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Research Department

The Fundamental Determinants of the Real Exchange Rate
of the U.S. Dollar Relative to Other G-7 Currencies

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Abstract

This paper provides a consistent theoretical framework to explain the fundamental determinants of the evolution in the medium to longer run of the real effective exchange rate of the United States relative to the currencies of the other G-7 countries. The fundamental determinants are productivity and thrift in the United States and the other major industrial countries. The real rate generated by these fundamentals is referred to as the natural real exchange rate (NATREX). Then, using cointegrating and error correction analysis, the paper examines the explanatory power of the NATREX model to explain the evolution of the real exchange rate of the U.S. dollar during the floating exchange rate period.

JEL Classification Numbers:

F20, F21, F31, F32, F14, F34

1/ Brown University. This paper was initiated when I was a visiting scholar in the Research Department. I am indebted to Peter B. Clark, G.P. Galli, and Guay C. Lim, for valuable suggestions concerning the application of the NATREX model.

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Summary

This paper provides a consistent theoretical framework to explain the evolution from the medium to the longer run of the equilibrium real effective exchange rate of the U.S. dollar relative to the currencies of the other major industrial countries. The equilibrium exchange rate abstracts from the effects of speculative capital flows and changes in reserves and requires both internal and external balance. Internal balance means that the rate of capacity utilization is at its stationary mean value. External balance requires convergence of real long-term rates of interest. The fundamental determinants of the equilibrium real exchange rate, called the Natural Real Exchange Rate (NATREX), are productivity and time preference (an inverse measure of thrift) in the United States and the other major industrial countries. Time preference is measured by the ratio of the sum of private and public consumption to GNP. Productivity, which reflects the q -ratio, is measured by the growth rate.

The paper explains theoretically how rises in either the rate of time preference or the q -ratio appreciate the real exchange rate in the medium run. In the longer run, an increase in time preference raises the foreign debt and depreciates the real exchange rate below its initial level. The long-run effect of a rise in the q -ratio is ambiguous. It eventually leads to a lower debt, which tends to appreciate the exchange rate; however, it also raises capital and output, thereby increasing imports and depreciating the exchange rate. The theoretical model can be shown to correspond to an estimating equation for the real exchange rate with three parts. The cointegrating part is the longer-run effect. This part plus the error correction component reflect the moving equilibrium. The third part is the disequilibrium component when there is neither internal nor external balance.

Nonlinear least squares are used to estimate the equation explaining the real exchange rate. This estimated equation serves two purposes. First, it shows that the model has explanatory power. Second, when the disequilibrium components are set equal to zero--which serves as the calibration method--the result is an estimate of the equilibrium real exchange rate. Comparing the actual with the estimated equilibrium real exchange rate yields a measure of the degree of misalignment. Moreover, the causes of the misalignment are shown. Finally, the NATREX is compared with the FEER/DEER concepts of equilibrium exchange rates.

I. Introduction

"In a world where exchange rates can fluctuate by 2 percent per day and 20 percent per year, economists are asked to evaluate the causes and consequences of such fluctuations. If we are to go beyond the Panglossian response that 'the market knows best,' we need some concept of an equilibrium rate of exchange as a standard against which to measure actual exchange rate changes." (Black (1994), p. 279). The second amendment of the IMF's Articles of Agreement (1977) states that the Fund shall exercise firm surveillance over the international monetary system and members' exchange rate policies. To do so requires a consistent theoretical framework, which is empirically implementable, to explain the fundamental determinants of the evolution of the equilibrium real effective exchange rate in the medium to longer run.

This paper attempts to provide such a framework. It focusses on the real exchange rate of the U.S. dollar relative to the currencies the other G-7 countries. In the theoretical framework adopted here the fundamental determinants of the equilibrium real exchange rate are measures of productivity and thrift in the G-7 countries. The equilibrium real exchange rate generated by these fundamentals, when short-term speculative and cyclical factors are ignored, is referred to as the natural real exchange rate (NATREX). Section II summarizes the stylized facts and explains why there is a need for a new model to explain real exchange rates. Section III explains the NATREX approach, discusses the structural equations, and relates the NATREX equations to other optimization approaches. Section IV describes the solution of the model. There is a medium-run solution when foreign debt and capital are given, and its trajectory to the longer run when debt and capital evolve endogenously. This explains why the real exchange rate is not stationary. Section V describes the data and the econometric results and Section VI shows how the NATREX explains the econometric results. Section VII shows (a) to what extent the actual real value of the U.S. dollar deviates from its medium-run and longer-run equilibrium values, and (b) what were the factors producing the medium- to the longer-run trends in the real value of the U.S. dollar. The NATREX analysis thus provides a response to the issues posed in the opening paragraph.

The NATREX is a benchmark for the medium- to longer-run equilibrium real exchange rate. We use the NATREX to estimate "misalignments" and to infer whether the observed real exchange rate movements are transitory or longer lasting. The NATREX, unlike Williamson's (1994) FEER or the DEER of Bayoumi et al. (1994), is a positive rather than a normative concept.

The NATREX is the real exchange rate which equates the current account to ex ante social saving less social investment generated by the fundamentals, productivity and thrift, when the U.S. fundamentals are corrected for cyclical variations in the rate of capacity utilization and incomplete adjustments in asset markets. Cyclical elements represented by the variation in the rate of capacity utilization from its stationary mean,

or deviations of the unemployment rate from the equilibrium rate of unemployment, are not considered among the sustainable fundamentals because they average out to zero. Speculative capital flows and changes in reserves are not considered in the determination of the NATREX because they are not sustainable, and have only short-run, but not medium-run effects on the real exchange rate. The excess of saving over investment, under these conditions, represents a capital outflow. The NATREX equates the current account to the nonspeculative capital outflow, evaluated at capacity output and when there is a complete adjustment in asset markets.

The real exchange rate $R(t)$ used here is the real effective exchange rate shown in equation (1) below, which is measured by the ratio of normalized unit labor costs, in a common currency, in the main tradable sector (manufacturing) in the United States relative to the other G-7 countries. The nominal effective exchange rate is denoted by $N(t)$, $w(t)$ represents U. S. normalized unit labor costs and $w'(t)$ is the counterpart in the rest of the G-7. A prime refers to the foreign variable. A rise in the exchange rate signifies an appreciation of the U.S. dollar. Equation (1) below thus defines the actual real exchange rate.

We show below that the NATREX at any time is a function of the capital intensity, $k(t)$, the foreign debt intensity, $F(t)$, and the fundamentals denoted by the vector $Z(t)$ of productivity and thrift in the United States and the other G-7 countries. The NATREX is represented by $R[k(t), F(t); Z(t)]$. The actual real exchange rate, $R(t)$, can be decomposed into three parts in equation (1a).

$$R(t) = N(t) w(t)/w'(t) \quad (1)$$

$$R(t) = [R(t) - R[k(t), F(t); Z(t)]] + [R[k(t), F(t); Z(t)] - R^*(Z(t))] + R^*(Z(t)) \quad (1a)$$

The first term, which is denoted by $e(t)=[R(t) - R[k(t), F(t); Z(t)]]$, refers to the transitory short-term factors. It is the deviation of the actual real exchange rate $R(t)$ from the NATREX. The deviation results from speculative forces, cyclical factors or from incomplete adjustments in asset markets. 1/ Our hypothesis is that the expectation of this deviation is zero, i.e., $E[e]=0$, so that the real exchange rate converges to the NATREX.

The fundamentals of productivity and social thrift, denoted by $Z(t)$, affect the NATREX directly, given endogenous capital and foreign debt, and also affect the rate of capital formation and the current account. The induced evolution of capital and debt produce a trajectory of the NATREX to a steady state $R^*(Z(t))$ conditional upon the fundamentals. The trajectory of the NATREX to the steady state is the second term, $[R[k(t), F(t); Z(t)] - R^*(Z(t))]$, and the steady state is the third term, $R^*[Z(t)]$. The actual real exchange rate moves over the medium run to the NATREX, and the latter moves over the long run to the steady state value.

1/ The speculative forces include anticipated changes in monetary policy.

The NATREX model does not address itself to the question of how much of the variation in the real exchange rate will occur through variations in the nominal exchange rate and how much through relative unit labor costs. The deviation $e(t)$ representing the short-run speculative and cyclical factors will go to zero but the model does not specify whether it will do so with a change in nominal exchange rates or from changes in nominal unit labor costs via differential wage inflation.

II. What Is To Be Explained

1. Stylized facts ^{1/}

There are several stylized facts that motivate the construction of the NATREX model. They explain why we do not use either the PPP hypothesis, the model with monetary dynamics and rational expectations, or the representative agent-intertemporal optimization model to determine what is the equilibrium real exchange rate. Many of these stylized facts are described in terms of their stationarity properties. A stationary variable is defined as one which reverts to a constant mean, i.e., whose mean and variance are finite and are independent of time. Ten stylized facts are considered below:

a. The real effective exchange rate of the U.S. dollar relative to the other G-7 currencies (USREUG7 in Figure 1), as well as the overall U.S. real effective exchange rate (USREU), are not stationary, as shown by the Dickey-Fuller (DF) and the augmented Dickey-Fuller (ADF) statistics in Table 1. Figure 1 plots, in normalized form, the real effective exchange rate (USREUG7), the nominal effective exchange rate (USNEUG7), and purchasing power parity (PPP) defined as the ratio of the G-7/U.S. GDP deflators. It is seen that the real and nominal rates move together and that the nominal value of the dollar is not related to the PPP during the floating rate period.

b. The U.S. real long-term interest rate (USRLT) and the real long-term interest rate in the rest of the G-7 (G6RLT) converge to each other. These real interest rates are defined as the nominal rate on long-term bonds less the change in the GDP deflator over the previous four quarters (see Figure 2). The real long-term interest rate differential (USRLT-G6RLT) is stationary with an expected value of zero.

^{1/} All of our data are from the International Monetary Fund, International Financial Statistics and the IMF Surveillance Data Bank (SDB) unless otherwise noted. In the IFS, the real effective exchange rate is denoted REU, and the nominal NEU. In the econometric work the real effective exchange rate relative to the other G-7 currencies is referred to as USREUG7.

Table 1. Description of the Data, Stationarity or
Nonstationarity Properties of Basic Variables:
DF/ADF Statistics

Variable	ADF/DF	
	I(1)	I(0)
(1975.1-1993.3)		
Real exchange rate (USREUG7)	(C,0) = -0.97	
U.S.-G-6 real long-term interest		(N,0) = -2.0*
U.S. time preference:		
moving average (MADISRAT)	(C,2) = -1.31	
U.S. growth		
moving average (MAUSGROW)		(C,2) = -3.8**
G-7 time preference		
moving average (MAG7DIS)		(C,2) = -2.85
G-7 growth		
moving average (MAG7GROW)		(C,1) = -3.87**
current account/GDP	(C,1) = -1.97	
U.S. net foreign assets/GNP	(C,1) = -1.13	
U.S. rate of capacity		
utilization (USCUR)		(C,1) = -3.39**
(1973.1 - 1994.1)		
Short-term capital flows		
total: (US77gd)		(C,1) = -3.2**
deposit money banks		(N,1) = -4.4**
Portfolio plus direct investment:	(C,1) = -1.76	
U.S. terms of trade		(C,1) = -3.27**
Real effective exchange rate		
(USREU)	(C,1) = -1.37	

Note: MacKinnon critical value, significant at 5 percent (*); 1 percent (**). The C or N indicates whether a constant (C) was or was not (N) used, and the integer refers to the number of lags. Variables, and line in IFS: four-quarter moving averages are denoted by prefix MA, and are used for USDISRAT (MADISRAT) and USGROWTH (MAUSGROWTH). (i) The index of time preference was derived as follows: call DISRAT or time preference the sum of total private (US96fc) plus public consumption (US91ffc-US93gfc) to GDP(US99bc). A four-quarter moving average is MADISRAT. For the G-7, the time preference G7DISRAT is government expenditures national income accounts plus private consumption divided by GDP. The four quarter moving average is MAG7DIS. (ii) The growth variables were growth of real GDP over the past year. For the United States we used a four-quarter moving average (MAUSGROW). (iii) The total short-term capital flow is US77gd, the component involving bank liabilities, called deposit money banks, is US77gbd. (iii) portfolio plus direct investment is (US77bbd + US77bad). (iv) The terms of trade are US74/US75. The foreign interest rate and price deflators refer to the G-7 less the United States. The foreign variables from national income accounts refer to all the G-7.

Figure 1.

United States: Nominal and Real Effective Exchange Rate
and Purchasing Power Parity

Figure 2.

Long-Term Real Interest Rates: United States vs. Other G-7 Countries
(In percent)

c. The ratio of net foreign assets to GNP is not stationary. 1/ The net foreign asset position is defined as total U.S. investment abroad less total foreign investment in the United States. The United States went from a creditor country with net foreign assets of 6.56 percent of GNP in 1950 and 10.66 percent in 1970, to a debtor position of -7.66 percent in 1990.

d. The ratio of the U.S. current account/GNP, or a four-quarter moving average of the current account/GNP, is not stationary. This nonstationarity is consistent with the stylized fact that countries vary their net creditor/debtor positions.

e. As found by Faruquee (1995), the net foreign asset position of the United States appears to be a significant determinant of the U.S. real exchange rate.

f. U.S. short-term capital flows are stationary with an expected value of zero. We use two measures of these flows. One is the balance of payments component described in the IFS as "other capital, deposit money banks" (US77gbd), which represents private capital flows reported by banks, and the second is the total short-term capital flow "other capital nie" (US77gd). 2/ Both are stationary and the expected value of the deposit money bank flow is zero.

g. The sum of portfolio investment (US77bbd) and direct investment (US77bad), and as a proportion of GNP (US99ac) are not stationary; their means are not constant over time.

h. During the period 1973:1-1994:1, the U.S. terms of trade were stationary, whereas the U.S. real effective exchange rate (USREU) was not stationary. Hence, the U.S. terms of trade (export prices/import prices) behave differently over time than the U.S. real effective exchange rate.

i. The U.S. ratio of private consumption plus total government consumption expenditure to GDP is not stationary. This ratio is referred to as the U.S. rate time of preference and a four-quarter moving average of this variable is labelled MADISRAT in Table 1. It does not have a mean that is independent of time during the period 1973:1-1994:2. The corresponding ratio in the other G-7 countries, called G-6 time preference and labelled MAG6DIS, is also not stationary.

j. The nominal deutsche mark/U.S. dollar is not cointegrated with the ratio of M3 to output in Germany divided by the ratio of M2 to output in the United States. 3/

1/ This variable is taken from Masson, et al. (1993), Table 4. Their annual data are for 1950-90.

2/ Unless otherwise noted, the term in parenthesis is the IFS mnemonic.

3/ The ADF (C,0) = -1.88 is not even significant at the 10 percent level. See Stein and Sauernheimer (1995).

2. The inadequacies of the existing models

There has been a large number of studies which have attempted to explain the substantial exchange rate movements observed in the post-1973 period. By now there is overwhelming evidence that the standard models are not able to explain the exchange rate movements of the G-7 countries. The two most popular models at present are (i) monetary dynamics with rational expectations, or (ii) the intertemporal optimization by a representative agent who makes a simultaneous saving-investment decision to optimize utility over an infinite horizon, subject to a constraint that the present value of the terminal debt be zero.

It is well-known that the model of monetary dynamics/rational expectations has foundered because the crucial structural equations are inconsistent with the evidence. These structural equations concern the stationarity of the real exchange rate and the assumptions of uncovered interest rate parity and rational expectations. The implied reduced form is that variations in the nominal exchange rate are primarily explained by variations in the current and rationally expected ratio of money to output.

The traditional PPP hypothesis assumes that the stable equilibrium real exchange rate is constant over time. Statistically, this implies that the real exchange rate is stationary at a constant (time invariant) mean, R^* , and that the actual real exchange rate converges to this constant mean in the medium run. This hypothesis is inconsistent with stylized fact (a), which is why PPP has failed to explain observed variations in the U.S. nominal exchange rate in the floating rate period. 1/ Stylized fact (j) indicates that a reduced form equation cannot explain movements in the nominal deutsche mark/dollar exchange rate by relative money stocks per unit of output over a longer period. These two variables are not cointegrated. Moreover, there is strong evidence that short-term capital flows are just

1/ See Breuer (1994), for a critique of the econometric analyses of PPP, and also Figure 1. Also see Clark et al. for a critique of PPP as an equilibrium rate of exchange.

noise, not reflections of rational expectations. 1/ The monetary dynamics/rational expectations models are inconsistent with stylized facts (a), (f), and (j).

The monetary dynamics model has been supplanted in the theoretical literature by a second type of model which we refer to as the representative agent intertemporal optimization models (RAIOM). These newer models make very restrictive and arbitrary assumptions. They do not lend themselves to straightforward objective empirical testing because their crucial variables are subjective anticipated or unanticipated productivity shocks. The few attempts to apply the RAIOM to explain the data have been unsuccessful. Rogoff (1992), has made one of the few attempts to see if these models have explanatory power. In Rogoff's study, the central proposition (p. 11) is that the percentage change in the real exchange rate from one period to the next is a linear combination of the unexpected shock to productivity in the traded goods sector, with a positive sign, and the unexpected shock to the productivity in the nontraded goods sector with a negative sign. The crucial variables, which are proxies for anticipated or unanticipated productivity shocks, turn out to be statistically insignificant and the one easily-measured variable--government consumption spending--has the wrong sign. 2/ Therefore, the RAIOM models have not been successful in explaining movements in the real exchange rate.

A major implication of the RAIOM is that the current account depends upon the deviations of interest rates, output, government spending and investment from their respective permanent levels. Since these deviations are supposedly stationary with an expectation of zero, the current account should be stationary. This hypothesis is rejected by stylized facts (c)-(e) and (g) above.

1/ The uncovered interest rate parity theory with rational expectations (UIRP/RE) is rejected for all major currencies and it cannot be rescued by invoking a nontautological risk premium. See Stein (1990), the Reserve Bank of Australia study by Blundell-Wignall, et al. (1994), and the Bank of Canada study by Boothe, Clinon, and Cote (1985). The UIRP/RE assumption has been one of the foundations of the models of monetary dynamics with rational expectations of the exchange rate. The Reserve Bank of Australia study summarized the literature in the following way. "No economic hypothesis has been rejected more decisively over more time periods, and for more countries than UIP[UIRP/RE]...Many researchers interpret rejection of UIP as evidence of a time varying risk premium, while still maintaining the assumption of rational expectations. However, as the risk premium is then typically defined to be the deviation from UIP, this interpretation is merely a tautology."

2/ The theoretical contribution of the RAIOM is that these anticipations are "forward looking," but the empirical measures of anticipations are just backward looking moving average processes (Rogoff: 1992, 24-25), which are just adaptive expectations.

Obstfeld and Rogoff (Section 4.2.2) are cognizant of the failures of these models. They wrote: "...it is unclear whether the intertemporal approach is simply false, or whether the many extraneous simplifications and maintained hypotheses imposed by the econometricians are to blame." I do not believe that the failure of these models to explain empirical reality is due to the demands of the econometricians. On a very general level the implications of the RAIOM are incredible. Using "reasonable" estimates of parameters, Obstfeld and Rogoff (Section 3.1.1) find that the theory implies that the steady state ratio of debt/GNP for a small open economy is 2000 percent. This incredible result is not the fault of the econometricians. We have therefore taken a less restrictive and more general approach than the standard models in our attempt to explain the behavior of the real exchange rate.

III. The NATREX Approach

Our optimization and structural equations are more general than the standard approach. First, we have independent saving and investment equations. There is no "representative agent" that describes a country, or whose decisions at time zero describe the pattern of consumption and saving over an infinite horizon. There is a market mechanism operating via the real exchange rate, that produces the ex post equality between social saving and the sum of investment and the current account. The "representative agent" models have no such market mechanism. Second, our optimization is based upon the view that agents know that they do not and cannot know the evolution of the fundamentals and do not have perfect knowledge of the structural equations of the system. As the real exchange rate and the underlying fundamentals vector Z are not stationary, as shown in Table 1, one cannot predict the values of $Z(t)$ in the future. At each point in time, our agents use all available information efficiently and there is a feedback control mechanism which ensures that the economy converges to the unknown and changing optimal steady state which depends upon the fundamentals. We explain the evolution of the debtor/creditor position over time to a dynamically stable steady state, where the trade balance is sufficient to pay the interest on the debt. When the ratio of net foreign assets/GDP converges to a constant steady state value, the present value of the steady state net foreign assets/GDP goes to zero as time goes to infinity. This requirement will be the NATREX "intertemporal budget constraint."

1. The structural equations

Equations (2)-(7) below describe the NATREX model for large economies. 1/ The endogenous variables are defined as follows, where a prime refers to the foreign country: R =real effective exchange rate,

1/ As the NATREX model and detailed optimization are discussed in Stein (1994), and Stein, Allen et al. (1995), our discussion here can be relatively terse and intuitive.

k (k') = capital intensity; F = foreign debt intensity (positive for debtor, negative for creditor), r (r') = real domestic long-term interest rate, C = social consumption per unit of effective labor and S = social saving/effective labor, $y = y(k; Z)$ capacity output/effective labor. Investment I is dk/dt . The current account deficit $-CA = Df/dt$, is the rate of change of the foreign debt. 1/ All variables except the exchange rate, interest rate and prices are measured per unit of effective labor. The exogenous variables are Z , the fundamentals: k' = G-6 capital intensity, productivity of capital is (u, u') , index of time preference is (g, g') .

a. Internal equilibrium in the goods market

In the NATREX model, investment less social saving ($I-S$) determines the nonspeculative capital flow (L). Investment depends upon the Keynes-Tobin q -ratio. Social saving depends upon wealth and time preference. In turn, investment less saving is determined by the capital intensity k , the debt intensity F , and the fundamentals Z . The equilibrium real exchange rate, R , is determined such that the current account $CA = N(R, k, F; Z)$, plus the nonspeculative capital flow $L = (I-S)(k, F; Z)$ sum to zero, at capacity output, as shown by equation (2). We ignore speculative factors in the determination of medium to long-run equilibrium. 2/

We write this equilibrium condition equating the current and capital accounts separately for the United States in equation (2) and for the rest of the G-7 in equation (3). The foreign variables are denoted by a prime.

$$(I - S)(k, F; Z) + N(R, k, F; Z) = 0 \quad (2)$$

$$(I' - S')(k', F; Z) - N(R, k, F; Z) = 0 \quad (3)$$

Equations (2)-(3) can be expressed in terms of the balance of payments. There is a set of real exchange rates and real long-term rates of interest which equates the current account to the capital flow $S-I$ in both the United States and the rest of the G-7. We shall refer to equation (2), graphed in the (R, r) space as the IX curve, and equation (3), graphed in the (R, r') space as the IX' curve in Figure 4. Additional equations involve the determinants of investment (7), saving (6), capital flows (8), and the current account $N(k, F, Z)$ in (2) and (3).

b. Portfolio balance: interest rate parity

The interest rate parity theory is that the real interest rate differential will be such that investors are indifferent between domestic and foreign assets. Investors have long horizons and contemplate both

1/ We measure the debt in U.S. dollars, and hence ignore changes in the value of the foreign debt due to changes in the exchange rate.

2/ Our concept of the equilibrium exchange rate is similar to Nurkse's. See also the discussion by Clark et al. (1994).

direct and portfolio investment. The expected real return on U.S. long-term assets is the real long-term rate of interest, r , plus the expected long-term real appreciation of the U.S. dollar, which is the average annual expected change in the exchange rate over a long horizon, denoted $E(DR)$. The expected return in the rest of the G-7 is the real long-term interest rate r' . Using all available information, the investors are assumed to know from Table 1 that the real effective exchange rate is integrated of order 1 such that the first differences are stationary. The equilibrium real exchange rate reflects the properties of the underlying fundamentals. As the fundamentals have been shown to be integrated $I(1)$, the real exchange rate is also $I(1)$. Therefore, in making their long-term investment decisions, investors in the United States and the foreign countries are assumed to know that $E(DR) = 0$ and therefore base their investment decisions only the real long-term interest rate differential $(r-r')$. Short-run exchange rate expectations are irrelevant for long-term investment decisions. Using all available information that the fundamentals and real exchange rate can be described as martingales $R(t) = R(t-1) + e(t)$, $E(e) = 0$, portfolio balance of long-term investment implies the convergence of real long-term rates of interest, $r=r'$.

$$r = r' \quad (4)$$

Figure 2 compares the foreign $\underline{1/}$ (G6RLT) and U.S. (USRLT) real long-term interest rates. As is clear from stylized fact (b), Figure 2, Table 1, the differential $(r-r')$ is stationary and its expected value is zero. The portfolio balance equation (4) implies that the foreign real long-term rate and the USRLT converge to each other, which is shown by the intersection of the IX and IX' curves in Figure 4. The world real rate of interest $r=r'$ is such that world saving less investment $(I-S) + (I'-S') = 0$ is zero, from equations (2)-(3).

c. Independent investment and saving decisions

In the NATREX model, firms make the investment decisions and households and government make saving decisions. We start with the fact that the crucial fundamentals $Z(t)$ are $I(1)$, such that there is no stationary distribution function and the uncertainty increases as one looks further into the future. The future is characterized by uncertainty rather than risk, in the sense of Frank Knight. $\underline{2/}$ The agents change over time with new generations, waves of immigration and new governments. In each case, the relevant agents, including the government, are assumed to adopt an optimizing feedback control algorithm that assures stability and convergence

$\underline{1/}$ Foreign refers to the rest of the G-7.

$\underline{2/}$ See Cartapanis (1994) pp. 107-14.

to the unknown steady state values. This is a major difference from the RAIOM discussed above. 1/

(1) The consumption and saving functions

The NATREX uses a consumption or saving function based upon intertemporal optimization which implies an intertemporal budget constraint that the foreign debt intensity stabilizes even though the future evolution of income is unknown. In our model, consumption and saving include both public plus private sector. From standard optimization theory, social consumption C is proportional to permanent income Y^* . The latter is based upon the expectation that current income will grow at the rate of growth of effective labor n , and it is discounted at the real long-term rate of interest r . This implies that social consumption is proportional to current capacity output $y=y(k;Z)$, where k is the capital intensity. Included in the exogenous vector Z is a parameter of productivity. 2/ This is the first part of consumption equation (5).

$$C/y = g - hF/y \quad (5)$$

When there is no foreign debt, the coefficient g is the ratio of private plus public consumption to capacity output. This is what we call time preference and reflects social tastes. A rise in g can occur in several ways: (i) a rise in the cyclically-adjusted government expenditures, not offset by a decrease in private consumption; (ii) a rise in private consumption induced by a tax cut, not offset by a decline in government expenditures; (iii) a change in the demographic composition of the population toward groups whose consumption to income ratio is high (the very young, and very old). Stylized fact (i) and Table 1 show that the ratio of social consumption to GNP is not stationary.

When the foreign debt rises to a level that the government realizes is unsustainable, then it changes its fiscal policy to reduce the deficit or increase government saving. This is a feedback control that is reflected in coefficient h . All of the decisions are based upon current measurements of

1/ The theory of the RAIOM is that agents are "forward looking." In the empirical work, the anticipations are based upon autoregressions (Rogoff, 1992, pp. 22-23). Hence in the empirical application of the RAIOM the anticipations is simply adaptive.

2/ The derivation is as follows, where n is the expected growth of effective labor and $r > n$ is the real rate of interest. Y^* is permanent income, and $y(k;u)$ is capacity GDP. $C(t) = c^* Y^* = c^* \int_0^\infty e^{-(r-n)v} dv = g y(t)$ where $t < v < \infty$. Parameter $g = c^*/(r-n) > 0$ is our measure of time preference, and c^* is derived from the underlying utility functions.

variables in a way that is guaranteed to drive the system to the steady state, regardless of the disturbances. 1/

Social saving is GNP less public plus private consumption. The general consumption function (5) implies saving function (6).

$$S = (1-g) y(k;Z) + (h - r) F = S(k,F;Z) \quad h > r, S_k > 0, S_F > 0 \quad (6)$$

Thus, social consumption, via the government budget deficit, is negatively related, and the social saving is positively related, to the foreign debt, $dS/F > 0$. This will satisfy our intertemporal budget constraint if feedback control h is greater than the real rate of interest r . A rise in capital increases wealth and permanent income, and we expect that to raise saving, $DS/dk > 0$.

(2) Investment equation

Equation (7) below is the independent investment equation. A similar investment equation applies abroad. In the standard optimal growth models, the Maximum Principle is used to derive the optimal rate of investment. This is an open loop control where we must have perfect knowledge. We must know with certainty the steady state capital intensity, denoted k^* , and the production function. Then the optimal growth path is a saddle point path. The slightest error in our knowledge, the slightest deviation from the path, will send the economy off on an errant trajectory; and the economy will never reach the steady state.

For this reason, Infante and Stein (1973), developed a closed loop suboptimal feedback control (SOFC) based upon dynamic programming. We proved that if the rate of investment is a nonlinear function of the difference between the current marginal product of capital and the social discount rate, then the trajectory of the rate of investment will be very close to the unknowable optimal trajectory. Our SOFC is guaranteed to home in on the unknowable optimal steady state capital intensity, and will be extremely close to the unknown optimum path. We have synthesized the optimal control for the realistic situation of imperfect knowledge and where the basic parameters are changing in an unknowable way. All that we require are observable measurements of the marginal product of capital.

1/ The hF/y term is missing from the RAIOM and, hence, there is no way to achieve the intertemporal budget constraint without perfect foresight. The budget constraint is simply assumed without any mechanism to produce it. We have a built in feedback control to achieve stability. In our case, as will be seen below, the rate of change of the foreign debt $dF/dt = I - S$. Using the saving function (6), we obtain: $dF/dt = I - (1-g)y + (r-h)F$. The stability comes from $\delta(dF/dt)/\delta F = (r-h) < 0$. A debt crisis will emerge if $r > h$. Eventually, governments take actions to increase h to exceed r . See Boltho (1994).

Infante and Stein analyzed the situation for an economy where the saving and investment decisions are identical. When saving and investment decisions are made independently, as in the NATREX model, and when the capital good is not the same as the output, then Stein (1994) and (1995, Chapters 2-3) made two changes in the SOFC. The optimal rate of investment is a nonlinear function of the Keynes-Tobin q-ratio based upon current measurements of variables, $dk/dt=J(q)$ in equation (7).

$$dk/dt=I= J(q) = J(k;Z) \quad J_k < 0 \quad (7)$$

The q-ratio is based on current measurements. The q-ratio is positively related to the current marginal physical product of capital $y'(k(t);Z(t))$, negatively to the current real rate of interest r , and to external factors such as the price of imported materials, contained in Z .

The rate of investment dk/dt will be positively related to the q-ratio so defined. When $q>1$, the rate of change of the capital intensity will be positive, and when $q<1$ the rate of change of the capital intensity will be negative. In the steady state, the capital intensity $k=k^*$, such that $q=1$. This function is designed to produce convergence to the unknown and changing optimal trajectory.

d. The rate of change of the foreign debt and the current account

The rate of change of the foreign debt, Df/dt , is given by equation (8), which is equal to investment less saving and equal to the current account deficit.

$$Df/dt = J(k;Z) - S(k,F;Z) = L(k,F;Z) \quad L_k=J_k-S_k<0; \quad L_F = -S_F<0 \quad (8)$$

Foreign debt is an endogenous variable, and countries may and do change from debtor to creditor and vice versa. The NATREX intertemporal budget constraint is that the value of the foreign debt converges to a steady-state value F^* . The system is dynamically stable: the debt does not explode. In the steady state, where the debt is constant, investment less saving equal to the current account deficit is equal to zero. Then, the trade balance B^* must equal the interest payments rF on the foreign debt.

The current account is the trade balance less interest payments on the foreign debt. The trade balance depends negatively upon the real exchange rate, positively upon foreign income and negatively upon domestic real GNP equal to $y(k;Z) - rF$, which is GDP less interest payments rF . We may therefore write the current account $CA=N(R,k,F,r;Z)$ in equations (2) and (3).

Equation (2) states that the real exchange rate adjusts to make the current account equal to saving less investment (S-I). Therefore the ex post current account will be as forward looking as are the saving and investment equations above.

IV. Solution of the Model

The equilibrium real exchange rate and real rates of interest are simultaneously determined in equations (2)-(4), using (5)-(8), as functions of capital, debt and the exogenous variables Z. This is described in Figure 4. The evolution of capital and debt are described by equations (7)-(8), described in phase diagram Figure 5. Debt and capital feed back on the real exchange rate described in Figure 4. This dynamic system is the NATREX model, which explains the medium to longer-run evolution of the real exchange rate in equation (1a).

The system can be analyzed graphically. 1/ The IX and IX' curves in Figure 4 are the sets of real effective exchange rates R and real rates of interest (r, r'), which equate the current account to the capital account, which is determined by planned saving less investment at capacity output in each country. The IX curve for the U.S. relates to R and r , and the IX' is related to R and r' in the rest of the world. Output in each country is equal to capacity output. The IX curve is negatively sloped because a rise in the real rate of interest r raises saving less investment (S-I), which is the capital outflow. To maintain the equality of the capital flow to the current account evaluated at capacity output, the real effective exchange rate must depreciate (decline) and thereby increase the current account. The foreign IS' curve is positively sloped because a rise in R is a depreciation of the foreign currency.

Given the IX and IX' curves, there will be portfolio adjustments between U.S. and foreign long-term assets when real long-term interest rates differ. At real exchange rate $R(B)$, the U.S. real rate of interest at point B is less than the foreign rate at point b . The excess demand for foreign assets and excess supply of U.S. assets will lead to a rise in the U.S. rate, a decline in the foreign rate and a depreciation of the dollar. The medium run NATREX is at point H in Figure 4, and is equation (9), and the world rate of interest is equation (10). At that point, in both the United States and the foreign countries, the current account plus the capital account is zero, world saving equals world investment, and there is portfolio balance such that real long-term rates of interest have converged. Neither country can control its real exchange rate or real long-term rate of interest; they are simultaneously determined.

1/ A mathematical solution is in Stein, Allen, et al. (Chapter 2), and Stein (1994), so a graphic solution will suffice here.

$$R(t) = R[k(t), F(t); Z(t)] \quad (9)$$

$$r(t) = r[k(t); Z(t)] \quad (10)$$

The foreign debt affects the real exchange rate in (9), but it does not affect the world rate of interest in (10) because the debt does not affect world saving and world investment.

The rate of change of the real exchange rate is equation (11), which depends upon the evolution of capital and debt. The rate of change of the debt is the current account deficit.

$$dR(t)/dt = R_k dk(t)/dt + R_F dF(t)/dt + R_Z dZ(t)/dt \quad (11)$$

When there is balance of payments equilibrium and portfolio balance at point H in Figure 4, the q-ratio need not be equal to unity, nor need investment less saving (equal to the capital inflow) be equal to zero. In that case, the capital and foreign debt will vary according to equations (7)-(8). These variations, as well as variations in the fundamentals, produce movements in the IX curve. Hence, the medium run NATREX in equation (9) is not stationary, and neither will be the observed real effective exchange rate.

Figure 5 describes equations (7) and (8), the endogenous movements in capital and debt. We have substituted the world rate of interest in equation (10) into the q-ratio. Hence the q-ratio depends upon capital $k(t)$ and exogenous forces $Z(t)$. The curve $dk/dt = J(k^*; Z) = 0$ indicates the capital intensity k^* , where the q-ratio is unity. Then the marginal return on capital is equal to the world real rate of interest given by equation (12) below. For k below (above) k^* , the q-ratio is greater (less) than unity. The capital stock converges monotonically to k^* in the direction of the horizontal vectors. This is the substance of equation (7).

$$J(k^*; Z) = 0 \quad (12)$$

The curve $L=0$ described by $Df/dt = L(k, F; Z) = 0$ is the relation between debt and capital such that investment less saving, which equals the capital account, is equal to zero i.e., $J-S=L=0$. Along the $L=0$ curve, the current account is zero, and the foreign debt is not changing. A rise in the foreign debt above the $L=0$ curve reduces wealth, which reduces consumption and raises saving. The rise in saving less investment produces a capital outflow, which reduces the foreign debt back to the $L=0$ curve. The vertical vectors describe the movement of the foreign debt. The element of stability $d(Df/dt)/Df < 0$, derived from our saving function, produces our intertemporal

budget constraint whereby the debt stabilizes at a value along the $L=0$ curve.

If, at the medium run NATREX point H in Figure 4, (i) the q-ratio exceeds unity and (ii) saving less investment (equal to the current account) is positive, then capital will increase and debt will decrease along one of the trajectories such as E0-E1 to the steady state at E1 in Figure 5. At the steady state position the following holds: the q-ratio is equal to unity, such that the marginal return on capital is equal to the world rate of interest; investment less saving, i.e., the current account, is zero; and the trade balance is equal to the interest payments. A zero current account equal to $S-I=0$, is described by equation (13), where the asterisk denotes a steady state value. ^{1/}

$$L(k, F^*; Z) = 0. \quad (13)$$

We may solve (9), (12), and (13) for the steady state values of the real exchange rate, the capital and debt intensities as functions of the fundamentals Z , which involve foreign and domestic productivity and thrift variables. The steady state value of the NATREX, R^* , is given by equation (14).

$$R^* = R(k^*, F^*; Z) = R[Z(t)] \quad (14)$$

This completes the explanation in an abstract way of the determination of the three components of the medium to longer-run real exchange rate in equation (1a).

V. Empirical Analysis

The NATREX model is suited to empirical analysis using the logic of the cointegration techniques. The observed real exchange rate consists of two large parts: a disequilibrium component and a moving equilibrium (NATREX). The disequilibrium effects of the deviation of the actual real exchange rate from the NATREX, given by the term $[R(t) - R(k(t), F(t); Z(t))]$, are associated with deviations of the rate of capacity utilization from its stationary mean

^{1/} To simplify the exposition of the model, we assume in the text that the growth of effective labor $n=0$. A true steady state requires that: (i) the growth of effective labor be the same in both countries and (ii) the discount rate in the instantaneous utility function of each country be equal to the common real rate of interest. As it is not clear how these conditions can occur, we use a less restrictive concept of the steady state.

and with disequilibrium in the asset markets, before real long-term rates of interest have converged. This is one measure of "misalignment." The NATREX itself consists of two parts. One part is the trajectory of the NATREX to the steady state, which describes how the endogenous evolution of capital and debt in Figure 5 move the IX and IX' curves in Figure 4 to generate a trajectory for the NATREX. The second part is the longer-run equation (14), which is based on IX curves associated with steady state values of capital (k^*) and debt (F^*). Formally, this can be described by a dynamic process econometric equation (15), which corresponds to equation (1a). The system is stable when $1 > a > 0$. The longer-run effect is term $BZ(t)$. The trajectory is the error correction term $[R(t-1) - BZ(t-1)]$. The disequilibrium effect is in impact term $[Z(t) - Z(t-1)]$.

$$R(t) = BZ(t) + a [R(t-1) - BZ(t-1)] + b [Z(t) - Z(t-1)] + e(t) \quad (15)$$

Equation (15), the sum of the moving equilibrium and disequilibrium elements, indicates the extent to which our model can explain the actual evolution of the real exchange rate. We have two concepts of the equilibrium real exchange rate. The first two terms concern the evolution of the equilibrium real exchange rate when there is portfolio balance and the rate of capacity utilization is at its stationary value. This is the medium-run equilibrium. We can therefore calibrate the moving equilibrium real exchange rate on the rate of capacity output and when there is portfolio adjustment. This calibration has been sought by Williamson (1994), Clark et al. (1994), Bayoumi et al. (1994) and Clark (1995) in attempting to determine the equilibrium rate of exchange. Hence we can separate the moving equilibrium from the disequilibrium effects. A comparison of the actual with the medium-run equilibrium is one measure of "misalignment." Finally, the first term represents the longer-run equilibrium which is just a function of the exogenous variables. The deviation of the actual real exchange rate from the longer-run equilibrium is the second measure of misalignment. In this part, we discuss the data and present the empirical results. In the next part, we explain the empirical results in terms of the model.

1. Data for the exogenous fundamentals

The endogenous variable is the real effective exchange rate of the United States relative to the rest of the G-7 (USREUG7). It is integrated of order I(1). The exogenous fundamentals Z are time preference and the productivity of capital in the United States and the other G-7 countries. While foreign debt is an important determinant of the real exchange rate in our model, we do not use it as an element in Z in our regressions because it is an endogenous variable in our model. ^{1/} The basic exogenous

^{1/} As noted in stylized fact (e), Faruquee did include the foreign debt in his regressions and obtained results that are consistent with ours.

fundamentals, productivity and thrift, and the real effective exchange rate are graphed in Figure 3.

We include in our regression equations two disequilibrium elements. One is the deviation of the rate of capacity utilization in manufacturing (USCUR) from its stationary mean value, which will influence the term $[R(t) - R(k(t), F(t); Z(t))]$ in equation (1a). Graphically, this disequilibrium is the vertical deviation of the real exchange rate from the IX curve of balance of payments equilibrium. The other is an impact term which reflects a temporary deviation between the United States and the foreign real long-term rates of interest $INTFIN = r - r'$. This is related to the horizontal distance between the U.S. IX and foreign IX' curves.

Our data are quarterly and at times there are marked seasonal variations. Generally, we use a four-quarter moving average of the variable Z, denoted as MA(Z). The NATREX model includes in Z four fundamental disturbances: the rate of time preference and the productivity of capital at home and abroad. The dependent variable is the real effective exchange rate. Table 1 describes the data and their stationarity or nonstationarity properties. We tried many different combinations of measures of the independent variables; time preference and productivity, and all yielded similar results.

a. Time preference

Time preference is concerned with the fraction of GDP that is consumed rather than invested. Theoretically this is (g, g') in the social consumption function (5) or saving function (6) in the model. We measure it empirically as the ratio of social consumption/GDP: private consumption/GDP plus government consumption/GDP. ^{1/} One measure of time preference is a four-quarter moving average of USDISRAT, denoted as MADISRAT. This is integrated I(1), and the ADF(C,2) = -1.31, which is not significant. For the other G-7 countries, we used a four-quarter moving average of the ratio of private consumption plus total government expenditure to GDP, denoted MAG7DIS. The ADF(C,2) = -2.85 is significant only at the 10 percent level.

There are two reasons why no distinction is made in the model between the private consumption and government consumption. One is that a decline in tax rates, or a rise in transfer payments, may increase private consumption without an offsetting effect on government consumption in the national income accounts. If we just used government consumption as an

^{1/} If we had accurate measures of the net liabilities to foreigners $F(t)$, then the time preference variable would be the residual of the ratio of social consumption/GNP on the net foreign liabilities/GNP. The model includes in net foreign liabilities both equity and debt, which are not easily and accurately measured.

Figure 3: Basic Variables

index of time preference, we would miss this effect. 1/ The other reason is that we have no need to get into the controversy whether there is or is not "Ricardian equivalence."

b. Productivity of capital

Ideally, we would like to use the q-ratio as the exogenous driving force behind capital formation. 2/ Unfortunately, this ratio bore no resemblance to the ratio of investment to GNP, and consequently it was not used in this study. 3/ Indirect measures are therefore adopted. The model claims that the q-ratio determines the growth of capital which determines the growth of real GDP. We therefore used a four-quarter moving average of the growth of real GDP in the United States, denoted MAUSGROW, as our measure of productivity in the United States. This is stationary $I(0)$ with an $ADF(C,2) = -3.8$. The stationary U.S. growth rate is positively related to the rate of capacity utilization in manufacturing, which is also stationary. 4/

For the other G-7 countries we also used a four-quarter moving average of the growth of real GDP as our proxy for productivity. This variable, denoted MAG7GROW, is stationary with a significant $ADF(C,1) = -3.87$.

c. Disequilibrium variables

There are two disequilibrium terms. They refer to the ephemeral determinants of the deviation of the actual real exchange rate from the NATREX, which is given by the term $R(t) - R[k(t), F(t); Z(t)]$ in equation (1a). One is the deviation of the rate of capacity utilization from its mean, denoted by DEVCUR. The second is the disequilibrium in the portfolio adjustment measured as the deviation between the real long-term rate of interest in the United States and in the rest of the G-7, denoted by INTDIF. The disequilibrium terms refer to the distance between the actual real exchange rate from the intersection of the IX and IX' curves in Figure 4.

1/ Alternatively, for the United States, we considered a four-quarter moving average of the ratio of government consumption/GNP to measure time preference. This is $I(1)$. We obtained the same basic results regardless of which measure of time preference was used.

2/ The q-ratio is measured as the ratio of industrial share prices to the producer prices of industrial goods (US62/US63a in the IFS).

3/ The q-ratio worked very well as a measure of productivity in our study of Germany; see Stein and Sauernheimer (1995).

4/ For the rate of capacity utilization the $ADF(C,1) = -3.39$ which is significant. The correlation coefficient between the U.S. growth rate and rate of capacity utilization is 0.72.

2. Econometrics 1/

We display two estimation methods. First (Table 2) is an ordinary least squares OLS estimate of the real exchange rate (USREUG7) regressed on the fundamentals and disequilibrium components. An AR(1) transformation was used to reduce the effects of serial correlation. The second method (Table 3) is a nonlinear least squares NLS estimate of equation (15). Similar results are obtained in both cases. The basic results are as follows: (1) A rise in U.S. time preference (MADISRAT) depreciates the equilibrium real value of the dollar; (2) an increase in foreign-time preference (MAG7DIS) appreciates the equilibrium value of the U.S. dollar. These effects are always significant; (3) neither the U.S. growth (MAUSGROW) nor the foreign growth variable (MAG7GROW) has a significant effect; (4) the disequilibrium deviation of the rate of capacity utilization from its stationary mean value (DEVCUR) significantly depreciates the real value of the dollar; (5) the disequilibrium deviation of the U.S. real long-term rate of interest from the foreign real long-term real rate of interest (INTDIF) significantly appreciates the dollar.

It is shown in Table 1 that the real effective exchange rate and U.S. rate of time preference are integrated I(1) and the other G-7 rate of time preference is close to I(1). 2/ On the other hand, the two growth variables are stationary I(0). Hence the longer-run movements in the real effective exchange rate are expected to be due primarily to the U.S. and other G-7 time preference variables. Since only the time preference variables and the disequilibrium variables were significant in Table 2, the NLS estimation in Table 3 uses for the fundamentals $Z=[MADISRAT, MAG7DIS]$ only the U.S. and other G-7 rates of time preference. The disequilibrium variables, reflected in the U.S. less the foreign real long-term rate of interest, INTDIF, and the deviation of the U.S. rate of capacity utilization from its stationary mean, DEVCUR, are the same as before. Figure 6, discussed below, plots the actual real exchange rate, the fitted value based upon Table 3 and the residuals. In terms of equation (15), the longer-run effects BZ are that a rise in U.S. time preference depreciates the dollar and a rise in other G-7 time preference appreciates the dollar. Coefficient a , of the error correction term $R(t-1)-BZ(t-1)$ in equation (15), is 0.78. Hence the system converges to a stable equilibrium. The disequilibrium effects are that a rise in the U.S. less foreign interest rate appreciates the dollar. A rise in the rate of capacity utilization above its stationary mean depreciates the dollar. The residual $e(t)$ from equation (15) is stationary at the 1 percent level. 3/ Moreover, it is normal and there is no heteroskedasticity. Using the Chow breakpoint test at 1980:2, there is no significant difference between the early and later subperiods.

1/ The econometric analysis draws on Lim (1995). The real effective exchange rate and the two time preference variables are I(1). The other variables are I(0).

2/ See the plots in Figure 3 for a graphic view of these results.

3/ The diagnostic tests are reported below each table.

Figure 4: Determination of Real Exchange Rate and Real Interest Rate

Table 2. OLS Regression of Real Effective Exchange Rate USREUG7
on Fundamentals and Disequilibrium Terms - AR(1) Transformation Used

Sample Period 1975:2-1993:3

Variable	Coefficient	Std. Error	T-Stat.	Prob.
CONSTANT	192.7397	118.0197	1.633115	0.1076
MADISRAT	-582.7147	107.0164	-5.445098	0.0000
MAUSGROW	-0.335879	0.532291	-0.631007	0.5304
MAG7DIS	473.9907	191.2641	-2.478200	0.0160
MAG7GROW	-0.183023	0.284273	-0.643830	0.5221
INTDIF	1.928176	0.409621	4.707219	0.0000
DEVCUR	-0.259351	0.324519	-0.799186	0.4273
AR(1)	0.825274	0.055837	14.78017	0.0000
R-squared	0.949495	Mean dependent var	78.02342	
Adj. R-squared	0.943699	S.D. dependent var	10.63447	
S.E. of regression	2.523319	Akaike info criterion	1.959802	
Sum squared resid	388.3956	Schwartz criterion	2.218829	
Log likelihood	-157.5199	F-statistic	163.8289	
Durbin-Watson stat	1.474129	Prob(F-statistic)	0.000000	

Note: Residuals: (1) From the LM test the F statistic has a probability of 0.05 indicating some serial correlation. (2) From de ARCH test, the F-statistic has a probability 0.85 thus indicating no heteroskedasticity. (3) The ADF(N,1) statistic for the residuals -4.42 is significant at the 1 percent level. The residuals are stationary.

Table 3. Nonlinear Least Squares Estimation

Sample Period 1975:2-1993:3

Variable <u>1/</u>	Coefficient	Std. Error	T-Stat.	Prob.
C(1)	335.9549	151.0658	2.223898	0.0297
C(2)	-660.0887	97.26102	-6.786776	0.0000
C(3)	370.8803	209.0170	1.774402	0.0808
C(4)	0.787841	0.052184	15.09745	0.0000
C(5)	0.404234	0.131753	3.068127	0.0032
C(6)	-0.420673	0.134209	-3.134453	0.0026
R-squared	0.945686	Mean dependent var	78.04067	
Adj. R-squared	0.941443	S.D. dependent var	10.55811	
S.E. of regression	2.554907	Akaike info criterion	1.957848	
Sum squared resid	417.7631	Schwartz criterion	2.150576	
Log likelihood	-161.8504	F-statistic	222.8684	
Durbin-Watson stat	1.475414	Prob(F-statistic)	0.000000	

Note: The residuals have the following characteristics: (1) From the LM test for serial correlation with two lags, the F-statistic is 2.99 with a probability of 0.057, so there is some serial correlation coming from the first lagged residual. (2) The ARCH test for heteroskedasticity with one lag has an F-statistic 0.45 with a probability 0.50. The White test for heteroskedasticity has an F-statistic 0.71 with a probability 0.73. Hence we do not find any heteroskedasticity. (3) The Jarque-Bera test for normality is 0.91 with a probability 0.91. Thus we cannot reject normality. (4) The ADF statistic for the residual from the entire equation (15) is -6.2 which is significant at the 1 percent level. The residuals are stationary. The Chow breakpoint was 1980:2. The F-statistic 1.00 has a probability 0.42. Hence there is no evidence of a structural break.

1/ USREUG7=C(1)+C(2)*MADISRAT+C(3)*MAG7DIS+C(4)*(USREUG7(-1)-C(1)-C(2)*MADISRAT(-1)-C(3)*MAG7DIS(-1))+C(5)*INTDIF+C(6)*DEVCUR.

VI. The NATREX Explanation of the Econometrics

In this part we use the NATREX model to explain the econometric results. In the concluding Section VII we then use the model and empirical results to answer the questions posed at the beginning of the paper concerning what is the equilibrium value of the U.S. dollar and what were the causes and degree of misalignment.

1. Impact and medium-run effects: a rise in U.S. time preference or the productivity of capital

Along the IX or IX' curve in Figure 7, the current account plus the nonspeculative capital inflow, equal to investment less saving, evaluated at capacity output, is equal to zero. At the intersection of the two curves there is portfolio balance and the real long-term rates of interest are equal in the United States and in the rest of the G-7. Let there be a rise in investment less saving, produced by either a rise in time preference or in productivity of capital in the United States. The rise in time preference could occur because there is a tax cut that increases private consumption, or a direct rise in government consumption that is not offset by private consumption. 1/ The rise in the investment occurs because of a rise in the q-ratio.

The IX curve shifts to IX(1) in Figure 7 because, given the real exchange rate $R(0)$ and the trade balance, a rise in the U.S. interest rate to H' is required to keep I-S unchanged. Initially, the U.S. real rate of interest will exceed the foreign rate by $H-H'$, and there will be a shift toward U.S. long-term assets and away from those in the foreign country. The real value of the dollar will rise to $R(1)$ and interest rate convergence at r_1 will occur at point H_1 . This is picked up by the disequilibrium effects in both the NLS estimation Table 3 and OLS estimate Table 2, where the differential between the U.S. and rest of the G-7 real rate of interest INTDIF leads to an appreciation of the U.S. dollar.

At point H_1 , there is an appreciated dollar $R(1) > R(0)$, a trade deficit equal to the capital inflow to finance the excess of investment less saving and the world real rate of interest that has risen to r_1 . This is the conventional medium-run effect.

2. Longer-run effects

The longer-run effects differ depending upon whether the rise in I-S equal to the nonspeculative capital inflow was produced by a rise in time preference or a rise in the productivity of capital.

1/ Private and public consumption were not cointegrated.

a. Rise in time preference

When the rise in time preference appreciates the real exchange rate to point H1 in Figure 7, there is a current account deficit and foreign debt rises. Moreover, insofar as there is a significant rise in the world real rate of interest at H1 compared to H, the q-ratio is reduced and the capital intensity tends to decline. This means that the economy is also at point E3 in phase diagram Figure 5. The economy travels along trajectory E3-E1 toward a steady state with a lower capital intensity and the current account deficits lead to a higher debt or lower net foreign assets. 1/

As the foreign debt increases along E3-E1 in Figure 5, U.S. wealth declines and foreign wealth rises. The excess demand for goods in the United States declines relative to capacity output because saving rises and the IX curve shifts downwards to IX(2). The rise in foreign wealth implies that their IX' curve shifts to the right to IX'(2). The new steady state point is H2 where the real value of the dollar declines to R(2). The real rate of interest does not change as a result of the redistribution of wealth from the United States to the rest of the G-7, since the redistribution does not change world saving and investment. 2/

The medium-run effect of a rise in U.S. time preference is to appreciate the real value of the dollar $R(1) > R(0)$. However, as the foreign debt rises along trajectory E3-E1, the real value of the dollar declines. The reason why the longer-run equilibrium real value R(2) is below its initial value R(0) is that the interest payments on the debt have increased and the real exchange rate must depreciate to generate a trade surplus sufficient to pay the higher interest. It is not necessarily paradoxical to argue that the rise in the U.S. dollar from 1980-83 and its decline from 1983-90 were both due to the Reagan tax policies. 3/

The sign of the U.S. time preference variable is negative in both the OLS Table 2, and in the NLS Table 3 for the cointegrating equation BZ. We interpret this result as indicating that the cointegrating equation, which is the longer-run relationship, is picking up the decline of the real exchange rate from H to H2 in Figure 7 as the economy travels along trajectory E3-E1 in Figure 5. Symmetrically, the sign of the foreign time

1/ Time preference is denoted by parameter g in the consumption function (6a). Then $dk^*/dg < 0$ and $dF^*/dg > 0$, where the steady state is denoted by an asterisk. The proof is in Stein (1994, Table 5) and in Stein, Allen (1995) equations (17)-(19) and Table 4.

2/ Equation (10) specifies that the world real rate of interest is independent of the U.S. foreign debt.

3/ This scenario does not occur in the RAIOM because the trajectory of the debt is part of the intertemporal optimization process. In these models, changes in the real exchange rate depend only on unanticipated disturbances to productivity. See Rogoff (1992), equation (18).

Figure 5: Dynamics of Capital and Debt Intensities

preference is positive, and thus a rise in the foreign time preference eventually leads to an appreciation in the real value of the U.S. dollar.

b. Rise in the productivity of capital

The other two exogenous variables in Z are estimates of the effects of an increase in the productivity of capital. If the productivity of capital rises in the United States, this raises the q -ratio and hence investment relative to saving. The medium-run effects are the same as before. The longer-run effects are different. We proved that a rise in the productivity parameter raises the steady state capital intensity, lowers the steady state debt intensity and produces an ambiguous effect upon the real exchange rate. ^{1/}

Unlike the case of a rise in time preference, when the real exchange rate appreciates from H to $H1$ in Figure 7, the capital inflows are now financing capital formation. This places the economy at point $E2$ in Figure 5. Both capital and debt are rising. This is a scenario involving stages of economic growth. Initially the debt rises as capital grows along trajectory $E2-N$. As the capital stock increases and labor productivity rises, saving increases due to the higher GDP, and investment declines as the capital stock rises. The rise in $S-I$ will lead eventually to capital outflows along $N-E1$. The debt will decline and capital will rise as the economy converges to the steady state.

The decline in the debt raises the IX curve in Figure 7. The corresponding effects in the rest of the $G-7$ shift the IX' curve to the left. The decline in the foreign debt (rise in net foreign assets) unambiguously appreciates the real exchange rate, as the trade balance must decline to offset the greater inflow of interest payments. The rise in capital per se raises imports and tends to depreciate the real exchange rate, so the net effect upon the real exchange rate is theoretically ambiguous.

In both the NLS Table 3 and OLS Table 2, neither the U.S. growth nor the $G-7$ growth variable is significant. ^{2/} It seems that the two effects of productivity on the real value of the dollar cancel each other.

^{1/} Let u an element of Z represent the rise in the productivity of capital which raises the q -ratio. We have shown that $dk^*/du > 0$ and $dF^*/du < 0$. See Stein (1994) Table 5, Stein, Allen et al. (1995), Chapter 2. We know that $R_F < 0$ such that a rise in debt lowers the IX curve, and $R_K < 0$ a rise in capital raises saving less investment and lowers the IX curve. The impact effect is $R_u > 0$. Hence $dR/du = R_u + R_K dk^*/du + R_F dF^*/du$ is ambiguous. The first and last terms are positive and the middle term is negative.

^{2/} The autoregressive transformation is important. Without it, the U.S. growth rate has a positive sign in an OLS estimation in Table 2.

VII. Conclusion: The Equilibrium Real Exchange Rate

The NATREX model and empirical analysis respond to the issues raised in the opening paragraph of this paper: they provide a theoretical explanation of the movement of the real rate of exchange in the medium to long run and an estimate of the degree of misalignment. The longer-run equilibrium exchange rate depends upon the fundamentals of social time preference at home and abroad. These fundamentals have not been stationary. Since the real exchange rate inherits the properties of the fundamentals, insofar as the fundamentals seem to be martingales, the observed real exchange rate will have the time series property of a martingale. This does not mean that the real exchange rate is a random variable unrelated to the theory. Quite the contrary: it is related to the fundamentals in the manner described by the NATREX model. Misalignment means that the exchange rates are not in line with the fundamentals when there is internal and external balance. To have internal balance, output should be at capacity output. To have external balance requires (i) there should be no deviation between the domestic and foreign real long-term real rates of interest, (ii) the real exchange rate should equate the current account to the nonspeculative capital outflow, and (iii) speculative capital flows and changes in reserves are excluded.

We now show how the NATREX model estimates the degree of misalignment. The nonlinear least squares estimates of the dynamic system, described by equation (15), are presented in Table 3. The model and econometric tests show that the actual real effective exchange rate of the U.S. dollar $R(t)$ depends upon the fundamentals of productivity and time preference at home and abroad, contained in vector $Z(t)$, an error correction mechanism driving the system to the longer-run equilibrium, and disequilibrium factors denoted DZ which represent the deviation between actual and capacity output ($DEVCUR$) and between the U.S. and the foreign real long-term rates of interest ($INTDIF$). The equilibrium real exchange rate is obtained when the disequilibrium vector is zero and there is no longer an error correction. We calibrate the equation in Table 3 such that output is at capacity ($DEVCUR=0$) and there is real long-term interest rate convergence ($INTDIF=0$). This calibration allows us to estimate the equilibrium and degree of misalignment.

Figure 6 compares the actual real exchange rate with the fitted value from nonlinear least squares estimates in Table 3, and also displays the stationary residuals. The regressors are the fundamentals and the disequilibrium terms. The fundamentals $Z(t)$ used in Table 3 and Figure 8 are time preference in the United States and the G-7 countries. The U.S. growth relative to its stationary mean is positively correlated with the deviation $DEVCUR$ between the actual rate of capacity utilization and its stationary mean. Hence the U.S. growth variable was not used as a fundamental in the empirical work, though it is a fundamental in the theoretical part. The foreign growth variable is also stationary and hence excluded from NLS Table 3. The residuals reflect speculative capital flows,

Figure 8.

United States: Real Effective Exchange Rate Relative to Rest of G-7
and Fit from NLS Estimate

Figure 7: Effects of Changes in Time Preference and Productivity

Figure 8.

United States: Actual and Equilibrium Real Effective Exchange Rate

since the cyclical elements are contained in the disequilibrium terms in the equation.

The misalignment estimates are shown in Figure 8, which compares the actual real effective exchange rate (USREUG7) with the longer-run equilibrium part. The latter is the first term $BZ(t)$ in equation (15), and is called the NATNLS 1/ in Figure 8. To obtain the equilibrium, the NATNLS, the error correction term and the disequilibrium terms are set equal to zero. We use NATNLS as our estimate of the equilibrium exchange rate in Figure 8. 2/ Misalignment is measured as the deviation ($UREUG7 - NATNLS$). The depreciation of the equilibrium value of the U.S. dollar since 1982 is indeed closely related to social time preference. 3/ As is seen in Figure 8, there were substantial misalignments from 1976-80 and from 1982-85. From then until the end of the sample period in 1993:3 the real exchange rate was close to its estimated equilibrium value.

The FEER/DEER models, of Williamson, Bayoumi et al., Clark et al. and Clark, calibrate the equilibrium exchange rate on an exogenously determined "desirable" current account. These are normative concepts. The NATREX model takes time preference as an exogenous variable without any normative significance, and the current account is an endogenous variable. The NATREX concept of equilibrium can be related to the FEER/DEER by setting our measure of time preference MADISRAT equal to their "desirable" time

1/ This is an acronym for the NATREX derived from the NLS regression.

2/ The difference between the estimates in the present paper and my 1994 paper in the Williamson volume are as follows: (1) Here I use the deviation DEVCUR between the actual and stationary mean rate of capacity utilization as a variable. It was absent from my 1994 paper. (2) Hence I now calibrate the equilibrium exchange rate on $DEVCUR=0$. This responds to Black's (1994) and Clark's (1995) criticisms of my 1994 paper that I did not calibrate the equilibrium exchange rate on an objective measure of internal balance. (2) The U.S. growth rate is correlated with DEVCUR and it has a stationary mean. Hence the inclusion of the disequilibrium DEVCUR replaces the U.S. growth rate variable used in my 1994 paper. (3) I now have direct measures of the G-7 rate of time preference which I did not have earlier. (4) The estimate of the equilibrium real exchange rate here is based upon the equilibrium $BZ(t)$ or NATNLS whereas it included the error correction term in the 1994 paper. (5) The data here refer to the U.S. relative to the G-7 and are consistent, whereas the data used in 1994 were less precise and referred to different concepts of the foreign country. (6) The present analysis permits many different decompositions of the exchange rate into disequilibrium, where there is neither internal (DEVCUR) nor external (INTDIF) balance, and I now have both explicit medium-run and longer-run equilibrium components.

3/ Compare the actual real exchange rate USREUG7 with our measure of time preference MADISRAT in Figure 3 to have a graphic feel for this result.

preference, and evaluating the equilibrium exchange rate with this component of vector Z. 1/

We have shown that the NATREX model has explanatory power and that it is useful for interpreting the behavior of exchange rates because it provides: (1) an estimate of misalignment between the actual real effective exchange rate and the equilibrium rate associated with internal and external balance; and (2) an estimate of how the fundamentals affect the equilibrium rate.

1/ The "Lucas critique" does not apply, since there are no anticipations involved in our concept of an equilibrium exchange rate.

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