Realignment Expectations, Forward Rate Bias, and Sterilized Intervention in an Adjustable Peg Exchange Rate Model with Policy Optimization

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Abstract

The paper models an adjustable peg exchange rate arrangement as a policy rule with an escape clause under which the timing and magnitudes of realignments are the outcomes of policy optimization decisions. Under the assumptions that market participants are rational, risk averse, and fully informed about the incentives of policymakers, the analysis focuses on the implications for relating realignment expectations to the state variables that enter the policy objective function, for modeling the bias in using forward exchange rates to predict future spot rates, and for characterizing the effectiveness of sterilized intervention.

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Summary

The paper models an adjustable peg exchange rate arrangement as a policy rule with an escape clause under which the timing and magnitudes of realignments are the outcomes of policy optimization decisions. Under the assumptions that market participants are rational, risk averse, and fully informed about the incentives of policymakers, the paper explores the implications (1) for relating realignment expectations to the variables that enter the policy objective function, (2) for modeling the bias in using forward exchange rates to predict future spot rates, and (3) for characterizing the effectiveness of sterilized intervention.

The analysis suggests that the co-movements of realignment expectations and the risk premium may be highly correlated, pointing to the possibility of bias in the existing methodology for constructing measures of realignment expectations. It also reconciles the view that macroeconomic fundamentals are relevant to realignment expectations, with the fact that the widening of constructed measures of realignment expectations for European exchange rates, during the summer of 1992, did not coincide with significant changes in macroeconomic conditions. In addition, the conceptual framework provides a model of the premium for bearing the risk of a policy decision to adjust the exchange rate peg—the type of event on which the peso-problem literature has focused—thereby integrating two of the proposed explanations of forward rate bias that are consistent with rational expectations.

The policy optimization framework provides useful insights on the effectiveness of sterilized intervention, including the perspective that its effects on the risk premium and on expected future exchange rates are not independent of each other. When market participants have complete information about the incentives of policymakers, the effectiveness of sterilized intervention depends critically on the relative weight attached to foreign exchange valuation losses in the policy objective function. Unless an increase in exposure to valuation losses makes the authorities more reluctant to realign, other things being equal, sterilized intervention will have no effect on rational assessments of realignment prospects. Furthermore, when sterilized intervention is effective, the time variation of realignment expectations and the risk premium may depend importantly on the extent to which there are explicit or implicit constraints on the use of sterilized intervention as a policy instrument. When completely unconstrained, intervention can be used to prevent time variation in the state of the economy from generating time variation in realignment expectations, while also greatly mitigating the time variation in the risk premium. Accordingly, in empirical attempts to relate realignment expectations or the risk premium to fundamental macroeconomic variables, it may be important to take account of institutional considerations governing the use of sterilized intervention.
I. Introduction

This paper draws on recent developments in the theory of monetary policy strategies to address three related issues in the literature on exchange rate behavior. An adjustable peg exchange rate arrangement is viewed as a policy rule with an escape clause, and the timing and magnitudes of realignments are described as the outcomes of policy optimization decisions. Under the assumption that market participants are rational and risk averse, the paper explores the implications (1) for relating realignment expectations to the state variables that enter the policy objective function, (2) for modeling the bias in using forward exchange rates to predict future spot rates, and (3) for characterizing the effectiveness of sterilized intervention. Since the objective of the paper is simply to illustrate that an explicit focus on the policy optimization problem may be constructive in refocusing the analysis of these issues, the conceptual framework is streamlined: in particular, the model of policy optimization is cast in terms of a one-dimensional problem of output stabilization; home-country output is also modeled in one-dimensional terms as a function of its relative price; and in contrast to the target zone literature, the band around the adjustable peg is assumed to have zero width.

The paper is organized as follows. Section II briefly reviews the three issues in the exchange rate literature. Section III develops the conceptual framework and applies it, first, to a case in which the policy authorities are assumed to control only the exchange rate peg, and second, to a case in which they are also free to vary their stock of foreign exchange reserves through sterilized intervention. Section IV concludes.

II. Some Issues in the Exchange Rate Literature

1. Realignment expectations

The analysis of realignment expectations has become an active area of empirically-oriented research associated with the growing literature on target zones. Among other things, this research has sought to verify that the realignments of exchange rates between European currencies in recent years were not the result of random self-fulfilling speculative

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1/ See Svensson (1992b) and Bertola (1993) for perspectives on the literature.
attacks, but were rather a result of pressures that had built up from fundamental macroeconomic imbalances. 1/

The empirical techniques used to obtain measures of realignment expectations have generally been based on the assumption of uncovered interest parity. 2/ Proponents of this approach recognize that "it is crucial whether the foreign exchange risk premium can be neglected or not," and have made a serious effort to explore whether the uncovered interest parity assumption can be justified. In particular, Svensson (1992a) extended the analysis of real and nominal risk premiums to the target zone framework and derived generous upper bounds on their empirical magnitudes, suggesting that they were "relatively small." Notably, this result was based on the assumption that the probability distribution of realignments is time invariant and not state contingent--in particular, not contingent on the state variables that tend to generate political pressures for realignments. To provide an alternative perspective, Svensson also calculated the real risk premium associated with a depreciation that is anticipated with probability one, suggesting that even when a realignment is expected with certainty the size of the risk premium is only about one-eighth the size of the expected realignment. 3/ The literature has thus continued to rely on the uncovered interest parity assumption on the grounds that the empirical results are not significantly affected by a small time-varying risk premium "unless its movements coincide with changing realignment expectations for some mysterious reason." 4/

As illustrated below, the analysis of realignment expectations in the context of a policy optimization problem establishes a strong and unmysterious possibility that the co-movements of realignment expectations and the risk premium are indeed highly correlated. This points to the possibility of bias in the existing methodology for constructing measures of realignment expectations, although it does not necessarily imply that the bias has been quantitatively large. The explicit focus on a policy optimization problem also provides an appealing conceptual framework for specifying a testable hypothesis about the nature of the relationship

1/ The two alternative hypotheses have generated different policy proposals. See Eichengreen and Wyplosz (1993), including the appended comments by Branson and Dornbusch, for a debate over the two alternative views and their policy implications. See Agénor, Bhandari, and Flood (1992) for a review of the literature on speculative attacks and balance of payments crises.

2/ See, for example, Bertola and Svensson (1993) and Lindberg, Söderlind, and Svensson (1993).

3/ Svensson (1992a), pp. 35-6. The calculation focuses on exchange rate jumps within the band but is also relevant to realignments anticipated with probability one.

4/ Rose and Svensson (1993), endnote 1, p. 34.
between realignment expectations and fundamental macroeconomic variables. 1/ In doing so, moreover, it reconciles the view that macroeconomic fundamentals are relevant to realignment decisions and expectations with the finding that constructed measures of realignment expectations for European exchange rates did not begin to widen before the summer of 1992, 2/ despite the fact that the macroeconomic conditions then prevailing had existed for some time.

2. Forward rate bias

One of the most widely tested propositions about exchange rate behavior is the hypothesis that forward premiums are unbiased predictors of changes in spot exchange rates. The empirical evidence strongly rejects this unbiasedness hypothesis, indicating as well that in many cases observed changes in spot rates are negatively correlated with ex ante forward premiums. 3/

Efforts to reconcile the evidence of forward rate bias with the hypothesis that market participants form rational expectations have focused on the possible importance of risk premiums, peso problems, rational bubbles, simultaneity bias, and incomplete information with rational learning. 4/ To date, however, the analysis of these possibilities has not provided a convincing explanation of the observed bias in forward rates, and models of a less than fully rational world with feedback trading have begun to emerge. 5/ The latter models are supported by evidence directly verifying the practice of feedback trading, 6/ along with other suggestive evidence. 7/ Even so, it is difficult for economists to constrain their theories about exchange rates unless they retain the hypothesis that some market participants form rational expectations based on future fundamentals. "If one does not assume rationality,... the behaviour of the exchange market is as much imposed by the theorist as anything else: the magician essentially pulls out of the hat the same rabbit the audience has seen him

1/ Although some recent studies of international interest rate differentials (for example, Caramazza (1993), Halikias (1993), Bartolini (1993), and Thomas (1993)) have made progress in relating constructed measures of realignment expectations to macroeconomic fundamentals, the policy optimization approach provides a less ad hoc approach to hypothesis specification.


stuff in a few minutes earlier." 1/ This strong attraction to the rationality hypothesis has focussed attention on the possibility that feedback trading represents rational behavior in an environment of incomplete information. 2/

The escape-clause framework employed in this paper and elsewhere 3/ suggests a direction for continuing the effort to reconcile the evidence on forward rate forecast bias with the rational expectations assumption. In particular, when the assumption of uncovered interest parity is abandoned, it provides a model of the premium for bearing the risk of a policy decision to adjust the exchange rate peg, which is the type of event on which the peso-problem literature has focused. Thus, the escape-clause framework integrates two of the proposed explanations of forward rate bias that are consistent with rational expectations, emphasizing that risk premiums vary over time with the state variables on which policymakers focus in deciding whether to realign, as well as with the relative asset stock and wealth variables that enter traditional portfolio balance models. 4/

3. The effectiveness of sterilized intervention

In the early 1980s, a comprehensive study of exchange market intervention, drawing on a variety of careful research efforts, reached the conclusion that sterilized intervention, unless supported by other policies, has at most a small and short-term effect on exchange rates. 5/ Surveys of the literature a decade later 6/ suggest that this conclusion stands largely unchanged, although the analytic framework for assessing the effectiveness of intervention has evolved. In particular, it is now postulated that, in addition to whatever effect sterilized intervention may have through the portfolio-balance or risk-premium channel, it may also have an influence on exchange rate expectations through a signalling channel. 7/

The discussion of signalling effects has suffered to some extent from the failure of many discussants to focus explicitly on how much market participants are assumed to know about the incentives of the policy authorities. Two cases can be distinguished. When market participants lack full information about the motives of the authorities, sterilized

4/ See Dooley and Isard (1983) for a traditional model of the risk premium.
7/ Mussa (1981) was one of the first to draw attention to the signalling effects of intervention.
Intervention may provide new information, signalling that the authorities intend to resist further exchange rate movement, and thereby leading market participants to revise their assessment of the conditional probability that the authorities will take action stronger than sterilized intervention if the exchange rate movement continues. By contrast, when market participants already have full information about the motives of the authorities and form rational expectations, the signalling effect cannot arise from clarifying the nature of the authorities' ultimate policy objectives, but rather depends on whether sterilized intervention influences the authorities' incentives to hold the exchange rate fixed in pursuit of their ultimate objectives. The signalling effect in this context has been characterized as the effect that the authorities can have when they put their money where their mouth is.\footnote{Mussa (1981), Obstfeld (1990).} By changing official foreign exchange holdings in a direction that exposes the authorities to the prospect of a greater valuation loss if the exchange rate continues to move, intervention conveys a signal that the authorities' incentive to resist further exchange rate movement has strengthened endogenously.

The latter effect is illustrated explicitly in the model developed below, which also suggests that the effect of sterilized intervention on the risk premium may be strongly correlated with its effect on exchange rate expectations. Within the context of an adjustable peg regime, sterilized intervention endogenously changes the ex ante probability that the authorities will choose to realign the peg following the realization of a future shock. It is shown, moreover, that the extent to which intervention affects ex ante assessments of realignment prospects depends both on the relative weight attached to foreign exchange valuation losses in the policy objective function and on the state of the macroeconomic variables that enter the objective function. An interesting insight from the policy optimization framework is that the effectiveness of sterilized intervention is directly proportionate to the "costs" that the authorities perceive when their policies lead to foreign exchange valuation losses. Institutional changes that induce the authorities to give greater weight to these costs can make them more reluctant to realign the exchange rate following an increase in their exposure to valuation losses, which in theory can make sterilized intervention more effective in influencing the realignment expectations of rational market participants. Obversely, in the limiting case in which the authorities pay no attention to the prospect of foreign exchange valuation losses, sterilized intervention has no effect on
expectations or perceptions of risk and only a minor portfolio-balance effect on the risk premium.

III. An Illustrative Model

1. Key assumptions and perspectives

The defining characteristic that distinguishes an adjustable peg system from a floating exchange rate regime is the explicit announcement of the policy authorities' intention to prevent the exchange rate from moving outside a target zone. This announced policy intention, along with the orientation of monetary policy toward stabilizing the exchange rate, provides an anchor for exchange rate expectations--particularly in the short run--that is not present in a floating exchange rate system. Of course, the credibility of announced policy intentions is not always high, and exchange rate pegs are sometimes adjusted. But there is abundant empirical evidence that the variability of nominal exchange rates is substantially lower under adjustable peg regimes than under floating rate systems. This obviously suggests that the authorities are motivated to behave differently--and are expected to behave differently--under adjustable peg regimes than under floating rate systems.

An important new direction for exchange rate analysis is to explicitly incorporate the optimization problem of the policy authorities into the set of information that is assumed to be known by market participants when they form their exchange rate expectations. In the spirit of recent new perspectives in the discussion of rules versus discretionary strategies for

1/ Mussa (1990).
2/ See Obstfeld (1991, 1993), Drazen and Masson (1993), Masson (1994), Cukierman, Kiguel, and Leiderman (1993), and Ozkan and Sutherland (1993) for steps in this direction. Most of these studies explore the links between realignment expectations and macroeconomic fundamentals, but with emphasis on different issues than this paper. Obstfeld (1991) is mainly concerned with the possibility of multiple equilibria and its implications for policy rules with escape clauses. Cukierman et al. (1993) focus to a large extent on the determinants of credibility in the context of private sector uncertainty about the nature of the policymaker. Drazen and Masson (1993) and Masson (1994) explore the relationship between credibility and macroeconomic fundamentals in a multiperiod context with uncertainty about the nature of the policymaker. Obstfeld (1993) and Ozkan and Sutherland (1993) are primarily interested in formulating new models of speculative attacks in which the prospect of running out of reserves does not play a central role in precipitating devaluations. All of these papers retain the uncovered interest parity assumption and do not focus on the risk premium; none of them addresses the effectiveness of sterilized intervention and its influence on realignment expectations.

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monetary policy, 1/ the choice of an adjustable exchange rate peg, like any other simple rule for monetary policy that tends to be overridden or modified in extreme circumstances, should be regarded as a strategy for balancing the credibility that can be gained from committing to a rule with the flexibility that is desirable when macroeconomic developments make the social costs of continuing to adhere to the rule very high. Such strategies have become known as "rules with escape clauses." 2/ As a proposition in positive economics, the prevalence of simple rules with implicit or explicit escape clauses—such as money supply growth targets and exchange rate pegs—reflects first, the fact that fully state-contingent rules are not a relevant possibility for monetary policy when knowledge about the structure of the economy and the nature of disturbances is incomplete, and second, the fact that partially state-contingent rules and discretion cannot be unambiguously ranked.

This section develops a simple model of the actual and expected behavior of the exchange rate in a two-country adjustable-peg system in which exchange rate policy is determined by the home country. The model focuses explicitly on the policy optimization problem, abandons the uncovered interest parity assumption, and ignores the band around the central parity. The latter assumption simplifies the task of illustrating how the probability of realignment and the magnitudes of realignment expectations and the risk premium depend on the state variables that enter the policymaker's loss function. 3/ The model also formalizes the effects of sterilized intervention in lowering the magnitudes of realignment probabilities and expectations, while emphasizing that, for intervention beyond a certain scale, the "direct benefits" of these effects are outweighed by the "indirect costs" of the greater prospects for overheating or overcooling the economy, given the lower probability that the authorities will choose to realign.

The analysis focuses on the optimizing behavior of the policy authority in the home country, who has two instruments—the logarithm of the exchange rate, s, and the level of foreign exchange reserves, X. The home-country interest rate adjusts endogenously to clear markets, given the settings of s and X. Attention concentrates first on the case in which the level of foreign exchange reserves is exogenously given, emphasizing that variation over time in the state of the economy generates time variation in realignment expectations and the risk premium. It next shifts to the opposite extreme—the case in which the authorities have the ability to vary their foreign exchange reserves without limit through sterilized

3/ It seems reasonable to conjecture that, if the analysis were extended to a target zone with non-zero width, realignment probabilities and expectations would also depend on the position of the exchange rate within the zone. Cukierman, Kiguel, and Leiderman (1993) provide insights into the case with non-zero bands.
intervention--emphasizing that optimal policy will then largely mitigate the extent to which time variation in the state of the economy generates time variation in realignment expectations and the risk premium.

To keep the model simple, it is assumed that each country produces a single good. The home-currency price of the home-country good, and the foreign-currency price of the foreign-country good, are assumed to be fixed and normalized to one. The relevant asset portfolios of private market participants are allocated between home-currency-denominated interest-bearing net claims on the home-country government and foreign-currency-denominated interest-bearing net claims on the foreign government. In the tradition of much of the portfolio balance literature, the analysis abstracts completely from the intertemporal budget constraints of the two governments. 1/

The policy objective of the home-country authorities at time $t$ is to minimize the expected value of a loss function with three components

\begin{equation}
L_{t+1} = (y_{t+1})^2 - 2a\alpha(s_{t+1}-s_t+r^*_t-r_t)X_t + c_{t+1}
\end{equation}

where: $y_{t+1}$ is the deviation of the logarithm of output from its full employment level; $s$ is the logarithm of the exchange rate in home currency per unit foreign exchange; $r_t$ and $r^*_t$ are the one-period home and foreign currency interest rates at time $t$; $s_{t+1}-s_t+r^*_t-r_t$ represents the return on holdings of foreign securities relative to that on holdings of domestic securities; $a$ is the elasticity of home-country output with respect to its relative price, as defined in condition (3) below; $\alpha$ is a parameter of the loss function, where $2a\alpha$ reflects the weight that society places on avoiding foreign exchange valuation losses relative to the objective of stabilizing output 2/; and $c_{t+1}$ is a fixed cost associated with realignment.

\begin{equation}
c_{t+1} = \begin{cases} 
2 & \text{if } s_{t+1} \neq s_t \\
0 & \text{if } s_{t+1} = s_t 
\end{cases}
\end{equation}

Deviations of home-country output from its full employment level are assumed to be persistent, but to respond positively to a decline in the relative

1/ This is perhaps a more innocent assumption in the two period setting of adjustable peg analysis than it would be in the analysis of a floating exchange rate system, in which market participants may require a longer time horizon for purposes of forming rational expectations.

2/ There is no loss of generality, and a significant gain in subsequent notational simplicity, from writing this weight as the product of $a$ and $2\alpha.$
price of home-country output, here equivalent to a depreciation of the home currency 1/  

\[ y_{t+1} = y_t + \alpha(s_{t+1} - s_t) + u_{t+1} \]  

where \( u_{t+1} \) is a serially uncorrelated disturbance. 2/ In this particularly simple case, home-country output changes only in response to changes in its relative price and random shocks. 3/  

The first component of the loss function—the squared deviation of output from its full employment level—has a long tradition. 4/ The fact that an inflation objective is not included in the loss function may limit the relevance of the example in some contexts, but presumably not for interpreting the pressures that have been imposed on exchange rates between European currencies during the past two years, when many European countries were not concerned about their inflation prospects in the short run.  

The second term in the loss function focuses on the one-period valuation gain on holdings of foreign exchange reserves, \((s_{t+1} - s_t + r_t^e - r_t)X_t\). The larger the volume of reserves accumulated (spent) to resist appreciation

1/ Recall that the home-currency price of home output and the foreign-currency price of foreign output are fixed and normalized to unity.  
2/ Given that deviations from full employment output are persistent, it would be desirable to extend the example to the case in which policymakers minimize the expected value of a multi-period loss function. Drazen and Masson (1993) and Masson (1994) consider multi-period frameworks, emphasizing that persistence in the process driving unemployment implies that adhering to a no-realignment pledge in the first period may reduce the credibility of the pledge in subsequent periods.  
3/ Efforts to extend the model might consider reducing the unit coefficient on lagged output and adding a link between output and the (real) interest rate, which is determined endogenously as described below. One could also consider models in which output responded only to the unexpected component of changes in its relative price. In general, these revisions would complicate the nature of the link between realignment expectations and deviations of output from its full employment level, but such a link would be present for all cases in which output deviations were at least partially persistent.  
4/ Early applications include Gray (1976) and Flood and Marion (1982). Aizenman and Frenkel (1985) derive this component as the optimal specification of the objective function for a world with sticky nominal wages when social welfare depends positively on consumption and negatively on labor. See Aizenman (1994), however, for a recent reconsideration based on an argument that the expected level of full-employment output is sensitive to the nature of the exchange rate regime. Barro and Gordon (1983) and others have added squared changes in the price level to the loss function, while noting the difficulty of justifying such an extension.
(depreciation), the greater will be the valuation losses associated with a subsequent decision to revalue upward (devalue) the home currency. This component of the loss function isolates a well-defined component of the costs of adjusting an exchange rate peg and plays a central role in making sterilized intervention effective in influencing exchange rate expectations and the risk premium in a rational expectations environment. Here it is implicitly assumed that the home country authorities could issue unlimited amounts of foreign-currency-denominated debt if they desired to do so, and thus—in contrast to much of the literature on speculative attacks 1/—that the prospect of running out of reserves is not a central factor in precipitating devaluations. 2/

The third component of the loss function is intended to capture the social costs of exercising discretion in the context of time consistency problems. 3/ Although the present example, by not including inflation in the policy loss function, departs from the traditional way of illustrating the time consistency problem for monetary policy, 4/ the model of real output determination introduces a temptation to override the announced exchange rate peg. This temptation is mitigated by the third component of the loss function—characterized as a fixed cost of exercising the escape clause—which can be viewed from the perspectives of both positive and normative economics. From a positive perspective, unless one assumes that the policy authorities perceive it to be costly to override announced policy rules, it is difficult to explain why exchange rates exhibit much lower short-term variability under fixed rate regimes than under floating rate regimes. And from both normative and positive perspectives, society can and does design institutional mechanisms— involving both accountability procedures and reward/penalty structures—for insuring that individual policy authorities do not exercise excessive discretion.

2. Optimal behavior and market-clearing conditions

The description of optimal behavior and market clearing conditions reflects the following sequence of actions and events, starting just after the determination of $s_t$ and continuing through the determination of $s_{t+1}$. First, $X_t$ is set (at an exogenous level in the first instance and an optimal

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1/ See Agénor, Bhandari, and Flood (1992).
2/ The literature on speculative attacks, emanating from the pathbreaking paper by Krugman (1979), was inspired by the literature on government price-fixing schemes in markets for exhaustible resources, including the analysis of gold prices by Salant and Henderson (1978). As argued by Obstfeld (1993) and Ozkan and Sutherland (1993), however, it has become difficult to defend the notion that countries with access to world capital markets have an exhaustible supply of foreign exchange reserves, which calls into question the central role that reserve adequacy has played in much of the literature on speculative attacks.
3/ This treatment follows Flood and Isard (1989).
4/ See, for example, Barro and Gordon (1983) and Rogoff (1985).
level in a second case) and the private sector forms its expectations and chooses its asset portfolio, given information about \( X_t \) and the ex ante probability distribution of \( u_{t+1} \). These actions generate the market clearing levels of the unobserved risk premium and the observed home-currency interest rate, given the fixed foreign-currency interest rate. Second, the realization of \( u_{t+1} \) occurs. Third, the authorities set \( s_{t+1} \) at the level that minimizes the policy loss \( L_{t+1} \), conditional on \( u_{t+1} \).

The private sector's portfolio choice problem involves allocating wealth between claims on the home government bearing the home-currency interest rate, and claims on the foreign government bearing the foreign-currency interest rate. The literature reflects several approaches to modeling this problem. One approach assumes that residents of the two countries have identical consumption preferences, often specified as Cobb Douglas functions of the amounts of each of the two goods consumed. Another approach assumes that home-country (foreign-country) residents have relatively strong preferences for the home (foreign) good, sometimes specifying each group's utility as a function of the mean and variance of its consumption. In addition to requiring a decision on how to represent consumer preferences, model specification requires a choice between ignoring non-portfolio income or specifying a model of the income earned from goods production.

With regard to these various modeling choices, the approach taken in this paper ignores non-portfolio income, assumes that residents of the two countries hold portfolios of both assets but have different consumption preferences, with each group consuming only the good produced in its own country, and postulates a mean-variance utility function, such that market clearing conditions depend in a simple approximate way on the perceived mean and variance of the one-period-ahead exchange rate. The objective of home-country (foreign-country) residents at time \( t \) is to maximize a function of the mean and variance of their one-period-ahead normalized wealth as valued in the home (foreign) currency after the realization of \( s_{t+1} \).

It is convenient to let \( W \) describe the portfolio wealth of home country residents and \( B \) and \( B^* \) their holdings of home and foreign bonds, with \( W^* \), \( \hat{B} \), and \( \hat{B}^* \) describing analogously the aggregate portfolio of foreign residents. Variables superscripted (not superscripted) with * are measured in foreign-currency (home-currency) units. Accordingly, for home-country

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1/ See, for example, Svensson (1992a).
2/ See, for example, Dornbusch (1983).
residents at time $t$, the levels of initial wealth and one-period-ahead wealth can be expressed as

\[
W_t = B_t + S_t B_t^*
\]

\[
W_{t+1} = B_t (1+r_t^*) + B_t^* (1+r_{t+1}^*) S_{t+1}
\]

\[
= W_t (1+r_t^*) + S_t B_t^* [(1+r_{t+1}^*) (S_{t+1}/S_t) - (1+r_t^*)]
\]

\[
= W_t (1+r_t^*) + S_t B_t^* [r_{t+1}^* + s_{t+1} - s_t - r_t^*]
\]

where $S$ denotes the level (and $s$ the logarithm) of the exchange rate. To simplify the notation, let

\[
\gamma_{t+1} = r_t - r_t^* - s_{t+1} + s_t
\]

\[
\phi_t = E_t \gamma_{t+1}
\]

\[
\sigma_t^2 = \text{Var}_t \gamma_{t+1} = \text{Var}_t s_{t+1}
\]

\[
b_t = S_t B_t^*/W_t
\]

where $E_t$ and $\text{Var}_t$ denote expected values and variances perceived at time $t$. Note that if the covered interest parity condition is approximately valid, conditions (6) and (7) can be re-expressed as

\[
\gamma_{t+1} \approx f_t - s_{t+1}
\]

\[
\phi_t \approx f_t - E_t s_{t+1}
\]

where $f_t = s_t + r_t - r_t^*$ is the logarithm of the forward exchange rate. The term, $\gamma_{t+1}$, represents the ex post measure of the forward rate forecast error. The risk premium, $\phi_t$, can be viewed as the expected excess yield on home-country securities that is required ex ante to compensate market participants for the risk of realignment. It also measures the forward-rate forecast bias, or more precisely, the expected value ex ante of the forward rate forecast error.

Assume that the utility function to be maximized is

\[
U = U(E_t(W_{t+1}/W_t), \text{Var}_t(W_{t+1}/W_t))
\]

Note that the arguments of $U$ can be expressed as

\[
E_t(W_{t+1}/W_t) = 1 + r_t - b_t \phi_t
\]

\[
\text{Var}_t(W_{t+1}/W_t) = (b_t)^2 \sigma_t^2
\]

Thus, for home-country residents the optimal choice of the portfolio share $b_t$ must satisfy the first order condition
where $U_1 > 0$ and $U_2 < 0$ are the first derivatives of $U$ with respect to its two arguments and $\theta = -2U_2/U_1 > 0$ reflects the degree of risk aversion.

The portfolio choice problem for foreign-country residents can be specified and solved analogously. The analogs of (5), (9), (11), (12), and (13) are

(5a) \[ \ddot{\omega}_{t+1}^* = \ddot{\omega}_t^*(1+r_t^*) + (\dot{b}_t/S_t)(r_t-r_t^*) s_{t+1} + s_t \]
(9a) \[ \ddot{b}_t = (\dot{b}_t/S_t)/\ddot{\omega}_t^* \]
(11a) \[ E_t(\ddot{\omega}_{t+1}/\ddot{\omega}_t^*) = 1+r_t^* + \ddot{b}_t \phi_t \]
(12a) \[ \text{Var}_t(\ddot{\omega}_{t+1}/\ddot{\omega}_t^*) = (\ddot{b}_t)^2 \sigma_t^2 \]
(13a) \[ \ddot{b}_t = \phi_t/\theta \sigma_t^2 \]

Accordingly, the market clearing condition for claims on the foreign public sector can be written as

(14) \[ \ddot{b}_t^* - X_t = b_t(\ddot{\omega}_t/S_t) + (1-\ddot{b}_t)\ddot{\omega}_t^* - \ddot{\omega}_t^* - \ddot{\omega}_t^* \phi_t/\theta \sigma_t^2 \]

where $\ddot{b}_t^*$ is the stock of such claims that would be pushed into private portfolios in the absence of any official holdings of foreign exchange reserves by the home-country authorities and $\ddot{\omega}_t^* = (\ddot{\omega}_t/S_t) + \ddot{\omega}_t^*$ is the initial level of aggregate private wealth valued in foreign currency. Note that the stock of non-interest-bearing monetary assets does not enter the analysis explicitly and thus is implicitly assumed to remain constant in this example; hence, all official intervention is sterilized. The analysis concentrates on the case in which market participants have full knowledge of $X_t$ at time $t$; intervention operations are not kept secret.

The next steps in solving the model are to derive the expected exchange rate and the market clearing domestic interest rate and risk premium that would emerge at any level of $X_t$. Note that, whenever it is optimal to change the exchange rate peg, the optimal value at which to set the new peg, $s_{t+1}^0$—after observing the realization of $u_{t+1}$—is determined by the first-order condition $3L_t s_{t+1}/\partial s_{t+1} = 0$; in particular, from (1) and (3)

(15) \[ s_{t+1}^0 = \frac{y_t^x}{\alpha} - \frac{u_t^x}{\alpha} s_t + \frac{u_{t+1}^x}{\alpha} \]
Whether or not a realignment to \( s_{t+1}^0 \) dominates a decision to maintain the peg at \( s_t \) depends on whether \( L_{t+1}(s_{t+1}^0) < L_{t+1}(s_t) \). From (1) and (3) it is straightforward to demonstrate that maintaining the peg at \( s_t \) is optimal if and only if \(-c \leq y_t-aX_t+u_{t+1} \leq c\). This condition provides the basis for characterizing the market clearing conditions that would emerge under rational expectations prior to the realization of \( u_{t+1} \), given information about \( y_t, X_t, a, c, \) and the ex ante probability distribution of \( u_{t+1} \).

To illustrate, consider a relatively simple example in which \( u_{t+1} \) is uniformly distributed on the interval \([-U,U]\). For this example, the set of possible combinations of initial conditions (in period \( t \)) and parameter values gives rise to four cases:

\[(16)\]

Case 1: \(-c < y_t-aX_t-U < y_t-aX_t+U < c\)

Case 2: \(y_t-aX_t-U \leq -c \leq y_t-aX_t+U \leq c\)

Case 3: \(-c \leq y_t-aX_t-U \leq c \leq y_t-aX_t+U\)

Case 4: \(y_t-aX_t-U < -c < c < y_t-aX_t+U\)

In case 1 there is no incentive to realign under any feasible realization of the shock. 1/ In case 4 there are incentives to depreciate the domestic currency under large positive realizations of \( u_{t+1} \) and to appreciate under large negative realizations. In cases 2 and 3 the incentives to realign are one-sided.

Consider case 3, in which a sufficiently large positive shock would overheat the home-country economy and provide an incentive to appreciate the home currency. 2/ In such a setting, market participants—with an understanding of this incentive and knowledge of the ex ante probability distribution of the shock—will rationally form expectations about \( s_{t+1} \) such that, from (15) and (16), using the notation \( z = y_t-aX_t+u_{t+1} \) 3/

\[(17)\]

\[
E_t s_{t+1} = s_t - \frac{1}{a} \int_{c}^{y_t-aX_t+U} (z/2U)dz
\]

\[
= s_t - \frac{1}{4aU} [(y_t-aX_t+U)^2 - c^2], < s_t
\]

1/ It is easily shown in this case that \( \phi_t = 0 \) and \( r_t = r_t^* \).
2/ The same incentive would exist if the model were extended to make home-country output a negative function of the domestic interest rate as well as a negative function of the relative price of home-country output.
3/ Recall that maintaining the peg at \( s_t \) is optimal if \( z < c \).
Similarly, the probability of realignment can be expressed as

\[ \text{Prob}(s_{t+1} < s_t) = \int_{c}^{y_t - aX_t + U} (1/2U)dz = m/2U \]  

where

\[ m = y_t - aX_t + U - c \]

Note that (17) gives a nonlinear (in this case, quadratic) expression for the magnitude of the expected realignment as a function of \( y_{t+1} \), which can be seen from (3) to represent the maximum degree to which the economy could be left overheated (i.e., the upper bound on \( y_{t+1} \)) in the absence of realignment. The level of \( X_t \) does not affect the conditional probability of \( y_{t+1} \) in the event of no realignment, but (when \( a > 0 \)) it does affect the authorities' incentives to realign and thereby influences the ex ante assessments of realignment probabilities and expectations by rational market participants.

From (6), (7) and (17), the risk premium—or equivalently, the expected error in using the forward rate as a predictor of the future spot rate—can be expressed as

\[ \phi_t = r_t - r^*_t + \frac{1}{4\alpha U} [(y_t - aX_t + U)^2 - c^2] \]

whereas (14) implies

\[ \phi_t = -\theta \sigma_t^2 [(B_t^* - X_t - \bar{\theta}_t^*)/\bar{\sigma}_t^*] \]

Thus,

\[ r_t = r^*_t - \frac{1}{4\alpha U} [(y_t - aX_t + U)^2 - c^2] - \theta \sigma_t^2 [(B_t^* - X_t - \bar{\theta}_t^*)/\bar{\sigma}_t^*] \]
In general, as can be seen from (21), the sign of the risk premium depends on the sign of \((B^*-X_t^*-\hat{W}_t^*)\). If the foreign country is an international creditor and the home country has positive net holdings of foreign exchange reserves, \(B^*-X_t^*-\hat{W}_t^*<0\) and \(\phi_t>0\).

Condition (21) indicates that time variation in the risk premium has two main sources: variation in portfolio stock variables and time variation in \(\sigma_t^2\), which in this particular example has the nonlinear form 1/

\[
\sigma_t^2 = \frac{1}{2\alpha^2} \left\{ \frac{m}{4}(m+2c)^2(1-\frac{m}{2U})^2 + \frac{m^3}{12} \right\}
\]

For case 3, as defined in condition (16), m ranges from 0 when \(y_t\) and \(X_t\) are such that \(y_t-aX_t+U=c\), to \(2U\) at \(y_t-aX_t-U=c\); and it is easily seen from (23) that \(\sigma_t^2=0\) at \(m=0\) and \(\sigma_t^2=U^2/3\alpha^2\) at \(m=2U\). As can also be seen from (18), the ex ante probability of realignment increases monotonically from 0 to 1 as m increases from 0 to 2U.

These results require careful interpretation. If, as one extreme case, the authorities had no freedom to vary \(X_t\), m would vary directly with \(y_t\) (given the parameter \(c\) and the probability distribution of the shock), and variation over time in the state of the economy would lead to time variation in the probability of realignment and the magnitudes of realignment expectations and the risk premium. There is a clear possibility, moreover, that the magnitude of realignment expectations would be strongly correlated with the perceived risk of realignment and hence the magnitude of the risk premium. 2/ At the other extreme, as evident from conditions (17)-(19) and (23), with complete freedom to vary \(X_t\) through sterilized intervention, the authorities—if they chose to do so—could prevent time variation in the state of the economy \(y_t\) from generating time variation in the ex ante probability of realignment and the magnitude of realignment expectations, while largely mitigating the time variation in the risk premium. 3/

The analysis now shifts to the case in which \(X_t\) is a policy choice variable. Before solving for the optimal level of foreign exchange

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1/ See the Appendix for a derivation.
2/ The magnitude of realignment expectations is a monotonically decreasing function of \(y_t\) (given \(X_t\)) in case 3, independently of the parameters \(c\) and \(U\). By contrast, the nature of the relationship between realignment risk and the state of the economy depends on the relative magnitudes of these parameters. Computer solutions show, for example, that the magnitude of realignment risk is a (near)-monotonically increasing function of \(y_t\) for \(c\leq U\), whereas a plot of \(\sigma_t^2\) against \(y_t\) has an approximately inverted-U shape for \(c=10U\).
3/ Recourse to sterilized intervention on such a scale would prevent time variation in the risk of realignment but not in the asset stock and wealth variables that enter condition (21).
reserves, consider the effects of sterilized intervention on the probability and expected magnitude of realignment. By differentiation of (17) and (18)

\[
\frac{\partial E_t s_{t+1}}{\partial X_t} = \frac{a}{2aU}(y_t - aX_t + U)
\]

\[
\frac{\partial \text{Prob}(s_{t+1} < s_t)}{\partial X_t} = -\frac{a}{2U}
\]

Thus, since \(y_t - aX_t + U\) is unambiguously positive in case 3, sterilized intervention purchases of foreign exchange (sales of home-currency securities) unambiguously reduce the expected magnitude and probability of an appreciation of the home currency when \(a > 0\), but have no effect on either unless the policy objective function attaches non-zero costs to foreign exchange valuation losses. Sterilized intervention is effective in influencing exchange rate expectations and the perceived risk of realignment if and only if \(a > 0\), and the effectiveness of intervention on the expected magnitude and probability of appreciation is directly proportionate to \(a\).

The next step is to derive the level of \(X_t\) that the policy authorities regard as optimal ex ante, before observing the realization of \(u_{t+1}\). This requires solving the first-order condition \(\frac{\partial (E_t L_{t+1})}{\partial X_t} = 0\) which, from (1), requires

\[
\frac{\partial E_t (y_{t+1})^2}{\partial X_t} + \frac{\partial E_t c_{t+1}}{\partial X_t} + 2a\alpha \frac{\partial \phi_t}{\partial X_t} = 0
\]

Using (2), (3), (15), (18), and (19), it is straightforward to establish that

\[
\frac{\partial E_t (y_{t+1})^2}{\partial X_t} = -\frac{a}{2U}(m+c)(m+c-2U)
\]

\[
\frac{\partial E_t c_{t+1}}{\partial X_t} = -\frac{ac^2}{2U}
\]

Substituting these expressions into (26), and simplifying, implies that the optimal stock of foreign exchange reserves must satisfy

\[
H + 2a\phi_t (1+\eta) = 0
\]

where \(\eta = (X_t/\phi_t) \frac{\partial \phi_t}{\partial X_t}\) and

\[
H = -(m/2U)(m+2c) + (m+c)
\]

Note that \(H\) corresponds to the sum of the expressions in (27) and (28), divided by the positive parameter \(a\). Thus, \(H > 0\) corresponds to a situation in which an increase in \(X_t\) reduces the expected loss associated with the
fixed costs of realignment by less than it increases the expected loss associated with the possibility of an overheated economy, given that it reduces the probability of a realignment to cool the economy. Note also, from (19), that at \( m=2U \), \( X_t=(y_t-U-c)/a \) and \( H=-c \); while at \( m=0 \), \( X_t=(y_t+U-c)/a \) and \( H=c \). Accordingly, for cases in which the term \( 2\alpha \phi (1+\eta) \) in (29), and hence \( H \), is smaller than \( c \) in absolute value, the optimal level of \( X_t \) must be such that \( 0<m<2U \). Such cases provide an example in which it is optimal for the authorities not to accumulate so large a quantity of foreign exchange reserves that they would be conditionally motivated to refrain from adjusting the exchange rate peg under even the largest possible realization of the shock (that is, not to increase \( X_t \) to the point at which \( m=0 \)). Rather, under the welfare parameters embodied in the policy objective function, the costs associated with an overheated economy make it important for the authorities to retain some willingness to exercise the escape clause (that is, to adjust the exchange rate peg) in response to large realizations of the shock.

IV. Concluding Remarks

This paper has argued that, for fixed-but-adjustable exchange rate arrangements, the analysis of exchange market pressures and interest differentials can usefully be extended by focusing explicitly on the policy optimization problem. In developing a simple illustrative model of exchange market pressures, it follows several other recent papers in characterizing the policy decision as a choice between adhering to a simple rule or exercising an escape clause. It departs from other studies of the relationship between realignment expectations and macroeconomic state variables, however, by relinquishing the assumption of uncovered interest parity and exploring the implications of the policy optimization approach for analyzing the behavior of the risk premium, the sources of forward-rate bias, and the effectiveness of sterilized intervention in influencing exchange rate expectations and the perceived risk of realignment.

The example developed in the paper has emphasized that if market participants are rational, their realignment expectations will depend on the fundamental macroeconomic state variables that enter the policy objective function, and quite likely in a nonlinear way. Moreover, the co-movements of realignment expectations and the risk premium may be strongly correlated, thus pointing to a possible bias in constructed measures of realignment expectations, although not necessarily a bias that is quantitatively large. The example also has emphasized that the relationship between realignment expectations and macroeconomic fundamentals may depend critically on the extent to which the authorities have political independence to conduct sterilized intervention operations, which expose taxpayers to the risk of foreign exchange valuation losses.

The escape-clause framework with policy optimization integrates two lines of thinking about the possible causes of forward rate prediction bias in a rational expectations environment. An important perspective in this
regard is that the incentives of the policy authorities create a peso problem for market participants: in each period the ex ante probability of a realignment is generally positive, even though realignments occur infrequently; and persistence in the fundamental macroeconomic state variables that influence the realignment decision leads, during intervals of no realignment, to positive serial correlation in realignment expectations and prediction errors. Related to this, when the uncovered interest parity assumption is abandoned, the escape-clause framework provides a model of the risk premium that integrates traditional concepts based on portfolio stocks and wealth variables with insights about the determinants of time variation in the risk of realignment within a peso-problem environment. Such an integrated model may be quite relevant for understanding forward rate prediction bias in adjustable peg regimes. It is not clear, however, whether the policy optimization framework can appropriately be extended to provide parallel perspectives on forward rate bias in floating exchange rate systems.

The policy optimization framework also provides useful perspectives on the effectiveness of sterilized intervention, including the perspective that the effects of intervention on the risk premium and on expectations about future exchange rates are not independent of each other. In analyzing the effectiveness of intervention, it is important to specify whether market participants are assumed to have complete information about the incentives of the policy authorities. Under the assumptions of complete information and rational expectations, sterilized intervention cannot signal new information about the authorities' ultimate policy objectives, but it can signal that the authorities have taken on a larger exposure to foreign exchange valuation losses and can rationally be regarded to have strengthened endogenously their incentives to resist a realignment. The strength of such a signal in influencing exchange rate expectations and the perceived risk of realignment depends critically on the costs of foreign exchange valuation losses. Society can design accountability procedures and reward/penalty structures that make it very costly for the authorities to incur valuation losses, thereby making them more reluctant to realign the exchange rate following an increase in their exposure to valuation losses. In theory, this could make sterilized intervention very effective in influencing market assessments of realignment prospects. By the same token, if the authorities perceive that valuation losses have no costs, sterilized intervention will have no effect on realignment expectations in a rational expectations environment with complete information about the authorities' incentives.

When valuation losses are perceived to be costly and sterilized intervention is effective in influencing market assessments of realignment prospects, the time variation of realignment expectations and the risk premium may depend importantly on the extent to which there are explicit or implicit constraints on the use of sterilized intervention as a policy instrument. If the authorities face no restrictions on their resort to sterilized intervention and regularly adjust their foreign exchange holdings to minimize an objective function that attaches costs both to variability in

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fundamental macroeconomic state variables and to foreign exchange valuation losses, their use of intervention as a policy instrument can prevent time variation in the state of the economy from generating time variation in realignment expectations, while also greatly mitigating the time variation in the risk premium. Thus, in empirical attempts to relate realignment expectations or the risk premium to macroeconomic state variables, it may be important, as noted earlier, to take account of institutional considerations governing the use of sterilized intervention.

As a final issue, it is useful to reflect on whether the analytic framework in this paper can help interpret the events that occurred in European exchange markets during 1992-93. Perhaps the most puzzling aspect of those developments is the fact that constructed measures of realignment expectations for European exchange rate pegs—and observed market pressures on those pegs—did not become significantly large until the summer of 1992, 1/ even though the macroeconomic conditions then prevailing had emerged much earlier. From one perspective, the fact that the rise in realignment expectations did not coincide with a significant change in macroeconomic fundamentals could suggest that the development of exchange market pressures represented a pure speculative attack that was unrelated to macroeconomic fundamentals.

A different contention, consistent with the example explored in this paper, is that realignment expectations remain subdued and unresponsive to macroeconomic imbalances until either the scope for sterilized intervention becomes limited or political support for (the weights in) the policy objective function erodes. It may be noted, in this connection, that market assessments of realignment possibilities began to change significantly after the Danish rejection of the Maastricht treaty in the referendum held in early June 1992, and in the wake of August opinion polls indicating a significant chance that French voters might also reject the treaty in a referendum scheduled for September 20. 2/ These political signals can be regarded as new information about the extent to which voters were unhappy with the existing state of macroeconomic fundamentals and hence, implicitly, with the "objective functions" that had been guiding the macroeconomic stabilization policies of their authorities. Consistently, the significant change that took place in measured realignment expectations can be regarded as a response to the new information about popular support for prevailing policies toward macroeconomic fundamentals—in particular, a reassessment by market participants of the probabilities that the policy makers or their objective functions might soon change in a manner that would raise the probabilities of realignment. 3/ Accordingly, even though fundamental

1/ Rose and Svensson (1993).
2/ See Research Department, IMF (1993).
3/ To infer that realignment expectations increased significantly is not to say that market participants attached high probabilities to prospects of realignment; indeed, most European exchange rate pegs resisted the summer 1992 pressures.
macroeconomic conditions did not change significantly at the time that realignment expectations began to mount, under this interpretation the increase in realignment expectations was ultimately related to macroeconomic conditions.
Derivation of Condition (22)

Define:

(A1) \[ z = y_t - aX_t + u_{t+1} \]

(A2) \[ z = y_t - aX_t + U \]

(A3) \[ m = z - c \]

(A4) \[ h = \frac{1}{4U} (z^2 - c^2) \]

Then using (15) and (17)

(A5) \[ s^0_{t+1} = s_t - \frac{z}{\alpha} \]

(A6) \[ E_t s_{t+1} = s_t - \frac{h}{\alpha} \]

So

(A7) \[
\sigma^2_t = E_t \left\{ (s_{t+1} - E_t s_{t+1})^2 \right\} = \frac{1}{2U\alpha^2} \int_c^z (h-z)^2 dz
\]

\[
= \frac{1}{2U\alpha^2} \left[ h^2 z - hz^2 + \frac{z^3}{3} \right] \bigg|_c^z
\]

\[
= \frac{1}{2U\alpha^2} \left\{ \frac{1}{16U^2} (z-c)^3 (z+c)^2 - \frac{1}{4U} (z-c)^2 (z+c)^2 + \frac{z^3}{3} \right\}
\]

\[
= \frac{1}{2U\alpha^2} \left\{ (z-c)(z+c)^2 \left( \frac{z-c}{4U} - \frac{1}{2} \right)^2 + (z-c)(z+c)^2 \left( \frac{z-c}{12} \right) - \frac{z(c-z)}{3} \right\}
\]

\[
= \frac{1}{2U\alpha^2} \left\{ \frac{m}{4}(m+2c)(1-\frac{m}{2U})^2 + \frac{m^3}{12} \right\}
\]
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