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Sustainable Plans and Mutual Default

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Abstract

This paper presents a model of optimal taxation in which both private agents and the government can default on their debt. We first consider Ramsey equilibria in which the government can precommit to its policies but in which private agents can default. We then consider sustainable equilibria in which both government and private agent decision rules are required to be sequentially rational. We show that when there is sufficiently little discounting and government consumption fluctuates enough, the Ramsey allocations and policies (in which the government never defaults) can be supported by a sustainable equilibrium.

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Summary

This paper presents a model in which both private borrowers and the government can default on their debt. It is easy to imagine conditions under which the government can, at least partially, default on its debt and, at the same time, have difficulty collecting debts owed to it by private citizens, such as students or farmers.

The analysis considers an extreme situation in which the government can enforce no private debt claims. Nevertheless, the analysis indicates that the government has good reasons not to default on its debt. If it defaults, private agents may never again lend to it. Moreover, since in the model it cannot enforce debt payments by private agents, the government cannot safely lend to the private sector. Thus, a situation arises in which, if the government ever defaults, it must thereafter continuously balance its budget. If forced to balance its budget annually, the government cannot smooth taxes, and this leads to a low level of welfare.

Thus, the losses attendant on banning the government from future borrowing are large. Under plausible assumptions, the analysis shows that these losses can be large enough to deter the government from ever defaulting.

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I. Introduction

A classic problem in the time consistency literature is that of government default on debt. In an early contribution, Prescott (1977) analyzed a simple infinite horizon economy in which the government finances a given stream of expenditures by raising distorting labor taxes and by selling debt. He found that if there is no technology for committing to its actions, the government will always default on outstanding debt to avoid levying distorting taxes. In the equilibrium of his model, the value of government debt is zero and the government runs a continuously balanced budget.

This early work presents a challenge to economists interested in explaining why governments do not default on their inherited debt. An intuitive answer is that governments fear that, if they default, private agents will be less willing to lend to them in the future. If the losses incurred by the government from future borrowing difficulties outweigh the current benefits from defaulting, the government will not default.

In this paper and a companion paper (Chari and Kehoe 1988), we explore this intuitive explanation in the context of a formal general equilibrium model. Our main insight is that whether or not this explanation can account for government debt depends crucially on the set of commitment and enforcement technologies available. In our earlier paper, we followed the standard setup of the time consistency literature in assuming that the only problems with commitment lie with the government. In that paper, the government had no way of committing to a specific set of policies; thus, it had no way of committing to honor its debt. There were, however, no problems in enforcing debt payments by private agents. Rather surprisingly, in that setup the intuitive explanation of why there is government debt does not hold. Briefly, the reason is as follows. In the model, the first part of the explanation does hold: there are equilibria in which, if the government ever defaulted on its debt, private agents would never let the government borrow from them again. However, the assumption that private agents always paid their debt implied the government could always safely lend to private agents. The ability to lend undercuts the force of the borrowing restrictions. In the model, the benefit of government debt is that it allows for the smoothing of tax distortions over time. It turns out that, even if the government could never borrow, the government by optimally lending could smooth taxes almost as well as it could under commitment. Hence, the losses incurred from banning the government from future borrowing were trivial and thus typically not sufficient to deter the government from defaulting on its debt.

In this paper, we consider a model in which both private agents and the government can default on their debt. The idea is that while it is easy to imagine environments in which the government can, at least partially, default on its debt--say, by taxation or inflation--it is also easy to imagine that certain debt owed to the government by private citizens, such as student loans or loans to farmers, is hard to collect. To keep the analysis tractable, we consider the extreme situation in which the government cannot enforce any private debt claims. We find that here, in stark contrast to our earlier work, the intuitive explanation does hold. First, there are equilibria in which, if the government defaults, private agents never let it borrow from them again. Moreover, since it cannot enforce debt payments by private agents, the government cannot safely lend to the private sector. Thus, there are equilibria in which, if the government ever defaults, it is stuck with continuously balancing its budget thereafter. If it is forced to balance its budget at each date, the government cannot smooth taxes

at all, and this leads to a rather low level of welfare. Thus, here the losses incurred from banning the government from future borrowing are large. Under plausible assumptions, we show that these losses are large enough to deter the government from ever defaulting.

Formally, our model is a variant of the optimal taxation models of Prescott (1977), Barro (1979), Lucas and Stokey (1983), and Persson, Persson, and Svensson (1987). As a benchmark, we consider an environment in which the government can commit to honoring its debt but private agents cannot. Since there is commitment by the government, the resulting equilibrium, called a Ramsey equilibrium, is a dynamic counterpart of the static equilibrium considered by Ramsey (1927). We characterize the Ramsey equilibrium as a solution to a planning problem. We then consider an environment in which neither the government nor the private agents can commit to honoring their debt. We allow the allocation rules of consumers and policy plans of the government to depend on the whole history of past government policies. We define a sustainable equilibrium as a set of allocation rules and policy plans that satisfy sequential rationality conditions for both the private agents and the government. We adapt some techniques from game theory to characterize completely the set of sustainable allocations and policies. We use this characterization to show that if the fluctuations in government spending never damp out and the discount rate is sufficiently small, then even the Ramsey outcomes can be supported by a sustainable equilibrium. We go on to show how the supportability of Ramsey outcomes depends critically on the nature of fluctuations in government spending. We provide examples showing that, if the fluctuations eventually damp out, then the Ramsey outcomes cannot be supported, regardless of the discount factor.

One question that arises in the analysis is, How is the notion of time consistency considered here related to the notion of perfection in game theory? We analyze this question by mapping our economy with budget constraints and competitive private agents into an anonymous game. We show that the symmetric perfect Bayesian equilibria of this game coincide with the sustainable outcomes in our economy.

This paper builds on several diverse strands of literature. First, it builds on the early literature on time consistency (for example, Kydland and Prescott 1977, Calvo 1978, and Fischer 1980), which extended the public finance approach of Ramsey (1927) to analyze the sequential design of policy in dynamic general equilibrium models. In this literature, the decision rules of the government and private agents were restricted to depend on physical state variables such as the money stock or the capital stock. In contrast, in our model, these rules are allowed to depend on the whole history of government policies. This difference implies that there are trigger-type equilibria in our model, while there were no such equilibria in the earlier models. Within this literature, the papers most closely related to ours are those of Prescott (1977) and Calvo (1988, 1989). As discussed, in Prescott's model, the government always defaults and in equilibrium there is no government debt. In contrast, in Calvo's models, there is government debt. This debt emerges not because of trigger strategies, but rather because there is a direct cost to default. The main focus of Calvo's work is to investigate how such costs can generate a multiplicity of equilibria of a very different kind from those considered here.

Second, this paper builds on some developments in game theory, especially the work on repeated games (Friedman 1971, Fudenberg and Maskin 1986, and Abreu 1988). From game theory, we borrow the idea of history-contingent

decision rules and, from Abreu's work, the idea of using the worst equilibrium to characterize the entire set of equilibrium outcomes. Our work differs from this literature in at least two ways. One is that, in that literature, the games consist of a finite number of players, each of whom has strategic power, whereas in our paper, there is one large agent, the government, and a large number of competitive private agents. The other difference is that the presence of government debt makes our model resemble a dynamic game. For both of these reasons, the standard results in repeated games, such as the folk theorem (Fudenberg and Maskin 1986), do not directly apply here.

Third, this paper is related to the recent literature on macroeconomic models of policy games (Barro and Gordon 1983, Backus and Driffill 1985, and Rogoff 1987). In that literature, the government often seems to be playing a game against a coalition of private agents, who may have different objectives than the government. In our model, the government maximizes the welfare of private agents, who behave competitively. Moreover, this literature analyzes models in which a static environment is repeated forever, while we analyze a dynamic model with a state variable that links periods.

Finally, this paper is related to the literature on international default (for example, Eaton, Gersovitz, and Stiglitz 1986; Grossman and Van Huyck 1986; Manuelli 1986; Atkeson 1988; Cole and English 1988; and Bulow and Rogoff 1989). The part of this literature most closely connected to our paper investigates whether a threat to cut off a defaulting country from future borrowing is credible and, if so, whether it can support positive borrowing by that country. Bulow and Rogoff find that such threats cannot support positive borrowing by the foreign country from the home country, whereas Grossman and Van Huyck find that they can. Although never discussed explicitly in these papers, the key to understanding these contrary results is the set of enforcement technologies available. Bulow and Rogoff assume there is some technology through which agents in one of the countries, the home country, can commit to servicing its debt, while there is no such technology in the foreign country. In this setup, no matter what it does, the foreign country can always safely lend to the home country's agents to smooth consumption. Bulow and Rogoff use this feature to prove that no equilibrium can have positive borrowing by the foreign country. In contrast, Grossman and Van Huyck explicitly rule out the possibility of the foreign country ever lending to the home country. In their equilibrium, if the foreign country defaults, it is forced into autarky for some (stochastic) length of time. Grossman and Van Huyck show that these threats are credible and that they can support borrowing by the foreign country. Thus, the link between enforcement technologies and the ability to support positive debt shows up in the open economy literature also.

This paper is organized as follows. Section II describes the economy. Section III considers an environment in which the government can commit to honoring its debt but private agents cannot. Section IV considers an environment in which neither the government nor private agents can commit to honoring their debt and introduces the notion of a sustainable equilibrium. Section V characterizes the sustainable outcomes for finite and infinite horizon economies. Section VI introduces uncertainty and establishes conditions under which the Ramsey outcomes are sustainable. Section VII relates the Ramsey equilibria and the sustainable equilibria to the perfect equilibria of certain anonymous games. Section VIII concludes.

II. The Economy

Consider a simple production economy populated by a large number of identical infinitely lived consumers. In each period t, there are two goods: labor and a consumption good. A constant returns-to-scale technology is available to transform one unit of labor into one unit of output. The output can be used for private consumption or for government consumption. The per capita level of government consumption in each period, denoted g_t, is exogenously specified. Let c_t and ℓ_t denote the per capita levels of private consumption and labor. Feasibility requires that

(1.1) $$c_t + g_t = \ell_t.$$

The preferences of each consumer are given by

(1.2) $$\sum_{t=0}^{\infty} \beta^t U(c_t, \ell_t)$$

where $0 < \beta < 1$ and U is increasing in consumption, decreasing in labor, and strictly concave and bounded. Assume also that consumption and leisure are normal goods.

Government consumption is financed through a proportional tax on labor income and with debt. Let τ_t denote the tax rate on labor income in period t. A unit of government debt at t is a claim to one unit of the consumption good at t + 1. Let $b_t \geq 0$ denote the number of claims purchased by consumers and q_t denote the price of a claim. Let $\delta_t \in [0,1]$ denote the default rate on government debt outstanding in period t. Here $\delta_t = 0$ corresponds to complete repayment, $\delta_t = 1$ to complete default, and $0 < \delta_t < 1$ to partial default. (One can think of δ_t as a tax on debt.) Let $d_t \geq 0$ denote the number of debt claims issued by the consumer at t and p_t the price of a claim. Let $\theta_t \in [0,1]$ denote the default rate on private debt outstanding in period t. The consumer's budget constraints can then be written as

(1.3) $$c_t - (1-\tau_t)\ell_t + q_t b_t - p_t d_t = (1-\delta_t)b_{t-1} - (1-\theta_t)d_{t-1} \quad \text{for } t = 0, \ldots, \infty$$

with $b_{-1} = d_{-1} = 0$. An allocation for consumers is a sequence $x = (x_t)_{t=0}^{\infty}$, where $x_t = (c_t, \ell_t, b_t, d_t, \theta_t)$.

The government sets labor tax rates, default rates, and debt prices to finance an exogenous sequence of government consumption. The government's budget constraints are

(1.4) $$\tau_t \ell_t - g_t + q_t b_t - p_t d_t = (1-\delta_t)b_{t-1} - (1-\theta_t)d_{t-1} \quad \text{for } t = 0, \ldots, \infty.$$

A policy for the government is a sequence $\pi = (\pi_t)_{t=0}^{\infty}$, where $\pi_t = (\tau_t, \delta_t, q_t, p_t)$.

III. Partial Commitment

Consider an environment with the following institutional structure. First, suppose there is an institution or commitment technology through which the government can bind itself to a particular policy once and for all at time zero. In particular, the government can commit to never defaulting on its debt. Next, suppose there is no technology for enforcing private debt repayments. In particular, let private agents be anonymous so that there is no way for anyone to keep track of an individual trader's payment history. Clearly, in such an environment, if a trader is lent a positive amount, the trader will default; thus, the current price of a claim to future payment is zero.

We model this institutional structure with two features. The government technology for commitment is formalized by having the government choose a policy once and for all and then having consumers choose their allocations. The anonymity of private agents coupled with the lack of an enforcement technology implies that private agents always default on their debt by setting θ_t identically equal to one. Hence, without loss of generality, we can set p_t and d_t identically equal to zero. From now on, we suppress θ_t, d_t, and p_t.

In this setup, a policy for the government is an infinite sequence of numbers $\pi = (\pi_t)_{t=0}^{\infty}$. Since the government needs to predict how consumers will respond to its policies, consumer behavior is described by rules that associate government policies with allocations. An allocation rule is a sequence of functions $f = (f_t)_{t=0}^{\infty}$ that maps policies into allocations. A Ramsey equilibrium is a policy π and an allocation rule f that satisfy

(i) Government maximization. The policy π maximizes

(2.1)
$$V(\pi,f) = \sum_{t=0}^{\infty}\beta^t U(c_t(\pi),\ell_t(\pi))$$

subject to

$$\tau_t\ell_t(\pi) - g_t + q_t b_t(\pi) = (1-\delta_t)b_{t-1}(\pi) \quad \text{for all t.}$$

(ii) Consumer maximization. For every policy π', the allocation $f(\pi')$ maximizes (1.2) subject to (1.3).

The allocations in a Ramsey equilibrium solve a simple programming problem called the Ramsey problem. We let R_t denote the value of the government surplus at t; namely, $R_t = U_c(\tau_t\ell_t-g_t)$. In any equilibrium, we can use the consumer's first-order conditions together with feasibility to write this as $R_t = U_c c_t + U_\ell \ell_t$. We have, then,

Proposition 1 (The Ramsey Equilibrium). The consumption and labor allocations, c and ℓ, in the Ramsey equilibrium solve the problem

$$\max \sum_{t=0}^{\infty} \beta^t U(c_t, \ell_t)$$

subject to

$$c_t + g_t = \ell_t \tag{2.2}$$

$$\sum_{t=0}^{\infty} \beta^t R_t = 0 \tag{2.3}$$

$$\sum_{t=s}^{\infty} \beta^t R_t \geq 0 \quad \text{for } s = 1, 2, \ldots \tag{2.4}$$

where $R_t \equiv U_c c_t + U_\ell \ell_t$ is the government surplus in period t in units of marginal utility.

Proof. The proposition is proved in several steps. First, note that by adding (1.3) and (1.4) we get (2.2), and thus feasibility is satisfied in equilibrium. Next, consider the allocation rule $f(\pi)$. For any policy π, the necessary and sufficient conditions for c, ℓ, and b to solve the consumer's problem are, by Weitzman's (1973) theorem, constraints (1.3) and (2.1) together with

$$\beta^t U_c(c_t, \ell_t) \leq p_t, \quad \text{with equality if } c_t > 0 \tag{2.5}$$

$$\beta^t U_\ell(c_t, \ell_t) \leq -p_t(1-\tau_t), \quad \text{with equality if } \ell_t > 0 \tag{2.6}$$

$$p_t q_t \geq (1-\delta_{t+1}) p_{t+1}, \quad \text{with equality if } b_t > 0 \tag{2.7}$$

$$\lim_{t \to \infty} p_t q_t b_t = 0 \tag{2.8}$$

where, for each t, p_t is the Lagrange multiplier on constraint (1.3). Multiplying (1.3) by p_t, summing over t, and using (2.7) and (2.8) gives

$$\sum_{t=0}^{\infty} p_t \left[c_t - (1-\tau_t) \ell_t \right] = 0 \tag{2.9}$$

where we use the fact that $b_0 = 0$. Likewise, summing from any t + 1 gives

$$p_{t+1}(1-\delta_{t+1}) b_t = \sum_{s=t+1}^{\infty} p_s \left[c_s - (1-\tau_s) \ell_s \right]. \tag{2.10}$$

Using (2.5) and (2.6), we can write (2.9) as

$$(2.11) \qquad \sum_{t=0}^{\infty} \beta^t \left[U_c c_t + U_\ell \ell_t\right] = 0$$

where, for convenience, we have suppressed the arguments of the derivatives. Equation (2.11) is the same as (2.3). We can also write (2.10) as

$$(2.12) \qquad \beta^{t+1} U_c (1-\delta_{t+1}) b_t = \sum_{s=t+1}^{\infty} \beta^s \left[U_c c_s + U_\ell \ell_s\right].$$

Since $b_t \geq 0$, the left side of (2.12) is nonnegative. Therefore, in any equilibrium, (2.4) must hold. Thus, (2.2)-(2.4) are necessary conditions that any Ramsey equilibrium must satisfy.

Finally, given any set of allocations, c and ℓ, that solve the Ramsey problem, it is clear we can construct sequences of debt holdings, tax policies, default rates, and debt prices such that these allocations solve the consumer's problem. Q.E.D.

Note that in the government's problem (2.1), there is no infinite horizon budget constraint. In the proposition, however, we use the equilibrium conditions to show that the Ramsey policies and allocations satisfy an infinite horizon constraint. The intuition for this result is as follows. Since the government cannot save by purchasing the private debt of consumers, the present value of spending is at least as large as the present value of revenues. Moreover, the government can borrow only as much as private agents are willing to lend it. The first-order conditions for consumers imply that private agents will only lend the government sequences of debt that have a present value of zero. Thus, Ponzi schemes are not equilibrium outcomes. Combining these features gives that, in equilibrium, the present value of government spending must equal the present value of government revenue.

IV. No Commitment

Consider an environment in which no commitment technologies are available to either the government or private agents. Formally, the government's lack of commitment is modeled by having the government choose policy sequentially: In each period, the government chooses its current policy, and then private agents make their decisions. The lack of commitment by private agents is summarized by imposing the requirement that private agents can save but not borrow.

In each period, the government and the consumers can vary their decisions, depending on the history of government policies up to the time the decision is made. At the beginning of period t, the government chooses a current policy as a function of the history $h_{t-1} = (\pi_s | s=0,...,t-1)$ together with a contingency plan for setting future policies for all possible future histories. Let $\sigma_t(h_{t-1})$ denote the time t labor tax rate, default rate, and price of debt chosen by the government when faced with history h_{t-1}. After the government sets current policy, consumers make their decisions. Faced with a history $h_t = (h_{t-1}, \pi_t)$, consumers choose time t levels of consumption, labor supply, and debt holdings, denoted

$f_t(h_t)$, together with a contingency plan for choosing future allocations for all possible future histories. (The reader may wonder why the histories do not include consumers' decisions. In an earlier version of this paper, we did define histories that way, but it turns out that we don't need to: no individual consumers perceive that the government or other consumers will change policies if individual decisions are changed. Also, see Section VI, where we show that deviation by consumers can be ignored in a game.)

To define a sustainable equilibrium, we need to explain how policy plans induce future histories. For any policy plan $\sigma = (\sigma_0, \sigma_1, ...)$, let $\sigma^t = (\sigma_t, \sigma_{t+1}, ...)$ denote the sequence of policy rules from time t onwards. For any t, call σ^t the continuation of σ. Let f^t denote the corresponding objects for the allocation rules, namely, the continuation of $f = (f_0, f_1, ...)$. Given a history h_{t-1}, the policy plan σ induces future histories by $h_t = (h_{t-1}, \sigma_t(h_{t-1}))$ and so on.

Consider the situation of the government in period t. Given some history h_{t-1} and given that future allocations evolve according to f, the government chooses a policy plan σ^t that maximizes the welfare of consumers subject to its budget constraint. Formally, given an allocation rule f, the policy plan σ is <u>rational for the government</u> after history h_{t-1} if the continuation of σ maximizes

(3.1) $$V_t(\sigma^t, f^t, h_{t-1}) = \sum_{s=t}^{\infty} \beta^s U(c_s(h_s), \ell_s(h_s))$$

subject to

(3.2) $$\tau_s(h_{s-1})\ell_s(h_s) - g_s + q_s(h_{s-1})b_s(h_s) = (1-\delta_s(h_{s-1}))b_{s-1}(h_{s-1})$$

where for all $s \geq t$ the future histories are induced by σ^t from h_{t-1}.

Next consider a private agent in period t. Given some history h_t and given that future policies evolve according to σ, a consumer chooses an allocation rule f^t to maximize welfare subject to the consumer's budget constraint. Given a policy *rule σ, an allocation rule f is <u>rational for consumers</u> after history h_t if the* continuation of f maximizes

(3.3) $$\sum_{s=t}^{\infty} \beta^s U(c_s(h_s), \ell_s(h_s))$$

subject to

(3.4) $$c_t(h_t) - (1-\tau_t)\ell_t(h_t) + q_t b_t(h_t) = (1-\delta_t)b_{t-1}(h_{t-1})$$

and, for s > t,

(3.5) $$c_s(h_s) - (1-\tau_s(h_{s-1}))\ell_s(h_s) + q_s(h_{s-1})b_s(h_s) = (1-\delta_s(h_{s-1}))b_{s-1}(h_{s-1})$$

where π_t is given in h_t and for all s > t the future histories are induced by σ^t from h_t. Then, a sustainable equilibrium is a pair (σ, f) that satisfies

(i) Sequential rationality by the government. Given the allocation rule f, the policy plan σ is rational for the government for every history h_{t-1}.

(ii) Sequential rationality by consumers. Given a policy plan σ, the allocation rule f is rational for consumers for every history h_t.

For later use, let $V(\sigma, f)$ denote the value of utility in a sustainable equilibrium. Notice that in the definition, we require that both the consumers and the government act optimally for every history of policies--even for histories not induced by the government's strategy or histories with violated feasibility. This requirement, which is analogous to the requirement of perfection in a game, guarantees that both the consumers and the government have an incentive to carry out their strategies in all contingencies.

V. Characterization of Sustainable Outcomes

1. Finite horizon

We start by considering a finite horizon version of the model. We use backward induction to show that in any sustainable equilibrium the government's budget constraint is continuously balanced and, thus, tax distortions cannot be smoothed over time. Consider, then, the following candidate policy plans and allocation rules, which we call the autarky policy plans and allocation rules. For any history, the government's policy plan specifies $\delta_t = 1$ and $q_t = 0$. The labor tax rate, denoted $\tau(g_t)$, is given by this solution to this problem: maximize $U(c, \ell)$ subject to

(4.1) $$c + g_t = \ell$$

(4.2) $$-\frac{U_\ell}{U_c}\ell = g_t.$$

Let $U^a(g_t)$ denote the maximized value of this problem. The allocation rules for consumers specify that, for every history, debt holdings are zero and consumption and labor supply maximize utility given the current tax rate. It is clear from the consumer's first-order condition $-U_\ell/U_c = (1-\tau_t)$ that (4.2) requires the government's budget to be balanced; namely, $\tau_t\ell_t = g_t$. We then have

Proposition 2 (The Autarky Equilibrium). The autarky policy plans and allocation rules are an equilibrium for the finite horizon economy. Furthermore, the equilibrium is essentially unique in the sense that in any equilibrium, the

value of newly issued debt, $q_t(h_{t-1})b_t(h_t)$, and the amount of inherited debt paid off, $(1-\delta_t(h_{t-1}))b_{t-1}(h_t)$, are identically zero, and the tax rate and allocations coincide with those in the autarky equilibrium.

Proof. The first part of the proposition is immediate. If consumers will not buy government debt at any price, the government must choose tax rates to balance its budget. The second part follows immediately from backward induction. In the last period, if the government inherits positive debt, it will default on it to minimize the amount of revenues it must raise through the distorting labor tax. Anticipating these policies, consumers will not buy any government debt at a positive price. As a result, the government must choose taxes to balance its budget. The same argument holds for earlier periods. Q.E.D.

2. Infinite horizon

In the infinite horizon model, the way to characterize the set of equilibria is not obvious. One way to proceed is simply to take the limits of a sequence of finite horizon equilibria. This technique will indeed yield an equilibrium. There are many other equilibria, however, that are not the limits of any sequence of finite horizon equilibria. In fact, the set of sustainable equilibria is very large and difficult to characterize. Fortunately, it is relatively easy to characterize the policies and allocations induced by such equilibria. Recall that a sustainable equilibrium (σ,f) is a sequence of functions that specify policies and allocations for all possible histories. Starting from the null history at date zero, a sustainable equilibrium induces a particular sequence of polices and allocations--prices, say, (π,x). We call this the outcome induced by the sustainable equilibrium.

In characterizing the set of such outcomes, we adapt techniques from Abreu's (1988) seminal work on repeated games. Let the autarky equilibrium in the infinite horizon model be defined exactly as in the finite horizon problem. The key to our characterization is to show that this equilibrium yields the lowest utility among all possible sustainable equilibria. We use this result to prove that a sequence of policies and allocations can be induced by some sustainable equilibrium if and only if the sequence can be induced by a particular sustainable equilibrium called the revert-to-autarky equilibrium. From this result, it will immediately follow that an arbitrary sequence (π,x) is an outcome of a sustainable equilibrium if and only if it satisfies two simple conditions. The first is that the sequence could be attained under commitment by some policy. The second is that the sequence must satisfy a set of simple inequalities.

We begin with a lemma that characterizes the worst equilibrium:

Lemma 1 (Autarky Is the Worst Sustainable Equilibrium). The autarky equilibrium is sustainable. Moreover, any sustainable equilibrium (σ,f) must have a utility level $V(\sigma,f)$ greater than or equal to the utility level $V(\sigma^a,f^a)$ of the autarky equilibrium.

Proof. First, from the same arguments as in Proposition 2, it follows that the autarky equilibrium is sustainable in an infinite horizon. To establish the rest of the proposition, we show that, for an arbitrary equilibrium (σ,f), the following inequalities hold: $V(\sigma,f) \geq V(\sigma^a,f) \geq V(\sigma^a,f^a)$. Both inequalities rely on the fact that in any sustainable equilibrium, the consumers, when confronted with the autarky policies, choose consumption and labor allocations to solve a simple static

problem. More precisely, it is immediate that for any history of the form $h_t = (h_{t-1}, \pi_t^a)$, the allocations $c_t(h_t)$ and $\ell_t(h_t)$ solve the problem of maximizing $U(c,\ell)$ subject to $c \leq (1-\tau)\ell$. From this result, it follows that a deviation by the government from σ to σ^a is feasible in that σ^a satisfies the government budget constraint for any equilibrium allocation rule f. Sequential rationality by the government then yields the first inequality. Next, from the same result, it follows that the allocations induced by (σ^a, f) are the same as the allocations induced by (σ^a, f^a); namely, $c_t = c(\tau(g_t), g_t)$ and $\ell_t = \ell(\tau(g_t), g_t)$, and thus the second inequality holds. Q.E.D.

In the next proposition, we investigate conditions under which an arbitrary outcome is sustainable. A necessary condition is that the outcome be attainable under commitment. Formally, an outcome (π, x) is <u>attainable under commitment</u> if it satisfies the government budget constraints (1.4) and x maximizes consumer utility at date zero, subject to the consumer budget constraints (1.3). It follows immediately that an outcome is attainable under commitment if the associated allocations satisfy feasibility, (2.2); the infinite horizon budget constraint, (2.3); and the nonnegativity constraints, (2.4). Likewise, if an allocation satisfies (2.2)-(2.4), then the associated outcome is attainable under commitment. Note that this concept does not require any type of optimality by the government. Intuitively, this requirement captures the limits on what the government could ever hope to achieve when faced with optimizing private agents, even if the government could commit to suboptimal policies.

To prove the proposition, we use a modified version of the autarky plans called the <u>revert-to-autarky</u> plans. For an arbitrary sequence of policies (π, x), the revert-to-autarky plans specify continuation with the candidate sequences (π, x) as long as the specified policies have been chosen in the past; otherwise, they specify revert to the autarky plans (σ^a, f^a). Thus, for example, at time t given a history h_{t-1}, this policy plan specifies this: Choose the policies π_t specified by π if the policies $(\pi_0, \ldots, \pi_{t-1})$ have been chosen according to π. If they have not, then revert to the autarky policy plan σ^a. The revert-to-autarky allocation rules are similarly defined. We then have

<u>Proposition 3</u> (Sustainable Equilibrium Outcomes). An arbitrary pair of sequences (π, x) is an outcome of a sustainable equilibrium if and only if (i) the pair (π, x) is attainable under commitment, and (ii) for every t, the following inequality holds:

$$(4.3) \qquad \sum_{s=t}^{\infty} \beta^s U(c_s, \ell_s) \geq \sum_{s=t}^{\infty} \beta^s U^a(g_s).$$

<u>Proof</u>. Suppose, first, that (π, x) is the outcome of a sustainable equilibrium (σ, f). Sequential rationality by the consumers requires that x maximize consumer welfare at date zero, while sequential rationality by the government implies that π satisfies the government's budget constraint. Thus, (π, x) is attainable under commitment. Next, by an argument similar to the one in Lemma 1, a deviation by the government to the autarky plan σ^a is feasible. Clearly, then, the utility of

the government must be at least as large as the right side of (4.3) for every period t. Thus (i) and (ii) hold.

Now suppose that some arbitrary pair of sequences (π,x) satisfies (i) and (ii). We show that the associated revert-to-autarky plans constitute a sustainable equilibrium. Consider, first, histories under which there have been no deviations from π up until t. Since x is optimal for consumers at date zero when they are faced with π, it is clear that the continuation of x is optimal for consumers at date t when they are faced with the continuation of π. This proves sequential rationality for consumers for such histories. Consider the situation of the government. If it deviates at date t to some other policy $\hat{\sigma}^t = (\hat{\sigma}_t, \hat{\sigma}_{t+1}, ...)$, then consumers will revert to the autarky allocation rules from time t onward. By construction, when the government is faced with the autarky allocation rule, it is optimal for it to choose the autarky policies. Hence, the most a deviation by the government at t can attain is the right side of (4.3). If the assumed inequality holds, then sticking to the specified plan is optimal.

Consider, next, histories in which there have been deviations from π before t. The revert-to-autarky plans then specify that both the government and the consumers pursue the autarky plans forever. By Proposition 2, such plans are sequentially rational. Q.E.D.

It is important to note that our theorem characterizes the set of sustainable outcomes rather than the set of equilibrium strategies. Equilibrium strategies can, of course, be quite different from the revert-to-autarky plans. Consider, for example, allocation rules and policies which specify that if the government ever deviates from a given sequence of policies, then consumers will not buy government debt for, say, two periods. Suppose these allocation rules and policies lead to a sustainable outcome. By definition, the outcome must be attainable under commitment, and, by Lemma 1, in the event of a deviation, the utility of the government must be at least as large as the right side of (4.3). More generally, for any equilibrium strategies we can dream up, the resulting outcomes must satisfy the conditions of Proposition 3. Conversely, if an arbitrary pair of policy and allocation sequences satisfies these conditions, then there exist some equilibrium strategies that support these sequences as a sustainable outcome. This implies that in checking whether a candidate outcome is sustainable by some equilibrium strategy, we need only check that this outcome satisfies the conditions of the proposition rather than searching over the complicated space of strategies. We exploit this feature in proving that, under certain conditions, the Ramsey outcomes are sustainable.

VI. Uncertainty

In this section, we extend the analysis of the previous sections to allow for stochastic government consumption. We are interested in developing conditions under which the Ramsey outcome is sustainable. To begin, we set up the model with uncertainty and discuss how the results of the previous sections extend to it. Then we discuss some characteristics of the Ramsey plan under the assumption that government consumption follows a positive Markov process. Finally, we show that the Ramsey plan is sustainable for large discount factors.

In particular, in Section V.1, we show that the Ramsey outcomes are sustainable if and only if, after every possible sequence of realizations of

government consumption, the discounted value of utility from then onward is higher under the Ramsey plan than it is under the autarky plan. We call these truncated discounted values of utilities the tail utilities of the Ramsey plan and the autarky plan. Since the autarky plan is simply a static plan repeated over and over, it is relatively easy to evaluate its tail utilities. In contrast, the Ramsey plan is inherently dynamic, and it is consequently more difficult to say much about its tail utilities. Indeed, the hard part of what follows is establishing that the Ramsey plan has enough stationarity so that comparisons between it and the autarky plan can be made.

In Section VI.2, we show that if government consumption follows a positive Markov chain, then the Ramsey plan exhibits a weak form of stationarity: after any sequence of realizations there is some following state into which zero debt is sold. In Section VI.3, we exploit this stationarity to show that as long as tax-smoothing is desirable, then for discount factors close enough to one, the tail utilities of the Ramsey plan are always larger than the tail utilities of the autarky plan.

In Section VI.4, we give some examples to show that if government consumption is nonstationary, in the sense that the fluctuations in it eventually damp out, then the Ramsey outcomes cannot be sustained, regardless of the discount factor.

1. General setup

Consider the model with stochastic government consumption. Let government consumption follow a given stochastic process for which the realizations up to and including time t are $g^t = (g_0,\ldots,g_t)$. The probability of observing any particular event g^t is $\mu(g^t)$. The initial realization g_0 is given. Each realization g_t is assumed to be in a finite set $\{\gamma_1,\ldots,\gamma_m\}$ with $\gamma_1 < \ldots < \gamma_m$. There is no other uncertainty in the economy, so the natural space of allocations is the space of infinite sequences $(c,\ell,b) = \{c(g^t),\ell(g^t),b(g^t) \mid \text{for all } t \text{ and } g^t\}$, where $c(g^t)$, $\ell(g^t)$, and $b(g^t)$ are contingent on the events g^t. As in Section II, here we suppress the debt sold by private agents. We define similar objects for the policy π. Notice, in particular, that debt is now assumed to be state contingent.

An allocation is feasible if

$$(5.1) \qquad c(g^t) + g_t = \ell(g^t) \quad \text{for all } t \text{ and } g^t.$$

The preferences of consumers are given by the expected utility function

$$(5.2) \qquad \sum_t \sum_{g^t} \beta^t \mu(g^t) U(c(g^t),\ell(g^t)).$$

The consumer's budget constraint at time t in event g^t is

(5.3) $$c(g^t) - (1-\tau(g^t))\ell(g^t) + q(g^t)b(g^t) = (1-\delta(g^t))b(g^{t-1}).$$

The government's budget constraint is similarly defined. A Ramsey equilibrium can be defined analogously to that in Section II. From the arguments in Proposition 1, it is immediate that the Ramsey equilibrium allocations maximize (5.2) subject to the constraints (5.1),

(5.4) $$\sum_{t=0} \sum_{g^t} \beta^t \mu(g^t) R(g^t) = 0$$

(5.5) $$\sum_{t=s}^{\infty} \sum_{g^t \ni g^s} \beta^t \mu(g^t) R_t(g^t) \geq 0 \quad \text{for all s and } g^s$$

where $R(g^t) = U_c c(g^t) + U_\ell \ell(g^t)$ and $g^t \ni g^s$ means all those states g^t whose first s + 1 components are g^s.

Next, we incorporate history into the environment without commitment by letting the histories include the past realizations of government spending. For such an environment, it is immediate to prove the analogue of Proposition 3: an arbitrary pair of contingent sequences (π,x) is an outcome of a sustainable equilibrium if and only if the pair is attainable under commitment and for each s and g^s the following inequality holds:

(5.6) $$\sum_{t=s}^{\infty} \sum_{g^t \ni g^s} \beta^t \mu(g^t) U(c(g^t),\ell(g^t)) \geq \sum_{t=s}^{\infty} \sum_{g^t \ni g^s} \beta^t \mu(g^t) U^a(g_t).$$

2. Some characteristics of the Ramsey outcome

In this section, we develop some characteristics of the Ramsey outcome under the assumption that government consumption follows a positive Markov chain. The main characteristic is that, after any string g^t, there is always some state g_{t+1} into which zero debt is sold. We will exploit this rather weak type of stationarity in showing that the Ramsey outcome is sustainable. Consider, then,

Assumption 1 (A Positive Markov Chain). Government consumption follows a Markov process with strictly positive elements. Let (μ_{ij}) denote the transition matrix of this Markov chain, so that $\mu_{ij} = \text{Prob}(g_{t+1}=\gamma_j | g_t=\gamma_i)$. Let $\mu > 0$ denote the minimum transition probability.

We begin with a simple lemma about the structure of the optimal surplus function under the Ramsey plan. This surplus function is defined by the first-order conditions to the Ramsey problem. Let λ_0 denote the Lagrange multiplier on the government's budget constraint (5.4) and $\lambda_s(g^s)$ the Lagrange

multiplier on the nonnegativity constraint (5.5). Then we can write these first-order conditions as (5.1), (5.4), (5.5), and

(5.7) $$U_c(c(g^t),\ell(g^t)) + U_\ell(c(g^t),\ell(g^t)) + \left[\lambda_0 + \alpha(g^t)\right]\left[R_c(g^t) + R_\ell(g^t)\right] = 0$$

where $\alpha(g^t) = \Sigma_{s=1}^{t}[\lambda_s(g^s) \mid g^t \ni g^s]$ and $R_c(g^t)$ and $R_\ell(g^t)$ denote the partial derivatives of $R(g^t)$.

In the lemma, we establish a property of the surplus function $R(g^t)$ which we will use to prove Proposition 4. In words, the lemma establishes the following. Consider two strings of government consumption that end in the same value. If the Lagrange multipliers have never bound on the first string but have bound on the second, then the surplus under the first string must be smaller than the surplus under the second. Formally, we have

Lemma 2. If, for any two strings g^t and g^s with $g_t = g_s$, the solution to the Ramsey problem has $\alpha(g^t) = 0$ and $\alpha(g^s) > 0$, then it also has $R(g^t) < R(g^s)$.

Proof. If (c,ℓ) solves the Ramsey problem with $\alpha(g^t) = 0$ and $\alpha(g^s) > 0$, then, with other allocations fixed, $(c(g^t),\ell(g^t))$ and $(c(g^s),\ell(g^s))$ must solve

(5.8) $$\max\ \beta^t\mu(g^t)U(c(g^t),\ell(g^t)) + \beta^s\mu(g^s)U(c(g^s),\ell(g^s))$$

subject to

(5.9) $$\beta^t\mu(g^t)R(g^t) + \beta^s\mu(g^s)R(g^s) + K_0 = 0$$

(5.10) $$\beta^s\mu(g^s)R(g^s) + K(g^r) \geq 0 \quad \text{for each } g^r \in g^s,\ r = 1, \ldots, s$$

where K_0 is the sum of the discounted value of surpluses at all nodes except for g^t and g^s and where for each $g^r \in g^s$, $K(g^r)$ is the sum of the discounted value of surpluses at all nodes following g^r except for g^s. Notice that (5.9) is simply (5.4), and (5.10) represents those constraints in (5.5) in which g^s appears. Notice also that, since $\alpha(g^t) = 0$, we can exclude the nonnegativity constraints in which g^t appears. However, since $\alpha(g^s) > 0$, at least one of the nonnegativity constraints in (5.10) must bind.

Consider for a moment a less constrained problem than (5.8), with the nonnegativity constraints in (5.10) dropped. From the resulting first-order conditions, it follows that the optimal allocations at g^t and g^s are identical, and thus the optimal surpluses--say, $R^*(g^t)$ and $R^*(g^s)$--satisfy

$$(5.11) \qquad R^*(g^t) = R^*(g^s) = -K_0 / \left[\beta^t\mu(g^t) + \beta^s\mu(g^s)\right].$$

Now if this solution were feasible for the original problem (5.8), then it would solve it. Moreover, at this solution, all the Lagrange multipliers for the nonnegativity constraints would be zero. The hypothesis, however, implies that one of these multipliers must be strictly positive, and thus these allocations and the resulting optimal surpluses--say, $R(g^t)$ and $R(g^s)$--must violate at least one of the nonnegativity constraints. If the solution in (5.11) violates at least one constraint in (5.10), it must be that

$$(5.12) \qquad \frac{-K_0}{\beta^t\mu(g^t) + \beta^s\mu(g^s)} < \max_{g^r \in g^s} \frac{K(g^r)}{\beta^r\mu(g^r)}.$$

Since at the optimal solution at least one nonnegativity constraint binds, the right side of (5.12) equals $R(g^s)$. This, together with the fact that at a solution (5.9) holds, implies that

$$(5.13) \qquad \frac{\beta^t\mu(g^t)R(g^t) + \beta^s\mu(g^s)R(g^s)}{\beta^t\mu(g^t) + \beta^s\mu(g^s)} < R(g^s).$$

From cross-multiplying in (5.13), it follows that $R(g^t) < R(g^s)$. Q.E.D.

We use this lemma to establish the following:

Proposition 4 (A Characteristic of Ramsey Outcomes). For every g^t there is some γ, possibly depending on g^t, such that $b(g^t,\gamma) = 0$.

Proof. In the proof we find it convenient to use the following notation. For any g^t, let

$$\Delta(g^t) = \left\{g^v \mid \alpha(g^t,g^v)=0, \ r=0,1,2...\right\}$$

where (g^t,g^v) denotes a t + 1 element string g^t starting from g_0 followed by an r + 1 element string g^v which may not start at g_0.

Now, for any string g^t, either the nonnegativity constraints have never bound for any $g^s \in g^t$, implying that $\alpha(g^t) = 0$, or a constraint has bound for at least one $g^s \in g^t$, implying that $\alpha(g^t) > 0$. Consider, first, a string g^t with $\alpha(g^t) = 0$. We eventually show that $b(g^t,\gamma_k) = 0$, where γ_k denotes g_0. To show this, we first establish a preliminary result: Suppose that $\alpha(g^t) = 0$; then for each g^r, $\alpha(g^r) = 0$ if and only if $\alpha(g^t,g^r) = 0$. Put differently, this preliminary result is that $\Delta(g_0) = \Delta(g^t,\gamma_k)$.

It turns out that establishing this preliminary result is the hard part of the proof. Consider, first, the necessity. By way of contradiction, suppose that $\alpha(g^t) = 0$ and $\alpha(g^r) = 0$, but $\alpha(g^t,g^r) > 0$. Consider the first point along the string (g^t,g^r) such that the nonnegativity constraint binds, say, (g^t,g^s). We will use Lemma 2 to argue that the debt sold from this point is strictly positive, thus contradicting the hypothesis that the nonnegativity constraint binds at this point. First, by Lemma 2, for all $g^v \in \Delta(g^s)$,

$$R(g^s,g^v) < R(g^t,g^s,g^v). \tag{5.14}$$

Next, the value of the debt at the point g^s is the weighted average of terms of the form $R(g^s,g^v)$ over strings $g^v \in \Delta(g^s)$. [Notice that, since zero debt is sold from states in $\Delta(g^s)$ to states outside of $\Delta(g^s)$, the sum of the present value of surpluses over all nodes (g^s,g^v) with $g^v \notin \Delta(g^s)$ is zero.] The value of the debt at the point (g^t,g^s) is at least as large as the weighted average of revenues over strings (g^t,g^s,g^v) with $g^v \in \Delta(g^s)$. [Notice that, since the debt sold into any point is nonnegative, the present value of surpluses starting at nodes (g^t,g^s,g^v) with $g^v \notin \Delta(g^s)$ is nonnegative.] Finally, by the Markov assumption, the weights in these sums are identical, so that (5.14) implies that the debt at (g^t,g^s) is strictly larger than the debt at g^s, which by construction is nonnegative. Thus, the debt at (g^t,g^s) is strictly positive, which contradicts the supposition that the nonnegativity constraint binds at this point. The proof of sufficiency is nearly identical.

We use this result to prove that $b(g^t,\gamma_k) = 0$. The value of this debt is equal to

$$\beta^{t+1} \sum_{v=0}^{\infty} \sum_{(g^t,g^v)} \left[\beta^v \mu(g^t,g^v) R(g^t,g^v) \mid g^v \in \Delta(g^t,\gamma_k)\right]. \tag{5.15}$$

The value of the debt at $g_0 = \gamma_k$ is equal to

$$\sum_{v=0}^{\infty} \sum_{g^v} \left[\beta^v \mu(g^v) R(g^v) \mid g^v \in \Delta(g_0)\right]. \tag{5.16}$$

We claim that (5.15) equals (5.16), which equals zero. First, from the above result, $\Delta(g_0) = \Delta(g^t,\gamma_k)$, so both sums are over the same values of g^v. Second, since the constraints do not bind along all such strings, it follows from the first-order conditions that the value of surpluses at g^v and (g^t,g^v) are equal. Third, from the Markov assumption, each probability weight in (5.15) is the product of $\mu(g^t)$ and the corresponding weight in (5.16). Finally, since the value

of debt at time zero equals zero, it follows that the debt at (g^t,γ_k) also equals zero.

Consider, next, a string with $\alpha(g^t) > 0$. Consider the last point in the string at which the nonnegativity constraint bound, say, at $g^s \in g^t$ with $g_s = \gamma_\ell$. The problem from this point on is exactly the same as the original problem with γ_ℓ replacing γ_k. We can, then, use exactly the same argument as before, treating γ_ℓ as the initial node to conclude that $b(g^t,\gamma_\ell) = 0$. Q.E.D.

3. Sustaining Ramsey outcomes

In this section, we investigate the sustainability of the Ramsey outcomes under an additional assumption which guarantees, basically, that some tax-smoothing is optimal for large discount factors. More specifically, we make

Assumption 2 (Tax-Smoothing). There are two states γ_i and γ_j such that $R(\gamma_i) < R(\gamma_j)$, where $R(\gamma_i)$ and $R(\gamma_j)$ are given in the solution to the following problem: choose (c_i,ℓ_i) and (c_j,ℓ_j) to solve

(5.17) $$\max U(c_i,\ell_i) + \pi_{ij}U(c_j,\ell_j)$$

subject to

$$c_k + \gamma_k = \ell_k \quad \text{for } k = i, j$$

$$R(\gamma_i) + \pi_{ij}R(\gamma_j) = 0$$

$$R(\gamma_i) \leq 0$$

where $R(\gamma_k) = U_c c_k + U_\ell \ell_k$.

We will show that, under Assumptions 1 and 2, the Ramsey outcomes are sustainable when there is sufficiently little discounting. We begin with a lemma which states that the difference in utility under the Ramsey and autarky plans becomes arbitrarily large as the discount factor approaches one. We let $V^r(\gamma,\beta)$ and $V^a(\gamma,\beta)$ denote the discounted expected utility under the Ramsey and autarky plans, respectively, starting from an initial state γ. We have

Lemma 3. Under Assumptions 1 and 2, for every constant M, there is a $\underline{\beta}$ in (0,1) such that, for all β in $(\underline{\beta},1)$ and all initial states γ,

(5.18) $$V^r(\gamma,\beta) - V^a(\gamma,\beta) \geq M.$$

Proof. Let the initial state $g_0 = \gamma$ for some $\gamma \in \{\gamma_1,...,\gamma_k\}$. Let γ_i and γ_j be the states that satisfy Assumption 2. Consider problem (5.17) with π_{ij} replaced by $\beta\pi_{ij}$. Let $c_k(\beta)$ and $\ell_k(\beta)$ for k = i, j denote the optimal allocations and $R_k(\beta)$ for k = i, j denote the optimal surpluses for such a problem. Since these optimal surpluses are continuous functions of β, Assumption 2 guarantees there is a neighborhood of one--say, $(\underline{\beta},1)$--such that, for all β in this neighborhood, $R_j(\beta) < 0$. Thus, there is some $\epsilon > 0$ such that, for all β in this neighborhood,

$$(5.19)\qquad \left[U(c_i(\beta),\ell_i(\beta)) + \beta\pi_{ij}U(c_j(\beta),\ell_j(\beta))\right] - \left[U(c_i^a,\ell_i^a) + \beta\pi_{ij}U(c_j^a,\ell_j^a)\right] > \epsilon$$

where (c_k^a,ℓ_k^a), k = i, j, denote the autarky solutions.

Consider the following plan. For strings g^{t+1} with $g_t = \gamma_i$ and $g_{t+1} = \gamma_j$, let the allocations at g_t and g_{t+1} be given by $(c_i(\beta),\ell_i(\beta))$ and $(c_j(\beta),\ell_j(\beta))$. For all other strings g^{t+1}, let the allocations at g_t and g_{t+1} be the autarky allocations. Denote the present value of the plan by $V(\gamma,\beta)$. It is clear that this plan does no better than the Ramsey plan; thus, $V^r(\gamma,\beta) \geq V(\gamma,\beta)$. Moreover, the difference between this plan and autarky is given by

$$(5.20)\qquad V(\gamma,\beta) - V^a(\gamma,\beta) \geq \beta\mu\epsilon + \beta^2\mu\epsilon + \beta^3\mu\epsilon + ...$$

where μ is the minimum transition probability. To understand (5.20), notice that for every string g^t there is a probability of at least μ of hitting γ_i at date t + 1. From (5.19), it follows that the expected discounted value at time zero of the difference between the above plan and the autarky plan from time t to time t + 1 is at least $\beta^t\mu\epsilon$. Since the right side of (5.20) does not depend on the initial state and $\beta\mu\epsilon/(1-\beta)$ converges to infinity as β converges to one, the proposition follows. Q.E.D.

We can combine Proposition 4 with Lemma 3 to establish the main result of this section: For large discount factors, the Ramsey outcomes are sustainable. The basic logic is as follows. Proposition 4 guarantees that after any string of government consumption, there is some state into which zero debt is sold. Notice that if zero debt is sold into a state, then the piece of the Ramsey problem defined over all strings emanating from that state has the same form as the original problem with the zero-debt state acting as an initial node. Lemma 3 guarantees that the difference in the tail utilities between the Ramsey and autarky plans goes to infinity after any zero-debt state. To prove the proposition, we show that this difference goes to infinity after any sequence of government consumption. The Markov assumption guarantees that after any such sequence the probability of never hitting a zero-debt state is small enough so that the utility difference between these plans is dominated by the utility difference after zero-debt states. Consequently, this difference goes to infinity as the discount factor approaches one. We have, then,

Proposition 5 (The Sustainability of Ramsey Outcomes). Under Assumptions 1 and 2, there is some $\underline{\beta}$ in (0,1) such that for all β in $(\underline{\beta},1)$ the Ramsey outcome is sustainable.

Proof. Let $U^r(g^t)$ denote the value of utility at g^t under the Ramsey plan. From (5.6), it follows that the Ramsey plan is sustainable if, for each g^s,

$$\sum_{t=s}^{\infty} \sum_{g^t \ni g^s} \beta^t \mu(g^t)\left[U^r(g^t) - U^a(g_t)\right] \geq 0. \tag{5.21}$$

We can divide this sum over all strings g^t which include g^s into two sets of strings. The first set consists of all those strings into which zero debt is sold together with strings that follow such a string. The second set consists of all those strings g^t for which positive debt has been sold into all substrings $g^r \in g^t$ for $r \geq s$.

Proposition 4 guarantees that for any string g^t, there is always some state at $t + 1$--say, $\gamma(g^t)$--into which zero debt is sold. Each such string $((g^t,\gamma(g^t))$ is like an initial node to a Ramsey problem starting at $\gamma(g^t)$. Lemma 3 then guarantees that the present value of the utility difference over strings following $(g^t,\gamma(g^t))$ goes to infinity as β approaches one. Thus, we are done if we can show that the utility difference over the second set of states is bounded. Since zero debt is always sold into some state, it follows that this utility difference is at most

$$\underline{u} + (1-\mu)\beta\underline{u} + (1-\mu)^2\beta^2\underline{u} + \ldots \tag{5.22}$$

where $\underline{u}$ is the maximal difference between any two utility values and μ is the minimum transition probability. Since for all β in (0,1) the sum in (5.22) is bounded by the constant $\underline{u}/\mu$, we are done. Q.E.D.

4. Some examples

In proving the Ramsey outcomes were sustainable, we made two assumptions. The first assumption guaranteed that the fluctuations in government consumption were sufficiently regular throughout time. Indeed, under this assumption, these fluctuations could never damp out. The second assumption guaranteed that there was a utility loss between autarky and the Ramsey plans over any two consecutive points in time. Taken together, these assumptions implied that the expected utility loss from deviating to autarky grows large for large discount factors.

The second assumption is fairly innocuous. It is easy to show that it is satisfied for standard utility functions like Cobb-Douglas, constant relative risk aversion, and quadratic. More important, without some such assumption, there would be no gains to tax-smoothing and the Ramsey plan would not involve selling any debt. Hence, there would be no time consistency problem to start with.

It is more interesting to investigate relaxing the first assumption. We present several examples of processes for government consumption that violate the

assumption in which the Ramsey outcome is not sustainable for any nonzero discount factor. In the examples, it is convenient to let the smallest level of government consumption be zero, so $\gamma_1 = 0$.

Example 1 (Eventually Constant Government Consumption). Let there be some T such that government consumption is zero after T; that is, $\mu(g^t,0) = 1$ for all $t \geq T$. As long as government consumption is positive for some $t < T$, it is easy to show that in the Ramsey equilibrium the inherited debt at T is positive and that tax rates are constant and positive for all $t \geq T$. In particular, for $g_t = 0$, the autarky allocation solves this problem:

$$\max_{c_t, \ell_t} U(c_t, \ell_t)$$

subject to

$$c_t \leq \ell_t.$$

Thus, for all $t \geq T$, all feasible allocations must satisfy this:

$$\sum_{s=t}^{\infty} \beta^{s-t} U(c_s, \ell_s) \leq \sum_{s=t}^{\infty} \beta^{s-t} U^a(0). \tag{5.23}$$

Combining (5.6) for $t \geq T$ with (5.23) implies that $b_t(g^t)$ and $\tau_t(g^t)$ are zero for all $t \geq T$. Recursively using consumer and government budget constraints, this implies that $b_t(g^t) = 0$ for all $t < T$.

We can explain this result intuitively as follows. Consider, first, the problem faced by the government at some $t \geq T$. If it inherits any positive debt, then, regardless of the policies of future governments, it is optimal to default on this debt, that is, set δ_t equal to one. Consider, next, the problem faced by the government at T - 1. The equilibrium price of any debt it issues is zero; hence, no revenues can be raised by selling debt. Consequently, if this government inherits any debt, it is optimal to default on it by setting δ_{T-1} equal to one. From repeating this argument for all $t < T$, it follows that all governments default and debt always has zero value. Thus, the Ramsey policy is not sustainable for such a process of government consumption, regardless of the discount factor. Similar arguments apply if government consumption eventually settles down to any constant amount. Notice that, in this example, date T acts like a finite endpoint, and then backward induction unwinds any strategy other than "autarky forever."

Example 2 (A Markov Chain With Absorbing States). Let government consumption be a Markov chain with an absorbing state of zero. It is easy to show that, for many utility functions, the Ramsey plan involves selling positive debt into all strings ending with a zero state. Every such string--namely, any $(g^{t-1},0)$--acts as a finite endpoint, and a backward induction argument similar to that of Example 1 from such an endpoint implies that the Ramsey plan is not

sustainable. In contrast to the certainty case, however, here the backward induction does not unwind all nonautarky plans. Indeed, it is easy to show that there are sustainable plans which involve selling positive debt into some nonzero states. Thus, in this example, the sustainable plan that yields the highest utility will lie strictly between autarky and the Ramsey plan, even for discount factors close to one.

Example 3 (Eventually Absorbing States). Let $g_t \in \{0,\gamma_1\}$ follow a time-dependent Markov chain with this transition matrix:

$$\begin{bmatrix} 1/t & 1-1/t \\ 1-\mu & \mu \end{bmatrix}.$$

For any discount factors, there is a large enough t so that we can use arguments similar to those in Example 2 to show that the Ramsey outcomes are not sustainable.

5. A parametric example

We present a parametric example to illustrate several features of sustainable outcomes and their associated utility levels. First, for low enough discount factors, the only sustainable outcome is autarky. Second, if a certain outcome is sustainable for some discount factor, then it is sustainable for a larger discount factor. Third, for a large enough value of the discount factor, all utilities between autarky and Ramsey are sustainable. In addition, we characterize the set of sustainable outcomes for a sequence of mean-preserving spreads of government consumption.

Let the utility function be

$$U(c,\ell) = c^{\alpha}(\bar{\ell}-\ell)^{1-\alpha}$$

with $\alpha = 0.33$ and $\bar{\ell} = 10$. Let government consumption be i.i.d. and take on two values γ_1 and γ_2 with probabilities $\mu_1 = \mu_2 = 0.5$. Let γ_1 be the initial state. We focus on stationary outcomes in which the allocations and the policies depend only on the current realization of government consumption. Thus, for example, $c(g^{t-1},\gamma_i) = c(\gamma_i)$ and $\ell(g^{t-1},\gamma_i) = \ell(\gamma_i)$ for all g^{t-1}.

Recall that an arbitrary pair of contingent sequences (π,x) is the outcome of a sustainable equilibrium if and only if it is attainable under commitment and satisfies the inequalities in (5.6). An outcome is attainable under commitment if it satisfies feasibility together with (5.4) and (5.5). For our stationary example, (5.4) and (5.5) reduce to

$$(5.24) \qquad R(\gamma_1) + \frac{\beta}{1-\beta}[\mu_1 R(\gamma_1) + \mu_2 R(\gamma_2)] = 0$$

and

$$(5.25) \qquad R(\gamma_2) + \frac{\beta}{1-\beta}[\mu_1 R(\gamma_1) + \mu_2 R(\gamma_2)] \geq 0.$$

The inequalities in (5.6) reduce to, for i = 1, 2,

$$(5.26) \qquad U(\gamma_i) + \frac{\beta}{1-\beta}[\mu_1 U(\gamma_1) + \mu_2 U(\gamma_2)]$$

$$\geq U^a(\gamma_i) + \frac{\beta}{1-\beta}[\mu_1 U^a(\gamma_1) + \mu_2 U^a(\gamma_2)]$$

where $U(\gamma_i) = U(c(\gamma_i),\ell(\gamma_i))$. We find it convenient to normalize the utility of an allocation by dividing by the utility in autarky. Thus, the normalized utility of an allocation is defined to be

$$(5.27) \qquad \frac{(1-\beta)U(\gamma_1) + \beta[\mu_1 U(\gamma_1) + \mu_2 U(\gamma_2)]}{(1-\beta)U^a(\gamma_1) + \beta[\mu_1 U^a(\gamma_1) + \mu_2 U^a(\gamma_2)]}.$$

In Figure 1, we plot the set of normalized utilities resulting from stationary sustainable allocations against the discount factor for $\gamma_1 = 2$ and $\gamma_2 = 1$. Three things are notable here. First, for $\beta \leq 0.7$, autarky is the only stationary sustainable allocation. Second, if a particular allocation is sustainable for some discount factor, then it is sustainable for a larger discount factor. Finally, for $\beta \geq 0.83$, all allocations with utilities between autarky and Ramsey are sustainable.

We now consider a sequence of mean-preserving spreads of government consumption from $\gamma_1 = \gamma_2 = 1.5$ up through $\gamma_1 = 2$ and $\gamma_2 = 1$. In Figure 2, we plot the set of stationary sustainable utilities against the variance of government consumption at $\beta = 0.9$. Note that the sustainable utility set is monotonically increasing in the variance. Intuitively, as government consumption becomes more variable, the benefits to tax-smoothing increase, so the punishment of reverting to autarky forever becomes more severe. This example highlights the connection between the variability of government consumption and the sustainability of good outcomes.

VII. Anonymous Games

In this section, we provide one rationalization of the equilibria considered in the previous sections, but in a game-theoretic context. In particular, we make precise the relationship between the equilibria of the economies with and without commitment and the perfect equilibrium of certain games. We first show that the Ramsey equilibrium is the unique subgame perfect equilibrium of a game with commitment. More importantly, we then show that the set of sustainable equilibria corresponds to the set of symmetric perfect Bayesian equilibria of a game with no commitment.

In the economies considered earlier, we modeled private agents as behaving competitively, in the sense that they assume policies are unaffected by their decisions. We capture this feature in a game using two assumptions. First, we assume there is a continuum of agents. Second, we assume individuals observe

only their own decisions and aggregate outcomes. A game with this feature is called an anonymous game (Green 1980, 1984).

In those earlier economies, it was notationally convenient to suppress an individual agent's decision to default and instead to build the consequences of private default directly into the equilibrium. While it is possible to do exactly the same here, we find it useful to instead work through the details of private default. To that end, we now let consumers decide on both how much government debt to buy and how much private debt to sell to the government. We let the government choose a price for the debt it sells and a price at which it is willing to buy private debt. Private agents can default on the debt they sell the government, and the government can default on the debt it sells to private agents.

1. General setup

There is a continuum of private agents represented by Lebesque measure λ on the interval [0,1] and a player called the government. A policy for the government is a vector π consisting of a default rate δ, a tax rate τ, a price for government debt q, and a price for private debt p. An action profile for private agents is a vector of measurable functions $x = (\ell,b,d,\theta)$ defined on the unit interval. The implied action for agent i is $x(i) = (\ell(i),b(i),d(i),\theta(i))$. For each i, $\ell(i)$ denotes the amount worked; b(i), the amount of government debt purchased; d(i), the amount of private debt sold; and $\theta(i)$, the default rate on private debt.

Now, for any t, let $L_t = \int \ell_t(i)\lambda(di)$ denote the aggregate amount of labor, and let B_t and D_t denote the aggregate amounts of government and private debt. Let $N_t = \int (1-\theta_t(i))d_{t-1}(i)\lambda(di)$ denote the aggregate net payment on private debt. The period t payoffs of agent i can then be written as

$$(6.1) \qquad V_i(\pi_t,x_t(i),x_{t-1}(i),x_t,x_{t-1}) = U(c_t(i),\ell_t(i)) + W(\pi_t,L_t,B_t,B_{t-1},D_t,N_t)$$

where $c_t(i) \equiv (1-\tau_t)\ell_t(i) + (1-\delta_t)b_{t-1}(i) - q_tb_t(i) - (1-\theta_t(i))d_{t-1}(i) + p_td_t(i)$ and where the function W equals zero if its arguments satisfy

$$(6.2) \qquad g_t \leq \tau_tL_t - (1-\delta_t)B_{t-1} + q_tB_t + N_t - p_tD_t$$

and some large negative number--say, -M--otherwise. The government's period t payoff is

$$(6.3) \qquad V_i(\pi_t,x_t,x_{t-1}) = \int V_i(\pi_t,x_t(i),x_{t-1}(i),x_t,x_{t-1})\lambda(di).$$

Recall that, in the usual definition of a game, there are no budget constraints. The function W incorporates the budget constraint of the government into its preferences in such a way that it will seek to balance the budget.

In what follows, we consider infinite horizon games in which agents maximize the discounted present value of the stream of single-period payoffs.

2. Commitment game

In a commitment game, the government first chooses an infinite sequence of policies $\pi = (\pi_t)_0^\infty$. A strategy for the government is thus just an infinite sequence of policies. Private agents, having seen π, then make their decisions. A strategy profile for private agents is a sequence of functions $f = (f_t)_0^\infty$ that map policies π into action profiles x. A strategy profile f naturally induces strategies for each agent of the form $f_t(i,\pi)$ for every period. For private agents, the payoffs over strategies are defined to be

$$(6.4) \qquad \sum_{t=0}^{\infty} \beta^t V_i(\pi_t, f_t(i,\pi), f_t(\pi), f_{t-1}(i,\pi), f_{t-1}(\pi)).$$

Likewise, the payoffs for the government are

$$(6.5) \qquad \sum_{t=0}^{\infty} \beta^t V(\pi_t, f_t(\pi), f_{t-1}(\pi)).$$

We can now define an equilibrium: A subgame perfect equilibrium for the commitment game is a strategy π for the government and a strategy profile f for private agents that satisfy two conditions: (i) For each agent i, given the strategies of other agents as specified by f and any policy π' for the government, the strategy $f(i,\pi')$ maximizes agent i's payoff; and (ii) given the strategy profile f, the strategy π maximizes the government's payoff. Comparing this definition with the Ramsey equilibrium of Section II, we immediately have

Proposition 6 (Equilibrium Outcomes for the Commitment Game). The subgame perfect equilibrium policies and allocations $(\pi, f(\pi))$ of the commitment game are identical (almost everywhere) to the Ramsey policies and allocations.

Proof. First, it is clear that in any equilibrium the value of private claims is zero. To see this, notice that if consumer i has borrowed a strictly positive amount, so $d_{t-1}(i) > 0$, then it is optimal for this consumer to default completely by setting $\theta_t(i)$ equal to one. Anticipating these decisions by consumers, the government sets the price it will pay for such private claims equal to zero; thus, $p_t = 0$. Using this fact, we can immediately see from the definitions that the set of sustainable outcomes almost everywhere equals the set of subgame perfect equilibrium outcomes. Q.E.D.

3. No-commitment game

Next, consider a game without a commitment technology. Let the timing of the moves be the same as in the no-commitment infinite horizon economy. In defining this game, we must be careful about what the players have observed when they make their decisions. We formalize what players have observed by defining histories both of the game and for the players. The history of the game is a complete description of all the actions chosen in the past by all the players. In particular, at the time of the government's decision in period t, the history of the game is $h_{t-1} = (x_s, \pi_s | s<t)$. At the time of the private agents' decisions in

period t, the history of the game is $h_{1t} = (h_{t-1},\pi_t)$. The government observes only aggregate outcomes; thus, a history for the government at the time it moves in period t is $H_t = (X_s,\pi_s | s<t)$, where $X_s = (L_s,B_s,D_s,N_s)$. Each individual observes aggregate outcomes together with the individual's own past decisions. Thus, a history for player i at the time this player moves in period t is $h_{1t}(i) = ((x_s(i),X_s,\pi_s | s<t),\pi_t)$. In what follows, players' histories correspond to information sets in the obvious way. For example, the individual history $h_{1t}(i)$ of player i at time t corresponds to the information set consisting of all histories of the game h_{1t} that are consistent with $h_{1t}(i)$.

Consider, next, the strategies for the players in the game. A strategy for the government is a sequence of functions $\sigma = (\sigma_t)_{t=0}^{\infty}$ which, for each t, map government histories H_t into policies π_t. A strategy profile for private agents is a sequence of functions $f = (f_t)_{t=0}^{\infty}$ which map histories of the game into action profiles. A strategy profile naturally induces strategies for each agent of the form $f_t(i,h_{1t})$. To be consistent with our informational restrictions, we require that, for each i, $f_t(i,\cdot)$ depends only on individual histories. [Technically, we require that $f_t(\cdot,\cdot)$ be measurable with respect to the σ-algebra generated by the individual histories.] Such profiles will be called <u>anonymous</u> profiles.

Payoffs for the players are naturally defined from the outcomes that the strategies induce. For example, the payoff for player i at date t, given a history of the game h_{1t}, is

$$W_{it}(\sigma,f(i),f;h_{1t}) = \sum_{s=t}^{\infty}\beta^s V_i(\pi_s,x_s(i),x_{s-1}(i),x_s,x_{s-1}) \tag{6.6}$$

where the future actions are induced from h_{1t} by f and σ. The payoff for the government at date t, given a history of the game h_{t-1}, is similarly defined:

$$W_t(\sigma,f;\ h_{t-1}) = \sum_{s=t}^{\infty}\beta^s V(\pi_s,x_s,x_{s-1}) \tag{6.7}$$

where the future actions are induced from h_t by f and σ.

Now we want to define some type of perfect equilibrium for this game. One approach would be to consider subgame perfect equilibrium. Given the informational restrictions, however, the only proper subgame is the original game itself; hence, any Nash equilibrium is subgame perfect. An alternative is to consider a type of Bayesian equilibrium. (See, for example, Fudenberg and Tirole 1988.)

A Bayesian equilibrium consists of strategy profiles together with a sequence of probability distributions. For every information set, there is a probability distribution over histories of the game consistent with that information set. Let $\mu(h_{1t} | h_{1t}(i))$ denote a probability distribution over the histories of the game h_{1t}

that are consistent with the information set associated with player i's history $h_{1t}(i)$. Likewise, let $\mu(h_{t-1} | H_{t-1})$ denote probability distributions at a government information set. Let μ denote the collection of these probability distributions. Given some collection of probability distributions μ and strategies σ and f, we can use (6.6) to write the expected utility of player i at the information set associated with history $h_{1t}(i)$ as

$$\int W_{it}(\sigma, f(i), f; h_{1t}) d\mu(h_{1t} | h_{1t}(i)).$$

We use (6.7) to define the expected utility for the government at the information set associated with a history H_{t-1} as

$$\int W_t(\sigma, f; h_{t-1}) d\mu(h_{t-1} | H_{t-1}).$$

In the equilibrium of Sections II and III, we used a representative agent to model the private agents. The standard interpretation is that the representative agent stands in for a large number of competitive private agents who, by construction, act identically in equilibrium. The way to model a game to keep the analysis parallel with a representative agent model is to require symmetry of the equilibria. In the commitment game, it is easy to see that all the equilibria are (almost everywhere) symmetric, so we did not need to impose symmetry. In the no-commitment game, there are typically asymmetric equilibria; hence, we require symmetry for that game.

A symmetric perfect Bayesian equilibrium is an anonymous strategy profile f, a government strategy σ, and a collection of probability distributions μ such that five conditions hold: (i) For each player i, period t, and history $h_{1t}(i)$, the continuation of the strategy f(i) maximizes player i's expected payoff; (ii) for each period t and history H_t, the continuation of σ maximizes the government's expected payoff; (iii) μ assigns probability one to histories of the game along the equilibrium path; (iv) the strategies of consumers are symmetric; and (v) μ assigns probability one to symmetric histories (both on and off the equilibrium path).

To understand condition (iii), consider, for example, a probability distribution over an information set of the government at date one. The history of the game along the equilibrium path is $h_1 = x_1 = f_1(h_{10})$, where h_{10} is a null history. This history of the game is a member of the government's information set corresponding to the history $H_1 = X_1 = \int f(i, h_{10}) \lambda(di)$. Condition (iii) requires that μ assign probability one to this history--namely, $\mu(h_1 | H_1) = 1$--and probability zero to any other history--$h_1' \neq f_1(h_{10})$--in this information set. Notice that condition (iii) places no restrictions on μ for histories of the game off the equilibrium path.

In condition (v), a symmetric history of the game for player i is one in which all private players other than i have chosen the same actions in the past. A symmetric history of the game for the government is one in which all private players have chosen the same action in the past. Thus, condition (v) requires, for example, that at any information set player i believes with probability one that all the other private agents have behaved symmetrically in the past.

Proposition 7 (Equilibrium Outcomes of the No-Commitment Game). The set of symmetric perfect Bayesian equilibrium policies and allocations of the no-commitment game is the same as the set of sustainable equilibrium policies and allocations.

The proof of this proposition is nearly identical to the one in our earlier paper (Chari and Kehoe 1989). Here, we present an intuitive explanation and refer readers to that paper for details. The essential difference between the definitions of a sustainable equilibrium and a symmetric perfect Bayesian equilibrium is that the latter requires rationality after histories with private deviations whereas the former does not even consider such histories. The main point of Proposition 7 is that the extra conditions in the symmetric perfect Bayesian equilibrium imposed after such histories are irrelevant to the set of outcome paths. Part of the proof relies on a straightforward result from game theory. Consider two equilibria for which the strategy profiles coincide for all histories in which there have not been simultaneous deviations in the past. The result is that these two equilibria generate identical outcome paths. The reason is simply that, in checking whether a deviation from a candidate equilibrium is profitable, the definition of equilibrium requires us to check only deviations by a single player. Intuitively, after any history in which there have been no simultaneous deviations, no player acting alone can induce future histories in which there will be simultaneous deviations. Thus, regardless of how we specify the continuation equilibrium after simultaneous deviations (as long as it is some equilibrium), we get the same outcome path.

In our anonymous game, the only type of deviations by private agents that can influence the future behavior of other private agents or the government (by affecting their information sets) are ones in which a positive measure of agents deviate simultaneously and change the aggregate outcomes. By this result, we can ignore such deviations, in the sense that no matter how we fill in the continuation equilibrium after such histories, we get the same outcome path. Moreover, given that no single private agent's deviation can affect the payoff or the information sets reached by other agents, we can ignore single deviations by private agents. Putting these results together and using the definitions of sustainable outcomes and symmetric perfect Bayesian outcomes, the result follows.

VIII. Conclusion

In this paper, we have analyzed the issue of default on the government debt in a model where private agents cannot be forced to honor their debt. We have completely characterized the set of sustainable outcomes and have shown that, if government consumption fluctuates enough, Ramsey outcomes are sustainable with sufficiently little discounting. We have also provided examples where Ramsey outcomes are not sustainable, regardless of the discount factor. In addition, we have related our work to recent developments in game theory.

Figure 1

Sustainable Utilities vs. The Discount Factor

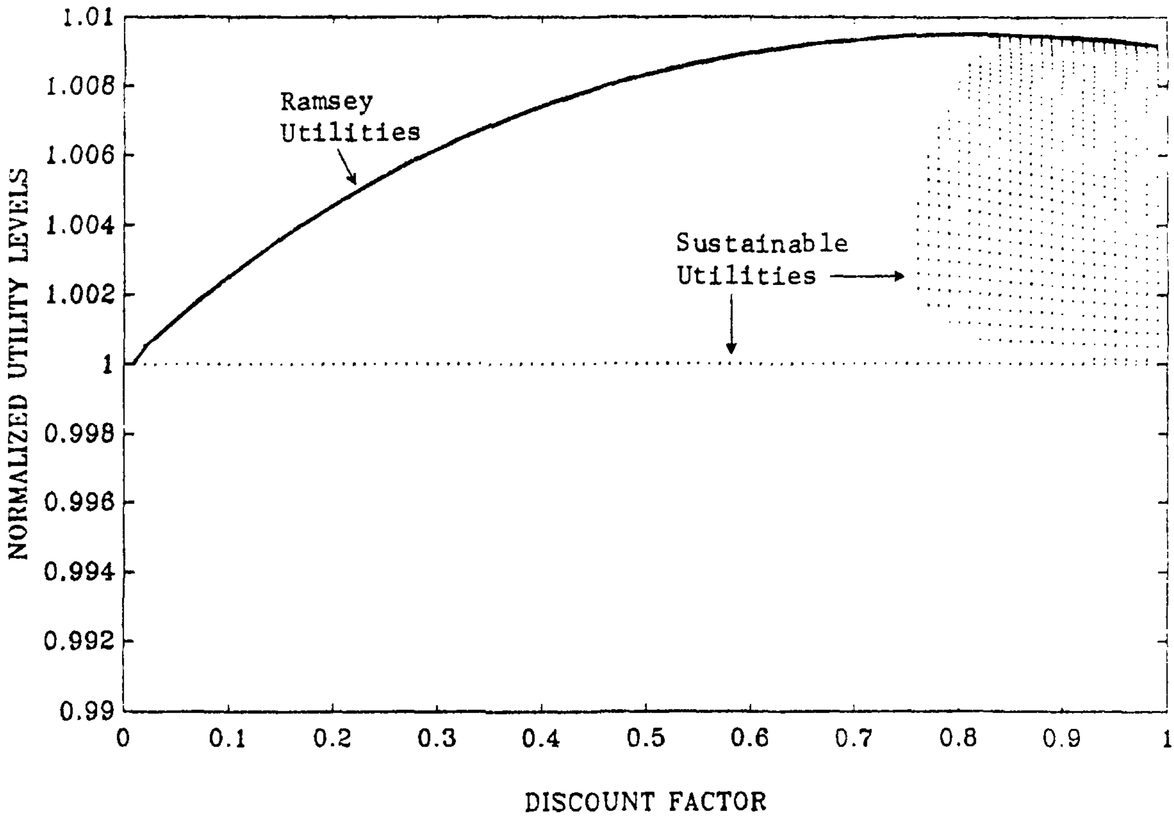
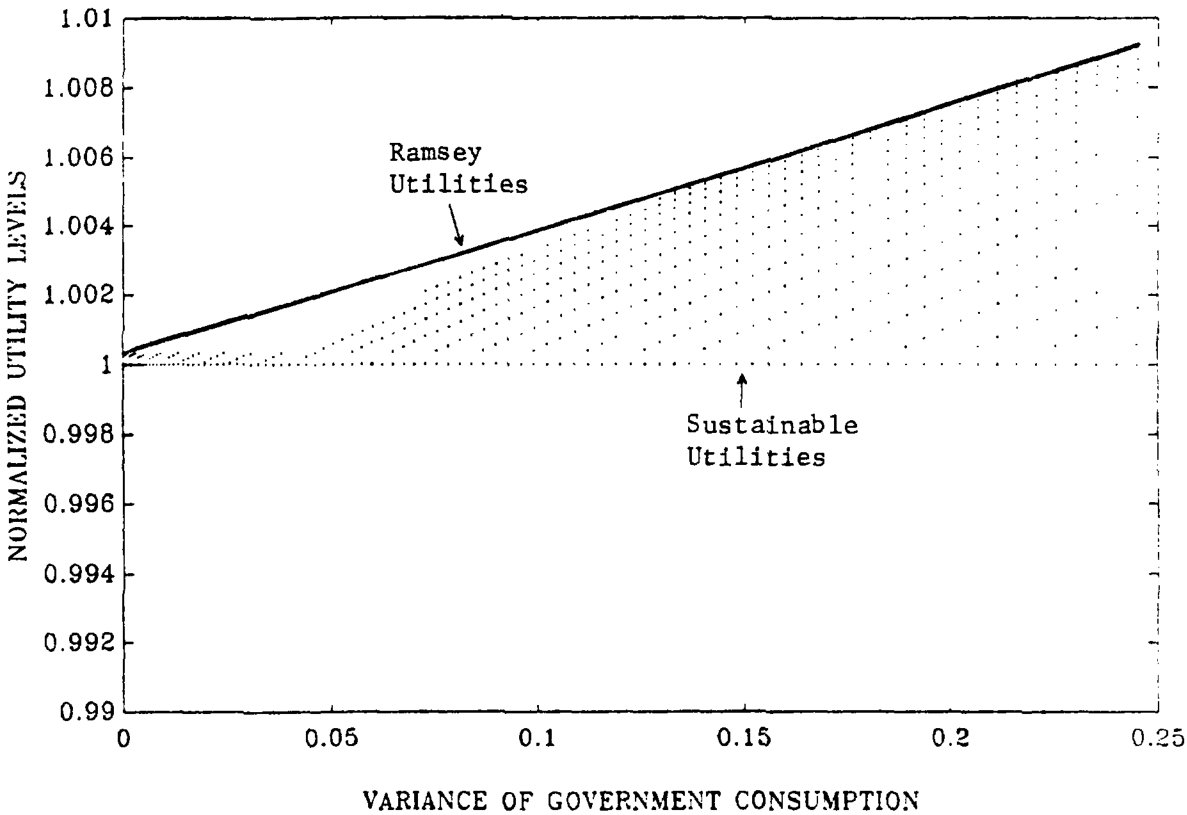


Figure 2

Sustainable Utilities vs. The Variance of Government Consumption



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