"Velocity of money is a discredited subject. The erratic behavior of the ratio of national income to money supply in the last twenty years has damaged, probably beyond repair, the usefulness of the circular velocity as a tool of short-term analysis. Money spending has emancipated itself vastly from money supply."

Doblin (1951)

Two conditional models of money demand ... have remained remarkably constant and otherwise well-specified in the presence of substantial data revisions and financial innovations .... Even so, the historical development of these models highlights that econometric models must be adaptive to the environment in the same way that agents are.

Hendry and Ericsson (1990)

I. Introduction

The search for a stable relationship between the stock of money and macroeconomic variables has engendered a vast literature in which successively more sophisticated models have been constructed over the dead bodies of their predecessors. Many econometricians, undaunted by the carnage, continue to be inspired by the importance of monetary policy and encouraged by the stability of trends in velocity over the longer run. The purpose of this paper is to evaluate and compare two approaches that have enjoyed a relative success and popularity in recent years: buffer stock and error correction models. Following this introduction, Section II provides some background on the development of those two approaches. Section III presents empirical estimates of the models, and Section IV compares their performance. The main conclusions are summarized in Section V.

Empirical studies of the demand for money can broadly be classified as either intensive or extensive. The former include those that examine in detail the demand for a single monetary aggregate, such as M1 in the United States, perhaps to discover additional determinants or new functional forms. In contrast, extensive studies typically look at more than one aggregate, perhaps in two or more countries or over different data periods, in order to evaluate the robustness of a single model. This paper does a bit of both, but it is primarily extensive: it compares the performance of the two approaches, and it makes that comparison over both broad and narrow definitions of money in each of the five largest industrial countries in order to shed light on whether the comparison itself is robust. It thus is complementary to studies that examine specific aggregates in more detail.

A large number of multicountry empirical studies of money demand have appeared during the past four decades, following Doblin's (1951) international survey of velocity trends. Early work generally established that similar functional forms could be estimated successfully over many countries; see Latané (1954, 1963), Kaufman and Latta (1966), Adekunle (1968), and Melitz and Correa (1970). Further work in the 1970s--Slovin and Sushka (1975), Coutière (1976), Coutière et al. (1976), Canarella and Roseman
(1978), and Al-Khuri and Nsouli (1978)—evaluated more detailed models and larger data sets, and in some cases made use of more sophisticated econometric techniques; again, most studies supported the view that the demand for money behaved similarly across countries.

More recent studies have incorporated data from the problematic 1970s and 1980s. Boughton (1981), Atkinson et al. (1984), and Fair (1987) performed a battery of stability tests on partial-adjustment equations, estimated using both narrow and broad aggregates, and generally found some evidence of structural instability; in addition, Fair compared the performances of the nominal and real partial adjustment models and found that the former provided superior results. Gandolfi and Lothian (1983) found evidence of second order serial correlation in broad money equations estimated for eight industrial countries; generalized least squares estimation of these equations helped eliminate stochastic shifts in the money-demand functions. Domowitz and Hakkio (1990) and Taylor (1986) appear to have been the first studies to apply the error correction approach to a multi-country framework; these studies found that the error correction model exhibits considerable uniformity across countries and provides stable equations. Subsequent work by Bomhoff (1990) and Leventakis (1990) provided estimates based on a Kalman filter approach and a currency substitution model, respectively; Leventakis' results on the applicability of the open-economy money-demand specification were inconclusive, while Bomhoff found that predictions based on the Kalman filter approach were able to forecast equally well in the 1970s and 1980s.

In Boughton and Tavlas (1990), we compared the forecasting performances of three models: partial adjustment, buffer stock, and error correction. The error correction predictions were superior within sample but the post-sample performances of the models were mixed. We also found that the buffer stock model outperformed the partial adjustment specification. Subsequently, Boughton (1991a, 1991b) found that although stable equations could be derived using the error correction approach, there was some doubt about the interpretation of the results and about the robustness of the specifications.

This paper provides an extension of our earlier papers in a number of respects. First, the buffer stock model evaluated here incorporates recent developments in specifying that model and ameliorates several restrictive assumptions in our earlier work (as explained below). Second, in addition to comparing the performances of the various models on the basis of the usual statistical criteria, tests are performed in order to evaluate the parameter stability and post-sample predictability of each model. We also present results of a more discriminating test of whether one model—i.e. buffer stock or error correction—encompasses the other. Finally, we evaluate and compare the economic implications of the equations, dividing the results into two broad categories: the dynamic adjustment processes and the steady-state properties of the demand for money.
II. Model Specification

1. Buffer stock models

The buffer stock approach was developed as an alternative paradigm to the conventional partial adjustment specification of money demand. In its simplest form, the partial adjustment model specifies the demand for real money balances in the following way:

\[ m_t - p_t = \beta_0 + \beta_1 y_t + \beta_2 i_t + \beta_3 (m_{t-1} - p_{t-1}) + \mu_t \]

where \( m_t \) is the logarithm of a measure of the money supply, \( p_t \) is the logarithm of a price index, \( y_t \) is the logarithm of a scale variable such as income or wealth, \( i_t \) is an opportunity cost variable, and \( \mu_t \) is an error term that may or may not be white noise, depending upon how model (1) is generated.

Among the attributes of model (1) that have been found troublesome in recent years are the following. First, almost all estimates of this model report very low short-run elasticities for income and interest rates and a coefficient close to unity on the lagged dependent variable. Since the absolute value of the long-run interest elasticity is higher than the short-run elasticity, interest rate overshooting must occur in the short run (Milbourne (1988); Swamy and Tavlas (1989a; 1989b)). However, such overshooting does not appear to be a feature of real-world money markets (Goodhart (1984)). Second, the transmission mechanics embedded in (1) appear to assume, if the model is found to be stable, that innovations in monetary policy have a predictable impact on prices, real income, and interest rates at each period. But such an outcome challenges the commonly accepted view that the dynamics underlying the move back to equilibrium are complex and unpredictable--i.e. the Friedman-Meiselman "long and variable lags" story (Boughton and Tavlas (1990)). Third, empirical estimates of (1) have yielded inaccurate predictions of real money balances. According to buffer stock proponents, one reason why the partial adjustment specification has fared poorly is that it does not account for the short-run impact of monetary shocks. Omission of such shocks, when they are determinants of real money holdings, implies that estimates based on model (1) are subject to specification bias.

The buffer stock model was devised to deal with each of these problems. The approach stresses the role of money as a shock absorber that temporarily smooths the response of the economy to monetary innovations. Such innovations are explicitly modeled as part of the determination of money demand. By so incorporating money shocks, the buffer stock model is able to deal with each of the three criticisms associated with the conventional model. First, the buffer stock model suggests that (positive) monetary innovations lead to an accumulation in cash balances in the short run as economic agents move off their short-run money demand curves. Consequently, cash balances—rather than interest rates—adjust in the short-run, implying that interest rate overshooting need not occur. Second, by modeling the effects on short-run money demand directly, the buffer stock model is able to deal more
realistically with the complicated nature of the monetary transmission mechanism. The lags are still deterministic rather than "variable," but they are at least modeled as being more complex than in the partial adjustment specification. Finally, inclusion of a money shock variable can help deal with the specification bias in the conventional model and, therefore, with the problem of parameter instability (assuming that the buffer stock model is the "true" model of money demand).

Three broad approaches to modeling buffer-stock money have been used in the empirical literature. One approach inverts the money-demand function, assuming the chosen dependent variable—i.e. the price level, output, or the interest rate—adjusts slowly to its long-run value (e.g. Artis and Lewis (1976) and Laidler (1980)). However, as Cuthbertson has noted, "the main problem with this approach is that only one argument may be chosen as the dependent variable whereas on a priori grounds one might expect all of the arguments of the demand function to adjust simultaneously" (1988, p. 112). A second approach involves the estimation of money shocks in large-scale econometric models. The key problem with this approach is that the estimated coefficients in the money demand equation are conditional on the correct specification of the entire model (Cuthbertson (1988, p. 112); Milbourne (1988, p. 200)).

The third, and perhaps most widely used approach, is the shock absorber model developed by Carr and Darby (1981). This approach has undergone considerable modification in order to take account of econometric problems in the original version. The shock absorber model formulates the demand for money as follows:

\[ m_t - p_t = \beta_0 + \beta_1 y_t + \beta_2 i_t + \beta_3 (m_{t-1} - p_{t-1}) + \alpha (m_t - m_t^*) + \mu_t \]

\[ m_t^* = gZ_t + \epsilon_t \]

where \( m_t^* \) is the anticipated component of the money supply, \( Z \) is a set of variables that agents assume have a systematic influence on the money supply, \( g \) is a vector of coefficients to be estimated, \((m_t - gZ_t)\) is the monetary innovation, and \( \epsilon_t \) is a white-noise error.

In Boughton and Tavlas (1990), we estimated the anticipated component of the innovation term by regressing the money supply on a polynomial distributed lag (second degree, twelve period) of its past values, similar to the approach used by Carr and Darby. However, as MacKinnon and Milbourne (1984) pointed out, OLS estimation of the Carr-Darby model leads to simultaneity bias (since the \( m_t \) component of the money innovation is correlated with the dependent variable). Accordingly, this paper uses instrumental variable estimation on \((m_t - m_t^*)\).

1/ Early work on the shock-absorber model also includes Laidler (1980) and Coats (1982).

2/ The specification of the autoregressive lag structure used by Carr and Darby is not revealed in their paper.
The foregoing approach is used as the point of departure with respect
to the shock absorber model estimated in this paper. We then extend the
model in several directions. First, we test for the impact of the lagged
inflation rate as a proxy for inflationary expectations, on the hypothesis
that money serves as a substitute for real assets (Tobin (1969)). Second,
we follow a suggestion proposed by Cuthbertson (1988) and Cuthbertson and
Taylor (1989) that monetary innovations need not be confined to money-supply
shocks, but can also appear as interest rate shocks. Specifically, in
periods when the monetary authorities are targeting interest rates, the
money supply is endogenous. Accordingly, interest rate innovations should
also be tested in the buffer-stock model as a proxy for unanticipated
changes in monetary policy.

Third, as noted by Goldfeld and Sichel (1990, p. 345), the use of only
the money supply to generate anticipations is overly restrictive. Accord-
ingly, we extend the information used by individuals in forming expectations
by incorporating other variables as determinants of anticipated money.
Fourth, as a variation of the shock absorber approach, we use a model of
expectations proposed by Cuthbertson and Taylor (1989) whereby agents
optimally update the coefficients of their expectations formation process in
light of new information.

Finally, we examine a variant of the buffer stock model that was
proposed by MacKinnon and Milbourne (1984). A simple formulation of the
model is

\[ m_t - p_t = \beta x_t + \alpha (m_t - m_t^*) + \mu_t \]  

where \( x \) is a vector of variables and \( \beta \) is a vector of coefficients (as in
equation (2), above). MacKinnon and Milbourne pointed out that the Carr-
Darby hypothesis implies that anticipated nominal money should not affect
the real demand for money, and that this is a testable proposition.

Equation (4) can be generalized as

\[ m_t - p_t = \beta x_t + \alpha (m_t - m_t^*) + \phi m_t^* + \mu_t. \]

If the Carr-Darby model is valid, one would expect \( 0 < \alpha \leq 1 \) and \( \phi = 0 \).
MacKinnon and Milbourne further propose transforming (5) to remove \( m_t \) from
the RHS; that gives

\[ m_t - p_t = \lambda (m_t^* - p_t) + \phi^* m_t^* + \beta^* x_t + \mu_t^*. \]
where \( \lambda = \alpha/(1-\alpha) \), \( \phi^* = \phi/(1-\alpha) \), \( \beta^* = \beta/(1-\alpha) \), and \( \mu_t^* = (1/(1-\alpha))\mu_t \). MacKinnon and Milbourne reject the model in this form, using U.S. data over 1953:1-1978:4, on the grounds that \( \phi^* > 0 \) and \( \lambda > 0 \). 

There are some potentially serious difficulties with the MacKinnon-Milbourne transformation. First, while \( m_t \) has been removed from the RHS, \( p_t \) has replaced it; thus the underlying problem of possible correlation of \( m_t \) with RHS coefficients has not been eliminated. Second, if \( \alpha = 1 \), all of the coefficients in equation (6) will be indeterminate. Third, two of the RHS terms--\( m_t^* \) and \( (m_t^*-p_t) \)--may be highly correlated, making the test statistics difficult to interpret.

An alternative to the MacKinnon-Milbourne transformation, which would eliminate the first problem and reduce the magnitude of the third, is to combine the two \( m_t \) terms in equation (6) and solve for \( m_t \):

\[
(7) \quad m_t = \beta^* x_t + \phi^* m_t^* + \alpha' p_t + \mu_t^*,
\]

where \( \phi' = (\phi-\alpha)/(1-\alpha) \) and \( \alpha' = 1/(1-\alpha) \). The main remaining limitation is the indeterminacy of the estimated coefficients when \( \alpha = 1 \). But it is also interesting to note a puzzling aspect of the model that is revealed by these transformations. In view of the very high correlation between \( m_t \) and \( m_t^* \), it would be surprising not to find \( \phi' = 1 \), even though the estimating equation is a simple algebraic transformation of the buffer stock model, under which one would expect to find \( \phi' < 0 \). This conflict in interpretation is examined empirically below.

2. Error correction models

There is now an extended literature on the rationale for the specification of error correction models of money demand (Boughton (1991b)). Two points may be emphasized here. First, the basis for the error correction model is very similar to that for the buffer stock model: the partial adjustment model has been demonstrated to be an inadequate representation of the way agents learn and respond to new information. Whereas the buffer stock model hypothesizes that news is captured by unanticipated changes in the supply of money, the error correction model hypothesizes that news is captured by disequilibrium in the long-run static demand function. That is, in this framework agents are postulated to respond to "errors" in the predictions from the long-run function by taking account of current and recent changes in the various arguments.

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1/ MacKinnon and Milbourne find (Table 2, p. 271) \( \lambda = 0.63 \) and \( \phi^* = 0.04 \), with t-ratios of 6.71 and 3.95, respectively. The implied underlying coefficients are \( \alpha = -1.7 \) and \( \phi = 0.01 \). But they seem to err in concluding that \( \phi \) is significantly positive; they do not derive the probability distribution on \( \phi \) (only on \( \phi^* \)), and it is not obvious that \( \phi = 0 \) can be rejected.
Second, this formulation may be viewed as a generalization of the buffer stock proposition. In both conceptions, agents adjust in response to unanticipated disturbances. In principle, these shocks could come from the supply or the demand side; the buffer stock model explicitly incorporates only the former, while the error correction model incorporates both. Because the two models may treat anticipations differently, however, estimates of the former may not be nested in the latter.

A fairly general formulation of the error correction model may be written as follows:

\[ m_t - p_t = \beta x_t + \mu_t \]  

\[ \Delta(m-p)_t = -\lambda \mu_{t-1} + \Gamma \Delta_k x_{t,j} + \epsilon_t, \quad i, k \geq 1, \quad j \geq 0; \]

where \( x \) is a vector of variables, \( \beta \) and \( \Gamma \) are coefficient vectors, \( \epsilon \sim \text{n.i.d.}(0, \sigma^2) \), \( \lambda > 0 \), and \( \mu \) does not have a unit root. Equation (8) is equivalent to the long-run solution to the models discussed above. In contrast to the partial adjustment and buffer stock models, however, the vector of differences on \( x \) in equation (9) may be as general as necessary to capture the dynamic adjustment process; that is, \( j \) and \( k \) are allowed to vary from one element of \( x \) to another. In other words, the form and content of the long-run function are largely dictated by theory, as in any other model, but the dynamic specification is dictated only by the restrictions that are present in the data.

In the most common approach for estimating the error correction model, after Engle and Granger (1987), an equation in the general form of (8) is estimated first to obtain \( \mu_t \). If \( \mu \) passes the standard tests for rejection of nonstationarity, \( \lambda \) that series is then used as an argument in equation (9), often with the constraint \( i = 1 \). Alternatively, that first stage may be skipped, and equation (9) may be estimated directly, with or without the constraint that each of the implicit components of \( \mu \) have the same lag. In other words, one may estimate the single, more general equation

\[ \Delta(m-p)_t = -\lambda [(m-p)_{t-1} - \beta x_{t-1}] + \Gamma \Delta_k x_{t,j} + \epsilon_t, \]

Evaluation of \( \mu \) is itself the subject of a large and growing literature; for a recent survey, see Dolado and Jenkinson (1987). The two most commonly applied criteria are a test devised by Sargan and Bhargava (1983), under which the Durbin-Watson statistic for equation (8) should be approximately 0.4 or higher; and a test devised by Dickey and Fuller (1981), under which the t-statistic on \( \mu_{t-1} \) in a regression of \( \Delta \mu \) on that variable--and possibly on lagged changes as well--should be approximately -3.5 or higher. Rejection of nonstationarity is equivalent to accepting the hypothesis that the variables included in equation (8) are cointegrated, conditional on the first differences of each of those variables being stationary.
where again the lags and differences may vary from one element of \( \chi \) to another. This single-stage generalization of the error correction model is adopted here. 1/

III. Estimation

The data base for this study comprises quarterly time series from 1963 through 1988 for the five largest industrial countries: the United States, Japan, the Federal Republic of Germany, France, and the United Kingdom. All data are seasonally adjusted, except for interest rates. There are ten monetary aggregates in all; one narrowly defined (M1) and one more broadly defined (M2 or M3). 2/ The strategy is to estimate the various models using data only through 1985:4 and then to test for post-sample stability using the final twelve observations; this is obviously an arbitrary choice of dates, but it is not possible to select a logical break point that would apply equally to the five countries.

1. Partial adjustment equations

Since the partial adjustment model served until recently as the most widely estimated model of money demand, consideration of estimates of this model provides a useful point of departure from which to compare the buffer stock and error correction results. In order to test common specifications of the partial adjustment model, the equations for both the narrow and broad aggregates incorporate only current values of real income and interest rates. With regard to interest rates, both short-term rates (denoted as \( i \) in Table 1) and long-term rates (denoted as \( r \)) were tested, but both were found to be significant only in the U.S. broad money equation; in most other equations it was the short-term rate that was found to be significant and dominant.

As reported in Table 1, the partial adjustment results appear to be satisfactory according to the usual statistical criteria. In only two instances were regressors found to be insignificant—those of real income in the narrow money equation for Japan and in the broad money equation for France. As with most previous estimates of the partial adjustment model,

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1/ The two methodologies are compared for these data in Boughton (1991b), where it is shown that in most cases the single-stage procedure works at least as well as the two-stage procedure. Yoshida (1990) compares the two for Japanese money demand equations and finds little overall difference.

2/ The composition of the aggregates is described in the Appendix to Boughton and Tavlas (1990). The scale variable is either real GNP (United States, Germany, and Japan) or real GDP (United Kingdom and France); price indexes are the corresponding deflators. Short-term interest rates apply to six-month commercial paper (United States), three-month interbank deposits (United Kingdom, Germany, and France), or gensaki bonds (Japan). Long-term rates are yields on government bonds except for Japan (telephone and telegraph bonds).
### Table 1. Partial Adjustment and Buffer Stock Models: Estimated Equations 1/

<table>
<thead>
<tr>
<th></th>
<th>United States</th>
<th>Mc2</th>
<th>Mc3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Partial adjustment (m-p)</td>
<td>(0.042y - 0.200i - 0.769\delta p_{-1} + 0.956(m-p)_{-1} - 0.030DUM80)</td>
<td>(0.179y - 0.585i - 0.365\delta p_{-1} + 0.813(m-p)_{-1} + 0.452\gamma)</td>
<td>(0.152y - 0.809i - 0.416\delta p_{-1} + 0.846(m-p)_{-1} + 0.332\gamma + 0.665(m-m^*))</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.967</td>
<td>0.998</td>
<td>0.998</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>0.81</td>
<td>0.689</td>
<td>0.669</td>
</tr>
<tr>
<td>(DW)</td>
<td>1.76</td>
<td>1.53</td>
<td>1.53</td>
</tr>
</tbody>
</table>

Buffer stock (m-p) | \(0.037y - 0.173i - 0.822\delta p_{-1} + 0.943(m-p)_{-1} - 0.024DUM80 + 1.28(m-m^*)\) | \(0.148y - 0.572i - 0.352\delta p_{-1} + 0.841(m-p)_{-1} + 0.464\gamma + 0.803(m-m^*)\) | \(0.152y - 0.809i - 0.416\delta p_{-1} + 0.846(m-p)_{-1} + 0.332\gamma + 0.665(m-m^*)\) |
| \(R^2\)       | 0.971         | 0.998           | 0.998           |
| \(\sigma\)    | 0.76          | 0.671           | 0.662           |
| \(DW\)        | 1.94          | 1.53            | 1.53            |

Buffer stock augmented (m-p) | \(0.032y - 0.299i - 0.792\delta p_{-1} + 0.969(m-p)_{-1} - 0.025DUM80 + 0.987(m-m^*)\) | \(0.152y - 0.809i - 0.416\delta p_{-1} + 0.846(m-p)_{-1} + 0.332\gamma + 0.665(m-m^*)\) |
| \(R^2\)       | 0.973         | 0.998           |
| \(\sigma\)    | 0.744         | 0.662           |
| \(DW\)        | 1.98          | 1.53            |

Buffer stock augmented (m-p) | \(0.037y - 0.173i - 0.822\delta p_{-1} + 0.943(m-p)_{-1} - 0.024DUM80 + 1.28(m-m^*)\) | \(0.032y - 0.299i - 0.792\delta p_{-1} + 0.969(m-p)_{-1} - 0.025DUM80 + 0.987(m-m^*)\) |
| \(R^2\)       | 0.971         | 0.973           |
| \(\sigma\)    | 0.76          | 0.744           |
| \(DW\)        | 1.94          | 1.98            |

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Table 1 (continued). Partial Adjustment and Buffer Stock Models:
Estimated Equations 1/

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<th>Country</th>
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<th>Buffer stock</th>
<th>Buffer stock augmented</th>
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<tr>
<td>Japan</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M1</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Partial adjustment</td>
<td>$(m-p) = 0.022y - 0.298i + 0.958(m-p)_{-1}$</td>
<td>$(m-p) = 0.013y - 0.286i + 0.964(m-p)_{-1} + 1.27(m-m^*)$</td>
<td>$(m-p) = 0.016y - 0.522i + 0.964(m-p)_{-1} + 1.22(m-m^*) + 0.32054i$</td>
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<td></td>
<td>$(0.227) (0.101) (0.028)$</td>
<td>$(0.031) (0.100) (0.030) (0.438)$</td>
<td>$(0.034) (0.149) (0.029) (0.422) (0.153)$</td>
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<tr>
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<td></td>
<td>$\sigma = 1.72$</td>
<td>$\sigma = 1.59$</td>
<td>$\sigma = 0.99$</td>
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<td>$DW = 1.53$</td>
<td>$DW = 1.38$</td>
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<td>M2</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Partial adjustment</td>
<td>$(m-p) = 0.123y - 0.629r - 0.926p_{-1} + 0.891(m-p)_{-1}$</td>
<td>$(m-p) = 0.103y - 0.555r - 0.376p_{-1} + 0.906(m-p)_{-1} + 1.89(m-m^*)$</td>
<td>$(m-p) = 0.091y - 1.030r - 0.273p_{-1} + 0.915(m-p)_{-1} + 1.61(m-m^*) + 0.59154i$</td>
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<tr>
<td></td>
<td>$(0.039) (0.157) (0.161) (0.028)$</td>
<td>$(0.037) (0.145) (0.159) (0.027) (0.636)$</td>
<td>$(0.034) (0.255) (0.156) (0.025) (0.594) (0.216)$</td>
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<td>$R^2 = 0.999$</td>
<td>$R^2 = 0.999$</td>
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<tr>
<td></td>
<td>$\sigma = 1.07$</td>
<td>$\sigma = 0.99$</td>
<td>$\sigma = 0.93$</td>
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<td>$DW = 1.19$</td>
<td>$DW = 1.52$</td>
<td>$DW = 1.58$</td>
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Estimated Equations 1/ Germany, Fed. Rep. of

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<th>$\sigma$</th>
<th>DW</th>
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<td><strong>M1</strong></td>
<td></td>
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<tr>
<td>Partial adjustment</td>
<td>$(m-p) = .242y - .526i + .785(m-p)_{-1}$</td>
<td>.995</td>
<td>1.35</td>
<td>2.07</td>
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<td>Buffer stock</td>
<td>$(m-p) = .214y - .491i + .808(m-p)_{-1} + .430(m-m^*)$</td>
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<td>1.33</td>
<td>2.03</td>
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<td>$(m-p) = .109y - .201i + .920(m-p)_{-1}$</td>
<td>.998</td>
<td>1.15</td>
<td>1.94</td>
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<tr>
<td><strong>M2</strong></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Partial adjustment</td>
<td>$(m-p) = .109y - .201i + .920(m-p)_{-1}$</td>
<td>.998</td>
<td>1.12</td>
<td>2.12</td>
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</table>

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Table 1 (continued). Partial Adjustment and Buffer Stock Models: Estimated Equations 1/

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<tbody>
<tr>
<td></td>
<td>M1</td>
<td></td>
</tr>
</tbody>
</table>
|                  | Partial adjustment | (m-p) = 0.141y - 0.698i + 0.947(m-p)_{-1}  
                      |                | (0.021) (0.098) (0.030)  |
|                  | Buffer stock   |                  |
|                  | (m-p) = 0.139y - 0.683i + 0.959(m-p)_{-1} + 0.644(m-m*)  
                      |                | (0.021) (0.099) (0.034) (0.521)  |
|                  | R² = 0.947     |                  |
|                  | σ = 1.78       |                  |
|                  | DW = 2.29      |                  |
|                  | M3             |                  |
|                  | Partial adjustment | (m-p) = 0.103y - 0.362r - 0.971Δp_{-1} + 0.971(m-p)_{-1}  
                      |                | (0.048) (0.288) (0.041) (0.041)  |
|                  | Buffer stock   |                  |
|                  | (m-p) = 0.098y - 0.331r - 0.391Δp_{-1} + 0.966(m-p)_{-1} + 0.600(m-m*)  
                      |                | (0.0467) (0.278) (0.345) (0.041) (0.396)  |
|                  | R² = 0.970     |                  |
|                  | σ = 2.36       |                  |
|                  | DW = 1.64      |                  |

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Table 1 (concluded). Partial Adjustment and Buffer Stock Models:
Estimated Equations 1/

<table>
<thead>
<tr>
<th>Country</th>
<th>Partial adjustment</th>
<th>Buffer stock</th>
</tr>
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<tbody>
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<td></td>
<td></td>
</tr>
<tr>
<td>M1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Partial adjustment</td>
<td>(m-p) = .138y - .266i + .708(m-p) _1</td>
<td>(m-p) = .140y - .365i + .683(m-p) _1 + 1.224(m-m*)</td>
</tr>
<tr>
<td></td>
<td>(.044) (.163) (.082)</td>
<td>(.043) (.150) (.083) (.348)</td>
</tr>
<tr>
<td></td>
<td>( R^2 = .867 )</td>
<td>( R^2 = .903 )</td>
</tr>
<tr>
<td></td>
<td>( \sigma = 2.86 )</td>
<td>( \sigma = 2.66 )</td>
</tr>
<tr>
<td></td>
<td>( DW = 2.30 )</td>
<td>( DW = 2.38 )</td>
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<tr>
<td>M2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Partial adjustment</td>
<td>(m-p) = .135y - .308r + .895(m-p) _1</td>
<td>(m-p) = .134y - .285z + .894(m-p) _1 + .768(m-m*)</td>
</tr>
<tr>
<td></td>
<td>(.115) (.144) (.079)</td>
<td>(.116) (.141) (.080) (.419)</td>
</tr>
<tr>
<td></td>
<td>( R^2 = .996 )</td>
<td>( R^2 = .996 )</td>
</tr>
<tr>
<td></td>
<td>( \sigma = 1.73 )</td>
<td>( \sigma = 1.66 )</td>
</tr>
<tr>
<td></td>
<td>( DW = 2.10 )</td>
<td>( DW = 2.21 )</td>
</tr>
</tbody>
</table>

1/ The sample period is 1965:1 - 1985:4. For notation, see text. Instrumental variable estimation was used on m-m*; the instruments were real income, the long-term interest rate, and the price level (all lagged t-1 through t-8). Numbers in parentheses in this and all subsequent tables are heteroskedastic-corrected standard errors. These and all subsequent equations included a constant term, not shown.
the lags were found to be long, with the coefficient on the lagged dependent variable ranging from 0.70 (narrow money for France) to 0.97 (broad money for the United Kingdom). In all equations for U.S. M1, a dummy variable was included for the temporary effects of the 1980 credit controls (DUM80 = 1 in 1980:2, -1 in 1980:3, and zero in all other periods); no other dummy variables were used.

2. Buffer stock equations

Table 1 also reports estimates of the buffer stock model. In order to deal with the Goldfeld-Sichel point—namely, that using only past values of the money supply to generate monetary anticipations is overly restrictive—the anticipated series was (in each instance) estimated by regressing the money supply on a polynomial distributed lag (second degree, eight lags) on past values of the money supply, the short-term interest rate, and the inflation rate. To correct for simultaneity, instrumental variable estimation was used on the money shock term (m-m*) using a uniform set of instruments (see notes to Table 1). 1/

As reported in Table 1, the addition of the monetary shock term was found to be positive and significant in eight of ten instances; in the equations for both narrow and broad money for the United Kingdom the coefficients were positive but marginally insignificant. 2/ The coefficients on the MS term range from 0.43 (narrow money, Germany) to 1.89 (broad money, Japan). Several of the equations (narrow money for the United States, France, and Japan, and broad money for Japan) have coefficients exceeding unity on the money shock terms; this would indicate overshooting following a monetary shock, which may be inconsistent with buffer stock theory. In all ten instances, the Durbin-Watson statistic improves as a result of the inclusion of the buffer stock term, implying the existence of specification bias in the partial adjustment equations. 3/ Finally, the standard error of the equations, a measure of within-sample forecasting performance, also improves (compared with the partial adjustment model) in all ten equations that incorporate the MS term.

As noted, the partial adjustment and buffer stock models were tested for the impact of price expectations, using the lagged value of the

1/ Different sets of instruments were used to estimate the anticipated money series and to perform instrumental variable estimation in order to help identify the equations.

2/ In our previous paper, based on restrictive estimation of anticipated money, insignificant money-shock coefficients were found in three equations: M1 for the United Kingdom, and M2 for France and Japan.

3/ This result would seem to support Laidler's (1982) finding that the error term in the partial adjustment model incorporates a money shock effect.
inflation rate as a proxy. The results were mixed; price expectations were found to be significant (for both models) in just four cases: the U.S. M1 equation and the broad money equations for the United States, Germany, and the United Kingdom.

In order to test for the impact of interest-rate shocks in the buffer stock model, a four-quarter moving standard deviation of the interest rate (S4t) was used as a proxy for such shocks. Theoretically, a positive interest rate shock would be expected to increase the demand for money, since a rise in the variance of interest rates increases the uncertainty associated with holding alternative financial instruments. Interest rate shocks were found to be significant in narrow money equations for the United States, Germany, and Japan; and in the broad money equations for the United States and Japan; providing some support for the hypothesis that individuals take account of more than just money-supply innovations in determining their real money holdings. The coefficients on the interest rate shock variable range from 0.15 (M1, United States) to 0.59 (M2, Japan).

Table 2 reports M1 equations in which the anticipated component of the money supply is based on sequential learning by agents. For these equations, we used recursive least squares estimation in deriving the data on monetary expectations, with the same regressors as before: past values of the money supply, the short-term interest rate, and the inflation rate. Recursive least squares estimation of anticipated money assumes that agents optimally update the coefficients of their monetary expectations each period, as new information becomes available. This approach differs from OLS estimation, whereby agents are implicitly assumed to form their predictions at each period on the basis of information in the entire sample period and with constant coefficients.

Because a number of data points are required for initialization of the recursive procedure, the estimation period for Table 2 begins in the early 1970s. For purposes of comparison, the previous equations (based on constant coefficients) were re-estimated over identical periods; these equations are denoted with the suffix a, and the recursive estimates are denoted with the suffix b. In four of five cases, recursive estimation of expectations provided superior results, in terms of the usual equation-performance criteria and in terms of increasing the t-ratio on the MS term. In the one remaining case (the United States), there was little difference between the two equations, except that the coefficient on MS is consistent with the Carr-Darby model only in the recursive estimate.

1/ The lagged inflation rate is obviously a crude proxy. It is preferred over the current rate to avoid confusion with the adjustment process. As Goldfeld (1976) showed, a significant coefficient on \( p_t - p_{t-1} \) could mean that the nominal money stock adjusts to a demand disturbance unless the coefficient is significantly different from the negative of the coefficient on lagged real balances.
Table 2. Buffer Stock Models for M1 with Recursive Expectations 1/

<table>
<thead>
<tr>
<th>Equation</th>
<th>Const.</th>
<th>y</th>
<th>i</th>
<th>(m-p)_1</th>
<th>m-m*</th>
<th>DUM80</th>
<th>σ</th>
<th>R^2</th>
<th>DW</th>
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<tr>
<td>US-a</td>
<td>-.224</td>
<td>.043</td>
<td>.211</td>
<td>.942</td>
<td>1.309</td>
<td>-.024</td>
<td>.86</td>
<td>.991</td>
<td>1.80</td>
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<td></td>
<td>(.056)</td>
<td>(.007)</td>
<td>(.051)</td>
<td>(.015)</td>
<td>(.280)</td>
<td>(.005)</td>
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</tr>
<tr>
<td>US-b</td>
<td>-.240</td>
<td>.045</td>
<td>.215</td>
<td>.939</td>
<td>.768</td>
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<td>.86</td>
<td>.991</td>
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<td>(.007)</td>
<td>(.047)</td>
<td>(.014)</td>
<td>(.180)</td>
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<tr>
<td>Ja-a</td>
<td>.048</td>
<td>.021</td>
<td>.319</td>
<td>.958</td>
<td>.993</td>
<td>1.63</td>
<td>.998</td>
<td>1.44</td>
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<tr>
<td></td>
<td>(.231)</td>
<td>(.033)</td>
<td>(.092)</td>
<td>(.029)</td>
<td>(.441)</td>
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<tr>
<td>Ja-b</td>
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<td>(.202)</td>
<td>(.029)</td>
<td>(.089)</td>
<td>(.025)</td>
<td>(.210)</td>
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<tr>
<td>Ge-a</td>
<td>-.853</td>
<td>.173</td>
<td>-.482</td>
<td>.852</td>
<td>.479</td>
<td>1.31</td>
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<td>(.231)</td>
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<td>(.038)</td>
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<tr>
<td>Ge-b</td>
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<td>.860</td>
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<td>(.042)</td>
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<tr>
<td>UK-a</td>
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<td>.193</td>
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<td>.605</td>
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<td>.992</td>
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<td>(.021)</td>
<td>(.511)</td>
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<tr>
<td>UK-b</td>
<td>-.454</td>
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<td>.993</td>
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<td>(.051)</td>
<td>(.097)</td>
<td>(.024)</td>
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<tr>
<td>Fr-a</td>
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<td>.710</td>
<td>.955</td>
<td>2.59</td>
<td>.934</td>
<td>2.38</td>
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<tr>
<td></td>
<td>(.131)</td>
<td>(.041)</td>
<td>(.135)</td>
<td>(.076)</td>
<td>(.317)</td>
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<td></td>
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<tr>
<td>Fr-b</td>
<td>.275</td>
<td>.130</td>
<td>-.349</td>
<td>.713</td>
<td>.968</td>
<td>2.44</td>
<td>.941</td>
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<tr>
<td></td>
<td>(.125)</td>
<td>(.039)</td>
<td>(.137)</td>
<td>(.071)</td>
<td>(.216)</td>
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</tr>
</tbody>
</table>

1/ The estimation period is 1971:1-1988:4, except for the United Kingdom where it is 1972:3-1988:4. Equations labeled (a) were estimated as in Table 1, but for this shorter data period; those labeled (b) were estimated using recursive least squares for the expectations term. Instrumental variables used to estimate m-m*, with instruments the same as used in equations reported in Table 1.
Finally, in order to test the Carr-Darby restriction—that anticipated money has no impact on money demand—we estimated M1 equations over the entire sample period, 1965:1-1988:4, based on a variant of equation (7), as summarized in Table 3. The final two columns in Table 3 show the derived structural coefficients, along with the derived standard errors. These results do not support MacKinnon and Milbourne’s contention that anticipated money matters in addition to unanticipated money: the implied coefficients on anticipated money ($\phi$) are insignificantly different from zero in every case. The implied coefficients on unanticipated money ($\alpha$), however, are uniformly negative. These estimates thus raise a puzzle: even if one constrains $\phi = 0$, in which case the transformed equation is identical to the original Carr-Darby model, one gets entirely different results by transforming the equation.

The effect of this transformation can be seen more clearly by comparing equation (4)—the original Carr-Darby model—directly with a variant of equation (7), constrained by setting $\phi = 0$:

\begin{equation}
(4) \quad m_t - p_t = \beta x_t + \alpha(m_t - m^*_t) + \mu_t
\end{equation}

and

\begin{equation}
(7') \quad m_t - p_t = \beta^* x_t + (1-\alpha')(m^*_t - p_t) + \mu^*_t,
\end{equation}

where, as before, $\beta^* = \beta/(1-\alpha)$ and $\mu^*_t = \mu_t/(1-\alpha)$. These two equations are identical in that one is simply an algebraic transformation of the other; and yet the direct and indirect estimates of the parameters of (4) are very different. The implication seems to be that the model is incomplete; in particular, either $x_t$ or $m^*_t$ (or both) is probably measured with error, resulting in biased and inconsistent coefficient estimates. This finding suggests that an extended specification may be required, and it thus lends additional importance to the development of detailed error correction models, as described below.

3. **Error correction equations**

The error correction estimates are shown in Table 4. The error correction term in each of these equations is simply the static terms taken as a group. For example, the equation for M1 in the United States could be rewritten in the form of equation (9): 

\begin{equation}
\Delta(m-p)_t = -.058\epsilon_{t-1} + \ldots,
\end{equation}

where $\epsilon_t = (m-p)_{t-1} - 1.5y_t + .5p_{t-2}$. There are two anomalies that become apparent with this formulation. First, the lag structure seems peculiar if this equation is conceived as a demand function, since it appears that real balances could be "causing" real income. Interpretation of these lags is
Table 3. Buffer Stock Model: Alternate Specification 1/

<table>
<thead>
<tr>
<th>Equation</th>
<th>$y$</th>
<th>$i$</th>
<th>$Ap_{-1}$</th>
<th>$(m-p)_{-1}$</th>
<th>$S4i$</th>
<th>DUM80</th>
<th>$m^*$</th>
<th>$p$</th>
<th>$\sigma$</th>
<th>$R^2$</th>
<th>$DW$</th>
<th>$\alpha$</th>
<th>$\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>0.113</td>
<td>-0.253</td>
<td>-0.456</td>
<td>0.217</td>
<td>0.235</td>
<td>0.019</td>
<td>0.706</td>
<td>-0.748</td>
<td>0.79</td>
<td>0.992</td>
<td>1.50</td>
<td>-2.97</td>
<td>-0.17</td>
</tr>
<tr>
<td>Japan</td>
<td>0.060</td>
<td>-0.160</td>
<td>0.234</td>
<td>0.191</td>
<td>0.738</td>
<td>-0.769</td>
<td>1.17</td>
<td>0.999</td>
<td>2.34</td>
<td>-3.33</td>
<td>-0.14</td>
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<td>Germany</td>
<td>0.127</td>
<td>-0.298</td>
<td>0.351</td>
<td>0.571</td>
<td>-0.588</td>
<td>1.19</td>
<td>0.997</td>
<td>1.94</td>
<td>-1.43</td>
<td>-0.04</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>France</td>
<td>0.062</td>
<td>-0.258</td>
<td>0.127</td>
<td>0.844</td>
<td>0.846</td>
<td>1.45</td>
<td>0.993</td>
<td>1.94</td>
<td>-5.48</td>
<td>-0.01</td>
<td></td>
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<tr>
<td>United Kingdom</td>
<td>0.038</td>
<td>-0.026</td>
<td>0.041</td>
<td>0.910</td>
<td>0.908</td>
<td>2.19</td>
<td>0.953</td>
<td>1.92</td>
<td>-9.70</td>
<td>0.03</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1/ The estimation period is 1965:1-1988:4; the dependent variable is $m-p$. Heteroskedastic-corrected standard errors are shown in parentheses.
Table 4. Error Correction Model: Estimated Equations

United States

\( \Delta(m-p) = -0.067(m-p)_{-2} + 0.098y_{-1} - 0.031p_{-3} - 0.545\Delta_{3}p - 0.522(\Delta r_{-1} + \Delta r_{-3}) \)
\( (0.014) \quad (0.010) \quad (0.050) \quad (0.073) \)
\(-0.218\Delta(m-p)_{-2} - 0.218\Delta_{3}i + 0.247\Delta_{3}y - 0.024DUM80 \)
\( (0.065) \quad (0.034) \quad (0.047) \quad (0.002) \)
\( R^2 = 0.79 \)
\( \sigma = 0.56 \)
\( DW = 1.97 \)

\( \Delta m = -0.147(m-p)y_{-4} - 0.111i_{-1} + 0.221\Delta_{4}y + 0.375\Delta_{2}p_{-2} \)
\( (0.031) \quad (0.033) \quad (0.036) \quad (0.057) \)
\(-0.503\Delta i - 0.158\Delta_{2}i_{-1} + 0.298\Delta r \)
\( (0.068) \quad (0.044) \quad (0.138) \)
\( R^2 = 0.67 \)
\( \sigma = 0.50 \)
\( DW = 1.73 \)

Japan

\( \Delta m = -0.387(m-p)_{-1} + 0.227y_{-1} - 0.067p_{-1} - 0.732r_{-3} - 0.464\Delta_{2}i_{-2} \)
\( (0.096) \quad (0.060) \quad (0.025) \quad (0.190) \quad (0.087) \)
\( R^2 = 0.72 \)
\( \sigma = 0.97 \)
\( DW = 2.00 \)

\( \Delta m = -0.053(m-p)_{-1} + 0.087y_{-3} - 0.034p_{-4} + 0.125i_{-4} - 0.318r_{-1} \)
\( (0.008) \quad (0.005) \quad (0.027) \quad (0.063) \)
\(+ 0.229\Delta(m-p)_{-1} + 0.075\Delta_{4}(m-p)_{-1} + 0.082\Delta_{4}p \)
\( (0.065) \quad (0.020) \quad (0.025) \)
\( R^2 = 0.88 \)
\( \sigma = 0.44 \)
\( DW = 1.85 \)
Table 4 (continued). Error Correction Model: Estimated Equations 1/

<table>
<thead>
<tr>
<th>German, Fed. Rep. of</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
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<td><strong>M1</strong></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>( \Delta(m-p) = - .112(m-p-y) - .795r_1 - .453\Delta_2(m-p) + 2.63\Delta_2y + .487\Delta_1 )</td>
<td>( (.037) )</td>
<td>( (.165) )</td>
<td>( (.065) )</td>
<td>( (.088) )</td>
<td>( (.055) )</td>
</tr>
<tr>
<td>( R^2 = .62 )</td>
<td>( \sigma = 1.11 )</td>
<td>( DW = 2.12 )</td>
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<tr>
<td><strong>M3</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Delta m = - .087(m-p) + .226y - .068p_1 + .151i_2 - .347r_1 )</td>
<td>( (.044) )</td>
<td>( (.043) )</td>
<td>( (.023) )</td>
<td>( (.056) )</td>
<td>( (.123) )</td>
</tr>
<tr>
<td>( + .275\Delta(m-p)_3 + .244(i_1+i_2-i_4) + .325\Delta p + .209\Delta y )</td>
<td>( (.073) )</td>
<td>( (.039) )</td>
<td>( (.157) )</td>
<td>( (.063) )</td>
<td></td>
</tr>
<tr>
<td>( R^2 = .49 )</td>
<td>( \sigma = 0.78 )</td>
<td>( DW = 2.28 )</td>
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</tbody>
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<table>
<thead>
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</tr>
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<tbody>
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<td><strong>M1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Delta m = - .036(m-p-3.85y) - .593i_4 - .607\Delta_4 i + .687\Delta r_3 + .166\Delta_3 p_1 )</td>
<td>( (.005) )</td>
<td>( (.083) )</td>
<td>( (.065) )</td>
<td>( (.189) )</td>
<td>( (.040) )</td>
</tr>
<tr>
<td>( R^2 = .62 )</td>
<td>( \sigma = 1.32 )</td>
<td>( DW = 2.41 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>M3</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Delta m = - .072(m-p)_1 + .230y - .027p_4 + .161\Delta_3(m-p)_1 )</td>
<td>( (.022) )</td>
<td>( (.049) )</td>
<td>( (.009) )</td>
<td>( (.040) )</td>
<td></td>
</tr>
<tr>
<td>( - .435\Delta_2 i_2 + .832\Delta r_3 )</td>
<td>( (.161) )</td>
<td>( (.406) )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( R^2 = .35 )</td>
<td>( \sigma = 1.72 )</td>
<td>( DW = 1.85 )</td>
<td></td>
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</table>
Table 4 (concluded). Error Correction Model: Estimated Equations 1/

<table>
<thead>
<tr>
<th></th>
<th>France</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Δm = - .308(m-p) -1 + .156y -4 - .323i -2</td>
</tr>
<tr>
<td></td>
<td>(0.069) (0.040) (0.174)</td>
</tr>
<tr>
<td></td>
<td>R² = 0.22</td>
</tr>
<tr>
<td></td>
<td>σ = 2.48</td>
</tr>
<tr>
<td></td>
<td>DW = 2.20</td>
</tr>
<tr>
<td></td>
<td>M2</td>
</tr>
<tr>
<td></td>
<td>Δm = - .088(m-p) -2 + .192y -1 - .037p -2 + .172Δ(m-p) -3 - .320Δi -2</td>
</tr>
<tr>
<td></td>
<td>(0.032) (0.054) (0.010) (0.070) (0.125)</td>
</tr>
<tr>
<td></td>
<td>R² = 0.24</td>
</tr>
<tr>
<td></td>
<td>σ = 1.29</td>
</tr>
<tr>
<td></td>
<td>DW = 1.93</td>
</tr>
</tbody>
</table>

1/ The sample period for the dependent variable is 1964:1 - 1985:4, except for M1 in Japan (1973:1 - 1985:4). The notation Δ₁x -j = x -j x -j -1; similarly, Δ₁Δx = (x -x -j) - (x -1 -x -1 -j). For other notation, see text.
not straightforward, however, because changes in these variables also appear in the equation. Furthermore, the lag structure (real balances preceding real income) makes more sense if the equation is conceived as a quasi-reduced form of a macromodel. Second, and more disturbingly, the long-run relationship is not homogeneous in the price level. That problem is examined further below.

The specification of these equations differs from one aggregate to another. In all cases, the initial specification was the same:

\[
(11) \Delta(m - p)_t = \beta_0 + \sum_{i=1}^5 \sum_{j=0}^4 \beta_{i+j} \chi_{i,t-j} + \mu_t
\]

where \( \chi = (m-p, p, y, i, r) \). Equation (11) was then reduced to a parsimonious form by sequentially eliminating insignificant terms and by imposing differencing restrictions that could not be rejected. Each regression was thus reduced to an equation in lagged levels plus current and lagged differences. As noted above, this specification search was conducted over the truncated sample period ending in 1985:4, in order that the predictive ability of the regression over the post-sample period could serve as a performance benchmark for sifting out valid restrictions from data mining.

All ten aggregates examined here do have an error correction representation, in that (a) there is a significant negative coefficient on the implicit lagged disequilibrium term, and (b) the long-run static equilibrium satisfies conventional priors with respect to the signs—and in most cases with respect to magnitudes—of the coefficients. 1/

In two cases, instabilities became apparent that required modifying the sample period. For M1 in Japan, there was no error correction representation when data prior to 1973 were included, and recursive estimation revealed a sharp break around the end of 1972; hence the specification search and final estimation covered only the sample 1973:1-1985:4. For M1 in the United Kingdom, instability in the mid-1980s led to anomalous results when the sample was truncated at end-1985; when the specification search was conducted over the full sample and then the final equation was re-estimated through 1985:4, the parameters and equation properties appeared satisfactory. Nonetheless, these problems suggest that some caution is warranted in interpreting the stability of those two regressions.

An interesting property of the equations in Table 4 is that in eight of the ten cases, the dependent variable is in terms of nominal changes (\( \Delta m \)), rather than real (\( \Delta(m - p) \)). In all cases, as noted above, the initial specification used changes in real balances as the dependent variable. In

1/ A third criterion is that the levels of the included variables should be stationary in first differences and should be cointegrated. Those properties are examined in Boughton (1991a), where it is shown that these null hypotheses are not rejected.
most cases, however, the coefficient on \( \Delta p \) was insignificantly different from \(-1\), and that coefficient was then imposed. The implication is that a shift in the demand for money results initially in an equal change in the nominal money stock.

This property, which affects only the dynamic and not the equilibrium characteristics of the equations, contradicts the view that monetary growth should be treated as exogenously controlled by the monetary authorities in the short run, and it supports the idea that money balances serve as a buffer stock. It may also imply that central banks have been practicing short-run interest rate targeting, or that they have been targeting a different monetary aggregate. One of the two exceptions is the equation for M1 in the United States, where \( \Delta p \) has a coefficient that is significantly negative but well below unity; U.S. M1 was targeted over a sizeable part of the sample period. The other exception, however, is the M1 equation for Germany, in which inflation does not affect the demand for money directly; although monetary targeting has been practiced forcefully in Germany since the early 1970s, M1 has never been an official policy target.

The total steady-state interest elasticity (the sum of the short- and long-rate elasticities) is nonpositive in all cases: negative in seven and zero in the other three (M1 in the United States and broad money in the United Kingdom and France). For narrow money, the coefficients are nonpositive everywhere on both short and long rates; in two cases, however, the short rate has a positive coefficient in the equations for broadly defined money (Japan and Germany), offset by a significantly larger negative coefficient on the long rate. Presumably these positive short-rate coefficients reflect a positive correlation with the interest rates that are paid on a portion of the broadly defined aggregates, so that the short rate acts like an own rate of return.

In addition to these effects from nominal interest rates, there are also a few cases in which the inflation rate has a negative effect on holdings of real balances (both aggregates in the United States and the broader aggregates in Germany and Japan). As noted above, this relationship is consistent with models in which real money balances serve as a portfolio substitute for real goods as well as for interest-bearing securities, but it could also reflect an adjustment process in which nominal rather than real balances respond to disturbances.

As noted in the above discussion of the augmented buffer stock model, the demand for money may also be affected by the volatility of interest

---

1/ That is, in estimating equation (11),

\[
\Delta (m-p) = \ldots + \beta_5 p - \beta_6 p - \beta_7 p - \ldots,
\]

it was found that \( \beta_7 = -\beta_6 = 1 \). These two terms were then eliminated by cancelling \( \Delta p \) from both sides of the equation.

2/ In two of these cases, the coefficient on inflation is positive but less than unity, and the dependent variable is nominal rather than real balances.
rates. In the error correction equations, interest rates could potentially affect money holdings as levels, first differences, or second differences, depending on how the lag coefficients are related. A significant coefficient on the second difference may be interpreted as a volatility effect. The only equations where such a response is evident are those for the United States: volatility of the short rate lowers M1, while volatility of the long rate raises M2. It seems unlikely that these relationships would be robust enough to be taken seriously as part of the process determining money holdings, but—as with the buffer stock equations—there is enough here to merit further investigation.

Finally, lagged changes in real balances (in addition to lagged levels) are found to have significant effects in more than half of the equations in Table 4. These include negative effects in two of the M1 equations (United States and Germany) and positive effects in all of the broad money equations except for the United States. There thus seems to be some autocorrelation in the data that is not otherwise explained by these equations.

IV. Model Comparisons

1. Statistical properties

Table 5 presents a number of statistics relating to the goodness of fit and intertemporal stability of the partial adjustment, buffer stock, and error correction models. Most of these statistics favor the error correction models over the other two. Notably, in every case the standard error of the estimate (within the estimation sample) is lowest for the error correction model, usually by substantial margins. This comparison suggests that the dynamic relationships are more complex than can be represented in the partial adjustment or the buffer stock model. If money demand equations are to be used for short-term forecasting or policy purposes, modeling these complex dynamics is important.

There is very little problem with serial correlation in any of these models. As indicated by the Durbin-Watson statistics, significant first-order serial correlation is a problem only in the partial adjustment and buffer stock estimates of the Japanese M1 equation, and in the partial adjustment model for M2 in Japan. In addition, there are a few cases of fourth-order serial correlation, indicating that seasonal factors have not been sufficiently modeled or eliminated through seasonal adjustment of the data.

Table 5 also presents two types of stability tests. The first is an N-step Chow test, derived from recursive least squares estimation. For this test, each equation is first estimated over a sample period ending in 1971:4, and an F test is computed for the consistency of that estimate with

1/ The comparison is not strictly valid for the Japanese M1 equation, because the sample periods differ.
Table 5. Statistical Comparisons of the Three Models

<table>
<thead>
<tr>
<th></th>
<th>In-Sample Statistics</th>
<th>Stability Tests</th>
<th>Post-Sample Predictions</th>
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<tbody>
<tr>
<td></td>
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<td>4th-order</td>
<td>Maximum Instability</td>
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<tr>
<td></td>
<td></td>
<td>serial correlation (F)</td>
<td>N-step Chow test</td>
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<tr>
<td>United States</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M1</td>
<td></td>
<td></td>
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<tr>
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</tr>
<tr>
<td>Buffer stock</td>
<td>0.74</td>
<td>1.98</td>
<td>0.76</td>
</tr>
<tr>
<td>Error correction</td>
<td>0.56</td>
<td>1.97</td>
<td>0.32</td>
</tr>
<tr>
<td>M2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Partial adjustment</td>
<td>0.69</td>
<td>1.53</td>
<td>1.46</td>
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<td>1.64</td>
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<td>1.73</td>
<td>0.03</td>
</tr>
<tr>
<td>Japan</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Partial adjustment</td>
<td>1.72</td>
<td>1.20</td>
<td>3.76*</td>
</tr>
<tr>
<td>Buffer stock</td>
<td>1.59</td>
<td>1.38</td>
<td>2.41</td>
</tr>
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<td>Error correction (1973-85)</td>
<td>0.97</td>
<td>2.00</td>
<td>1.36</td>
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<tr>
<td>M2</td>
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<tr>
<td>Partial adjustment</td>
<td>1.06</td>
<td>1.19</td>
<td>6.04**</td>
</tr>
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<td>0.04</td>
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<td>M3</td>
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<tr>
<td>Partial adjustment</td>
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<td>1.94</td>
<td>1.34</td>
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<td>2.12</td>
<td>1.69</td>
</tr>
<tr>
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<td>0.78</td>
<td>2.28</td>
<td>0.22</td>
</tr>
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<td>M1</td>
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<tr>
<td>Partial adjustment</td>
<td>1.75</td>
<td>2.29</td>
<td>0.84</td>
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<td>Buffer stock</td>
<td>1.77</td>
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<td>1.06</td>
</tr>
<tr>
<td>Error correction</td>
<td>1.32</td>
<td>2.41</td>
<td>1.26</td>
</tr>
<tr>
<td>M3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Partial adjustment</td>
<td>2.36</td>
<td>1.57</td>
<td>3.78</td>
</tr>
<tr>
<td>Buffer stock</td>
<td>2.36</td>
<td>1.64</td>
<td>3.47*</td>
</tr>
<tr>
<td>Error correction</td>
<td>1.72</td>
<td>1.85</td>
<td>2.72</td>
</tr>
<tr>
<td>France</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>M1</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Partial adjustment</td>
<td>2.66</td>
<td>2.30</td>
<td>1.06</td>
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<tr>
<td>Buffer stock</td>
<td>2.66</td>
<td>2.38</td>
<td>1.59</td>
</tr>
<tr>
<td>Error correction</td>
<td>2.48</td>
<td>2.20</td>
<td>1.68</td>
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<tr>
<td>M2</td>
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<tr>
<td>Partial adjustment</td>
<td>1.73</td>
<td>2.10</td>
<td>1.32</td>
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<tr>
<td>Buffer stock</td>
<td>1.66</td>
<td>2.21</td>
<td>2.45</td>
</tr>
<tr>
<td>Error correction</td>
<td>1.29</td>
<td>1.93</td>
<td>15.05**</td>
</tr>
</tbody>
</table>

1/ For F statistics, * and ** indicate rejection of the null hypothesis at the .05 or .01 level, respectively.

2/ Standard error of the estimate (x100). Because the partial adjustment models were estimated in levels and the error correction model in differences, the R² statistics are not comparable, but the σ's are. The σ’s are comparable between equations estimated in Δm and Δ(m−p), on the assumption that prices are exogenous.

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the remainder of the sample (through 1988:4). The test is repeated sequentially with one quarter added to the initial sample each time, through the end of the available data. What is presented in Table 5 is the maximum value of the F statistic and the truncation date at which that value occurs. For example, for the U.S. M1 equation, when the equations are estimated through 1986:4, the Chow test strongly rejects the hypothesis that the remaining eight observations are generated by the same process. At every other cutoff point, the equations are relatively more stable, so that date is the most likely break point.

By this recursive Chow test, eight of the ten cases show some instability, and there are no systematic differences among the three models. The only cases where all three models are stable over the entire sample period are the broad money equations for Japan and Germany. For three of the eight problem cases, however, the difficulty is confined to the early 1970s, while the more recent period is stable. Thus only about half of the cases show any significant instability in the 1980s.

The second stability test shown in Table 5 is an N-step Chow test for the twelve quarters that were not used for the original estimation of each model. This test is especially relevant for the error correction models, owing to the potential for data mining in the process of reducing each equation to a parsimonious form. In other words, the error correction equations were specified (not just estimated) using data only through the end of 1985; if the remaining observations fit that same model, then a strict test has been passed.

The results of this second test are also quite mixed. On the plus side for the error correction models, in most cases (seven out of ten), those models pass the post-sample stability test. Furthermore, in six of ten cases, the root mean squared errors (RMSEs) over the post-sample period are lowest for the error correction models. On the other hand, in eight of the ten cases, the value of the Chow F statistic is higher for the error correction model than for the two simpler models. Overall, the tests summarized in Table 5 generally support the view that the demand for money has been reasonably stable since around the mid-1970s, but they provide little basis for choosing among the various models.

The next set of tests is more discriminating. Table 6 presents a very simple test of whether one model encompasses the other. This test, devised by Davidson and MacKinnon (1981), asks whether the predictions from one model add significantly to the fit of a second model. If so, and if the predictions from the second model do not add significantly to the first, then the first model may be said to encompass the second. In eight of the ten cases, the error correction model encompasses the buffer stock model by this test, and in the other two cases there is no significant difference.

1/ Owing to software limitations on the number of allowed regressors, the more sophisticated encompassing tests that are incorporated into PC-GIVE could not be performed with these equations.
Table 6. Davidson-MacKinnon Encompassing Tests

(F statistics) 1/

<table>
<thead>
<tr>
<th></th>
<th>Error Correction vs. Buffer Stock</th>
<th>Buffer Stock vs. Error Correction</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>United States</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M1</td>
<td>0.02</td>
<td>40.95**</td>
</tr>
<tr>
<td>M2</td>
<td>2.28</td>
<td>30.99**</td>
</tr>
<tr>
<td><strong>Japan</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M1</td>
<td>5.65*</td>
<td>28.87**</td>
</tr>
<tr>
<td>M2</td>
<td>0.12</td>
<td>3.95*</td>
</tr>
<tr>
<td><strong>Germany</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M1</td>
<td>4.49*</td>
<td>13.64**</td>
</tr>
<tr>
<td>M3</td>
<td>6.37*</td>
<td>24.97**</td>
</tr>
<tr>
<td><strong>France</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M1</td>
<td>5.23*</td>
<td>5.86*</td>
</tr>
<tr>
<td>M2</td>
<td>0.57</td>
<td>3.58*</td>
</tr>
<tr>
<td><strong>United Kingdom</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M1</td>
<td>0.07</td>
<td>1.26</td>
</tr>
<tr>
<td>M3</td>
<td>0.04</td>
<td>20.09**</td>
</tr>
</tbody>
</table>

1/ Test of the significance of adding the predictions from the "additional" model to the regression of the basic model; see Davidson and MacKinnon (1981). The null hypothesis is that the information from the additional model is already in the basic model. The symbols * and ** indicate rejection of the null hypothesis at the .05 or .01 level, respectively.
The implication of these results is that the buffer stock models fail to capture important dynamic and systematic influences on the short-run demand for money. The Davidson-MacKinnon test thus provides strong support for developing relatively complex dynamic models.

2. Economic properties

The economic implications of the equations presented in this paper can be divided into two broad categories: the dynamic adjustment processes and the long-run (steady-state) properties of the demand for money. The former are difficult to compare, because the dynamics depend on assumptions about the nature of the shocks to which the demand for money is subjected. This is especially true for the error correction models, because of the very complex dynamics that are present in most of the equations listed in Table 4. Nonetheless, it is worth noting that the adjustment coefficients—the rate at which the money stock would return toward its initial equilibrium after a disturbance, with no change in any argument in the demand function—are generally no larger in the error correction equations than in the other two models. 1/ In most cases, these coefficients are no more than a few percent per quarter, although relatively rapid adjustment is estimated for U.S. M2, for the M1 equations in Germany and France, and for M1 in Japan when the sample is truncated to begin in 1973 (Table 4). In sum, the problem with the partial adjustment—and, to a lesser extent, the buffer stock—model is not that misspecification produces an undue degree of autoregressivity; rather, it is that the simple models omit additional significant dynamic elements.

The second category of economic properties concerns the long-run elasticities (Table 7). 2/ Within this classification, there are three key issues: whether the price elasticity is unity, whether the real income elasticity is greater or less than unity, and whether the level of interest rates has a negative effect in the steady state. The first issue determines whether the long-run demand function is homogeneous in real values, while the second and third issues have implications for the likely long-run trend in velocity.

1/ For the partial adjustment and buffer stock models, the adjustment coefficients relate to real money balances and are measured as $1 - \beta_1$, where $\beta_1$ is the coefficient on the lagged dependent variable in Table 1. For the error correction models, the adjustment coefficients might relate either to real or nominal balances and are measured as $-\beta_1$, where $\beta_1$ is the coefficient on lagged real money balances in Table 4. In some of the error correction models, adjustment in this sense begins only after a lag of one to three quarters.

2/ For this paper, long-run elasticities have been computed at the steady-state solution to each model, using data through 1988:4. Inflation rates have been set to zero; that decision affects only the semi-elasticities with respect to nominal interest rates. The choice to use the full rather than the truncated data sample had little practical relevance, since the differences all turned out to be insignificant.
Table 7. Steady-State Elasticities for the Three Models 1/  

<table>
<thead>
<tr>
<th>Country</th>
<th>M1</th>
<th>M2</th>
<th>M3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Price Level</td>
<td>Real Income</td>
<td>Nominal Income</td>
</tr>
<tr>
<td>United States</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M1</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Partial adjustment</td>
<td>1.0</td>
<td>0.77</td>
<td>0.88</td>
</tr>
<tr>
<td>Buffer stock</td>
<td>1.0</td>
<td>0.72</td>
<td>0.86</td>
</tr>
<tr>
<td>Error correction</td>
<td>0.54</td>
<td>1.46</td>
<td>1.0</td>
</tr>
<tr>
<td>M2</td>
<td></td>
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<tr>
<td>Partial adjustment</td>
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<td>1.03</td>
<td>1.02</td>
</tr>
<tr>
<td>Buffer stock</td>
<td>1.0</td>
<td>0.95</td>
<td>0.98</td>
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<tr>
<td>Error correction</td>
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<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Japan</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M1</td>
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<td></td>
</tr>
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1/ Elasticities listed as 1.0 were imposed.

2/ Computed as the average of the elasticities with respect to the price level and real income.
All theoretical models of the demand for money postulate that the demand for real balances should be independent of the price level, and most empirical models—including the partial adjustment and buffer stock models estimated above—simply impose that condition at the outset. But there are reasons to doubt its empirical validity; notably, price indexes may be inadequate proxies for the aggregate of prices by which agents implicitly deflate money holdings, and the construction of monetary aggregates may introduce aggregation bias into estimated coefficients. Therefore, the error correction models have been specified so as to allow the long-run elasticities to differ from unity. 1/ As Table 7 shows, only four of the ten error correction equations have unitary long-run elasticities; the other six are all significantly less than one. With the exception of Japan (where both aggregates have less-than-unitary price elasticities), for each country one aggregate has a unitary elasticity while the other is lower.

Tests were also performed for the validity of price-level homogeneity in the buffer stock and partial adjustment equations. 2/ It was found that unitary long-run price elasticity does not hold for either the partial adjustment or buffer stock model in the narrow money equations for Japan (less than unity) and France (greater than unity), and broad money equations for the United States (greater than unity) and Japan (less than unity). These results are consistent with the error correction models only for Japan, but they do confirm the lack of generality in the proposition that money demand equations are homogeneous in the price level.

The most likely explanation for this phenomenon would appear to be aggregation bias. That is, it seems likely that transactions and investment balances have quite different real income elasticities and that the composition of the aggregates has changed markedly during the sample period; under those conditions, as Theil (1954) demonstrated, there could be a substantial bias in the estimated aggregate coefficients. But this is a tentative explanation that requires further research.

In most cases, in spite of the different treatment of price elasticities, the three models give congruent estimates regarding whether the real income elasticity is greater or less than unity. The most striking aspect of these results is the high level of most of the income elasticities for the broad aggregates and even for some of the narrow aggregates. Except for M1 in France, Japan, and possibly the United States, there is no evidence here supporting the Baumol-Tobin view of agents economizing on cash balances relative to income as income rises. In some cases, the evidence goes in quite the opposite direction: the aggregates have behaved more as "luxury goods."

1/ The partial adjustment model also constrains the short-run price elasticity to be unity by including only real balances on both sides of the equation.
2/ These results are not shown in Table 7, but they are available on request.
The interest rate effects are generally conventional, with short- and/or long-term rates being negatively related to money demand. Three less conventional implications are worth noting. First, there are term structure effects in three cases. As noted above, two of the error correction equations—the broad money equations for Japan and Germany—show short rates positively related to money demand, while the long rate has a larger negative effect; and for U.S. M2, the partial adjustment and buffer stock models find the opposite configuration, but still with the expected overall negative relationship. Second, there are three cases where the error correction equations imply that interest rates affect the dynamic processes but not the steady state; more generally, the error correction models tend to yield relatively small long-run interest elasticities, perhaps because they allow for a wider range of dynamic effects. Third, the differences between narrow and broad aggregates tend to be minor; there is no significant tendency for interest elasticities to be smaller for broad than for narrow aggregates.

V. Conclusions

On balance, the empirical findings in this paper are encouraging, in that the estimated money demand equations are reasonably stable and display mostly conventional economic properties. The dynamic adjustment processes, however, are found to be complex and difficult to identify; in that sense, these results support the cautious conclusion reached by Hendry and Ericsson that was quoted at the top of this paper: the models estimated here are different from those that would have been estimated optimally even a few years ago, and it would be premature to argue that a definitive model of money demand exists, much less that one has been identified.

What is abundantly clear is that the partial adjustment model can be safely buried; the buffer stock and/or error correction models outperform it in all of the tests performed here. Furthermore, the buffer stock model is demonstrated to be a useful extension of the partial adjustment model. In most cases, unanticipated shocks to the nominal stock of money have a significant positive effect on real money balances, while anticipated changes do not.

Comparison between buffer stock and error correction models is less clear, at least partly because of the wide range of dynamic specifications derived through the error correction approach. Some of the error correction equations have properties that raise doubts about their interpretation as models of the demand for money; notably, several equations have long-run price elasticities that are significantly less than unity. But the error correction equations also reveal the presence of significant dynamic properties—such as term structure effects, effects from the volatility of interest rates, and adjustment lags that differ markedly between variables—that are normally ignored in the specification of conventional partial adjustment or buffer stock models. Within the estimation period, these equations generally outperform and encompass the buffer stock equations by the usual statistical criteria. The encompassing tests are especially
revealing, in that they clearly indicate the omission of important relationships in the buffer stock equations. On the other hand, in post-sample tests, the buffer stock equations in some cases do better. Therefore, although the partial adjustment and buffer stock models are both shown to be overly restrictive, and the error correction model offers promise as a more general approach, it would seem that some means must yet be found to identify restrictions that will help to tie down the error correction approach without sacrificing its compelling advantages.