Germany	22.3	21.5	43.8	1987
Spain	19.8	20.9	40.7	1987
United Kingdom	6.55	6.661	13.2	1987
United States	25.27	26.53	51.8	1987
Numbers are percentage of total employment.				
<sup>1</sup> <i>H</i> is Total Hirings, <i>S</i> is Separation				
<i>WT</i> is Worker Turnover				
Source: Burda and Wyplosz (1994) and CEPR (1994)				

section we argue that rough uniformity of turnover rates (Table 2) and marked difference in unemployment flows (Table 1), imply large job to job movements in Europe. While this seems intuitive, results in Tables (2) and (3) are not sufficient, by themselves, to justify such a claim. The reason being that inflow data in Table (1) can not distinguish between different inflow source, i.e. between people who enter unemployment from the employment pool (the *EU* flow), or from out of the labor force (the *NU* flow). Similar problems affect the outflow rates. Thus, at least in principle, Europe could be characterized by large movements between employment and out of the labor force (large *EN* and *NE* flows), and the large turnover data would not reflect large job to job movements, but large flows between employment and out of the labor force.

To check that this possibility is not the case, we can rely on two sources. First, existing estimates of job to job movements in the U.S. and the U.K. suggest that job to job movements are much more important in the U.K. For the U.S., Blanchard and Diamond (1989) estimates suggest that 20 percent of total hiring is a direct job to job movements. Conversely for the U.K., Gregg and Wadsworth (1994) find that approximately 40 percent of new hires represents direct job to job movements. Second, we look in more details at the countries in which unemployment surveys allow to match individual data over time, and we obtain separate estimates of the flows *UE*, *UN* and *EU*. Given the uniformity of the turnover rate, we can obtain residual estimates of job to job movements in Europe and in the United States. Table (4) reports detailed information on unemployment inflows and outflows, as well as transition from out of the labor force for four countries: Canada, Spain, United Kingdom and the United States.

The first column in each of the sub-panel of Table 4 suggests that the flows *UE*, *EU*, and *NE* are larger in North American labor markets than in Spain and in the U.K. The fact that the *UE* flow is so low in Spain may not be surprising, given the negative GDP growth in Spain and the positive GDP growth in the other three countries. However, for the same reason, we would expect a high *EU* flows in Spain. Table (4)

suggests that transitions between employment and unemployment are much lower in Spain than in U.S. and Canada, despite opposite GDP growth.

Table 4. Unemployment Inflows and Outflows

Unemployment Outflows					
Labor Force Status at $t + 1$ of Unemployed people at time $t$					
	Employed	Unemployed	N	Year	Growth
	$UE$	$UU$	$UN$		
	(1)	(2)	(3)	(4)	
Canada	48.4	23.1	28.5	1989-90	2.0
Spain <sup>5</sup>	23.7	59.5	15.5	1992-93	-1.1
United Kingdom	29.8	53.4	16.9	1992-93	2.2
United States	49.8	25	25	1992-93	3.1
Unemployment Inflows					
Labor Force Status at $t + 1$ of Unemployed people at time $t$					
	Employed	Unemployed	N	Year	Growth
	$EU$	$UU$	$NU$		
	(1)	(1)	(1)	(4)	
Canada	55.5	22.4	21.2	1989-90	2.0
Spain <sup>5</sup>	30.4	48.9	18.5	1992-93	-1.1
United Kingdom	33.7	48	18.5	1992-93	2.2
United States	48.2	28.8	22.9	1992-93	3.1
Exit from the Labor Force					
Labor Force Status at $t$ of Out of the Labor Force at time $t + 1$					
	Employed	Unemployed	N	Year	Growth
	$NE$	$NU$	$NN$		
	(3)	(3)	(3)	(4)	
Canada	24.8	6.3	68.9	1989-90	2.0
Spain <sup>5</sup>	5.4	4.1	90.3	1992-93	-1.1
United Kingdom	10.5	4.8	84.7	1992-93	2.2
United States	16.6	5.3	78.2	1992-93	3.1
Numbers are percentage of total employment.					
<sup>1</sup> $UE$ is Unemployment-Employment flow, $UU$ are people remaining unemployed					
<sup>1</sup> $UN$ is Unemployment-Non Participation flow.					
<sup>2</sup> $EU$ is Employment-Unemployment flow, $NU$ is Unemployment Non participation flow					
<sup>3</sup> $NE$ is Out of the labor force-Employment flow, $NN$ are people remaining out of the labor force					
<sup>4</sup> Real GDP growth					
<sup>5</sup> Number for Spain do not add up to 100, as the full sample include people in the military service.					
Source: OECD (1995)					

If the flows  $UE$ ,  $NE$  and  $EU$  are larger in the United States and Canada, and if we assume that the flows  $EN$  are roughly similar to the flows  $NE$ , uniformity of turnover rate between North America and Europe necessarily implies larger job-to-job movements in Europe.<sup>5</sup> To sum up, even though direct evidence is not yet available, the existing information let us infer that, in reality, job to job movements are more important in highly regulated European markets. The next section looks at job security provisions in Europe, and argues that worker protection legislation is conducive to direct job to job movements.

### III. MANDATORY NOTICE AND JOB SECURITY PROVISIONS

Worker dismissals, or employer-initiated job separations, can be restricted in several ways. The most applied form of restrictions in OECD countries is the requirement to provide workers with severance payments, i.e. with a fixed monetary compensation equal to several months of salary. With severance payments, firing can take place at any time, but it involves a monetary transfer equals to  $F$ . Advance notice, the other common form of firing restrictions, requires firms to provide the worker with several weeks/months of advance warnings, and it forces the firm to keep the worker employed during the entire notice period. From the firm stand point, if all that matters is the present value of the workers' cost during the notice period, the effect of advance notice on labor demand is very similar to the effect of severance payments. As a consequence, modeling the multidimensional aspect of workers' restrictions as a fixed firing costs, as most of economic literature does (Bentolila and Bertola, 1990, Bertola, 1990) is certainly a good first-order approximation.

However, advance notice transforms the job separation process into a time consuming activity and, to the extent that a transition into unemployment is welfare reducing, it provides the workers with an incentive to search on-the-job, and to quit before the expiration of the advance notice.<sup>6</sup> In reality there are several restrictions, different than advance notice, that make firing a time consuming process, much more complex than a simple monetary transfer. In some countries, worker dismissal requires authorization from a third party before dismissal can become effective, and in most European countries workers are given the provisions for appeal against unfair dismissals. Finally, mass firing often requires a bargaining between the union and the firm, and ex-ante firms do not know the exact timing of the bargaining process. All these forms of job security provisions, albeit relevant in reality, are not traditionally captured by the fixed firing rule assumed in the economic literature. To the extent that job to job

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<sup>5</sup>If two countries have similar  $WT$ , and, from footnote 4  $UE$ ,  $NE$  and  $EU$  are larger in a one country, then uniformity of  $WT$  implies that  $EE$  and  $EN$  must be larger in the other country.

<sup>6</sup>For the United States there is ample evidence that notified workers experience shorter unemployment spells. Even though in the United States there is no legislated mandatory notice, approximately 15 percent of the layoffs workers receive layoff notification. See Addison and Blackburn (1997), and references therein.

movements are not central to the analysis, neglecting the time-consuming property of job security provisions may certainly be analytical convenient. However, with reference to international comparison of worker turnover, these form of firing restrictions may well be important, and represent an potential channels for large job to job movements in highly regulated markets.

International comparison of job security provision has received great attention in the literature (OECD 1994b, Grubb and Wells, 1993, Bertola, 1990), and several quantitative indicators of the strictness of job security have been proposed. Roughly speaking, all indicators proposed rank European countries (Italy, France, Spain and Germany) among the countries with the toughest restrictions, and North American countries among the countries with the weakest restrictions.<sup>7</sup>

Table 5. Strictness of Job Security in Selected Countries

	<i>OECD</i> "Regulation "	<i>IOE</i> "Obstacles"	Notice Period
	(1)	(2)	(3)
France	9.5	2.5	2.0
Germany	12	2.5	4.5
Italy	14.25	3	0
Spain	15	3	3
United Kingdom	6	0.5	0
Canada	1.65	0.6	0.25
United States	0.36	0.4	0
<sup>1</sup> OECD Index. Weighted average of regulatory constraints, including procedural inconveniences, notice and severance pay and difficulty of dismissal <sup>2</sup> The <i>IOE</i> classifies regulatory constraints as insignificant (scored 0), minor (scored 1), serious (scored 2), or fundamental (scored 3) <sup>3</sup> Legislated maximum notice period Source: OECD (1994b)			

Table 5 reports three quantitative measures of firing restrictions. Among the several indicators available in the literature, Table 5 selected the indicators that best describe the time-consuming property of firing restrictions. The last column of Table 5 reports the legislative maximum notice, the most quantifiable dimension of job security provisions. On average, advance notice appears higher in European countries, and can be as high as four months in the case of Germany. However, Table 5 identifies countries (like Italy) that have very stringent job security provisions but do not impose a legislated maximum notice. However, there exist other dimensions, albeit less directly

<sup>7</sup>Scandinavian countries, not analyzed in this papers, lie somewhat between the two extreme of the scale.

measurable, that make Italy a country in which firing is extremely time consuming. Table 5 relies on two other measures proposed by the OECD and by the International Organization of Employers (IOE). In Italy, for example, the regulatory constraints are defined as “fundamental” by the International Organization of Employers. Similarly, the OECD indicator of Table 5 suggests that Italian procedural inconveniences are among the toughest in the OECD. Clearly, all these regulatory inconveniences transform firing into a time-consuming property, and all indicators in Table 5 suggest that the U.S. and Canada are the countries in which firing is fastest. In the rest of the paper, we address explicitly the case of advance notice, but the discussion in this section should make clear that there are other dimensions, albeit not directly observable, that make firing a time-consuming process.

#### IV. DESCRIPTION OF THE MODEL

We consider an economy populated by a homogeneous mass of risk-neutral workers, normalized to one for simplicity. Each worker can be in two states, employed or unemployed. Unemployed income is exogenous and is formally indicated with  $b$ . Since our focus is on jobs, rather than on firms, we assume that each firm is made up of only one job. As in conventional search equilibrium models, a job is a productive opportunity owned by a firm, and is capable of producing output only when is matched to a job seeker worker. If the job is vacant the firm actively searches for a worker, and offers the job to the first worker who meets. Opening a vacancy does not involves any fixed costs but searching for a worker and keeping a vacancy open entails a flow cost equal to  $\gamma$ . When the job is filled and producing firms do not search.

We depart from the traditional search equilibrium model with non-cooperative wages along four dimensions. First, existing jobs can be in two states, good or bad. A good job produces a net flow of production equals to  $y_g$ , whereas a bad job yields a flow of production equal to  $y_b$ , with  $y_g > y_b$ . Good jobs are hit by idiosyncratic adverse business conditions and turn bad according to a Poisson process with arrival rate equal to  $\lambda$ . Furthermore, we assume that the outside income of the unemployed,  $b$ , is larger than the productivity of a bad job, so that  $b > y_b$ . As a result, in a world with no job security provisions bad jobs would be immediately destroyed.

Second, we assume that vacant firms have the option to select the best jobs in the market and, accordingly, they decide to create only good jobs. However, once a job has been created its technology becomes irreversible and, once hit by an idiosyncratic shock, the firm can not adjust its productivity.

Third, we assume that firing can take place only upon the expiration of an institutionally determined advance notice. A worker can not be made unemployed unless he or she has been given an advance notice exogenously set to  $\tau^*$ . The requirement to give advance notice applies only to employer initiated separations, and is

consistent with a voluntary quit. Since the productivity of the bad job is lower than the outside income of the unemployed ( $y_b < b$ ), bad jobs represent a deadweight loss imposed by labor market institutions.<sup>8</sup>

Fourth, similarly to Pissarides (1994), we let workers search on-the-job, and we give them the opportunity to change job without experiencing an unemployment spell. Since all vacant jobs are good jobs, a worker employed in a good firm has no incentive to search on the job. Conversely, a worker employed in a bad firm has the incentive to search on the job and look for a vacant good job.

Even though job separation requires mandatory notice, a firm has the option to decide the timing of the notice. In what follows, we assume that the firm incurs a productivity loss equal to  $\delta$  whenever it issues the advance notice. There are several reasons that justify such assumption. First, notified workers are likely to reduce their commitment on the job. Second, workers may start looking for other jobs, thus taking away productive resources from the job. Given these technological constraints, firms may choose between alternative notice strategies. In the first strategy firms issue notice only after the realization of the bad shock, and they incur the cost  $\delta$  only when conditions are bad. In the second strategy notices are issued in a precautionary way. Firms may be willing to suffer a productivity loss equal to  $\delta$  when conditions are still good, so as to reduce the expected duration of a bad job. Section V and Appendix II show that only if the cost  $\delta$  is high enough the firm will issue the advance notice upon the realization of the shock  $\lambda$ .

The existence of search costs, which are sunk when the firm and the worker meet, endow existing matches with local monopoly power and produce a *pure economic rent*. To close the model we need a wage determination rule and, in what follows, we assume that the surplus in a good job is constantly split in fixed proportion. Wage determination in bad jobs is more subtle. In reality, advance notice legislation seems to imply that during the notice period the worker should be paid its pre-notice wage. Thus, the legislation tries to insure that the effect of the advance notice is not undone by threats of wage cuts. If workers and firms could sign long-term contracts it would be possible to write a model which incorporate this institutional feature. However, in the spirit of the traditional search literature, wages are continuously renegotiated and adverse shocks are conducive to lower wages (Mortensen and Pissarides, 1994). In the model of this paper bad jobs have negative productivity and there is no surplus to be shared. In what follows, we adapt the following solution. Firms are forced to give the worker an advance notice equal to  $\tau^*$ , while simultaneously they have to pay a wage that is at least as high as  $\bar{w}_{min}$ . In addition, we assume that the minimum wage is at least as high as  $b$ , the income of the unemployed ( $\bar{w}_{min} \geq b$ ). If the firm and the worker have the same knowledge about the outside opportunities to the worker, a bad firm will never set

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<sup>8</sup>Most job security provisions impose an inefficiency in labor market allocations. See Lazear (1988) for an extensive analysis and section VII for a discussion.



a wage higher than  $\bar{w}_{min}$ . In the limit case, when  $\bar{w}_{min} = b$ , workers are just indifferent between working in a bad firm and being unemployed.

The number of contacts between searching firms and job seekers is given by the matching technology

$$x = x(v, u + n_b), \quad (2)$$

where  $x$  is the total number of matches in a given instant,  $v$  is the number of vacancies,  $u$  is the unemployment rate, and  $n_b$  is employment in bad jobs, a measure of the employed job seekers.<sup>9</sup> In equation (2) all measures are expressed as fractions of the fixed labor force. The matching technology  $x$  is assumed homogeneous of degree one and increase and concave in both of its arguments. The transition rates from different labor market states are derived from equation (2), after dividing the number of total contacts by the relevant stock of job seekers. Thus, the instant probability that a vacant job meets a job seekers is given by

$$\frac{x(v, u + n_b)}{v} = x(1, \frac{u + n_b}{v}) = q(\theta); \quad \theta \equiv \frac{v}{u + n_b}, \quad (3)$$

where  $\theta$  is a measure of market tightness from the firm stand point, and  $q(\theta)$  is a decreasing function of  $\theta$ , so that  $q'(\theta) < 0$ . Making use of (3), the total number of contacts between unemployed job seekers and vacant jobs is

$$\frac{u}{u + n_b} x(v, u + n_b) = u \frac{v}{u + n_b} \frac{x(v, u + n_b)}{v} = u\alpha(\theta); \quad \alpha(\theta) = \theta q(\theta),$$

where  $\alpha(\theta)$  is the probability that any job seeker meets a vacant job.  $\alpha(\theta)$  is increasing in  $\theta$ , so that  $\frac{\partial}{\partial \theta} \alpha(\theta) > 0$ . Finally, the total number of contacts between workers employed in bad jobs and vacant firms is simply

$$\frac{n_b}{u + n_b} x(v, u + n_b) = n_b \alpha(\theta).$$

An equilibrium in this model is a a distribution of employment at jobs of different quality, consistent with a free entry condition on the job market, and a profit maximizing notice strategy.

## V. THE MODEL

We present and solves the model in three steps. The next section presents and solves job creation and wage determination under the assumption that firms issue advance notice only when business conditions turn bad. Section B and Appendix II derive a set of conditions that insures that a firm may not profitable deviate from the notice strategy assumed in section A. Finally, section C solves for the aggregate equilibrium.

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<sup>9</sup>Equation (2) implicitly assumes that there is no on-the-job search by workers employed in good job. As we will show below, this is true in equilibrium.

### A. Job Creation and Wage Determination

Wage determination in good jobs plays a key role for the workers' decision to search on-the-job. In order to derive the equilibrium wage, we need to specify the worker and firm expected returns.

The firm's expected profit from operating a vacant job of good quality is denoted by  $V_g$ , and reads

$$rV_g = -\gamma + q(\theta)(J_g - V_g), \quad (4)$$

where  $r$  is the firm (and worker) discount rate,  $J_g$  is the firms' value of a good job and  $q(\theta)$  is the firm's probability of finding a job seeker, either unemployed or employed in bad jobs. Equation (4) describes the return of a vacant job as the sum of a flow cost  $\gamma$  and a probability  $q(\theta)$  of a capital gain equal to  $J_g - V_g$ . Since there are no fixed costs in creating a vacancy, firms will open up new job opportunities until the full exhaustion of rents, and, as Pissarides (1990) has shown, free entry in the job market implies  $V_g = 0$ . Substituting for  $V_g = 0$  in (4) yields

$$J_g = \frac{\gamma}{q(\theta)}. \quad (5)$$

Equation (5) is the first key equation of the model, and implies that the value of a new job is equal to the expected search costs. If the firm issues the mandatory notice when business conditions turn bad,  $J_g$  satisfies the following asset valuation

$$rJ_g = y_g - w_g + \lambda(J_b(0) - J_g), \quad (6)$$

where  $w_g$  is the wage paid in good jobs and  $\lambda$  is the arrival rate of adverse business conditions. Equation (6) shows that a good job yields a net dividend equals to the difference between the value of the labor product and the wage, and an expected capital loss equal to the difference between the value of a bad and a good job. Equation (6) implies that a good firm issues the advance notice only when the shock  $\lambda$  arrives, and the firm's productivity permanently drop to  $y_b$ .<sup>10</sup> The expression  $J_b(0)$  keeps track of the value of bad jobs at notice time  $h = 0$ , when the maximum notice period is equal to  $\tau^*$ . A worker employed in a good job does not search on-the-job, earns a wage equal to  $w_g$  and his asset valuation function  $E_g$  reads

$$rE_g = w_g + \lambda(E_b(0) - E_g), \quad (7)$$

where  $E_b(0)$  is the value of a bad job at the beginning of his notice period. Finally, unemployed workers receive an unemployed income equal to  $b$ , and face an instantaneous probability  $\alpha(\theta)$  of finding a good job. Thus, the present value of being unemployed reads

$$rU = b + \alpha(\theta)(E_g - U). \quad (8)$$

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<sup>10</sup>The formal conditions that induce firms to give notice only after the realization of the shock  $\lambda$  are discussed in section B and Appendix II.

The total surplus from a good job is the sum of the the worker's and the firm's present discount values, net of their outside options

$$W_g = J_g + E_g - (U + V_g) = J_g + E_g - U, \quad (9)$$

where the second equality in (9) made use of the free-entry condition in (5). Equation (9) implies that when negotiations break down severance notice is not paid and the firm can immediately post a new good vacancy. If workers receive a fraction  $\beta$  of the total surplus from the job, equation (9) implies

$$(1 - \beta)(E_g - U) = \beta J_g. \quad (10)$$

Substituting (6), (7) and (8) into (9), yields the following expression for the surplus in a good job

$$rW_g = y_g - b + \lambda [J_b(0) + E_b(0) - (E_g + J_g)] - \beta\alpha(\theta)W_g, \quad (11)$$

where  $\beta\alpha(\theta)W_g$  is the unemployed surplus from finding a good job. From equation (11) it is clear that in order to solve for  $W_g$ , we need first an expression for the value of bad jobs at notice time 0. In general, if we indicate with  $J_b(h)$  the value of bad job at notice time  $h$ , when the required notice is  $\tau^*$ , its asset valuation function reads

$$rJ_b(h) = y_b - (\delta + \bar{w}_{min}) + \frac{\partial}{\partial h}J_b(h) - \alpha(\theta)J_b(h) \quad \forall h \leq \tau^*, \quad (12)$$

where  $\bar{w}_{min}$  is the wage in bad jobs,  $\delta$  is the productivity loss associated with the mandatory notice,  $\frac{\partial}{\partial h}J_b(h)$  is the capital gain associated with the elapsing of notice time, and the last term reflects the probability that the workers quits to a good job. Similarly, the value to the worker of being employed in a bad job of notice time  $h$  reads

$$rE_b(h) = \bar{w}_{min} + \frac{\partial}{\partial h}E_b(h) + \alpha(\theta)(E_g - E_b(h)) \quad \forall h \leq \tau^*, \quad (13)$$

where  $(E_g - E_b(h))$  reflects the capital gain associated with the quit. Since the wage in bad firm can not be lower than  $\bar{w}_{min}$ , and the worker's participation constraint must be satisfied, the (negative) surplus in a bad job at notice time  $h$  is

$$F_b(h) = J_b(h) + E_b(h) - U \leq 0, \quad E_b(h) \geq U, \quad (14)$$

where  $F_b(h)$  is the deadweight loss induced by advance notice. Making use of equation (14), the surplus in a bad job is the solution to the following differential equation in notice time

$$(r + \alpha(\theta))F_b(h) = y_b - (b + \delta) + \frac{\partial}{\partial h}F_b(h). \quad (15)$$

Since a bad job at notice time  $h = \tau^*$  has zero value, the boundary condition for solving (15) is  $F_b(\tau^*) = 0$ , and the solution to the differential equation (15) reads

$$F_b(h) = \frac{y_b - (b + \delta)}{r + \alpha(\theta)} (1 - e^{-(r + \alpha(\theta))(\tau^* - h)}) \leq 0 \quad \forall h \leq \tau^*. \quad (16)$$

Equation (16) shows that the value of a bad job is a negative function, monotonically increasing in notice time.<sup>11</sup> The surplus is discounted by two factors, the pure discount rate  $r$  and the probability that the worker finds a good job  $\alpha(\theta)$ . Appendix I reports simple comparative static results. Intuitively, the (negative) value of a bad job decreases monotonically with the length of the maximum notice  $\tau^*$  and the wage in the bad sector. Furthermore, since  $F_b(h) < 0$ , the deadweight loss increases with the interest rate. However, the most important comparative static result is that

$$\frac{\partial F_b(h)}{\partial \theta} > 0.$$

Thus, an increase in market tightness decreases the cost of job separation. Since workers employed in bad jobs search for employment in good jobs, the higher is the number of vacant jobs (i.e. the higher is  $\theta$ ) the higher is the probability that a worker under advance notice is matched to a good job and quits before the expiration of the notice time. As a result, the value of a bad job becomes less negative as  $\theta$  increases. Figure 1 plots the value of a bad job against market tightness  $\theta$ , and shows how higher notice time  $\tau^*$  reduces  $F_b$  at low market tightness.<sup>12</sup> Figure 1 shows also that, as  $\theta$  increases,  $F_b$  tends to zero and the marginal effect of longer notice time  $\tau^*$  disappears.

By substituting into (11) the value of (16) evaluated at  $\tau = 0$ , the surplus in a good job reads

$$[r + \lambda + \beta\alpha(\theta)] W_g = y_g - b + \lambda \frac{y_b - (b + \delta)}{r + \alpha(\theta)} \left(1 - e^{-(r + \alpha(\theta))\tau^*}\right), \quad (19)$$

or simply

$$[r + \lambda + \beta\alpha(\theta)] W_g = y_g - b + \lambda F_b(0, \tau^*). \quad (20)$$

Equation (19) is the second key equation of the model, and shows that a good job yields a dividend made up of three components: the productivity of a good job, the outside income flow of the unemployed, and the expected value of a bad job at zero tenure time, a measure of firing costs. While the first term increases the surplus of the job, the remaining two components reduce it. From equation (19) it is clear that what enters into the surplus determination is the present value of the bad jobs. From the firm's

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<sup>11</sup>From the worker's standpoint, the (positive) surplus from being employed in a bad is

$$\tau [E_b(h) - U] = \frac{\bar{w}_{min} - b}{r + \alpha(\theta)} \left(1 - e^{-(r + \alpha(\theta))(\tau^* - h)}\right), \quad (17)$$

which is positive as long as  $\bar{w}_{min} \geq b$ . Conversely, for the firm, the negative value of having a bad job is

$$r J_b(h) = \frac{\bar{y}_b - (\bar{w}_{min} + \delta)}{r + \alpha(\theta)} \left(1 - e^{-(r + \alpha(\theta))(\tau^* - h)}\right), \quad (18)$$

<sup>12</sup>The value of the other parameters used in the simulation of figure 1 are specified in Table 6.

stand point, a firing tax equal to  $F_b(0)$  would have the same effect on the value of the job, in the sense that it would reduce the total surplus from the job in a way similar to the effect of advance notice. However, a fixed firing costs would not introduce time to fire in the job separation process, and would not affect the worker decision to search on the job.<sup>13</sup>

The relationship between  $W_g$ ,  $\theta$  and  $\tau^*$  is non-linear, and it deserves some discussion. When  $\tau^* = 0$ , higher  $\theta$  reduces the value of a job. In a standard matching model, an increase in market tightness increases the outside option of the unemployed and reduces the local monopoly rent of a given match. In this paper, with  $\tau^* > 0$ , the effect of higher  $\theta$  on  $W_g$  depends crucially on the relationship between  $F_b(\theta)$  and  $\theta$ , and the total derivative of  $W_g$  with respect to  $\theta$  reads

$$\frac{\partial W_g}{\partial \theta} = \frac{1}{r + \lambda + \beta\alpha(\theta)} \left[ -\beta\alpha'(\theta)W_g + \lambda \frac{\partial F_b(\theta)}{\partial \theta} \right] \quad (22)$$

In general, since  $\frac{\partial F_b}{\partial \theta} \geq 0$ , the sign of (22) is ambiguous. Numerical simulations show that different values of  $\tau^*$  change the sign of the derivative of equation (22), for very low values of  $\theta$ . Conversely, as  $\theta$  grows,  $\frac{\partial F_b}{\partial \theta}$  tends to zero and the sign of the derivative in equation (22) becomes negative.<sup>14</sup> Figure 2 plots  $W_g$  against  $\theta$  for different values of  $\tau^*$ , and it shows different curves' shape for low values of  $\theta$ . However, figure 2 suggests that, for a given  $\theta$ ,  $W_g$  shifts downward when  $\tau^*$  is increased.<sup>15</sup> Thus, an increase in  $\tau^*$  unambiguously shifts downward the curve  $W_g$  in figure 2.

To complete the description of firm behavior we need to specify the wage. Substituting equation (19) into (7), making use of equation (8), the wage in a good firm reads

$$w_g = rU + \frac{\beta(r + \lambda)}{r + \lambda + \beta\alpha(\theta)} \left[ y_g - b + \frac{\lambda(y_b - b - \delta)}{r + \alpha(\theta)} (1 - e^{-(r + \alpha(\theta))\tau^*}) \right]. \quad (24)$$

The wage in a good job is the sum of the permanent income of the unemployed job seeker, augmented by a fraction  $\beta$  of the dividend enjoyed by being matched with a

<sup>13</sup>A viable labor market requires that  $W_g > 0$ , and in what follows, we assume that

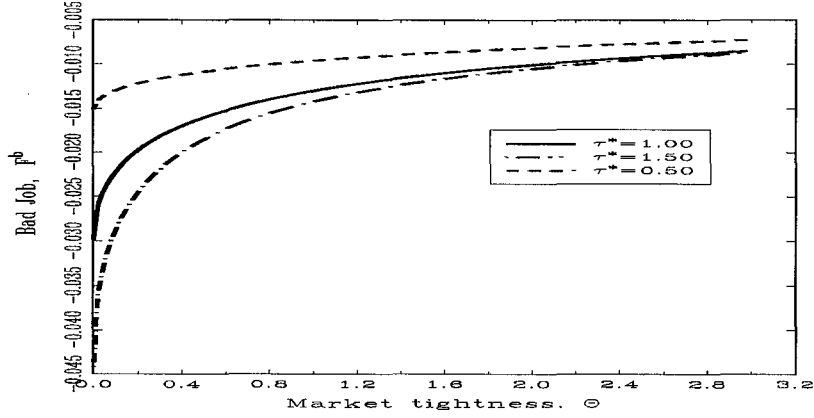
$$(y_g - b + \lambda F_b(0, \tau^*)) > 0 \quad \forall \theta, \quad \forall \tau^*. \quad (21)$$

<sup>14</sup>The effect of advance notice on the match surplus is similar to the effect of firing costs on natural turnover, as it is illustrated in the model of Saint-Paul (1995), where firms use quits as a way to reach costless workers reductions.

<sup>15</sup>Differentiating  $W_g$  with respect to  $\tau^*$ , for a given value of  $\theta$ , yields

$$\left. \frac{\partial W_g}{\partial \tau^*} \right|_{\theta=\bar{\theta}} = \frac{\lambda \frac{\partial F_b}{\partial \tau^*}}{r + \lambda + \beta\alpha(\bar{\theta})} \geq 0. \quad (23)$$

Figure 1. Bad Jobs over market tightness for different  $\tau^*$



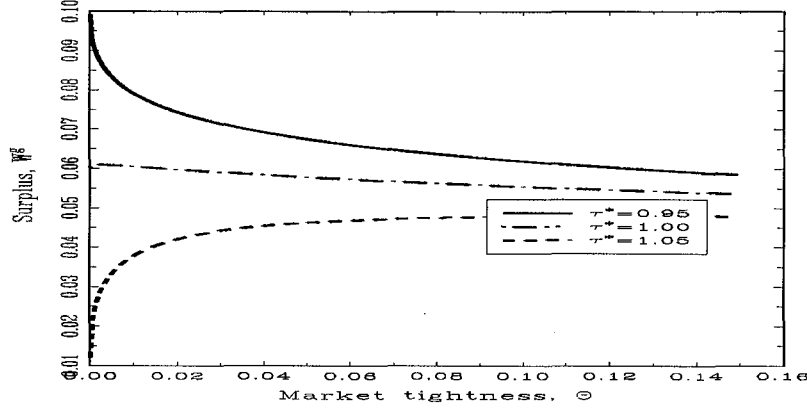
good job. Since the expression in the square bracket of equation (24) is positive, the wage in a good job is higher than the permanent income of the unemployed.

### B. On The Timing of Advance Notice

The previous section solved the firm's problem under the assumption that firms give the advance notice only when conditions turn bad. This strategy, however, may not necessarily be optimal. Even though a firm is forced to give mandatory notice before separation can take place, in the model, as well as in reality, the firms may *endogenously choose the moment in which to give notice*. If issuing notice was completely costless, firms would continuously put workers on notice, just to ask them to ignore such warnings if business conditions remain favorable. In reality, however, giving notice is likely to entail productivity costs, and this section explicitly tries to account for these costs. We assume that the cost of giving notice is formally incurred by the firm, but since the worker and the firm share the outcome of their labor products, it would not make differences if the costs were formally incurred by the workers.<sup>16</sup> In the model, *the decision on whether to issue notice, at any point in time, is an endogenous variable*.

<sup>16</sup>In this section, as in the rest of the paper, we assume that business conditions are perfectly known to the firm and the worker, and we rule out any signaling problem associated with mandatory notice. Kuhn (1992) solves a partial equilibrium model in which business conditions are not perfectly known to the worker, and mandatory notice act as a signaling device.

Figure 2. Surplus  $W^g$  over market tightness for different notice times,  $\tau^*$



In what follows we shall consider the value functions of a firm that has already issued the notice, even though its business conditions are still good. In what follows  $h$  shall indicate the *time since notice*, or the time elapsed since a firm has last issued a notice warning. If  $\tilde{J}_g(h)$  is the value of a good firm at notice time  $h$ , its asset valuation reads

$$r\tilde{J}_g(h) = y_g - \delta - \tilde{w}_g(h) + \lambda [J_b(h) - \tilde{J}_g(h)] + \frac{\partial}{\partial h} \tilde{J}_g(h) \quad \forall h \leq \tau^*, \quad (25)$$

where  $J_b(h)$  is the value of a bad job with notice time equal to  $h$ , and its value is given by equation (12). With respect to equation (6), equation (25) features the flow cost  $\delta$ , and an extra capital gain term in notice time. As  $h$  grows, the expected firing costs if conditions turn bad fall, and the value of  $\tilde{J}_g(h)$  grows. For a worker employed in a good job with notice, the capital gains associated with the elapsing of  $h$  are more subtle, since we need to specify the workers' outside options. In this section we assume that vacant firms do not give notice, and behave in a way consistent with the analysis of the previous section. This is equivalent to considering an unilateral (match specific) deviation from the strategy of issuing notice only when business conditions are bad. In a rational expectation equilibrium, such deviations should not be profitable. While we refer to the appendix for a formal derivation of the conditions, this section reports only the results, and it provides the intuition with the help of a simple chart. If  $\tilde{E}_g(h)$  is the worker's value of being employed in a good job under notice, its asset valuation reads

$$r\tilde{E}_g(h) = \tilde{w}_g(h) + \lambda [E_b(h) - \tilde{E}_g(h)] + \alpha(\theta) [Max(E_g; \tilde{E}_b(h)) - \tilde{E}_g(h)] + \frac{\partial}{\partial h} \tilde{E}_g(h), \quad (26)$$

where the *max* operator reflects the worker's choice on whether to accept a vacant job. There are two forces that affect the worker's capital gains. On the one hand, as  $h$  grows, the worker's expected income if conditions turn bad is lower ( $E_b(h, \tau^*) \leq E_b(0)$ ), and finding a good job with no notice (a permanent job) increases the worker's expected income. On the other hand, expected firing costs in a good job in which notice is given are lower than in a good job without notice, and the overall surplus from the job may be higher. Since the worker and the firm share the surplus in fixed proportion, a worker in a good job with notice may be better off than in a job with no notice. If the value of being unemployed is given by (8), the surplus from a good job with notice,  $\tilde{W}_g(h)$ , solves

$$\begin{aligned} [r + \lambda + \beta\alpha(\theta)] \tilde{W}_g(h) = & y_g - b - \delta + \lambda F_b(h) + \\ & + \alpha(\theta) [Max(\beta W_g, \beta \tilde{W}_g(h))] - \alpha \beta \tilde{W}_g(h) + \frac{\partial}{\partial h} \tilde{W}_g(h) \end{aligned} \quad (27)$$

To solve the mapping in equation (27) we need to solve the maximization between  $\beta W_g$ , and  $\beta \tilde{W}_g(h)$ , which obviously depends on the value of  $\delta$ . In the Appendix we solve the maximization in equation (27) for each of the two possibilities. Nevertheless, independently of the solution to the maximization in (27), it is true that

$$\frac{\partial \tilde{W}_g(h)}{\partial h} \geq 0. \quad (28)$$

Thus, as notice time elapses, the expected firing costs decrease and the “match” experiences a positive capital gain. For the strategy of giving notice in bad times to be an equilibrium, it is necessary that the cost  $\delta$  be larger than a minimum cut-off value,  $\delta^*$ , where  $\delta^*$  solves

$$\delta^* : W_g = \tilde{W}_g(h) \Big|_{h=0}. \quad (29)$$

The appendix shows that the necessary and sufficient conditions that satisfy (29) require that the cost  $\delta$  be higher than the capital gain associated with the elapsing of notice time. Formally, the necessary and sufficient conditions on  $\delta^*$  are derived as the solution to

$$\delta^* = \frac{\partial \tilde{W}_g(h)}{\partial h}. \quad (30)$$

The cut-off value  $\delta^*$  for the sufficient condition reads<sup>17</sup>

$$\delta^* = \frac{\frac{\lambda(y_b - b)}{\lambda - \alpha(\theta)} [e^{-(r+\lambda)\tau^*} - e^{-(r+\alpha(\theta))\tau^*}]}{1 + \frac{\lambda}{\lambda - \alpha(\theta)} [e^{-(r+\lambda)\tau^*} - e^{-(r+\alpha(\theta))\tau^*}]} \quad (31)$$

By virtue of (31), giving notice when business conditions turn bad is optimal as long as  $\delta$  is higher than  $\delta^*$ . Since the cut-off level  $\delta^*$  depends also on  $\theta$ , the sufficient condition requires  $\delta > \delta^*(\theta)$ .<sup>18</sup>

<sup>17</sup>See Appendix II for the formal derivation of the result

<sup>18</sup>See section C for the derivation of the equilibrium  $\theta$ .



Figure 3. Value of a Good Job over Notice Time  $h$ ;  $\delta = \delta^*$

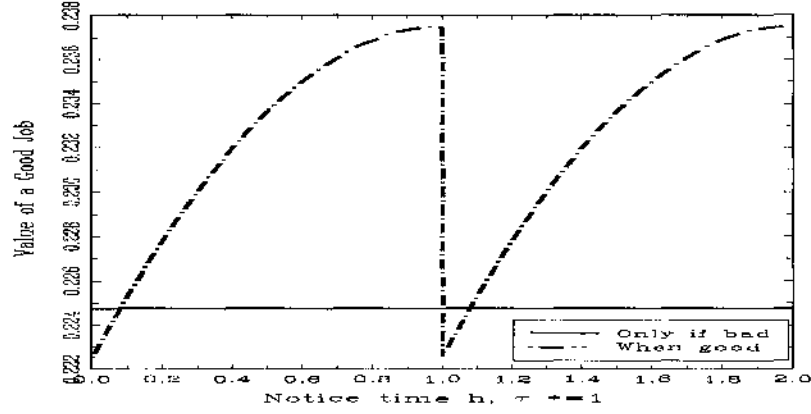


Figure 3 plots the value of a good job against notice time.<sup>19</sup> The constant line is the value of a good job under the policy described in the previous section. Conversely, the dotted line is the value of a good job when notice is issued in a precautionary way, and its formal expression is given by equation (XII) in Appendix II. Figure 3 assumes that as soon as the notice time is completely elapsed (at  $\tau^* = 1$  in the figure) the firm issues another notice. Clearly, the firm's value function reaches a maximum at  $h = \tau^*$ , immediately before the firm's value discontinuously jumps. As the notice time elapses, the precautionary benefits linked to the last notice warning are exhausted, and the firm faces the same decision problem. Since the two strategies should be compared at  $h = 0$ , (and  $h = \tau^*$ , when a new notice is given) figure 3 (plotted with  $\delta = \delta^*$ ) implies that issuing notice when conditions are bad is optimal, and there is no profitable deviation.<sup>20</sup> In the rest of the paper we assume that the cost  $\delta$  satisfies the conditions of equation (31), and firms issue notices only when conditions turn bad.

### C. Equilibrium with Advance Notice

Firm's behavior is described by the job creation condition (5), and by the surplus of a good job (19). To complete the equilibrium's description, we need a value of  $\theta$  that is consistent with (5) and (19), and a productivity loss  $\delta$  that satisfies the condition (31).

<sup>19</sup>The value of the other parameters used in the simulation of figure 3 are specified in Table 6.

<sup>20</sup>An alternative policy of issuing notice at interval lower than  $\tau^*$  is obviously sub-optimal, since it would increase the cost and it would reduce the benefit.

Making use of (9), equation (5) can be written as

$$W_g = \frac{\gamma}{(1 - \beta)q(\theta)}, \quad (32)$$

so that equations (32) and (19) form a system to be solved for  $\theta$  and  $W_g$ . Figure 4 shows the equilibrium determination of  $W_g$  and  $\theta$ . From equation (32), the relationship between  $\theta$  and  $W_g$  is unambiguously positive. From the job creation condition, an increase in  $W_g$  causes more firms to come to the market. From equation (19), the sign of the total derivative between  $W_g$  and  $\theta$  is ambiguous, and we can not rule out the existence of multiple equilibria. In the rest of the paper, however, we select parameters that are consistent with a downward sloping surplus curve, and with a unique equilibrium  $\theta$ .<sup>21</sup> Figure 4 plots the equilibrium value of  $\theta$  and reports the effect of an increase of  $\tau^*$  on the equilibrium  $\theta$ .

An increase in the maximum notice period  $\tau^*$  unambiguously reduces market tightness  $\theta$ . Figure 4 shows that the job creation condition, for given values of  $W_g$  is not affected by the increase in  $\tau^*$ . Conversely, for a given value of  $\theta$ , an increase in  $\tau^*$  shifts downward the surplus from the job. Thus, the surplus curve shifts along the job creation condition and  $\theta$  and  $W_g$  unambiguously fall. Intuitively, for given number of job seekers, an increase in the length of the advance notice  $\tau^*$  reduces the surplus from a good match. As a result, fewer vacancies are created and  $\theta$  falls.

To close the model we need to keep track of the distribution of employment between good and bad jobs and, in particular, of the distribution of bad jobs at different tenure time  $\tau$ . In what follows, we shall indicate with  $N_b(t, \tau)$ , the time  $t$  number of workers employed in bad jobs with notice period less or equal than  $\tau$ . Over time, it must be true that

$$\frac{dN_b(t, \tau)}{dt} = -\frac{dN_b(t, \tau)}{d\tau} - \alpha(\theta)N_b(t, \tau) + \lambda N_g, \quad (33)$$

where the first term refers to the number of workers whose duration becomes higher than  $\tau$ , the second term refers to the fraction of workers who found a good job, and the last term refers to the number of good jobs that issue a new notice warning at time  $t$ . In steady state, the distribution of bad jobs at different notice time must be constant and, in equation (33),  $\frac{dN_b(t, \tau)}{dt} = 0$ . The steady state density of unemployment duration solves the following differential equation in notice time  $\tau$

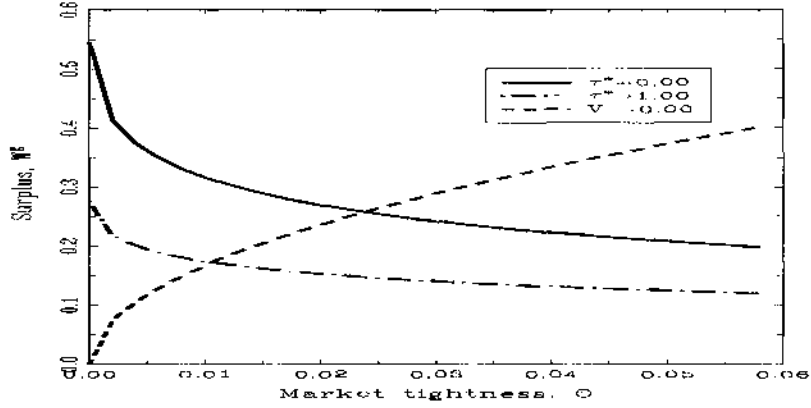
$$\frac{dN_b(\tau)}{d\tau} = -\alpha(\theta)N_b(\tau) + \lambda N_g, \quad (34)$$

where the time  $t$  index, irrelevant in steady state, has been omitted for analytical

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<sup>21</sup>Garibaldi (1998b) studies the possibility of multiple equilibria in a dynamic version of the model presented in this paper.

Figure 4. Equilibrium market tightness  $\theta$  for different notice times,  $\tau^*$



convenience. The solution to (34) reads

$$N_b(\tau) = \frac{\lambda N_g}{\alpha(\theta)} (1 - e^{-\alpha(\theta)\tau}). \quad (35)$$

The dynamics of unemployment is given by the difference between inflows and outflows. Unemployment outflows are simply given by  $\alpha(\theta)u$ , whereas unemployment inflows are given by the proportion of bad jobs that reach maximum notice  $\tau^*$ . If we indicate with  $\dot{u}$  the time derivative of unemployment, its dynamic motion reads

$$\dot{u} = \left. \frac{dN_b(t, \tau)}{d\tau} \right|_{\tau=\tau^*} - \alpha(\theta)u_t, \quad (36)$$

where  $\left. \frac{dN_b(t, \tau)}{d\tau} \right|_{\tau=\tau^*}$  is the fraction of bad workers that reach duration  $\tau^*$  at time  $t$ . In steady state unemployment is constant, and the distribution of bad jobs at duration  $\tau$  is time invariant. Making use of the derivative of (34) evaluated at  $\tau^*$ , steady-state unemployment reads

$$u = \frac{\lambda n_g e^{-\alpha(\theta)\tau^*}}{\alpha(\theta)}. \quad (37)$$

Equation (37) is the first of the three equations that determines the aggregate steady state variables  $u$ ,  $n_b$  and  $n_g$ . We need two other equations. The first one is the law of motion of  $n_g$ . In steady state, jobs flow into the good state at rate  $\alpha(\theta)$  from the

unemployment pool and from the stock of bad jobs, and jobs flow out of the good state at rate  $\lambda$ . Since inflows must be equal to outflows,  $n_g$  solves

$$\lambda n_g = \alpha(\theta)(n_b + u). \quad (38)$$

The last equation is the labor force constraint, which implies that  $n_g, u$  and  $n_b$  must continuously sum up to one. At each instant it must be true that

$$u + n_g + n_b = 1. \quad (39)$$

The model is recursive and is fully described by five equations. For a given value of  $\delta$ , equations (32) and (19) uniquely yield a  $\theta$ . If  $\delta^*(\theta) \leq \delta$ , the computed  $\theta$  describes the general equilibrium of the model. Given  $\theta$ , equations (37), (38) and (39), uniquely solve for  $u, n_g$  and  $n_b$ . Solving this simple system the equilibrium number of good jobs is

$$n_g = \frac{\alpha(\theta)}{\lambda + \alpha(\theta)}, \quad (40)$$

while the equilibrium number of bad jobs is

$$n_b = \frac{\lambda(1 - e^{-\alpha(\theta)\tau^*})}{\lambda + \alpha(\theta)}. \quad (41)$$

Finally, equilibrium unemployment reads

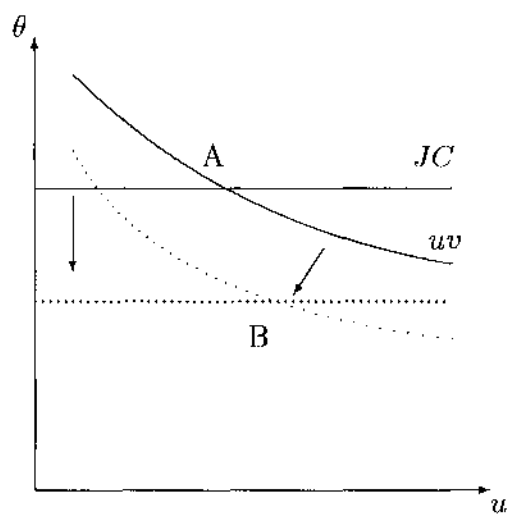
$$u = \frac{\lambda e^{-\alpha(\theta)\tau^*}}{\lambda + \alpha(\theta)}. \quad (42)$$

Equation (42) is the Beveridge curve and implies a negative relationship between  $\theta$  and unemployment. If there is no notice period, (i.e.  $\tau^*$  is equal to zero) the number of bad jobs from equation (41) is equal to zero and the Beveridge curve approaches the vacancy-unemployment relationship of the traditional matching model (Pissarides, 1990). Figure 5 describes equilibrium unemployment in a  $(\theta, u)$  space. The curve labeled  $JC$  gives the equilibrium  $\theta$  from equations (4) and (19), while the curve labeled  $uv$  is the Beveridge curve from equation (42). Since the Beveridge curve is downward sloping, equilibrium unemployment is unique, and it is labeled point A in the figure. Figure 5 reports also the comparative static effect of an increase in the maximum notice time  $\tau^*$ . Since an increase in  $\tau^*$  unambiguously reduce market tightness, the  $JC$  curve shifts downward. However, for given  $\theta$ , the beveridge curve of equation (42) shifts downward and the new equilibrium unemployment can be above or below the previous level, depending on the magnitude of the relative shift of the two curves.

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Figure 5. The Effect of an increase in  $\tau^*$  on equilibrium unemployment

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## VI. SIMULATIONS

This section presents the results of a simulation of the model for different maximum notice time  $\tau^*$ . The simulations are meant to be suggestive, rather than fully realistic, and are performed for understanding the relationship between  $\tau^*$ , the unemployment stock, job creation, unemployment inflows and outflows, and on-the-job search. Table 6 reports the values of the parameters used in the simulation. The parameters characterize a representative OECD economy, and they are in line with the parameters values used by Mortensen and Pissarides (1994) for simulating the behavior of the U.S. manufacturing flows on the quarterly basis.

The most important parameter to set is the value of the notice length  $\tau^*$ , and Table 6 shows that throughout the simulations we let  $\tau^*$  varying from a minimum value of zero to a maximum value of 1.5. The question is how long is a notice length of, say, 1. With  $\lambda$ , the arrival rate of adverse shock, equal to 0.06 the average duration of a good job is approximately equal to 17 quarters. In general, a yearly job turnover rate of 22 percent (in line with the cross country number in Table 2) implies an average duration of each job equal to 22 quarters. In this respect, an advance notice of 1 quarter and a duration of a job of 22 quarters are roughly consistent with the cross country evidence provided in Table 2 and with the legislated maximum notice presented in Table 5.

Figure 6 plots job creation, unemployment inflows and job quits for different durations  $\tau^*$ .<sup>22</sup> A quarterly job creation of 5 percent, as shown in figure 6 is similar to the average quarterly job creation, as reported by Davis Haltiwanger and Shuh (1996). Since the simulations refer to steady state figures, job turnover is simply twice the value of job creation. Table 7 reports the detailed results of the simulations statistics for a steady state economy whose maximum notice time is given in each column of Table 7. The results show that an increase in  $\tau^*$  leaves approximately constant job turnover, but dramatically affect unemployment inflows and job to job movements. In the rest of this section we look in more details at the different statistics.

Figure 7 reports equilibrium unemployment for different notice values. Consistent with the comparative static results of the previous section, figure 7 shows that the relationship between unemployment and the notice length is non linear, and

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<sup>22</sup>Job creation in the simulation is simply

$$JC = JD = \alpha(\theta)(u + n_b).$$

Unemployment inflows are equal to unemployment outflows and read

$$U_{inf} = \alpha(\theta)u.$$

Finally, quits are given by

$$Quits = \alpha(\theta)n_b.$$

Figure 6. Job Flows for different notice times,  $\tau^*$

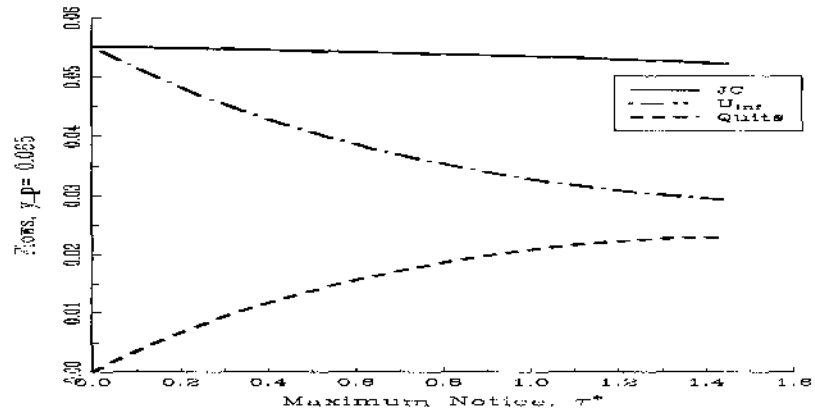
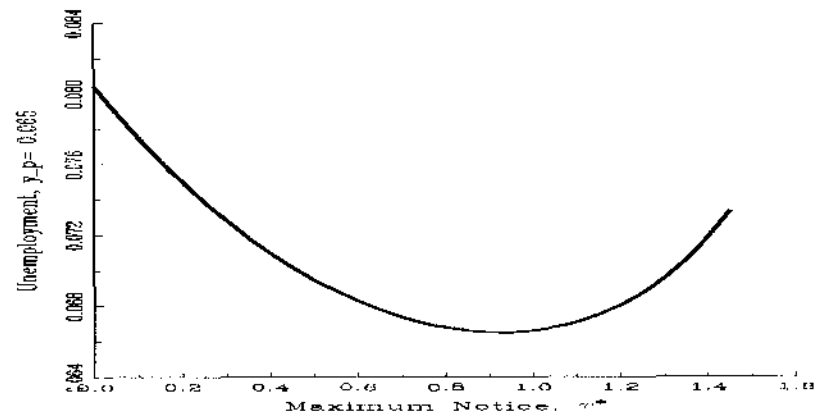


Figure 7. Unemployment for different notice times,  $\tau^*$







existence of bad jobs and advance notice, this result should not be surprising. Conversely, as the length of advance notice increases, unemployment inflows becomes smaller, and job-to-job movements increase. More formally, the effect of advance notice on turnover is driven by two opposing forces. On the one hand, an increase in the notice period  $\tau^*$  reduces  $\theta$ . Since job creation is given by  $\alpha(\theta)(u + n_b)$ , lower  $\theta$  tends to reduce turnover. On the other hand, an increase in the notice period increases  $n_b$ , the number of job-searching employees. The increase in  $n_b$  increases job to job movements, and since unemployment is approximately constant as  $\tau^*$  increases, the higher number of job seekers tends to increase turnover. As a result, the turnover in regulated markets can be as high as the turnover in unregulated market, while the corresponding unemployment flows much smaller.

Accordingly, the average duration of unemployment, which is equal to two quarters in an economy with no advance notice, increases to three quarters as  $\tau^*$  reaches 1.5. Even though advance notice does not effect the stock of unemployment, it unambiguously reduces the average duration of unemployment. Overall, results in Table 7 are consistent with cross country evidence provided in section II, and with the view that job-to-job movements are more important in European labor markets.

However, various simulation exercises suggest that the relationship between unemployment and notice length depends crucially on the productivity in the good jobs,  $y_g$ . Table 8 reports simulation statistics for unemployment and job flows for different productivity values. A relatively high value of  $y_g$  is consistent with a negative relationship between unemployment and  $\tau^*$ , whereas the opposite seems true when the value of  $y_g$  is relatively low. In previous theoretical and empirical studies (Bentolila and Bertola, 1990, Bentolila and Saint Paul, 1994, Lazear 1987) the relationship between unemployment and various measure of firing costs is overall ambiguous, and the simulations of this paper are consistent with this ambiguity, suggesting that the relationship is non linear. Finally, Table 7 suggests that advance notice substantially reduces wages in good jobs. As the firms anticipate future firing costs, the total surplus from the job is lower, and this is translated into lower wage at good jobs.

Table 8. Robustness Check

	notation	$\tau^* = 0.0$	$\tau^* = 0.3$	$\tau^* = 0.6$	$\tau^* = 0.9$	$\tau^* = 1.2$	$\tau^* = 1.50$
$y_p = 0.065$							
Unemp. (a)	$u$	8.0	7.3	6.8	6.7	6.8	7.5
Job Creat. (a)	$\alpha(\theta)(u + n_b)$	5.5	5.5	5.4	5.4	5.3	5.2
Unemp. Dur. (b)	$\frac{1}{q(\theta)}$	1.5	1.6	1.8	2.0	2.2	2.6
$y_p = 0.075$							
Unemp. (a)	$u$	7.3	6.4	5.7	5.3	4.9	4.8
Job Creat. (a)	$\alpha(\theta)(u + n_b)$	5.6	5.5	5.5	5.5	5.4	5.4
Unemp. Dur. (b)	$\frac{1}{q(\theta)}$	1.3	1.4	1.5	1.7	1.8	1.9
$y_p = 0.055$							
Unemp. (a)	$u$	9.0	8.5	8.5	9.2	11.1	22.6
Job Creat. (a)	$\alpha(\theta)(u + n_b)$	5.5	5.4	5.3	5.2	5.0	4.3
Unemp. Dur. (b)	$\frac{1}{q(\theta)}$	1.6	1.9	2.1	2.5	3.2	6.6
(a), Numbers are in percentage of the labor force							
(b), Numbers of quarters							
Source: Authors' calculation							

## VII. DISCUSSIONS

The influential work of Lazear (1987, 1990) has shown that side payments and contractual arrangements between a job-worker match could prevent deadweight regulation from having any effect on employment and wages. In this paper, as in most of the existing literature on firing costs,<sup>23</sup> we have assumed that regulation is binding, and we have studied the macroeconomic effects of a binding advance notice. In theory, as Lazear (1990) points out in his seminal paper, we should not expect any difference between advance notice and severance payments, since an employer could treat notice requirements as severance pay, simply by telling the worker not to report during the notice period, and paying him anyway. Our theoretical analysis ruled out this possibility, and assumed that the firm must necessarily keep the worker employed during the entire notice period.

However, in the cross-country analysis of Lazear, advance notice affects employment and labor force participation rates more than severance payments, suggesting that "something else may be going on"<sup>24</sup>. Furthermore, Burda (1992) and Lazear (1990) suggest that what matters for employment and wage determination is not severance payment per-se, but rather the difference between the firing cost incurred by the firm and the severance benefits received by the worker. In the model of Burda, such difference is referred to some general "red tape" costs that are dissipated or received by third parties. In reality, it is difficult to think to the effect of regulation on employment determination as univocally linked to the existence of a loosely defined "red tape" cost. Other studies, and Millard and Mortensen (1997) in particular, model employment

<sup>23</sup>Bentolila and Bertola, 1990; Bentolila and Saint-Paul 1994; Saint-Paul, 1995; Bertola and Caballero, 1994

<sup>24</sup>See Lazear (1990), pp. 712

protection legislation as a firing tax to be paid to third parties when firing takes place. However, looking at the existing labor market legislation, it is difficult to find the empirical counterpart of a pure firing tax.

If firms issue the advance notice only when conditions are bad, in way similar to what we described in the paper, the existence of advance notice introduces a deadweight loss in the job termination process. However, this deadweight loss is not due to the "red tape" cost mentioned by Burda (1992), nor to the "firing tax" analyzed by Mortensen and Millard (1997). In this paper, the deadweight loss is associated with the simultaneous existence of advance notice and a negative surplus in the bad jobs. With a simple severance payment, firing would take place as soon as the job turns bad, and there would be no deadweight loss associated with job security provisions. As a result, this paper has outlined a potential difference between notice requirements and severance payments on employment determination. In this respect, the fact that Lazear finds, empirically, a different employment effect between advance notice and severance payment could be linked to the mechanism outlined in this paper, namely the simultaneous existence of advance notice and a negative surplus in bad jobs. Of course, this mechanism would not exist if the firms could simply tell workers not to report for work, and paying them anyway during the notice period. The reason why this contractual arrangement does not happen in reality was not modeled in the paper, and it will be the focus of future research.

## VIII. CONCLUSIONS

This paper was motivated by two empirical findings on labor market flows across countries. The first observation concerns the behavior of unemployment flows. Unemployment inflows and outflows are much larger in North America than in Europe, and consequently, the incidence of long-term unemployment is much smaller in North America than in Europe. These marked difference in unemployment flows appear consistent with the view that high job security provisions in Europe reduce unemployment turnover, and increase the average duration of unemployment. The second observation concerns the behavior of job and worker turnover. On average, turnover in highly regulated European markets appear similar to the turnover of North American markets, suggesting that European labor markets create a number of jobs similar to the North-American markets. This paper has argued that the existence of advance notice in European labor market, along with other institutions that transform the job separation into a time-consuming process, can potentially be consistent with both sets of observations.

Time to fire in the job separation process appears an important determinant of the relationship between job security provisions, labor market flows and unemployment duration. Even though advance notice reduce firms' incentive to create new jobs, it

provides workers with an opportunity to switch to high productivity jobs without experiencing an unemployment spell. In a search unemployment environment, where jobs are subject to idiosyncratic uncertainty, firms would give advance notice also when business conditions are good, as a way to reduce expected firing costs if conditions turn bad. We have shown that such policy are not pursued in equilibrium only if giving notice entails sufficiently high productivity losses. Furthermore, we have shown that an increase in advance notice causes two effects on turnover in the labor market. On the one hand, higher advance notice causes firms to reduce the number of vacancies, thus inducing a reduction in turnover. On the other hand, higher notice increases the number of workers who search on the job, causing an increase in job turnover. As a result, turnover in high regulated market can be as high as turnover in unregulated market. However, an increase in advance notice reduces the number of people who enter and exit from unemployment, causing an overall increase in the average duration of unemployment.

### Some Comparative Static Results

An increase in  $\tau^*$  unambiguously reduces the value of a bad job  $F_b(\theta, \tau^*)$ . The value of a bad job at notice time  $h = 0$  reads

$$F_b(\tau, \tau^*) = \frac{y_b - b - \delta}{r + \alpha(\theta)} \left( 1 - e^{-(r+\alpha(\theta))\tau^*} \right). \quad (I)$$

Differentiating equation (I) with respect to  $\tau^*$  yields

$$\frac{\partial F_b(\theta, \tau^*)}{\partial \tau^*} = (y_b - b - \delta) e^{-(r+\alpha(\theta))\tau^*}. \quad (II)$$

Clearly

$$\text{sign}\left(\frac{\partial F_b(\theta, \tau^*)}{\partial \tau^*}\right) = \text{sign}(y_b - b - \delta).$$

Since  $y_b < (b + \delta)$ , it immediately follows that

$$\frac{\partial F_b(\theta, \tau^*)}{\partial \tau^*} < 0$$

An increase in  $r$  unambiguously increases the value of a bad job

Differentiating (I) with respect to  $r$  yields

$$\frac{\partial F_b(\theta, \tau^*)}{\partial r} = \frac{(y_b - b - \delta) \tau^* e^{-(r+\alpha(\theta))\tau^*} [r + \alpha(\theta)] - (y_b - b - \delta) (1 - e^{-(r+\alpha(\theta))\tau^*})}{[r + \alpha(\theta)]^2} \quad (III)$$

If we let  $\Gamma = (r + \alpha(\theta))\tau^*$ , where  $\Gamma > 0$  for positive notice time  $\tau^*$ , equation (III) can be written as

$$\frac{\partial F_b(\theta, \tau^*)}{\partial r} = \frac{(y_b - b - \delta)}{(r + \alpha(\theta))^2} (e^{-\Gamma}(\Gamma + 1) - 1) \quad (IV)$$

Since  $\Gamma \geq \ln(\Gamma + 1)$ , it follows that

$$(e^{-\Gamma}(\Gamma + 1) - 1) \leq 0 \quad (V)$$

Making use of (V) it immediately follows that

$$\text{sign}\left(\frac{\partial F_b(\theta, \tau^*)}{\partial r}\right) = \text{sign}(-y_b + b + \delta).$$

Since  $y_b < (b + \delta)$  it follows that

$$\frac{\partial F_b(\theta, \tau^*)}{\partial r} \geq 0$$

An increase in  $\theta$  increases the value of a bad job

Differentiating (I) with respect to  $\theta$  yields

$$\frac{\partial F_b(\theta, \tau^*)}{\partial \theta} = \frac{(y_b - b - \delta) q(\theta) (1 - \eta(\theta))}{(r + \alpha(\theta))^2} \left( e^{-(r+\alpha(\theta))\tau^*} (r + \alpha(\theta)) \tau^* - 1 + e^{-(r+\alpha(\theta))\tau^*} \right), \quad (VI)$$

where  $\eta(\theta) \leq 1$  is the elasticity of the matching function with respect to  $\theta$ . Since  $\Gamma = (r + \alpha(\theta))\tau^* > 0$ , and

$$(e^{-\Gamma}(\Gamma + 1) - 1) \leq 0,$$

it immediately follows that

$$\text{sign}\left(\frac{\partial F_b(\theta, \tau^*)}{\partial \theta}\right) = \text{sign}(-y_b + b + \delta).$$

Since  $y_b < (b + \delta)$ , it follows that

$$\frac{\partial F_b(\theta, \tau^*)}{\partial \theta} \geq 0.$$

### Productivity Losses and Advance Notice

This section provides necessary and sufficient conditions that insure that the strategy of issuing notice when conditions are bad is part of a fully consistent equilibrium. If we first assume that all firm issue notice when conditions turn bad, the surplus from a good job is

$$[r + \lambda + \beta\alpha(\theta)] W_g = y_g - b + \lambda \frac{y_b - b - \delta}{r + \alpha(\theta)} \left( 1 - e^{-(r+\theta q(\theta))r^*} \right). \quad (\text{VII})$$

Let us consider a match that unilaterally deviates from the strategy underlying equation (VII). At notice time  $h$ , the value of a good job with notice,  $\tilde{W}_g(h)$ , reads

$$\begin{aligned} (r + \lambda + \beta\alpha(\theta)) \tilde{W}_g(h) &= y_g - b - \delta + \lambda F_b(h) + \\ &+ \alpha(\theta) \left[ \text{Max}(\beta W_g, \beta \tilde{W}_g(h)) \right] - \alpha \beta \tilde{W}_g(h) + \frac{\partial}{\partial h} \tilde{W}_g(h). \end{aligned} \quad (\text{VIII})$$

We look for a set of necessary and sufficient conditions on the value of  $\delta$  that

$$\delta : W_g > \tilde{W}_g(h) \Big|_{h=0}. \quad (\text{IX})$$

Since the condition (IX) is part of the maximization in (VIII), we need to proceed in two steps. First, we assume that  $W_g > \tilde{W}_g(h)$  and we obtain a necessary condition on  $\delta$ . Second, we assume that  $W_g < \tilde{W}_g(h)$  and we obtain a sufficient condition on  $\delta$ . As we show below, the two conditions are not identical, and in the text we concentrate on the sufficient condition.

### Necessary Conditions

If we assume that  $W_g > \tilde{W}_g(h)$  equation (IX) reads

$$(r + \lambda + \beta\alpha(\theta))\tilde{W}_g(h) = y_g - b - \delta + \lambda \frac{(y_b - b - \delta)}{r + \alpha(\theta)} \left[ 1 - e^{(r+\alpha(\theta))(\tau^*-h)} \right] + \frac{\partial}{\partial h} \tilde{W}_g(h). \quad (X)$$

Since at  $h = \tau^*$ , the expected firing costs if conditions turn bad are zero, the boundary condition for solving equation (X) is

$$\tilde{W}_g(\tau^*) = \frac{y_g - b - \delta}{r + \lambda + \beta\alpha(\theta)} \quad (XI)$$

and the solution to the differential equation (X) reads

$$\begin{aligned} \tilde{W}_g(h) = & \frac{\lambda(y_b - b - \delta)}{(r + \lambda + \alpha(\theta)\beta)(\lambda - \alpha(\theta)(1 - \beta))} e^{-(r + \lambda + \alpha(\theta)\beta)(\tau^* - h)} + \frac{y_g - b - \delta}{r + \lambda + \alpha(\theta)\beta} + \\ & + \frac{\lambda(y_b - b - \delta)}{(r + \alpha(\theta))(r + \lambda + \alpha(\theta)\beta)} - \frac{\lambda(y_b - b - \delta)}{(r + \alpha(\theta))(\lambda - \alpha(\theta)(1 - \beta))} e^{-(r + \alpha(\theta))(\tau^* - h)}. \end{aligned} \quad (XII)$$

Differentiating equation (XII) with respect to  $h$  yields

$$\frac{\partial \tilde{W}_g}{\partial h} = \frac{\lambda(y_b - b - \delta)}{\lambda - \alpha(\theta)(1 - \beta)} \left[ e^{-(r + \lambda + \alpha(\theta)\beta)(\tau^* - h)} - e^{-(r + \alpha(\theta))(\tau^* - h)} \right] \geq 0. \quad (XIII)$$

Thus, when  $W_g > \tilde{W}_g(h)$ , issuing notice in good times implies a surplus that is increasing in notice time  $h$ . Making use of equations (19) in the text and (XII), the necessary condition on  $\delta$ , for  $W_g > W_g(h)|_{h=0}$  to be satisfied is simply

$$\delta_{nec}^* > \frac{\lambda(y_b - b - \delta_{nec}^*)}{\lambda - \alpha(1 - \beta)} \left[ e^{-(r + \lambda + \alpha(\theta)\beta)(\tau^* - h)} - e^{-(r + \alpha(\theta))(\tau^* - h)}, \right] \quad (XIV)$$

where  $\delta_{nec}^*$  is the cut-off value on  $\delta$ , and reads

$$\delta_{nec}^* = \frac{\frac{\lambda(y_b - b)}{\lambda - \alpha(\theta)(1 - \beta)} \left[ e^{-(r + \lambda + \beta\alpha(\theta))\tau^*} - e^{-(r + \alpha(\theta))\tau^*} \right]}{1 + \frac{\lambda}{\lambda - \alpha(\theta)(1 - \beta)} \left[ e^{-(r + \lambda + \beta\alpha(\theta))\tau^*} - e^{-(r + \alpha(\theta))\tau^*} \right]}. \quad (XV)$$

### Sufficient Conditions

To find a  $\delta_{suff}^*$  such that  $W_g > \tilde{W}_g(h)\big|_{h=0}$  we proceed as follows. We first assume that  $W_g < \tilde{W}_g(h)$ , we solve equation (VIII). We then obtain a value of  $\delta = \delta_{suff}^*$  that implies that  $W_g < \tilde{W}_g(h)\big|_{h=0}$ . Thus,  $\delta_{suff}^*$  is the cut-off point for the sufficient condition. If  $W_g < \tilde{W}_g(h)\big|_{h=0}$ , equation (VIII) reads

$$(r + \lambda)\tilde{W}_g(h) = y_g - \delta - b + \lambda F_b(h) - \alpha\beta W_g + \frac{\partial}{\partial h}\tilde{W}_g(h). \quad (\text{XVI})$$

At  $h = \tau^*$ , the expected firing costs are zero and equation (XVI) satisfies the following boundary condition

$$\tilde{W}_g(\tau^*) = \frac{y_g - b - \delta}{r + \lambda} - \frac{\alpha(\theta)\beta}{r + \lambda} W_g, \quad (\text{XVII})$$

and the solution to the differential equation is

$$\begin{aligned} \tilde{W}_g(h) = & \frac{y_g - b}{r + \lambda + \alpha(\theta)\beta} - \frac{\delta}{r + \lambda} + \frac{\lambda(y_b - b - \delta)}{(\lambda - \alpha(\theta))(r + \alpha(\theta))} e^{-(r + \lambda)(\tau^* - h)} + \\ & + \frac{\alpha(\theta)\beta\lambda(y_b - b - \delta)}{(r + \alpha(\theta))(r + \lambda + \alpha(\theta)\beta)(r + \lambda)} [1 - e^{-(r + \alpha)(\tau^* - h)}] - \frac{\lambda(y_b - b - \delta)}{(\lambda - \alpha(\theta))(r + \alpha(\theta))} e^{-(r + \alpha(\theta))(\tau^* - h)} + \frac{\lambda(y_b - b - \delta)}{(r + \alpha(\theta))(r + \lambda)} \end{aligned} \quad (\text{XVIII})$$

Differentiating equation (XVIII) with respect to  $h$  yields

$$\frac{\partial}{\partial h}\tilde{W}_g(h) = + \frac{\lambda(y_b - b - \delta)}{\lambda - \alpha(\theta)} \left[ e^{-(r + \lambda)(\tau^* - h)} - e^{-(r + \alpha(\theta))(\tau^* - h)} \right] \geq 0, \quad (\text{XIX})$$

which is strictly positive for any  $h \geq 0$ . Thus, even when  $W_g < \tilde{W}_g(h)\big|_{h=0}$ , the surplus in good jobs with notice increases in notice time. Making use of (XVIII),  $W_g < \tilde{W}_g(h)\big|_{h=0}$ , is equivalent to

$$\delta_{suff}^* \leq \frac{\lambda(y_b - b - \delta_{suff}^*)}{\lambda - \alpha(\theta)} \left[ e^{-(r + \lambda)\tau^*} - e^{-(r + \alpha(\theta))\tau^*} \right]. \quad (\text{XX})$$

Making use of equation (XX), the sufficient condition for having  $W_g > \tilde{W}_g(h)\big|_{h=0}$ , is a value of  $\delta > \delta_{suff}^*$ , where  $\delta_{suff}^*$  solves

$$\delta^* = \frac{\frac{\lambda(y_b - b)}{\lambda - \alpha(\theta)} \left[ e^{-(r + \lambda)\tau^*} - e^{-(r + \alpha(\theta))\tau^*} \right]}{1 + \frac{\lambda}{\lambda - \alpha(\theta)} \left[ e^{-(r + \lambda)\tau^*} - e^{-(r + \alpha(\theta))\tau^*} \right]}. \quad (\text{XXI})$$

Finally, making use of (XXI) and (I), is easy to show that

$$\delta_{suff}^* \geq \delta_{nec}^*$$



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