Working Paper
The Optimal Rate of Money Creation in an Overlapping Generations Model: Numerical Simulations for the U.S. Economy

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Abstract

This paper develops a large scale overlapping generations model and calibrates it for the U.S. economy. Simulations with the model show that the steady state welfare maximizing inflation rate may be positive, although the numerical results are not robust. It is also shown, however, that increases in the inflation rate are never Pareto efficient because during the transition to the new steady state at least some generations are made worse-off. Using an optimality criterion that takes into account the welfare of all generations, it is found that implementing Friedman's rule is a Pareto superior policy, and that the efficiency gains derived from implementing such rule could be substantial.

JEL Classification Numbers:
E42; E52

1/ This paper is based on my Ph.D. dissertation at Boston University. I have benefitted from very helpful comments by Larry Kotlikoff, Christophe Chamley, Russell Cooper, Jorge Marquez-Ruarte, Miguel Savastano, and Jose de Gregorio. All remaining errors are my sole responsibility.
## Contents

<table>
<thead>
<tr>
<th>Summary</th>
<th>iii</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Introduction</td>
<td>1</td>
</tr>
<tr>
<td>II. The Model</td>
<td>3</td>
</tr>
<tr>
<td>III. Baseline Scenario</td>
<td>14</td>
</tr>
<tr>
<td>IV. The Optimal Rate of Monetary Growth</td>
<td>20</td>
</tr>
<tr>
<td>V. Conclusion</td>
<td>31</td>
</tr>
<tr>
<td>Appendix A: A Definition of the φs</td>
<td>33</td>
</tr>
<tr>
<td>Appendix B: Interest Elasticity of the Money Demand</td>
<td>34</td>
</tr>
<tr>
<td>Appendix C: The Price Equation</td>
<td>35</td>
</tr>
<tr>
<td>References</td>
<td>36</td>
</tr>
</tbody>
</table>

### Tables

1. Baseline Scenario                          | 17  |
2. Steady State Sensitivity Analysis          | 18  |
3. The Tobin Effect in the Baseline Scenario  | 22  |
4. Dynamic Adjustment following a Permanent Increase in μ from 0% to 3% | 25  |
5. Relative Efficiency of Alternative Monetary Regimes | 28  |
6. Efficiency Costs of Inflation              | 30  |

### Figures

1. Age-Consumption and Age-Money Profiles     | 16a |
2. Age-Income Profile                         | 16b |
3. Intergenerational Distribution of the Capital Stock | 16c |
4. Marginal Propensities to Consume out of Total Wealth | 16d |
5. Steady State Welfare Maximizing Inflation Rate | 22a |
6. Welfare Effects of a Switch to the Steady-State Welfare Maximizing Inflation Rate | 24a |
Summary

This paper examines the optimal rate of money creation in a first-best world. For this purpose, a multiperiod overlapping-generations model is constructed and calibrated for the U.S. economy. The model is first used to find the inflation rate that maximizes welfare in the steady state. Because of the positive relationship between inflation and the capital stock, the steady-state welfare-maximizing inflation rate is not the rate of deflation recommended by Friedman; instead, an annual inflation rate of 3 percent seems to be optimal. However, because a wide range of inflation rates leads to similar welfare outcomes, the paper finds this rate not to be robust. Moreover, it is shown that, when starting from a lower rate of inflation, raising the inflation rate to 3 percent is not a Pareto-efficient policy because the transition to the new steady state leads to a reduction in the welfare of some generations. In other words, the increase in steady-state welfare brought about by the increase in inflation is achieved at the expense of some of the generations alive during the transition to the new steady state.

The issue of the optimal rate of money creation is then re-examined, using an optimality criterion that takes into account the welfare of all generations. The objective of this exercise is to find a Pareto-superior inflation rate such that, starting from any inflation rate, the transition to the new regime imposes no welfare losses on any generation and permits all successive generations to enjoy higher levels of welfare in the new steady state. It turns out that implementing Friedman's rule is the most efficient policy and that the efficiency gains obtained with such a rule are quite substantial.
I. Introduction

In his famous 1968 essay on the optimal quantity of money, Friedman concluded that a full liquidity rule should be followed by the monetary authority. The rule consisted in creating a rate of deflation equal to the real rate of interest; as a result, the cost of holding money would be driven to zero and the economy would demand its satiation level of money balances. Friedman estimated that the benefits of implementing such rule in the United States could be around 0.3 percent of GDP. 1/

The optimality of Friedman’s full liquidity rule in a first best world has been the subject of much debate. 2/ The primary reason for the non-optimality of this rule is that in models in which money is not superneutral there might be welfare gains associated with increasing the rate of money creation, at least for certain values of the inflation rate. 3/ This argument has been developed more recently in the context of a type of Overlapping Generations (OG) models that exhibit a positive relationship between inflation and the steady state capital stock. When this effect (usually referred to as the Tobin effect) is present, faster rates of money creation have two opposite effects on welfare in the steady state: a positive income effect as the individual’s lifetime resources increase because of the higher capital intensity, and a negative substitution effect stemming from the distortion of the individual’s marginal decisions. Weiss (1980) has proven that the first effect dominates the second one whenever the rates of money creation are low. He provides an intuitive explanation of this result by relating the negative substitution effect to the deadweight loss associated with conventional taxation, which is generally a second order effect. For faster rates of money creation, however, the net effect may be negative.

Weiss’ result has been criticized by others, who have questioned both the set of policy tools used in the determination of the optimal rate of money creation and his optimality criterion. Abel (1987), for example, uses a welfare function in which the lifetime utilities of all future generations are discounted using a social discount rate. Abel contends that in order to consider the issue of the optimal rate of money creation as a first best policy, other non-distortionary policy instruments available to the government should be taken into account and an overall package should be analyzed. By allowing the government to distribute resources across all generations alive at a given time through a pay-as-you-go system of lump-sum transfers, the author finds that under the optimal package, the steady state...

1/ Friedman (1968), p. 44.
2/ The following discussion concentrates on the optimal inflation rate in a world in which lump-sum taxation is available. For a discussion on the optimal rate of inflation in a second-best world see, for example, Phelps (1973) and Chamley (1986).
3/ See, for example, Summers (1981).
of the economy is characterized by a zero nominal interest rate and by monetary contraction at the rate of intergenerational discount.

In a related study, McCallum (1987) defines a different criterion of optimality in the context of an otherwise identical model. He sets up the problem of the determination of an optimal monetary rule as an optimization problem in which the lifetime utility of an individual is maximized subject to the standard budget constraints and to constrained lifetime utilities for all subsequent generations. Unlike Abel, McCallum does not consider the use of other policy instruments. The result is, again, that optimality can be achieved by implementing Friedman's rule.

This paper follows the line of work just described and re-examines the issue of the optimal rate of money creation in the context of a large-scale OG model calibrated for the U.S. economy. Using the lifetime utility of a representative individual in steady state as the optimality criterion, an optimal inflation rate is derived. As claimed by Weiss, the steady state welfare maximizing inflation rate is not the one recommended by Friedman; instead, an annual inflation rate of about 3 percent seems to be optimal. However, it is shown that raising the inflation rate from any level below 3 percent toward that level is not a Pareto efficient policy because the transition is characterized by a reduction in the welfare levels of the generations alive at the time of the change in monetary policy. A few generations born soon after the change in policy also experience a level of welfare lower than the one they would have enjoyed if no change in policy had taken place. In other words, the increase in steady state welfare levels is achieved at the expense of some of the generations alive during the transition to the new steady state. For this reason, a search for a Pareto efficient monetary regime is carried out. The objective is to find an inflation rate such that starting from any given inflation, the transition to the optimal regime does not impose welfare losses on any generation and permits all successive generations enjoy higher levels of welfare in the new steady state. It turns out that Friedman's rule is the most efficient rule, despite the existence of a Tobin effect. The numerical results obtained for the U.S. economy are compared with those obtained in other studies and it is found that the efficiency gains associated with the implementation of the Friedman rule are likely to be substantial.

The paper is organized as follows: the next section explains the basic model and discusses the simulation methodology; section three shows the calibration exercise and presents a baseline scenario; section four presents the simulation results of alternative monetary rules and their welfare implications; section five concludes.

1/ As it will be seen in sub-section IV.2, this is not a robust result since a wide range of values for the inflation rate lead to similar welfare gains.
II. The Model

The economy is composed of three sectors: a household sector, a production sector, and a government sector.

1. Household behavior

The assumptions that characterize the behavior of the household sector are the following: (i) there is one individual per generation who lives for 55 periods; 1/ (ii) this individual works for the first 45 years of his life, supplying inelastically one unit of time every year, and fully retires at age 46; (iii) since no bequests or other forms of intergenerational transfers are allowed, newly born generations initially hold neither money nor assets; and (iv) there is no population growth.

The instantaneous utility function \( V \) of an individual of age \( t \) born in year \( s \) is given by the isoelastic function:

\[
V_{s,t} = \theta \frac{1-\gamma}{1-\gamma} c_{s,t}^{1-\gamma} + (1-\theta) \frac{1-\gamma}{1-\gamma} m_{s,t}^{1-\gamma}
\]

where \( c_{s,t} \) and \( m_{s,t} \) represent consumption and real money balances at the end of period \( s+t \) of an individual born in period \( s \); \( \theta \) is a distributional parameter denoting the intensity of the household's preference for consumption as opposed to money; and \( \gamma \) is a preference parameter representing the inverse of the elasticity of substitution between the levels of consumption (or real money balances) of any two consecutive years. 2/

The introduction of money in the utility function is motivated by the assumption that money demanded at the end of period \( t \) provides instant liquidity services. Assuming that the individual discounts the annual

1/ The reason for using a 55-period model is essentially demographic, since for the United States 55 constitutes a good approximation of the number of years that elapse from the time an individual enters the labor force until he dies.

2/ Different values of \( \gamma \) could have been used for \( c \) and \( m \); however, as will be seen in the next section, the selection of a common value yields plausible results in the numerical simulations. Furthermore, a common value of \( \gamma \) ensures that the preferences represented by (1) are homothetic; as a result, the welfare analysis developed in section IV is greatly simplified.

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utilities given by (1) at the subjective rate $\delta$, his lifetime utility can be written as: 1/

$$U_s = \sum_{t=0}^{54} (1+\delta)^{-t} v_{s,s+t}$$  \hspace{1cm} (2)

New generations entering the labor force do not possess any real assets or money and, consequently, their wealth, measured at the end of their first year of life, is simply the sum of their human wealth ($h$) and the present discounted value (PDV) of annual government transfers ($\psi$):

$$h_{s,s} = \sum_{t=0}^{44} R_{s,s+t} w_{s,s+t}$$  \hspace{1cm} (3)

$$\psi_{s,s} = \sum_{t=1}^{54} R_{s,s+t} \sigma_{s,s+t}$$  \hspace{1cm} (4)

where $w$ is the wage rate, $\sigma$ is the annual government transfer and $R_{s,s+t}$ represents a market real discount factor between years $s$ and $t$ and is defined as follows:

$$R_{s,s+t} = \begin{cases} 
1 & \text{if } t=0 \\
\prod_{j=1}^{t} (1+r_{s+j})^{-1} & \text{if } t>0 
\end{cases}$$

Money demanded at the end of a given year is carried over to the next. For this reason, individuals can save either in real assets (a) 2/ which yield a real interest rate ($r$), or in money (m) which, in addition to be subject to an inflation tax, does not yield a real interest. Thus, the budget constraint of an individual born in year $s$ is, in real terms:

---

1/ The term $m_{s,s+55}$ is actually dropped from the lifetime utility function. For a discussion about the plausibility of this assumption see: Orphanides and Solow (1990).

2/ The term "real asset" is used here as a synonym of claims on physical capital.
The term \( \frac{1}{1+i} \) represents the marginal cost of the liquidity services provided by money during period \( t \). Notice that when choosing to hold money at the end of year \( t \) the individual gives up the real interest that he could earn on the real asset and also becomes subject to an inflation tax. Therefore, the cost of holding money during a given period is \( im \) and the PDV of such cost measured at the end of the previous period (when money is demanded) is \( im/(1+i) \).

The representative individual born in year \( s \) is supposed to choose a sequence of real consumption levels \( c_s = (c_s, s, \ldots, c_s, s, s+54) \) and money demands \( m_s = (m_{s+1}, \ldots, m_{s+54}) \) that maximize his lifetime utility (2) subject to the budget constraint (5). The resulting consumption demand for his first year of life is:

\[
c_{s, s} = \phi_{s, s} [ h_{s, s} + \psi_{s, s} ]
\]  

where \( \phi \) denotes the fraction of the individual's wealth that will be allocated to consumption in his first year of life. The parameter \( \phi \) is a function of the entire path of inflation and real interest rates over the individual’s lifetime. Thus, when the economy is not in steady state, \( \phi \) will vary with the age of the individual as well as with time.

Once the first year level of consumption has been determined the remaining consumption and money demands can be obtained using the first order conditions from the individual’s utility maximization problem:

\[
c_{s, s} = \left[ \frac{1+R_{s, s}+t}{1+i} \right]^{1/\gamma} c_{s, s} + t - 1, \ldots, 54.
\]  

\[
m_{s, s} = \beta \xi_{s, s}^{-1/\gamma} c_{s, s} + t, \quad t = 0, \ldots, 53;
\]  

\[\beta = [(1-\theta)/\theta]^{1/\gamma}.
\]

---

1/ The nominal interest rate is defined as: \( i = (1+r)(1+\pi) - 1 \), and \( \pi_t \) is the inflation rate: \( p_t/p_{t-1} - 1 \).

2/ See Appendix A for an complete derivation of \( \phi \).
Equations (7) and (9) represent Euler equations for consumption and money, respectively. Notice that the individual's optimal decision regarding the intertemporal distribution of money balances depends not only on the real interest rate, but also on the marginal cost of holding money in consecutive periods. Equation (8) results from equating the marginal rate of substitution between contemporary levels of consumption and money balances to their relative prices. 1/

2. The production sector

There is a single production sector which is assumed to behave competitively. Both labor (L) and capital (k) are assumed to be homogeneous and the latter is non-depreciating. Furthermore, the absence of population growth, together with the assumption that retirement occurs at age 46, determines that the labor force is always composed by 45 individuals, each supplying inelastically one unit of labor. The production technology is assumed to be of the Cobb-Douglas type, under constant returns to scale.

Choosing units for the individual labor supplies in such a way that the total labor supply in each period is equal to one, the economy's production function can be written as:

$$y_t = k_t^\alpha$$  \hspace{1cm} (10)

Perfect competition in the production sector will equate factor prices to their marginal productivities:

$$w_t = (1-\alpha) k_t^\alpha$$  \hspace{1cm} (11)

$$r_t = \alpha k_t^{\alpha-1}$$  \hspace{1cm} (12)

Notice that $w$ represents the economy's total wage income; therefore, the wage income received by a single working individual during a given year, $w$, is $1/45$ of $w$. Equations (11) and (12) will ensure that factor markets will always be in equilibrium in the sense that the sequence of demands for labor and capital will be equal to their supplies.

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1/ Of the three equations one is obviously redundant. All three are shown, however, with the purpose of facilitating the discussion.
3. The government

The government is assumed to select exogenously the rate of monetary creation ($\mu$) and to follow the monetary rule:

$$M_{t+1} = M_t (1 + \mu_t)$$  \hspace{1cm} (13)

For positive values of $\mu$ the government collects seigniorage ($S$). The proceeds from seigniorage finance annual transfers ($\sigma$) to the private sector. Therefore, the government's budget constraint is simply:

$$S_t = \frac{M_{t+1} - M_t}{P_t} = \sum_{s=t-54}^{t-1} \sigma_{s,t}$$  \hspace{1cm} (14)

It is further assumed that the transfers are proportional to the individual monetary balances held at the beginning of the period: 1/

$$\sigma_{s,t} = S_t \frac{P_t m_{s,t}}{M_t} = \mu_t m_{s,t}$$  \hspace{1cm} (15)

Obviously, the selection of negative values of $\mu$ would amount to a policy of continually reducing the money supply via the collection of taxes $\sigma$ and destroying the proceeds.

4. Equilibrium in the money market

Aggregation of the demands for money of all generations yields the economy's aggregate money demand, $m^*$. In equilibrium, $m^*$ must equal the real quantity of money available at the end of the period:

$$m^*_{t+1} = \sum_{s=t-53}^{t} m_{s,t+1} = \frac{M_{t+1}}{P_t}$$  \hspace{1cm} (16)

Under the monetary policy rule defined by (13), this equilibrium condition leads to the following price equation:

1/ This assumption is made in order to prevent an intergenerational redistribution of resources. Further discussion on this issue is provided in section IV.1.
5. **Equilibrium in the capital market**

Equilibrium in the capital market requires that the sum of all individual asset demands be equal to the existing capital stock. Denoting by $a_{s,t+1}$ the amount of assets held at the end of period $t$ by an individual born in period $s$, the equilibrium condition for the capital market can be written as:

$$ k_{t+1} = \sum_{s=t-53}^{t} a_{s,t+1} \tag{18} $$

In order to illustrate the implications of (18), consider the demand for real assets by the youngest 54 generation as a function of the current levels of wages and interest rates, the current government transfers, and the individual holdings of assets and money:

$$ a_{t,t+1} = \omega_t - c_{t,t} - m_{t,t+1} \tag{19} $$

$$ a_{t-s,t+1} = \omega_t + a_{t-s,t} + a_{t-s,t}(1+r_t) + \frac{m_{t-s,t} + t}{1+r_t} - c_{t-s,t} - m_{t-s,t+1}; \quad s = 1, \ldots, 45. \tag{20} $$

$$ a_{t-s,t+1} = a_{t-s,t} + a_{t-s,t}(1+r_t) + \frac{m_{t-s,t}}{1+r_t} - c_{t-s,t} - m_{t-s,t+1}; \quad s = 46, \ldots, 53. \tag{21} $$

Equation (19) shows a budget identity for a newly born individual: his demand for the real asset has to be equal to the difference between his first period saving $(\omega - c)$ and his purchase of real money balances. Equation (20) characterizes the behavior of the other individuals in the labor force. For them, the change in their holdings of real assets will equal their saving $(\omega + \sigma + ra - c)$ minus their accumulation of real monetary balances. For those individuals already retired, their patterns of capital accumulation are similar to those described by (20) except that they do not receive a salary and, therefore, their current saving amounts to $(ra + \sigma - c)$.
Substituting equations (19)-(21) into the equilibrium condition (18) and using knowledge of the fact that all generations will spend all their resources on consumption before dying yields:

\[ k_{t+1} = \omega_t + (1 + r_t) k_t - c^*_t \]  \hspace{1cm} (22)

where \( c^* \) represents aggregate consumption. Expression (22) represents the economy-wide budget constraint. Given the assumption of constant returns to scale, it can be written in the more conventional national income identity form, in order to show that current income is either consumed or invested:

\[ y_t = c^*_t + k_{t+1} - k_t \]  \hspace{1cm} (23)

6. **The steady state**

As a consequence of the absence of population growth and technological progress, the steady state of this economy will be characterized by a constant capital stock and, given the monetary rule described by (13), by a constant inflation rate. This section derives the steady state values of \( k \) and \( \pi \) by combining their steady state properties with the equilibrium conditions in the capital and money markets. The task is facilitated by the fact that in the steady state the optimal vectors \( c \) and \( m \) exhibit the following property:

\[ c_s = c_{s+h}, \quad h = 0,1,2, \ldots \]  \hspace{1cm} (24)

\[ m_s = m_{s+h}, \quad h = 0,1,2, \ldots \]  \hspace{1cm} (25)

In other words, the level of consumption of an individual of a given age will always be constant in steady state, and the same is true for his money demand. Thus, the optimal sequences \( c \) and \( m \) chosen by a single individual in steady state can be interpreted as the values of \( c \) and \( m \) chosen by all individuals currently alive. This property facilitates greatly the task of aggregation across generations required for the implementation of the equilibrium conditions (16) and (18). Similarly, since in the steady state factor prices are constant, the values of \( h \) and \( \psi \) at the end of the individual's first year of life will also be constant. This implies that in the steady state the only relevant difference between individuals is their age, and this is the only sub-index that will be employed.
From equations (6) and (8), the steady state consumption and money demands of the youngest individual are:

\[ c_0 = \phi_0 \left[ h_0 + \psi_0 \right] \quad (26) \]

\[ m_1 = \beta \xi^{-1/\gamma} c_0 \quad (27) \]

and the value of the current consumption and money demands for the other individuals currently alive can be obtained from equations (7) and (9), in the following way:

\[ c_s = \left( \frac{1+r}{1+\delta} \right)^{1/\gamma} c_{s-1}, \quad s=1,\ldots,54. \quad (28) \]

\[ m_s = \left( \frac{1+r}{1+\delta} \right)^{1/\gamma} m_{s-1}, \quad s=2,\ldots,54. \quad (29) \]

Again, notice that although these expressions represent Euler equations for a single individual's optimization problem, they are being used here to relate the contemporary levels of consumption of individuals from different generations. Aggregation across individuals of their current money demands yields:

\[ m^* = \beta \xi^{-1/\gamma} \phi_0 \left[ h_0 + \psi_0 \right] \sum_{j=0}^{53} \left( \frac{1+r}{1+\delta} \right)^{j/\gamma} \quad (30) \]

Expression (30) shows that the steady state aggregate demand for money has the standard properties regarding scale and cost variables, i.e., it is directly related to wealth and inversely related to the nominal interest rate.

Finally, aggregating the current levels of consumption and substituting them into the steady state version of equation (23) leads to the following steady state relationship:

\[ y = c^* \quad (31) \]

which denotes that, given the constant capital stock in steady state, all income is consumed.
Equations (26)-(30) constitute the core of the algorithm used in the numerical solution of the model in the steady state. The algorithm proceeded in the following manner: an initial guess for the capital stock \( k_0 \) was used to calculate first round values for the factor prices \( -w_0 \) and \( r_0 \) which permitted the determination of \( h_0 \) according to equation (3). Furthermore, these factor prices, together with the value of \( \mu \) provided exogenously, were used to compute first round values for \( i, \xi \) and \( \phi \), the latter according to Appendix A. All these values, along with a first round guess for the transfers \( \sigma \), were then combined in order to determine the first year consumption and money demands of the economy's youngest individual, as described by equations (26) and (27). Once these demands had been obtained, the Euler equations (28) and (29) were used with the purpose of deriving the consumption and money demands of the other 54 individuals. The individual demands were then used to compute the levels of \( c^* \), and \( m^* \). These aggregate variables were used to update the original guess for the capital stock in the following way: the value of \( c^* \) along with the factor prices implied by the initial guess for the capital stock were substituted in the steady state version of equation (23):

\[
k_1 = \frac{1}{r_0}(c^*_0 - w_0)
\]

Likewise, the first round money demands were used to determine the government transfers according to equation (15). The updated values of \( k \) and \( \sigma \) were weighted with the old guesses in order to obtain a new guess for the next round of calculations. This process was repeated until convergence was achieved.

7. Dynamics

The characterization of the model provided in the preceding sections has been based on the behavior of a single individual. This is particularly useful for analyzing the economy's steady state, because in that case the behavior of one individual throughout his lifetime can be used to infer the behavior of other individuals currently alive. The analysis of the dynamics of the model is more complicated, however, because out of the steady state both age and time matter, i.e., the consumption and money demands of individuals of a given age will differ across generations. This increases the complexity of the model and makes it impossible to derive its analytical solution or present it in terms of reduced form equations. For this reason, this section will simply provide an intuitive explanation of the dynamic behavior of the model based on the methodology used in the numerical calibration.

\[1/ \] The algorithm is similar to the one used by Auerbach and Kotlikoff (1983).
In order to illustrate the dynamics of the model, consider an economy that has been in a steady state forever, and where a policy change is announced in year T. This will imply that starting at T every individual, except for the one born in year T and the one who will die at the end of that year, will have to revise his optimal plans for his remaining life horizon. In practice, that means that the middle 53 generations alive at the time of the announcement will have to be treated as if they had been born again, except for two major differences: (i) they are now endowed with some real capital and some nominal money holdings; and (ii) they will have different lifetime horizons.

Formally, the problem faced by each of the middle 53 generations is that of selecting sequences of consumption and money demands that maximize their utility over their remaining years of life, given their current levels of wealth. Thus, for an individual of age s in year T, the problem is to maximize:

\[
\sum_{t=0}^{54+T-s} (1+\delta)^t V_{T-s,T+t}
\]

where \( V \) is defined as in (1). The relevant budget constraint for such individual is now:

\[
\sum_{t=0}^{54+T-s} R_{T+t} c_{T-s,T+t} + \sum_{t=1}^{54+T-s} \xi_{T+t} m_{T-s,T+t} = \left[ a_{T-s,T} (1+r_T) + \frac{M_{T-s,T}}{P_T} + h_{T-s,T} + \psi_{T-s,T} \right]
\]

Notice that the summations run until the year 54+T-s, at the end of which the individual will die. The RHS of equation (33) represents the individual's total wealth. Since those individuals born before T-44 are already retired, the term \( h_{T-s,T} \) for them would be equal to zero. For example, consider an individual of age 3 at the beginning of year T. This individual will die at the end of year T+51 and, therefore, will have to maximize his utility over the remaining 52 years of his life. He will do that by selecting \( c_{T-3,T} = \{ c_{T-3,T}, \ldots, c_{T-3,T+51} \} \) and \( m_{T-3,T} = \{ m_{T-3,T+1}, \ldots, m_{T-3,T+51} \} \) in order to maximize (32) subject to (33) for s=3.

The solution to the optimization problem just described, along with the standard solutions obtained earlier, lead to the following set of consumption demands at the end of period T:
where, the term $\phi_{T-s,T}$ is defined in Appendix A, and $h_{T-s,T}$ is zero for $s > 44$. The demands for money at the end of year $T$ can be obtained by using the first order condition (7):

$$m_{T-s,T+1} = \beta \xi_{T+1}^{-1/\gamma} c_{T-s,T}; \quad s = 0, \ldots, 53. \tag{37}$$

All these money demand equations can then be aggregated and the resulting expression substituted in equation (17) in order to solve for the equilibrium price level. The solution is:

$$P_T = P(M_T; \mu_T; \alpha_{T-s,T}; M_{T-s,T}; h_{T-s,T}; \psi_{T-s,T}), \quad \text{for } s = 0, \ldots, 53. \tag{38}$$

The equilibrium price level of period $T$ depends on the stock of nominal money available at the end of that period, the marginal propensities $\phi$, and the way in which total wealth (both human and non-human) is distributed among individuals of different generations. Given the definition of the $\phi_s, \psi_s$ and $h_s$, the price equation can be written as: 1/

$$P_T = P(M_T; M_{T-s,T}; \alpha_{T-s,T}; h_{T-s,T}; \psi_{T-s,T}; \omega_{T+n}; r_{T+s}; \pi_{T+s}) \tag{38}$$

for: $s = 0, \ldots, 54; \quad n = 0, \ldots, 44.$

This equation shows that knowledge of the current levels of $M$ and $\mu$ along with the distribution of wealth and of the time path of factor prices and inflation over the next 54 years fully determines $P_T$. Obviously, equation (38) is not a reduced form equation for the price level. It can be clearly seen now, that an algebraic derivation of the reduced form equations for $k$ and $P$ is impossible. Equation (38), however, constitutes the first step of the methodology used for the numerical solution of the model. With

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1/ See Appendix C for the explicit form of the price equation used in the simulations.
an initial guess of the current and future factor prices and inflation rates, as well as initial conditions for the distribution of wealth and the money stock, the initial $P$ can be computed. This value of $P$ can then be used to determine the real value of money balances held by each of the 54 generations. This, along with the $\phi$s obtained from the first guess vectors of factor prices and inflation rates, will determine the current levels of the consumption demands according to equations (34)-(36). These consumption levels can be aggregated in order to obtain $c^*$ which can then be used in equation (22) to obtain the next period's capital stock. In the model's calibration, this procedure was followed for 150 consecutive years, a period considered long enough for the economy to have completed its adjustment to the new steady state.

It is important to point out the nature of the endogenous variables in the model. The capital stock, and therefore factor prices $w$ and $r$, are predetermined variables, i.e., their values will not change immediately after a policy change is implemented or announced. Likewise, the nominal money supply and the individual holdings of money and assets at the beginning of any year also constitute predetermined variables. This implies that the simulation of dynamic transition paths will require the specification of initial conditions for these variables. The price level, on the other hand, is not a predetermined variable and it might jump in response to such policies. Since at the end of a given period the optimal consumption decision of the older 54 individuals depends directly on the real value of their wealth, $c^*$ and $m^*$ will typically jump when $P$ does, because $P$ affects the wealth of individuals via changes in the real value of money balances. The resulting deviation of private consumption from its steady state value will then determine the time path of the capital stock towards its new steady state according to equation (22), provided the system is stable.

III. Baseline Scenario

In order to solve the model numerically, values have to be chosen for the household preference parameters $\theta$, $\gamma$ and $\delta$, and for the technological parameter $\alpha$. The selection of values for those parameters was based on recent empirical evidence from the U.S. economy. A detailed explanation of this calibration exercise as well as of the construction of a baseline scenario is provided next.

1. Parameter values

Empirical estimations of $\gamma$ in models that include money in the utility function could not be found. However, the model developed in section 2 portrays an important relationship between $\gamma$ and the economy's money demand, because the elasticity of the latter with respect to the nominal interest rate is approximately $1/\gamma(1+i)$. For this reason, the selection of $\gamma$ is based on empirical evidence. The exact value of the elasticity is presented in Appendix B.
particular value of $\gamma$ is based on two criteria: a realistic value for the intertemporal elasticity of substitution and a reasonable interest elasticity of the demand for money.

Empirical estimates of the intertemporal elasticity of substitution for the U.S. economy vary quite considerably. Hall (1980) obtained very low elasticities (between 0.04 and 0.06). Higher values were found by Mankiw (1981), (1985), with estimates that ranged between 0.25 and 0.44; while, Mankiw, Rotemberg and Summers (1985) found elasticities greater than one. Recently, however, Campbell and Mankiw (1989) have found that consistent estimation of the intertemporal substitution parameter requires taking into account a sector of liquidity constrained consumers. Within such framework, they find robust but quite low estimates of elasticities of substitution (between 0.016 and 0.089) for those individuals that do not face liquidity constraints.

As for the interest elasticity of the money demand, empirical estimates for the U.S. economy until 1973 were consistently found to lie around 0.16, as reported in the works of Goldfeld (1973), Judd and Scadding (1982), and Goldfeld and Sichel (1990). After 1973, however, there have been two episodes that have apparently introduced instability in the money demand: "The Case of the Missing Money" described by Goldfeld (1976) and the significant decline in velocity occurred in the early eighties. These episodes affected the stability of the parameters in the traditional formulations of the demand for money. However, Baba, Hendry and Starr (1987) have recently attributed the empirical difficulties present in the post 1973 data to misspecification of the money demand function. They try to correct for this problem by using a richer dynamic structure and by addressing the issue of financial innovation with a more sophisticated computation of the costs of holding money. Their estimated equation passes the tests of parameter constancy over the periods of apparent instability mentioned above, and yields an interest elasticity that ranges from -0.57 to -0.3.

Based on this evidence, and in the context of a nominal interest rate of around 11 percent, a value of 4 was selected for $\gamma$. This yields an intertemporal elasticity of 0.25 and an interest elasticity of the money demand of around -0.23. The value chosen for the parameter $\delta$ is 0.015, which corresponds to the one used by Auerbach and Kotlikoff (1987), and the distributional parameter $\theta$ was given two alternative values, 0.99985 and 0.999998, which lead to money to output ratios similar to the historical values of the M1-GDP and money base(MB)-GDP ratios for the United States. 1/

The remaining parameter ($\alpha$) is related to the production technology and, in the context of the model presented in section 2, represents the

---

1/ The historical averages for the period 1970-90 are 17.6 percent for the M1-GDP ratio and 6.5 percent for the money base-GDP ratio.
share of capital income on output. The value of this parameter was set at 0.25 based on the historical average of that share in the U.S. economy.

2. **Main features of the model in the baseline scenario**

The model was solved numerically using the parameter values just described. The results are shown in Table 1. Notice that in both cases (Ml and MB) the resulting capital output ratio is around 4.3, somewhat larger than the estimated rate of 3.5 for the U.S. economy. 1/ The resulting real interest rate is 5.8 percent, which is slightly below the average real return to equity during the period 1948-1981: 6.58 percent. 2/

The main features of the model in the baseline scenario are illustrated in Figures 1-3, which show the age profiles of consumption and money demands, income, and the intergenerational distribution of the capital stock. These figures reveal two important features of the model: first, an increasing age profile of money, indicating that at any point in time most of the money supply is in the hands of the elderly; second, the figures reveal typical life cycle patterns. A comparison of Figures 1 and 2 shows that while in the labor force, the individual's level of consumption is consistently below his income (wage plus capital income). After retirement, however, consumption will always be higher than his current income. The implications of this behavior are reflected in Figure 3 which shows the distribution of the capital stock. The holdings of capital increase with age while the individual is working, but after retirement, the individual starts depleting his wealth. Therefore, the capital stock is concentrated among those individuals who are close to retirement or have retired recently. Notice also that even though money is part of the individual's wealth at any point in time, overall it represents a very small part of it. Nevertheless, changes in the price level will induce a "money balances" effect which will, in turn, affect the economy's aggregate consumption. The magnitude of this effect for each individual will depend not only on the size of his money holdings but also on the value of his current \( \phi \), the marginal propensity to consume out of wealth. Figure 4 shows the values of the \( \phi \)s in the baseline scenario (for the case of Ml). Notice that the value of \( \phi \) rises very quickly with age; thus, changes in the price level can have strong effects on the elderly not only because they hold most of the money, but because the older they are the faster they deplete their wealth.

The baseline scenario just described constitutes the starting point of the policy exercises developed in the following section. For this reason, it is necessary to evaluate how robust the results reported in Table 1 are to marginal changes in the values of the parameters \( \alpha, \delta, \gamma, \) and \( \theta \). This is done in Table 2, where the effects on a few key variables of changes in these parameters is examined.

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1/ Auerbach and Kotlikoff (1987), p.64.
Figure 1: Age-Consumption and Age-Money Profiles
Figure 2: Age-Income Profile

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Figure 3: Intergenerational Distribution of the Capital Stock
Figure 4: Marginal Propensities to Consume out of Wealth
### Table 1. Baseline Scenario

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Table 2. Steady State Sensitivity Analysis

\((\pi-5\%)\)

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<th>(\delta)</th>
<th>(\gamma)</th>
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The first row of Table 2 represents the baseline scenario (MI). The following two rows show the effects of changes in α on the variables mentioned before. An increase in α from 0.25 to 0.27 leads to a capital stock about 6 percent larger and an a real interest rate one-half of a percentage point above the one corresponding to the base case. Nevertheless, the money-output ratio and the interest elasticity of the money demand (η) are not affected significantly. An analogous result is obtained when a larger change in α is considered. The third row in Table 2 corresponds to a value of α of 0.15, 40 percent smaller than the one used in the base-case. As a result, the capital stock is 23 percent smaller and the interest is 2 1/4 percentage points lower. As before, the monetary variables experience very small changes.

In the case of the distributional parameter θ, the situation is the opposite: changes in θ result in relatively large changes in the money-output ratio compared to the changes in the real variables. Notice, however, that although the changes in θ shown in Table 2 are small, the resulting changes in β, which determines the marginal rate of substitution between money and consumption, are significant. For this reason, a small reduction in θ (of about 0.1 percent) leads to a relatively large increase in β (60 percent), a significant increase in the money to output ratio and reduction in the inflation rate. Neither the real variables nor η experience significant changes.

As one would have expected, an increase in the time preference rate, δ, leads to a lower capital stock, as individuals tend to consume more earlier in their lives. The opposite is obviously true in the case of reductions in the value of δ. These changes are also reflected in the real interest rate; however, the money-output ratio experiences very small changes. This is particularly clear in the case of an increase in δ from 0.015 to 0.05, which results in a reduction in the capital stock of about 35 percent and an increase in the real interest rate of 2.7 percentage points. Despite this large change in the real variables, both the money to output ratio and η remain almost invariant.

Changes in γ produce relatively large changes in all variables. This, however, is a direct consequence of the adoption of a common γ for both consumption and money balances in the utility function. As a result, changes in γ lead to simultaneous changes in the intertemporal elasticity of substitution, the interest elasticity of the money demand and the marginal rate of substitution between consumption and money. For example, an increase in γ from 4 to 5 leads to a reduction in the capital stock of about 13 percent as well as an effect on the money-output ratio comparable to the one obtained from a reduction in θ to 0.999. Conversely, a relatively large fall in γ, from 4 to 2, results in a significant increase in the capital stock and a sizable reduction in the money-output ratio. The negative relationship between γ and the capital stock is due to the fact that larger values of γ reduce the intertemporal elasticity of substitution and flatten the age consumption profile, with the resulting reduction in savings. The large effect on the money-output ratio of changes in γ is explained by the
fact that the interest elasticity of the money demand changes in inverse proportion to changes in \( \gamma \), and that the value of \( \beta \) increases with \( \gamma \). In this case, the reduction in \( \gamma \) from 4 to 2 leads to a 100 percent increase in the interest elasticity -at similar inflation rates- and a 93 percent reduction in \( \beta \). The result is, obviously, a significantly smaller money to output ratio.

In summary, Table 2 shows that the baseline results shown in Table 1 are reasonably robust to changes in the parameters \( \delta \) and \( \alpha \). Nevertheless, changes in the parameters \( \theta \) and, especially \( \gamma \), lead to relatively large changes in the results. This is somewhat disturbing because, as mentioned before, the existing estimates of \( \gamma \) for the US economy tend to vary quite considerably. This problem could be addressed by allowing different elasticities of substitution for money and consumption in the utility function. Such task, however, is left for future research.

IV. The Optimal Rate of Monetary Growth

1. Money growth and capital formation in the steady state

An important feature of the model developed in section II is the non-superneutrality of money. In fact, in that model faster rates of money creation lead to an increase in the capital stock. This result, usually referred to as the "Tobin effect", was first found to be present in OG models similar to the one developed here by Stein (1971), and has been analyzed by Weiss (1980), Drazen (1981) and more recently in the surveys by McCallum (1990) and Orphanides and Solow (1990). There are basically two reasons why an increase in the rate of money creation may lead to a change in the steady state capital stock in this type of models: first, the increase in the rate of inflation raises the cost of holding money and induces a recomposition of the individuals' portfolio away from money and towards capital. The second channel is related to the intergenerational redistribution of resources implicit in the collection and re-allocation of seigniorage. As it was explained in the previous section, money holdings vary with age. For this reason, if the proceeds from seigniorage are transferred back to the private sector in proportions different from the ones in which seigniorage was paid, a net intergenerational transfer occurs and capital formation is affected.

In order to obtain a quantitative estimate of the impact of the Tobin effect in the present model, the second channel was avoided. The government was assumed to set a fix rate of money creation and to return the proceeds from seigniorage in a way proportional to the money holdings of the different individuals. This rule ensured that the policy was "compensated" in the sense of maintaining unchanged the resources available to the private sector as a whole, and also precluding any distributional effects among generations. In other words, the policy amounts to a simple change in the relative prices of consumption and money balances.
Table 3 shows the results of the policy exercise just described. Increases in $\mu$ do have a positive effect on the capital stock, although the magnitude of this effect is small and decreasing with further increases in $\mu$. For example, an increase in $\mu$ from 0 percent to 1 percent increases output by 0.05 percent, but the increase in output is about half of this when $\mu$ is raised from 4 percent to 5 percent. The increase in the capital stock associated with faster rates of money creation leads to higher wage rates, a lower real interest rate and, therefore, a larger PDV of lifetime resources for every generation in the steady state. Also, notice that the substitution effect of higher inflation rates is stronger than the Tobin effect and therefore the aggregate money demand falls as inflation increases. This indicates that the Tobin effect embedded in this model is quite small, corroborating the findings of other authors. However, in spite of the small magnitude of the income effect, it is not clear a priori how an individual's lifetime utility in the steady state is affected by changes in $\mu$. In fact, it is possible that, as claimed by Weiss (1980), increases in $\mu$ will lead to increases in lifetime utility.

2. Money growth and welfare: steady state utility as optimality criterion

Weiss' claim that in the presence of a Tobin effect the optimal rate of inflation may be positive is examined here. The following experiment is carried out: the steady state lifetime utility of a representative individual is evaluated at different inflation rates and a welfare maximizing inflation rate, $\pi^*$, is identified. Using the steady state corresponding to $\pi^*$ as a benchmark, an equivalent variation measure ($\lambda$) is computed with the purpose of quantifying the welfare cost of monetary regimes with inflation rates different from $\pi^*$. The results are shown in Figure 5. The value of $\lambda$ corresponding to the welfare maximizing inflation rate is obviously 0, and negative in every other case. The steady state welfare maximizing inflation rate is 3 percent in the baseline scenario (M1). This value falls in the range of optimal inflation rates computed by Summers (1981) for the U.S. economy using a model that exhibited the Tobin effect (-4.4 percent to 17.2 percent).

1/ Two other studies for the U.S. economy have found either a quantitatively insignificant Tobin effect (Danthine et al. (1987)) or money superneutrality (Geweke (1986)).

2/ The equivalent variation measure $\lambda$ indicates the proportion by which the lifetime resources of a newly born individual would have to be adjusted under an inflation rate $\pi_0$, for him to obtain the same utility that he would have enjoyed if inflation were $\pi^*$.

3/ The same rate is obtained when the money base is the relevant monetary aggregate.

4/ It should be pointed out that Summers' results stem from a model in which the Tobin effect is practically imposed by the assumption of a constant savings rate in a world with two assets: money and capital.
Table 3. The Tobin Effect in the Baseline Scenario (M1)

<table>
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<th>$\mu$ (%)</th>
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<th>w</th>
<th>r (%)</th>
<th>c*</th>
<th>m*</th>
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<td>1.629</td>
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</table>
Figure 5: Steady-state welfare maximizing inflation rate
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The results shown in Figure 5 seem to support Weiss' contention. For relatively low inflation rates, increases in inflation may actually result in increases in lifetime utility, because the Tobin effect outweighs the negative substitution effect. Higher levels of inflation, however, lead to larger marginal distortions in the relative prices faced by the individual and also to weaker income effects. As a result, when the inflation rate exceeds $\pi^*$, the lifetime utility of a representative individual in steady state will decrease with further increases in $\mu$.

An interesting feature revealed by Figure 5 is the lack of robustness of the 3 percent rule. Indeed, Figure 5 shows that the welfare cost of not having an inflation rate equal to 3 percent is very small, especially when the inflation rate is sufficiently close to 3 percent. Inflation rates that range between -1.5 percent and 13 percent carry a welfare cost of no more than 0.1 percent of an individual's lifetime resources. Notice also that, although small, the welfare cost tends to increase rather quickly with further reductions in the inflation rate below 3 percent. This is explained by the fact that the largest income effects associated with increases in the inflation rate occur in the range of inflation rates below 3 percent.

An important issue omitted from the discussion so far is the transition from one steady state to the next. This is particularly relevant in the discussion of the optimal rate of money creation because, from a theoretical point of view, it is entirely possible that switching to a regime characterized by $\pi^*$ may actually hurt some generations. In fact, this is exactly what happens in this model whenever the initial inflation rate is below $\pi^*$. Figure 6 shows the welfare effects of switching to the welfare maximizing rate of money creation (3 percent), starting from two different inflation rates. In the first case, the original steady state is characterized by an inflation rate of 0 percent and in the second case the original inflation rate is 10 percent. Figure 6 plots two sets of values for $\lambda$, one for each transition path. In other words, for each generation represented in the horizontal axis there are two values of $\lambda$, each representing the welfare effect on that particular generation of a change in the rate of money creation from either 0 percent or 10 percent to 3 percent. Obviously, the two sets of values of $\lambda$ are not directly comparable because they are expressed as percentages of an individual's lifetime resources in the original steady states, which are different.

In both cases, the value of $\lambda$ is zero for the generation born in year -55 because that generation dies the year before the change is implemented, and therefore is unaffected by it. Notice that even though switching to a 3 percent regime leads to a steady state welfare gain in both cases, Figure 6 reveals an important asymmetry: when the inflation rate in the original steady state is below the optimal rate, the increase in steady state lifetime utility is achieved at the expense of some of the generations alive during the transition. In fact, those individuals alive at the time of the increase in $\mu$ (generations born in years -54 to -1) experience a reduction in their lifetime utility levels, a reduction that is bigger the younger they are in year 0. The lifetime utility of those individuals born
between years 0 and 9 is also lower than the one they would have enjoyed if no change in policy had taken place. When the original inflation rate is above $\pi^*$, however, there is no reduction in welfare. As a matter of fact, the same generations that would suffer from the increase in $\pi$ from 0 percent to 3 percent would enjoy large increases in welfare in this case.

In order to understand why the generations alive at the time of the policy change experience relatively large changes in lifetime utility despite the fact that the changes in $\mu$ are compensated, it is necessary to analyze the dynamics of the price level and the capital stock during the transition. This is shown in Table 4 which illustrates the economy's transitional dynamics following a permanent increase in the rate of money creation from 0 percent to the steady state welfare maximizing rate of 3 percent. Notice that in year 0, when the policy is implemented, both aggregate consumption and the aggregate money demand fall. The cause of this initial fall has largely been omitted from the discussion about the optimal rate of money creation, although it turns out to be a crucial element in the determination of a Pareto efficient dynamic path (in section IV.3). The fall in the demands for consumption and money reflects a negative wealth effect associated with an unusually high erosion of the existing monetary balances. Such erosion occurs because of a significant increase in the price level in period 0 (14 percent) and is a result of the decline in the equilibrium real value of money prompted by the permanent increase in $\mu$. In a model with perfect foresight and price flexibility, this adjustment takes place almost immediately. This result is independent from the amount of seigniorage collected by the government because the proceeds are immediately returned to the private sector. Notice, however, that in year 0 the inflation rate is significantly higher than the rate of money creation. This implies that for each individual the reduction in the real value of money is larger than the increase in the money transfers made by the government in year 0, with the consequent negative effect on their wealth. In other words, all individuals entering period 0 with positive money balances face an unexpected negative money balances effect.

As mentioned before, it is because of this negative wealth effect that the aggregate demands for both consumption and money fall. This initial reduction in private consumption will in turn permit an accumulation of capital that will continue until the economy reaches its new steady state equilibrium. It should be stressed that the reason for the increase in the capital stock following a permanent increase in the rate of money creation is different from the one relating such increase solely to the existence of a Tobin effect. The Tobin effect argument relies on a substitution effect by which higher inflation rates induce all individuals to restructure the composition of their savings, substituting capital for money. However, in the model developed in section II, and in particular from equation (22), it is clear that starting from a steady state equilibrium, the capital stock can increase only if there is a reduction in private consumption. Such reduction, however, cannot be brought about only by the substitution effect associated with a permanent increase in the inflation rate. As shown in the first order conditions (7) to (9), in steady state the level of inflation...
Figure 6: Welfare Effects of a Switch to the Steady State Welfare Maximizing Inflation Rate
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Table 4. Dynamic Adjustment Following a Permanent Increase in $\mu$ from 0% to 3%

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<th>$\pi(%)$</th>
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<th>$c^*$</th>
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<td>6.924</td>
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</tr>
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<td>0.327</td>
</tr>
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<td>2.99</td>
<td>6.928</td>
<td>1.621</td>
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<td>0.327</td>
</tr>
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</tr>
<tr>
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<td>3.00</td>
<td>6.954</td>
<td>1.624</td>
<td>0.327</td>
</tr>
</tbody>
</table>

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does not affect the intertemporal allocations of consumption or money, although it induces contemporary substitution away from money towards consumption. Therefore, the reduction in aggregate consumption in year 0 shown in Table 4 occurs only because of the negative wealth effect described above. In fact, as it will be seen in the next section, if such effect were to be eliminated through a system of lump-sum transfers the increase in the inflation rate from 0 percent to 3 percent would not lead to capital accumulation or an increase in welfare.

Exactly the opposite result would be obtained if a permanent reduction in \( \mu \) from 10 percent to 3 percent were implemented. The permanent reduction in \( \mu \) would lead to a positive wealth effect associated with an increase in the real value of money. This increase would be validated through an actual reduction in the price level in period 0. Even though the higher wealth would lead to an increase in consumption and eventually a reduction in the capital stock, the new steady state would be characterized by a higher level of lifetime utility, reflecting the benefits of a higher degree of monetization.

One conclusion that can be drawn from the asymmetry just described relates to the relative efficiency, in a Pareto sense, of the 3 percent regime compared to the original one, once the entire dynamic process leading to the new steady state is taken into account. In the case of switching from 10 percent to 3 percent no generation is made worse off and, in fact, every generation is made better off. In other words, switching from 10 percent to the 3 percent inflation regime is in this sense a Pareto efficient policy. However, the same is not true in the case of switching from a 0 percent to 3 percent regime, because some generations are hurt during the transition. In both cases, the difference is due to the wealth effect associated with immediate changes in the value of money that occur when a permanent change in \( \mu \) is implemented. Notice, however, that even in the case of a reduction in \( \mu \) from 10 percent to 3 percent there is not a simple way of assessing the efficiency gains of such policy change. A methodology that permits to make this type of assessment is developed in the next section.

3. Money growth and welfare: Pareto efficiency as optimality criterion

This section re-examines the issue of the optimal rate of money creation, bringing two new elements into the analysis. First, an optimality criterion different from simple steady state utility maximization is defined. Second, the government is allowed to use other instruments in order to obtain a first best solution to the problem. The optimality criterion will consist on the comparison of two lifetime utility levels: the one prevailing before the policy change, \( U_0 \), and an endogenously determined, uniform level of utility enjoyed by all generations born since year 0, \( U_1 \). This comparison is made possible by including in the simulations a Government Agency (GA) that uses its lump sum taxes and payments in order to transfer resources among generations with the purpose of determining the two
lifetime utility levels $U_0$ and $U_1$. 1/ This amounts to allowing the government to participate in the capital market, although in a restricted sense, one that simply facilitates the computation of efficiency gains. This particular way of assessing optimality is chosen for its analytical simplicity given the purpose of quantification of the efficiency gains: on the one hand, there is no need to specify a social discount rate and, on the other, having only two utility levels to compare implies that a single gauge of efficiency gains (or losses) can be computed through an equivalent variation measure. 2/

The methodology was applied to the baseline scenario in the following way. The initial steady state selected was that corresponding to the baseline scenario ($\mu_0=5$ percent). The value of $\mu$ was then changed to some constant value, $\mu_1$, and the value of $U_1(\mu_1)$ was computed. Based on $U_0$ and $U_1(\mu_1)$, a measure of efficiency gains associated with a permanent change in the rate of monetary creation from $\mu_0$ to $\mu_1$ was calculated from the following equation: 3/

$$\lambda(\mu_0,\mu_1) = \left[ \frac{U_1}{U_0} \right]^{1/\gamma - 1} - 1. \quad (39)$$

This procedure was repeated for different values of $\mu_1$, and the value of $\mu_1$ that maximizes $\lambda(\mu_0,\mu_1)$ was identified. The results of this exercise are reported in Table 5.

Before explaining the results presented in Table 5 a few words of caution regarding this methodology are necessary. Since the GA incurs in borrowing and lending activities in its attempt to equate the lifetime utility of the two demographic groups ($s=-54, \ldots, -1$ and $s=0, \ldots, 0$) to $U_0$ and $U_1$ respectively, its actions will affect the level of the capital stock in the new steady state. Moreover, the magnitude of this effect will depend not only on the particular regime being implemented, i.e., the value of $\mu_1$, but also on the characteristics of the original steady state. For this reason, there will not be a unique final steady state associated with a given value of $\mu_1$. Depending on the values of $\mu_0$ and the capital stock in the initial steady state, different values of $k$ will be obtained when the rate of money creation is changed to $\mu_1$ using the GA. However, the fact that the value of $\lambda(\mu_0,\mu_1)$ depends also on the characteristics of the steady state associated with $\mu_0$ does not preclude the possibility of obtaining a

1/ The GA is assumed to break even in the sense that the PDV of all its transfers must equal the PDV of the taxes it collects.

2/ This methodology is an extension of the one used by Auerbach and Kotlikoff (1987); a detailed explanation of it can be found in chapter 3 of Hamann (1992).

3/ As before, $\lambda$ represents an equivalent variation measure, expressed as a percentage of an individual’s lifetime resources in the original steady state. It is a function of the ratio $U_1/U_0$ because the utility function (1) is homothetic.
Table 5. Relative Efficiency of Alternative Monetary Regimes

<table>
<thead>
<tr>
<th>$\mu_i(%)$</th>
<th>$k$</th>
<th>$i(%)$</th>
<th>$c^*$</th>
<th>$m^*$</th>
<th>$\lambda(5%, \mu_i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4.0</td>
<td>7.198</td>
<td>1.46</td>
<td>1.638</td>
<td>0.511</td>
<td>0.0174</td>
</tr>
<tr>
<td>-3.0</td>
<td>7.163</td>
<td>2.54</td>
<td>1.636</td>
<td>0.446</td>
<td>0.0148</td>
</tr>
<tr>
<td>-2.0</td>
<td>7.133</td>
<td>3.61</td>
<td>1.634</td>
<td>0.407</td>
<td>0.0124</td>
</tr>
<tr>
<td>-1.0</td>
<td>7.106</td>
<td>4.69</td>
<td>1.632</td>
<td>0.383</td>
<td>0.0103</td>
</tr>
<tr>
<td>0.0</td>
<td>7.079</td>
<td>5.76</td>
<td>1.631</td>
<td>0.365</td>
<td>0.0083</td>
</tr>
<tr>
<td>1.0</td>
<td>7.055</td>
<td>6.83</td>
<td>1.630</td>
<td>0.350</td>
<td>0.0065</td>
</tr>
<tr>
<td>2.0</td>
<td>7.030</td>
<td>7.91</td>
<td>1.628</td>
<td>0.338</td>
<td>0.0047</td>
</tr>
<tr>
<td>3.0</td>
<td>7.008</td>
<td>8.98</td>
<td>1.627</td>
<td>0.328</td>
<td>0.0031</td>
</tr>
<tr>
<td>4.0</td>
<td>6.987</td>
<td>10.05</td>
<td>1.626</td>
<td>0.320</td>
<td>0.0015</td>
</tr>
<tr>
<td>5.0</td>
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<td>11.11</td>
<td>1.625</td>
<td>0.312</td>
<td>0.0000</td>
</tr>
<tr>
<td>6.0</td>
<td>6.948</td>
<td>12.19</td>
<td>1.624</td>
<td>0.306</td>
<td>-0.0014</td>
</tr>
</tbody>
</table>

Note: the values of $i$, $c^*$ and $m^*$ correspond to the new steady state.
"most efficient" regime. In fact, such regime would be characterized by a rate $\mu^*$ such that $\lambda(\mu_0, \mu^*)$ would be positive for any value of $\mu_0$, although the particular value of $\lambda(\mu_0, \mu^*)$ would certainly depend on $\mu_0$. In practice, the search for such $\mu^*$ was facilitated by the fact that a switch to a regime with a lower rate of money creation was found to be always efficient.

Table 5 shows only the results obtained when the 5 percent rule was used as the starting steady state, but they are sufficiently general to illustrate the basic message: a reduction in the rate of money creation is always an efficient policy. Moreover, the values of $\lambda$ show that the efficiency gains of reducing the rate of money creation are larger the lower the value of $\mu^*_1$. This result suggests that implementing Friedman's rule of a zero nominal interest rate would in fact be the most efficient policy. Notice, however, that the lowest value of $\mu^*_1$ reported in the table is -4 percent, which corresponds to a nominal interest rate of 1.5 percent. There are two reasons for this. First, Friedman's rule cannot be actually implemented in this model because there is not a finite value of monetary balances with which an individual would achieve monetary satiation; and second, values of the nominal interest rate significantly closer to zero cannot be implemented computationally because the algorithms used for simulation tend to become unstable. Thus, the argument about the optimality of Friedman's rule is based only on an approximate sense.

The message of Table 5 is not only that the methodology employed here leads to a different conclusion than the one obtained from steady state calculations but also that the efficiency gains associated with the implementation of the optimal rate of money creation can be substantial. Indeed, a reduction in the rate of inflation from 5 percent to -4 percent leads to efficiency gains of about 1.7 percent of lifetime resources. The magnitude of the efficiency gains is reduced significantly, however, if the definition of money used in the calibrations is the money base rather than $M_1$. In such case, the efficiency gains are scaled down by approximately the value of the ratio of $M_1$ to $MB$: 2.5.

The methodology on efficiency gains developed in this section can be used to compare the results of the present model with those obtained recently by Cooley and Hansen (1989). In their model the relationship between inflation and capital formation is negative and, therefore, the steady state utility maximizing inflation rate is the one recommended by Friedman. In order to analyze the welfare effects of inflation, the authors remove the technology shock from their real business cycle model and evaluate the steady state lifetime utility of a representative individual under different rates of money creation.

The comparison is based on deviations from a -4 percent inflation rate and are reported in Table 6. The three cases reported are: Cooley and

---

1/ Given the parameterization of their model, these authors find an optimal inflation rate of -4 percent.
Table 6. Efficiency Costs of Inflation

<table>
<thead>
<tr>
<th></th>
<th>Inf = 0%</th>
<th>Inf = 10%</th>
</tr>
</thead>
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<tr>
<td></td>
<td>C&amp;H</td>
<td>M1</td>
</tr>
<tr>
<td>$k_0$</td>
<td>100.0</td>
<td>100.0</td>
</tr>
<tr>
<td>$k_1$</td>
<td>99.0</td>
<td>98.3</td>
</tr>
<tr>
<td>$y_0$</td>
<td>100.0</td>
<td>100.0</td>
</tr>
<tr>
<td>$y_1$</td>
<td>99.0</td>
<td>99.6</td>
</tr>
<tr>
<td>$c_0$</td>
<td>100.0</td>
<td>100.0</td>
</tr>
<tr>
<td>$c_1$</td>
<td>99.0</td>
<td>99.6</td>
</tr>
</tbody>
</table>

welf. cost (% of y) 0.11% 0.25% 0.09% 0.39% 0.67% 0.23%
Hansen's (C&H) I., and the two cases considered in this paper's baseline scenario (Ml and MB). The initial steady state values of c*, y and k have been normalized to 100 in order to facilitate the comparison. Also, the welfare cost of inflation is now expressed in terms of output rather than an individual's lifetime resources. The estimates of the costs of inflation obtained with Ml as the relevant monetary aggregate are larger than the ones reported by C&H. The welfare cost of a 0 percent inflation rate is 0.25 percent of output, against the 0.11 percent reported by C&H. Similarly, the welfare cost of a 10 percent inflation rate, 0.67 percent of output, is also almost twice as large as C&H's estimate. In fact, the figures obtained with Ml are somewhat larger than the ones found by other authors. Using Ml also as the relevant monetary aggregate, Lucas (1981) found that the cost of a 10 percent inflation rate is 0.45 percent of GDP, and McCallum (1989) found that the cost is 0.3 percent.

In all cases higher inflation leads to comparable reductions in both output and consumption. However, the reasons for these reductions in output and consumption when inflation increases are entirely different in the two models. In C&H an increase in inflation leads to a substitution of leisure for consumption as the latter becomes implicitly taxed. As a result of increased leisure output falls and so does consumption. On the other hand, in the model presented in section II, the increase in inflation leads to substitution of consumption for money and to a negative money balances effect. Nevertheless, since the GA compensates the initial generations so that their lifetime utility is unaffected by the policy change, they receive a transfer from future generations. Under these circumstances the increase in consumption by the initial generations leads to a crowding out of the capital stock and to lower sustainable levels of consumption in the new steady state.

V. Conclusion

This paper has developed a 55-period overlapping generations model, and has calibrated it for the U.S. economy. The model has been used to analyze the issue of the optimal rate of money creation under different optimality criteria. When steady state lifetime utility is the optimality criterion, the optimal inflation rate is 3 percent, although this is not a very robust number. Nevertheless, it is argued that using steady state lifetime utility as the optimality criterion can be misleading, because it ignores important developments that take place during the transition. In particular, it is shown that although an increase in the rate of inflation to 3 percent will always lead to a higher steady state lifetime utility, it will do so at the expense of some generations. Increases in the rate of

1/ The results reported for C&H are those corresponding to a quarterly cash-in-advance constraint (top panel of their table 2, p. 743). In that case, their implicit money to output ratio is 18.6 percent for the steady state corresponding to an inflation rate of -4 percent, compared to a 25.4 percent in this model's baseline scenario (Ml).
inflation are, therefore, not Pareto efficient; moreover, because of the different impact of an increase in the rate of inflation on all generations, it is difficult to assess the welfare effect of changes in the inflation rate. These problems are addressed by designing a methodology for efficiency calculations that takes into account the welfare of all generations involved both during the transition and in the new steady state. The methodology is based on lump-sum taxes and transfers implemented by a government agency (GA) with two purposes: (i) leaving the generations alive at the time of the change in policy with the same level of lifetime utility they would have enjoyed had there been no change in policy \( U_0 \), and (ii) equating everybody else's lifetime utility to a common level \( U_f \) such that the PDV of all lump-sum taxes and transfers equals zero. In this context it is found that Friedman's rule is the best policy.

The optimality of Friedman's rule in a model that exhibits a Tobin effect reflects the fact that the benefits of immediate monetization of the economy are of great relative importance. Indeed, once the rate of money creation is lowered all individuals alive face both a positive wealth effect and a reduction in the relative price of money, the net effect of which is an increase in their lifetime utilities. Since the optimality criterion used here requires that these 54 individuals end up with a level of lifetime utility equal to the one they would have enjoyed in the case of no change in the rate of money creation, practically all of the increase in their wealth is transferred to the GA at the end of year 0. Therefore, the GA becomes a net creditor in year 0 and from then on it transfers resources to future generations in order to raise their lifetime utilities to the uniform level \( U_f \). This amounts to an intergenerational transfer of the initial wealth effect away from generations born before year 0 towards all subsequent generations. It is well known that in an OG model such transfer leads to an increase in capital formation. It turns out that this effect is always quantitatively more important than the negative Tobin effect of a lower inflation rate. The result is that a reduction in \( \mu \), whenever implemented along with the GA's system of lump-sum transfers, leads to both an increased capital stock and a higher level of monetary balances.

The simulations results show that the magnitude of the welfare gains associated with implementing Friedman's rule in the United States would be considerable. With a GDP of US$6 trillion, the benefits of lowering the inflation rate from 5 percent to -4 percent are of the order of US$29 billion if M1 is considered the relevant monetary aggregate, and of US$12 billion if the monetary base is used instead. As indicators of the deadweight loss involved, these figures are quite significant.
Definition of $\phi$

The marginal propensities to consume $\phi$ can be written as:

$$\phi_{T-s,T} = \begin{cases} \sum_{j=0}^{53-s} \phi_{j,T}^1 \phi_{j,T}^2 + \phi_{54-s,T}^2 & s=0,\ldots,53 \\ 1 & s=54 \end{cases}$$

where the functions $\phi^1$ and $\phi^2$ are defined as:

$$\phi_{j,T}^1 = \begin{cases} (1 + \beta \xi_{T+j+1}^{1/\gamma-1}) & j=0,\ldots,53 \\ 1 & j=54 \end{cases}$$

$$\phi_{j,T}^2 = \begin{cases} 1 & j=0 \\ \prod_{s=1}^{j} (1+r_{T+s})^{1/\gamma-1} & j=1,\ldots,54 \\ \frac{(1+\delta)^{1/\gamma}}{(1+\delta)^{1/\gamma}} & \end{cases}$$

Notice that in the logarithmic case when $\gamma=1$, the $\phi$s are not functions of factor prices, but only of the parameters $\theta$ and $\delta$. In other words, they would depend on the age of the individual but not on time.
The Interest Elasticity of the Money Demand

The interest elasticity of the money demand, \( \eta \), is defined as follows:

\[
\eta_{m*,i} = \left. \frac{\partial \ln m}{\partial \ln i} \right|_{k=k} = -\frac{1}{1+i} \left[ \frac{1/\gamma + \beta \xi^{1-1/\gamma}}{1 + \beta \xi^{1-1/\gamma}} \right]
\]

For values of \( \xi \) sufficiently close to zero, the elasticity can be approximated by \(-1/\gamma(1+i)\).
The Price Equation

The explicit form of the equation for the price level corresponding to expression (38) is:

\[
P_t = \frac{M_t (1+\mu_t) - \beta \xi_t^{-1/\gamma} \sum_{s=1}^{53} \phi_{t-s, t} M_{t-s, t}}{\beta \xi_t^{-1/\gamma} \left[ \sum_{s=0}^{44} \phi_{t-s, t} h_{t-s, t} + (1+\tau_t) \sum_{s=1}^{53} \phi_{t-s, t} a_{t-s, t} + \sum_{s=0}^{53} \phi_{t-s, t} \psi_{t-s, t} \right]}
\]
References


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