Working Paper

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Volatility in financial markets has forced economists to reexamine
the validity of the efficient markets hypothesis, and new empirical
approaches have been applied to the study of this important issue in
recent years. Many of the recent studies have found evidence of
excessive volatility. In the aftermath of the stock market crash of
1987 and the perceived increase in market volatility, some economists
have advocated additional market regulations. Are these proposed
regulations necessary and would they serve to reduce market volatility?
This paper presents a review of recent studies on financial market
volatility and examines the proposed regulations.

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# Table of Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction</td>
<td>1</td>
</tr>
<tr>
<td>I. A Review of the Economic Theory</td>
<td>2</td>
</tr>
<tr>
<td>1. Valuation models</td>
<td>2</td>
</tr>
<tr>
<td>2. Prices, fundamental value, and the efficient markets hypothesis</td>
<td>6</td>
</tr>
<tr>
<td>3. Speculative bubbles and noise</td>
<td>7</td>
</tr>
<tr>
<td>II. Market Volatility and Noise: A Review of the Empirical Research</td>
<td>10</td>
</tr>
<tr>
<td>1. The variance bounds tests and criticism</td>
<td>11</td>
</tr>
<tr>
<td>2. The second generation of tests</td>
<td>12</td>
</tr>
<tr>
<td>3. Long horizon returns and mean reversion in the stock market</td>
<td>15</td>
</tr>
<tr>
<td>4. Tests that incorporate discount rate variability</td>
<td>17</td>
</tr>
<tr>
<td>5. Interpretation of the empirical results</td>
<td>19</td>
</tr>
<tr>
<td>6. Stock market volatility and margin requirements</td>
<td>22</td>
</tr>
<tr>
<td>7. Prices, interest rates, and market fundamentals in the bond market</td>
<td>25</td>
</tr>
<tr>
<td>8. Synthesis, summary, and conclusions</td>
<td>27</td>
</tr>
<tr>
<td>III. The Stock Market Crash of 1987 and the Implications for Market Regulations</td>
<td>30</td>
</tr>
<tr>
<td>1. Program trading</td>
<td>32</td>
</tr>
<tr>
<td>2. The stock market crash of 1987</td>
<td>36</td>
</tr>
<tr>
<td>3. Proposals for new market regulations</td>
<td>38</td>
</tr>
<tr>
<td>IV. Summary and Conclusions</td>
<td>41</td>
</tr>
<tr>
<td>Appendix: Expectation Theories of the Term Structure</td>
<td>43</td>
</tr>
<tr>
<td>References</td>
<td>45</td>
</tr>
</tbody>
</table>

**List of Figures:**

- Figure 1. Changes in Real Dividends                                  22a
- Figure 2. Changes in Real Earnings                                   22b
- Figure 3. Percentage Changes in Real Dividends                       22c
- Figure 4. Percentage Changes in Real Earnings                        22d
- Figure 5. Ratio of Margin Credit to NYSE Value                       22e
- Figure 6. NYSE, Price-Dividend Ratios Risk Premium Models            36a
Introduction

Volatility in financial markets, particularly stock markets, is an issue that concerns government policy makers, market analysts, corporate managers, and economists. During the 1980s, new financial markets developed around the world, and exchanges introduced futures and option contracts on interest rates, stock indexes, and foreign exchange rates. These markets experienced impressive growth until the crash of stock markets around the world in October of 1987. After the stock market crash, serious questions were raised concerning volatility in financial markets and the role of the new financial futures and options. As a result, a number of reforms for financial markets have been proposed, and some of these have been instituted.

While the financial markets were experiencing dramatic growth and asset price volatility was becoming more noticeable, economists were reexamining the efficient markets hypothesis, the theory that states that financial markets always price securities correctly. The evidence that has accumulated over the last ten years suggests that there may be excess volatility in stock markets and that stock prices regularly deviate from their fundamental values. The empirical results have stimulated several alternative views to explain the observed volatility; these include the speculative bubbles model, fads, and noise trading. The stock market crash of 1987 forced economists to reassess the validity of the efficient markets hypothesis, and it is fair to say that it is currently an open issue. The purpose of this paper is to survey the economic theory and the empirical evidence on pricing and volatility in financial markets, to synthesize the literature, and to address the necessity for some of the market reforms that have been recently proposed. Section I of the paper is a review of the relevant economic theory of how securities are priced in financial markets and it includes the efficient markets hypothesis as well as recent models which incorporate either speculative bubbles or noise trading. Section II is a survey of the empirical evidence on excess volatility and noise trading. The principal issue is whether asset prices deviate significantly from fundamental value. In Section III, the various proposals for new market regulations are examined from the perspective of the empirical evidence discussed in Section II. The proposals covered include program trading, margin requirements, circuit breakers and price limits, and transactions taxes.

The review of the empirical evidence on financial market volatility reveals evidence that stock prices deviate from fundamental value, but there is no evidence that prices deviate from fundamental value in another important market, the bond market. At a superficial level, the evidence of excess volatility and deviations of prices from fundamental value in the stock market would imply the need to impose additional restrictions and regulations on stock markets and related markets such as stock index futures markets. A careful analysis of the proposed market regulations, however, suggests that many of these changes are unnecessary and would serve no purpose at all.
I. A Review of the Economic Theory

A careful analysis of financial market volatility requires a theory of how capital assets are valued in the marketplace. The theory must explain how the prices are determined, and in a dynamic world, it must explain how and why prices change. Without a theory or an explanation for asset price changes, one can do little more than catalogue the sequence of events. A theory of price changes is essential for determining appropriate forms of market regulations; in Section III, the different implications for market regulation that follow from the various theories are discussed. There are several models available for pricing capital assets and several competing explanations or theories for the price changes that are observed in financial markets. Asset pricing models are used to determine what is called the market fundamental or fundamental value, the asset value based on economic fundamentals only. A more complete, rigorous definition is presented below. The alternative theories for price changes deal with changes in the fundamental value and deviations of market prices from the market fundamental. These theories are organized under the following broad categories: the efficient markets hypothesis, speculative bubbles, noise, and overreaction. The efficient markets hypothesis is the rational pricing theory, and the last three can be classified as irrational pricing theories. The irrational pricing theories can, however, incorporate selected aspects of rational expectations and the rational determination of fundamental value, but these theories take the position that market prices deviate from rationally determined fundamental values.

1. Valuation models

The model that investment analysts typically use for estimating fundamental value is the familiar present value model that was developed by John B. Williams in 1938:

\[
V = \sum_{t=1}^{\infty} \frac{D_t}{(1+k)^t}
\]

(1)

where \( V \) is the value of the asset, \( D_t \) is the future cash-flow or dividend, and \( k \) is the required rate of return or the discount rate for the asset. In words, the fundamental value of an asset is equal to the discounted present value of the future cash-flows. If the future cash-flows are risky, \( D_t \) is replaced with its expected value \( E(D_t) \). In the case of bonds, there is a fixed maturity and a schedule of promised cash-flows (interest plus principal). For relatively safe bonds the cash-flows are easy to predict, and the investment analysts typically set the present value equal to the current price in order to compute yields. In the case of stocks, the analyst must forecast future dividends, including a long-term growth rate, and determine an appropriate discount rate in order to value the asset. Some observers have noted that earnings are more important than dividends in the valuation of common stocks, and there is a temptation to value earnings
directly. If one uses a valuation of earnings approach, there must be an
adjustment for the retained earnings. Williams and others have shown that
if the analyses are done correctly the two approaches should produce the
same answer. Indeed, a dividend forecast should start from an earnings
forecast.

In recent years, more sophisticated asset pricing models have developed
in the financial economics literature. The familiar present value model can
be extended to incorporate random variation in both cash-flows and discount
rates. Let $k_t$ be the discount rate from period $t+1$ back to period $t$. Then
the present value model becomes

\[ V_t = E_t \left[ \sum_{j=1}^{\infty} \frac{D_{t+j}}{(1+k_{t+1})^j} \right]. \]  

Another approach is to start with a dynamic equilibrium asset pricing model,
like the model of Lucas (1978). The fundamental asset pricing equation in
Lucas's model is

\[ V_t U'(C_t) = \beta E_t[U'(C_{t+1})[V_{t+1}+D_{t+1}]], \]

where $U'(C_t)$ is the marginal utility of consumption at time $t$ and $\beta$ is a
discount factor from the representative agent's intertemporal utility
function. $\beta$ is a number between zero and one. At this point it is
necessary to make a distinction between real variables and nominal
variables. This distinction is not necessary in models (1) and (2) if one
remembers to discount nominal cash-flows with nominal rates and real cash-
flows with real discount rates. The Lucas model is derived from an
intertemporal consumption-investment problem, and the cash-flows and prices
are all denominated in consumption units. For this reason, the model
formally applies to real interest rates, real cash-flows, and real prices,
with consumption and wealth in real terms. The model is a first order
difference equation and the solution which ignores the bubble solution is

\[ V_t = \sum_{j=1}^{\infty} \beta^j E_t \left[ \frac{U'(C_{t+j})}{U'(C_t)} D_{t+j} \right]. \]  

This expression simplifies under the assumption of risk neutrality, a linear
utility function,

\[ V_t = \sum_{j=1}^{\infty} \beta^j E_t(D_{t+j}). \]
This last model states that fundamental value is equal to the discounted present value of expected future dividends, where $\beta$ is the discount factor, which is related to the discount rate used above via the relation $\beta = 1/(1+k)$. In this case the discount rate, $k$, is the same for all assets regardless of their risk. The more complicated expression in equation (3) incorporates risk aversion by weighing the cash-flow with the ratio of marginal utility of future consumption to marginal utility of consumption today. 1/

A more general version of this model is one in which marginal utility of consumption is replaced with marginal utility of wealth:

$$V_t = \sum_{j=1}^{\infty} \beta^j E_t \left[ \frac{U'(W_{t+j})}{U'(W_t)} D_{t+j} \right], \quad (5)$$

where $U'(W)$ now represents marginal utility of (real) wealth. In the Lucas model, the intertemporal utility function is separable over time and marginal utility of wealth equals marginal utility of consumption. In models that relax the assumption of time separable utility, marginal utility of wealth is no longer equal to marginal utility of current consumption only, and models like equation (5) must be used to determine fundamental value for assets. This model can be extended to price or value assets in nominal terms. Let $p_t$ be the consumption price deflator at time $t$ and replace $V_t$ and $D_{t+j}$ with their deflated nominal counterparts: $V_t = p_t V_t$ and $D_{t+j} = p_{t+j} D_{t+j}$. After some rearrangement,

$$V_t = \sum_{j=1}^{\infty} \beta^j E_t \left[ \frac{U'(W_{t+j}) p_{t+j}}{U'(W_t) p_t} \delta_{t+j} \right].$$

Now define a new variable, $A_t = U'(W_t) p_t$. Drop the $-$ on $V_t$ and $\delta_t$ and simply interpret these variables as nominal quantities:

---

1/ Note that

$$E_t\left[ \beta^j \frac{U'(C_{t+j})}{U'(C_t)} D_{t+j} \right] = E_t\left[ \beta^j \frac{U'(C_{t+j})}{U'(C_t)} \right] E_t(D_{t+j}) + Cov_t\left[ \beta^j \frac{U'(C_{t+j})}{U'(C_t)}, D_{t+j} \right]$$

where $Cov_t$ is the covariance conditional on information at time $t$. $E_t[\beta^jU'(C_{t+j})/U'(C_t)]$ is equal to the price of a default free asset that pays one unit of consumption at time $t+j$; it corresponds to a long term real interest rate. The first term on the right side represents the expected dividend or cashflow discounted at this real interest rate. The second term represents an adjustment for the risk.

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This asset pricing model is applied to nominal cash-flows and nominal prices. $\lambda_{t+j}/\lambda_t$ is the marginal rate of substitution (MRS) between a $ at time $t+j$ and a $ at time $t$, and the model is sometimes called a MRS model.

There are other capital asset pricing models (CAPM's) in the finance literature: the Sharpe-Lintner CAPM, Ross's arbitrage pricing theory, Merton's intertemporal CAPM, and Breeden's version of the consumption based CAPM. The principal results for these models are expressions for equilibrium expected returns, and none of these models directly address the problem of determining equilibrium asset prices. In fact these models provide very little insight into the determination of equilibrium asset prices. Take for example, the Sharpe-Lintner CAPM; its basic result is the following statement about equilibrium expected returns:

$$E(R_A) = E(R_F) + \beta_i(E(R_M) - E(R_F)),$$

where the expected return on the market and the risk free rate are determined outside the model. By using the identity,

$$1 + E(R_{i,t+1}) = \frac{E_t(V_{i,t+1} + D_{i,t+1})}{V_{i,t}},$$

one can derive the following expression for $V_{i,t}$:

$$V_{i,t} = \frac{E_t(V_{i,t+1}) + E_t(D_{i,t+1})}{1 + R_{F,t+1} + \beta_i[E_t(R_{M,t+1} - R_{F,t+1})]},$$

where the risk free rate, $R_{F,t+1}$, is known at time $t$. To solve this difference equation for the asset price, one must place some additional structure on changes in the risk free interest rate and the market risk premium, $E_t(R_{M,t+1}) - R_{F,t+1}$. Some investment analysts use the Sharpe-Lintner CAPM and Ross's arbitrage pricing theory to calculate expected returns, which are then used to determine discount rates for valuing assets with equation (1). These equilibrium expected return models are not directly useful in the analysis of fundamental value for capital assets, but they are useful for comparing returns across different assets and different portfolios over time.

The continuous time asset pricing of Cox, Ingersoll, and Ross (1985a) is another more recent model that does address the issue of pricing assets. This model goes beyond the analysis in the continuous time models of Merton and Breeden; it derives endogenously the equilibrium interest rate and a method for determining equilibrium asset prices is developed. Cox, Ingersoll, and Ross show that prices of capital assets and related contingent claims must satisfy a fundamental partial differential equation plus a set of boundary conditions, and the solution is a risk adjusted
expectation of the cash-flow discounted by an integral of the instantaneous interest rate. This model is extremely useful in the valuation of contingent claims and bonds that have finite lives, but it has not been used to value assets with extremely long (or infinite) lives like common stocks.

2. Prices, fundamental value, and the efficient markets hypothesis

The financial models discussed above serve as precise statements or models of market fundamentals. An important issue in financial markets is the relationship between market prices and fundamental value. First, do market prices reflect fundamental value only? If there are deviations from fundamental value, are the deviations large? Second, does the market incorporate all relevant information in forming the expectations that determine fundamental value? These two questions are closely related, but there are some important subtle distinctions. The second issue, or question, concerns the informational efficiency of financial markets and it is an issue that has been studied extensively in finance for the last 30 years. If financial markets do not incorporate all relevant information in the formation of expectations, then there is available information, on which traders can act to earn either arbitrage profits or excess profits. The first issue goes deeper and concerns the relationship between market prices and fundamental value. It is possible to have markets in which all information is reflected in current prices, that is expectations are formed rationally, and the market price can deviate substantially from fundamental value. Examples are contained in the theories discussed below.

The perfect markets theory is commonly known as the efficient markets hypothesis (EMH). There are several versions of the EMH; the most frequently cited version of this theory is the one presented by Fama (1976, Chapter 5) which is a revision, with some corrections, of his original treatment of the EMH. According to Fama, the EMH states that asset prices reflect all available information. In forming expectations about next period’s price or rate of return, the market uses the correct probability distributions and all available information. Formally, let \( F_M(P_{t+1}) \) be the market’s subjective probability distribution function for next period’s price. \( E(P_{t+1} | I_H_t) \) is the market’s subjective expectation and there are corresponding definitions for rates of return, \( F'_M(R_{t+1}) \) and \( E(R_{t+1} | I_H_t) \). Now consider the actual or objective distributions, \( F(P_{t+1}) \) and \( F'(R_{t+1}) \), as well as the objective expectations, \( E(P_{t+1} | I_t) \) and \( E(R_{t+1} | I_t) \). In the EMH, the market uses all the relevant information and the market’s subjective distributions equal the objective distributions: \( F_M = F \), \( F'_M = F' \), and \( I_H_t = I_t \). The market uses the correct distributions in forming expectations and arriving at equilibrium or market clearing prices. The empirical implications are that price changes and rates of returns should possess the fair game property. Let \( \epsilon_{t+1} = P_{t+1} - E(P_{t+1} | I_H_t) \) and \( \epsilon'_{t+1} = R_{t+1} - E(R_{t+1} | I_H_t) \). The fair game property implies that the innovations, \( \epsilon_{t+1} \) and \( \epsilon'_{t+1} \), cannot be predicted using any available information at time \( t \). If an empirical researcher could find information in \( I_t \) which is useful in predicting \( \epsilon_{t+1} \) or \( \epsilon'_{t+1} \), he would have a rejection of the EMH. The difficult aspect of this
empirical research is specifying the behavior of expected returns or expected prices. A variety of models have been used to test this version of the EMH, but when the empirical tests result in rejection, it is not possible to determine whether the model for expected returns or the EMH has been rejected by the data.

Fama provides a review of the early studies through the early 1970s. Most studies, at least those cited by Fama, generally support Fama's definition of an efficient market. Specifically, stock returns have very little serial correlation and it is difficult to find variables that are useful in predicting future returns. Or from an investor's perspective, there are no simple trading rules that can produce above normal profits. There are, however, some exceptions. For a brief review of these exceptions, see Section VII of LeRoy's (1989) survey. First, there is a small amount of serial correlation in stock returns, but Fama notes that plausible variation in expected returns can induce a small amount of serial correlation in stock returns. More recent evidence on the predictable variation in stock returns can be found in the work of Fama and French (1988a, b), Poterba and Summers (1988), and Lo and MacKinlay (1988). There is a large finance literature on the January effect, a size effect, and a price-earnings ratio effect. The principal findings of this literature are that stock returns tend to be high in January for small stocks and a trading strategy based on price-earnings ratios (buy stocks with the lowest P/E ratios) can outperform the market. More recently DeBondt and Thaler (1985, 1987) and Lehmann (1990) have presented profitable trading strategies that suggest overreaction in the stock market.

These studies of stock return behavior and the numerous studies on market efficiency in the finance literature can be interpreted as empirical studies of the informational efficiency of financial markets, and these studies represent tests of the implications of Fama's definition for market efficiency. The other version of the EMH states that market prices are always equal to fundamental value. This view of the EMH is implicit in much of the finance literature, and an unequivocal statement can be found in Sharpe (1990, page 79). According to the efficient markets view, market prices reflect fundamental value and the market is very quick and efficient in the way in which new information is incorporated and reflected in market prices. Thus the EMH implies a theory for determining market prices and the dynamics for price changes. Market prices always reflect fundamental value. As new information comes to the market, it is quickly incorporated and reflected in a new set of prices. Price changes can be explained by the arrival of new information, which causes changes in the expectations of future dividends or cash-flows.

3. Speculative bubbles and noise

The other theories focus on the relationship between market prices and fundamental value. The theory of rational bubbles is an example of a model in which expectations are formed rationally, the market is informationally
efficient, but there are large deviations between market prices and fundamental value. This point can be most easily demonstrated in a model with risk neutrality and constant discount rates. The fundamental dynamic asset pricing relation in such a model is the difference equation:

\[ P_t = \beta [E_t(P_{t+1}) + E_t(D_{t+1})], \quad (7) \]

where \( \beta \) can be restated as \( 1/(1+k) \). The market fundamental presented back in equation (4) is only one possible solution for this difference equation. Since this is a first order difference equation, one can add an arbitrary solution as follows

\[ P_t = \sum_{j=1}^{\infty} \beta^j E_t(D_{t+j}) + \frac{A_t}{\beta}, \quad (8) \]

where \( A_t \) is a martingale: \( E_t(A_{t+k}) = A_t \) for any \( k > 0 \). It is easy to verify that equation (8) for prices also satisfies the difference equation in (7). The term \( A_t/\beta \) is also known as a rational bubble. Because \( 1/\beta \) is greater than one, the bubble term is expected to grow, and one can construct bubble processes that simultaneously satisfy the martingale property and each period have a small probability of experiencing a large drop, or crash. This theory is unsettling because it suggests that there is no unique equilibrium price and there can be large deviations from the market fundamental. The price solution in equation (8) satisfies Fama’s definition for market efficiency because all information that is relevant for forming expectations on future dividends and future paths for the bubble is incorporated in the current price. The bubble satisfies the fair game property. At this point it is worth noting that tests on price changes or rates of returns may be able to identify information inefficiencies in the market, but they cannot detect deviations of prices from market fundamentals. The rates of return from the price process in equation (8) are serially uncorrelated and any empirical test that relies on the predictability of rates of return would have absolutely no power to detect the bubble or the deviations from the market fundamental. Some financial economists have suggested the use of runs tests on rates of returns to look for evidence of speculative bubbles, but the model here implies that such exercises are useless. \( 1/ \) Even though the bubble, \( A_t/\beta \), is expected to grow, it can be random and experience the same kind of variation that we attribute to rates of return in the EMH.

All of the analysis presented here on bubbles in a model with constant discount rates can be extended to the intertemporal models that do not have this restriction. The solution in equation (5) can be modified as follows:

\[ 1/ \text{At least two well-known financial economists have discussed the use of runs tests for detecting bubbles. The references have been omitted to protect the innocent.} \]
where $A_t$ again satisfies the martingale property. With risk aversion, $U'(W_t)$ decreases as wealth grows and this revised bubble term also has the property that it is expected to grow. The consumption-based version of this model follows by replacing $U'(W_t)$ with $U'(C_t)$. A similar result holds for the MRS model in equation (6).

At the theoretical level, there are a variety of arguments that can be used to rule out bubbles in some models. In terms of mathematically modeling, the price must satisfy a first order difference equation but there are not enough boundary conditions to pin down a unique price. If the difference equation (7) is taken from Lucas’s model, then there is a transversality condition from the representative agent’s dynamic optimization problem which can be used to rule out the possibility of a bubble. This follows from the infinite horizon in the agent’s intertemporal utility function; intuitively the agent considers the long run consequences of the bubble and knows that it cannot be sustained. Tirole (1982, 1985) has examined this issue in two papers and he notes that bubbles arise if agents have myopia; otherwise rational bubbles cannot develop in markets with agents who pay attention to the long run. Ad hoc models, in which agents maximize a utility of wealth function over a finite time horizon, can produce difference equations for asset prices like equation (7), but these models do not have the necessary boundary conditions to rule out bubbles. Most of the trading in financial markets is done by institutional investors, firms that manage funds for individuals and other organizations, and a myopic utility function may very well represent an accurate description of their behavior. Even though there are conditions under which rational bubbles can be theoretically eliminated, there are models in which rational bubbles can develop.

More recent models or explanations for departures of prices from market fundamentals have been offered by Shiller (1984, 1989a), Summers (1986), and Black (1986). These theories are less formal in the sense that they are not derived from careful models of optimizing behavior by individuals. In these models, prices include both the fundamental and a noise term:

$$P_t = \sum_{j=1}^{\infty} \beta^j E_t (D_{t+j}) + N_t.$$  

Shiller has promoted the idea that investors in the marketplace ignore fundamental value and follow fads or popular trends. Summers has presented a simple model in which the noise term, $N_t$, is driven by an autoregressive process with extremely slow mean reversion. He noted that in this model there can be large deviations of prices from fundamental value for long periods of time and that the conventional tests for market efficiency have
extremely low power for detecting these departures. More recently in work with Cutler and Poterba (1989, 1990) and De Long, Shleifer, and Waldman (1989, 1990), he has developed models which demonstrate that it is possible for noise traders to influence market prices without being driven out by individuals who trade on fundamentals only. In De Long, Shleifer, Summers, and Waldman (1990), a simple model is presented in which rational trading by fundamental traders can push the price even further away from fundamental value. In the noise trading models, there are noise traders and rational speculators and the equilibrium price contains a noise component. In these models the noise traders are not driven out of the market and the noise, the deviation from fundamental value, persists. Black has also presented a model in which prices include some noise around the fundamental value. One important issue concerns the size or magnitude of a potential noise term. If noise accounted for 5 percent of the variation in asset prices, it could be considered innocuous. If noise accounted for over half of the variation in asset prices, one could make a strong case for imposing a variety of regulations and restrictions on financial markets. In the next section the empirical work that addresses these issues is reviewed.

II. Market Volatility and Noise: A Review of the Empirical Research

This section presents a review of the empirical literature on financial market volatility, particularly the work on excess volatility and noise. This literature includes studies of volatility in both the stock market and the bond market, but most of the work has focused on the stock market. The empirical tests for both markets are important and at the end of this section some interesting insights are obtained by contrasting the results for these two markets. There have also been studies of volatility in foreign exchange markets, and at least one paper, Meese (1986), has attempted a test for bubbles in foreign exchange markets. The major difficulty in applying these tests to foreign exchange markets is the specification of market fundamentals and there seems to be little agreement on the appropriate form. For this reason, this paper concentrates on bond and stock markets. For stock and bond markets, there is general agreement on the form of the market fundamental; the disagreements arise over the specification of discount rates.

The review of the literature is organized as follows. First the original variance bounds tests on the stock market and the subsequent criticism are covered. This is followed by a review of what some have called the second generation tests, which were developed in response to the criticism. The discussion then turns to the tests for mean reversion, the tests on long horizon returns, and some recent tests which incorporate discount rate variability. Some recent work on the connection between stock market volatility and margin requirements is briefly reviewed and then the tests that have been applied to the bond market are reviewed. There are already several good surveys of the literature on bubbles and excess volatility. These include West (1988), Chapter 4 of Shiller's Market
Volatility (1989), LeRoy (1989), and Camerer (1990). In addition to the research covered in these surveys, this paper adds some tests presented in recent working papers by Mankiw, Romer, and Shapiro (1989), Durlauf and Hall (1988, 1989), and Scott (1990), and some recent research on stock market volatility and margin requirements is incorporated.

1. The variance bounds tests and criticism

The variance bounds literature began with the original work of Shiller (1981b) and LeRoy and Porter (1981). These two papers examined the variance restrictions that are implied by the present value model of stock prices. Define the ex post market fundamental as follows

\[ P^*_t = \sum_{j=1}^{\infty} \beta^j D_{t+j}, \]

where \( P^*_t \) is based on actual dividends or cash-flows. If asset prices are determined by market fundamentals alone, as in equation (4) of Section I, and expectations are rational, then \( P_t = E_t(P^*_t) \) and \( P^*_t - P_t + e_t \), where \( e_t \) is a forecast error which should be uncorrelated with anything in the time \( t \) information set, \( I_t \). This observation implies the following variance relations:

\[ \text{Var}(P^*_t) = \text{Var}(P_t) + \text{Var}(e_t) \]

\[ \text{Var}(P^*_t) \geq \text{Var}(P_t). \]

Shiller and LeRoy and Porter present tests of this variance restriction. Shiller constructed a time series for \( P^* \), and computed sample variances for detrended versions of \( P^* \) and \( P \). LeRoy and Porter estimated bi-variate time series models and used the parameter estimates to calculate the relevant variances. Shiller's sample variance for \( P \) was so much greater than the sample variance for \( P^* \) that he did not bother with formal statistical tests. LeRoy and Porter also found dramatic rejection of the variance restrictions in their point estimates, but many of their tests were not statistically significant at conventional levels. Shiller concluded that the stock market was too volatile. LeRoy and Porter suggested several possible explanations: (1) the market could be too volatile, (2) the present value model with constant discount rates has been rejected, or (3) the tests are invalid. Subsequent papers by LeRoy and LaCivita (1981) and Michener (1982) presented intertemporal models with risk aversion as possible explanations for the volatility of stock prices. These intertemporal models with risk aversion are models in which the discount rates vary over time.

The original tests sparked a very lively debate as critics focused on some of the weaknesses of the original tests. For example, if \( P \) and \( P^* \) are not stationary time series, then the variances do not exist and the corresponding sample variances are meaningless. Shiller removed a
deterministic time tend from his data series before computing the sample variances. LeRoy and Porter looked at their data, earnings and stock prices with an adjustment for retained earnings, and concluded that the data appeared to be stationary. Much of the criticism, particularly that of Kleidon (1986a, b) and Marsh and Merton (1986), has focused on this part of the analysis. If the dividends and stock prices need to be differenced in order to have stationary time series, then tests based on Shiller's method of detrending are invalid. The issue is ultimately related to the recent debate in the econometrics literature concerning unit roots in time series, and several tests have been conducted to determine whether there are unit roots in dividends and stock prices. For the reader who is unfamiliar with the literature on unit roots, here is a brief introduction. Consider a time series, $y_t$, with the following representation: $y_t = \rho y_{t-1} + u_t$, where $u_t$ is a stationary time series that can have some serial correlation. One of the roots for $y$ is $1/\rho$, and if $\rho = 1$, $y$ has a root on the unit circle and is a nonstationary time series. Kleidon and Marsh and Merton show that the variance bounds tests are extremely sensitive with respect to assumptions about stationarity. These authors use plausible models for dividends and stock prices, in which the growth rates are stationary time series, and show that the original tests are extremely biased. Kleidon presents Monte Carlo simulations in which the present value model holds and the variance bounds tests lead to rejection. Marsh and Merton, with a logarithmic random walk for dividends and Shiller's method for computing the terminal value in the $P^*$ series, show that the variance bound inequalities are reversed if the present value model holds. In his original tests, Shiller calculated the $P^*$ series recursively by starting at the end of the sample and working backwards:

$$P^*_{t} = \beta (P^*_{t+1} + D_{t+1}), \quad t = 1, \ldots, T.$$ 

For the terminal value, $P^*_{T+1}$, he used the sample mean from the price series, and this procedure is the one criticized by Marsh and Merton. Formally, the important feature of the model, $P_t = E_t(P^*_t)$, no longer holds if $P^*$ is calculated in this manner. A better method for calculating $P^*$ is to use the terminal price, as was done in the paper by Grossman and Shiller (1981). If the terminal stock price is used for $P^*$ at the end of the sample, then the relation $P_t = E_t(P^*_{t})$ is preserved.

2. The second generation of tests

The criticism motivated a second round of tests, the second generation tests, which are contained in papers by Mankiw, Romer, and Shapiro (1985), Scott (1985), Campbell and Shiller (1987), and West (1988a). All of these papers address the statistical problems that arise when the levels for $P$ and $P^*$ are not stationary time series. Mankiw, Romer, and Shapiro develop a straightforward extension of the variance bounds test by considering the variability of $P$ and $P^*$ relative to a naive forecast, $P^0$ in their notation:

$$E(P^*_t - P^0_t)^2 = E(P^*_t - P_t)^2 + E(P_t - P^0_t)^2.$$ 

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Their test is based on the following two inequalities:

\[ E(P^*_t - P^0_t)^2 \geq E(P^*_t - P_t)^2 \]
\[ E(P^*_t - P^0_t)^2 \geq E(P_t - P^0_t)^2. \]

For their naive forecast, they assume that dividends follow a random walk and they use \( P^0_t = (\beta/(1-\beta)) D_t \). Sample variances can be applied directly to these time series; the use of the naive forecast eliminates the need to detrend or to remove sample means. There is a possibility that the variances of these time series are growing (a heteroskedasticity problem), and the authors run a second set of tests on the data series, deflated by the price, \( P_t \). Their test results imply rejection of the present value model.

West (1988) has also developed a revised variance bounds test by considering the variability of innovations in the dividend process. He uses a subset, \( H_t \), of the information set \( I_t \) used by the market to derive the following inequality:

\[
E \left[ \sum_{j=1}^{\infty} E(\beta^j D_{t+j} | H_t) - E(\sum_{j=1}^{\infty} \beta^j D_{t+j} | H_{t-1}) \right]^2 \geq E(P_t + D_t - E(P_t + D_t | I_{t-1}))^2.
\]

The variance of the forecast error for the left side of the inequality is greater because less information is used to form the forecast. By specifying a time series model for dividends, West calculated the variance for the upper bound, the left side, by using the methods of Hansen and Sargent (1980). Since \( P_{t-1} = \beta E(P_t + D_t | I_{t-1}) \), the variance for the right side can be calculated by using the price series and an estimate for \( \beta \). This relationship holds even if one needs to difference the dividend series to make it stationary. This variance inequality is also rejected by the data, but both Mankiw, Romer, and Shapiro and West note that rejection of their inequalities is not nearly as dramatic as the rejection in the original tests. West also presents some Monte Carlo simulations for his test and he finds that there is a small bias, but the bias is not large enough to explain the rejections observed in the actual data.

Scott (1985) and Campbell and Shiller (1987) present alternative tests of the present value model which are not based on variance inequalities. The test developed by Scott is based on a simple regression interpretation of the present value model. As pointed out above, if stock prices reflect market fundamentals only, then the following relationship between prices and ex post market fundamentals must hold: \( P^*_t = P_t + e_t \). The stock price should be an unbiased predictor of \( P^*_t \) and the series \( (P^*_t - P_t) \) should not be correlated with any variables in the time \( t \) information set. A simple test is to run the least squares regression

\[ P^*_t = a + b P_t + e_t, \]
and test whether $a=0$ and $b=1$. The test can be easily modified if growth rates in dividends and prices are the relevant stationary time series: deflate the time series $P^*_t$ and $P_t$ by a measure of dividends at time $t$. 1/ Scott used $\bar{D}$, dividends summed over the previous year. The resulting regression is

\[(P^*_t/\bar{D}_t) = a + b \frac{P_t}{\bar{D}_t} + e_t.\]

If the growth in dividends, $(D_t/D_{t-1})$, is a stationary time series and the mean value for this rate is less than the discount rate, then $P^*/\bar{D}$ and $P/\bar{D}$ are also stationary time series. If $(D_t/D_{t-1})$ and $P_t/\bar{D}_t$ are not stationary time series, then the infinite sums of discounted dividends do not converge and there is no solution for the market fundamental. In this regression it is necessary to account for the serial correlation in the error term when constructing the test statistics, and Scott uses a spectral method for his calculation. This particular test exploits two implications of the present value model: (1) stock prices should be unbiased predictors of $P^*$ and (2) there should be some positive covariation between $P^*$ and $P$. Scott applies this test to stock price data and finds that the model restrictions are strongly rejected by the data. His results suggest that there is little or no covariability between the price series and the ex post market fundamental. He runs a Monte Carlo simulation for this test to verify that there are no biases against the present value model; he does find a small bias in the point estimates of the slope coefficient, but it is not large enough to bias the tests based on individual t statistics.

Campbell and Shiller (1987) use the theory of cointegration to derive testable implications of present value models. Their results on the bond market are discussed below in the subsection on tests for the bond market. The present value model for stock prices implies that $P^*$ and $P$ are cointegrated. Campbell and Shiller define a spread variable, which is $S_t = P_t - \beta D_t/(1-\beta)$; if $\Delta D_t$ is stationary, then $S_t$ and $\Delta P_t$ are also stationary time series. They specify and estimate a vector autoregression (VAR) for $\Delta D_t$ and $S_t$. 2/ The present value model applied to the spread variable implies that $S_t = E_t(S^*_t)$ where

---

1/ If one must difference the logarithms of dividends and prices to get stationary time series, then the corresponding growth rates are stationary time series.

2/ The form of the VAR is

\[
\begin{bmatrix}
\Delta D_t \\
S_t
\end{bmatrix} = \begin{bmatrix} a(L) & b(L) \\
c(L) & d(L)
\end{bmatrix} \begin{bmatrix}
\Delta D_{t-1} \\
S_{t-1}
\end{bmatrix} + \begin{bmatrix}
u_{1t} \\
u_{2t}
\end{bmatrix},
\]

where $a(L)$, $b(L)$, $c(L)$, and $d(L)$ are polynomials in the lag operator.
This implication produces a nonlinear restriction on the coefficients in the VAR for $\Delta D_t$ and $S_t$. Campbell and Shiller also use the unrestricted VAR to calculate some interesting test statistics. They define the theoretical spread,

$$S'_t = 1/(1+\beta) \sum_{j=1}^{\infty} \beta^j E(\Delta D_{t+j}|H_t),$$

where $H_t$ indicates that the expectations of future dividend changes are calculated from the unrestricted VAR. If the present value model holds, then $S$ and $S'$ should differ only by some sampling error, which implies that $\text{Var}(S)/\text{Var}(S') = 1$ and $\text{Corr}(S,S') = 1$. Campbell and Shiller use two different discount rates, or values for $\beta$, in their tests on stock prices: in one case the discount rate is set equal to the average rate of return, 8.2 percent, and in the other case the discount rate is estimated from the cointegrating regression for an estimate of 3.2 percent. They find that the nonlinear restriction on the VAR is rejected at conventional significance levels and that the variance of $S$ is much greater than the variance of $S'$, the theoretical spread from the unrestricted VAR. The correlation between $S_t$ and $S'_t$ is negative with the higher discount rate, but it is 0.911 with the lower discount rate. In the latter case $S_t$ seems to track $S'_t$, but in the first case it does not. Their results are somewhat mixed and depend on the discount rate used, but they generally conclude that the restrictions implied by the present value model are rejected by the stock price data.

3. Long horizon returns and mean reversion in the stock market

A review of the excess volatility literature would not be complete without some mention of some recent tests promoted by Fama and French (1988a) and Poterba and Summers (1988) for long horizon returns. Fama and French have found that serial correlation in stock returns is much greater if we calculate the returns over longer time horizons. Their procedure is to calculate autocorrelations by running the following regressions on rates of return:

$$R(t,t+T) = \alpha(T) + \beta(T) R(t-T,t) + \epsilon(t,t+T),$$

for different return horizons, $T$, that range from one year out to ten years. They calculated these autocorrelations for a wide variety of stock portfolios and found a very interesting pattern. The autocorrelations are

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1/ Recall that $\beta$ is related to the discount rate $k$ as follows: $\beta = 1/(1+k)$. 

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close to zero for short horizons (one year or less), but at two years they are consistently negative and the magnitude reaches a peak at around five years at values between -0.3 and -0.6. As the horizon extends out to ten years, the autocorrelations return to zero. Fama and French emphasize that these results have two possible interpretations: (1) the results are consistent with the model of Summers (1986) in which there is a noise component that has slow mean reversion; or (2) the results are consistent with variation in expected returns.

Poterba and Summers (1988) examine both autocorrelation tests like those of Fama and French and a variance ratio test. The variance ratio test compares variances for rates of return over different time horizons:

$$\text{VR}(k) = \frac{\text{Var}(R^k_t)}{k} \div \frac{\text{Var}(R^{12}_t)}{12},$$

where the returns are monthly returns and the benchmark is the variance for one-period returns. If there is no serial correlation in the return series, then all of the variance ratios should be close to one. If there is some mean reversion in the return series, then the variance ratios will drop below one as the horizon is extended. Poterba and Summers argue that the variance ratio test has more power and they apply this test to aggregate stock return series for the United States and 17 other countries. Their tests generally show evidence of positive serial correlation over short horizons (less than one year) and they find evidence of mean reversion as the variance ratios for long horizon (eight years) returns drop well below one. Poterba and Summers tend to emphasize the noise interpretation of their results. These tests provide evidence that there is definitely some serial correlation in stock returns, but it has already been noted that serial correlation in stock returns is not necessarily evidence of noise or bubbles in asset prices. Campbell and Shiller, in several of their recent papers, have commented on the connection between these tests on long horizon returns and the previous tests in the excess volatility literature. Several of the previous tests can be reinterpreted as tests on the predictability of the series, $P^*_t - P_t$. If the present value model is correct this series should be uncorrelated with any variable in the time $t$ information set. The test in Scott can be rearranged as the following regression:

$$P^*_t - P_t = a + b P_t + e_t,$$

where $a$ and $b$ should be zero and the coefficient on any time $t$ auxiliary variable should also be zero. Scott's results indicate a negative $b$ coefficient that is very significant. Look closely at $P^*_t - P_t$; it is a very long horizon return and it should be no surprise that the results of Fama and French and Poterba and Summers are very similar to the results found earlier in the excess volatility literature.
4. Tests that incorporate discount rate variability

Many economists have tried to rationalize these results by observing that all of these tests have a maintained hypothesis that discount rates are constant, or equivalently that expected returns are constant. How much variation in interest rates and discount rates is necessary in order to explain the observed variability of stock prices? Shiller (1981b) presented some analysis that his original results could not be explained by real interest rate variability, but his analysis was based on a linear approximation and some important second order effects may have been omitted. Several papers have presented tests which attempt to account for some interest rate variability. These papers include Grossman and Shiller (1981), Campbell and Shiller (1988a, b), and recent working papers by Flood, Hodrick, and Kaplan (1986), Mankiw, Romer, and Shapiro (1989), and Scott (1990). Grossman and Shiller used a consumption based CAPM to incorporate some discount rate variability; their model was essentially equation (3) in Section I. The corresponding ex post market fundamental is

\[ P^*_t = \sum_{j=1}^{\infty} \beta^j \frac{U'(C_{t+j})}{U'(C_t)} D_{t+j}. \]

Grossman and Shiller used a constant relative risk aversion utility function, \( U(C) = C^{-\gamma}/(1-\gamma) \), with values for \( \gamma \), the risk aversion parameter, that range from one to four. Their approach was to present the calculations for \( P \) and \( P^* \) graphically and they did not calculate any sample variances or other statistics. They found that a large risk aversion parameter increases the variability of \( P^* \), but large persistent deviations of \( P \) from \( P^* \) remain. The results presented by Grossman and Shiller are only suggestive.

Flood, Hodrick, and Kaplan (1986) suggest a procedure that can be best described as falling somewhere between the Fama-French serial correlation tests and the Grossman-Shiller application of the consumption based CAPM. Their idea is to estimate iterated Euler equations by using the econometric techniques of Hansen and Singleton (1982). Using the constant relative risk aversion utility function, they estimate the \( \beta \) and \( \gamma \) parameters by applying the following restrictions to real consumption and real stock return data:

\[ E_t [\beta (C_t/C_{t+j})^{1-\gamma} (P_{t+j}+D_{t+j})/P_t - 1] = 0, \quad \text{for } j=1,2,\ldots \]

The overall fit of these models can be tested by applying a \( \chi^2 \) goodness of fit test. As they increase \( j \), they effectively increase the return horizon over which the returns are calculated and tested. Their approach can be interpreted as an analysis similar to Fama and French, adjusted for changes in the marginal rate of substitution as measured by ratios of the marginal utility of consumption. Their test incorporates a form of discount rate variability, or variability in expected returns. They find that rejection by the \( \chi^2 \) test becomes progressively worse as the time horizon is extended, and they interpret the results as overwhelming rejection of the consumption
based CAPM. The consumption based CAPM has a long history (10 years) of failing miserably in empirical tests.

Another method, more in the spirit of standard finance models, is to model discount rate variability as a function of the short term interest rate. One simple model is to assume that the required rate of return on stock, the relevant one-period discount rate, is equal to the short term interest rate plus a constant risk premium. This approach has been applied by Campbell and Shiller (1988a, b), Mankiw, Romer, and Shapiro (1989), and Scott (1990). Scott also presents some tests in which the risk premium is allowed to vary with volatility of one-period stock returns. The review of these models begins with Scott's test. When discount rates vary, one must use a model of the following form for ex post market fundamentals:

\[
P^*_t = \left[ \sum_{j=1}^{\infty} \frac{D_{t+j}}{(1+k_{t+j})} \right],
\]

and if prices reflect market fundamentals only then \( P_t = E_t(P^*_t) \). \( k_t \) is the required or expected rate of return, \( E_t(R_{t+1}) \), and it is modeled as follows: \( k_t = R_{F,t+1} + R_P t \), where \( R_{F,t+1} \) is the risk free rate known at time \( t \), and \( R_P \) is a risk premium. If the risk premium is constant it can be easily estimated from the sample mean for \( (R_t - R_{Pt}) \). Scott and Mankiw, Romer, and Shapiro also note that if nominal interest rates and nominal discount rates are used, then the tests can be applied directly to nominal prices and cashflows and there is no need to deflate by a consumption price index. Scott considers both a constant risk premium model for the discount rate and one in which the risk premium varies with stock return volatility. This latter model is motivated by recent research in which expected returns on stock market aggregates are linked to the underlying return volatility. \(^1\)

In both cases a regression test is applied with the new \( P^*/D \) regressed on

---

\(^1\) Scott uses the following GARCH model, which was one of several models presented in French, Schwert, and Stambaugh (1987):

\[
\begin{align*}
R_{M,t+1} - R_{F,t+1} &= \alpha + \beta \sigma_{t+1} + \epsilon_{t+1} - \theta \epsilon_t \\
\sigma_{t+1}^2 &= a + b \sigma_t^2 + c_1 \epsilon_t^2 + c_2 \epsilon_{t-1}^2 \\
R_P &= E_t(R_{M,t+1} - R_{F,t+1}) = \alpha + \beta \sigma_{t+1} - \theta \epsilon_t.
\end{align*}
\]

In this model the risk premium varies with volatility, as measured by the GARCH model, and there is an adjustment for the small amount of serial correlation that remains in the excess return. This model is estimated by the method of maximum likelihood and the \( P^* \) series is calculated with the estimated discount rates. GARCH is the acronym for generalized autoregressive conditional heteroskedasticity. This technique has been useful in modeling changes in conditional variances.
P/D, where again the series are deflated by dividends. The estimated coefficients for the price-dividend ratio should be close to one, but the estimates are negative and the t statistics for the test that \( b = 1 \) indicate rejection at conventional significance levels. The particularly striking feature of the regression is that the R^2s are very close to zero, which implies that the price series is an extremely poor forecaster of the ex post market fundamental. This revised ex post market fundamental now accounts for dividend variability and some discount rate variability. Shiller has noted that the regression can be interpreted as a test of the predictability of very long horizon returns. Consider the regression of \( P^*_{t} - P_t \), deflated, on the price-dividend ratio. These results imply that the price-dividend ratio is a good predictor of this long horizon return, even after an adjustment is made for discount rate variability.

Mankiw, Romer, and Shapiro (1989) use the constant risk premium model to recalculate the \( P^* \) series and they apply their variance test and the regression test to the data. They also find that the model continues to be rejected when they incorporate discount rate variability. Campbell and Shiller (1988a, b) have developed some additional tests based on their dividend-price ratio model and this model can be easily adapted to handle discount rate variability. Their dividend-price ratio model follows from a linear approximation for the logarithm of the holding period return, and the model produces the following relationship when the expected return is modeled as the short term interest rate plus a constant risk premium:

\[
\ln(D_t/P_t) = r_t + \sum_{j=1}^{\infty} \beta^j E_t(r_{t+j} - \Delta \ln D_{t+j}) + c
\]

where \( r_t \) is the short term interest rate and \( c \) is a constant. In Campbell and Shiller (1988a) they apply the econometric tests of their 1987 paper to a VAR that includes \( \ln(D_t/P_t) \), \( (\Delta \ln D_t - r_t) \), and a third variable which is the log of the ratio of a 30 year moving average of earnings over the price. This last variable is useful in forecasting future dividends and future dividend-price ratios. They find that the restrictions on the VAR are rejected by the data. They also consider different time horizons for the dividend-price ratio model and the rejection becomes more significant as the time horizon is extended.

5. **Interpretation of the empirical results**

All of the tests that have been discussed to this point are tests of the present value model for stock prices, interpreted as tests of the null hypothesis that stock prices reflect market fundamentals only. The alternative hypotheses that are supported by the empirical results are varied. One view is that there are serious specification errors in the models used for market fundamentals. The alternative view is that stock prices contain a large noise component or a bubble. Flood and Hodrick (1986) have shown that none of these tests can be interpreted as evidence of
"rational" bubbles in stock prices. In most of the tests, the empirical researchers use the terminal price as the starting point in the recursive calculation for \( P^* \), the ex post market fundamental series. This procedure is consistent with the underlying null hypothesis that stock prices reflect fundamental value, but if there is a rational bubble in the stock prices this rational bubble is inserted into the \( P^* \) series as well. As Flood and Hodrick show, the net effect is that the tests presented above should have no power in detecting a rational bubble, and they interpret these test results as evidence against a rational bubble. This, of course, does not rule out the alternative hypothesis that stock prices contain either a large noise component or a near-rational bubble.

One exception to this criticism is the specification test for bubbles developed by West (1987). West sets up equations for returns, dividends, and prices and performs a specification test for the nonlinear across equation restrictions implied by the null hypothesis of no bubbles, which is essentially the present value model for stock prices. He notes that under the null hypothesis of no bubbles, all of the parameters are estimated consistently if the equations for returns and dividends have been specified correctly. If there is a rational bubble in stock prices, the parameters of the return and dividend equations can be estimated consistently, but the parameter estimates for the stock price equation are inconsistent because of the missing variable, the bubble, which may be correlated with dividends. The restrictions are tested with a \( \chi^2 \) statistic, and the behavior of this test statistic under the alternative hypothesis with bubbles in stock prices cannot be determined so that the power of this test is unknown. West runs a variety of diagnostic tests on the return and dividend equations to check for specification errors and he finds that the equations pass the battery of tests. The specification test for the restrictions on the parameters in the stock price equation are strongly rejected by the data. This test, however, can be viewed as another test of the present value model constructed carefully so that a rational bubble can be incorporated as part of the alternative hypothesis. One can also view these results in the same manner.

\[\begin{align*}
P_t &= \beta (P_{t+1} + D_{t+1}) + u_{t+1} \\
\Delta D_{t+1} &= \mu + \sum_{i=1}^{r} \phi_i \Delta D_{t+1-i} + v_{t+1} \\
\Delta P_{t+1} &= m + \sum_{i=1}^{r} \delta_i \Delta D_{t+1-i} + w_{t+1}.
\end{align*}\]

The present value model implies that the coefficients \( m, \delta_1, ..., \delta_r \) are functions of the parameters \( \beta, \mu, \phi_1, ..., \phi_r \), and West uses a \( \chi^2 \) test for this restriction.
that the other results on tests of the present value model have been interpreted.

Since the initial variance bounds tests of Shiller, LeRoy, and Porter, numerous tests and variations on tests have been developed to test whether stock prices reflect market fundamentals. These tests have been motivated by econometric issues raised by the critics. The issue of stationary time series is obviously important because (1) some form of stationarity in the time series is necessary in order to have reliable large sample properties for the test statistics; and (2) the different transformations needed to obtain stationary time series have very different implications for the potential variability of the series. Much of the early debate was focused on unit root tests for the dividend series. The subsequent tests, which account for unit roots in the dividend process, have shown that the present value model continues to be rejected, but the rejections are not nearly as dramatic as those of the original tests. Several of the papers have presented tests for unit roots in the dividend series, and in most cases the null hypothesis of a unit root is not rejected for the level of dividends, but it is rejected for the change in dividends. Another possibility for nonstationarity is that the variance of the change in dividends is growing over time. If this were true, one would need to work with percentage changes or growth rates, instead of first differences. The tests presented by Mankiw, Romer, and Shapiro (1985, 1989), Scott (1985, 1990), and Campbell and Shiller (1988a, b) are tests that follow from the assumption that growth rates in dividends and earnings are stationary. Some time series plots of the U.S. data are presented in Figures 1-4. The data are Shiller's annual time series from 1890 to 1985, reproduced in his book Market Volatility. The earnings series is included because most firms set dividends as a proportion of their earnings and earnings are useful in forecasting future dividends. Figures 1 and 2 contain plots of changes in real dividends and changes in real earnings, respectively. The dividend and earnings have been deflated by the consumption deflator. Figures 3 and 4 contain plots of the percentage changes in real dividends and real earnings. In all four graphs, the series have the appearance of stationary time series.

A skeptic may, however, raise the issue that even the growth rates are not stationary time series. 1/ This claim is countered with two observations. One, the growth rates must be restricted or else there is a risk that fundamental value is either infinite or undefined; in such an economy there would be no market fundamental and asset prices would bounce around without any meaningful variation. A growth rate for dividends and earnings that exceeds the discount rate, or the interest rate, for the economy makes no sense at all. If the return on capital were that great,
then competition in capital markets would push interest rates and discount rates up. Finally take the identity for returns,

$$1 + R_t = \frac{(P_t + D_t)}{P_{t-1}} = \frac{P_t}{P_{t-1}} + \frac{D_t}{P_{t-1}}. \quad \text{The return is a combination of the growth rate in the price and the dividend yield. If one wants to argue that growth rates in dividends, earnings, and prices are not stationary, then all of the research of the last 30 years on returns in the finance literature must also be discarded.}

6. **Stock market volatility and margin requirements**

Since the stock market crash of 1987, there has been a renewed interest in the behavior of stock market volatility and margin requirements. A series of papers on the connection between volatility and margin was stimulated by results published in a paper by Hardouvelis (1988). Hardevoulis examined this relationship by running regressions of stock market volatility on a set of explanatory variables that included initial margin requirements. The other variables, variability of industrial production and bond returns plus a measure of recent stock price movements, were included as control variables. Hardouvelis found a statistically significant negative relation between stock market volatility and initial margin requirements. The policy implication is that one can reduce volatility by increasing margin requirements. Several observations are necessary. First the initial margin requirements, set by the Federal Reserve Board in the United States, are not changed frequently and the time series resembles a step function. Second, the initial margin requirement is not really a measure of the amount of margin that investors and speculators are actually using in the stock market; the initial margin requirement is a limit only on the amount of borrowing when an investor initially buys the stock. The exchanges set the maintenance margin requirements which determine margin calls. This one paper stimulated additional work by Salinger (1989), Schwert (1989), Kupiec (1989), and Hsieh and Miller (1990). A good review of this recent work can be found in Section 3.1 of Roll (1989b). The additional research reexamined the regression analysis and it introduced a more relevant variable, actual margin credit, specifically the ratio of margin credit to total value on the New York Stock Exchange (NYSE); if the use of margin credit affects volatility, there should be a positive relation between margin credit and volatility. The subsequent papers have shown that whether initial margin requirements or a measure of margin credit is used, there is no significant effect of margin on volatility if either the regressions are run on first differences (changes) or the 1930s are eliminated from the data set.

Numerous regressions have been run on this issue, but the real insight can be seen in a graph of the ratio of margin credit to value on the NYSE. Such a graph is reproduced in Figure 5 and the data are from Table 1 in Salinger’s paper for the period 1926 to 1987. Schwert has also constructed a similar data series and his graph begins in 1917; he finds that this ratio varied between 15 percent and 28 percent for the period 1917 to 1929. Prior to the Crash of 1929, investors in the U.S. used much more margin in
FIGURE 1

Changes in Real Dividends

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FIGURE 2

Changes in Real Earnings

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Figure 3
Percentage Changes in Real Dividends

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FIGURE 4
Percentage Changes in Real Earnings

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FIGURE 5

Ratio of Margin Credit to NYSE Value

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financing their stock portfolios than they have used since the Depression of
the 1930s. During the 1930s when margin regulations and other reforms were
first introduced at the federal level, there was a sharp reduction in the
use of margin. Since World War II, the margin credit ratio has been very
stable with values fluctuating between 1 and 2 percent. Stock market
volatility was high during the 1930s and it increased for short periods
during the 1974-75 downturn and after the Crash of 1987. A regression of
volatility on margin credit will produce a positive relation if we include
the 1930s in the data set. Otherwise there is no significant relationship
in the data. The most interesting feature of the data is the sharp decrease
in the relative use of margin during the 1930s and its low level since then.

Hardouvelis (1990), in a recently published paper, has presented some
additional evidence on the relationship between margin requirements and
excess volatility. In the first half of this paper, he claims to show that
his additional work continues to produce a significant negative relation
between initial margin requirements and volatility, but a careful analysis
of his results does not support this claim. First consider a simple
description of the transmission of the effect of margin requirement changes
on volatility. An increase in the initial margin requirement reduces the
use of margin credit. As margin credit is restricted, speculators leave the
market and volatility decreases. If the change in the initial margin
requirement is to have an effect on volatility, then it should produce a
change in the actual use of margin. Under this hypothesis, a positive
relation between the use of margin credit and volatility should be observed.
If there is no relationship between initial margin requirements and the
actual use of margin credit, then it would be difficult to argue that
initial margin requirements have an effect on market volatility. By using
ordinary regressions of volatility on initial margin requirements, the
margin credit ratio, and several control variables, Hardouvelis continues to
produce significant negative coefficients on the initial margin requirement
variable. He corrects for the serial correlation in the residuals, but he
does not include lagged values of volatility, the dependent variable. There
is evidence of persistence in volatility: there is some mean reversion in
volatility and this period's volatility is a good predictor for next
period's volatility. A time series approach that accounts for this serial
dependence in the data is a more appropriate technique for analyzing the
relationship among these variables, and in his Table 4D, Hardouvelis
presents the results of vector autoregressions on a system for volatility,
initial margin requirements, real stock returns, and the margin credit
ratio. Hardouvelis claims that this analysis also supports the negative
relationship between margin requirements and volatility, but a careful
analysis of Table 4D reveals that this is an overstatement. In the VAR for
volatility, the sum of the coefficients on the initial margin requirement
variable is negative and significant at the 5 percent level, but a joint
test that all of the coefficients are zero cannot be rejected at the
5 percent level. The $\chi^2$ statistic for this test is, however, significant at
the 5.9 percent level. The sum of the coefficients on the margin credit
ratio is also negative and significant at the 5 percent level, but the $\chi^2$
test that all of the coefficients are zero is not significant at the 10 percent level. There is, however, strong evidence that lagged values of volatility and lagged values of stock returns are useful in predicting volatility. In the time series regression, the effect of the margin variables on volatility is weak. In the VAR for the margin credit ratio, the coefficients on the initial margin requirements are significantly different from zero, but the sum of the coefficients is effectively zero. Changes in initial margin requirements are useful in predicting changes in the margin credit ratio, but the long-run effect is zero. The sample for the VAR's are monthly data from 1935 to 1987. Initial margin requirements were set at 45 percent in 1934, increased to 55 percent in 1936, and then lowered to 40 percent in 1937 where they remained until 1945. Since 1945, the initial margin requirement has fluctuated between 50 percent and 100 percent. Recall from Figure 5 that the margin credit ratio was higher during the 1930s, when the initial margin requirements were at historically low levels. Since 1945, the margin credit ratio has been low while the initial margin requirements have been generally higher. If the 1930s were removed from the sample, the significant coefficients on initial margin requirements in the margin credit ratio equation might disappear. The hypothesis that initial margin requirements affect the use of margin credit and margin credit affects volatility is not strongly supported in the VAR analysis.

In the second half of the paper, Hardouvelis does present some evidence that there is a connection between initial margin requirements and excess volatility. The most interesting results are his regressions of long horizon returns on price-dividend ratios, in which he reexamines the results of Fama and French (1988b). A large proportion, roughly 30 percent to 50 percent, of the variation of long horizon returns (two to five years) can be predicted by the price-dividend ratio. Hardouvelis introduces dummy variables for high and low margin requirements and finds that the predictable component in stock returns (as predicted by price-dividend ratios) is smaller when margin requirements are higher. These results are evidence of a relationship between stock return predictability and margin requirements, and suggest a possible connection excess volatility and margin requirements. Hardouvelis concludes that the Federal Reserve has been effective in using margin requirements to dampen the effects of destabilizing speculation. It would be worthwhile to extend this analysis with actual margin credit.

There is additional research on the behavior of volatility over time. Schwert (1989a) has constructed monthly estimates of stock return volatility in the U.S. that go back to 1859, and from this data he has presented some interesting stylized facts regarding stock return volatility. Stock return volatility was much higher during the Depression of the 1930s, and it tends to increase during recessions. Most of the macroeconomic time series were more volatile during the Depression, but none of the economic variables experienced increases in volatility that were similar to the increase in stock return volatility; stock return volatility was two to three times
greater during the Depression. There seems to be some association between stock return volatility and the volatility in macroeconomic variables and there is some association between stock return volatility and financial leverage, but all of these effects are weak. There is a strong positive correlation between volume and volatility, and Schwert finds that the number of trading days has a small positive effect on stock return volatility. French and Roll (1986) have also found a trading day effect by studying the volatility of daily returns during the second half of 1968 when the stock market was closed on Wednesdays; they found that variances of stock price changes were smaller on Wednesdays when the market was closed even though the information flow of the economy was the same. Schwert confesses that he is unable to explain changes in aggregate stock market volatility with simple valuation models.

7. Prices, interest rates, and market fundamentals in the bond market

Tests for excess volatility have also been applied to bond markets by Shiller (1979), Singleton (1980), and Campbell and Shiller (1987). These papers have examined the following present value relation for interest rates:

\[
R_t(N) = \frac{1 - \gamma}{1 - \gamma^N} \sum_{j=0}^{N-1} \gamma^j E_t(r_{t+j}) + \phi_N,
\]  

(9)

where \( r_t \) is the short term, one period, interest rate, \( R_t(N) \) is the long rate (the yield-to-maturity on an N period bond), and \( \phi_N \) is a constant liquidity premium. \( \gamma = 1/(1+R) \) where \( R \) is the coupon rate. 1/ This model is derived from a linear approximation of the holding period return on a bond, and the relationship represents a version of the expectations theory of the term structure of interest rates that places more weight on expected short rates in the near future and less weight on the distant future. The typical statement of the expectations theory is

\[
1 + R_t(N) = \left[ 1 + r_t \right] \left[ 1 + E_t(r_{t+1}) \right] \cdots \left[ 1 + E_t(r_{t+N-1}) \right]^{1/N},
\]  

(10)

where the long rate is a geometric average of the corresponding expected short rates. This relationship applies when \( R_t(N) \) is the yield-to-maturity on an N period discount bond. These relationships are not exact and do not follow directly from asset pricing models, even if we assume risk neutrality. 2/ Equation (9), which has been tested by Shiller and others, is an approximation at best, and if it is rejected by the data it is possible that the approximation error may be responsible.

1/ Because the coupon rates on the bonds in his samples change over time, Shiller uses the sample mean of \( R_t(N) \) for \( R \).

2/ This point is demonstrated in the appendix.
A brief review of the empirical work on volatility in the bond market is presented here, and a more complete survey can be found in Chapter 15 in Shiller's *Market Volatility*. The first variance bounds tests on the relationship in equation (9) were presented by Shiller (1979) and Singleton (1980). Define the ex post series,

\[ R^*_t(N) = \frac{1-\gamma}{1-\gamma^N} \sum_{j=0}^{N-1} \gamma^j r_{t+j} \]

Then \( R_t(N) = E_t[R^*_t(N)] + \phi_N \), where \( \phi_N \) is assumed to be a constant. Shiller showed that the model implies the following set of variance restrictions:

\[ \text{Var}(R) < \text{Var}(R^*) < \text{Var}(r) \]

In his initial work, he found that the long rates \( R_t \) were too volatile when compared with the corresponding \( R^*_t \) series, and for some of his series the sample variance for \( R_t \) exceeded the sample variance for the short rate, \( r_t \). His general conclusion was that long-term bond rates were too volatile. Singleton extended this work by constructing formal statistical tests of the variance bounds and he found that the variance of the long rate exceeded the upper bound measured by the variance of \( R^* \).

Flavin (1983) subsequently demonstrated that the small sample properties of this interest rate model are questionable. Using a simple autoregressive process for the short rate with an autoregressive coefficient of 0.95, she found that there is a serious bias in this variance test if one uses sample sizes comparable to those used by Shiller and Singleton. In chapter 13 of *Market Volatility* (1989), Shiller presents more recent tests with two very long data sets: one for the United States from 1857 to 1988 and one for the United Kingdom from 1824 to 1987. The results for these longer data sets differ from the results of the initial tests. First, he finds that the observed variability of \( R \) is consistent with the variability of \( R^* \). He also runs the following simple regression:

\[ R^*_t - R_{t-1} = a + b (R_t - R_{t-1}) + e_t, \]

where \( b \) should equal one and \( a \) is an estimate of the negative of the constant liquidity premium. The estimate of the slope coefficient in the U.S. data is 1.156, and the test for \( b = 1 \) is not rejected. The estimated slope coefficient for the U.K. data is 0.347 and the test for \( b = 1 \) is rejected at conventional significance levels. The results are mixed but Shiller concludes that the evidence generally supports this expectations model for the bond market. He attributes the earlier rejections of the model to the smaller sample sizes used in the initial studies and the small sample biases discussed by Flavin.

Additional results on the term structure were presented in Campbell and Shiller (1987), where the theory of cointegration was applied to the present value model for stocks and the expectations theory in equation (9) above. For bonds, they define the spread variable to be \( R_t(N) - r_t \) and they estimate...
a vector autoregression (VAR). The model for the term structure implies a set of nonlinear restrictions on the coefficients in the VAR. Campbell and Shiller find that these restrictions are rejected by the data, but they do find a high correlation between the theoretical spread computed from the unrestricted VAR and the actual spread. Although the model restrictions are rejected, there is evidence that long-term rates move with rational forecasts of future short term rates.

In a recent working paper, Scott (1990) presents some tests of the present value model applied directly to bond prices. He uses the following model, with discount rates that vary with short term interest rates, to calculate the $P^*$ series for bond prices:

$$P_t^* = \sum_{j=1}^{N} \frac{C}{\prod_{i=0}^{j-1} (1+k_{t+i})} + \frac{100}{\prod_{i=0}^{N-1} (1+k_{t+i})}.$$

In one case he sets $k_{t+i}$ equal to $r_{t+i}$ and in a second case he sets $k_{t+i}$ equal to $r_{t+i}$ plus a risk premium that declines as time to maturity decreases. His sample consists of monthly prices on short, medium, and long-term U.S. Treasury bonds for the period 1932 to 1985, and the corresponding $P^*$ series is calculated for each bond. He applies a regression test by regressing $P_t^*$ on $P_t$ and a constant, and he is unable to reject this present value model for any of the bonds in his sample. He also finds that the sample variance of $P^*$ is greater than the sample variance of $P$ for all of his bond series. The results of these recent tests on the term structure models suggest that there is no evidence of excess volatility or noise in the bond market.

8. Synthesis, summary, and conclusions

The empirical results reviewed here suggest that the prices reflect market fundamentals in the bond market, but not in the stock market. For the stock market, the present value model is usually rejected by the data and there is some evidence of serial correlation and mean reversion in returns. This evidence on stock prices has several interpretations. As previously noted, the evidence on serial correlation is consistent with the EMH, coupled with intertemporal variation in expected returns. The evidence on the present value models with variation in discount rates cannot be so easily dismissed. One possibility is that there is a very large noise component in stock prices. Another is that there is a serious specification error in the models for fundamental value. Diba and Grossman (1988) and

1/ The model for the VAR is

$$\begin{bmatrix} \Delta r_t \\ R_t(N) - r_t \end{bmatrix} = \begin{bmatrix} a(L) & b(L) \\ c(L) & d(L) \end{bmatrix} \begin{bmatrix} \Delta r_{t-1} \\ R_{t-1}(N) - r_{t-1} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}.$$
Durlauf and Hall (1988, 1989) have presented useful interpretations of these models. Diba and Grossman introduce a specification error so that:

\[ P_t = E_t(P^*_t) + u_t, \]

where \( u_t \) is the specification error and they suggest omitted tax effects as a possible specification error. Durlauf and Hall present an alternative view that is observationally equivalent:

\[ P_t = E_t(P^*_t) + N_t, \]

where \( N_t \) is a noise component. They show how to estimate lower bounds for the variability of the noise term and they suggest that we measure the usefulness of our models by the relative size of the noise term. Their analysis involves regressions of the following form:

\[ P^*_t - P_t = a + b P_t + c' \mathbf{x}_t + e_t. \]

\( P^*_t - P_t \) is equal to the forecast error, \( P^*_t - E_t(P^*_t) \), plus the noise term. The forecast error should be uncorrelated with any information variable dated at time \( t \), but the noise term (or the specification error) can be correlated with time \( t \) variables, particularly the stock price which would contain the noise term. If there is no noise, then all of the coefficients in the regression should be zero. If there is noise then some of the coefficients will be nonzero, and the variance of the fitted values will represents an estimate of the lower bound for the variance of the noise term.

These regressions are very similar to the regression tests used by Scott and more recently by Shiller. The Durlauf and Hall noise measure can be easily calculated for the regressions in Scott (1990). Two sets of calculations are presented. The first set is for two stock price models in which the discount rates are functions of the short term interest rate plus a risk premium. The data for the regressions are monthly prices and dividends for the value-weighted NYSE portfolio, 1927-87. The regression equation is

\[ (P^*_t - P_t)/\bar{D}_t = a + b (P_t/\bar{D}_t) + e_t, \]

where the series \( P^*_t \) and \( P_t \) are deflated by annual dividends \( \bar{D}_t \). The results are summarized as follows: 1/

1/ The estimate for \( \text{Var}(N_t) \) is \( b^2 \text{Var}(P_t/\bar{D}_t) \), which is the variance for the fitted value of the regression.
In both cases, 68 percent of the variance of \((P^*_t - P_t)/\bar{D}_t\) is explained or predicted by the price-dividend ratio. \(P^*_t - P_t\) is a long horizon return, adjusted for discount rate changes, and the series should be unpredictable given information at time \(t\). The estimates for the noise variability suggest that the noise term is relatively large and that it accounts for most (68 percent) of the variability in \(P^*_t - P_t\); the noise also accounts for most of the price variability. Of course one can interpret these numbers as estimates for the variability of the specification error, with the conclusion that the specification error for the model is quite large. Now contrast these numbers with the same calculations for long-term bond prices (15-30 year Treasury bonds) in the following regression:

\[ P^*_t - P_t = a + b P_t + e_t. \]

The sample consists of monthly bond prices for the period 1932-85.

<table>
<thead>
<tr>
<th>Constant Risk Premium Model</th>
<th>Time Varying Risk Premium Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b) (Standard Error)</td>
<td>(-1.1514) (0.3802)</td>
</tr>
<tr>
<td>(t(b=0))</td>
<td>(-3.03)</td>
</tr>
<tr>
<td>(R^2)</td>
<td>(0.68)</td>
</tr>
<tr>
<td>(\sigma(N_t/\bar{D}_t))</td>
<td>(7.42)</td>
</tr>
<tr>
<td>(\sigma((P^*_t-P_t)/\bar{D}_t))</td>
<td>(9.00)</td>
</tr>
<tr>
<td>(\sigma(P_t/\bar{D}_t))</td>
<td>(6.45)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Zero Risk Premium</th>
<th>Risk Premium as a Function of Maturity</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b) (Standard Error)</td>
<td>(-0.1024) (2.5600)</td>
</tr>
<tr>
<td>(t(b=0))</td>
<td>(-0.04)</td>
</tr>
<tr>
<td>(R^2)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>(\sigma(N_t))</td>
<td>(0.9828)</td>
</tr>
<tr>
<td>(\sigma(P^*_t-P_t))</td>
<td>(23.8557)</td>
</tr>
<tr>
<td>(\sigma(P_t))</td>
<td>(9.5974)</td>
</tr>
</tbody>
</table>

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In the bond regressions, none of the coefficients are statistically significant, and the resulting estimates for the noise variance are not statistically significant. Noise or specification error is relatively small for the present value model applied to bond prices. The present value models work reasonably well for bonds, but not for stocks.

Do stock prices deviate substantially from fundamental value? The evidence from the numerous studies reviewed suggests that there are significant deviations of stock prices from market fundamentals. It should be noted that almost all of these studies have concentrated on data for the U.S. market. The study of mean reversion by Poterba and Summers (1988) did present evidence of serial correlation in the stock markets of 18 different countries. The inability of economists to explain either stock price movements or changes in stock market volatility provides additional support for this view. The challenge to defenders of the EMH is to explain the failure of the valuation models applied to the stock market. The different empirical tests have incorporated dividend variability, interest rate variability, and variability in different measures of the risk premium, but the various modifications have been unable to rationalize the observed variability of stock prices. Why do the models work reasonably well for bond prices, but not for stock prices? One answer, from the discussion of bubbles in Section I, is the presence of irrational or near-rational bubbles in stock prices, but not in bond prices. The evidence also supports the notion of noise or fads in stock prices, but noise trading or fads models should not be applied to all markets because the empirical evidence does not warrant the application of these models to the bond market. Stephen LeRoy (1989, page 1616) recently summarized his position on the efficient markets hypothesis as follows. "The most radical revision in efficient markets reasoning will involve those implications of market efficiency that depend on asset prices equaling or closely approximating fundamental values. The evidence suggests that, contrary to the assertion of this version of efficient markets theory, such large discrepancies between price and fundamental value regularly occur."

III. The Stock Market Crash of 1987 and the Implications for Market Regulations

On October 19, 1987, stock markets around the world crashed. In the United States, the NYSE composite lost 20.4 percent of its value, and the market was down by roughly 35 percent from its peak achieved during the previous week. There were no events or developments in the U.S. economy that could explain this sudden and rapid decline in the stock market. This drop of over 20 percent surpassed by a wide margin the record one-day declines that were set back in October of 1929. The Crash was equally dramatic in other stock markets: over the two days, October 19 and 20, 1987, stock prices dropped by 18 percent in Canada, 13 percent in Germany,


The Stock Market Crash of 1987 has also generated renewed interest in market regulations and several reforms have been proposed. "Circuit breakers" and trading halts have already been introduced on several exchanges, and some critics have requested regulations prohibiting certain trading activities commonly known as program trading. Other proposed changes include a transactions tax and changes in margin requirements for the stock market and the futures markets. The different theories of how financial markets function all result in different assessments of the need for market reforms. For an example, suppose that noise traders dominate markets and that this noise trading has produced an increase in volatility. Markets could be improved by introducing reforms that would restrict the trading activities of noise traders. One reform supported by Summers and Summers (1989) and others is a transactions tax that would be applied to stock and bond markets. Other reforms would include increases in margin requirements, particularly on futures contracts, restrictions on program trading, and the prohibition of some futures contracts that promote speculation. Now contrast this very negative, suspicious, view of financial markets with the efficient markets hypothesis. If the EMH is true, then there is no need for any market regulations because they would simply impede the efficiency of the market. Individuals should be allowed to trade on anything and the exchanges should be allowed to set their own margin.

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requirements. Government regulations should be minimal and there would be no purpose served by the imposition of a transactions tax. This section begins with a review of the Stock Market Crash and then the various proposed market reforms are discussed. In Section II evidence that stock prices deviate from fundamental value was presented. It is not clear whether the deviations are the result of speculative bubbles, noise trading, or fads, but it is clear that there is no evidence of large deviations from fundamentals in bond markets. Whether one interprets the deviations in the stock market as a bubble or noise, it is clear that the bond market, and possibly other markets, should be treated separately.

1. Program trading

Much of the current debate concerning the Crash and market reforms involves program trading. The Brady report, for example, concentrated on program trading and market structure issues. Although there are potentially many program trading strategies, there are two prominent forms of program trading that need to be distinguished: stock index arbitrage and portfolio insurance. Program trading is the generic name for trading strategies that are mechanical; in most cases they are programmed on a computer and the computer actually sends buy and sell orders to the exchanges. A computer facilitates program trading, but it is not necessary. Some forms of program trading, stock index arbitrage for example, involve the buying or selling of large portfolios of stocks.

Stock index arbitrage is the simultaneous trading of stocks and index futures to take advantage of a temporary price discrepancy in the two markets. Stock index arbitrageurs literally use an arbitrage formula to do the trading. For any market, stocks, bonds, commodities, or foreign exchange, there is a simple relationship for determining the futures price if the underlying spot asset trades. The formula for stocks is

\[ f = S \left(1 + R - \frac{D}{S}\right) , \]

where \( f \) is the futures price, \( S \) is the spot price for the stock, \( R \) is the short term interest rate and \( D/S \) is the dividend yield. The formal arbitrage relation is easy to establish for forward prices. Suppose the forward price \( F \) is too high, \( F > S(1+R-D/S) \). Then buy the stocks and sell forward. The portfolio has no risk and requires an investment in the stock only. The initial value of the portfolio is \( S \) and the opportunity cost is the risk free interest rate. The rate of return on the portfolio is \((F+D-S)/S\); simply collect the dividends and sell the stock at the forward price. If \( F > S(1+R-D/S) \) then \((F+D-S)/S\), the rate of return on the arbitrage portfolio, is greater than \( R \), the risk free rate of return. Arbitrageurs will shift from risk free assets, Treasury bills, into this arbitrage portfolio. The arbitrage profits disappear when \( F = S(1+R-D/S) \). If \( F < S(1+R-D/S) \), the arbitrage is reversed: buy forward and short-sell
the stock. For those traders who can short sell without restrictions, there is an arbitrage opportunity. 1/

The formal arbitrage relationship applies exactly for forward prices and approximately for futures prices. Futures contracts are different because they are settled each day on the exchanges; they are marked to the market. There are cashflows each day between the longs and the shorts that are determined by the changes in the settlement prices. This cashflow difference can lead to a small difference between forward and futures prices and we apply the formula above as an approximation for futures prices. To take advantage of the difference between the futures price and the theoretical price on the right side, an arbitrageur must be able to finance the daily cashflows on the futures contract. Because of marking to market and the costs of transacting in these markets, there are some small differences between futures prices and the theoretical prices. In fact, the empirical studies find that stock index futures prices stay within a band around the theoretical price. 2/ This model for futures prices is commonly known as the cost of carry model, but when it is applied to forward exchange rates it is known as covered interest rate arbitrage.

Stock index arbitrage occurs whenever the difference between the futures price and the theoretical price becomes too large. The stock index arbitrage effectively enforces this pricing relationship. Some market observers have expressed concern that the stock index futures market drives the stock market because they observe the futures market moving ahead of the stock market. The observation that futures markets lead the stock market is correct, but it does not imply causality. Suppose that rational market traders perceive correctly that value has decreased for the entire stock market. It is cheaper for these traders to sell futures instead of selling a large stock portfolio. As a result they sell stock index futures and the futures price drops first. At some point where $f$ is less than $S(1+R-D/S)$ the computers of the stock index arbitrageurs commence: they buy futures and sell stocks. The effect in the market, if the lower price is correct, is a drop in the stock market. This is a situation in which the futures market leads the stock market, but there is no causality running from the futures market to the stock market. If there had been no futures market, the same traders would have sold their stocks, and the price would have dropped anyway. Stock index arbitrage is an innocuous trading activity that serves to keep the futures prices in line with the prices in the stock market.

The other common form of program trading is portfolio insurance, which is considerably more complex than stock index arbitrage. The principal

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1/ The arbitrageurs who can do this form of trading are pension funds and brokerage firms who hold inventories of stocks. They simply sell the stock and buy futures.

2/ For some examples, see Cornell and French (1983) and Modest and Sundaresan (1983).
objective of portfolio insurance is to insure that the value of one's stock portfolio does not drop below some minimum value. One way to insure a portfolio would be to buy a put option on the portfolio of stocks, specifically a European put with a maturity that matches the desired time horizon of the investment manager. European options are options that can be exercised only at expiration. In exchange markets, there are puts traded on stock indexes, but these puts are generally short term (three months or less), and they are American options which can be exercised early. The early exercise feature is of no use to a portfolio insurer, and the longer term European puts, if they were available, would actually be cheaper than the short term American puts. Instead of buying the more expensive American puts on the exchanges, portfolio managers actually used synthetic puts to create portfolio insurance. Synthetic puts are trading strategies that use option pricing theory to create dynamic portfolios that have payoffs that mimic the payoffs on real puts. The trading strategy uses positions in stocks and short term bonds. The size of the stock position is determined by the put delta, the partial derivative of the put price with respect to the underlying stock price. Since the put delta varies with the underlying stock price, the size of the stock position varies and it must be managed on a daily basis. To mimic a put option, one must take short positions and the users of portfolio insurance typically took short positions in stock index futures because the transactions costs are lower. When the portfolio insurance is set up, a short position in stock index futures is established, but this position must be adjusted on a regular basis. If stock prices increase, the put delta decreases in magnitude and the size of the short position in stock index futures must be reduced. If stock prices decline, the magnitude of the put delta increases and the short position in the futures market must be increased. As the market goes up, portfolio insurers are buying back futures contracts. As the market goes down, portfolio insurers are selling more futures contracts. With portfolio insurance, there is a potential for the trading strategy to exacerbate a market move. Prior to the Stock Market Crash of 1987, $60 to $80 billion invested in the U.S. stock market was protected by portfolio insurance. The trading strategy performed very poorly during the Crash and many investment managers terminated their portfolio insurance programs. More recently a new market has developed in portfolio puts. Here a financial institution, a well capitalized investment house, sells a put option to a pension fund that has funds invested in the stock market. The put option has a payoff that is tied to a market index and it is customized for the client; it can, for example, be a one year European put on a market index. In this case the financial institution is short a put option and it can hedge this position by engaging in the same dynamic trading strategy that portfolio insurers were using prior to the Crash. This newer portfolio put market has been estimated at only $2 billion. 1/

Program trading was a significant proportion of the trading volume on October 19, 1987. The Brady report reveals that program trading accounted for roughly 20 percent of the combined volume in the stock market (NYSE) and the futures market (the S&P 500 index futures, CME). The volume for this day was $21 billion on the NYSE and the equivalent of $20 billion in stock index futures at the CME. Portfolio insurers executed sell orders of $2.3 billion on the NYSE and $4 billion in the futures market. Portfolio insurance actually accounted for less than 20 percent of the volume, but when the $1.73 billion of sell orders executed on the NYSE by stock index arbitrageurs are added the percent of the volume attributed to program trading comes up to roughly 20 percent. The Brady report notes that three of the four largest sellers on this day were portfolio insurers; the other one was a mutual fund. Mutual funds were also large net sellers, reacting to redemptions by their clients, primarily individual investors. Portfolio insurers were not the only sellers, as 80 percent of the volume was attributed to other investors and traders. Portfolio insurance played a role during the Crash, but it should not be identified as the cause of the Crash. To stimulate the sell orders by portfolio insurers, there must be some initial drop in the stock market. Portfolio insurance, per se, did not cause the Crash, but it did play a role in the speed at which the decline occurred. Without portfolio insurance trading strategies, the decline would have occurred anyway, but it would not have occurred as rapidly.

It is possible that portfolio insurance played a role in the increase in stock prices that preceded the Crash. The market rose during the first nine months of 1987 and portfolio managers continued to buy because they wanted to participate in the run-up. One could argue that a portfolio manager would have been penalized in performance reviews for being out of the market during the increase. A manager would have been compelled to remain in the market even if he or she had believed it to be overvalued. With portfolio insurance, a portfolio manager could participate in the run-up and be protected against the ultimate drop. 1/ This story is plausible and attractive, but it does not provide a complete explanation for the Crash. The important lesson from the Crash was that the dynamic trading strategy behind portfolio insurance may not always work, particularly when it becomes most critical. Because the strategy failed to protect investment managers during the Crash, many of the programs were canceled, and today the portfolio insurance market is only a fraction of the $60 to $80 billion market that existed prior to the Crash. Some institutions now use a combination of real index put options and short positions in the futures market. In place of the old market, a new form of portfolio insurance has emerged: portfolio puts sold by investment bankers. As mentioned earlier, this new market for portfolio puts is much smaller ($2 billion), but dynamic trading strategies will continue to be used by some investors.

1/ Leland and Rubinstein (1988) present this scenario, but they dismiss it as an explanation for the Crash.
One final issue pertaining to portfolio insurance needs to be addressed. Did portfolio insurance increase volatility in the stock market? Several papers, including Greenwald and Stein (1988), have shown that, with the exception of the period during and immediately following the Crash, stock market volatility during the 1980s was at a normal level. The concept of portfolio insurance was introduced during the 1980s and the market grew, but there was no noticeable increase in market volatility prior to October of 1987. It is also worth noting that there were no stock index futures markets, and no stock index arbitrage or portfolio insurance, during the stock market crash of 1929 and the sharp downturn of 1974-75. 1/ Stock market collapses occurred long before the introduction of stock index futures markets and program trading. A comparison with the bond market is also useful in this case. Futures contracts on Treasury bills and bonds began trading before the creation of stock index futures, and there is also arbitrage between the spot Treasury market and the futures market, that is similar to stock index arbitrage. Investment managers also employ dynamic trading strategies to hedge their bond portfolios; one example is tactical asset allocation in which the percentages of stocks and bonds in a large portfolio are adjusted on a daily basis. There is, however, no evidence of excess volatility in the bond market and almost no criticism has been directed at program trading strategies employed in the bond markets.

2. The stock market crash of 1987

One cannot make a convincing argument for program trading as the cause of the Crash of 1987, and most economists have noted that were no fundamental economic changes large enough to explain the price decline of the Crash. 2/ The argument that there was a temporary breakdown or temporary loss of liquidity in the market is also not a convincing explanation. During the six months following the Crash the market did not recover. It did return to its previous record levels during the first half of 1990, after corporate earnings, as measured by Standard & Poor's, increased by 70 percent. The bubble hypothesis does provide an explanation for the Crash in the U.S.: the stock market rose dramatically during the first nine months of 1987 and after the Crash the aggregate indexes were back to their levels as of January 1987. Just prior to the Crash, the price-dividend ratio was at a level of almost 40, which is near a record high for the U.S. market. The price-earnings ratios were also near record highs. In Figure 6, a graph of the price-dividend ratio for the value weighted NYSE from 1927 to 1987 is presented. The graph also includes the

1/ During the recession of 1974-75, the stock market dropped by roughly 50 percent in real terms.
2/ One exception is the paper by Mitchell and Netter (1989), which argues that the introduction of a tax bill in the U.S. Congress triggered the Crash. The tax bill would have penalized corporate takeovers. Roll (1989b) has taken the position that this is not a plausible explanation for the 20 percent decline of the entire U.S. market. Nor is it an explanation for the worldwide crash.
FIGURE 6
NYSE, PRICE-DIVIDEND RATIOS
RISK PREMIUM MODELS

\[ \frac{P_t}{D_t} \]
Constant Risk Premium

\[ \frac{P_t^*}{D_t} \]
Time-Varying Risk Premium

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ex post market fundamentals, $P_t^*/D_t$, calculated by Scott (1990) with two different risk premium models that incorporate discount rate variability. There have been periods when the price-dividend ratio has been in the range of 35 to 40, but the average is somewhere between 20 and 25. It is difficult to determine an appropriate range for either the price-dividend ratio or the price-earnings ratio, but given the evidence that stock prices seem to deviate from market fundamentals, extremely high values for these ratios would indicate an overvalued market. It should also be noted that there is evidence in the finance literature that trading strategies based on price-earnings ratios outperform the market. Fama (1989), the chief proponent of the efficient markets hypothesis, has argued that the Crash of October 1987 is evidence of the efficiency and quickness of the market in moving to a new equilibrium, but he confesses that he is unable to explain the increase that preceded the Crash. Roll (1989b) has noted that the bubbles explanation has its weaknesses: it might explain the Crash in the U.S. market, but not in all of the other stock markets around the world that also experienced crashes. In another paper, Roll (1989a) did find a negative correlation across countries between the size of the market crash and the prior increases that preceded October 1987, and this piece of evidence does support the notion of overvalued markets. King and Wadhani (1990) have recently presented a "contagion" model in which shocks or mistakes in one stock market are transmitted to other stock markets around the world and they present some initial empirical evidence to support their model. Roll also presents some supporting evidence. He shows that prior to the Crash the correlation across international stock markets was relatively small and unstable, but during the month of October of 1987 when volatility suddenly increased the correlations across markets also increased. This phenomenon that the correlations increase with an increase in volatility is predicted by King and Wadhani's contagion model. This contagion model supports the idea that the crash of a bubble, or the collapse of an overvalued market, in one country could be transmitted to other markets.

The empirical evidence suggests that there are regularly deviations, and sometimes large deviations, of stock market prices from fundamental value. During the first nine months of 1987 there was a steady increase in the U.S. stock market so that indicators like the price-earnings ratio and the price-dividend ratio, were at their highest levels historically. The U.S. market was overvalued, and numerous security analysts expressed similar opinions in the financial press during the early weeks of October 1987. Other stock markets, Japan, France, Germany, and the U.K., also experienced impressive increases prior to the Crash. During the week preceding October 19, the U.S. market began dropping, and over the weekend of October 17-18, stock mutual finds received redemption notices from their clients. On Monday, October 19, the market opened down and the portfolio insurance programs hit the exchanges with large sell orders. Portfolio insurance was not the cause of the Crash, but it contributed to the speed at which the market dropped. On Monday and Tuesday, chaos emerged as the

1/ See the graphs in the International Monetary Fund (1989), page 63.
exchanges reacted to the large price swings and concerns developed over the ability of the exchanges to handle the large volume and cashflows. Eventually the U.S. stock market stabilized at a much lower level and after a period of several months market volatility settled down.

3. Proposals for new market regulations

The Brady report recommended several changes in the structure of U.S. financial markets. It recommended that one agency, the Federal Reserve, be responsible for regulating markets and setting margin requirements. The report also emphasized the need to make margins on stock index futures more consistent with margins in the stock market, which usually means higher margin requirements in the futures markets. The report also recommended the use of circuit breakers, specifically prearranged trading halts if prices move too much, and a unified clearing and settlement system. The U.S. stock markets, including the stock index futures and options markets, have over ten different clearinghouses which serve to settle transactions, transfer funds, and provide some insurance that transactions will be completed. / Circuit breakers have been established in some markets, and the other proposals remain under discussion. In addition to these changes, proposals for transactions taxes and abolition of program trading have been introduced.

Although margin requirements are low for stock index futures relative to margin requirements for stock purchases, it is not clear that there is a need to raise margin requirements in the futures markets. It is important to remember that margin requirements in futures markets are different and serve an altogether different purpose. Margin requirements in the stock market set a limit on the amount that an investor can borrow with the stock as collateral. The funds for these loans come from the brokerage firms and ultimately from the banking system. In futures contracts the underlying asset or index is almost never purchased. Cash is transferred between longs and shorts each day as futures prices change. The margin is a deposit of cash or liquid assets to guarantee that each party in a futures contract will be able to meet his or her obligations. In most cases the daily cash transfers come out of the posted margin; if the margin drops below the maintenance margin requirements set by the exchanges and their clearinghouses, then a margin call is issued. The exchanges and the clearinghouses have very strong incentives to set margin requirements high enough to ensure that all participants will meet their obligations. The clearinghouses effectively guarantee the performance of the futures contracts. The Chicago futures exchanges, for example, have instituted a system in which margin requirements are automatically adjusted up or down as price volatility changes. During the volatile week of the Stock Market Crash, the futures exchanges held cash settlements in the middle of the trading day as well as at the end, when settlements normally occur. Futures

/ See Bernanke (1990), particularly page 135, for a discussion of clearing and settlement in the United States.

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markets are used by both hedgers and speculators. If margin requirements were raised above what is necessary to meet daily settlements, then the additional costs, in the form of larger cash deposits, would be imposed on both of these users of futures markets.

Margin requirements on stock serve a different purpose. When stock is purchased on margin, there is a loan with the stock serving as collateral. In many cases, banks are the ultimate lenders when stock is bought on margin. It has been noted that stock prices are volatile, and some of this risk is transferred to the banking system. In a country like the U.S., deposit insurance plays an important role in the banking system, and excessive use of stock market credit could have a direct impact on the government. With stock being purchased on margin, the risk of stock market volatility is borne by brokerage firms, and banking system, and the deposit insurance agency as well as the speculators. The primary issue is whether there is a need for additional government regulations. Will the restrictions imposed by brokerage firms and banks be sufficient? A good description of the history of Federal Reserve margin requirements in the U.S. can be found in Garbade (1982). He notes that the U.S. Congress instituted federal regulation of margin requirements during the 1930s for two reasons: (1) to limit the use of credit for stock market speculation so that more credit would be available for productive uses; and (2) to reduce stock market fluctuations that might develop from the use of margin credit. As noted back in Section II, the evidence on the effectiveness of margin requirements are mixed, and the need for government regulation of initial margin requirements on stock purchases remains an open issue.

The Brady report and the Securities and Exchange Commission (SEC) have advocated the use of circuit breakers and price limits in the stock market and the stock index futures markets. Circuit breakers have been established on the NYSE, and the options and futures exchanges have established a variety of circuit breakers, temporary trading halts, and price limits for the stock index contracts.1 The experience with circuit breakers and trading halts has been brief and limited. Circuit breakers did go into effect on October 13, 1989, when the U.S. stock market experienced a sudden drop and the Dow Jones Industrial Average closed down 190 points, or 6.9 percent. The SEC's Division of Market Regulation examined the trading activity around this market break and concluded that circuit breakers were effective.2 By contrast, the Commodity Futures Trading Commission (CFTC) examined this same period and concluded that the circuit breakers and trading halts had no effect on the market. The CFTC staff looked at price volatility around the imposition of the trading halts and concluded that the trading halts did not serve to reduce volatility.3 Many of the

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1/ For a more complete description, see Appendix C of the SEC's, "Trading Analysis of October 13 and 16, 1989" (May 1990).
financial futures contracts do not have daily price limits, but most of the commodity futures do. In a recent study of price limit moves in futures markets, Ma, Rao, and Sears (1989) conclude that price limits are associated with reductions in volatility, but in their comments Miller (1989) and Lehmann (1989) suggest that these conclusions may be premature. The results on the effectiveness of price limits and circuit breakers are mixed, and it is too early to draw any conclusions. If a sudden drop in the stock market were a movement back to fundamental value, circuit breakers and price limits would impede the adjustment of the market. Some of the trading in these markets is done to hedge stock portfolios, and trading halts prevent hedgers from making adjustments.

Another proposed change for U.S. financial markets is the transactions tax. Transactions taxes are used in a number of financial markets around the world, but some of these taxes will be eliminated soon. 1/ By using the argument that noise trading plays an influential role in financial markets, Summers and Summers (1989) present a convincing case for a transactions tax. There are, however, some weaknesses in their "cautious case" for a transaction tax. First, they recommend applying the tax to stock and bond markets. It has already been noted that there is no empirical evidence to support the notion of noise or noise trading in bond markets. Much of their case for the tax in the stock markets rests on their argument that the tax would reduce unnecessary trading volume, noise trading in particular, and volatility. Imposing the tax in bond markets would serve no purpose. Another part of their argument relies on the existence of transactions taxes in the other major financial markets; the U.S. could impose a tax without running the risk of losing trading volume to foreign markets. If other countries were to reduce or eliminate their transactions taxes as some are currently doing, then the argument for a transactions tax in the U.S. becomes much weaker. From a broader perspective, should industrialized countries impose transactions taxes on their financial markets? There is some evidence that trading activity generates volatility, but imposition of transactions taxes would require international coordination and there would be incentives for individual countries, and markets, to compete for business by lowering their taxes. Problems in the clearing and settlement of transactions in financial markets emerged during the Crash of 1987, and the Brady report recommended several structural changes to address these problems. During the week of the Crash, the clearinghouses in both the stock and futures markets were confronted with technological problems as they attempted to process unprecedented levels of transactions. The worst problems mentioned by

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1/ For a discussion of the transactions tax proposal, see Stiglitz (1989) and Summers and Summers (1989). Summers and Summers include a summary of transactions taxes in use around the world as of late 1989. For a discussion of recent changes in transactions taxes, see White, Kupiec, and Duffee (1990). The Netherlands has abolished its tax, and the U.K. and Germany have made plans to remove their taxes.
Bernanke (1990) in his discussion of clearing and settlement were the two and a half hour shutdown of the Fedwire facility on Tuesday, October 20, and the difficulties throughout the week in transferring funds between New York and Chicago. The Federal Reserve, however, exercised its role as lender of last resort and temporarily increased liquidity in the banking system. 1/ Despite the unprecedented levels for volume and daily settlements, the futures exchanges were able to collect all payments due and none of their customers lost funds. The exchanges have responded to the communications problems and concerns over financial solvency, and a number of improvements in the clearing and settlement systems have been made.

IV. Summary and Conclusions

Do asset prices reflect fundamental value in financial markets? Is there too much volatility in financial markets? The empirical studies reviewed in Section II provide evidence that stock prices do regularly deviate from fundamental value. The early studies of excess volatility in the stock market presented by Shiller, LeRoy, and Porter were criticized and challenged, but more recent studies, which relax some of the restrictive assumptions in the initial studies and address some of the econometric issues, continue to produce evidence against the efficient markets hypothesis and the notion that stock prices reflect fundamental value. The results of the more recent studies are not as dramatic, but they do imply that there are significant deviations of stock prices from fundamental value. By contrast, the recent work on interest rates and prices in bond markets has been unable to uncover any significant evidence of deviations from fundamental value in the bond market. Robert Shiller, who generated some of the initial evidence of excess volatility in long-term interest rates, has recently reversed his position with respect to the bond market. 2/

In Section I of this paper, several alternative models for the formation of prices in financial markets were discussed: the efficient markets hypothesis, the speculative bubbles model, and noise trading models. The evidence of excess volatility in the stock market provides support for the alternatives to the efficient markets hypothesis, and there has been a recent increase in research on noise trading models and other alternative views in the financial economics literature. The Stock Market Crash of 1987 also stimulated an increase in this line of research as most financial economists have been unable to explain the Crash in terms of changes in fundamental value or fundamental economic factors. 3/ In his recent survey on efficient capital markets, LeRoy has concluded that it will be necessary to revise the way that efficient markets reasoning is applied.

1/ For a good discussion of clearing and settlement during the week of the Crash and an analysis of the events, see Bernanke (1990).
3/ See the papers by Fama (1989) and Roll (1989a, b).
Despite the evidence that stock prices do not always reflect fundamental value and there may be noise and occasional market crashes, it is not clear that a wide range of restrictions on financial markets are necessary. Financial markets do serve an important role in the allocation of capital and the goal of any new regulations should be to promote reforms or changes that serve to move stock prices closer to their fundamental values. Imposing circuit breakers and trading halts will not necessarily promote a more efficient market, and it is quite possible that these restrictions could impede attempts by markets to return to prices that are closer to fundamental value. Available empirical evidence suggests that restrictions on various trading strategies like stock index arbitrage and portfolio insurance are unnecessary because these strategies do not contribute directly to potential noise or deviations from fundamental value in financial markets. Dynamic trading strategies, including portfolio insurance, are in some cases hedging strategies for portfolio managers. The other form of program trading, stock index arbitrage, is an innocuous form of arbitrage that links the stock index futures market with the stock market. Margins in futures markets do not serve the same purpose that margins in the stock market serve, and there is no need to raise margins in the futures markets. During the week of the Stock Market Crash of 1987 there were no defaults on the futures exchanges. Many of the traders in financial futures markets use these markets as an inexpensive vehicle for hedging bond and stock portfolios, and increasing margin requirements would raise the transactions costs unnecessarily. Transactions taxes would serve no purpose in bond markets, but it might deserve further consideration for stock markets. There is, however, an incentive for individual countries to eliminate transactions taxes in order to promote the international use of their financial markets. If one could make a strong case for applying transactions taxes in stock markets, it would be necessary to apply the tax across all major stock markets and to have some form of international cooperation. In addition, numerous administration problems with the transactions tax would need to be resolved.
Expectation Theories of the Term Structure

This appendix demonstrates the difficulty in deriving expectation theories of the term structure of interest rates. To see this apply the asset pricing model in equation (6) of Section I, the MRS model, to price default-free bonds. For a bond with a coupon payment $C$ and par value of $100$, the price is

$$P_t = \sum_{j=1}^{N} C \cdot E_t \left( \frac{\beta^j \lambda_{t+j}}{\lambda_t} \right) + 100 \cdot E_t \left( \frac{\beta^N \lambda_{t+N}}{\lambda_t} \right).$$

For a discount bond that pays $1$ at the end of period $t+N$, the price is

$$P_t = E_t \left( \frac{\beta^N \lambda_{t+N}}{\lambda_t} \right) = \frac{1}{[1+R_t(N)]^N},$$

where $R_t(N)$ is the per period yield-to-maturity for this discount bond. Let $r_t$ be the one period interest rate so that

$$\frac{1}{1+r_t} = E_t \left( \frac{\beta \lambda_{t+1}}{\lambda_t} \right).$$

Now define $(\eta_t - 1)$ as the innovation in the MRS so that $E_t(\eta_{t+1}) = 1$ and

$$\frac{\beta \lambda_{t+1}}{\lambda_t} = \frac{\eta_{t+1}}{1+r_t}.$$

$\eta_{t+1}$ is a strictly positive variable because marginal utility of real wealth and ratios of the consumption price deflator should be positive. By using the MRS written as a combination of the short interest rate and the innovation, one can rewrite the equation for the discount bond price as follows:

$$E_t \left( \frac{\beta^N \lambda_{t+N}}{\lambda_t} \right) = E_t \left( \frac{\beta \lambda_{t+1}}{\lambda_t} \cdot \frac{\beta \lambda_{t+2}}{\lambda_{t+1}} \cdots \frac{\beta \lambda_{t+N}}{\lambda_{t+N-1}} \right)$$

and

$$\frac{1}{[1+R_t(N)]^N} = E_t \left[ \frac{1}{1+r_t} \cdot \frac{\eta_{t+1}}{1+r_{t+1}} \cdots \frac{\eta_{t+N-1}}{1+r_{t+N-1}} \right].$$
In general, $\eta_{t+j}$ and $r_{t+j}$ are correlated, so that risk aversion plays a role in the determination of long term interest rates. Risk neutrality is not sufficient to generate an expectations theory for the term structure because there can be correlation between unanticipated changes in inflation and interest rates. If one makes the assumption that $\eta_{t+j}$ and $r_{t+j}$ are uncorrelated for all $j$'s then the expression for long term interest rates can be simplified as follows:

$$\frac{1}{(1+R_t)^N} = E_t \left\{ \prod_{j=0}^{N-1} \frac{1}{(1+r_{t+j})} \right\}.$$ 

But there is no direct relationship between this last expression and equations (9) or (10) in the paper.

$$(1+R_t)^N \approx E_t \left\{ \prod_{j=0}^{N-1} (1+r_{t+j}) \right\} \approx \prod_{j=0}^{N-1} E_t (1+r_{t+j}).$$

Hence equations (9) and (10) are approximations.
References


