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Government Spending and Inflationary Finance: A Public Finance Approach

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Abstract

This paper analyzes the relationship between inflation tax and the level of government spending in a public finance context. The key feature of the model developed is that it recognizes the possibility that conventional taxes, such as the consumption tax, may carry increasing marginal collection costs. As a result, and unlike previous findings in the literature, the inflation tax becomes an increasing function of government spending. Furthermore, the more inefficient the tax collection system, the larger the increase in the inflation tax for a given increase in government spending. A numerical analysis of the model provides additional insights into these relationships.

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Contents

I. Introduction

The study of the inflation tax in a public finance context has received increasing attention in the literature ever since Phelps (1973) pioneered this approach (see Kimbrough (1986) and the references therein). Given that lump-sum taxes are not feasible in the real world and that the inflation tax is only one of several taxes that a government can resort to, it is certainly arguable that analyzing the problem of inflationary finance in a public finance context is the appropriate way of dealing with this important issue (see, for instance, Frenkel (1987)). Alternative ways of approaching the optimum quantity of money rule have either rely on partial equilibrium models or ignore distortions existing elsewhere in the economy (i.e., outside the money market). 1/

If money is modeled as reducing transaction costs, Kimbrough (1986) reaches the conclusion that, even if distortive taxes have to be used, it is not optimal to resort to inflationary finance. This result is not surprising in view of the well known result, in the public finance literature (surveyed by Auerbach (1985)), that it is not optimal to tax intermediate goods. 2/ Since money can be viewed as acting as an input into the production function in Kimbrough's (1986) setup, the optimality of adhering to a zero nominal interest rate (which requires setting a rate of deflation that exactly offsets the real interest rate) follows immediately from public finance considerations. 3/ Kimbrough's result is in sharp contrast to Phelps's finding that a positive nominal interest would be part of an optimal tax package. 4/ The key to the different results lies in the fact that Phelps (1973), by including real money balances as an argument in the utility function, models money as a final good. Therefore, since it is optimal to tax all final goods, money should also be taxed.

1/ See Friedman (1968), Johnson (1970), Bewley (1980), and Jovanovic (1982), among others.

2/ The results holds either under a constant-returns-to-scale technology or, in the presence of decreasing returns to scale, if all pure profits are taxed away.

3/ The nominal interest rate will be also referred to as the inflation tax rate or, simply, inflation tax. The consumer's expenditure on holding money accrues to the government because by printing money the government avoids interest costs on the public debt. Phelps (1973) discusses alternative definitions of the concept "inflation tax".

4/ Others have also derived results similar to Phelp's (1973); see the references in Kimbrough (1986).

If one agrees with the view that money is not a final good since it is not wanted per se but rather as a means towards an end,

Kimbrough's (1986) result cannot explain the widespread use of inflationary finance that takes place especially in developing countries (see, for instance, Fischer (1982) and Ramirez-Rojas (1986) for evidence). A characteristic of the tax administration system that is particularly prevalent in developing countries, however, can explain the existence of inflationary finance; namely, alternative taxes are costly to collect (Aizenman (1987), Végh (1987a)). 1/ A key feature of this result is that the inflation tax (i.e., the nominal interest rate) does not depend on the level of government spending. In particular, when the only other available tax is, for instance, a consumption tax, the optimal inflation tax is totally determined by the collection costs parameter, as formally shown in Végh (1987b). It is generally believed, however, that higher government spending and/or higher budget deficits lead to higher inflation rates as the government seeks additional revenues.

The purpose of this paper is to take a comprehensive look at the relationship between government spending and inflationary finance in a public finance context. Several important issues concerning this relationship are addressed both analytically and numerically. Assuming that alternative taxes are costly to collect, it is shown that the lack of influence of government spending on the inflation tax is due to the assumption of constant marginal collection costs (as in Aizenman (1985) and Végh (1987b)). If marginal collection costs are an increasing function of tax revenues, the inflation tax is an increasing function of government spending. 2/ 3/ Furthermore, the more inefficient the tax system, as indicated by higher collection costs

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1/ Végh (1987b) shows that the presence of currency substitution can also render the use of inflationary finance optimal. The reason is that the foreign nominal interest rate distorts the consumption/leisure choice faced by the consumer because it acts as an indirect tax on consumption. A positive inflation rate can be shown to reduce that distortion.

2/ Aizenman (1983) derives a similar result. However, Aizenman's model differs sharply from the paradigm used in this paper in that it is not a public finance setup because the government makes use of no distortionary taxes other than the inflation tax.

3/ Instead of incorporating collection costs, Aizenman (1986) assumes that consumption taxes are not feasible and studies the optimal combination of capital controls, tariffs, and inflation in financing government spending.

for a given level of tax revenues, the larger the increase in the inflation tax for a given increase in government spending. Interestingly enough, however, although the optimal nominal interest rate is an increasing function of government spending, the share of revenues from the inflation tax turns out to be a decreasing function of government spending. The analysis also shows that multiple equilibria may arise; namely, there may be both a high and a low nominal interest rates associated with a given level of government spending that are part of general equilibrium solutions of the model. No extra conditions are needed, however, to rule out multiple equilibria. It is shown that even if both equilibria are, by definition, relative maxima of the optimal taxation problem faced by the government, there is only one global maximum, which corresponds to the low-inflation equilibrium.

The paper proceeds as follows. In Section II, the consumer's and the government's optimization problems are presented. Section III deals with the solution of the model. The analysis in this section abstracts from multiple equilibria and concentrates on the basic results regarding the effects of an increase in government spending on the inflation and consumption tax. It also deals with the effects of parametric changes in the marginal collection costs schedule. To gain further insights into the model, Section IV presents a numerical analysis that suggests some new results. First, the nominal interest rate is a convex function of government spending; namely, the higher the initial level of government spending, the higher the increase in the nominal interest rate for a given increase in government spending. Second, the analysis shows that the relative importance of revenues from the inflation tax decreases as government spending increases. Third, the possibility of multiple equilibria is clearly established and is related to the analysis of the previous section. Section V contains some concluding remarks.

II. The Model

In this section, the consumer's optimization problem and the government's optimal taxation problem are introduced. The presentation of the consumer's problem will be brief since it does not differ basically from that of Kimbrough (1986) or Végh (1987a).

1. The consumer's problem

Consider a small open economy with flexible exchange rates. 1/ There is only one (tradable and nonstorable) good. Labor is the only factor of production and is taken to be the numeraire. The consumer may hold two assets: domestic money and an internationally traded bond whose real rate of return is constant and equal to r. Transacting is a costly activity in this economy in that it requires the use of shopping time (s_t). The consumer is endowed with one unit of time per period so that his time constraint is $s_t+n_t+h_t=1$; where n denotes labor and h leisure. The good is produced under a constant-returns-to-scale technology given by $y_t=n_t$; where y denotes production of the good and units have been so defined that producing one unit of the good requires one unit of labor.

The transactions technology is given by:

$$(1) \quad s_t=v\left\{\frac{m_t}{c_t(1+\theta_t)}\right\} c_t(1+\theta_t); \qquad v'(.)\leq 0, \ v''(.)>0;$$

where m denotes real money balances, c stands for consumption, and θ represents the consumption tax. The function v(.) indicates the amount of time that the consumer spends shopping per unit of consumption expenditure. (For notational simplicity, we will define $X=m/c(1+\theta)$ and refer to X as relative money balances.) Additional relative money balances bring about positive but diminishing reductions in shopping time. There is a level of relative money balances, $X=X^*$, such that $v'(X^*)=0$; namely, gains from additional liquidity are exhausted. For simplicity, it is also assumed that $v(X^*)=0$.

1/ The analysis can be readily reinterpretated as applying to a closed economy where a domestic bond is issued.

The consumer faces the following maximization problem:

$$\max_{\{c_t, h_t, m_t\}_{t=o}^{\infty}} \sum_{t=o}^{\infty} \beta^t U(c_t, h_t)$$

subject to:

$$(2) \quad b_{-1} + \sum_{t=o}^{\infty} (1/1+r)^t (1-h_t) = \sum_{t=o}^{\infty} (1/1+r)^t \{(1+\theta_t)c_t + s_t + [i_t/(1+i_t)]m_t\};$$

where the utility function, U(.), is strictly increasing in both arguments and strictly concave; β is one over one plus the rate of time preference; b_o denotes initial holdings of the internationally traded bond, i is the domestic nominal interest rate, and s is given by (1). The optimality conditions for this problem (assuming an interior solution) are:

$$(3) \quad \frac{U_c(c_t, h_t)}{U_h(c_t, h_t)} = (1+\theta_t)[1 + v(X_t) - v'(X_t)X_t] \qquad t=0,1,..$$

$$(4) \quad -v'(X_t) = I_t \qquad t=0,1..$$

where $I \equiv [i/(1+i)]$. 1/ Equation (3) shows that the consumer equates the marginal rate of substitution between consumption and leisure to the relative price of consumption, which consists of its direct price, $1+\theta$, plus the increase in transactions costs associated with consuming an additional unit of the good, given by the term in square brackets in (3). Equation (4) states that the consumer holds relative money balances up to the point where their marginal benefit, in terms of reduced transaction costs, equals their opportunity cost.

2. The government's optimal taxation problem

The government faces an exogenously given path of spending $\{g_t\}_{t=o}^{\infty}$, which will be taken to be constant over time; namely, $g_t = g_o$ for all t. Before formally stating the government's problem, it is necessary to define the equilibrium price functions, which follow implicitly from equations (3) and (4):

$$(5) \quad (1+\theta_t) \equiv y^{\theta}(c_t, h_t, m_t) = \frac{U_c(c_t, h_t)}{U_h(c_t, h_t)} \left\{1 + v(X_t) - v'(X_t)X_t\right\}^{-1}$$

$$(6) \quad I_t \equiv y^{I}(c_t, h_t, m_t) = -v'(X_t),$$

1/ Use has been made of the condition $\beta = 1/1+r$. This condition is needed to ensure the existence of a steady state. It also implies that there are no intrinsic dynamics in the model, in the sense of Obstfeld and Stockman (1985).

where $X_t \equiv m_t/c_t(1+\theta_t) \equiv m_t/c_t y^{\theta}$. 1/ As shown below, the government chooses an optimal sequence $\{c_t, h_t, m_t\}_{t=o}^{\infty}$. By plugging these optimal values into (5) and (6), the optimal sequence of taxes $\{\theta_t, I_t\}_{t=o}^{\infty}$ follows. This procedure ensures that the social optimum can be the outcome of a competitive equilibrium.

The government's problem is given by: 2/

$$\underset{\{c_t, h_t, m_t\}_{t=o}^{\infty}}{\text{Maximize}} \quad \sum_{t=o}^{\infty} U(c_t, h_t)$$

subject to:

$$(7) \quad \sum_{t=o}^{\infty} (1/1+r)^t \, g_o = \sum^{\infty} (1/1+r)^t \, [(1-\phi)\theta_t c_t + I_t m_t]$$

1/ By implicit differentiation of equations (5)- (6), the following espressions obtain:

$$\frac{\partial y^{\theta}}{\partial m} = \frac{v''(X)\ X}{c} \; \frac{1}{1 + v(X) - v'(X)X + v''(X)X^2}$$

$$\frac{\partial y^{I}}{\partial m} = \frac{-v''(X)}{cy^{\theta}} \; \frac{1 + v(X) - v'(X)X}{1 + v(X) - v'(X)X + v''(X)X^2}$$

where $X \equiv m/cy^{\theta}$. By substituting these expressions into the optimality condition for real money balances from the government's optimization problem, equation (9) obtains.

2/ It will be assumed that the government acts "honestly" in the sense of Auernheimer (1974). This implies, as pointed out by Calvo (1978), that problems of time-inconsistency are assumed away.

$$(8)\quad \sum_{t=o}^{\infty} (1/1+r)^{t}(1-h_{t}) = \sum_{t=o}^{\infty} (1/1+r)^{t}\ [(1+\phi)\theta_{t}c_{t} + g_{o} + v(X_{t})c_{t}(1+\theta_{t})],$$

where $\phi=\phi(\theta c)$ and $\phi'(\theta c)\geq 0$. Marginal collection costs, represented by ϕ, are a non-decreasing function of total revenues from the consumption tax, θc. In particular, if $\phi'(\theta c)=0$ (namely, $\phi=\phi_{o}$, where ϕ_{o} is a paramater whose value lies between zero and unity), marginal collection costs are constant. For notational simplicity, θ_{t} and I_{t} figure in (7) and (8) instead of $[y^{\theta}(c_{t},h_{t},m_{t})-1]$ and $y^{I}(c_{t},h_{t},m_{t})$, respectively, but it should be kept in mind that (c_{t},h_{t},m_{t}) are the only variables that appear in the government's problem. Equation (7) is the intertemporal government's budget constraint. Equation (8) is the intertemporal resource constraint for the economy as a whole.

Carrying out the government's maximization problem, substituting into these optimality conditions the partial derivatives of the equilibrium price functions, and resorting to the consumer's first order conditions, the general equilibrium of this economy can be represented by the following set of five equations in five unknowns: c, h, X, θ, I. 1/ 2/

$$(9)\quad -v'(X) + \frac{v''(X)\ X}{1+v(X)-v'(X)+v''(X)X^{2}}\left[-\phi(\theta c)-\phi'(\theta c)-v(X)+v'(X)X\right]=0$$

$$(10)\quad g_{o}= [1-\phi(\theta c)]\theta c + IXc(1+\theta)$$

$$(11)\quad 1-h = c(1+\theta) + v(X)c(1+\theta) + IXc(1+\theta)$$

1/ In what follows, time subscripts will be dropped. Since government spending is constant over time and undergoes only unexpected and permanent changes, the adjustment of the economy will be instantaneous because, as indicated earlier, there are no intrinsic dynamics in the model.

2/ For simplicity, it has been assumed that $b_{-1}=0$.

$$(12)\quad \frac{U_c(c,h)}{U_h(c,h)} = (1+\theta)\{1+v(X)-v'(X)X\}$$

$$(13)\quad -v'(X) = I$$

The interpretation of equation (9) will be addressed in detail below. Equation (10) is the government's budget constraint. (Recall that this economy is always in the steady state; therefore, borrowed funds cannot be a source of revenue.) The first term on the right-hand side of (10) represents net revenues from the consumption tax and the second one revenues from the inflation tax (note that Im can be rewritten as $IXc(1+\theta)$). Equation (11) is the steady-state consumer's budget constraint. The left-hand side shows the consumer's source of revenue (labor income) while the three terms in the right-hand side indicate the consumer's uses of funds: expenditure on consumption (which includes the consumption tax), transaction costs (or shopping time), and the cost of the inflation tax, respectively. Equations (12) and (13) are simply the consumer's first-order conditions (3) and (4), which are reproduced for convenience. This system fully describes the general equilibrium of the economy. The parameters of the system are g_o and the marginal collection costs function $\phi(\theta c)$. In the general form presented in equations (9)-(13), however, the system is highly non-linear and an analytical solution is not feasible.

Equation (9) is derived from the government's optimality condition for real money balances. It can be viewed as determining the optimal level of relative money balances, X, for a given value of θc. Existing results in the literature are a particular case of this model. Suppose that marginal collection costs were constant; namely, $\phi(\theta c)=\phi_o$, where $0<\phi_o<1$. In that case, equation (9) reduces to:

$$(14)\quad -v'(X) + \frac{v''(X)\,X}{1+v(X)-v'(X)+v''(X)X^2}\left[-\phi_o-v(X)+v'(X)X\right]=0.$$

It is immediately apparent that this equation implicitly defines the optimal level of relative money balances, X^o, as a function of the collection costs parameter, ϕ_o; namely, $X^o=X(\phi_o)$. Given X^o, equation

(13) yields the optimal inflation tax, I^o. Furthermore, it can be shown that $I^o(\phi_o=0)=0$ (Kimbrough (1986)) and that $I^o(\phi_o>0)>0$ and $[d(I^o)/d(\phi^o)]>0$ (as first indicated by Aizenman (1985) and proved in the present framework by Végh (1987b)). 1/ The intuition behind these results is as follows. When collection costs are zero, the use of the inflation tax would entail a loss of resources due to positive transaction costs. The use of the consumption tax, instead, carries no extra cost in terms of resources. Therefore, it is optimal to use only the consumption tax to finance government spending. When collection costs are positive, however, the use of the consumption tax also implies a loss of resources to the economy. Hence, it is optimal to use both taxes. Two aspects of these results are worth noticing. First, the optimal inflation tax does not depend on the level of government spending. Second, from a mathematical point of view, constant marginal collection cost makes the analysis fairly simple because, as just shown, equation (9) becomes a reduced form equation for X, given implicitly by (14), which is all that is needed to determine, via (13), the optimal inflation tax. In the general case where there exist increasing marginal collection costs, equation (9) ceases to be a reduced form for X, which complicates the task of determining the optimal inflation rate. The next section addresses this analytical task.

III. Solution of the Model

For reasons just mentioned, it proves necessary to simplify the model to make progress in the analysis of the optimal inflation tax in the presence of increasing marginal collection costs. Two assumptions will be made: first, the utility function takes the logarithmic form $U(c,h)=\log(c)+\log(h)$; second, the transaction costs technology can be represented by a quadratic function, i.e., $v(X)=X^2-X+d$. Under these assumptions, equations (9) and (11)-(13) become:

$$(15)\quad 1 - \frac{2X}{1+d+X^2}\left[1 + 2d + \phi(\theta c) + \phi'(\theta c)\theta c\right] = 0$$

1/ Since $I=i/1+i$ is an increasing function of i, it makes no difference whether one refers to I or i.

$$(16)\quad \frac{h}{c(1+\theta)} = 1 + d - X^2$$

$$(17)\quad 1-h = c(1+\theta) + (X^2-X+d)c(1+\theta) + IXc(1+\theta)$$

$$(18)\quad X = (1/2)(1-I)$$

Equation (10) remains unchanged. Combining equations (16) and (17), and making use of equation (18), it follows that h=1/2. Owing to the logarithmic nature of the utility function, the amount of leisure (and hence the amount of resources available to this economy in each period) is constant. Substituting h=1/2 into equation (16), the following obtains:

$$(19)\quad c(1+\theta) = (1/2)\frac{1}{1 + d - X^2}$$

which is a quite useful expresion because it expresses total expenditure on consumption solely as a function of relative money balances, X. Substituting (19) into the government's budget constraint, given by (10), equation (20) obtains. Equation (20), together with equation (15) (reproduced for convenience below as equation (21)), form a two-equation system in two unknowns: θc and X. For notational convenience, we will denote gross revenues from the consumption tax, θc, as z; namely $z \equiv \theta c$.

$$(20)\quad [1-\phi(z)]z + (1/2)\ \frac{X - 2X^2}{1 + d - X^2} = g_o$$

$$(21)\quad 1 + d + X^2 - 2X\left[1 + 2d + \phi(z) + \phi'(z)z\right] = 0.$$

This system determines equilibrium values of z and X, and hence, via equation (18), of z and I. Figure I illustrates the joint determination of I and z. Schedule FE (for Fiscal Equilibrium), which depicts equation (20), is drawn for a given value of government

spending (g_o). It slopes downwards because an increase in z (gross revenues from the consumption tax) brings about an increase in net revenues so that, for a given level of spending, the inflation tax has to be reduced in order to decrease revenues from money creation. The analytical expression for the slope of the FE schedule is:

$$(22)\quad \frac{dz}{dI} = (1/4)\left[\frac{X^2-(1+d)4X+(1+d)}{(1+d-X^2)^2}\right]\left[\frac{1}{1-\phi(z)-\phi'(z)z}\right]<0$$

Under certain assumptions, this expression is negative. 1/ As the numerical analysis will later reveal, however, these assumptions are not necessary since the optimum will always occur at the downward portion of FE were this schedule to slope upwards for some values of I. To see why, rewrite (22) as:

$$(23)\quad [1-\phi(z)-\phi'(z)z]dz + (-1/2)\frac{X^2-(1+d)4X+(1+d)}{(1+d-X^2)^2}\,dI = 0$$

The coefficient of dz indicates the marginal increase in net revenues from the consumption tax that results from an increase of one unit in gross revenues. It seems clear that, at an optimum, this coefficient

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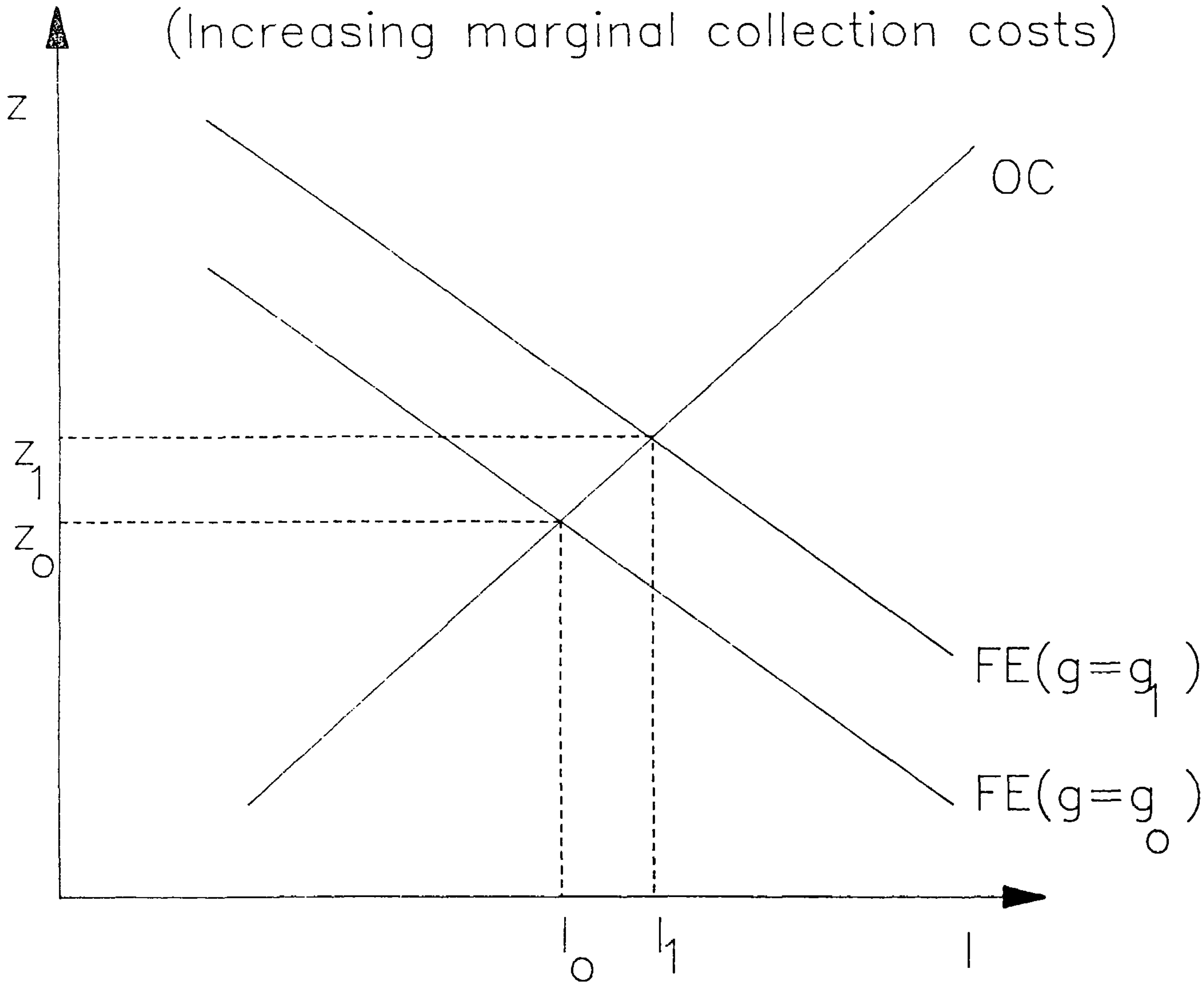
1/ The following assumptions, which remain in effect for the rest of this section, are sufficient to ensure that expression (24) is negative. Suppose that $\phi(z)=\phi_o+kz$, where $k>0$ is a parameter, and that $\phi_o+k<3/4$. Then, $1-\phi(z)-\phi'(z)z=1-\phi_o-2kz$. Taking into account that $2z<1$ (because z has to be less than available resources which are one half), the condition $\phi_o+k<3/4$ is sufficient to ensure that $1-\phi(z)-\phi'(z)z>0$ and thus that the second term in square brackets is positive. With respect to the first term in square brackets in (22), it follows from the optimality condition (21) that $X>(1/2)(1/1+\phi_o+k)$. Taking d to be 1/4, it follows that the expression $X^2-(1+d)4X+(1+d)$ is negative for the range of possible values of X, which makes this term negative.

Figure I: Determination of l and z
(Increasing marginal collection costs)



will not be negative because it would mean that net revenues from the last unit of gross revenues are negative. A lower z would not only increase net revenues but also increase, ceteris paribus, private consumption. A similar argument applies to the coefficient of dI in (23). This coefficient represents the marginal increase in revenue from money creation when I is increased by one unit. Clearly, at an optimum, this coefficient will not be negative because it would mean that, at the margin, revenues from money creation would be negative. Reducing I would both increase revenues from money creation and decrease transaction costs.

The schedule given by (21) is depicted in Figure I as OC (for Optimality Condition). This condition can be interpreted as follows. As shown in Kimbrough (1986), the inflation tax also acts as an indirect tax on consumption because it taxes an intermediate good, money, which is used to decrease the effective cost of consumption. The consumer is indifferent, therefore, to different combinations of the consumption and the inflation tax that imply the same effective tax on consumption. Thus, the government can be seen as choosing, for a given effective tax on consumption, that combination of the consumption and the inflation tax that minimizes the resource loss. Hence, schedule OC can be interpreted as the locus of points (I,z) where the loss of resources that results from collecting a given amount of revenue is minimized. This implies, roughly speaking, equating at the margin the resources lost in collecting both taxes. Schedule OC slopes upward, then, because an increase in I, for given z, raises the marginal transactions costs, as follows from (13) while an increase in z, for given I, raises the marginal collection costs associated with the consumption tax. The analytical expression for the slope of the OC schedule is given by:

$$(24) \quad \frac{dz}{dI} = (1/2) \frac{[X^2-(1+d)/X]}{-4X\phi'(z)} > 0$$

The numerator is negative since $X \leq 2d$ (note that $v'(2d)=v(2d)=0$; that is, $X^*=2d$) and $d=1/4$. As shown in Figure I, the intersection of the $FE(g=g_o)$ schedule and the OC schedule determines the equilibrium values of I and z, denoted by I_o and z_o. Note that if marginal collection costs are constant (which implies that $\phi'(z)=0$), the schedule OC would become a vertical line at point I_o in Figure I (not drawn). Since, at the margin, the loss of resources, derived from

the presence of collection costs is constant, there is only one value of I which equates both margins.

1. Effects of an increase in government spending

Consider an increase in government spending from g_o to g_1. Refer first to the benchmark case where marginal collection costs are constant ($\phi(z)=\phi_o$). In that case, as already indicated, the schedule OC becomes a vertical line at I_o in Figure I. The upward shift in the FE schedule implies that the new equilibrium would occur at the same nominal interest rate, I_o. Analytically, the change in z is given by $(dz/dg)=(1/1-\phi_o)$. Net revenues from the consumption tax, therefore, increase by the same amount that government spending does. In other words, the increase in government spending is solely financed by additional revenues from the consumption tax. 1/

Figure I illustrates the case where marginal collection costs are increasing. The increase in government spending from g_o to g_1 causes the nominal interest rate to rise to I_1. Given that the coefficient of dI in (23) is positive, revenues from the inflation tax also increase. Gross revenues from the consumption tax become z_1. The analytical expressions for these changes are given by:

$$(25)\quad \frac{dI}{dg} = - \frac{8X\phi'(z)}{\Delta} > 0$$

1/ The result that the nominal interest rate remains the same is a general one, as indicated earlier. The particular specification of the model adopted here, however, also implies that, when marginal collection costs are constant, revenues from the inflation tax remain unchanged (i.e., real money balances are independent of g). This feature may not be robust to alternative specifications.

$$(26) \quad \frac{dz}{dg} = \frac{X^2 - (1+d)}{X\Delta} > 0$$

where:

$$\Delta \equiv [1-\phi(z)-\phi'(z)][(X^2-(1+d))/X] + 2X\phi'(z)\{[X^2-4(1+d)X+(1+d)]/(1+d-X^2)^2\} < 0.$$

The sign of the different expressions follows from the assumptions made to ensure a unique equilibrium; in particular, recall that $X \leq 2d$ and $d=1/4$.

Intuitively, the increase in government spending means that, for a given I, higher revenues from the consumption tax are needed, which increases marginal collection costs. A rise in I, which increases marginal transaction costs, is therefore needed to equate both margins again. Note that, conceptually, it would be incorrect to interpret the increase in the inflation rate along the following lines: if revenue needs increase, it seems reasonable that both sources of revenue be activated to meet them. The case of constant marginal collection costs clearly refutes this interpretation. It is the fact that the marginal collection costs of the consumption tax rise that triggers the increase in the optimal nominal interest rate. 1/

2. Effects of different marginal collection costs schedules.

Consider now the effects of parametric changes in the marginal collection costs schedule on the increase in the optimal nominal interest rate that results from a rise in government spending. Suppose that the marginal collection costs function is given by $\phi(z)=kz$, where $k \geq 0$ is a parameter, and consider the initial equilibrium that results

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1/ If the (steady state) budget deficit is defined as government spending minus net revenues from the consumption tax, if follows from the analysis that higher government spending leads to larger budget deficits and higher nominal interest rates.

from no government spending (g_o=0). 1/ It is useful to concentrate on this initial equilibrium because this equilibrium is independent of the parameter k, as it can be readily verified, and the translation of the FE schedule is also independent of the value of k, which allows for a meaningful analytical and graphical interpretation. As will be verified later, however, the results generalize to any initial value of g.

Figure II illustrates the effects of different values of k. When k is zero, the OC schedule coincides with the vertical axis. The higher is k, the flatter is the OC schedule. The reason is that a higher value of k implies that a given increase in gross revenues from the consumption tax results in a larger rise in marginal collection costs; therefore, a higher rise in marginal transaction costs is needed, which requires a larger increase in I. The initial equilibrium is at the origin, where the three OC schedules intersect the FE(g=0) schedule. Since there is no government spending, both the consumption and the inflation tax are zero. If government spending rises, the FE schedule shifts upwards. The horizontal shift (i.e., holding I constant) is dz=dg. When k=0, the new equilibrium is (I=0,z_o). When k is positive, say $k_1>0$, the new equilibrium is (I_1,z_1). A higher k, for instance $k_2>k_1$, delivers (I_2,z_2) as the new equilibrium. Thus, for a given increase in government spending, the increase in the inflation rate is higher, the larger the parameter k is. Analytically, the corresponding expressions are:

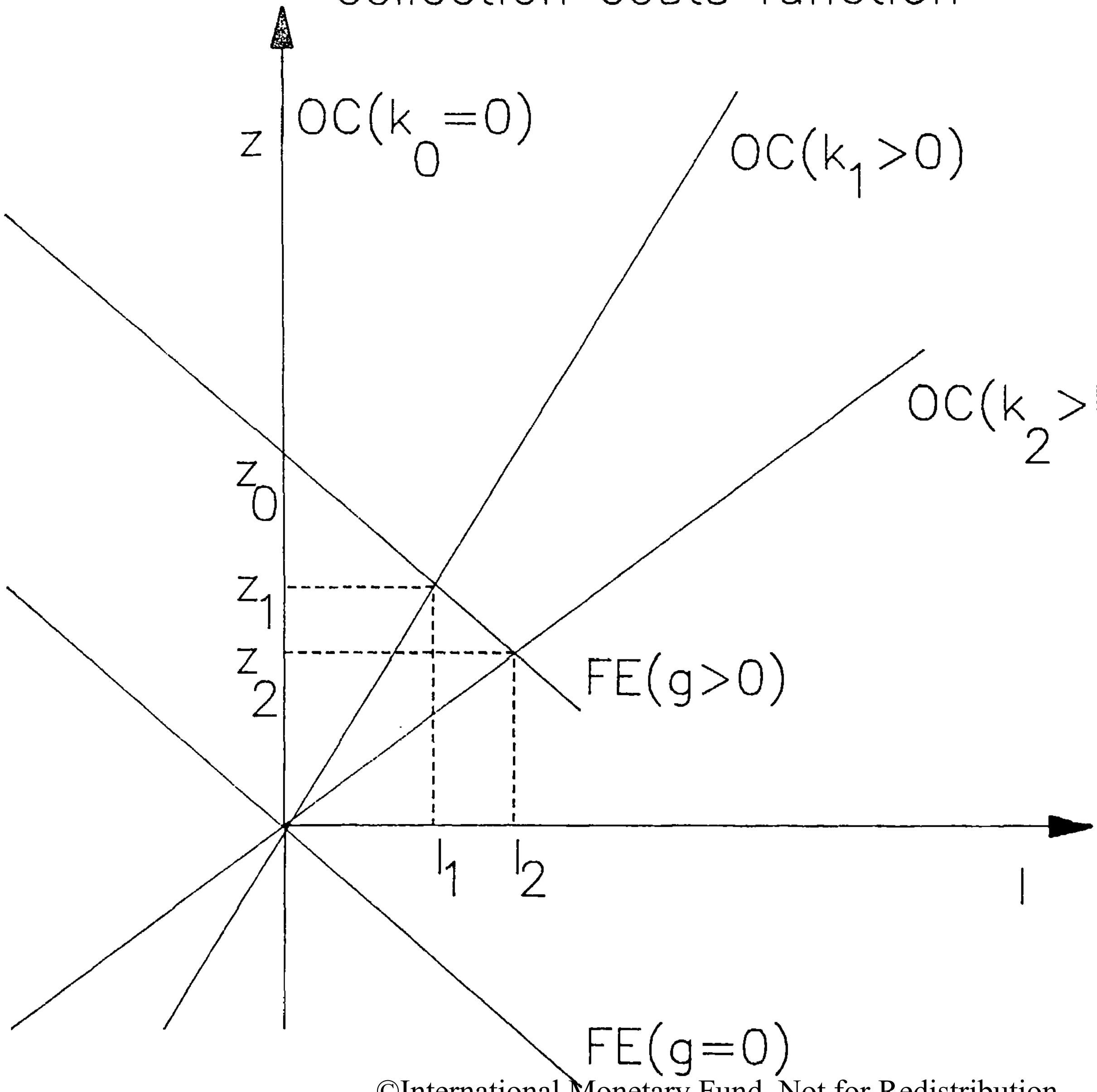
$$(27) \quad \frac{dz}{dg} = \frac{2X^{*} - 2(1+2d)}{\Delta(g=0)} > 0$$

$$(28) \quad \frac{dI}{dg} = - \frac{8X^{*}k}{\Delta(g=0)} > 0$$

where Δ(g=0) denotes the determinant of the system evaluated at the initial equilibrium, whose expression is given by:

1/ Owing to the consideration of this particular initial equilibrium, there is no need to restrict the value of k, as before.

Figure II: Effects of parametric changes in the marginal collection costs function



$$\Delta(g=0) = 2X^{*}-2(1+2d) + 2X^{*}k \frac{X^{*2}-(1+d)4X^{*}+(1+d)}{(1+d-X^{*2})^{2}} <0$$

The signs of the different expressions follows from the fact that $X^{*}=1/2$ and $d=1/4$.

The last expression indicates that $\Delta(g=0)$ is a decreasing function of k. It follows that expression (27) is a decreasing function of k while expression (28) is an increasing function of k (note that a rise in k increases the numerator by more, in proportional terms, than it does the absolute value of the denominator), which confirms the graphical representation. Given that, at the initial equilibrium, z=0, the increase in net revenues from the consumption tax also represents the rise in gross revenues in all three cases examined. Since a higher value of k can be interpreted as representing less efficient tax collection systems, the model predicts that a given increase in government spending will increase the nominal interest rate the most in countries with the least efficient tax collection systems.

IV. Numerical Analysis of the Model

In this section, a numerical analysis makes it possible to gain further insights into the workings of the model. The issue of multiple equilibria is addressed first. As mentioned earlier, the analysis suggests the possibility of multiple equilibria; this section shows that multiple equilibria can indeed arise. The simulation also suggests additional properties of the model. First, the nominal interest rate is a convex function of government spending; in other words, the higher the level of government spending, the larger the increase in the nominal interest rate that results from a given increase in government spending. Second, the more inefficient the tax collection system is, the higher is the optimal nominal interest rate for a given level of government spending. Third, the share of revenues (i.e., relative to total revenues) of the inflation tax is a decreasing function of government spending.

In the previous section, the restriction that $\phi_o+k<3/4$ was imposed on the marginal collection costs schedule. This condition was crucial to ensure a unique equilibrium. The question arises, however, as to whether the analysis of the previous Section III remains relevant in the case where that condition is relaxed and multiple equilibria results. The numerical analysis that follows suggests that this is indeed the case. When there are multiple equilibria (two, in this case), even though both equilibria are, by definition, relative maxima of the optimal taxation problem faced by the government, only one of them is a global maximum. As will be shown, the global maximum is the one that occurs along the downward portion of the FE schedule so that the analysis of the previous section continues to apply.

For the purposes of this section, it proves convenient to work in the (X,z) plane rather than in the (I,z) plane, as was done in the last section . Since X totally determines I (recall that I=1-2X), it makes no difference which plane one works with. It should be kept in mind, however, that because a lower X implies a higher I, the slope of the FE and OC schedules is reversed relative to the previous section; namely, in the (X,z) plane, FE slopes upwards and OC downwards. In what follows, the marginal collection costs schedule is taken to be given by $\phi(z)= \phi_o + kz$ but the assumption $\phi_o+k<3/4$ is dropped. 1/

Using the fact that I=1-2X, it follows from equation (22) that:

$$(29) \quad \frac{dz}{dX} = -(1/2) \frac{\{X^2-4(1+d)X+(1+d)/(1+d-X^2)^2\}}{1 - \phi_o - 2kz}$$

1/ We choose the simplest specification (i.e., linear) for the marginal collection costs to pursue matters as far as possible. It also shows that even the simplest specification can generate more than one equilibria (in this case two equilibria); hence more complicated schedules could generate more than two equilibria. It is not clear, however, what the economic meaning of a marginal collection cost schedule of order higher than one would be. Note that a linear marginal collection costs function implies that total collection costs are a convex function of gross revenues, which has a familiar economic interpretation. If the marginal collection costs were, say, quadratic, it would mean that total collection costs have non-zero third derivatives which have no obvious economic interpretation.

Examining equation (29), which shows the slope of the FE schedule, we observe two features that could generate multiple equilibria. (Note that, under the assumptions of the previous section, this expression is positive.) First, assuming that the denominator is positive, low values of X can make the numerator turn positive, which would make FE slope downwards. This could mean that the downward sloping OC schedule could intersect the FE schedule twice. Second, note that when $z=(1-\phi_o)/2k$, the denominator is zero which means that the function $z=z(X)$ is not defined at that point (by the implicit-function theorem). For values of $z<(1-\phi_o)/2k$, however, the implicit-function theorem tells us that we have another well defined function $z=z(X)$, in addition to the one defined for $z>(1-\phi_o)/2k$. This is not surprising when we observe that the function $F(X,z)$ defined by (20) is quadratic in z when $\phi(z)=\phi_o+kz$. Two equilibria could now arise if the OC schedule intersects once each of the functions. Figure III shows that it is the second possibility that takes place. Figure III shows the actual schedules FE (both of them) and the OC schedule. 1/ The parameter values are $\phi_o=0$, $g=0.16$ (which represents 32% of GDP since total output equals 1/2) and $k=2$. 2/ There are two equilibria denoted by E_o and E_1, both of which are feasible in the sense that the non-negativity constraint of consumption is not violated. At E_0, X is higher (i.e., I is lower) and z is lower than at E_1. Given that a higher I means higher resources lost to transaction costs, and a higher z implies higher resources lost to collection costs, it seems intuitively clear that the the global optimum is E_0 and that the equilibrium E_1 can be ruled out with the simple, but persuasive, argument that if the government is assumed to be maximizing the consumer's welfare, it will choose the global maximum when faced with two relative maxima.

1/ Note that, in equilibrium, neither X nor z can exceed one-half (which explains the scales chosen for figures III and V). When $d=1/4$, $X^*=1/2$, so that this is the maximum value of X that can be chosen without the non-negative constraint of the nominal interest rate being violated. On the other hand, z, gross revenues from the consumption tax, cannot exceed total output, which is one-half.

2/ Lower values of k have the effect of moving upwards the upper branch of FE thus rendering it irrelevant in the sense that the intersection denoted by E_1 occurs at non-admissible values (i.e., the intersection takes place outside the box depicted in Figure III).

The proof that at E_0 consumer's welfare is higher than at E_1 is straightforward. It follows from (16) that:

$$(30) \quad c = (1/2) \frac{1}{1 + d - X^2} - z$$

Therefore, both a higher X and a lower z imply a higher private consumption. Since leisure always equal one-half, utility is higher at E_0 than at E_1.

Figure IV shows the optimal nominal interest rate as a function of government spending. There is a range of values of g for which the schedule is a correspondence rather than a function; namely, there are two equilibrium interest rates for a given level of government spending which are solutions of the model. (The reason why the backward bending portion of the correspondence does not go farther is that the numerical analysis indicates that the non-negativity constraint on consumption would be violated.) Note that there is a limit to the amount of government spending that can be financed (in this case just below 0.18, which is equivalent to 36% of GDP). Naturally, the maximum amount of government spending that can be financed depends on the magnitude of k. Lower values of k imply that higher amounts may be financed. In the limiting case where k=0, for instance, the government could finance a level of spending of 100% of GDP. 1/

The simulation tends to confirm the intuition contained in Section III in the sense that the equilibrium (or the relevant one if there is more than one) will occur at the upward sloping portion of the FE schedule (downward sloping portion of FE in the (I,z) plane). (In fact, the assumption of Section III restricted our attention to that portion of the FE schedule). Figure V shows the actual FE and OC schedules in the (X,z) plane for the case where k=1 and g=0.1. The upper branch of FE corresponds to non-feasible values of the endogenous variables; this explains the only equilibrium. It is seen

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1/ Or, more precisely, assuming that marginal utility tends to infinity when consumption approaches zero, any level arbritarily close to 100% of GDP.

Figure III: Multiple equilibria (k=2, g=0.16)

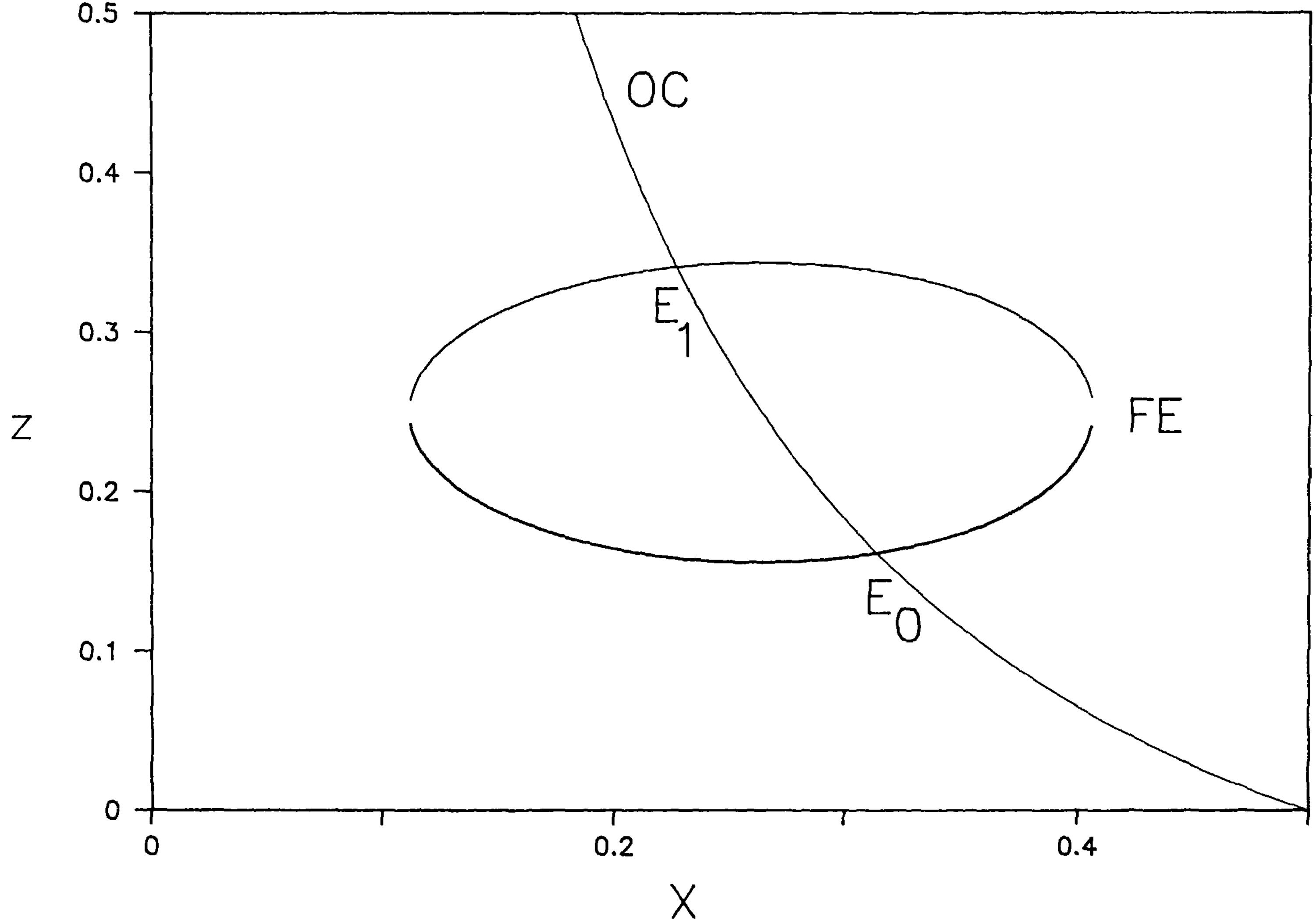


Figure IV: Optimal nominal interest rate as a function of government spending (k=2)

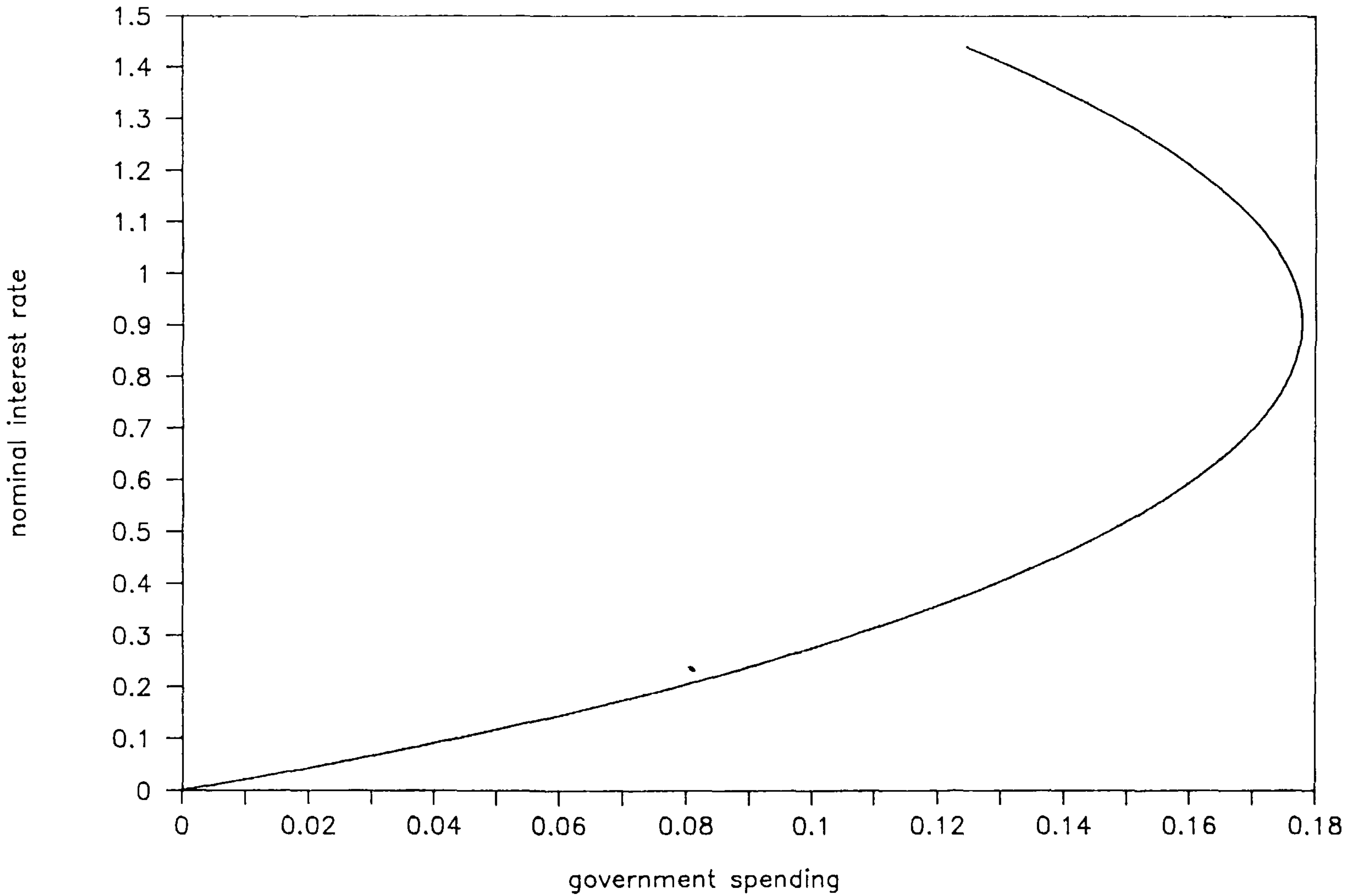
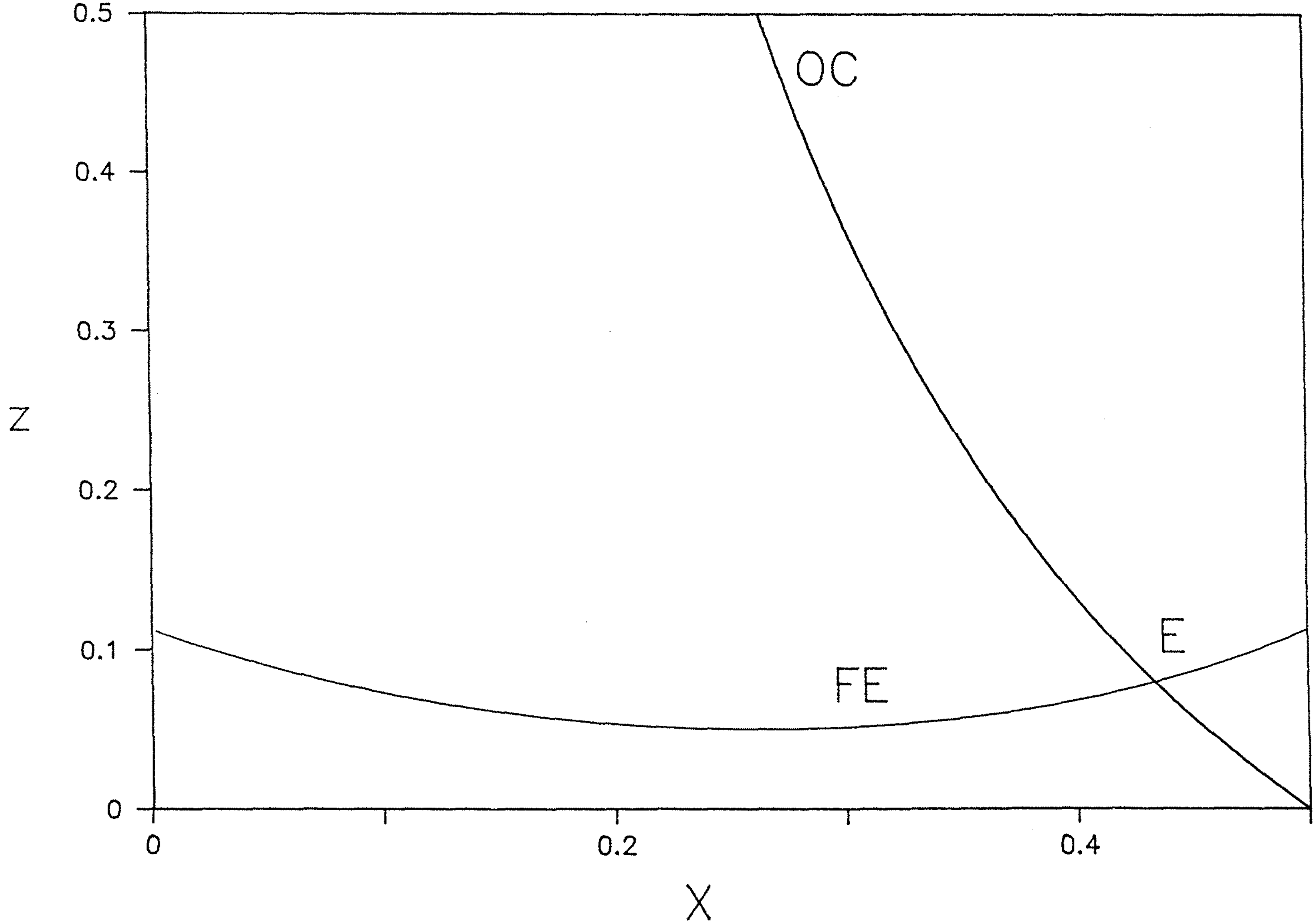


Figure V: Determination of X and z (k=1, g=0.1)



that the equilibrium also occurs in the upward sloping portion of FE. (Other simulations for different values of k, not shown in the paper, confirm this fact).

The simulation of the model suggests additional interesting results. Figure VI shows the optimal nominal interest rate as a function of government spending for different values of the parameter k. (Note that the independent variable, government spending, is shown along the Y-axis.) Two conclusions emerge from the picture. First, as regards the influence of k on the optimal level of the nominal interest, Figure VI shows that, for a given level of government spending, the optimal nominal interest rate is higher, the larger is k. Therefore, if the value of k were to be reduced by rendering more efficient the tax collection system, the nominal interest would fall, even if the level of government spending remained unchanged. Second, as regards the effects of k on the changes of the nominal interest rate, Figure VI shows that, for a given increase in government spending, the increase in the nominal interest rate is larger, the higher is k. This result was obtained analytically in the previous section for values of government spending close to zero; the simulation thus suggests that the result remains valid for any initial value of government spending. 1

Figure IV, together with Figure VI, also suggest that the optimal nominal interest rate is a convex function of government spending. Therefore, a given increase in government spending leads to a higher increase in the nominal interest rate, the larger is the initial level of government spending.

Finally, the numerical analysis also reveals that the share of total revenues accounted for by the (net) revenues from the consumption is an increasing function of government spending while the opposite is true of the share of revenues from the inflation tax. Interestingly enough, different values of k were considered and all yielded the same result; namely, the share of revenues from the inflation tax decreases as government spending rises. As one would expect, higher values of k imply a larger share of revenues from money creation. It is also the case, however, that higher values of k result in a more rapid fall of the share of revenues from the inflation tax.

1/ Note that there are also multiple equilibria for k=1.5.

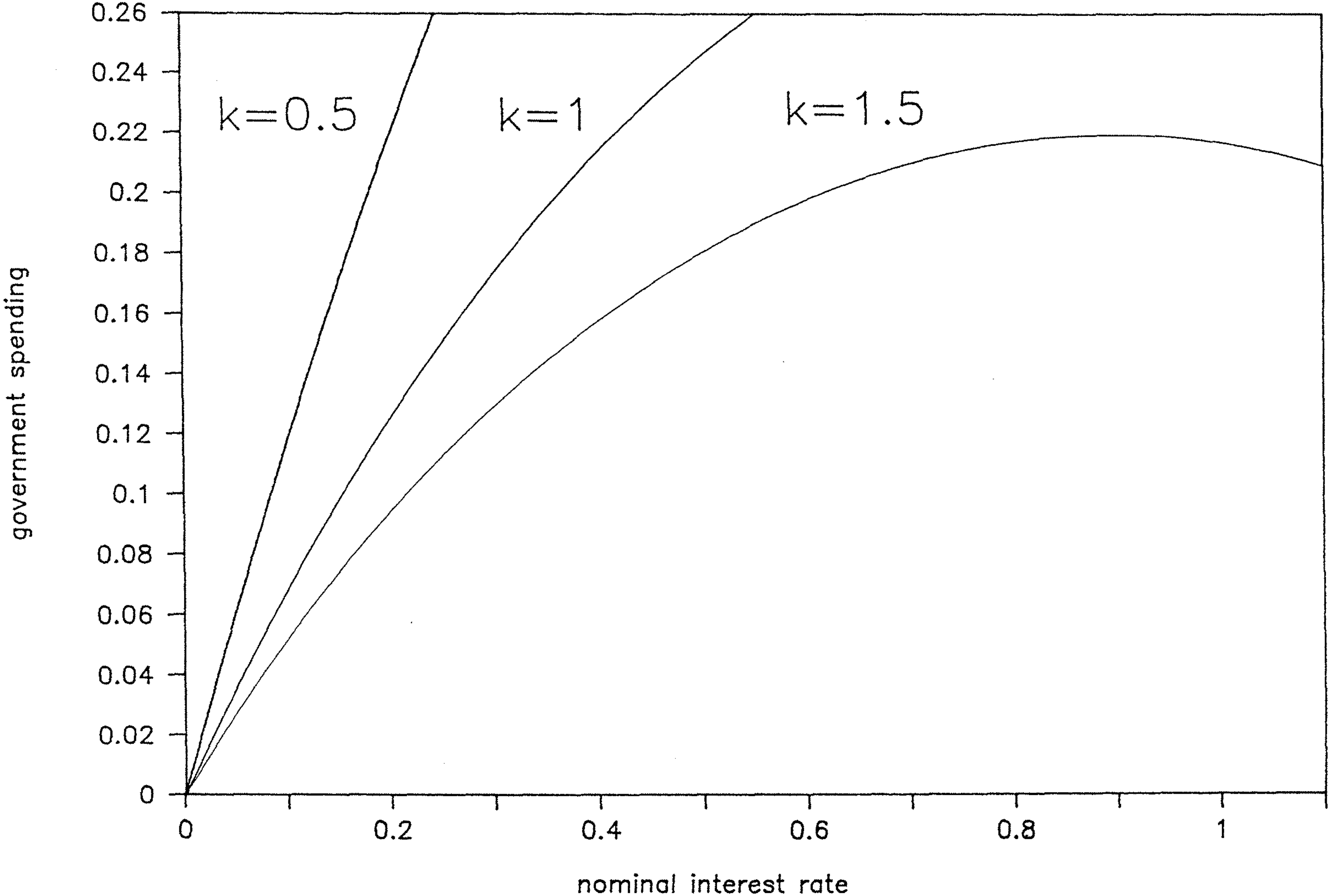
IV. Conclusions

This paper has analyzed the relationship between government spending and inflationary finance in a a public finance framework. It has been shown that, when marginal collection costs are increasing, a rise in government spending leads to an increase in the optimal nominal interest rate. A more inefficient tax collection system implies a higher increase in the nominal interest rate for a given increase in government spending. The simulation of the model illustrated the possibility of multiple equilibria but it was shown that the high nominal interest rate equilibrium can be discarded. The simulation also suggested additional results. First, the nominal interest rate is a convex function of government spending; namely, the higher is the level of government spending, the larger is the increase in the nominal interest rate for a given increase in government spending. Second, the share of revenues from the inflation tax decreases as government spending increases.

From a policy perspective, the model lends support to the notion that, especially in less developed countries, a reduction in government spending should lead to a reduction in the inflation rate because the ultimate reason for the existence of a high inflation rate lies in the government's need for revenues. An improvement in the efficiency of the tax collection system, however, should also be viewed as an important component in any attempt to reduce the reliance on the inflation tax.

Owing to the absence of intrinsic dynamics in the model, the economy studied in this paper is always in the steady state. Therefore, the analysis cannot consider the possibility of borrowed funds as a source of government revenue. An interesting extension of this work would be to introduce sources of dynamics in the model, thus giving the government the possibility of not balancing its budget in each period, and study the optimal policy response to temporary and permanent changes in government spending. One source of dynamics could be, for instance, the assumption that the consumption tax cannot be adjusted instantaneously to its new level because of, say, the need for congressional authorization and other necessary administrative changes. This could explain a higher reliance on the inflation tax in short run (in the event of a war, for instance).

Figure VI: Optimal i as a function of g
for different values of k



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