Working Paper

INTERNATIONAL MONETARY FUND

WP/92/91

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Research Department

The Savings Trap and Economic Take-off

Prepared by Carlos M. Asilis* and Atish R. Ghosh*

Authorized for distribution by Malcolm Knight

November 1992

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Abstract

We develop an overlapping generations model of a developing economy in which 'culture' and technology interact to determine savings, investment and growth. Investment is assumed to involve intermediation or other costs which may, in each period, result in either of two stable equilibria for the savings rate. At the "good" equilibrium, savings and growth are higher than at the "bad" equilibrium. Whether the country attains the good or bad equilibrium in any period depends on each individual's belief about the savings behavior of other agents in the economy. The model implies that fiscal policy or public activities to facilitate private investment can influence saving. In particular, a sustained period of fiscal restraint can shift the economy onto a higher savings and growth path.

JEL Classification Nos.

D50, D91, H30

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*International Monetary Fund and Princeton University, respectively. The views expressed herein do not necessarily reflect those of the institutions with which the authors are affiliated. We would like to thank William Branson, Willem Buiter, Malcolm Knight, and Masao Ogaki for useful discussions and comments.

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<u>Contents</u>

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I. Introduction

> The central problem in the theory of economic development is to understand the process by which a community which was previously saving 4 or 5 percent of its income or less, converts itself into an economy where voluntary saving is running at about 12 to 15 percent of national income or more.
>
> Arthur Lewis (1954)

It has long been recognized that savings behavior is crucial to economic development. Virtually without exception, those countries that have achieved high savings rates have also enjoyed high levels and growth rates of per capita income--although the direction of causality between savings and income growth remains a matter of debate. If one looks at crude cross-sectional scatter diagrams of developing countries' gross domestic savings rates against either the level or the rate of growth of per capita real income, one is struck both by the positive relationships and by the apparent clustering (see Charts 1 and 2). Broadly speaking, there appear to be two distinct groups of developing countries (admittedly, with numerous outliers): countries whose domestic savings rate 1/ is below 10 percent of GDP, whose per capita real income is below US$400 and whose per capita growth rate is below one percent per annum; and countries whose savings rate is above 15 percent and who enjoy both substantially positive per capita growth rates and high levels of income. 2/

The idea that economic development requires a discrete change in savings behavior also appears to receive support from time series studies of specific countries. Indeed, out of the various success stories of economic development a fairly clear pattern emerges: An initial period of low savings--perhaps 5 percent of GDP--and slow economic growth; the "take-off" period during which, in the space of just a few years, the savings rate increases dramatically to more than 10, and perhaps as much as 20 or 30 percent of GDP; and the 'mature economy' phase, marked by declining savings and growth rates. Breaking out of the initial phase may be the most important step towards economic development (Rostow (1960)). Yet reaching

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1/ Defined as the ratio of gross domestic savings (GDP minus total private and government consumption) to GDP.

2/ The Charts use World Bank data for annual average growth of real per capita income (1965-90) and levels of per capita income in 1990 US$ (World Bank Atlas method); the savings rate is gross domestic savings as a fraction of GDP, and the sample covers 97 developing countries for which data are available. Some countries in the sample went from low savings rates and low growth rates to much higher savings and growth rates during this period, so that the picture of two distinct groups may be clouded by these observations.

the take-off point may not be so straightforward. Writing some forty years ago, Ragnar Nurske described the "savings trap" into which many developing countries fall (Nurske (1953), p.5):

> On the supply side, there is a small capacity to save, resulting from the low level of real income. The low real income is a reflection of low productivity, which in its turn is due largely to the lack of capital. A lack of capital is the result of the small capacity to save, and so the circle is complete.

Standard neoclassical models, such as the Diamond (1965) overlapping generations model, are not particularly useful in describing this trap. Indeed, the typical neoclassical model does not really allow for the possibility of a savings trap. In such models a low per capita capital stock implies a very high marginal product of capital, so that even a modest rise in the level of savings and net investment leads to a sharp increase in wages. Therefore, even if savings are equal to a fixed fraction of labor income, the rise in wages ensures subsequent increases in the level of savings. This effect is reinforced when savings levels depend positively on the rate of return on savings, since the initially high marginal product of capital also implies a high real interest rate and a strong inducement to save. In sharp contrast, most stagnating economies have experienced negative--not highly positive--real interest rates. 1/

Another shortcoming of neoclassical growth models is that they do not admit any role for the cultural and institutional factors that are emphasized by development economists. Yet some societies clearly save more than others. The importance of cultural and institutional factors in determining economic behavior remains an open question, of course, but until economists have a much better understanding of the development process it seems premature to dismiss them out of hand. 2/ And it seems at least plausible that a past record of high savings rates may create a high savings 'culture' which then tends to be self perpetuating.

In this paper we develop a model of savings and growth in which a financially closed economy can remain trapped indefinitely in a low savings equilibrium. (In our particular formulation the low savings equilibrium also exhibits negative real interest rates, but this is not essential to the

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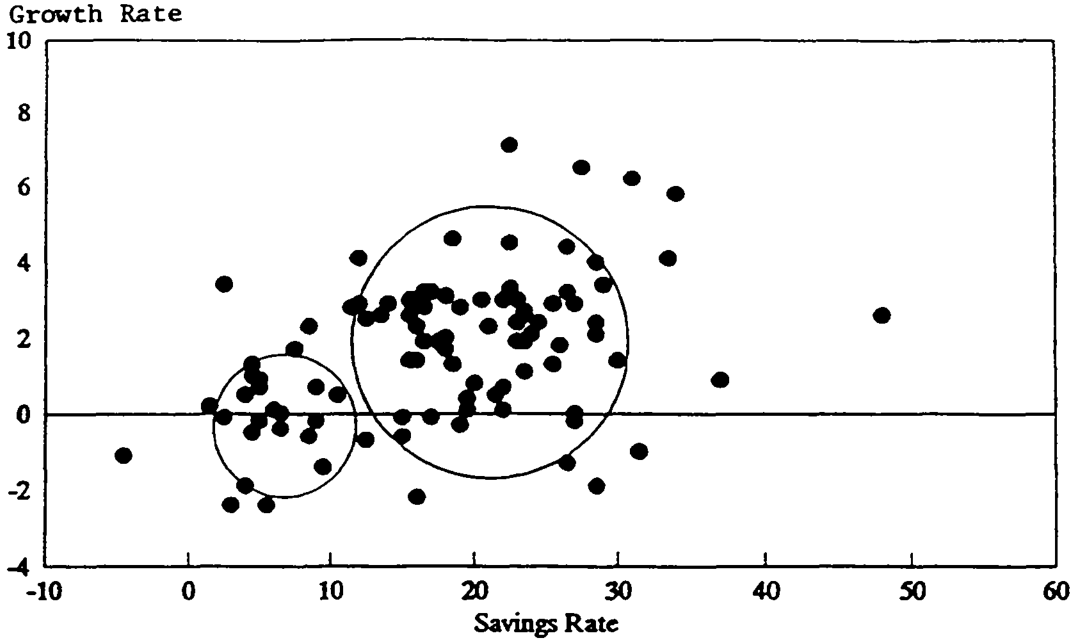
1/ An IMF study of 21 developing countries reports a strong association between negative real interest rates and low GDP growth rates (IMF (1983)); see also Khan and Villanueva (1991) and Polak (1989).

2/ There is, indeed, a large body of literature that explores the link between "cultural factors" and economic development, including the works of Max Weber and Gunnar Myrdal; Packard-Winkler (1989) provides a more recent discussion.

Chart 1

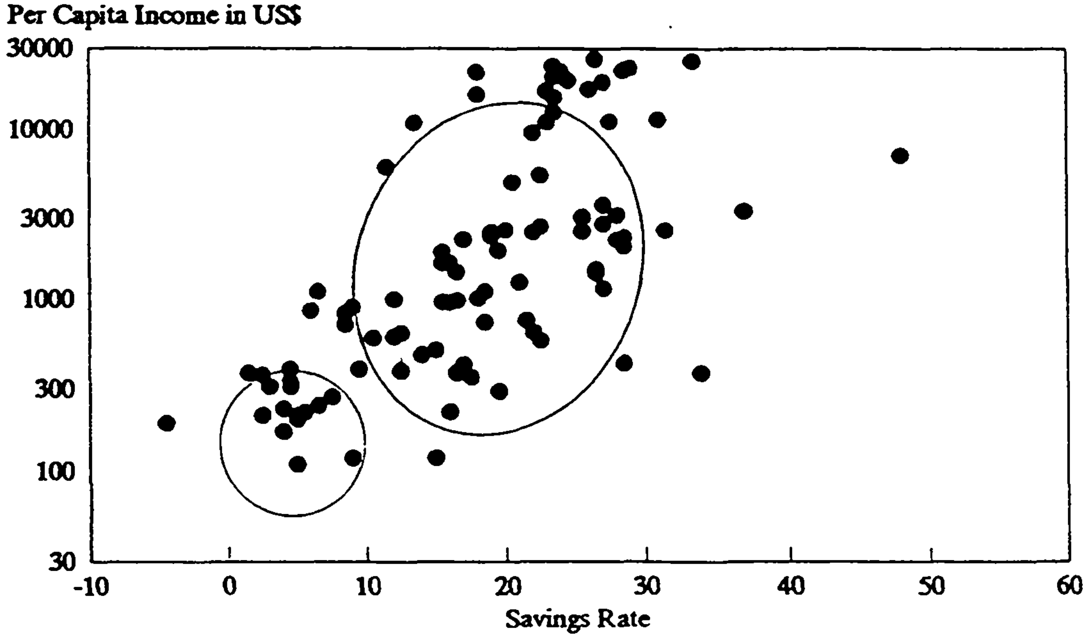
Savings Rate and Growth in Per Capita Income



Averages for 1965-90
Source: World Bank

Chart 2

Savings Rate and Per Capita Income



Savings Rate: Average 1965-1990
Per Capita Income: 1990 US$
Source: World Bank

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story). In our model the economy's history of savings behavior affects current savings decisions so that the culture, in the sense of the customary beliefs and norms of the society, has real economic consequences. Thus an economy in which savings have been low in the past--what one might call a "low savings" culture--creates a self-fulfilling expectation that savings will be low in the current period. Conversely, in countries where savings have traditionally been high, this culture helps to sustain the high savings rate.

Our point of departure from the Diamond model is our assumption that there is a cost to undertaking investment. This cost makes the rate of return an (initially) increasing function of aggregate investment. These increasing returns to investment could arise, for example, from diminishing marginal costs of intermediation or indivisibilities in capital equipment. Certainly there are many investment projects, especially those involving social overhead investment, which have very substantial fixed costs. Such costs, moreover, are likely to be particularly important during the early stages of development when they are large relative to both the existing capital stock and the level of labor income.

We assume that the increasing returns to investment exist at the level of the aggregate production function. 1/ This could either reflect the presence of positive spill-overs among individual investment projects or the assumption that investment projects are large relative to the savings of the individual investor. For example, the rate of return to building a factory will presumably be much higher if the country has already invested in power generation, transportation, and telecommunications. Alternatively, the increasing returns to aggregate investment could result from the high cost of capital equipment. This would mean that the savings of many individual investors need to be channeled into investment projects before any positive rate of return is possible. In either case, the important assumption is that the rate of return to an individual agent depends upon the aggregate level of savings.

The idea that increasing returns to investment may be important in developing countries extends at least as far back as the writings of Young (1928), Rosenstein-Rodan (1943) and Scitovsky (1959). 2/ As Chenery (1979), who also provides empirical support for this idea in Latin America, writes:

1/ The term "aggregate" production function need not refer to the national level. If there is little borrowing and lending across regions then the aggregate production function, as used here, can simply mean the regional production function.

2/ For an excellent recent treatment of the Rosenstein-Rodan model see Murphy, Shleifer and Vishny (1989).

> In his celebrated article on the "Problems of Industrialization of Eastern and Southeastern Europe" Rosenstein-Rodan suggested that a group of investments that would be profitable if considered together may separately appear unprofitable and may not be undertaken by an individual investor who does not take advantage of external economies.

In the Solow (1956) model, the assumption of (initially) increasing returns to investment leads to multiple <u>steady states</u> (not multiple instantaneous equilibria), but the steady state associated with the low capital/labor ratio is not dynamically stable (see Branson (1989) for a discussion). 1/ If population growth is an increasing function of income, however, it is possible to have a dynamically stable low-steady-state level of per capita income (Buttrick (1958) and Nelson (1956)). 2/ While these models imply the (possible) existence of a savings trap, they also imply that the savings <u>function</u> is the same regardless of whether a country is on a path towards the low steady state or the high steady state. This property seems overly mechanical, however, since it allows <u>only</u> history to matter: given an initial capital stock, all economies are assumed to have the same dynamics. Krugman (1991) has emphasized that in models with increasing returns both history and expectations should matter. 3/

As a description of a savings trap, moreover, these models suffer from the shortcoming that the savings rate does not change dramatically even when the economy breaks out of the low equilibrium. But as many commentators have claimed, there seems to be something <u>qualitatively</u> different about the behavior of an economy as it passes into the take-off phase, which is distinct from normal economic fluctuations or growth. 4/

We restrict our analysis to a closed economy context. While opening the economy to foreign capital may in principle alleviate the problem of

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1/ For reasonable specifications of the production function, the low steady state implies a capital/labor ratio of zero; that is, the economy necessarily "implodes" at the bad steady state.

2/ Becker, Murphy and Tamura (1990) provide a more modern treatment of this idea.

3/ Krugman (1991) and Matsuyama (1991) discuss the role of multiple equilibria in models where the action of moving from one sector to another is costly for labor, but increasing returns prevail in the manufacturing sector. Accordingly, only if each agent believes that many others will move to manufacturing in the future will it be worthwhile for him to move into the manufacturing sector himself.

4/ Schumpeter, in fact, defines economic development as "that kind of change arising from within the system which so displaces its equilibrium point that the new one cannot be reached from the old one by infinitesimal steps" Schumpeter (1959) p. 64.

the "savings trap," in practice, particularly since the Second World War, net inflows of external savings have generally been quite modest as a percentage of total investment requirements (this holds both for industrialized countries and developing countries; see Feldstein and Horioka (1980) and Dooley, Frankel and Mathieson (1987)). 1/ Moreover, even in case where foreign capital is more freely available, the savings trap is likely to manifest itself as an "investment trap" in which neither domestic nor foreign residents are willing to invest in a developing country without the knowledge that other investors will invest simultaneously. Thus while savings by domestic residents may no longer be low when they can invest abroad, there is no guarantee that the country itself will enjoy high levels of investment and strong economic growth.

In the model developed here we neglect international capital flows and assume that domestic investment must be financed from domestic savings. 2/ We also explicitly assume that the savings rate is an increasing function of the rate of return. 3/ The model further assumes that the rate of return on an individual's savings depends upon the level of aggregate savings, and each individual's optimal savings depend upon the economy-wide interest rate. When each agent thinks that all other agents will undertake a low level of savings he knows that, because the fixed cost of investment will be spread over a small quantity of savings, the real return to savings will be low. Accordingly, he will undertake a low level of savings himself (we assume that the wage rate--out of which savings must be financed--is sufficiently low that no individual can save enough to ensure a high rate of return when no one else in the economy has a high savings rate). Conversely, the expectation of a high aggregate savings rate induces a high level of savings by each individual. Thus multiple equilibria may emerge in each period.

Obviously, a crucial element in this story is savers' knowledge that there is a fixed cost of investment, combined with the beliefs held by each agent about the savings that will be undertaken by all other agents. We assume that these beliefs follow a Markov process, which means that over time the economy builds up a high-savings or a low-savings 'culture'.

1/ Between 1960 and 1985, for example, net flows of foreign direct investment into developing countries averaged only 0.5 percent of GDP, and portfolio investment another 0.45 percent, so that the total inflow averaged less than one percent of GDP. (Griffin (1989), p. 83).

2/ Despite the increased role of capital flows in recent years many developing countries, particularly the poorer ones, still have only limited access to international credit markets.

3/ Theoretically, savings can be either an increasing or a decreasing function of the rate of return, depending upon whether income or substitution effects dominate. There is a large body of literature which examines the relation between savings and interest rates in developing countries; Fry (1982) surveys some of these studies and concludes that real interest rates have a positive effect on savings.

Although there are two possible equilibria in every period, (where period length, in our model, corresponds to the length of a generation, approximately twenty years) the assumption of Markov beliefs means that the equilibria will be correlated over time. Indeed, in the absence of policy interventions a country that starts out with a low (high) equilibrium will continue to follow the dynamic path implied by the low (high) equilibrium. On the other hand, there are policies--such as certain types of fiscal measures or the establishment of savings cooperatives or development banks--that may succeed in lifting the economy from the low equilibrium path to the high equilibrium path. One particularly interesting aspect of the model is that if such a policy intervention or positive exogenous shock causes the country to break out of the savings trap, then the savings rate can jump to a higher level, in a way that is often observed in countries during their take-off stage. 1/

Our model is related to, though distinct from, the "endogenous growth" models of Romer (1986), Lucas (1988) and others. 2/ That literature seeks to explain the persistent differences in rates of per capita output growth across countries, whereas our concern is the savings trap into which many developing countries appear to have fallen. 3/ Thus our concern here is not in explaining the differences in growth rates between, say, Japan and the United States, but in analyzing why and how some countries have had such a dramatic jump in savings and growth rates in the past 25 years, while many

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1/ There is a substantial body of literature that seeks to link savings behavior in developing countries with either the level or the growth rate of per capita income (Gersovitz (1988) provides a survey). Usually these studies focus exclusively on the effect of income on savings, and their results are somewhat mixed: Leff (1969) and Modigliani (1970) find that the rate of growth of income affects savings, while the level of per capita income is a much less significant determinant. Ram (1982), in contrast, finds fairly strong support for the level of per capita income affecting savings. In the model developed here, of course, savings, and both the level and rate of growth of income, are all endogenous. Nevertheless, the "good" dynamic path will be associated with higher levels of savings and income, as well as a higher growth rate of income, as compared to the "bad" dynamic path. It is not clear that linear regressions that incorporate either the level or growth rate of income will be able to yield very robust results on the link between savings and income if economic development does indeed involve a discrete jump from one dynamic path to another of the type considered here.

2/ See, for example, Sala-i-Martin (1990), Rebelo (1991), Grossman and Helpman (1991) and the papers in the special issue of the Journal of Political Economy, October 1990.

3/ The closest model to ours is the human capital model of Becker, Murphy and Tamura (1990). In their model, multiple steady states can occur but not multiple equilibria within a period. Azariadis and Drazen (1990) develop a model in which sufficient human capital must be acquired before the economy can take off.

other developing countries have suffered from both low savings rates and low growth rates of per capita income. 1/

The plan of the paper is as follows. Section II outlines the model and describes its main features. Since the model is highly nonlinear, however, it is difficult to derive all of the properties of the equilibria. In section III, therefore, we consider a parametric case which yields simple analytic results. In section IV we discuss the various policy measures that can be undertaken to launch the economy onto the good equilibrium path. Section V provides some brief concluding remarks.

II. The Model

We consider an economy populated by an infinite sequence of overlapping generations. In each period a new generation consisting of N agents is born. As in Diamond (1965), agents are assumed to live for two periods, earning labor income and saving while young and consuming their savings when old. For every agent, preferences are defined over two-period consumption streams and are time separable. Thus the *i*th agent's preferences are given by:

$$u^i(c^i_t) + \beta^i u^i(c^i_{t+1}) \qquad i = 1,\ldots, N \tag{1}$$

where β^i denotes the *i*th agent's discount factor so that $0 \leq \beta^i \leq 1$. We assume that $u^i = u$, $\beta^i = \beta$ for all $i = 1, \ldots, N$. To obtain analytic results we impose some additional structure on preferences; to wit, the utility function is assumed to be of the form:

$$u^i(c^i) = \frac{(c^i)^{(1 - \sigma^i)}}{(1 - \sigma^i)} \tag{2}$$

where we assume that the coefficient of risk aversion, σ^i, is the same across individuals and less than unity: $\sigma^i = \sigma < 1$, $i = 1,\ldots, N$.

The production technology is a time invariant Cobb-Douglas function, given by:

$$Y\,(K_t, L_t) = K^{\alpha}_t\, L^{(1 - \alpha)}_t \qquad 0 < \alpha < 1 \tag{3}$$

1/ The sudden spurt in growth is, as in our model, a property of the Gerschenkron model (Gerschenkron (1962)).

where K_t denotes the aggregate capital stock at the beginning of period t, determined in period t - 1, while L_t denotes the aggregate labor supplied in period t. For simplicity, we assume labor to be inelastically supplied; each agent, moreover, is endowed with (1/N) units of labor. Workers are paid the marginal product of labor.

We assume that there is a cost to undertaking investment that arises, for example, from costs of intermediation or the need for investment in a minimum level of domestic infrastructure. The important assumption is that the cost is a function of the aggregate level of investment (or savings). This cost is assumed to be borne entirely by owners of capital so that there is a wedge, F, between the rate of return paid to owners of capital and the marginal product of capital. 1/ The rate of return to undertaking savings between periods t and t+1 is therefore:

$$r_{t+1,t} = \frac{(S_t - F)}{S_t} \alpha K_{t+1}^{(\alpha-1)} - \delta \tag{4}$$

where S_t is the level of aggregate savings (which equals investment, in equilibrium), δ is the constant rate of depreciation and where:

$$K_{t+1} = (1-\delta)K_t + S_t \tag{5}$$

The ith agent's optimization problem is to choose s_t^i to maximize (1) subject to:

$$c_t^i = \frac{(1 - \alpha)K_t^\alpha}{N} - s_t^i \tag{6}$$

1/ This is an analytically convenient way to specify increasing returns to investment. The implied cost of investment is given by:

$$C(S,K) = \int_0^S (F/\hat{S})f_k(K(1 - \delta)+\hat{S})\, d\hat{S}$$

therefore, $C_S > 0$ and $C_K < 0$, that is, the cost of undertaking investment is an increasing function of the amount of investment and a decreasing function of the capital stock in place.

$$c^i_{t+1} = s^i_t(1 + r_{t+1,t}) \tag{7}$$

taking as given K_t, and the conjectured savings of all other agents,

$\tilde{S}^{\backslash i}_t = \sum_{\substack{j=1 \\ j\neq i}}^{N} \tilde{s}^j_t$, where the tilde denotes a conjectured value. The

resulting first-order condition implies:

$$s^i_t = \frac{\beta^{(1/\sigma)}(1+r_{t+1,t})^{(1/\sigma-1)}\left\{\frac{(1-\alpha)K^\alpha_t}{N}\right\}}{1 + \beta^{(1/\sigma)}(1+r_{t+1,t})^{(1/\sigma-1)}} \tag{8}$$

We call (8) the *i*th agent's savings condition. An <u>equilibrium</u> consists of an initial capital stock, and sequences $\{(\tilde{S}^{\backslash i}_t)^N_{i=1}\}^\infty_{t=0}$ and $\{(S^{\backslash i}_t)^N_{i=1}\}^\infty_{t=0}$ such that $\tilde{S}^{\backslash i}_t = S^{\backslash i}_t$, and (8) holds for all i and all t, together with (4) and (5). Equilibrium consists of a dated sequence of Nash equilibria in savings levels since, for each t, each agent takes as given the savings decisions of all other agents when choosing his own savings.

Imposing symmetry across agents and multiplying (8) by N yields aggregate savings as a function of the interest rate:

$$S_t = S(r) = \frac{\beta^{(1/\sigma)}(1+r_{t+1,t})^{(1/\sigma-1)}\left\{(1-\alpha)K^\alpha_t\right\}}{1 + \beta^{(1/\sigma)}(1+r_{t+1,t})^{(1/\sigma-1)}} \tag{9}$$

and from (4) and (5) we obtain a second relation between the interest rate and the aggregate level of savings:

$$r_{t+1,t} = r(S) = \frac{(S_t - F)}{S_t} \alpha ((1 - \delta)K_t + S_t)^{(\alpha-1)} - \delta \tag{10}$$

Solving (9) and (10) simultaneously yields the instantaneous equilibrium. As we show in the appendix, $S(r)$ is increasing in r (under the assumption, made above, that $\sigma < 1$) with $S(-1) = 0$, and $r(S)$ is increasing in S for $S < S^*$ and decreasing thereafter, where S^* is the positive root of:

$$Q(S) = S^2 + \frac{F(\alpha - 2)}{(1 - \alpha)} S - \frac{KF(1 - \delta)}{(1 - \alpha)} \tag{11}$$

and $r(0) = -\infty$.

Using these properties we can sketch the two relations (9) and (10). From Figure 1 it is evident that there can be two equilibria: the bad equilibrium (S^b), at which savings are low and the interest rate is low, and the good equilibrium (S^a) with both high savings and a high interest rate. We have drawn the good equilibrium, S^a, as occurring on the backward bending portion of the r(S) curve, though it can also occur on the upward sloping portion. Figure 2 illustrates the determination of the instantaneous equilibria in period t-1. We start with a candidate level of savings, S^a_{t-1}. Given this level of savings, the SE quadrant gives the capital stock in period t. Given a capital stock, the NE quadrant determines the net rate of return, $r_t = [(S-F)/S]f_k - \delta$. Given an interest rate, we obtain the savings level which satisfies condition (9). Finally, this level of savings must be consistent with the original S^a_{t-1}. 1/ Again, we see that there can be two equilibria, denoted S^a and S^b. The intuition behind the existence of these two equilibria is straightforward. Suppose that each agent expects all other agents to undertake a low level of savings. He is also assumed to know that, because of the fixed cost of investment, a low savings outcome will mean that the net rate of return is low as well which, in turn, validates the low level of (individual and aggregate) savings. Conversely, if each agent expects a high savings rate by all other agents, the high level of savings yields a high interest rate which is consistent with the high savings rate.

1/ Notice that the good equilibrium, S^a, can exist either on the upward or the downward sloping portion of the r(S) curve in the NE quadrant. Corresponding to Figure 1, however, even when the good equilibrium occurs on the downward sloping portion of the r(S) curve it necessarily entails a higher interest rate.

Figure 1
Savings and Investment Functions

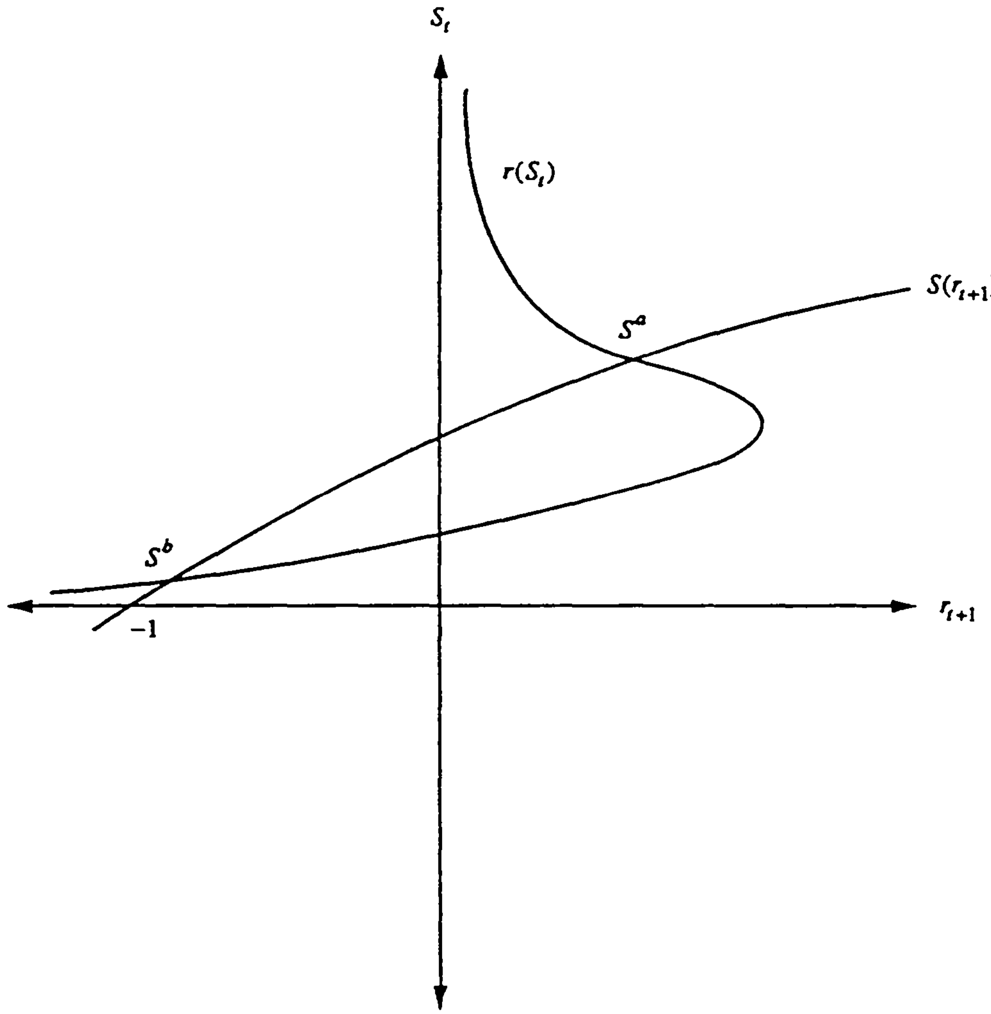
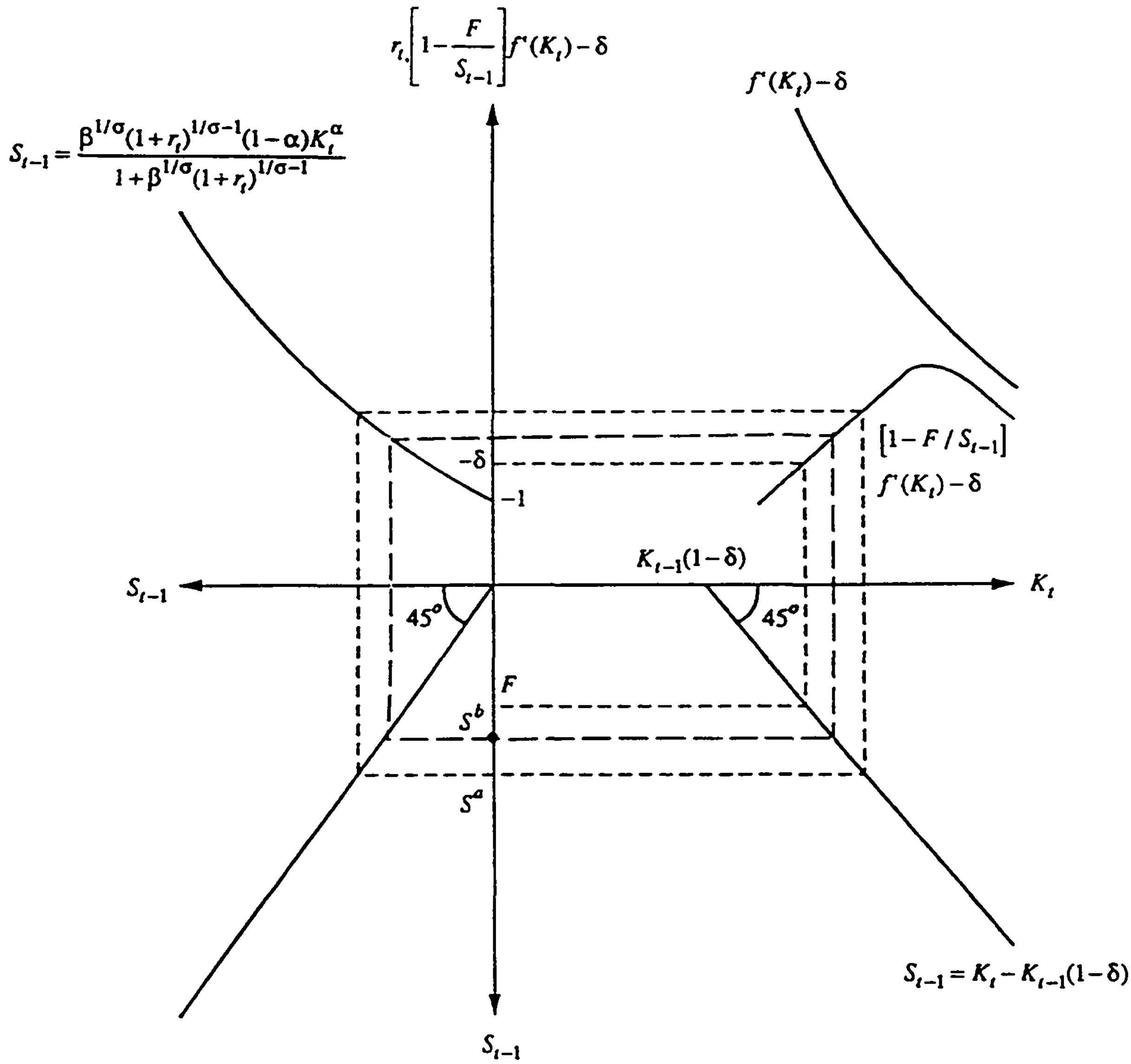


Figure 2
Determination of Instantaneous Equilibrium



Characterizing these equilibria (and, indeed, proving that they exist) is difficult because of the highly nonlinear forms of S(r) and r(S). Our strategy, therefore, will be to establish the most important properties for a special case.

III. Properties of the Equilibria and the Savings Trap

Our goal in this section is to characterize the two equilibria and derive their properties. Three questions, in particular, need to be addressed. First, since the equilibria depend upon each agent's conjecture about the aggregate savings rate, the resulting Nash equilibrium must be game-theoretically stable. That is, a perturbation in any agent's beliefs should not alter the equilibrium. Second, we would like to know the behavior of the model as income rises. Is it possible, for example, to "grow" out of the bad equilibrium? The third, related, question is whether a dynamically stable steady state associated with the bad equilibrium exists, so that once an economy enters the savings trap it remains there indefinitely.

The conditions for the existence of two (instantaneous) equilibria turn out to be intuitively appealing: if the fixed cost, F, is very small (relative to the aggregate wage bill) then only the good equilibrium exists since, in the limit $F \Rightarrow 0$, we return to the standard Diamond model. 1/ Conversely, an instantaneous equilibrium with positive savings will not exist if F is too large (again, relative to the aggregate wage bill) since a net rate of return of -1 would imply zero savings. (The overlapping generations model, with its strong life-cycle savings motive, will have positive savings even when the net rate of return is negative, as long as it is greater than -1). A steady state will not exist if the fixed cost is so large that savings can never cover depreciation of capital. 2/

Rather than attempt to characterize the model in its full generality we focus on a special case, but one that we believe captures the elements of the full model. To wit, we now assume that both the coefficient of risk aversion, σ, and the share of capital, α, are equal to one-half. While a coefficient of risk aversion of (1/2) is eminently plausible, the empirical literature suggests that the share of capital is generally lower, 0.35 being a more typical figure for a developing country.

We begin by characterizing the instantaneous savings equilibrium. For the moment we shall ignore the question of how conjectures are formed. We define a bad (good) equilibrium as one in which each agent holds the

1/ It is the aggregate wage bill which is of relevance here because in the overlapping generations model savings are undertaken by the young and therefore must be financed out of wages.

2/ That is, if $S(\underline{K}) < \delta \underline{K}$ where $\underline{K}$ is the lowest capital stock.

rational belief that the aggregate savings rate will be low (high). In period t, the state variable is then the inherited capital stock, K_t; S_t and $r_{t+1,t}$ are determined in the current period. With the assumption $\alpha = \sigma = (1/2)$, our two conditions for equilibrium become:

$$S_t = S(r) = \frac{\beta^2(1+r_{t+1,t})\left\{(1/2)K_t^{(1/2)}\right\}}{\{1 + \beta^2(1 + r_{t+1,t})\}} \tag{12}$$

$$r_{t+1,t} = r(S) = \frac{(S_t - F)}{S_t}(1/2)((1-\delta)K_t + S_t)^{-(1/2)} - \delta \tag{13}$$

which is convenient because substituting (13) into (12) yields (after algebraic manipulations) a polynomial in S and K, x(S,K):

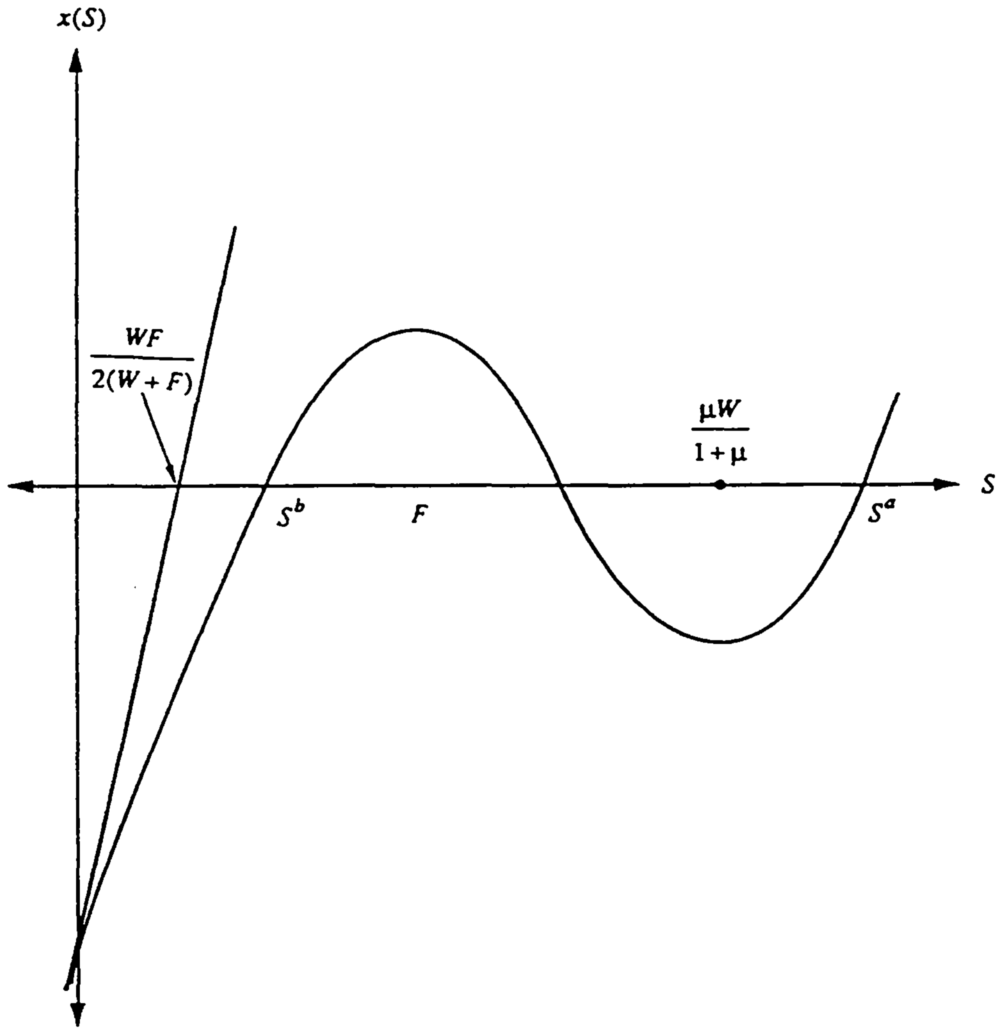
$$x(S,K) = S^2[K(1-\delta) + S]\,[S(1+\mu) - \mu W]^2 - \left\{\frac{S - F}{2}\right\}^2 \beta^4 (W - S)^2 \tag{14}$$

where $\mu = \beta^2(1 - \delta) < 1$, W is the aggregate wage bill, equal to $(1/2)K_t^{(1/2)}$ and where we have dropped time subscripts.

Figure 3 sketches the polynomial x(S,K) for a given inherited capital stock, K. In drawing the figure, we impose the assumption that $F < \frac{\mu W}{(1+\mu)}$. In terms of Figure 1, this condition essentially ensures that the S-intercept of S(r) is above the S-intercept of r(S), so that the two functions intersect. If the fixed cost were too large, then no equilibrium with positive levels of savings and investment could exist. 1/ If F is taken as a datum of the economy, then this condition implies a lower bound on the feasible capital stock, $\underline{K}$:

1/ The condition $F < \mu W/(1+\mu)$ is sufficient, though not necessary, for equilibrium to exist.

Figure 3
Determination of Savings Equilibrium



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$$\underline{K} = \frac{(2\ (1 + \mu)\ F)^2}{\mu^2} \tag{15}$$

We show in the appendix that this assumption on F is sufficient to ensure that two instantaneous equilibria exist, corresponding to the low and the high root of x(S,K): the center root of x(S,K) does not correspond to an intersection of r(S) and S(r) and can be ignored. The low savings equilibrium, S^b, satisfies:

$$S^b\ (K) \simeq \frac{WF}{2(W + F)} < F \tag{16}$$

and the high equilibrium satisfies:

$$S^a(K) > \frac{\mu W}{(1 + \mu)} \tag{17}$$

Consider, next, an increase in the inherited capital stock. From Figure 4 the savings function S(r) rotates upward, while r(S) shifts upwards. It is shown in the appendix that, at the <u>low</u> equilibrium, a sufficient condition for savings to <u>decrease</u> in response to an increase in the inherited capital stock is:

$$F > \frac{\beta^2}{(1 + \mu)} \tag{18}$$

At the <u>high</u> equilibrium, savings are an <u>increasing</u> function of the inherited capital stock. This condition is also sufficient to ensure that $|\ S_k^b\ | < (1 - \delta)$ and $|\ S_k^a\ | < (1 - \delta)$, where S_K denotes the derivatives of the (optimal) savings function at each equilibrium.

The property $S_K^b < 0$ and $S_K^a > 0$ has very important implications for the dynamics of the economy, since it implies that the country cannot simply "grow" out of the bad equilibrium. Increases in the capital stock would be associated with a decline in savings so that the economy would remain trapped at the bad equilibrium. Before proving that a steady state

associated with the low savings equilibrium exists, however, we need to establish the game-theoretic stability of each instantaneous equilibrium.

Recall that each root, S^a and S^b, represents a Nash equilibrium in savings decisions since each agent takes as given the aggregate level of savings. These Nash equilibria will be stable if the composite reaction function is a contraction, in the sense that:

$$\sum_{j \neq i}^{N} \left| \frac{\partial s_t^i}{\partial \tilde{s}_t^j} \right| < 1 \text{ for } i = 1, \ldots, N \tag{19}$$

where $\tilde{s}^j$ is the savings of agent j, as conjectured by agent i. This condition is no more than the N person analog to the familiar diagram of stable reaction functions in a two-person game. To verify that this condition holds, notice that, using the chain rule, $\partial s_t^i / \partial \tilde{s}_t^j$ can be written:

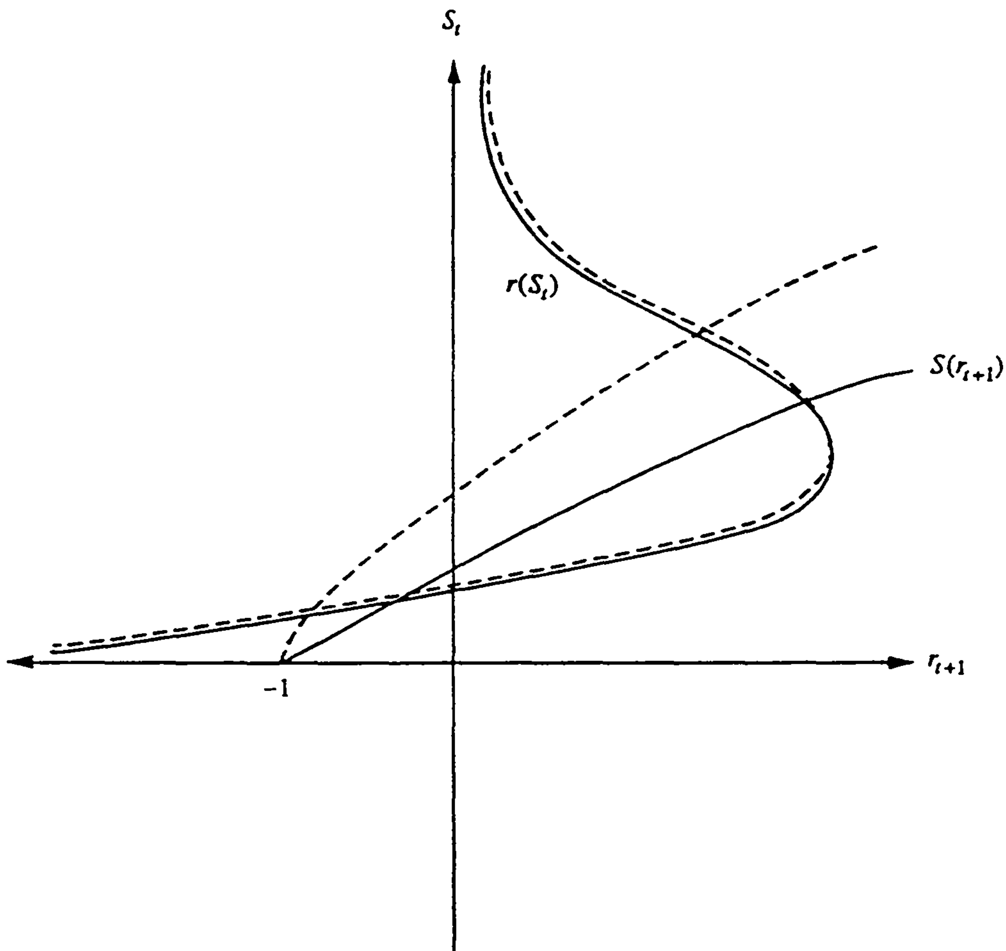
$$\frac{\partial s_t^i}{\partial \tilde{s}_t^j} = \frac{\partial s_t^i}{\partial S_t} \bullet \frac{\partial S_t}{\partial K_t} \bullet \frac{\partial K_t}{\partial \tilde{K}_{t+1}} \bullet \frac{\partial \tilde{K}_{t+1}}{\partial \tilde{s}_t^j} < \frac{1}{(N - 1)} \tag{20}$$

Since $S_t = Ns_t^i$, and $\tilde{K}_{t+1} = (1 - \delta)K_t + \sum_{j=1}^{N} \tilde{s}_t^j$, the condition for game theoretic stability becomes:

$$| S_K | < \frac{N}{(N - 1)} (1 - \delta) \tag{21}$$

but as claimed above and proved in the appendix, both $|S_K^a|$ and $|S_K^b|$ are less than (1- δ); therefore, both roots represent stable Nash equilibria.

Figure 4
Savings Equilibrium:
Effect of an Increase in Inherited Capital Stock



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Suppose, next, that the economy passes through a <u>sequence</u> of either good or bad equilibria:

$$\{S^a(K_0), S^a(K_1), S^a(K_2), S^a(K_3), \ldots, S^a(K_t), \ldots\} \tag{22}$$

$$\{S^b(K_0), S^b(K_1), S^b(K_2), S^b(K_3), \ldots, S^b(K_t), \ldots\} \tag{23}$$

Our next question is whether the economy attains a <u>steady state</u> associated with the good savings equilibrium after a sequence $\{S^a(K)\}$ and a steady state associated with the bad savings equilibrium after a sequence $\{S^b(K)\}$. These steady states will exist as long as the root of the law of motion of capital eventually lies within the unit circle:

$$dK_{t+1}/dK_t = |(1-\delta) + S_K| < 1 \tag{24}$$

The standard Inada conditions ensure that this condition is met for sufficiently large K at the good equilibrium. At the bad equilibrium,

$S_K^b < 0$ and $|S_K^b| < (1-\delta)$ so that this condition is necessarily met as well;

however, this is not quite enough to guarantee the existence of a steady state associated with the bad equilibrium. Recall from (15) that we have a lower bound on $\underline{K}$; for a feasible steady state to exist therefore requires that $S(\underline{K}) > \delta\underline{K}$, as illustrated in figure 5. Otherwise, savings can never cover depreciation and the economy "implodes". 1/ Using (16) this condition becomes:

$$\frac{\underline{W}F}{2(\underline{W} + F)} > \delta\underline{K} \tag{25}$$

Substituting from (15) and simplifying yields a condition on F:

$$F < \frac{\mu^2}{8\delta(1 + 2\mu)} \tag{26}$$

1/ That is, the model no longer provides a sensible description of the economy.

Thus as long as F satisfies (18) and (26), a nontrivial steady state associated with the low savings equilibrium will exist. 1/

Thus far we have shown that in any period there exist two possible savings equilibria which are functions of the inherited capital stock: $S_t^a(K_t)$ and $S_t^b(K_t)$. Moreover, a sequence of good or bad equilibria results in an associated steady state: K_{ss}^a and K_{ss}^b. Yet what is it that determines whether, each period, it is the good or the bad equilibrium that emerges? Clearly, if all agents in the economy believe that aggregate savings will be high, then each agent expects a high interest rate and accordingly undertakes a high savings rate, thus fulfilling the beliefs. Conversely, pessimism about the aggregate savings rate, and therefore the interest rate, will be self-fulfilling as well. How are beliefs themselves formed? In general, economists have little to say on this crucial matter. We consider that it is sensible to think that a 'culture' forms whereby, in the absence of any external factors, a history of high (low) savings leads to beliefs that savings will be high (low) in the current period. By 'culture' here we mean a set of norms, traditions, or common expectations: Factors often stressed by development economists but largely ignored in the standard neoclassical literature.

In order to formalize this idea we posit a system of Markovian conjectures h_t such that agent *i's* conjecture of everyone else's aggregate savings for period t, $\tilde{S}_t^{\backslash i}$, is conditional on period t-1's aggregate savings: 2/

$$\tilde{S}_t^{\backslash i} = h(S_{t-1}) \tag{27}$$

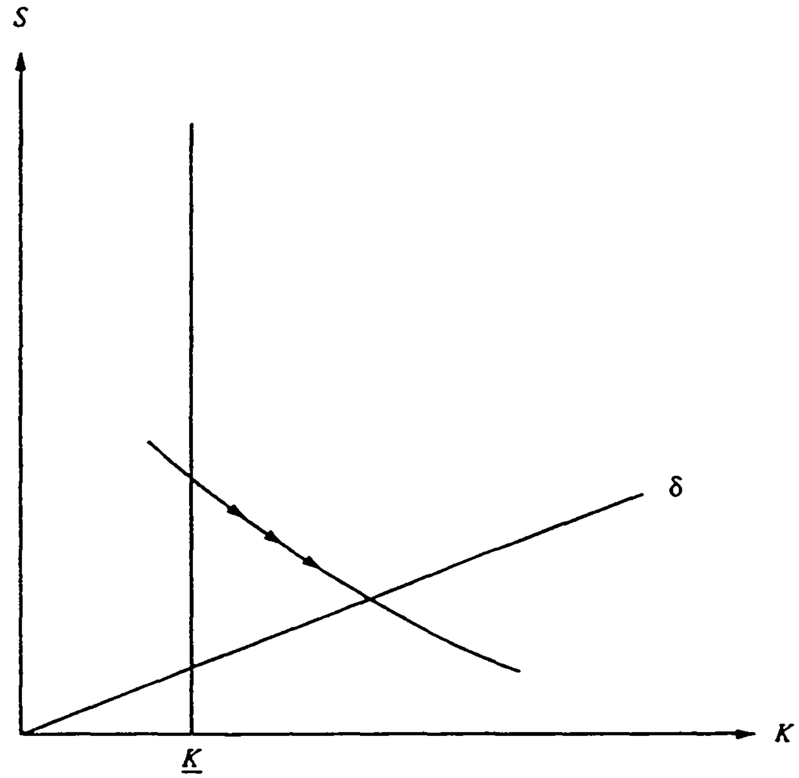
$$\begin{cases} [(N-1)/N]S_t^j(K_t) \text{ w.p.1 if } S_{t-1} = S_{t-1}^j,\ j = a,b \\ [(N-1)/N]S_t^a(K_t) \text{ w.p. } \pi \text{ and } [(N-1)/N]S_t^b(K_t) \text{ w.p. } (1-\pi) \text{ otherwise } 3/ \end{cases}$$

1/ Condition (18) is very much a sufficient condition rather than a necessary one. Simulations suggest the existence of a considerably lower bound on the minimum F but an analytic proof is difficult.

2/ For models embodying Markovian conjectures in the industrial organization literature see e.g. Friedman (1977) (chapter 5). In the following section we introduce taxes and government revenues; for simplicity we assume that all such revenues are invested in the aggregate production function so S_t denotes the sum of private and public savings.

3/ For simplicity, with (correlated) probability across agents.

Figure 5
Bad Equilibrium: Steady State Dynamics



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Thus given a low (high) savings equilibrium in period t-1, the *i*th agent conjectures a low (high) savings equilibrium for period t also. Of course, this system of beliefs does not specify how the first-period equilibrium is chosen, except to assign a probability π to the good equilibrium and a probability $(1-\pi)$ to the bad equilibrium. That is, history chooses whether the economy starts out on the good or bad equilibrium path, but thereafter beliefs are correlated over time. In our opinion, this is quite realistic in that historical accidents can be major determinants of the initial conditions of an economy. 1/

By construction, the system of Markovian conjectures, h, sustains the sequences $\{S^a(K_t)\}$ and $\{S^b(K_t)\}$ as equilibrium paths. It suffices to note that $S_t^j(K_t)$, $j = a,b$, constituting period t equilibria, implies that, faced with conjectures $[(N-1)/N]S_t^j$, the *i*th agent will find it optimal to choose S_t^j/N, $j = a,b$, thus validating the conjecture.

Hence, once the economy starts out in the bad savings equilibrium it will be trapped there indefinitely, eventually reaching the steady state associated with the bad equilibrium. Conversely, if the economy is able to start out along the good savings equilibrium it will continue along a sequence of good equilibria until the steady state is reached. In the next section we examine what policies the government can undertake to push the economy from the bad equilibrium path to the good equilibrium path.

IV. Government Policies

This model accords a potentially important role to fiscal policy in influencing aggregate savings and economic growth. Since ours is an overlapping generations model, a one-period fiscal policy is implicitly defined as a fiscal stance that is maintained consistently for a number of years. The policy instruments usually assumed in overlapping generations models consist of lump-sum taxes or transfers imposed either on the young or the old. Lump-sum taxes have real effects because they change the time profile of the agents' incomes, thereby affecting their optimal savings decisions. If the economy is in the high savings equilibrium, lump-sum taxes yield the standard effects: a tax on the young, for example, would lower savings since the individual agent need undertake less savings to

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1/ As Becker, Murphy and Tamura (1990) write, "Many attempts to explain why some countries and continents have had the best economic performance during the past several centuries give too little attention to accidents and good fortune." p. S14.

provide for his retirement years. More interestingly, if the economy is in the low savings equilibrium lump-sum instruments can have implications for economic growth. That is to say, around the bad equilibrium the effects are exactly the opposite: a decrease in income earned by the young leads to an increase in savings. Letting τ^W denote a lump-sum wage tax, and τ^o a lump-sum tax on the old. The representative agent's budget constraints become:

$$c_t^i = w_t^i(1 - \tau^W) - s_t^i \tag{28}$$

$$c_{t+1}^i = s_t^i(1 + r_{t+1,t})(1 - \tau^o) \tag{29}$$

where $w_t^i = (1 - \alpha)K^\alpha/N$. The first-order condition for the ith agent's optimization problem yields:

$$s_t^i = \frac{\beta^{(1/\sigma)}(1+r_{t+1,t})^{(1/\sigma-1)}(1-\tau^o)^{(1/\sigma-1)}\ w_t^i(1-\tau^W)}{1 + \beta^{(1/\sigma)}(1+r_{t+1,t})^{(1/\sigma-1)}(1-\tau^o)^{(1/\sigma-1)}} \tag{30}$$

so the evolution of the economy in the presence of government intervention is given by: 1/

$$S_t = \frac{\beta^{(1/\sigma)}(1+r_{t+1,t})^{(1/\sigma-1)}(1-\tau^o)^{(1/\sigma-1)}\ W_t(1-\tau^W)}{1 + \beta^{(1/\sigma)}(1+r_{t+1,t})^{(1/\sigma-1)}(1-\tau^o)^{(1/\sigma-1)}} \tag{31}$$

$$r_{t+1,t} = \frac{(S-F)}{S}\ \alpha\ [(1-\delta)K_t + S_t]^{(\alpha-1)} - \delta \tag{32}$$

1/ For the moment we do not impose the government's budget constraint.

$$K_{t+1} = (1-\delta)K_t + S_t \tag{33}$$

A decrease in the wage tax, $d\tau^W < 0$, leads to an outward shift in the savings function (31) while keeping the interest rate function, r(S), unchanged. This decrease in taxes leads to a fall in savings around the bad equilibrium and an increase in savings around the good equilibrium (Figure 6). The increase around the good equilibrium, moreover, depends upon whether the equilibrium point occurs on the positively sloped portion of the r(S) curve or the negatively sloped portion. Intuitively, at point S_1^a there are increasing returns to savings, so an increase in S raises the interest rate, leading to a further increase in savings. Conversely, at point S_2^a the increased supply of savings lowers the interest rate which offsets part of the increase in savings.

Savings decrease at the bad equilibrium because a decrease in the interest rate leads to a larger fall in the supply of savings, S(r), than in the demand for savings, as given by r(S); that is, S(r) cuts r(S) from below at S^b. The decrease in the tax on wages raises the supply of savings at the original interest rate; to restore equilibrium the interest rate must fall. Since the demand for savings is very unresponsive to interest rate changes, however, the interest rate must fall below its original value in order to restore equilibrium. 1/ At this lower interest rate, in turn, the equilibrium level of savings is lower than in the original equilibrium.

A decrease in taxes on the old, $d\tau^o$, has exactly the opposite effects when $\sigma < 1$. The S(r) function shifts inward, as agents save less because a given level of savings provides a larger income in the second period:

$$\partial S_t/\partial \tau^o = \frac{(1/\sigma-1)\{\beta^{(1/\sigma)}(1+r_{t+1,t})^{(1/\sigma-1)}(1-\tau^o)^{(1/\sigma-2)}\ W_t(1-\tau^W)\}}{\left\{1+\beta^{(1/\sigma)}(1+r_{t+1,t})^{(1/\sigma-1)}(1-\tau^o)^{(1/\sigma-1)}\right\}^2} \tag{34}$$

Accordingly, the equilibrium level of savings decreases at the good equilibrium and increases at the bad equilibrium.

1/ Along the interest rate function (32), $dr/dS \Rightarrow -\infty$ as $S \Rightarrow 0$ so $dS/dr \Rightarrow 0$.

Although these policy instruments can affect the roots $S^a(K)$ and $S^b(K)$, they do not shift the economy from the bad equilibrium to the good equilibrium (or vice versa). The reason that such taxes and subsidies do not move the economy to the high savings equilibrium (though they may increase the savings rate around the bad equilibrium) is that the savings trap arises in this model from a lack of coordination of savings decisions; the appropriate form of intervention must be one which ensures that all agents in the economy know that the rate of return will be high (i.e., aggregate savings will be large).

To see what forms of government policy may be useful, consider the following "extensive form" of period t moves by the private sector and the government. Inherited in period t are the capital stock, K_t, and the history of aggregate savings which are relevant for the period t conjectures, $h(S_{t-1})$. Individual agents choose s_t^i optimally on the basis of $h(S_{t-1})$ and K_t. The government chooses a lump-sum wage tax of τ_t^W. Agents then consume. The government invests the tax proceeds and redistributes the total returns, $\tau_t^W(1+r_{t+1,t})$ to the old in period t+1. One point that bears emphasizing is that the government need not undertake the investment itself. Rather, the government could raise taxes and use them to fund an investment bank which would then lend to private firms in the economy. 1/ In period t+1 the investment bank is repaid and the government distributes the proceeds to the old.

Suppose the economy evolves along the low equilibrium path $\{S^b\}$ for the first t-1 periods. It is then straightforward to construct a government policy $\{0,\ldots,0,\tau_t^W,0,0,\ldots\}$ which entails positive lump-sum wage taxes in period t and zero in all other periods and which launches the economy onto the high equilibrium growth path $\{S^a(K)\}$ from period t onwards (Figure 7). Thus the perturbed equilibrium path would be given by:

$$\{S^b(K_0), S^b(K_1), \ldots, S^b(K_{t-1}), S^a(K_t), S^a(K_{t+1}), \ldots\} \tag{35}$$

The tax that launches the economy onto the high growth path is, simply:

$$\tau_t^W = S^a(K_t) - S^b(K_t) \tag{36}$$

Such a tax is clearly feasible, since it is equal to the difference between the savings which would have been chosen optimally by individual agents had they been optimistic about the aggregate savings and the level of savings chosen in the bad equilibrium. Private savings equal $S^b(K_t)$ since,

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1/ In particular, there is no need for the government to be involved in production, only in coordinating savings by the private sector.

Figure 6
Effects of a Decrease of the Tax on Wages

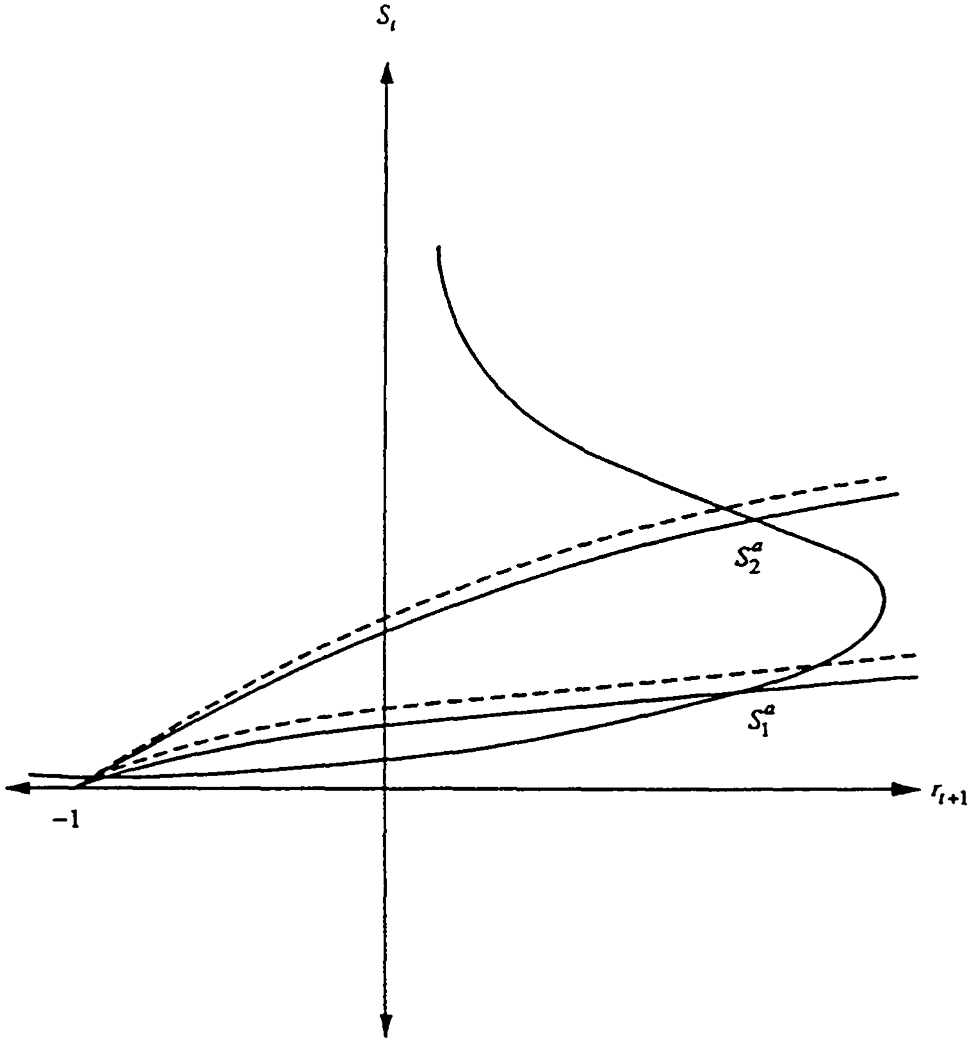
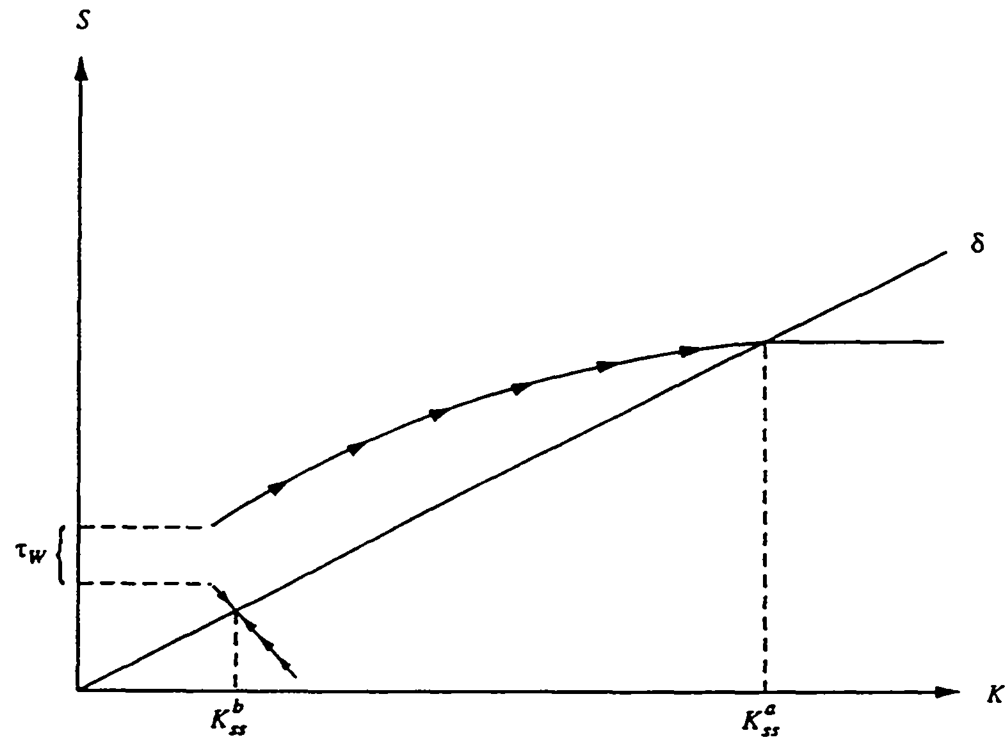


Figure 7
Take-off



by hypothesis, the government did not tax or invest in period t-1, so the individual's optimal savings are:

$$S^i_t|_{h(S^b_{t-1})} = \frac{S^b(K_t)}{N} \tag{37}$$

and aggregate savings, which equal investment in equilibrium, are

$S^b(K_t) + \tau^W_t = S^a(K_t)$. This provides an analytical basis which suggests

that sustained fiscal policies can have an effect in moving the economy from low savings to high savings. Moreover, this effort by the government, though lasting for only one period (one generation), launches the economy onto the high growth path permanently by creating a high savings 'culture'. 1/ Of course, it bears emphasizing that in the context of an overlapping generations model such as that used here, "one period" refers to an interval of about twenty years. Thus in period t+1 conjectures are given by:

$$\tilde{S}_{t+1} = h(S^b(K_t) + \tau^W_t) = h(S^a(K_t)) \tag{38}$$

and the economy proceeds along the high savings path towards the good steady state. Notice that the savings rate jumps from $S^b(K_{t-1})$ to $S^a(K_t)$ between periods t-1 and t; the sort of jump that is frequently observed in economies during their take-off stage. While the model is obviously a stylized representation, it is consistent with patterns of fiscal policy, savings, and growth that have been evident over certain periods in the economic history of such countries as Korea and Japan.

1/ That the culture is altered within half a generation is, of course, a result of our assumption on Markov beliefs. More generally it may take longer to create a high savings culture if beliefs depend upon more distant savings rates as well. Presumably no government would deliberately move the economy from the good equilibrium path to the bad equilibrium path though theoretically it is possible by raising sufficiently high taxes and then spending revenues on government consumption rather than using them for investment.

V. Conclusions

Many developing countries appear to exhibit extended periods of low growth, low savings rates, and low or negative real interest rates. On the other hand, there are also a number of countries that have exhibited persistent high savings rates over extended periods of time. Yet standard neoclassical models seem unable to capture either the longer-term "savings trap" cases or the high-savings patterns of a number of developing countries.

In this paper we have drawn on the ideas of Rosenstein-Rodan, Scitovsky, and Rostow to develop a model of the savings trap in which a lack of coordination may result in a low savings equilibrium. Once pessimistic beliefs have been formed, moreover, a low savings 'culture' emerges and in the absence of policy interventions or other exogenous shocks the economy remains in this equilibrium indefinitely. Conversely, if the economy is able to break out of the low savings equilibrium there will be a discrete jump in the savings rate--a phenomenon commonly observed in developing countries which have achieved the conditions of Rostow's "take-off" stage. In particular, our model suggests that fiscal policy can play an important role in permanently pulling the economy from a low savings/low growth path onto a high savings/high growth path. The latter can be accomplished through a sustained period of tight fiscal policy over a number of years (a "generation" in the context of our model).

'Stage' theories of economic development often suffer from a linear conception of history in which every country is assumed to go through the same 'stages'. In our opinion, such a view is overly simplistic; rather, exogenous historical accidents may be important determinants of whether a country rapidly achieves a high level of development or suffers through long periods of economic stagnation. In the model developed here, the initial path of the economy is determined exogenously and stochastically. Some countries may be fortunate in starting out along the high growth equilibrium, while others fall into the savings trap. Yet once the economy is on the path of stagnating growth confidence about the growth prospects of the country diminish with self-fulfilling pessimism about savings and growth rates.

Government policies may be able to coordinate savings and lift the economy out of the low growth equilibrium. We explored only one, rather crude, policy instrument: A direct tax that is used to fund a development bank which in turn finances private investment. There may, however, be numerous possibilities at a more microeconomic level as long as the costs of investment are not too great. In nineteenth century Germany, for example, local savings banks and cooperatives appear to have been instrumental in collecting the savings of individual investors and using them to finance industrial growth. The important point is that economic development cannot take place without harnessing the savings of many individuals in the economy; and these individuals may be less inclined to save unless they know

that a "critical mass" of aggregate savings exists which will ensure high rates of return.

Our model yields the policy implication that the adoption of tight fiscal discipline over a sustained period of time can assist in positioning a low savings/low growth economy onto a high savings/high growth path.

In the text we make a number of assertions which are proved here.

Proposition 1

For $\sigma < 1$ $S_r(r) > 0$, and a sufficient condition for $S_{rr}(r) < 0$ is $\sigma \geq (1/2)$

Proof

Differentiating (9) yields:

$$S_r = \frac{(\frac{1}{\sigma} - 1)\beta^{(1/\sigma)}(1+r)^{(1/\sigma-2)}(1-\alpha)K^{\alpha}}{\left\{1 + \beta^{(1/\sigma)}(1+r)^{(1/\sigma-1)}\right\}^2}$$

Moreover,

$$S_{rr} = \frac{(\frac{1}{\sigma} - 1)\beta^{(1/\sigma)}(1+r)^{(1/\sigma-2)}(1-\alpha)K^{\alpha}}{\left\{1 + \beta^{(1/\sigma)}(1+r)^{(1/\sigma-1)}\right\}^2} \times$$

$$\left\{\frac{2\beta^{(1/\sigma)}(1-1/\sigma)(1+r)^{(1/\sigma-2)}}{1 + \beta^{(1/\sigma)}(1+r)^{(1/\sigma-1)}} + \frac{(1/\sigma - 2)}{(1 + r)}\right\}$$

so $\sigma \geq (1/2)$ is sufficient for $S_{rr} < 0$.

Proposition 2

$r_S(S) > 0$ for $0 < S < S^*$ where S^* is the positive root of:

$$Q(S) = S^2 + \frac{F(\alpha-2)}{(1-\alpha)} S - \frac{K(1-\delta)F}{(1-\alpha)}$$

Proof

Differentiating (10) gives:

$$r_S(S) = \frac{\alpha(K(1-\delta) + S)^{(\alpha-1)}}{S} \left\{ \frac{F}{S} + \frac{(\alpha-1)(S-F)}{(K(1-\delta) + S)} \right\}$$

so $\mathrm{sgn}(r_S(S)) = -\mathrm{sgn}(Q(S))$ where:

$$Q(S) = S^2 + \frac{F(\alpha-2)}{(1-\alpha)} S - \frac{K(1-\delta)F}{(1-\alpha)}$$

and the positive root of $Q(S)$ is given by:

$$\hat{S}(K) = \frac{(2-\alpha)F}{2(1-\alpha)} + \frac{1}{2} \sqrt{\frac{(F(\alpha-2))^2}{(1-\alpha)^2} + \frac{4K(1-\delta)F}{(1-\alpha)}}$$

and $Q(S) < 0$ for $0 < S < \hat{S}(K)$ so $r_S(S) < 0$ for $S < \hat{S}(K)$. Moreover, $\lim_{S \to 0} r(S) = -\infty$.

Proposition 3

For $\sigma = \alpha = (1/2)$ the intersections of $S(r)$ and $r(S)$ satisfy the polynomial

$$x(S,K) = S^2\{(K(1-\delta)+S)\}\{S(1+\mu) - \mu W\}^2 - (1/4)\beta^4(S-F)^2(W-S)^2$$

Proof

Substituting (10) into (9) yields:

$$S = \frac{\beta^{(1/\sigma)}\{(1-\delta) + \frac{(S-F)}{S}\,\alpha(K(1-\delta)+S)^{(\alpha-1)}\}^{(1/\sigma-1)}W}{\left\{1 + \beta^{(1/\sigma)}\{(1-\delta) + \frac{(S-F)}{S}\,\alpha(K(1-\delta) + S)^{(\alpha-1)}\right\}}$$

Let $\sigma = \alpha = (1/2)$ then S becomes:

$$S = \frac{\beta^{2}\{(1-\delta) + \frac{(S-F)}{S}\,(1/2)(K(1-\delta)+S)^{-(1/2)}\}W}{\left\{1 + \beta^{2}\{(1-\delta) + \frac{(S-F)}{S}\,(1/2)(K(1-\delta)+S)^{-(1/2)}\right\}}$$

Thus,

$$\{S^2 + \beta^2(1-\delta)S^2 + \beta^2(1/2)S(S-F)(K(1-\delta)+S)^{-(1/2)}\}$$

$$= \beta^2\{(1-\delta)S + (1/2)(S-F)(K(1-\delta)+S)^{-(1/2)}\}W$$

Multiplying by $(K(1-\delta)+S)^{(1/2)}$ yields:

$$S^2(1+\beta^2(1-\delta))(K(1-\delta)+S)^{(1/2)} + \beta^2(1/2)S(S-F)$$

$$= \beta^2(1-\delta)S(K(1-\delta)+S)^{(1/2)}W + (1/2)(S-F)\beta^2W$$

Let $\mu = \beta^2(1-\delta)$

$$S\{(K(1-\delta)+S)^{(1/2)}\}\{S(1+\mu) - \mu W\} = (1/2)\beta^2(S-F)(W-S)$$

Squaring both sides and subtracting yields:

$$x(S,K) = S^2\{(K(1-\delta)+S)\}\{S(1+\mu) - \mu W\}^2 - (1/4)\beta^4(S-F)^2(W-S)^2$$

Proposition 4

For $F < \frac{\mu W}{(1+\mu)}$ then exist two savings equilibria.

Proof

Consider the polynomial x(S,K). Then,

$$x(0,K) = - \beta^4 W^2 F^2/4 < 0$$

$$x(F,K) = F^2[K(1-\delta)+F][F(1+\mu) - \mu W]^2 > 0$$

$$x(\frac{\mu W}{(1+\mu)}, K) = -(\beta^4/4)(\frac{\mu W}{(1+\mu)} - F)^2 (W - \frac{\mu W}{(1+\mu)})^2 < 0$$

$$x(W,K) = (K(1-\delta)+W)W^4 > 0$$

Thus there must be one root satisfying $0 < S^b < F$ and one root satisfying $\frac{\mu W}{(1+\mu)} < S^a < W$. The center root of x(S,K) does not correspond to an intersection of S(r) and r(S). To obtain an approximate value of the lower root, we take a linear approximation given by:

$$0 = x_S(0)S^b + x(0)$$

Using $x_S(0) = (\beta^4/2)(WF)(W+F)$ and $x(0) = -(F^2/4)\beta^4W^2$ we obtain:

$$S^b = \frac{WF}{2(W+F)}$$

Proposition 5

i. $F > \frac{\beta^2}{(1+\mu)}$ is sufficient for $0 > S_K^b > -(1-\delta)$ and

ii. $(1-\delta) > S_K^a > 0$.

i. Consider S^b_K

From the polynomial $x(S^b, K) = 0$, we have:

$$S^b_K = \frac{-x_K}{x_S}$$

First note that $x_S > 0$:

$$x_S(S^b,K) = (S^b)^2[S^b(1+\mu)-\mu W]^2 + 2S^b(K(1-\delta)+S^b)(S^b(1+\mu)-\mu W)^2$$

$$+ 2(1+\mu)(S^b)^2(K(1-\delta)+S^b)(S^b(1+\mu)-\mu W) - (\beta^4/2)(S^b-F)(W-S^b)(w+F-2S^b)$$

Since $0 < S^b < F < W$, only the third term is negative. Therefore:

$$x_S(S^b,K) > 2S^b(K(1-\delta)+S^b)(S^b(1+\mu)-\mu W)\{2S^b(1+\mu)-\mu W\}$$

but $2S^b(1+\mu) = (1+\mu)WF/(W+F)$ so

$$x_S(S^b,K) > 2S^b(K(1-\delta)+S^b)(S^b(1+\mu)-\mu W)\{W(F-\mu W)/(W+F)\} > 0$$

Consider, next, $x_K(S^b,K)$.

$$x_K(S^b,K) = (1-\delta)(S^b)^2(S^b(1+\mu)-\mu W)^2 - \mu W_k 2(S^b)^2(K(1-\delta)+S^b)(S^b(1+\mu)-\mu W)$$

$$- (\beta^4/2)(S^b-F)^2(W-S^b)W_k$$

The first two terms are positive, while the last term is negative. Therefore,

$$x_K(S^b,K) > \mu 2(S^b)^2(\mu W-(1+\mu)S^b)(K(1-\delta)+S^b)W_k - (\beta^4/2)(S^b-F)^2(W-S^b)W_k$$

but the minimum value of W is $(1+\mu)F/\mu$ and the minimum value of S if the steady state is approached from below is δK. Hence,

$$x_K(S^b,K) > \mu 2(S^b)^2(1+\mu)KW_k - (\beta^4/2)(F-S^b)(W-S^b)W_k$$

but $WW_k = (1/8)$ and $KW_k = (1/2)W$,

$$x_K(S^b,K) > \mu(S^b)^2(1+\mu)W - (\beta^4/16)F$$

The minimum value of S is F/4 and the minimum value of W is $(1+\mu)F/\mu$ so

$$x_K(S^b,K) > (1+\mu)^2F^2 - \beta^4$$

hence, $F > \dfrac{\beta^2}{(1+\mu)}$ is sufficient for $x_K(S^b,K)$ to be positive.

Finally, assuming $x_K(S^b,K) > 0$, the maximum value of $x_K(S^b,K)$ is

$$x_K(S^b,K) < (1-\delta)(S^b)^2(S^b(1+\mu)-\mu W)^2 - \mu W_k 2(S^b)^2(K(1-\delta)+S^b)(S^b(1+\mu)-\mu W)$$

while $x_S(S^b,K)$ is larger than:

$$x_S(S^b,K) > (S^b)^2[S^b(1+\mu)-\mu W]^2 + 2S^b(K(1-\delta)+S^b)(S^b(1+\mu)-\mu W)^2$$

$$+ 2(1+\mu)(S^b)^2(K(1-\delta)+S^b)(S^b(1+\mu)-\mu W)$$

Thus $Z = x_K(S^b,K) - (1-\delta)x_S(S^b,K)$ is smaller than:

$$Z < - \mu W_k(S^b)^2(S^b(1+\mu)-\mu W) - (1-\delta)S^b(S^b(1+\mu)-\mu W)(2S^b(1+\mu)-\mu W)$$

Or,

$$Z < \mu W_k S^b - (1-\delta)(\mu W - 2S^b(1+\mu))$$

Using $S^b = \frac{WF}{2(W+F)}$ gives:

$$Z < \mu W_k S^b - (1-\delta)\ \frac{W(\mu W-F)}{(W+F)}$$

and $W > \frac{(1+\mu)F}{\mu}$:

$$Z < \mu W_k S^b - (1-\delta)\ \frac{W\mu F}{(W+F)}$$

but $S^b < F$ and $W/(W+F)$ is increasing in W, so:

$$Z < W_k - (1-\delta)\ \frac{W}{(W+F)} < W_k - (1-\delta)\ \frac{(1+\mu)}{(1+2\mu)}$$

W_k is decreasing in K, so its highest value is at $K = \{2(1+\mu)F/\mu\}^2$

$$Z < \frac{\mu}{8(1+\mu)F} - (1-\delta)\ \frac{(1+\mu)}{(1+2\mu)} \text{ or } Z < \frac{\beta^2}{8(1+\mu)F} - \frac{(1+\mu)}{(1+2\mu)}$$

Thus

$$F > \frac{\beta^2(1+2\mu)}{8(1+\mu)^2}$$

is sufficient for $\dfrac{x_K(S^b,K)}{x_S(S^b,K)} < (1-\delta)$. Since $(1+2\mu)/(1+\mu) < 8$, the

condition $F > \frac{\beta^2}{(1+\mu)}$, is sufficient for $\frac{x_K(S^b,K)}{x_S(S^b,K)} < (1-\delta)$ as well.

ii. Consider, next, S_K^a. Differentiating (12) with respect to r and K and differentiating (13) with respect to S and K yields:

$$S_K^a = \frac{(S_r r_K + S_K)}{(1 - S_r r_S)}$$

where S_r and S_K are the derivatives of (12) with respect to r and K, and r_S and r_K are the derivatives of (13) with respect to S and K. $S_r > 0$ and $S_K > 0$ and at the good equilibrium, $S^a > F$ so $r_K < 0$. If $r_S < 0$ then $S_K^a < S_K$. If $r_S > 0$ then the minimum value of $(1 - S_r r_S)$ occurs at the maximum value of r_S. It can be shown that the maximum value of r_S occurs at S = F which occurs at $K = \underline{K}$. Since S_K is decreasing in K, the maximum value of S_K^a occurs at S = F (when $r_S > 0$) so when $r_S > 0$ we need only show that $S_K^a < (1-\delta)$ at $K = \underline{K}$.

Suppose, first, that $r_S < 0$, then $S_K^a < S_K$:

$$S_K^a < S_K = \frac{\beta^2(1+r_{t+1,t})\left\{(1/4)K_t^{-(1/2)}\right\}}{1 + \beta^2(1+r_{t+1,t})} < (1/4)K_t^{-(1/2)}$$

But the minimum value of $\underline{K} = \left\{\frac{2(1+\mu)F}{\mu}\right\}^2$ so

$$S_K^a < (1/4)\left\{\frac{\mu}{2(1+\mu)F}\right\}$$

Therefore $S_K^a < (1-\delta)$ as long as $F > \frac{\beta^2}{8(1+\mu)}$. Next, suppose that $r_S > 0$,

value occurs at $S^a = F$ and this value is given by:

$$r_S < (F/S^{a2})(1/2)K^{-(1/2)} = (1/F)(1/2)K^{-(1/2)}$$

But $S_r = \dfrac{\beta^2 W}{(1+\beta^2(1+r))^2}$ so that

$$S_r r_S < \frac{1}{4F} \frac{\beta^2}{(1+\beta^2(1+r))^2}$$

but the minimum value of F imposed above is $F > \dfrac{\beta^2}{(1+\mu)}$ so

$$S_r r_S < (1/4)(1+\mu)/(1+\beta^2(1+r))^2 < (1/2)$$

Thus $S_K^a < 2S_K$ which is less than $(1-\delta)$ for $F > \dfrac{\beta^2}{4(1+\mu)}$.

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