Exchange Restrictions and Devaluation Crises

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Abstract

This paper develops a model of devaluation crises for an economy where foreign exchange restrictions lead to the emergence of a parallel market. The devaluation rule relates the size of the parity change to the spread between the official and parallel exchange rates. The mechanism that triggers the devaluation relates credit policy and the inflation tax. A credit expansion leads to an increase in the spread and possibly to a fall in inflation tax revenue, as agents switch away from domestic currency holdings. A devaluation reverses temporarily the process of erosion of the tax base if the associated fall in the premium raises the credibility of the new parity.

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Summary

This paper develops a model of devaluation crises for an economy where foreign exchange controls lead to the emergence of a parallel market for foreign exchange. The devaluation rule relates the size of the parity change to the level of the spread between the official and parallel exchange rates. Although several rationales can be provided for the mechanism that triggers a devaluation, the paper focuses on the relationship between credit expansion and the yield of the inflation tax under currency substitution. An excessive rate of growth of domestic credit generates expectations of depreciation of the parallel market exchange rate, which give agents an incentive to switch away from domestic money holdings. This shift reduces the tax base and proceeds from the inflation tax if the degree of substitution between domestic and foreign currency holdings is high relative to the rate of credit expansion. If the credibility of the official exchange rate is inversely related to the level of the premium, a devaluation of the domestic currency may be successful in recapturing the tax base. A devaluation reduces the level of the parallel market premium on impact and lowers the probability of a future exchange rate adjustment. This, in turn, reduces the expected rate of depreciation of the parallel rate, and provides an incentive to increase domestic currency holdings. However, the model suggests that if underlying macroeconomic factors are not altered, periodic devaluations will need to be implemented in order to maintain the yield of the inflation tax.

The basic model is extended to consider model-consistent devaluation expectations, uncertainty regarding the central bank’s policy response, intensification of foreign exchange controls, and an endogenous credit policy rule. The major conclusions of the analysis are the following. First, in the presence of a quasi-legal parallel market for foreign exchange where transaction costs are low and currency substitution effects play an important role, monetary financing of fiscal deficits may be self-defeating because the rise in the rate of depreciation of the parallel market rate has a negative impact on the yield of the inflation tax. Second, a devaluation --which is here related to the level of the parallel market premium-- may provide a temporary increase in the inflation tax base. Third, foreign exchange restrictions are not a substitute for fundamental reforms in macroeconomic policy to prevent crisis situations.

The concluding section summarizes the empirical regularities that are likely to be observed in periods surrounding devaluation crises, as predicted by the model, and discusses possible extensions of the analysis.
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I. Introduction

Recent developments in the literature on exchange rate and balance of payments crises has examined the consequences of incompatible monetary, fiscal and exchange rate policies for the balance of payments of a small open economy. In Krugman's (1979) early analysis, domestic credit creation in excess of money demand growth has been shown to lead to a gradual loss of reserves, and ultimately to a speculative attack against the currency that forces the abandonment of the fixed exchange rate regime.

Krugman's analysis has been extended in several directions, most notably by Flood and Garber (1984) who incorporate uncertainty about future budgetary policies in a stochastic framework and demonstrate the existence of a forward discount on currencies subject to the threat of a speculative attack. 1/ Although the focus of the literature has been mainly on the transition from a fixed exchange rate to a post-collapse floating exchange rate, several studies have examined the case where the central bank devalues instead of floating the currency. Blanco and Garber (1983) consider the case where recurrent devaluations occur, as a result of the central bank's policy of selling reserves to defend the existing rate until a critical lower bound is reached, at which point a new exchange rate is established. Cumby and Wijnbergen (1989) apply a similar model to a crawling peg regime, and emphasise the uncertainty regarding the lower limit on reserves that triggers the parity change.

The process leading to a balance of payments crisis or an exchange rate collapse in a developing economy has also recently been investigated by Edwards (1989), who characterizes a typical crisis as follows. 2/ In most developing countries, fiscal imbalances are to a large extent financed by money creation. Domestic credit expansion above money demand growth typically translates into an excess demand for goods, traded and nontraded, as well as financial assets. The excess demand for tradables generates a loss of foreign reserves for the central bank, while the excess demand for nontradables translates into higher prices for those goods. As a consequence, the real exchange rate appreciates. If fiscal and monetary policies are not altered, the central bank will eventually "run out" of reserves, and a


2/ See also Edwards and Montiel (1989) who consider, in addition, devaluation crises triggered by an adverse terms of trade shock.
balance of payments crisis will ensue. The econometric evidence produced by Edwards --based on an analysis of 39 devaluation episodes that took place between 1962 and 1982-- supports the view that the probability of a devaluation is strongly affected by the evolution of foreign assets of the central bank, the real exchange rate, and fiscal policy. Essentially, for Edwards, a nominal devaluation is seen as an attempt by the authorities to correct the real exchange overvaluation produced by the expansionary fiscal and credit policies. 1/

This paper develops a stochastic model of recurrent devaluation crises in an economy where foreign exchange controls on both trade and capital transactions lead to the emergence of a parallel market for foreign currency. 2/ The devaluation rule relates the size of the parity change to the level of the spread between the official and parallel exchange rates. Although there are several rationales that can be provided for the mechanism that triggers a devaluation --such as loss of reserves, distortions created by a growing spread, etc-- the paper focuses on the relationship between credit expansion and the yield of the inflation tax under currency substitution. In order to focus the analysis, we consider a rationing scheme which prevents any adjustment in reserves, and which precludes therefore speculative attacks. The major result is that in an economy where a parallel market for foreign exchange plays an important role and where currency substitution is prevalent, a policy of deficit financing through money creation may lead to a "forced" devaluation. Portfolio shifts away from domestic currency may reduce the proceeds from the inflation tax to a level deemed unacceptable by the authorities. In this context,

1/ Calvo’s (1987) optimizing model of collapsing fixed exchange rates also characterizes the process leading to a devaluation crisis by a progressive appreciation of the real exchange rate. Edwards’ work builds on an earlier paper by Rodriguez (1978), who develops a model of a small open economy with no capital mobility, and where inconsistent fiscal policies lead to a real overvaluation of the exchange rate, losses in reserves, and eventually to a devaluation crisis. As long as the devaluation is not accompanied by a reversal of the unsustainable fiscal policies, the nominal devaluation can only generate a temporary improvement in the balance of payments; without fiscal restraint, there will be recurrent balance of payments crises that will lead to a devaluation-inflation spiral. Rodriguez, however, does not explicitly consider the role of devaluation expectations in his analysis.

2/ The literature on balance of payments crises and speculative attacks has not considered the type of foreign exchange restrictions examined here. Wyplosz (1986) and Dellas and Stockman (1988), for example, consider only temporary restrictions on capital flows, while the focus of the present study is on the impact of permanent restrictions --a pervasive feature in many developing countries. As it turns out, the policy implications regarding the effectiveness of controls differ considerably in the present framework.
the devaluation can be considered as a way to reverse the process of erosion of the tax base. The analysis also suggests that intensification of foreign exchange controls in an effort to postpone the exchange rate realignment will only make matters worse.

The plan of the paper is as follows. Section II sets out the analytical framework. Section III determines the probability of a devaluation crisis, and examines the evolution of the official exchange rate before and after the parity change. Extensions of the basic model are discussed in Section IV. Section V presents a summary of the results and identifies the empirical regularities predicted by the model.

II. The Analytical Framework

The basic model presented in this paper draws on the linear stochastic extensions of Krugman's analysis by Blanco and Garber (1983), and Flood and Garber (1984). It differs, however, considerably from the above papers in its assumptions, notably in its explicit introduction of foreign exchange controls and rationing, currency substitution, and a premium-based devaluation rule.

We consider an economy operating under a dual exchange rate system in which an official market for foreign exchange with a pegged nominal exchange rate coexists with a quasi-legal parallel market for foreign currency. Excess demand is then satisfied in the parallel market at the free exchange rate, which is determined by the interactions between supply and demand for foreign exchange. The existence of tariffs, exchange controls, and a positive parallel market premium leads to illegal trade activities. The rationing scheme considered here assumes that all foreign currency receipts by the central bank are allocated administratively (at the official exchange rate) to the private sector. Official reserves (expressed in foreign currency) are thus protected, and remain constant.

Domestic output, which consists of a tradable good, is taken as exogenous. The domestic price of the good is given by the purchasing power parity condition. Only two substitutable financial assets are available, domestic money and foreign money, both being non-interest bearing assets. Agents form their expectations and make their portfolio decisions at the end of each period.

1/ The model can be easily modified to analyse the case of a crawling peg regime.
1. The Devaluation rule

The central bank, having fixed the exchange rate at $\bar{e}$ at time $t$, must decide at the end of period $t+1$, on the basis of some viability criterion, whether or not to adhere to the official parity. We assume in what follows that the authorities will maintain the fixed exchange rate prevailing at $t$ as long as the level of the parallel market premium is not "too high". If, however, the premium rises above a critical upper bound during $t+1$, $\bar{p}$, the official exchange rate will be devalued at the end of that period, and the central bank will establish a new fixed exchange rate, $\bar{e}_{t+1}$, using a time-invariant policy rule.

The rule for setting the post-devaluation exchange rate will in general be a function of the structure of the economy. As the new fixed exchange rate that would be set after a devaluation, $\bar{e}_{t+1}$ must be "viable" in the following sense: it must be such that the condition $\bar{p}_{t+1} \leq \bar{p}$ is satisfied ($\bar{p}_{t+1}$ denoting the post-collapse premium), since otherwise the post-devaluation official rate is not sustainable. A particular rule that fulfills this condition is

$$
\bar{e}_{t+1} = \bar{e} + \left( \frac{1}{1 - \delta} \right) (\bar{p}_{t+1} - \bar{p}). \quad 0 < \delta < 1
$$

Since the coefficient $\delta$ is positive, the devaluation rule described by equation (1) produces a post-devaluation exchange rate greater than the minimum viable rate, that is, the rate required to set $\bar{p}_{t+1} = \bar{p}$. This implies that the post-devaluation premium falls below the critical upper bound. \footnote{Ideally, the devaluation rule should be the outcome of an optimization process, involving a government objective function, a trade-off parameter between different sources of financing of the budget deficit, or different "viable" fixed rates. So far, however, limited experiments have yielded rather disappointing results (see Blanco and Garber, 1983), and we follow here the alternative procedure which consists in specifying what amounts to a "plausible" rule, given the specification of the model described below.}

Several rationales can be provided for a devaluation rule of the type described by equation (1): concerns over the degree of under-invoicing of exports/over-invoicing of imports, the level of the real exchange rate, or the distortions created by a widening differential between the official and parallel exchange rates. An alternative mechanism that may trigger the rule is the inflation tax, as discussed below.

\footnote{The stability properties of a rule similar to (1) is examined by Kharas and Pinto (1989).}
2. Prices and the Money Market

The level of the premium given in the rule (1), is determined by the structure of the economy. Formally, the equations describing the equilibrium of the money market and domestic prices are given by the following log-linear system:

\[
\begin{align*}
    m_t &= p_t + a_0 - a_1(E_t b_{t+1} - b_t) \quad a_1 > 0 \\
    m_t &= \theta(e_t + \bar{R}) + (1 - \theta)d_t \quad 0 < \theta < 1 \\
    d_t &= d_{t-1} + \mu + \epsilon_t \quad \mu > 0 \\
    p_t &= e_t + \sigma(b_t - e_t) \quad 0 \leq \sigma \leq 1
\end{align*}
\]

where \( m_t \) denotes the nominal money stock, \( d_t \) domestic credit, \( p_t \) the domestic price level, \( \bar{R} \) the stock of net foreign assets held by the central bank, \( e_t \) the officially fixed exchange rate, and \( b_t \) the parallel market rate. All variables are measured in logarithms, except \( \epsilon_t \) which denotes a random variable with zero mean, whose distribution is specified below. \( E_t z_{t+1} \) denotes the value of \( z \) agents expect to prevail at the end of period \( t+1 \), given information available at the end of period \( t \).

Equation (2) describes the money market equilibrium relationship in the presence of currency substitution and foreign exchange rationing. The demand for domestic currency is inversely related to the (one-period ahead) expected rate of depreciation of the parallel market exchange rate. \( \epsilon_t \) Equation (3) is based on a log-linear approximation and defines nominal money supply as the sum of net foreign assets (which are constant in foreign currency terms since the authorities allow no changes in reserves) and domestic credit. Equation (4) shows that the rate of growth of domestic credit is explained by deterministic factors (\( \mu \), which reflects the need to finance the fiscal deficit), and by unpredictable elements, captured

\( 1/ \) Portfolio preferences are similarly defined in Kharas and Pinto (1989). We assume throughout that real wealth is constant, and that \( a_0/a_1 \geq E_t b_{t+1} - b_t \).

\( 2/ \) Alternatively, it could be assumed that the authorities allow foreign reserves to rise --or fall-- by a given proportion in each period. As long as this rate of change is predetermined, the results of this paper would not be affected.
by the random variable $\epsilon_t$. Although unknown to agents at the beginning of the period, the credit shock takes a realized value during period $t$.

Equation (5) indicates that the price level depends on the official exchange rate and the level of the premium, defined as the difference between the parallel rate and the official exchange rate. This results from the assumption that trade takes place through both official and illegal channels, with the price of smuggled imports reflecting the marginal cost of foreign exchange -- that is, the parallel market exchange rate. Equation (5) derives from the assumption of purchasing power parity at a composite exchange rate, with the foreign price level set to unity for simplicity. 1/ Since the domestic and foreign good are perfectly substitutable, marginal cost pricing implies that $\sigma$ should be close to 1. 2/

3. The Premium and the Inflation Tax

Before deriving a behavioral equation for the parallel market exchange rate from the above system, the process by which agents assess the viability of the official exchange rate must first be described. If, in agents' minds, the official exchange rate is pegged at a "credible level", its expected rate of depreciation will always be zero. However, agents do not necessarily believe that the official parity will be adhered to by the authorities in all circumstances. Instead, when forming expectations about the next period's official exchange rate, they will usually assess the credibility of the current parity, based on the information available. A convenient and fairly general formulation is to postulate that the degree of credibility of the official exchange rate is inversely related to the current level of the premium. When the premium is perceived as being "too high", the fixed exchange rate will appear impossible to defend, and the credibility of the official parity will be undermined. Formally, it may be assumed that agents attach the probability $q_t$ to the eventuality of a devaluation occurring at the end of the next period, and the probability $1 - q_t$ to adherence to the current parity. If a devaluation occurs, agents expect a depreciation of the official exchange rate by a proportion $\eta$, which is taken as constant for the moment. The rate of depreciation of the official exchange rate expected by the public between the end of period $t$ and $t+1$ is

1/ More precisely, equation (5) is derived from a price equation in which the domestic price level is defined as the product of the foreign price level by a weighted (geometric) average of the official and parallel exchange rates.

2/ This also requires that the risk factor (or the probability of being caught) associated with the illegal nature of smuggling be small.
therefore given by
\[ \eta_{t+1} = E_t e_{t+1} - e_t = t \eta, \quad 0 \leq t \eta \leq 1, \quad \eta > 0. \] (6)

The devaluation probability (or, equivalently, the degree of credibility of the official parity) \( t \eta \) formed by portfolio holders may depend, in general, on a number of factors. The crucial feature of the model is that \( t \eta \) is taken to be an increasing function of the premium, that is, \( \partial_t \eta / \partial \rho_t > 0 \). If agents are fully confident that the current official parity will be adhered to, then \( t \eta = 0 \), which occurs only for \( \rho_t = 0 \). In addition, it is also assumed that \( t \eta_{t+k} = t \eta \forall k = 1, 2, \ldots \infty \) (the devaluation probability is independent of the expectational horizon), and that \( \lim_{t \to \infty} t \eta = 1 \) for \( \rho_t \to \infty \). 1/

From equations (2), (3) and (5),
\[ d_t = c_0 + (1 - \frac{\sigma}{1 - \theta}) e_t + (c_1 + \frac{\sigma}{1 - \theta}) b_t - c_1 E_t b_{t+1}, \]
where \( c_0 = (a_0 - \theta \bar{R}) / (1 - \theta) \), and \( c_1 = a_1 / (1 - \theta) \).

A convenient procedure to find a solution to this equation is the method of undetermined coefficients. Conjecturing the solution form \( b_t = \lambda_0 + \lambda_1 d_t + \lambda_2 e_t \), and substituting this trial solution in (7) yields, using (3) and (6),
\[ b_t = c/s + [(d_t - (1 - s)e_t)/s], \]
where \( c = c_1 [\mu + t \eta (s - 1)] / s - c_0 \), and \( s = \sigma / (1 - \theta) > 1 \), since by assumption \( \sigma \) is close to 1 and \( \theta < 1 \). Defining the parallel market premium \( \rho_t \) as the difference between the parallel and official exchange rate yields
\[ \rho_t = c/s + (d_t - e_t)/s, \]
(8b)

1/ The assumption that the devaluation probability is independent of the expectational horizon is made simply for convenience. The implications of the model are unaltered if it is assumed that the degree of credibility of the official exchange rate falls with the passage of time -- an assumption that would indeed be consistent with the above framework.
Equations (8a) and (8b) describe the evolution of the parallel exchange rate and the premium when agents attach a probability $q_t$ to a devaluation of the official exchange rate occurring next period. The higher this probability is, the higher will be $b_t$ and $p_t$. For given expectations, a rise in the stock of domestic currency (through credit expansion) is --for a given official exchange rate-- associated with a rise in the free exchange rate which reduces the expected rate of depreciation of the unofficial price of foreign exchange, and helps restore portfolio equilibrium. From equation (8a),

$$E_t b_{t+1} - b_t = [\mu + q_t \eta(s - 1)]/s, \quad (9)$$

which implies that an increase in the rate of expansion of domestic credit raises, by the proportion $\Gamma/s = [1 + q_t(s - 1)(\partial q_t/\partial \mu)]/s$, the expected rate of depreciation of the parallel exchange rate. As a consequence, the demand for domestic currency falls. Credit growth and the rise in the parallel rate raise anticipations of a future increase in the free exchange rate, reduce the credibility of the current official exchange rate, and thereby cause a reduction in domestic currency holdings.

The above description of the relationship between an expansion in domestic credit and the parallel market premium has an important bearing on what happens to government revenues from the issuance of money, namely the inflation tax. Assume for the moment that $e_t = 0$ for all $t$. The yield of the inflation tax --assuming no devaluation takes place-- is given by the product of the rate of domestic money growth, $(1 - \delta)\mu$, and the real money stock. Since, as shown above, an increase in the rate of expansion of domestic credit is associated with a fall in the real stock of domestic currency held in private agents' portfolios, (the logarithm of) the proceeds from the inflation tax will fall or rise according to whether the derivative of $\log[(1 - \delta)\mu] + m_t - p_t = \log[(1 - \delta)\mu] + a_0 - a_1[\mu + q_t \eta(s - 1)]/s$, with respect to $\mu$ is positive or negative. For $a_1 > s/\mu \Gamma$, that is, for a high enough degree of currency substitution, inflation tax revenue may actually fall as a result of an expansionary credit policy. The higher the value of $\mu$ is, the more likely it is that the above condition will be satisfied.

The general implication of this result is that the higher the rate of expansion of the nominal stock of domestic credit, the higher will be the expected rate of depreciation of the parallel exchange rate, and, consequently, the lower the proportion of domestic currency.

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1/ From equation (8b) and the definition of $q_t$, $\partial q_t/\partial \mu = [(\partial q_t/\partial \rho_t)(\partial \rho_t/\partial \mu)] > 0$, so that $\Gamma > 0$. 

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held in agents' portfolios. As the public shifts away from domestic to foreign currency, the relative importance of the monetary base, and hence the financing that can be achieved through money creation, is reduced for any given rate of growth of domestic credit. If the degree of currency substitution is "high" enough, the gain in revenue from accelerating credit growth is more than offset by the erosion of the tax base that results from the decline in holdings of real balances, due to the now higher expected rate of depreciation of the parallel market exchange rate. As a consequence, the flow of seignorage from the inflation tax falls. 1/

What can the government do to prevent the erosion of its inflation tax base? Obviously, the "first best" strategy is to reduce the rate of growth of domestic credit by a sufficient amount to ensure that the condition \( \mu < s/a_1 \) is satisfied. This may not be, however, a feasible policy option in the presence of fiscal rigidities. The key assumption made in this paper is that when, as a result of a sustained expansionary credit policy, revenues from the inflation tax have fallen to a critical level, the authorities may devalue the official exchange rate --using the rule given by equation (1)-- in an attempt to reverse the process. From equations (8a), (8b) and (9), a once-and-for-all devaluation implemented at the end of period \( t \) implies a depreciation of the parallel exchange rate (in the proportion \( [s - 1]/s \)), a fall in the level of the premium (in the proportion \( 1/s \)) and in the probability that a devaluation will occur in the following period, and therefore a fall in the anticipated rate of return on foreign currency holdings. The devaluation "works" by (partly) restoring the credibility of the fixed exchange rate regime, and this reduces the expected rate of return on foreign money for the next period, since expectations are formed at the end of period \( t \). This provides an incentive to switch back into domestic currency assets. Domestic money balances will increase (by the proportion \( a_1 \eta(s - 1)[\delta_{f,q}/\partial p_{t+1}] \)), and the yield of the inflation tax will rise.

However, the fall in the expected rate of depreciation of the parallel exchange rate will only be temporary since, in the next period, the level of the premium will resume its upward course, if monetary and fiscal policies are unaltered, implying that the devaluation probability formed by asset holders will rise again. A once-and-for-all devaluation, therefore, can only provide a temporary increase in the tax base, a proposition whose implications are further discussed below.

1/ The loss of seignorage associated with a switch from domestic to foreign currency was stressed by Fischer (1982). This result can also readily be interpreted in terms of the conventional analysis of the inflation tax, according to which there exists a revenue-maximizing equilibrium rate of monetary expansion (and corresponding rate of inflation), and attempts to inflate above this rate would result in a decline in total revenues.
In a country where domestic residents can hold foreign money balances, currency substitution can become an effective way of avoiding the inflation tax on the holdings of domestic cash balances. If, at the same time, government deficits are high, the notion that the authorities may have to protect --or recapture, through a devaluation-- its inflation tax base has some credence.  

An alternative rationale for government intervention, which can be referred to as a "sovereignty" argument, is to assume that the government will not permit the complete collapse of its currency. This implies that although there exists a rate of domestic credit expansion --and, therefore, an expected rate of depreciation-- that is high enough to drive agents to liquidate the whole stock of domestic money in their portfolios, in practice agents will not be able to substitute away completely from the domestic currency. Through institutional constraints (legal requirements that domestic money be used for specific transactions, for example), the authorities can prevent the national currency to be driven out of existence. In this context, a devaluation may be viewed as an alternative means to prevent a complete "dollarization" of the economy. Although this argument is also quite appealing, in what follows we will stress the inflation tax mechanism as the major factor underlying the decision to devalue.

In light of the foregoing discussion, it will be assumed that the official exchange rate is maintained only if the proceeds from the inflation tax do not fall as a result of an increase in the rate of growth of domestic credit. Generalizing the condition given above, the actual yield of the inflation tax (which is known only after the realization of the domestic credit shock) rises following a credit expansion only if \( \mu + \epsilon_{t+1} < s/a_1 \Gamma \). Since, from equation (8b), the actual rate of change of the premium between \( t \) and \( t+1 \) --assuming no devaluation takes place in the interval, so that \( e_{t+1} = e_t = \bar{e} \)-- is equal to \((\mu + \epsilon_{t+1})/s\), the above condition is equivalent to

\[ \rho_{t+1} - \rho_t < 1/a_1 \Gamma. \]

Concerns over the yield of the inflation tax imply, therefore, that the authorities will maintain the fixed exchange rate prevailing at \( t \) as long as the actual rate of increase of the parallel market premium is not "too large". If, however, the rate of change of the

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1/ See for instance Calvo's (1986) discussion of the experience of Argentina in the late 1970s. For most developing countries, the inflation tax is an administratively attractive method of collecting revenue as well as a relatively efficient source, because of the higher collection costs of alternatives.

2/ More generally, it could be assumed that the authorities allow a fall in revenue from the inflation tax by a given amount. This would have no effect on the results and would simply change the definition of \( \rho \).
premium rises above a critical upper bound during \( t+1 \) (implying an unacceptable fall in the yield of the inflation tax), the official exchange rate will be devalued at the end of that period, and the central bank will establish a new fixed exchange rate. A devaluation implemented at the end of \( t+1 \) will lower the level of the premium, and (since expectations are formed at the end of every period) will reduce the probability of a further devaluation occurring next period as well as the expected rate of depreciation of the parallel exchange rate. This will imply an instantaneous shift away from foreign currency holdings at the end of \( t+1 \), as discussed previously.

Since the value of the premium at \( t \) is common knowledge at the end of period \( t \), a viability criterion which states that the rate of increase of \( \rho \) between \( t \) and \( t+1 \) should be lower than a given value is equivalent to the requirement that the level of the premium during \( t+1 \) remains below a given upper bound, \( \bar{\rho} \). \(^1\) However, although agents may know \( \bar{\rho} \) with certainty (or, more generally, they may form a prior probability distribution over the possible values that \( \bar{\rho} \) may take on), the exact timing of the collapse of the official parity cannot be predicted at \( t \), because of the stochastic nature of the credit shock.

The devaluation rule (1) ensures that the post-collapse premium falls below the critical upper bound, implying a fall in the devaluation probability formed by asset holders and in the expected rate of depreciation of the parallel rate for \( t+2 \), a rise in domestic currency holdings, and an increase in revenue from the inflation tax.

III. The Process of a Devaluation Crisis

This section provides a formal derivation of the devaluation probability and examines the behavior of the official exchange rate under the possibility of a regime collapse.

1. The Devaluation Probability

As argued in the previous section, if the officially fixed exchange rate proves to be unsustainable due to the rise in the premium, the authorities will devalue the domestic currency in an attempt to maintain the yield of the inflation tax. Let \( \pi_{t+1} \) denote the probability, as of the end of period \( t \), that the official exchange

\(^1\) From the above discussion, \( \rho = \rho_t + (1/a_1\pi_t) \). However, if the minimum yield from the inflation tax deemed acceptable by the authorities is unknown to private agents, the definition of \( \bar{\rho} \) will be more complicated. The case of a stochastic upper bound is analyzed below.
rate will be devalued at the end of $t+1$. The probability that the current exchange rate will be maintained is therefore given by $(1 - \pi_{t+1})$. The unconditional expected official exchange rate at $t+1$ can therefore be expressed as

$$E_t e_{t+1} = \pi_{t+1} E_t (\rho_{t+1} > \bar{\rho}) + (1 - \pi_{t+1}) \bar{e},$$

(10)

where $E_t (\rho_{t+1} > \bar{\rho})$ denotes the exchange rate that would prevail if the authorities devalue at the end of period $t+1$.

The probability as of the end of period $t$ that a devaluation will occur at the end of period $t+1$ is simply the probability that, under the current parity, the parallel market premium rises above $\bar{\rho}$ during $t+1$. From equation (8b), this probability depends in turn on the probability that a sufficiently large realization for domestic credit occurs. Formally,

$$\pi_{t+1} = \text{Prob}[(c + d_{t+1})/s - \bar{\rho} > 0],$$

(11)

or, using (4),

$$\pi_{t+1} = \text{Prob}[(c + \mu + d_{t+1} + \epsilon_{t+1})/s - \bar{\rho} > 0],$$

(11')

where now $c = c_1[\mu + \eta(s - 1)]/s - c_0 - \bar{e}$. The credit shock $\epsilon_t$ is assumed to obey

$$\epsilon_t = -1/\lambda + \nu_t,$$

(12)

where $1/\lambda^2$ denotes the variance of $\epsilon_t$, and $\nu_t$ a random variable distributed exponentially with an unconditional probability density function: $2/\$

---

$1/$ While $q_t$ refers to the devaluation probability formed by asset holders, $\pi_{t+1}$ denotes the "aggregate" probability of a parity change, once the behavior of the authorities is taken into account.

$2/$ The implication of an exponential distribution in the present context is that it is always possible that a particularly sharp increment to credit causes a rise in the premium so large that a devaluation occurs immediately. This can happen even if the current level of the premium is much below $\bar{\rho}$. There is, therefore, always a non-zero probability that a devaluation could occur. The case where the rate of growth of domestic credit follows a uniform distribution is briefly discussed in Appendix II.
To ensure that the rate of growth of domestic credit remains positive, we will assume that \( \mu > 1/\lambda \). Using (12) and (13), we can write (11'), using the variable \( k_t \) for notational ease, as

\[
\pi_{t+1} = 1 - F(k_t) = \text{Prob} \left[ v_{t+1} > k_t \right],
\]

where \( k_t = s - (c + \mu) + 1/\lambda - d_t \), and \( F(.) \) is the cumulative distribution function associated with \( f_v(v_t) \). Using the density function (13), the cumulative probability of a devaluation is therefore

\[
\pi_{t+1} = \int_{k_t}^{\infty} \lambda \exp(-\lambda v_{t+1}) dv_{t+1}, \quad k_t \geq 0.
\]

Integrating (15) yields:

\[
\pi_{t+1} = \begin{cases} 
\exp(-\lambda k_t), & k_t \geq 0, \\
1, & k_t < 0.
\end{cases}
\]

Equation (16) shows that the higher the upper limit on the premium \( \bar{\rho} \) is, or the higher \( \sigma \) (since \( ds/d\sigma > 0 \)), the lower the probability of a devaluation. A rise in \( \sigma \), which measures the sensitivity of the price level to changes in the premium, reduces the impact of the expected rate of depreciation of the parallel market rate, dampens the shift away from domestic currency, and reduces therefore the effect of a random increase in domestic credit growth on the premium. By contrast, the higher the rate of expansion of domestic credit, the higher the probability of a collapse of the fixed rate. The collapse probability depends also on the degree of credibility asset holders attach to the official exchange rate. Specifically, it is easy to show from (16) that \( \partial \pi_{t+1}/\partial q > 0 \): the lower the degree of credibility of the official parity (the higher \( q \)), the higher will be the economy-wide probability of a devaluation occurring at the end of period \( t+1 \). 1/ A devaluation occurs with certainty if \( k_t < 0 \). 2/

1/ Expectations of private agents have therefore a "self-fulfilling" character in this model. But note that \( q = 1 \) is neither necessary, nor sufficient to precipitate a collapse.

2/ If the process driving domestic credit were completely deterministic, the solution of \( k_t = 0 \) would define the exact level of
\( k_t \geq 0 \), the probability of a collapse can be completely eliminated only when \( \lambda \rightarrow +\infty \). Since \( 1/\lambda^2 \) is the variance of the exponentially distributed random domestic growth term, this condition implies that agents must know with certainty the credit rule.

The solution procedure leading to equation (16) can be used to calculate not only the probability of a parity change occurring at the end of period \( t+1 \) but also, more generally, the probability at \( t \) that the fixed exchange rate regime collapses in period \( t+n \) (neither earlier nor later) and the probability, at \( t \), that it collapses no later than period \( t+n \). It can be shown that the devaluation probability increases with the length of the interval \( (t,t+n) \), and that whatever the length of this interval, a rise in the deterministic component of the rate of growth of domestic credit always increases the probability of an exchange rate crisis. The proofs of these propositions are contained in Appendix I.

Finally, an important aspect of the solution (16) is that the devaluation probability is independent of the devaluation rule. As shown below, this depends critically on the assumption that asset holders do not know the policy reaction function of the authorities and/or do not use that information in forming their expectations about the future official exchange rate.

2. The Conditional Exchange Rate

Consider now the determination of the conditional exchange rate expected to prevail at the end of period \( t+1 \). From the above discussion, \( E_t(\bar{e}_{t+1} | \bar{\rho}_{t+1} = \bar{\rho}) = E_t(\bar{e}_{t+1} | v_{t+1} > k_t) \). Using the devaluation rule (1) and equations (4) and (8b), the conditional expectation can be expressed as

\[
E_t(\bar{e}_{t+1} | v_{t+1} > k_t) = \frac{(e - \bar{\rho})}{1 - \delta} + \frac{1}{s(1 - \delta)} \left[ (c + \mu) + d_t + E(v_{t+1} | v_{t+1} > k_t) - 1/\lambda \right].
\]

To find \( E(v_{t+1} | v_{t+1} > k_t) \), we form the conditional probability density function over \( v_{t+1} \), where the conditioning information is \( v_{t+1} > k_t \). As shown by Flood and Garber (1984, pp. 12-13), this conditional density function is:

\( \ldots \) (continued from page 13) domestic credit above which the collapse is certain to happen.
\[ g(v_{t+1}) = \begin{cases} \lambda \exp[\lambda(k_t - v_{t+1})], & k_t \geq 0, \\ \lambda \exp(-\lambda v_{t+1}), & k_t < 0. \end{cases} \]  

Hence, for \( k_t \geq 0 \),

\[ E(v_{t+1}|v_{t+1} > k_t) = \int_{k_t}^{\infty} \lambda v_{t+1} \exp[\lambda(k_t - v_{t+1})]dv_{t+1} = k_t + 1/\lambda. \]  

Substituting this result in (17) yields

\[ E_t(\bar{e}_{t+1}|v_{t+1} > k_t) = (\bar{e} - \frac{\bar{\rho}}{1 - \delta}) + \frac{1}{s(1 - \delta)}[(c + \mu) + (d + k_t)], \]  

or, using the definition of \( k_t \),

\[ E_t(\bar{e}_{t+1}|v_{t+1} > k_t) = \bar{e} + \frac{1}{\lambda s(1 - \delta)}. \]  

Equation (21) shows that the higher \( \delta \), the higher will be the post-devaluation exchange rate, and the lower the post-devaluation premium. Also, the higher the elasticity of domestic prices with respect to the premium, the lower will be the post-collapse exchange rate. However, the post-devaluation exchange rate does not depend on the degree of credibility attached by asset holders to the existing official parity.

Finally, substituting (21) in (10) and using (16) yields the unconditional forecast of the exchange rate for \( t+1 \):

\[ E_t e_{t+1} = \bar{e} + \exp(-\lambda k_t)\left[\frac{1}{\lambda s(1 - \delta)} \right]. \quad k_t \geq 0 \]  

The unconditional expected exchange rate will depend on all factors affecting the devaluation probability and the post-collapse, conditional exchange rate.

If a collapse indeed occurs, the premium will fall, generating a temporary increase in domestic currency holdings. If credit policy remains expansionary, however, the premium will resume its upward movement, eventually leading to another "forced" devaluation. The initial drop and subsequent rise in the premium following a devaluation is consistent with the empirical evidence provided by

\[ \frac{\bar{\rho}_{t+1}}{s}, \text{ the value of the premium immediately after the devaluation, } \rho_{t+1} \text{, will be equal to } \rho_{t+1} - (\bar{e}_{t+1} - \bar{e})/s. \]

IV. Extensions of the Basic Model

We now consider several modifications of the above framework and examine how the process of a devaluation crisis is altered. To avoid unnecessary complications, these extensions are introduced in the basic model one at a time, although several of them could be dealt with simultaneously.

1. Consistent Devaluation Expectations

It has been assumed in the previous discussion that agents, uncertain about whether the fixed exchange rate will be adhered to, learn over time about the viability of the official parity by observing the behavior of the authorities and the parallel market premium, and infer whether domestic credit and fiscal policies are consistent with the current exchange rate. At the end of each period, agents make their portfolio decisions based on their assessment of the probability that the official exchange rate will be devalued at the end of the following period. Their actions, in turn, affect the economy-wide probability of a devaluation.

Consider now the case where expectations about the future rate of devaluation of the official exchange rate are not exogenously given, but depend on agents' expectations about the monetary authorities' policy reaction function. That is, agents know the exchange rate rule (9) and use this information to form consistent predictions. The expected rate of depreciation of the official exchange rate is now given by

\[ \eta_{t+1} = (1 - \tau q)\bar{e} + \tau q \left[ \bar{e} + \frac{1}{1 - \delta} (E_{t} \rho_{t+1} - \bar{\rho}) \right] - \bar{e}, \]

\[ = \tau (E_{t} \rho_{t+1} - \bar{\rho}), \quad \tau = \frac{q}{1 - \delta} \]

where \( 0 < \tau q < 1 \), measures the degree of credibility agents attach to the official parity. \(^{1/}\) By the definition of \( \rho \),

\[ E_{t} b_{t+1} - b_{t} = E_{t} \rho_{t+1} - \rho_{t} + \eta_{t+1}. \]

\(^{1/}\) As before, \( \tau q \) is assumed to be positively related to the level of the premium.
Combining this expression with (23) yields

$$
\eta_{t+1} = \tilde{\tau}[(E_t b_{t+1} - b_t) + (\rho_t - \bar{\rho})], \quad \tilde{\tau} = \tau/(1 + \tau) \quad (23')
$$

Equation (23') shows that, with endogenous expectations, a rise in the level of the premium affects the rate of devaluation expected by asset holders through two channels: the effect of $\rho_t$ on $\tilde{\tau}q$ (and therefore $\tilde{\tau}$) as before, and the effect of $\rho_t$ on the rate of devaluation agents expect the authorities to implement, should they decide to devalue. A rise in the expected rate of depreciation of the parallel exchange rate raises also the expected rate of depreciation of the official rate, but less than proportionally ($\tilde{\tau} < 1$).

Using (2)-(5) and (23'), and positing as before the solution form $b_t = \lambda_0 + \lambda_1 d_t + \lambda_2 e_t$, it can be shown that the coefficients $\lambda_k$ are solution of the following system:

$$
\lambda_0 = [c_1(\lambda_1 \mu - \bar{\rho} \lambda_2 \tilde{\tau}) - c_0(1 - \lambda_2 \tilde{\tau})]/[s - \lambda_2(s + c_1)\tilde{\tau}], \quad (24a)
$$

$$
\lambda_1 = (1 - \lambda_2 \tilde{\tau})/[s - \lambda_2(s + c_1)\tilde{\tau}], \quad (24b)
$$

$$
\lambda_2 = [(s - 1) - \lambda_2 \tilde{\tau}(s + c_1 - 1)]/[s - \lambda_2(s + c_1)\tilde{\tau}]. \quad (24c)
$$

In the particular case of "full" credibility by asset holders ($\tilde{\tau}q = 0$), equations (8a) and (24a)-(24c) yield the same solution for the parallel market exchange rate. In the general case, however, (24a)-(24c) form a non-linear system in the coefficients $\lambda_k$, with $\lambda_0$ and $\lambda_1$ solving residually once $\lambda_2$ is determined from equation (24c). The quadratic equation (24c), given that its discriminant $\Delta$ is positive, has two real and positive roots (by Descartes' rule of signs), given by

$$
\lambda_2 = [(\tilde{\tau}(s + c_1 - 1) + s) \pm \Delta^{1/2}]/2\tilde{\tau}(s + c_1).
$$

However, it can easily be shown that both roots cannot be lower than unity. Consistency between the properties of the model and "stylized facts" requires selection of the only root that is lower than one, denoted $\lambda_2$. 1/ Simple manipulations yield now

1/ Otherwise, the parallel market premium does not fall following a devaluation.
implying that as before, a devaluation, by reducing the level of the premium and lowering expectations of a future devaluation, reduces the expected rate of return on foreign currency holdings.

Using equations (24a)-(24c), the devaluation probability is now given by

\[ \pi_{t+1} = \exp(-\lambda k_t), \quad k_t \geq 0 \]

where \( k_t = (\rho - \lambda_0 - \lambda_2 \delta)/\lambda_1 - (\mu + \delta_{t2}) \), and \( \lambda_0 \) and \( \lambda_1 \) denote the values of \( \lambda_0 \) and \( \lambda_1 \) associated with \( \lambda_2 \). As before, the collapse probability depends on the degree of confidence agents attach to the official exchange rate. It is tedious but straightforward to show from (25) that \( \frac{\partial \pi_{t+1}/\partial \rho > 0}{} \): the lower the degree of credibility of the official parity, the higher will be the probability of a devaluation at the end of period \( t+1 \). In addition, the devaluation probability depends also on the parameter characterizing the devaluation rule, the coefficient \( \delta \). The higher the rate of devaluation in the policy rule, (the higher is \( \delta \)), the higher the likelihood that a parity change will occur at the end of \( t+1 \), because of its induced effect on expectations formed by asset holders.

2. Uncertain Central Bank Policy Response

Up to this point it has been assumed that a devaluation is known to be imposed with certainty when the premium becomes greater than \( \rho \). The analysis, however, can be extended to deal with situations in which the policymaker's response is uncertain. 1/ Two cases are considered here. First, uncertainty relative to the value of the premium that triggers the devaluation; and second, uncertainty regarding the parity change itself, as well as its size.

A fairly general way to introduce uncertainty regarding the critical upper bound is to consider the case where the central bank randomly changes the value of the premium that will trigger the

1/ In the literature on balance of payments crises, several authors have examined the impact of uncertain central bank behavior. Willman (1989) for instance considers the case where agents do not know the threshold level of reserves, which is taken alternatively as stochastic or fixed but unknown to private agents. A similar case is analyzed by Cumby and Wijnbergen (1989).
devaluation. This has important implications concerning the timing of the parity change. For a given growth rate of domestic credit, even though agents may know the general form of the authorities' decision rule, they cannot calculate with certainty the critical level of the premium that will result in a discrete parity change. Formally, the probability of a devaluation becomes

\[ \pi_{t+1} = \text{Prob}(\rho_{t+1} > \bar{\rho} + \zeta_{t+1}) \tag{26} \]

where \( \zeta_{t+1} = -1/\nu + \omega_{t+1} \) denotes a random variable distributed with zero mean and variance \( 1/\nu^2 \), and \( \omega_t \) is distributed exponentially with mean \( 1/\varphi \) and density \( 1/\varphi \).

Using (4), (8b) and (27), equation (26) can be written as

\[ \pi_{t+1} = \text{Prob}[v_{t+1} - s\omega_{t+1} > k_t], \quad k_t \geq 0 \tag{28} \]

where now \( k_t = s\bar{\rho} - (c + \mu) + 1/\lambda - d_t - s/\varphi \). To evaluate the cumulative probability of a collapse requires now to set up a convolution integral. Let \( \chi_{t+1} = s\omega_{t+1} \), so that \( f_{\omega}(\chi_{t+1}) = (1/s)f_{\omega}(\chi_{t+1}/s) \), and define \( z_{t+1} = v_{t+1} - \chi_{t+1} \). If \( v_{t+1} \) and \( \omega_{t+1} \) are independent random processes, the probability density function of \( z_{t+1} \) is (see for example Kendall and Stuart, 1977, p. 280):

\[
f_z(z_{t+1}) = \int_0^\infty (1/s)f_v(z_{t+1} + \chi_{t+1})f_{\omega}(\chi_{t+1}/s)d\chi_{t+1} \]

\[ = \int_0^\infty (\lambda\varphi/s)\exp(-\lambda z_{t+1})\exp[-(\lambda + \varphi/s)\chi_{t+1}]d\chi_{t+1}, \]

which becomes, after integrating,

\[ f_z(z_{t+1}) = [\lambda\varphi/(\varphi + \lambda s)]\exp(-\lambda z_{t+1}), \quad \text{for } 0 \leq z_{t+1} \leq \infty. \tag{29} \]

---

1/ It is also assumed that \( \rho > -1/\nu \). This specification ensures that the upper critical bound, although stochastic, is always positive.
The collapse probability (28) can therefore be written as

\[ t^\pi_{t+1} = \int_{k_t}^\infty f_z(z_{t+1}) dz_{t+1}. \] (30)

Integration yields

\[ t^\pi_{t+1} = \begin{cases} \frac{\varphi}{(\varphi + \lambda s)} \exp(-\lambda k_t), & k_t \geq 0, \\ 1, & k_t < 0. \end{cases} \] (31)

Equation (31) shows that the higher the degree of uncertainty on the level of the premium that triggers the devaluation rule, that is, the lower \( \varphi \) is, the higher the probability of the devaluation \( (\partial_t^\pi_{t+1}/\partial \varphi < 0) \).

As an alternative way to introduce uncertainty over policy response in the model, consider now the case where the upper bound \( \bar{\rho} \) is perfectly known to the public. Suppose further that the public is uncertain about whether, if the viability condition is violated (that is, \( \rho_{t+1} \geq \bar{\rho} \)), the central bank will effectively devalue or not at the end of period \( t+1 \). Formally, agents consider that a devaluation will be imposed with probability \( \alpha \). Then the exchange rate the public expects to prevail at the end of \( t+1 \) is

\[ E_t e_{t+1} = t^\pi_{t+1} \mathbb{E}_t(e_{t+1}|\rho_{t+1} > \bar{\rho}) + (1 - \alpha) \bar{e} + (1 - t^\pi_{t+1}) \bar{e}, \]

or, using (21),

\[ = t^\pi_{t+1} \alpha \mathbb{E}_t(e_{t+1}|\nu_{t+1} > k_t) + \bar{e}(1 - t^\pi_{t+1}) \alpha, \]

The probability of not devaluing is thus \((1 - \alpha)\). If the policymaker could credibly choose the probability of devaluing the currency (which depends on things like the publicly perceived importance it attaches to the current parity), the optimal choice would involve setting \( \alpha = 0 \). If, however, announcements of future policies are not fully credible, and no precommitment mechanism is available, the public will assess the value of \( \alpha \) by evaluating the policymaker's objective function under the two policy options. For simplicity, we take \( \alpha \) as exogenous here.

An alternative model examining the credibility of a devaluation threat is discussed by Calvo (1989).
The unconditional exchange rate will now depend not only on the distribution of the domestic credit shock, the level of domestic credit and the official exchange rate, but also on the probability that the public attaches to the authorities' intentions to devalue --if they have to. The higher $\alpha$ is --that is, the higher the probability that a devaluation will effectively take place if the premium overshoots its viable level-- the higher will be the expected post-collapse exchange rate.

Finally, one can consider uncertainty about the size of the devaluation --that is, uncertainty about $\delta$. Without recourse to formal derivations, it is intuitively clear that under the assumptions of the basic model, stochastic movements in $\delta$ have no effect on the devaluation probability if the rate of depreciation of the official parity asset holders expect to prevail in the event of a devaluation is exogenous --although this does not hold if devaluation expectations are consistent with the devaluation rule-- and will also have no effect on the conditional exchange rate if they are independent of credit shocks.

3. Controls and the Postponement of the Crisis

Consider now the case where the authorities try to postpone the parity change and intensify foreign exchange controls when the premium reaches the level $\rho_{\text{min}}$. 1/ For convenience, let us assume that the central bank will maintain the fixed exchange rate as long as the premium remains in the interval $(\rho_{\text{min}}, \rho_{\text{max}})$, where $\rho_{\text{max}} > \rho_{\text{min}}$, and will devalue (with probability one) only when $\rho_{t+1} > \rho_{\text{max}}$, using rule (1).

Although exchange restrictions have not been explicitly modeled here, a heuristic discussion of the issue may be sufficient to describe the process at work. An intensification of controls --measures, for instance, that tend to make it more costly to substitute foreign for domestic money in the official market-- is likely to increase excess demand for foreign currency at the official exchange rate, with a consequent spill-over effect on the parallel market. The result will be a depreciation of the free exchange rate, an increase

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1/ Evidence that governments in developing countries have in several cases resorted to exchange and capital controls (limitations on conversion of domestic currency into foreign currency, taxes on foreign currency holdings, etc.) in their efforts to postpone macroeconomic and exchange rate adjustment is provided by Edwards (1989), and Edwards and Montiel (1989).
in the premium, a rise in the devaluation probability formed by asset holders, and a rise in the expected rate of return on foreign currency holdings which will hasten --assuming credit policy remains unchanged--the attainment of the upper limit $\rho_{max}$. Therefore, in this model, a reinforcement of foreign exchange controls in an effort to postpone exchange rate or fiscal adjustment only makes matters worse, by increasing the likelihood that a "forced" devaluation will eventually take place. The "credibility effect" created by the rise in the level of the premium defeats the very purpose of the controls.

4. **Endogenous Credit Policy Switches**

As is typical in models of balance of payments crises where credit policy is exogenous, the basic framework of this paper shows that the devaluation crisis is the outcome of excessive monetary expansion. The analysis implies that a fixed exchange rate regime cannot be viable indefinitely --although the transition to a new official exchange rate is diffuse, due to the stochastic nature of the credit growth process. A further implication of the model is that as long as credit policy follows an expansionary path, and as long as the authorities maintain their objective of not allowing the premium to rise above a critical bound (that is, as long as the expected yield of the inflation tax is not allowed to fall below a critical level), the economy will undergo a succession of devaluation crises. Typically, the premium will fall following the devaluation --implying a temporary switch away from foreign currency holdings-- and then will resume its upward movement, until a new devaluation takes place.

---

1/ A simple way to formalize this argument is to note that controls may lead to an exogenous, downward shift in $a_0$, since agents may reduce their domestic currency holdings now in anticipation of higher future parallel market premia. Equation (14) therefore implies that $\frac{\partial \pi_{t+1}}{\partial a_0} < 0$, which is the result stated in the text.

2/ "Irrational" beliefs have been excluded from the solution of the premium in equation (6). Actually, if we were to consider a solution that is not bubble-free, even if the authorities hold the nominal stock of domestic money constant (that is, with no changes in "fundamentals"), the belief that the value of the domestic currency will depreciate in the future can lead agents to substitute foreign currency for domestic money in their portfolios, initiating a self-fulfilling devaluation crisis. Note that a similar result obtains by considering exogenous shifts in $q$.

3/ In Ize and Ortiz (1987), fiscal rigidities are shown to be associated not only with exchange rate crises, but also with capital flight.
The apparently ineluctable character of a devaluation crisis, as well as its recurrent nature, runs into the same kind of conceptual difficulties encountered in models of collapsing fixed exchange rate regimes in which the "excessive" rate of domestic credit creation is the result of an exogenously determined fiscal deficit; namely, why is it that the authorities do not attempt to prevent the crisis by adjusting their fiscal and credit policies? Indeed, the probability of collapse derived earlier is non-zero simply because it has been assumed that the authorities maintain some flexibility with respect to domestic credit creation. Fiscal discipline, that is, a reduction in unpredictable changes in credit behavior, can significantly reduce the devaluation probability.

Moreover, there is nothing in the model "forcing" the central bank to devalue at the moment the premium reaches its critical upper bound; instead, the authorities could as well change their monetary policy rule to ensure viability of the official parity. Some recent models of balance of payments crises have considered this type of endogenous changes in monetary policy. For Drazen and Helpman (1988) for instance, the assumption that the authorities initially choose the exchange rate as a stabilization instrument, instead of altering underlying macroeconomic policy that is inconsistent with the existing exchange parity, may serve as a first step in a two-stage process. But ultimately, if the new exchange rate regime is inconsistent with the underlying policy process, there will be a need for a new policy regime. A similar argument can be made in the case considered here.

One simple way to introduce endogenous credit policy adjustment is to consider, instead of (4), a credit rule with feedback:

\[ d_{t+1} = (1 - \alpha) \epsilon_{t+1} - \alpha \mu + \epsilon_{t+1} \quad 0 \leq \alpha \leq 1 \]

Note that in the above model, setting \( \mu = 0 \) is not a sufficient condition for eliminating the probability that a collapse will occur; A low value of \( \mu \) will slow the increase in the premium, but does not offset hikes attributable to random credit shocks.

Obstfeld (1986b) has shown that the belief that the authorities will adopt an expansionary credit policy rule following the occurrence of a regime collapse could fuel crisis expectations and generate a self-fulfilling speculative attack.

An alternative possibility would be to assume that the central bank is expected to devalue with probability \( \alpha \), and to change its monetary policy rule, that is, to set \( \mu = 0 \), with probability \( 1 - \alpha \) while retaining the official parity \( e \). This would have no effect on the conditional exchange rate but would affect the aggregate devaluation probability. Formally, instead of (4), the credit rule would be written as

\[ d_{t+1} = \alpha (\mu + \epsilon_{t+1}) + (1 - \alpha) \epsilon_{t+1} - \alpha \mu + \epsilon_{t+1} \quad 0 \leq \alpha \leq 1 \]
according to which the authorities vary the average rate of growth of
domestic credit in \( t+1 \) inversely with the level of the premium at \( t \).
The solution procedure is now slightly more involved. Positing as
before a solution of the form and using (2)-(5)
and (33) to solve with respect to the premium, it is easily shown that
the coefficients \( \lambda_k \) are solutions of the following system:

\[
\begin{align}
\lambda_1^2 & = \frac{1}{c_1 \gamma^2} - 1, \\
\lambda_2 & = - \frac{1}{\sigma + c_1 \gamma}, \\
\lambda_0 & = \frac{\sigma + c_1 \gamma \lambda_2 - c_0}{\sigma + c_1 \gamma}.
\end{align}
\]  

(34a)  
(34b)  
(34c)

Equations (34b) and (34c) solve for \( \lambda_2 \) and \( \lambda_0 \) residually once \( \lambda_1 \)
is determined from (34a). Because equation (34a) is quadratic, and
since its discriminant is positive, there are two real and distinct
possible values for \( \lambda_1 \), given by

\[
\lambda_1 = \frac{-\sigma \pm (\sigma^2 + 4c_1 \gamma)^{1/2}}{2c_1 \gamma}.
\]  

(35)

By Descartes' rule of signs, one root is positive and the other
negative. A simple way to choose among the two roots is to consider
the case where \( \gamma \to 0 \) -- the "no-feedback" rule. In this case,
\( 2c_1 \gamma \lambda_1 \to 0 \), and \( (\sigma^2 + 4c_1 \gamma)^{1/2} \to \sigma \). Therefore, for the above equality
to be satisfied when \( \gamma \to 0 \), the only meaningful solution is the
positive root, denoted below \( \lambda_1^* \). It is easy to see from (35) that
\( d\lambda_1^*/d\gamma < 0 \), so that a higher feedback coefficient reduces the impact
of the rate of growth of domestic credit on the premium.

Solving the basic model as before yields

\[
\pi_{t+1} = \exp(-\lambda k_t), \quad k_t \geq 0
\]  

(34)

2/ (continued from page 23) Assuming that the probability \( \alpha \) is
inversely related to the level of the exchange rate spread would yield
the same kind of ambiguous result regarding the effect of the premium
on the devaluation probability as discussed below.
where now $k_t = (\bar{\lambda} - \lambda_0 - \lambda_2 \bar{e})/\lambda_1 - (d_t + \mu - \gamma \rho_t)$, and $\bar{\lambda}, \lambda_0$ and $\lambda_2$ denote the values of $\lambda$, and $\lambda_0$ associated with the positive root of (34a). Again, tedious calculations indicate that $\partial \pi_{t+1}/\partial \gamma < 0$; the higher the feedback coefficient is, the lower the probability of a devaluation. In the "perfect feedback" case, that is, for $\gamma \rightarrow \infty$, the devaluation probability falls to zero. But now, for a given $\bar{\rho}$, the impact of the premium on the collapse probability is ambiguous. A rise in $\rho_t$ has a negative impact on $d_{t+1}$, which reduces the possibility that the next realization of credit growth forces the devaluation. But a rise in $\rho_t$ also lowers the credibility of the current fixed exchange rate now and raises the expected rate of return on foreign currency holdings, which gives agents an incentive to shift away from domestic currency, and increases the likelihood that the authorities will implement a devaluation at the end of $t+1$.

V. Conclusions

The purpose of this paper has been to extend the literature on balance of payments crises to consider the case of an economy where permanent exchange restrictions give rise to a parallel market for foreign currency. The monetary authorities are assumed to pursue a given rate of expansion of domestic credit, and a fixed parity. These policies are not necessarily consistent. While most of the attention in models of balance of payments crises has been focused upon the switch from a fixed exchange rate regime to a flexible regime or a devalued official rate with a foreign reserve constraint acting as the policy trigger, the model developed here focuses on parity changes that are based on a viability criterion for the fixed exchange rate. The authorities are assumed to follow a devaluation rule whereby the size of a parity change depends on the difference between the parallel market premium and a given threshold level. The paper highlights one particular trigger mechanism for such a rule: the yield of the inflation tax --which can be levied only on domestic money balances-- in an economy where agents make portfolio choices between domestic and foreign currencies. An excessive rate of growth of domestic credit pushes the parallel market premium up, lowers the credibility of the official exchange rate and raises the probability formed by asset holders that a devaluation will occur in the future. This, in turn, raises the expected rate of return on foreign currency holdings and gives agents an incentive to switch away from domestic money. In an attempt to recapture the tax base, the authorities may devalue the domestic currency. The downward jump in the level of the premium which results from the parity change raises the credibility of the new parity, lowers the expected rate of return on foreign currency, and generates a portfolio shift into domestic currency. However, if underlying macroeconomic factors are not altered, periodic devaluations will need to be implemented to preserve the tax base.
because a rising premium steadily erodes the confidence of the public in the fixed official exchange rate.

While many of the conclusions drawn in the preceding sections are specific to the model considered, there are several broad lessons to be drawn from the analysis. First, monetary financing of a budget deficit is, in a sense, self-defeating when domestic agents have the option of shifting away from the domestic currency. It is generally associated with a rise in the rate of depreciation of the parallel market rate, which makes it increasingly attractive for domestic residents to hold foreign currency. As a consequence of this portfolio shift, the ability to collect any seignorage from money issuance will be severely impaired. Second, while traditionally devaluation decisions are viewed as resulting from concerns over the behavior of "external sector" indicators (foreign reserves and the real exchange rate), the analysis in this paper suggests that a premium-based devaluation rule can be triggered by another mechanism, namely the preservation of the yield of the inflation tax. Third, foreign exchange controls are not a substitute to fundamental macroeconomic policy reforms to prevent exchange rate crises.

Despite its simplicity, the model predicts several a number of empirical regularities that are likely to be observed in periods surrounding devaluation episodes in developing countries. First, in the years preceding a devaluation, fiscal deficits and credit policy should indicate a significantly expansionary stance. Second, following a devaluation, the parallel market premium should experience a sharp downward movement, before eventually rising again, if fiscal and monetary policies are maintained on an expansionary course. Third, if devaluation decisions are motivated at least in part by concerns over the yield of the inflation tax, and if currency substitution is prevalent, the data should indicate, in the periods preceding a devaluation, a sharp drop in revenue from money creation—that is, the government's ability to finance deficits through money growth should be diminishing. At the same time, one should observe a sharp increase in the velocity of money, as agents tend to reduce their holdings of domestic currency. In the periods following the parity change, the opposite phenomenon should be observed.

The first two empirical predictions of the analysis have been well documented by Edwards (1989) and Kamin (1988). However, there is no systematic evidence available so far on the third proposition,

1/ The shift away from domestic currency may also take the form of international capital movements. Edwards (1989) for instance shows that in a number of developing countries, capital flight intensified in the periods preceding a devaluation. This suggests that an additional element which may be useful in assessing the extent of the shift away from domestic currency holdings is to examine the behavior of foreign currency deposits held abroad by domestic residents.
which applies to countries where foreign exchange controls are prevalent, a large parallel market exists, and where currency substitution effects are deemed important. Tests of these hypotheses, using data pertaining to devaluation episodes in developing countries, would constitute an important complement to the present analysis.

Finally, there are several extensions of the model, other than those already considered here, that may be worthwhile investigating. First, the analysis does not explicitly consider the demand for foreign currency or domestic financial wealth, which represents a budget constraint as agents swap domestic against foreign money. Second, the analysis of alternative rationing schemes, whereby the authorities may allow some endogenous changes in foreign reserves, may increase the realism of the model. Edwards (1989) has shown that devaluation episodes have usually been preceded by a sharp drop in international reserves. Moreover, the devaluation rule used in this paper can be motivated by leakages between the official and the parallel market for foreign exchange, as a result of under-invoicing of exports. The addition of endogenous reserve losses would allow the current model to relate inter-market behavior to the process of a balance of payments crises.

1/ However, although a discrete devaluation does in fact reduce real wealth, nothing of substance would be altered by incorporating changes in wealth into the analysis. The assumption of constant real wealth and instantaneous exchanges between domestic and foreign currency holdings simplifies the model considerably.

2/ Otani (1989) has recently developed a model which explains endogenously both the loss in reserves and a discrete exchange rate realignment. In his model, private agents evaluate the imminence of the devaluation crisis on the basis of the amount of reserves the authorities presently have.
In the text, only the case where the devaluation occurs at the end of period $t+1$, conditional on information available at $t$, has been considered. More generally, let $\pi_{t, t+n}$, where $n \geq 1$ is an integer, be the probability, at $t$, that the fixed exchange rate regime collapses in period $t+n$ (neither earlier nor later) and let $\Pi(t, t+n)$ be the probability, at $t$, that it collapses no later than period $t+n$.

Formally, \[ \text{(Ala)} \]
$$
\pi_{t, t+n} = \Pr(\rho_{t+n} > \bar{\rho} \text{ and } \rho_{t+j-1} < \bar{\rho} \text{, } 1 \leq j < n)
$$

\[ \Pi(t, t+n) = \sum_{j=t+1}^{t+n} (\pi_{t+j}) = \Pr(\rho_{t+n} > \bar{\rho}). \text{ (Alb)} \]

$\Pi(t, t+n)$ is therefore the probability density function of the "collapse interval" $(t, t+n)$. We now derive the expectation of this distribution -- that is, the mean duration of the interval until the

1/ The last equality in (Alb) holds because $\rho$ is strictly increasing in $n$ (see below, equation A4). Take, for instance, $n=2$; the probability that the official exchange rate will be devalued in no more than two periods is given by

$$
\Pi(t, t+2) = \pi_{t+1} + \pi_{t+2} = \Pr(\rho_{t+1} > \bar{\rho}) + \Pr(\rho_{t+2} > \bar{\rho}) - \Pr(\rho_{t+1} > \bar{\rho} \text{ and } \rho_{t+2} > \bar{\rho}).
$$

But since $\rho_t$ is monotonically increasing in $n$,

$$
\Pr(\rho_{t+1} > \bar{\rho} \text{ and } \rho_{t+2} > \bar{\rho}) = \Pr(\rho_{t+1} > \bar{\rho}),
$$

$$
\Pr(\rho_{t+2} > \bar{\rho} \text{ and } \rho_{t+1} > \bar{\rho}) = \Pr(\rho_{t+2} > \bar{\rho} | \rho_{t+1} > \bar{\rho}) \cdot \Pr(\rho_{t+1} > \bar{\rho}) = \Pr(\rho_{t+1} > \bar{\rho}),
$$

so that

$$
\Pi(t, t+2) = \Pr(\rho_{t+2} > \bar{\rho}).
$$
devaluation takes place. From equation (8b) in the text,

\[ \rho_{t+n} = c/s + (d_{t+n}/s), \]  

(A2)

where \( c = c_1[\mu + \gamma_s(s - 1)]/s - c_0 - \bar{e} \). From equations (4) and (12), recursive substitution yields

\[ d_{t+n} = d_t + n(\mu - 1/\lambda) + \sum_{j=1}^{t+n} u_{t+j}. \]  

(A3)

Substituting (A3) in (A2) yields

\[ \rho_{t+n} = c/s + (1/s)[d_t + n(\mu - 1/\lambda) + u_t(n)], \]  

(A4)

where \( u_t(n) = \sum_{j=1}^{t+n} u_{t+j} \). Following the same procedure as before, it follows that

\[ \Pi(t, t+n) = \text{Prob}(\rho_{t+n} > \bar{\rho}) = \text{Prob}(u_t(n) > k_t(n)), \]

where \( k_t(n) = s\bar{\rho} - c - n(\mu - 1/\lambda) + d_t \). Since the \( u_{t+j}, j = 1, 2, \ldots, n \) are independent random variables each having an exponential distribution with parameter \( \lambda \), it follows that \( u_t(n) \) has the gamma density \( \Gamma(n, \lambda) \), defined by

\[ \Gamma(n, \lambda) = \begin{cases} \frac{\lambda^n}{(n-1)!} u_t(n)^{n-1}\exp[-\lambda u_t(n)] & u_t(n) \geq 0, \\ 0 & u_t(n) < 0. \end{cases} \]  

(A5)

\( \Pi(t, t+n) \) is then given by integrating \( \Gamma(n, \lambda) \) in the interval \([k(t, t+n), \infty)\):

\[ \Pi(t, t+n) = \begin{cases} \int_{k_t(n)}^{\infty} \frac{\lambda^n}{(n-1)!} u_t(n)^{n-1}\exp[-\lambda u_t(n)]du_t(n) & k_t(n) \geq 0, \\ 1 & k_t(n) < 0. \end{cases} \]  

(A6)

Using well-known properties of Gamma functions (see for instance Hogg and Craig, 1978, p. 105), this simplifies to
\[ \Pi(t, t+n) = \begin{cases} \sum_{j=0}^{n-1} \left[ (\lambda k_t(n))^j / j! \right] \exp[-\lambda k_t(n)], & k_t(n) \geq 0, \\ 1, & k_t(n) < 0. \end{cases} \]  \hfill (A7)

For \( n=1 \), equation (A7) yields the solution (16) given in the text. Having calculated \( \Pi(t, t+n) \), it is easily shown that, whatever the length of the interval \( n \), a rise at \( t \) in the deterministic component of the rate of domestic credit growth, \( \mu \), increases the probability of a devaluation occurring during an interval of \( n \) periods after the policy change. First, note that

\[ d\Pi(t, t+n)/d\mu = \left[ d\Pi(t, t+n)/dk_t(n) \right] \left[ dk_t(n)/d\mu \right]. \]

From equation (A7),

\[ d\Pi(t, t+n)/dk_t(n) = \sum_{j=0}^{n-1} \left[ \lambda k_t(n) \right]^j / j! \exp[-\lambda k_t(n)] \left[ (j/k_t(n)) - \lambda \right] \]

\[ = -\lambda (\lambda k_t(n))^{n-1} / (n-1)! \exp[-\lambda k_t(n)] < 0. \]  \hfill (A8)

From the definition of \( k_t(n) \) given above, \( dk_t(n)/d\mu < 0 \). Combining this result with (A8) yields \( d\Pi(t, t+n)/d\mu > 0 \).

It can also be shown that the probability of a devaluation occurring no later than \( t+n \) is an increasing function of \( n \), that is, \( \Pi(t, t+n+1) > \Pi(t, t+n) \). First, observe that holding \( k_t(n) \) constant, increasing \( n \) by one period adds a positive term to the sum in (A7). Second, from the definition of \( k_t(n) \) given above, an increase in \( n \) will be associated with a decrease in \( k_t(n) \) if \( (\mu - 1/\lambda) > 0 \). This is precisely the condition for positive credit growth as discussed in the text, and is assumed to hold. Finally, as shown by (A8), \( d\Pi(t, t+n)/dk_t(n) < 0 \). Combining these results implies that an increase in \( n \) always raises the devaluation probability \( n \)-step ahead.
APPENDIX II

A Uniformly Distributed Stochastic Credit Shock

This Appendix considers briefly the case where the stochastic shock in the credit policy rule is specified as a uniform rather than an exponential distribution, in the basic model. Instead of (4), consider the following credit rule:

\[ d_{t+1} = d_t + \mu_{t+1} \]  \hspace{1cm} \text{(A1)}

where \( \mu_{t+1} \), the rate of growth of domestic credit, is assumed to follow a uniform distribution with a minimum of \( \underline{\mu} \), and a maximum of \( \bar{\mu} \), so that the mean value is \( \frac{\mu + \bar{\mu}}{2} \). Let

\[ c = \left[ c_1 (\mu + \bar{\mu})/2 - c_0 \right]/\sigma; \]

from (7b), the equation describing the evolution of the premium is

\[ \rho_{t+1} = c + (d_t + \mu_{t+1})/s, \]  \hspace{1cm} \text{(A2)}

where \( c = c_1 [\mu + \eta (s - 1)]/s - c_0 - \bar{\epsilon} \). The probability of a devaluation at the end of period \( t+1 \), \( \pi_{t+1} \), is equal to the probability that \( \rho_{t+1} > \bar{\rho} \), so that, using (A2),

\[ \pi_{t+1} = \text{Prob}(\mu_{t+1} > k_t), \quad \mu \leq k_t \leq \bar{\mu} \]  \hspace{1cm} \text{(A3)}

where now \( k_t = s(\bar{\rho} - c) - d_t \). Evaluating this probability, using the uniform distribution assumption for \( \mu_{t+1} \), yields

\[ \pi_{t+1} = \frac{\mu}{k_t} \left[ 1/(\mu - \underline{\mu}) \right] d\mu_{t+1} = (\bar{\mu} - k_t)/(\mu - \underline{\mu}), \quad \mu \leq k_t \leq \bar{\mu} \]  \hspace{1cm} \text{(A4)}

---

\( ^{1/} \) Dornbusch (1987) provides a stochastic formulation of the Flood-Garber model in terms of a uniform distribution. The exponential and the (discrete) uniform distributions are well characterized in Hogg and Craig (1978).
From the probability expression (A4), it appears that up to a level of domestic credit \( d = s(\rho - c) - \mu \), the probability of a devaluation at the end of period \( t+1 \) is zero. As domestic credit rises above that lower limit the probability rises, reaching an upper limit of 1 for \( k_t = \mu \), or \( d = s(\rho - c) - \mu \).

Using the devaluation rule (9), the conditional exchange rate is now given by, since \( E_t(\tilde{e}_{t+1} | \rho_{t+1} > \bar{\rho}) = E_t(\tilde{e}_{t+1} | \mu_{t+1} > k_t) \),

\[
E_t(\tilde{e}_{t+1} | \mu_{t+1} > k_t) = \bar{e} + \frac{c - \bar{\rho}}{1 - \delta} + \frac{1}{s(1 - \delta)} \left[ d_t + E_t(\mu_{t+1} | \mu_{t+1} > k_t) \right].
\]

From the properties of the uniform distribution, the conditional expectation \( E_t(\mu_{t+1} | \mu_{t+1} > k_t) \) is simply \( (\mu + k_t)/2 \). From the definition of \( k_t \), one gets

\[
E_t(\tilde{e}_{t+1} | \rho_{t+1} > \bar{\rho}) = \bar{e} + \frac{c - \bar{\rho}}{1 - \delta} + \frac{1}{s(1 - \delta)} \left[ s(\rho - c) + \frac{\mu - k_t}{2} \right]. \tag{A5}
\]

From (10), (A3) and (A5), the unconditional exchange rate is therefore given by

\[
E_t(e_{t+1}) = \bar{e} + \frac{t_\pi t+1(\mu - k_t)}{2s(1 - \delta)}. \tag{A6}
\]

Using (A4) and (A6), comparative static exercises similar to those discussed in the text can be conducted.
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