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Interest Rate Targeting in a Small Open Economy: The Predetermined Exchange Rates Case*

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Abstract

An important hurdle in analyzing interest rate targeting is that standard models usually lead to price level or inflation rate indeterminacy. This paper develops a simple framework in which such problems do not arise because the bonds whose interest rate is controlled provide liquidity services. This framework is used to examine interest rate targeting in a small open economy under predetermined exchange rates. A permanent increase in the interest rate has no real effects. In contrast, a temporary increase in the interest rate leads to higher consumption and to a current account deficit that worsens over time.

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I. Introduction

The interest rate enjoys a unique position among macroeconomic adjustment policies. Historically, it played a central role during the Gold Standard era (despite the economic theorist insistence on specie-flow mechanisms). Currently, it is one of the most watched variables among the G7 countries. In developing countries, interest rates have been manipulated, among other things, to provide cheap credit to the government, to increase savings and investment, and to try to quell raging inflation. This high reputation of interest-rate policy among economic practitioners contrasts sharply with the one it has among their more academic peers. Wicksell (1965) thought that pegging the interest rate, for example, might lead to instability of the inflation rate—a conjecture which also emerges from standard adaptive-expectations models—while Rational Expectations theory shows the possibility that, unless there is some additional nominal anchor, an interest rate policy may lead to indeterminacy of the price level and/or of the rate of inflation (see, for instance, Calvo (1983), Canzoneri, Henderson, and Rogoff (1983), Gagnon and Henderson (1989), McCallum (1981, 1986), and Sargent and Wallace (1975)).

Given the importance that policy-makers sometimes attach to controlling interest rates in order to achieve certain objectives, it would seem important to develop an analytical framework within which interest rate policy can be analyzed. The present paper is an attempt in that direction; it develops a model in which interest rate policy is free from the above-mentioned indeterminacy or instability problems. This

1/ Hume (1752) provided the classical exposition of the price-specie-flow mechanism. Governments' frequent disregard for the "rules of the game" under the Gold Standard is the focus of the seminal work of Bloomfield (1959). At the time, observers were well aware of the active use of interest rates by Central Banks. Keynes (1924), for instance, argued that one of the main characteristics of the British monetary system under the Gold Standard was "the use of the bank rate for regulating the balance of immediate foreign indebtedness (and hence the flow, by import and export, of gold)" (p. 19).

2/ Batten et al (1989) examine the implementation of monetary policy in the G5 countries and conclude that monetary authorities focus on influencing key short-term interest rates.

3/ See Fry (1988) for a discussion of interest rate policies in developing countries.

4/ McCallum (1986) distinguishes between price level "indeterminacy" (the model does not determine the value of any nominal magnitude) and "non-uniqueness or multiplicity" of price level solutions (there are multiple paths of real money balances).

5/ Barro (1989) suggests that interest rates should play a major role in a positive theory of monetary policy because "central bankers, including those at the Federal Reserve, seem to talk mainly in terms of controlling or targeting interest rates" (p.3).
is achieved by postulating that the interest rate which is controlled by the Central Bank is, in fact, not a pure rate, but rather the interest rate borne by bonds that possess some of the characteristics of domestic money. It is shown that just a "pinch of liquidity" in the assets whose interest rate is controlled by the Central Bank is in general enough to restore full monetary determination. This is an interesting result because it exhibits the relative weakness of the Rational Expectations case against interest rate policy given that Central Banks usually deal with government assets that are not substantially different from high-powered money--see, for example, the discussion in Bryant and Wallace (1979). This constitutes the first central message of the paper.

The analysis concentrates on a small open economy under predetermined exchange rates in which price flexibility and full employment prevail. In addition, and to abstract from other well-known effects, we make sufficient assumptions to guarantee inflation super-neutrality and Ricardian equivalence. These assumptions are strong and militate against getting almost any effect from monetary policy (see Calvo (1989b)). They are responsible in part for the absence of real effects stemming from permanent changes in monetary and interest rate policy. 1/ These assumptions, however, help to highlight the role of temporary policies--what actual policy-making is almost all about--because temporary policies are shown to have real effects even under those strong pro-neutrality conditions. This is the second central message of the paper.

We develop a simple model which proves useful to understand some of the basic issues raised by interest rate policy. The discussion is carried out in terms of a representative consumer subject to a liquidity constraint. To introduce the liquidity component into domestic bonds, we assume that the consumer "produces" liquidity by means of high-powered money and bonds (which we identify with interest-bearing checking accounts). In the present context, therefore, increasing the interest rate on bonds is equivalent to offering a higher return on money (defined as the sum of high-powered money and interest-bearing checking deposits). Hence, it should not come as a big surprise that an interest rate hike could have expansionary effects contrary, however, to the predictions of more ad-hoc models. This is the third central message of the paper. We feel that this message could be very relevant to countries that attempt to influence aggregate demand by increasing the rate of interest. This policy tends to reduce the cost of holding liquid assets and may, therefore, put no downward pressure on aggregate demand. 2/

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1/ By real effects, we mean effects on consumption--which determines welfare. Permanent policies, however, will affect the real money supply. 2/ Another independent, and more familiar, reason for the counterproductive nature of an interest rate hike in high-inflation countries is that in many cases the government is one of the main borrowers. Thus, the policy tends to worsen the fiscal situation even further.
It is shown that a temporary increase in the rate of interest is equivalent to a temporary decrease of the rate of devaluation. They are both expansionary because they reduce the opportunity cost of holding money, and consequently tend to stimulate consumption. The resulting increase in money demand induces an initial accumulation of reserves at the Central Bank. The current account, however, deteriorates from the first moment, and reserves are eventually lost. The country is eventually poorer and, more surprisingly, the representative individual also feels poorer from the very start. The policy has, therefore, no redeeming social value.

The paper proceeds as follows. Section II develops the basic model and shows that permanent changes in the nominal interest rate have no real effects. Section III studies the effects of a temporary increase in the nominal interest rate. Section IV closes the paper by relating the present analysis to interest rate policy in more complex models.

II. The Model

Consider a small open economy that operates under predetermined exchange rates. There exists only one (non-storable) good in the world whose price is given and equal to unity. The representative consumer is endowed with a constant flow of the good denoted by \( y \).

The utility function of the representative consumer is given by

\[
\int_0^\infty u(c_t)\exp(-\beta t)dt,
\]

where the instantaneous utility function, \( u(.) \), is assumed to be increasing, twice-continuously differentiable, and strictly concave; \( c_t \) denotes consumption; and \( \beta \) is the positive and constant subjective discount rate.

The consumer is subject to a "liquidity-in-advance" constraint. The requirement that a liquid asset (money) be used in order to purchase goods is by now a common feature of monetary models. 1/ A more novel feature introduced into the present analysis—which explains the use of the term "liquidity-in-advance"—is that the consumer is posited to use two distinct liquid assets to carry out his or her purchases. In addition to cash \( (H) \), the consumer makes use of demand deposits \( (Z) \) that earn interest at a rate \( i \) (for instance, NOW or Money Market Accounts). 2/

This formulation is intended to capture, in a simple way, the stylized fact that consumers usually resort to both types of assets to carry out

1/ The use of the cash-in-advance constraint in continuous-time models is discussed in Feenstra (1985).

2/ The sum of cash and demand deposits will be referred to as "money" \( (M) \); that is, \( M=H+Z \).
their transactions. \textsuperscript{1} Specifically, the liquidity-in-advance constraint is given by

\begin{equation}
(2) \quad c_t \leq \ell(h_t, z_t), \quad \ell_h > 0, \quad \ell_z > 0, \quad \ell_{hh} < 0, \quad \ell_{zz} < 0, \quad \ell_{hz} > 0,
\end{equation}

where $h_t$ and $z_t$ stand for real balances of cash and demand deposits, respectively; and $\ell(\cdot)$ is a concave, homogenous of degree one, twice-continuously differentiable function, which can be viewed as a liquidity-services production function. \textsuperscript{2} \textsuperscript{3} The liquidity-in-advance constraint (2) requires that consumption do not exceed the liquidity services produced by the use of both cash and demand deposits.

The consumer also holds an internationally traded bond, $f$, whose rate of return is given and equal to $r$. The consumer's lifetime budget constraint is therefore

\begin{equation}
(3) \quad a_0 + \int_0^\infty (y + r_t) \exp(-rt) dt = \int_0^\infty [c_t + I_t h_t + (I_t - i_t) z_t] \exp(-rt) dt,
\end{equation}

where $a_0 = f_0 + h_0 + z_0$ denotes initial real assets; $r_t$ stands for government lump-sum transfers; and $I_t = r + \pi_t$, $\pi_t$ being the rate of inflation, stands for the "pure" nominal interest rate. \textsuperscript{4} $I_t$ and $(I_t - i_t)$ (both of which will be assumed to be positive) represent the opportunity cost of holding $h$ and $z$, respectively.

\textsuperscript{1} The incorporation of demand deposits into the cash-in-advance constraint can be found in Walsh (1984). Brock (1989) assumes that both assets reduce transaction costs or "shopping" time. In both cases, the two assets are posited to be imperfect substitutes. Englund and Svensson (1988) distinguish between "cash" goods and "check" goods, both of which are subject to liquidity constraints. Demands for both cash and demand deposits have been derived in a Baumol-Tobin context by Santomero (1979) and Whitesell (1989). Given the different costs of using cash versus debitable accounts, cash is used for small transactions while debitable accounts are used for large transactions (see Whitesell (1989)).

\textsuperscript{2} A subscript $i$ on a function denotes the partial derivative with respect to its $i$th argument.

\textsuperscript{3} The assumption $\ell_{hz} > 0$ is equivalent to ruling out perfect substitutability between $h$ and $z$. (Notice that if $\ell_{hz} = 0$, then, by Euler's theorem, $\ell_{hh} = \ell_{zz} = 0$, in which case $\ell(h, z)$ is a linear function.) As discussed below, the analysis still applies to the case of perfect substitution but it involves a corner solution in which only demand deposits are used.

\textsuperscript{4} $I_t$ will be referred to as the pure nominal interest rate to distinguish it from $i_t$, which will be referred to simply as the nominal interest rate.
The consumer chooses an optimal sequence \((c_t, h_t, z_t)_{t=0}^{\infty}\) to maximize (1) subject to (2) and (3). In addition to (2), holding with equality, and (3), the other first-order conditions for this problem are

\[(4)\quad u'(c_t) = \lambda [1 + I_t/p_h(h_t, z_t)],\]

\[(5)\quad \ell_z(h_t, z_t)/\ell_h(h_t, z_t) = (I_t - i_t)/I_t\]

where \(\lambda\) is the (constant) multiplier associated with constraint (3). 1/ Equation (4) has the familiar interpretation "marginal utility of consumption equals marginal utility of wealth times price of consumption". The effective price of consumption--given by the term in square brackets on the right-hand side (RHS) of equation (4)--consists of the direct price, unity, plus the marginal cost of producing the liquidity needed to purchase that unit of consumption. (We return below to the critical issue of how changes in \(i\) affect the effective price of consumption).

Equation (5) equates the marginal rate of substitution between \(h\) and \(z\) to the ratio of their opportunity costs. Because \(\ell(h, z)\) is homogenous of degree one, \(\ell_z(h, z)\) and \(\ell_h(h, z)\) are homogenous of degree zero and thus equation (5) implicitly defines the demand for \(h\) relative to \(z\) as a decreasing function of the opportunity cost of \(z\) relative to that of \(h\):

\[(6)\quad z_t/h_t = \phi [(I_t - i_t)/I_t],\]

where

\[(7)\quad \phi' [(I_t - i_t)/I_t] = \frac{\ell_h(1, \phi)}{\ell_z(1, \phi) - [(I_t - i_t)/I_t]\ell_h(1, \phi)} < 0.\]

An increase in \(I\), which raises proportionately more the opportunity cost of \(z\) than that of \(h\), induces the consumer to decrease the ratio of \(z\) to \(h\). An increase in \(i\), which lowers the opportunity cost of \(z\), increases the ratio of \(z\) to \(h\).

1/ It has been assumed that \(\beta = r\) to ensure the existence of a steady-state. This implies that there are no intrinsic dynamics in the model so that all dynamics will result from the implementation of temporary policies.

2/ The constraint (2) holds with equality at an optimum given that it has been assumed that both \(I\) and \((I - i)\) are positive.
Combining the liquidity-in-advance constraint (2)—holding with equality—with equation (6) yields the demand for cash, demand deposits, and money (see Appendix I):

\[(8) \quad h_t = \psi^h[c_t, (I_t - i_t)/I_t], \]

\[(9) \quad z_t = \psi^z[c_t, (I_t - i_t)/I_t], \]

\[(10) \quad m_t = \psi^m[c_t, (I_t - i_t)/I_t], \]

where \(m = h + z\) and a sign under an argument denotes the sign of the corresponding partial derivative. An increase in \(i\) reduces the demand for cash and raises the demand for demand deposits. The effect on the demand for money is positive because the reduction in the demand for cash is less than the rise in the demand for demand deposits. 1/

The function \(\phi(.)\), given by (6), can be used to express the effective price of consumption \((p)\), given by the term in square brackets on the RHS of (4), as a function of \(I\) and \(i\):

\[(11) \quad p(I_t, i_t) = 1 + \frac{I_t}{\ell_h(1, \phi[(I_t - i_t)/I_t])}. \]

where

\[(12) \quad p(I_t, i_t) = \frac{\ell_h(1, \phi) - \ell_{HZ}(1, \phi)(i_t \phi'/I_t)}{\ell^2_h(1, \phi)} > 0, \]

1/ To see how this increased demand for money comes about, consider the following example. Suppose that the liquidity-services production function is Cobb-Douglas with equal weights; that is, \(\ell(h, z) = (hz)^{\frac{1}{2}}. \) Then, \(c = (hz)^{\frac{1}{2}}\) and \(z/h = I/(I-i)\) so that \(z > h.\) An increase in \(i\) results in \(z\) increasing by the same proportion than \(h\) falls because consumption is given. Because \(z > h,\) the demand for \(z\) rises by more than the demand for \(h\) falls thus increasing the demand for \(m.\)
An increase in I raises the effective price of consumption because it raises the opportunity cost of both h and z, as indicated by (12). Less familiar, but critical to the whole analysis, is the way in which a rise in i affects the effective price of consumption. As follows from (13), a rise in i decreases the effective cost of consumption by making it less costly to hold interest-bearing demand deposits, which are used to produce liquidity services. 1/

The other actor in this economy is the government. To keep the model simple, we abstract from the banking system and assume that the government issues two nominal liabilities, high-powered money (H) and interest-bearing demand deposits (Z). There is no government consumption so that any revenues left after paying interest on demand deposits are transferred back to the consumer in a lump-sum fashion. Formally, the present value of government transfers is given by

\[ \int_0^\infty \tau_t \exp(-rt) dt = k_0 + \int_0^\infty [\dot{h}_t + \dot{z}_t + \pi_t (h_t + z_t) - i_t z_t] \exp(-rt) dt, \]

where \( k_0 \) stands for government's initial holdings of bonds and a dot over a variable denotes its time derivative. The government collects revenues from the creation of both high-powered money \( (\dot{h}_t + \pi_t h_t) \) and demand deposits \( (\dot{z}_t + \pi_t z_t) \). The rate of inflation is given by the rate of devaluation because foreign inflation is taken to be zero.

The government is assumed to control the interest rate paid on H by giving up the control over the composition of its liabilities, H and Z. 2/ In other words, the composition of its liabilities is demand-determined. In order to use this model to think about the real world, it is useful to keep in mind the following interpretation. The government issues bonds

1/ Notice that if \( \ell_{hz} = 0 \), equation (13) does not apply because h and z would be perfect substitutes, as indicated earlier. In that case, the consumer uses only demand deposits because they have a lower opportunity cost than cash. The effective price of consumption is \( 1 + I - i \) (assuming \( \ell(h,z) = h + z \)) so that an increase in i decreases the effective price of consumption.

2/ Alternatively (as shown below), the authorities can be viewed as issuing only high-powered money and determining the interest rate paid on demand deposits by controlling reserves requirements, given that I is exogenously given.
for instance, treasury bills) yielding a rate \( i \) which are entirely acquired by financial institutions. Financial institutions issue in turn demand deposits to consumers which, in a competitive equilibrium with costless banking, will also yield \( i \). It is as though the financial institutions "broke up" in small pieces the government bonds and sold them to consumers as NOW or Money Market Accounts. This is the channel through which the nominal interest rate determined by the government affects the nominal yield of a portion of the consumer's money holdings and thus the consumer's consumption path.

The combination of (3) and (14) yields the economy's lifetime resource constraint (provided, naturally, that the transversality conditions \( \lim[m_t \exp(-rt)] = 0 \) and \( \lim[h_t \exp(-r_t)] = 0 \) as \( t \to \infty \) hold):

\[
(15) \quad b_0 + \int_0^\infty y \exp(-rt) dt = \int_0^\infty c_t \exp(-rt) dt,
\]

where \( b_0 = f_0 + k_0 \) are the economy's initial bond holdings. Equation (15) simply says that the present value of consumption equals the present value of tradable resources.

For computational simplicity, we assume that \( u(c) = \log(c) \). 1/ Making use of (4) and (15), the expression for the equilibrium value of the multiplier follows: 2/

\[
(16) \quad \lambda = \frac{1}{y/r + b_0} \int_0^\infty \frac{\exp(-rt) dt}{1 + I_t/I_t + \delta_t (1, \phi [(I_t - i_t)/I_t])}.
\]

Replacing (16) into (4) yields the equilibrium consumption path:

1/ See, for instance, Obstfeld (1983, 1985) for a similar research strategy.
2/ For notational simplicity, no superscripts are introduced to denote equilibrium values.
This expression is key to the whole analysis. The ratio on the RHS can be viewed as the equilibrium marginal propensity to consume (MPC) out of permanent income, $y + rb_0$. As will become clear below, the numerator can be interpreted as the average effective price over the interval $[0,\infty)$. Therefore, the equilibrium MPC is the ratio of the average effective price to the current effective price. In equilibrium, all that matters is the current effective price relative to the average effective price because changes in the effective price of consumption have no wealth effects so that only substitution effects remain. Hence, if the effective price path is constant over time, the equilibrium MPC is unity for all $t$ and consumption is constant over time and equal to permanent income. In other words, there are no reasons for the economy to engage in intertemporal price substitution. In contrast, if the effective price is lower today than it will be in the future, the equilibrium MPC is above unity and consumption is higher today than in the future.

Finally, the current account path can be derived as follows. Private asset accumulation is given by

$$c_t = \frac{1}{1 + I_t/\lambda_h(1, \phi[(I_t - i_t)/I_t])}$$

This expression is key to the whole analysis. The ratio on the RHS can be viewed as the equilibrium marginal propensity to consume (MPC) out of permanent income, $y + rb_0$. As will become clear below, the numerator can be interpreted as the average effective price over the interval $[0,\infty)$. Therefore, the equilibrium MPC is the ratio of the average effective price to the current effective price. In equilibrium, all that matters is the current effective price relative to the average effective price because changes in the effective price of consumption have no wealth effects so that only substitution effects remain. Hence, if the effective price path is constant over time, the equilibrium MPC is unity for all $t$ and consumption is constant over time and equal to permanent income. In other words, there are no reasons for the economy to engage in intertemporal price substitution. In contrast, if the effective price is lower today than it will be in the future, the equilibrium MPC is above unity and consumption is higher today than in the future.

Finally, the current account path can be derived as follows. Private asset accumulation is given by

$$x_t = y + rf_t + i_t z_t + \pi_t(h_t + z_t) - c_t.$$  

The transfer level is assumed to adjust so that the expansion of domestic credit is $\pi_t(h_t + z_t)$; domestic credit expansion just compensates the consumer for the real depreciation of $h$ and $z$. Formally,

$$\tau_t = \pi_t(h_t + z_t) + r k_t - i_t z_t.$$  

Combining (18) and (19) and taking into account that credit rule (19) implies that $h_t + z_t = k_t$ yields

Note that a constant path of the effective price constitutes the only case in which the equilibrium MPC is unity for all $t$. If the effective price varies over time, the equilibrium MPC may be unity for some $t$ but it cannot be unity for all $t$. 

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As expected, the accumulation of net foreign assets (i.e., the current account balance) equals GNP minus absorption.

We are now ready to show that a permanent change in the nominal interest rate has no real effects. For this purpose, assume a constant devaluation rate so that $\pi_t$ is constant at $\pi$ and thus $I-r+\pi$. 1/ It is assumed here--and throughout the paper--that, prior to the disturbance (which takes place at $t=0$), the economy is in a stationary state. At the initial steady state, $i_t = i^1$. Then it follows from (17) that $c = y + rb_0$; namely, consumption equals permanent income. Consider an unanticipated and permanent increase of $i$ from $i^1$ to $i^2$ at $t=0$. It can be readily verified from (17) that consumption remains unchanged at the permanent income level, $y + rb_0$. The intuition follows from the interpretation of equation (17). Recall that a change in $i$, whether temporary or permanent, has no wealth effects in equilibrium, as can be seen from (15). 2/

Therefore, consumption will change only as a result of intertemporal price substitution effects. The permanent change in $i$, however, does not affect the equilibrium MPC, which continues to be unity, because the average price decreases by the same amount as the current price does. The fact that the price path remains flat—even if at a lower level because the rise in the nominal interest rate reduces the effective price of consumption—implies that there is no change in consumption.

The rise in the interest rate increases the demand for demand deposits by more than it decreases the demand for cash; that is, the demand for money rises, as follows from equation (10). This implies that there is a gain in reserves as the consumer exchanges bonds for money at the Central Bank.

Interestingly, although there is no change in the pure real interest rate, $I-\pi$, this experiment raises the real interest rate on the liquid bond, $i-\pi$. Thus, in the present context there is no necessary connection between the real interest rate on the liquid asset and real economic activity.

1/ Calvo (1986, 1989b) makes the same point in the context of temporary stabilization policies based on the control of the money supply or the exchange rate.

2/ If cash and demand deposits were modeled as reducing transaction costs, as in Brock (1988), then changes in $i$ would affect available resources. In this paper, by adopting the liquidity-in-advance specification, we abstract from such effects in order to concentrate exclusively on intertemporal price speculation effects.

\[ (20) \quad \delta_t = y + rb_t - c_t. \]
III. Temporary Increase in the Interest Rate

This section focuses on the central experiment of the paper: a temporary increase in i. Suppose then that at time \( t=0 \) (the "present"), the authorities increase the interest rate from \( i^1 \) to \( i^2 \). At \( t=T \), the interest rate is brought back to \( i^1 \). More formally, for some \( T>0 \),

\[
(21a) \quad i_t = i^2 \quad 0 \leq t < T \\
(21b) \quad i_t = i^1 \quad t \geq T
\]

where \( i^2 > i^1 \). Replacing the new policy, given by (21), into (17), we obtain the new consumption path:

\[
(22a) \quad c_t = (y+rb_0) \frac{1}{(1/p^2)[1-\exp(-rT)] + (1/p^1)\exp(-rT)} \\
(22b) \quad c_t = (y+rb_0) \frac{1}{(1/p^2)[1-\exp(-rT)] + (1/p^1)\exp(-rT)}
\]

where \( p^1 = 1 + I/I_0(1,\phi((I-i^1)/I)), \)

\( p^2 = 1 + I/I_0(1,\phi((I-i^2)/I)). \)

For notational convenience, \( p^1 \) and \( p^2 \), where \( p^2 < p^1 \) (recall equation (13)), stand for the effective price of consumption associated with \( i^2 \) and \( i^1 \), respectively. In order to make clear that, as already suggested, the numerator of (22) can be interpreted as the average effective price, rewrite equations (22) as:

\[
\text{For notational convenience, } p^1 \text{ and } p^2, \text{ where } p^2 < p^1 \text{ (recall equation (13)), stand for the effective price of consumption associated with } i^2 \text{ and } i^1, \text{ respectively. In order to make clear that, as already suggested, the numerator of (22) can be interpreted as the average effective price, rewrite equations (22) as:}
\]

\[1/ \text{ It should be clear that, given that permanent changes in } i \text{ have no real effects, an anticipated decrease in } i \text{ would have the same real effects as a temporary increase in } i.\]
where $0 < \Phi(T) < 1$; $\Phi(T+0) \rightarrow 0$; $\Phi(T-\infty) \rightarrow 1$; and $\Phi'(T) > 0$ (see Appendix II). Hence, equations (23) illustrate the notion of the equilibrium MPC being the ratio of the average effective price to the current effective price.

Figure 1 illustrates the consumption path that results from the policy given by (21). Equations (23) indicate that consumption jumps upwards upon the announcement and remains constant up to time $T$. At $t = T$, consumption jumps downwards and remains constant thereafter at a level below initial permanent income. When the nominal interest rate is increased, there is a once-and-for-all decrease in the average price. The fall in the average price, however, is more than offset by the reduction in the current price. Therefore, the equilibrium MPC increases above unity (as can be verified from (23a)) which raises consumption. The fact that the current price remains unchanged until $t = T$ implies that consumption stays constant as well. At $t = T$, the current price increases back to $p^1$ which decreases the equilibrium MPC below unity so that consumption jumps downwards at that point. This anticipated discontinuity in the consumption path is feasible under predetermined exchange rates because with the price level, and hence the exchange rate, given at $t = T$, there are no profit opportunities to be taken advantage of.  

The path of the current account—which is illustrated in Figure 2—is given by equation (20) after equations (23) are substituted into it. Due to the increase in consumption at $t = 0$, the current account jumps into deficit. It then deteriorates steadily between 0 and $T$ because, although the gap between GDP and consumption remains constant, interest payments on net foreign assets decline throughout. At $t = T$, the current account becomes balanced and the stock of foreign assets stops declining. In the new steady state, net foreign assets are lower than initially.

1/ In the context of anticipated devaluations, an anticipated discontinuity is possible only if there is no capital mobility (see Calvo (1989a)).
Figure 1: Consumption Path
Figure 2: Current Account Path
As far as the stock of reserves is concerned, note that both the rise in \( i \) and the rise in consumption increase money demand (recall equation (10)). Therefore, there is an initial gain in Central Bank reserves at \( t=0 \). Reserves remain constant between 0 and \( T \) but when time \( T \) arrives there is a sharp drop (because both the interest rate and consumption fall) which will more than offset the initial gain. This follows from the fact that the steady-state stock of reserves decreases. The reason is that, in the new steady state, the interest rate is back at its initial level but consumption is lower, which implies, by equation (10)), that money demand is lower. Since real domestic credit remains constant, this implies a lower level of reserves.

This policy experiment illustrates even more sharply the lack of necessary relationship between the real interest rate on the liquid asset and economic activity. Since the effect of the policy described by (21) is independent of the level of \( i \) at time zero, the real interest rate on the liquid asset could have gone up or down during \([0,T)\), and still the real effects would be the same. It is, therefore, not the level, but the expected change of the real (or nominal) interest rate on the liquid asset that really matters.

Equations (23), together with the earlier finding that permanent changes in \( i \) do not affect consumption, may be used to examine the role played by \( T \). When a temporary rise in the nominal interest rate is considered, \( T \) measures the time during which the interest rate remains at the higher level; namely, the degree of "temporariness" of the policy. Figure 3 depicts the consumption path for different values of \( T \). To examine the initial jump in consumption (and hence the level of consumption for \( 0 \leq t < T \)), consider (23a) as a function of \( T>0 \). Equation (23a) indicates the level of consumption until the interest rate decreases at time \( T \). If \( T=0 \) (no change in policy), \( c=y+rb_0 \). It follows from (23a) that \( c(T) \) is discontinuous at \( T=0 \) because as \( T \to 0 \) \( \lim[c(T)]= (y+rb_0)(p^T/p^0) \). Furthermore, \( c(T) \) is a decreasing function of \( T \) and as \( T \to \infty \lim[c(T)]= y+rb_0 \). Thus, when the rise in the interest rate gets arbitrarily large, consumption before \( T \) tends to its permanent (as of \( t=0 \)) income level. Equation (23b), as a function of \( T \), yields the level of consumption for \( t=T \) (and hence for \( t \geq T \)) for those cases in which \( T<\infty \). It follows from (23b) that (a) consumption at \( T \) is a decreasing function of \( T \); (b) as \( T \to 0 \) \( \lim[c(T)]=y+rb_0 \); and (c) as \( T \to \infty \) \( \lim[c(t)]= (y+rb_0)(p^T/p^0) \). Note that the latter limit does not coincide with the value of consumption when "\( T=\infty \)" (i.e., when the rise in interest rates is permanent); this situation is analogous to the discontinuity of \( c(T) \) at \( T=0 \). Intuitively, as long as at some point in time, no matter how far away it may be, the effective price of consumption increases, consumption will decrease at that time. Thus, as Figure 3 illustrates, the shorter the rise in the interest rate remains in effect (\( T=0 \)), the larger the initial increase in consumption (and hence the larger the initial current account deficit) and the higher the level of consumption after \( T \). Intuitively, the price path is flat except for a very short period of time which implies that the intertemporal price speculation effects alluded to above get exacerbated. In contrast, the
longer the change in policy \((T \rightarrow \infty)\), the smaller the initial increase in consumption and the higher the decrease in consumption at \(T\).

To illustrate the effects of a higher ratio of demand deposits to cash on the response of the effective price of consumption (and thus of consumption) to a rise in \(i\), consider the case in which \(\ell(h,z)\) exhibits fixed proportions. Formally, let the liquidity-in-advance constraint be given by:

\[
\begin{align*}
(24a) & \quad m_t = \alpha c_t, \\
(24b) & \quad h_t = q m_t, \quad z_t = (1-q)m_t,
\end{align*}
\]

where \(\alpha > 0\), \(0 < q < 1\). The effective price of consumption is then (taking into account policy \((21)\)):

\[
(25) \quad p(I, i_j^I) = 1 + \alpha [q I + (1-q)(I-i_j^I)], \quad j=1,2.
\]

Figure 4 illustrates the effects of a rise in the interest rate for different values of \(1-q\). Clearly, if \((1-q)=0\), there is no channel through which the interest rate can affect the effective price of consumption so that the consumption path remains flat. A given increase in the nominal interest rate results in a lower effective price the larger is \((1-q)\). Thus, a larger \((1-q)\) leads, other things being equal, to higher consumption between 0 and \(T\) and a correspondingly lower consumption after \(T\). (Compare the paths corresponding to \(1-q_1\) and \(1-q_2\), where \((1-q_1)>(1-q_2)\), in Figure 4.) This suggests that economies in which interest-bearing liquidity plays an important role will exhibit a larger response to temporary interest rate increases. 1/

Consider now the welfare implications of a temporary rise in interest rates. To begin with, observe that such a policy is not Pareto-optimal: a planner interested in maximizing \((1)\) subject to \((15)\) would choose a constant level of consumption equal to permanent income. Therefore, the optimal policy is to choose a constant interest rate---the level is irrelevant---which induces the consumer to choose the Pareto-optimal consumption path. It seems intuitively clear that the welfare cost of a given temporary rise in interest rate increases with the proportion of demand-deposits held by the consumer. The reason is that, as argued above

\[1/\] In the Cobb-Douglas case \((c=h^{\alpha}z^{1-\alpha})\), where \(z/h=[(1-\alpha)/\alpha][I/(I-i)]\), it can be shown that the fall in the effective price of consumption for a given increase in \(i\) is greater the larger is \(1-\alpha\) until \(1-\alpha\) reaches a value of 0.684, at which point the ratio \(z/h\) equals 4.3 (assuming \(I/(I-i)=2\)). (The liquidity-services production function needs to be specified for this exercise because third derivatives are involved.)
Figure 3: Consumption Path For Different Degrees of Temporariness
Figure 4: Consumption Path as a Function of The Proportion of Demand-Deposits Held

Note: \((1 - q_1) > (1 - q_2)\)
and is illustrated in Figure 4, higher values of \((1-q)\) result in a "less smooth" consumption path and thus--one would expect--lead to higher welfare losses. (Simulations of the model suggest that this is indeed the case.) In contrast, welfare does not change monotonically with \(T\) because of the following. First, recall that consumption remains unchanged when \(T=0\) or \(T=\infty\); second, the welfare loss converges to zero when \(T=0\) or \(T=\infty\); and third, the welfare loss is positive for all \(T\) such that \(0<T<\infty\). 1/ Therefore, there exists a \(T\in(0,\infty)\) at which the welfare loss reaches a maximum.

It should be noted that this set-up could be used to examine a temporary stabilization program under predetermined exchange rates (i.e., a temporary reduction in the devaluation rate), as in Calvo (1986). Suppose that \(\pi\) is reduced during the period from \(t=0\) to \(T\). It follows from (17) that a reduction in \(\pi\) (which reduces \(I\)) is equivalent to an increase in \(i\) because both affect consumption through the same (and only) channel: the effective price of consumption. Therefore, the same effects on consumption and the current account obtain for a temporary decrease in the inflation rate.

Finally, note that the authorities could control the interest rate paid on demand deposits by manipulating reserve requirements. 2/ To show that this policy is equivalent to the one studied above, note that, under competitive and costless banking, the zero profit condition implies that

\[
(26) \quad i_t = (1-\gamma)I_t,
\]

where \(0<\gamma<1\) denotes the required reserve ratio. Since \(I\) is exogenously given, controlling \(\gamma\) is tantamount to controlling \(i\). Thus, a temporary reduction in \(\gamma\) by temporarily reducing the effective price of consumption--has an expansionary effect on consumption.

IV. Conclusions

Policy-makers have frequently manipulated nominal interest rates in order to achieve different macroeconomic goals. An analytical hurdle for studying the effects of such policies has been the fact that interest rate targeting usually results in price level or inflation rate indeterminacy. This paper provides an analytical framework in which a meaningful examination of interest-rate policy is feasible. The key feature is that the authorities control the interest rate borne by a

1/ Naturally, the discontinuity of \(c(T)\) at \(T=0\) does not imply that the welfare loss is discontinuous at that point. In technical terms, the integral is continuous if the integrand is piecewise continuous.

2/ This equivalence would not hold under flexible exchange rates, however, because the pure nominal rate is not exogenously given.
liquid bond—which we identify as interest-bearing demand deposits—rather than the rate of the non-liquid bond. In this context, raising interest rates lowers the cost of holding money. If the interest rate hike is unexpected and permanent, there are no real effects because the effective price of consumption remains constant over time. If the interest rate rise is temporary, the effective price of consumption is lower today compared to the future and thus an expansionary effect in consumption results.

We have thus isolated one specific—and often disregarded—channel through which higher nominal interest rates may work. This is an important first step, from a conceptual point of view, towards understanding the effects of interest rate policy in more realistic (and thus more complex) models. Even in considerably more complex models, this channel will still be present.

We have dealt with the simplest possible scenario: a small open economy with full employment, flexible prices, and predetermined exchange rates. The simplest extension is to consider the case of flexible prices and analyze the effectiveness of raising interest rates in fighting inflation (see Calvo and Végh (1990b)). In that context, it is still the case that only temporary changes in the interest rate have any effects on the economy. The expansionary effect on consumption that results from a temporary increase in the interest rate necessitates a fall on impact in the price level to increase the real money supply. The increase in consumption is accompanied, however, by an increase in the rate of inflation. Furthermore, the inflation rate increases exponentially thereafter. The conclusion, therefore, is that raising interest rates seems hardly appropriate in the context of a flexible-prices model as a means of fighting inflation.

The expansionary effects that result from an increase in the interest rate in the flexible-prices models run counter to conventional wisdom (according to which raising interest rates should be contractionary). This has led us to examine interest rate policy in sticky-prices model. In Calvo and Végh (1989, 1990c), we analyze interest rate policy in a closed-economy, staggered-prices model. The main message is that the conventional (i.e., contractionary) effects re-emerge in that context. Inflation is brought down—at the cost of a sharp fall in output—both when the interest rate is raised temporarily and when it is raised permanently. When the rise is temporary, however, the initial fall in inflation is followed by an upsurge in inflation over and above its initial level. The expansionary effect of a higher interest rate isolated in this paper is still present in the sticky-prices model, in the sense that the consumer would like to increase consumption. The key difference, however, is that the increased consumption cannot be effected because the real money supply cannot increase instantaneously due to the fact that (a) the economy is closed, and (b) the price level cannot jump.

Having realized that the absence of capital mobility might be crucial to the results obtained in the closed-economy, staggered-prices model, the
The next logical (and final) step in our quest for understanding interest rate policy is to open the sticky-prices model to trade in goods and assets, suspecting that—in the Mundell-Fleming spirit—predetermined rates coupled with perfect capital mobility would dramatically affect the results. To this effect, we plan to use the open-economy, staggered-prices model developed in Calvo and Végh (1990a) to study interest rate policy. One would expect that, under predetermined exchange rates, the expansionary effects reappear—even in the presence of sticky-prices—because the public may increase its real money holdings through the Central Bank window. Therefore, a temporary increase in the interest rate may have expansionary effects in both the traded and non-traded goods sector. Under flexible prices, however, one would expect to find that the traded-goods sector expands while the non-traded goods sector contracts. Flexible exchange rates prevent, once again, the public from increasing real money balances.
Derivation of demand for cash, demand deposits, and money

From equation (2), holding with equality, and equation (6), it follows that

\( h_T[c_T, (I_T-i_T)/I_T] = c_T/\ell[1, \phi((I_T-i_T)/I_T)], \)

\( z_T[c_T, (I_T-i_T)/I_T] = c_T\phi((I_T-I_T)/I_T)/\ell[1, \phi((I_T-i_T)/I_T)], \)

\( m_T[c_T, (I_T-i_T)/I_T] = c_T[1+\phi((I_T-I_T)/I_T)]/\ell[1, \phi((I_T-i_T)/I_T)]. \)

Clearly, all three functions are increasing in \( c \). Denoting \((I-i)/I\) by \( R \), we have

\( h_R[c_T, R_T] = \frac{-c_T\ell Z [1, \phi(R_T)] \phi'(R_T)}{\ell^2 [1, \phi(R_T)]} > 0, \)

\( z_R[c_T, R_T] = \frac{c_T\phi'(R_T)(\ell [1, \phi(R_T)] - \phi(R_T)\ell Z [1, \phi(R_T)])}{\ell^2 [1, \phi(R_T)]} < 0, \)

\( m_R[c_T, R_T] = \frac{c_T\phi'(R_T)(\ell [1, \phi(R_T)] - [1+\phi(R_T)]\ell Z [1, \phi(R_T)])}{\ell^2 [1, \phi(R_T)]} < 0. \)

The sign of the numerator of equations (I.5) and (I.6) follows from Euler's theorem and the fact that, by equation (5), \( \ell_h > \ell_Z \).
Derivation of $\Phi(T)$

To illustrate the idea that the equilibrium MPC can be thought of as the ratio of the average effective price to the current effective price, the function $\Phi(T)$ was used in equations (23). This function and its properties are now derived. Let the function $\Phi(T)$, $0<T<\infty$, be defined in implicit form, by

$$\Phi(T) = \frac{1}{p^2\Phi(T) + p^1[1-\Phi(T)]} = \frac{1}{(1/p^2)[1-\exp(-rT)] + (1/p^1)\exp(-rT)}$$

where the RHS is the numerator of equations (22). For notational simplicity, denote the denominator on the RHS of (II.1) by $\Gamma(T)$. Using (II.1), $\Phi(T)$ can be solved for:

$$\Phi(T) = \frac{1}{\Gamma(T) - p^1}[1/(p^2 - p^1)].$$

Clearly, $\Phi(T)$ is continuous for $T \in (0, \infty)$ since $p^2 wp^1$. Recalling that $p^2 < p^1$, it follows from (II.2) that (a) $\Phi'(T) > 0$, (b) as $T \to 0$ $\lim[\Phi(T)] = 0$, and (c) as $T \to \infty$ $\lim[\Phi(T)] = 1$. Hence, $0 < \Phi(T) < 1$. 

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