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IMF Working Paper

Uncertainty, Financial Frictions and Nominal Rigidities: A Quantitative Investigation

by Ambrogio Cesa-Bianchi and Emilio Fernandez-Corugedo

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IMF Working Paper

Research Department

Uncertainty, Financial Frictions and Nominal Rigidities: A Quantitative Investigation

Prepared by **Ambrogio Cesa-Bianchi and Emilio Fernandez-Corugedo¹**

Authorized for distribution by Benjamin Hunt

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Abstract

Are uncertainty shocks a major source of business cycle fluctuations? This paper studies the effect of a mean preserving shock to the variance of aggregate total factor productivity (macro uncertainty) and to the dispersion of entrepreneurs' idiosyncratic productivity (micro uncertainty) in a financial accelerator DSGE model with sticky prices. It explores the different mechanisms through which uncertainty shocks are propagated and amplified. The time series properties of macro and micro uncertainty are estimated using U.S. aggregate and firm-level data, respectively. While surprise increases in micro uncertainty have a larger impact on output than macro uncertainty, these account for a small (non-trivial) share of output volatility.

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Online appendix available at https://sites.google.com/site/ambropo/CF_Uncertainty_OnlineAppendix.pdf.

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I. INTRODUCTION

Are uncertainty shocks a major source of business cycle fluctuations? Economists and policy makers alike have debated this question in recent years as the widespread and heightened levels of uncertainty may have been one of the key factors behind the unusual depth and duration of the “Great Recession”, as well as its slow recovery. This paper provides a quantitative assessment of whether and how uncertainty shocks can affect macroeconomic dynamics.

Economists have long understood the mechanisms by which uncertainty affects key economic variables.¹ Credit market imperfections can create additional channels through which fluctuations in uncertainty can affect macroeconomic outcomes. For example, when firms choose their scale before observing (uninsurable) shocks and bear the risk of a costly default, high uncertainty can lead to a reduction of factor inputs (Arellano, Bai, and Kehoe, 2012); or when the relation between lender and borrower is subject to asymmetric information (agency and/or moral hazard problems) an increase in uncertainty will in general raise the cost of external finance (Christiano, Motto, and Rostagno, 2014; Gilchrist, Sim, and Zakrajsek, 2014).

The vast majority of the recent literature has modelled uncertainty as changes in the variance of the exogenous processes driving the model economy.² In turn, this definition of uncertainty has been used with two different notions: (i) uncertainty about *aggregate* shocks, such as the time-varying variance of the economy-wide total factor productivity; and (ii) uncertainty about *idiosyncratic* shocks, such as the cross-sectional dispersion of firm-level productivity in models with heterogeneous firms. In this paper we consider both notions of uncertainty and we refer to the former as “macro uncertainty” and to the latter as “micro uncertainty”.³

¹For example, Leland (1968), Kimball (1990) and Carroll and Kimball (2007) show the theoretical conditions needed for (future) uncertainty to affect consumption, later quantified empirically by Carroll and Samwick (1995) and others (see eg Carroll and Kimball). Hartman (1976), Abel (1983), Bernanke (1983), Caballero (1991), and Dixit and Pindyck (1994) show the theoretical conditions needed for uncertainty to affect investment. Recently Bloom (2009) has shown that uncertainty can have sizeable effects on firms’ demand for factor inputs.

²A notable exception is the literature on ambiguity, see among others Ilut and Schneider (2014).

³An increasing body of research has studied the role of micro and macro uncertainty using dynamic stochastic general equilibrium (DSGE) models. For uncertainty about aggregate shocks see Justiniano and Primiceri (2008), Fernandez-Villaverde and Rubio-Ramirez (2010), Fernandez-Villaverde and others (2011), Basu and Bundick (2017), Born and Pfeifer (2014), Fernandez-Villaverde and others (2015), Gourio (2012), and Leduc and Liu (2015). For uncertainty about idiosyncratic shocks see Dorofeenko, Lee, and Salyer (2008), Gilchrist, Sim, and Zakrajsek (2014), Arellano, Bai, and Kehoe (2012), Christiano, Motto, and Rostagno (2014). Finally, Bloom (2009), Bloom and others (2012), Bachmann and Bayer (2013), and Balke, Martinez-Garcia, and Zeng (2017) consider both notions of uncertainty.

The aim of this paper is threefold. First, to investigate the transmission of both micro and macro uncertainty shocks to the real economy. Second, to explore different mechanisms through which uncertainty shocks are amplified in the model, such as the severity of the credit frictions, price stickiness, the monetary policy response, and consumer preferences. Third, and finally, to quantify the role of uncertainty shocks in driving business cycle fluctuations.

We use a general equilibrium model with sticky prices and credit frictions in the spirit of [Bernanke, Gertler, and Gilchrist \(1999\)](#) (BGG henceforth). We characterize the cyclical fluctuations in macro uncertainty using aggregate total factor productivity (TFP) data for the U.S. business sector; and the cyclical fluctuations in micro uncertainty using the cross-sectional dispersion of establishment-level TFP from the Census panel of manufacturing establishments. We then feed the estimated processes into the model solution to compute the response of the model economy to a mean-preserving shock to the variance of aggregate productivity (a “macro uncertainty shock”); and to the variance of idiosyncratic productivity (a “micro uncertainty shock”). When doing so, we also consider the role of the different mechanisms that affect their propagation. Finally, we gauge the importance of uncertainty shocks (and of the different channels through which these shocks are transmitted) by computing business cycle statistics from simulated data. Specifically, we compare the baseline model with variants of the model where we introduce the shocks and/or the amplifying mechanisms one at the time.

Our paper contributes to the growing literature on the role of uncertainty shocks as a driver of business cycle fluctuations. We highlight three key contributions. First, to the best of our knowledge this is the first paper that studies both macro and micro uncertainty in an environment with credit frictions and sticky prices.⁴ This allows us to deepen our understanding of how different uncertainty shocks propagate to the real economy in such an environment.

Second, we investigate the key ingredients in the transmission of both micro and macro uncertainty shocks, exploring the different mechanisms through which shocks are amplified in the model. In doing so, we are able to explain why the recent literature has found contrasting results concerning the importance of uncertainty shocks in driving business cycle fluctuations. The differences can be explained by the different physical environments and frictions consid-

⁴Other papers considered both micro and macro uncertainty shocks, but in different environments relative to ours (see [Bloom \(2009\)](#), [Bloom and others \(2012\)](#), and [Bachmann and Bayer \(2013\)](#)). While writing this paper we became aware of a paper by [Balke, Martinez-Garcia, and Zeng \(2017\)](#) who also consider both micro and macro uncertainty in a set up similar to ours. The key difference lies in the estimation of the time series properties of micro uncertainty, as we explain in more detail below.

ered (e.g., financial frictions, nominal rigidities, zero lower bound on monetary policy, among others).⁵

Finally, our estimates of the cyclical fluctuations in micro uncertainty contribute to the debate on how to parametrize micro uncertainty processes in this class of financial accelerator models. In a recent paper, [Christiano, Motto, and Rostagno \(2014\)](#) recover the time series properties of micro uncertainty (which they label a “risk shock”) from macroeconomic and financial aggregate data through the estimation of a richer version of [Bernanke, Gertler, and Gilchrist \(1999\)](#)’s original model.⁶ Differently, in this paper we use disaggregated establishment-level data from [Bloom and others \(2012\)](#) to estimate the time series properties of micro uncertainty. Despite the different estimation procedure, our estimate of the volatility of micro uncertainty shocks is smaller, but comparable to [Christiano, Motto, and Rostagno \(2014\)](#)’s estimate.⁷

Our results are as follows. We first analyze the conditional impact of uncertainty shocks. Impulse responses show that both macro and micro uncertainty shocks have a contractionary impact on the economy. Both shocks propagate to the rest of the economy via sticky prices and the financial accelerator mechanism, which not only are crucial for generating comovement between consumption and investment, but also amplify the impact of both shocks on output. The monetary policy response and the specification of households’ preferences are other important determinants of the impact of both micro and macro uncertainty shocks on output. We also find that the impact of macro and micro uncertainty shocks on economic activity is quantitatively different: a one standard deviation shock to micro uncertainty leads to a 0.8 percent fall in total output, over 30 times larger than a one standard deviation shock to macro uncertainty.

We then study the unconditional implications of uncertainty shocks. Business cycle statistics obtained from a model *without* uncertainty shocks (i.e., where ‘standard’ aggregate TFP is the only driving force of the economy) are almost identical to those obtained from our baseline

⁵For example [Bachmann and Bayer \(2013\)](#), [Born and Pfeifer \(2014\)](#), [Gilchrist, Sim, and Zakrajsek \(2014\)](#), and [Chugh \(2016\)](#) find little evidence of uncertainty shocks being a major driver of business cycle fluctuations. In contrast, [Christiano, Motto, and Rostagno \(2014\)](#) find that a large share of output fluctuations can be explained by (micro) uncertainty shocks. [Bloom and others \(2012\)](#) show that the conditional impact of (large) uncertainty shocks can be economically significant. [Fernandez-Villaverde and others \(2011\)](#) and [Basu and Bundick \(2017\)](#) also show that, when the monetary policy is constrained by the zero lower bound, the conditional impact of uncertainty shocks can be sizable.

⁶In a similar spirit, [Balke, Martinez-Garcia, and Zeng \(2017\)](#) estimate their model with the simulated method of moments.

⁷Our results are closer to the few “micro” estimates available in the literature. [Chugh \(2016\)](#) uses the disaggregated plant-level data constructed by [Cooper and Haltiwanger \(2006\)](#) from the Longitudinal Research database; [Gilchrist, Sim, and Zakrajsek \(2014\)](#) use disaggregated data from Compustat on firms’ net sales; [Bachmann and Bayer \(2013\)](#) use firm-level German data from USTAN.

model, suggesting that uncertainty shocks are unlikely to be a major driver of business cycle fluctuations. When introduced alone, micro uncertainty shocks can only explain a small, but non-trivial share of the total volatility of output (around 20 percent). These results echo the findings of [Bachmann and Bayer \(2013\)](#), who argue (within an environment with physical adjustment as opposed to financial frictions) that uncertainty shocks are not a key driver of business cycle fluctuations.

Our estimates of the importance of micro uncertainty shocks fall in between the estimates obtained in previous studies. Using different variants of our baseline model we are able to reconcile the available evidence on the importance of micro uncertainty shocks found in the literature: the simulated business cycle statistics show that the amount of output volatility generated by uncertainty shocks is particularly sensitive to the degree of price stickiness, the severity of the credit friction, the specification of households preferences and the reaction function of monetary policy.

The remainder of the paper proceeds as follows: section [II](#) presents the model, including the sources of uncertainty; section [III](#) discusses the choice of the model's parameters, the estimation of the time series properties of micro and macro uncertainty, and the solution method employed; section [IV](#) presents the key results from the impulse response analysis, while section [V](#) reports the results from the unconditional analysis. Section [VI](#) concludes.

II. MODEL

This section outlines the model used in our analysis. It is a variant of the BGG model formulated by [Faia and Monacelli \(2007\)](#) comprising optimizing households; intermediate-goods producing firms that use households' labor and finished entrepreneurial capital as inputs for production; perfectly competitive firms that assemble a continuum of intermediate goods into a final good; capital producers that transform output into unfinished capital goods; entrepreneurs that purchase this capital, rent it to firms and are subject to a credit friction; financial intermediaries that channel households' savings into loans for entrepreneurs; and a policy maker that sets interest rates. A graphical overview of the model is provided in [Figure 1](#). In what follows we consider the problems faced by each agent.

Figure 1. SIMPLIFIED STRUCTURE OF THE MODEL ECONOMY

A. Households

There is a continuum of households, each indexed by $i \in (0, 1)$. They consume a composite final good, invest in safe bank deposits, supply labor, and own shares of a monopolistic competitive sector that produces differentiated varieties of goods. The representative household chooses the set of processes $\{C_t, N_t\}_{t=0}^{\infty}$ and one-period nominal deposits $\{D_t\}_{t=0}^{\infty}$ held at financial intermediaries (described below), taking as given the set of processes $\{P_t, W_t, (1 + R_t^n)\}_{t=0}^{\infty}$ and the initial condition D_0 to maximize:

$$\max_{\{C_t, N_t, D_t\}_{t=0}^{\infty}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t U(C_t, N_t), \quad (1)$$

subject to the sequence of budget constraints:

$$P_t C_t + D_{t+1} \leq (1 + R_t^n) D_t + W_t N_t + \Pi_t, \quad (2)$$

where $U(C_t, N_t)$ denotes the household's preferences, C_t is workers' consumption of the final good, P_t are consumer prices, W_t is the nominal wage, N_t is total labor hours, R_t^n is the nominal net interest rate paid on deposits, and Π_t are the nominal profits that households receive from running production in the monopolistic sector.

The first order conditions of the above problem read:

$$\begin{aligned} U_{c,t} &= \beta(1 + R_t^n) \mathbb{E}_t \left[U_{c,t+1} \frac{P_t}{P_{t+1}} \right], \\ \frac{W_t}{P_t} &= -\frac{U_{n,t}}{U_{c,t}}, \end{aligned} \quad (3)$$

where $U_c = \partial U / \partial C$, $U_n = \partial U / \partial N$, together with $\lim_{j \rightarrow \infty} D_{t+j} / (1 + R_t^n) = 0$ and the budget constraint (2) holding with equality. Our baseline model assumes that preferences are as in [Greenwood, Hercowitz, and Huffman \(1988\)](#) (GHH preferences henceforth):

$$U(C_t, N_t) = \frac{1}{1 - \rho} (C_t - \tau N_t^{1+v})^{1-\rho}, \quad (4)$$

so that:

$$\begin{aligned} U_{c,t} &= (C_t - \tau N_t^{1+v})^{-\rho}, \\ U_{n,t} &= -\tau(1+v) N_t^v (C_t - \tau N_t^{1+v})^{-\rho}. \end{aligned} \quad (5)$$

GHH preferences remove wealth effects from the labor supply decision and thereby prevent labor supply shifts in response to movements in consumption, which mitigate the impact of uncertainty shocks (see section IV and the Appendix for more).

B. Unfinished Capital Producers

A competitive sector of capital producers combines investment I —which is expressed in terms of the final goods and hence with price P_t — and the existing (depreciated) capital stock to produce unfinished capital goods, K . This activity entails physical adjustment costs. The corresponding constant return to scale production function is $\phi(I_t/K_t)K_t$ where $\phi(\cdot)$ is increasing and convex. We assume the following functional form:

$$\phi\left(\frac{I_t}{K_t}\right)K_t = \left[\frac{I_t}{K_t} - \frac{\phi_k}{2} \left(\frac{I_t}{K_t} - \delta \right)^2 \right] K_t, \quad (6)$$

so that capital accumulation obeys:

$$K_{t+1} = (1 - \delta)K_t + I_t - \frac{\phi_k}{2} \left(\frac{I_t}{K_t} - \delta \right)^2 K_t. \quad (7)$$

Defining Q_t as the re-sell price of the capital good, capital producers maximize profits:

$$\max_{I_t} Q_t \left[I_t - \frac{\phi_k}{2} \left(\frac{I_t}{K_t} - \delta \right)^2 K_t \right] - P_t I_t, \quad (8)$$

implying the following optimality condition:

$$Q_t \left[1 - \phi_k \left(\frac{I_t}{K_t} - \delta \right) \right] = P_t. \quad (9)$$

C. Entrepreneurs

The activity of risk-neutral entrepreneurs is at the heart of the credit friction. At the end of period t , each entrepreneur j purchases unfinished capital from the capital producers at the price Q_t and transforms it into finished capital to be used for production in $t + 1$.

The transformation of unfinished capital into finished capital is performed with a technology that is subject to idiosyncratic productivity shocks ω_{t+1}^j . As it is commonly assumed in the literature, these shocks are assumed to be independently and identically distributed (*i.i.d.*) across entrepreneurs and time, and to follow a log normal distribution, $\omega \sim \log \mathcal{N}(1, \sigma_\omega^2)$, with cumulative distribution function denoted by $F(\omega)$.⁸ For the solution of the entrepreneur-ial problem, the variance of ω is taken as a given parameter, although as we shall see below, allowing for time variation in σ_ω^2 in the solution of the model will constitute a major source of uncertainty in our economy (that we label micro uncertainty).⁹

To finance the purchase of unfinished capital, entrepreneurs employ internal funds but also need to acquire an external loan from a financial intermediary (banks). The relationship with the lender is modelled assuming asymmetric information between entrepreneurs and banks and a costly state verification as in [Townsend \(1979\)](#) and [Gale and Hellwig \(1985\)](#). Specifically, the idiosyncratic shock to entrepreneurs is private information for the entrepreneur. To observe the shock, the lender must pay an auditing cost that is a fixed proportion $\mu \in [0, 1]$ of the realized gross return to capital held by the entrepreneur. The optimal loan contract will induce entrepreneurs to not misreport their earnings and will minimize the expected auditing costs incurred by the lender. Under these assumptions, the optimal contract is a standard

⁸This assumption is in contrast with ‘micro’ measures of productivity, which are typically quite persistent in the data. However, assuming persistence in the idiosyncratic productivity shock would require having to track the distribution of entrepreneurs through time and thereby complicate the solution of the optimal debt contract.

⁹Note that other papers in the earlier literature have considered a similar definition of time-varying uncertainty (or “risk”) as the one used here. See, for example [Christiano, Motto, and Rostagno \(2003\)](#), [Dorofeenko, Lee, and Salyer \(2008\)](#), [Christiano, Motto, and Rostagno \(2010\)](#), and [Christiano, Motto, and Rostagno \(2014\)](#).

debt contract with costly bankruptcy. If the entrepreneur does not default, the lender receives a fixed payment independent of the realization of the idiosyncratic shock; but if the entrepreneur defaults, the lender audits and seizes whatever is left. For this reason the interest rate on entrepreneurial loans will be given by a spread over the risk free rate. The derivation of the optimal contract is shown below.

1. The Optimal Loan Contract

There are two agents, entrepreneurs and banks. At the end of period t , an entrepreneur j holds nominal net worth NW_{t+1}^j and acquires credit, B_{t+1}^j , to finance capital purchases:

$$B_{t+1}^j = Q_t K_{t+1}^j - NW_{t+1}^j. \quad (10)$$

Before defining the entrepreneurs' problem we first define the expected *nominal income from holding one unit of finished capital*, which we denote by $\mathcal{Y}_{t+1}^k$. This is composed of the rental rate of capital (Z_{t+1}) plus the re-sell price of capital, net of depreciation and physical adjustment costs:

$$\mathcal{Y}_{t+1}^k \equiv Z_{t+1} + Q_{t+1} \left[(1 - \delta) - \frac{\phi_k}{2} \left(\frac{I_{t+1}}{K_{t+1}} - \delta \right)^2 + \phi_k \left(\frac{I_{t+1}}{K_{t+1}} - \delta \right) \frac{I_{t+1}}{K_{t+1}} \right]. \quad (11)$$

This implicitly defines the return to entrepreneurs from holding a unit of capital:

$$1 + R_{t+1}^k \equiv \mathcal{Y}_{t+1}^k / Q_t. \quad (12)$$

The idiosyncratic shock is realized before the beginning of period $t + 1$. Entrepreneur j will repay his loans only if $\omega_{t+1}^j \mathcal{Y}_{t+1}^k K_{t+1}^j \geq B_{t+1}^j (1 + R_{t+1}^L)$ where R_{t+1}^L is the lending rate paid on loans. Therefore, the above expression defines the cut-off value of the idiosyncratic shock that separates bankrupt and non-bankrupt entrepreneurs. An entrepreneur who experiences an idiosyncratic shock equal to:

$$\omega_{t+1}^j < \bar{\omega}_{t+1}^j = \frac{B_{t+1}^j (1 + R_{t+1}^L)}{\mathcal{Y}_{t+1}^k K_{t+1}^j}, \quad (13)$$

will default on his debt and the bank will seize all his remaining assets after paying the monitoring cost.

On the other hand, banks operate only if the following condition is satisfied:

$$\mathcal{Y}_{t+1}^k K_{t+1}^j \left(\Gamma(\bar{\omega}_{t+1}^j) - \mu G(\bar{\omega}_{t+1}^j) \right) \geq (1 + R_t^n) B_{t+1}^j, \quad (14)$$

where $G(\bar{\omega}_{t+1}^j) \equiv \int_0^{\bar{\omega}} \omega_{t+1}^j dF(\omega)$ and $\Gamma(\bar{\omega}_{t+1}^j) \equiv \left[1 - \int_0^{\bar{\omega}} dF(\omega) \right] \bar{\omega}_{t+1}^j + G(\bar{\omega}_{t+1}^j)$. As in BGG, $\Gamma(\bar{\omega}_{t+1}^j)$ is the share of finished capital going to banks and symmetrically, $1 - \Gamma(\bar{\omega}_{t+1}^j)$ is the share of finished capital going to entrepreneurs, with $G(\bar{\omega}_{t+1}^j)$ being the average value of the idiosyncratic shock for bankrupt entrepreneurs.

The optimal contract is derived by maximizing over $\{\bar{\omega}_{t+1}^j, B_{t+1}^j\}$ entrepreneurial profits:

$$\max_{\{\bar{\omega}_{t+1}^j, B_{t+1}^j\}} \mathcal{Y}_{t+1}^k K_{t+1}^j \left(1 - \Gamma(\bar{\omega}_{t+1}^j) \right), \quad (15)$$

subject to (10) and (14) holding with equality. By equalizing the Lagrangian multipliers of the first order conditions of the above problem and using the definition of the nominal income from holding one unit of finished capital (11) we get:

$$\frac{1 + R_{t+1}^k}{1 + R_t^n} = \psi_t, \quad (16)$$

where

$$\psi_t = \left(\frac{\left(1 - \Gamma(\bar{\omega}_{t+1}^j) \right) \left(\Gamma'(\bar{\omega}_{t+1}^j) - \mu G'(\bar{\omega}_{t+1}^j) \right)}{\Gamma'(\bar{\omega}_{t+1}^j)} + \left(\Gamma(\bar{\omega}_{t+1}^j) - \mu G(\bar{\omega}_{t+1}^j) \right) \right)^{-1}, \quad (17)$$

is the *external finance premium*. As in BGG, $\psi_t = f(\bar{\omega}_{t+1}^j)$ with $f'(\bar{\omega}_{t+1}^j) > 0$. Moreover, the ratio between the lending rate and the risk free rate gives the *risk premium*, which can be computed from the zero profit condition as:

$$\frac{1 + R_{t+1}^L}{1 + R_t^n} = \frac{\psi_t}{\bar{\omega}_{t+1}^j} \left(1 - \frac{NW_{t+1}^j}{Q_t K_{t+1}^j} \right), \quad (18)$$

where $NW_{t+1}^j / Q_t K_{t+1}^j$ is the inverse of the leverage ratio. Equation (18) shows that, in the presence of credit market imperfections, the premium paid on the risk free interest rate for a loan depends on the entrepreneur's balance-sheet condition, so that the higher the leverage, the higher is the premium charged on entrepreneurial risky loans.

Finally note that from the zero-profit condition, a demand function for capital that is increasing in net worth but decreasing in price can be derived:

$$K_{t+1}^j = \left(\frac{1}{1 - \psi_t \left(\Gamma(\bar{\omega}_{t+1}^j) - \mu G(\bar{\omega}_{t+1}^j) \right)} \right) \frac{NW_{t+1}^j}{Q_t}. \quad (19)$$

2. Evolution of Net Worth

To ensure that entrepreneurs do not accumulate enough funds to finance their expenditures on capital entirely with net worth, we assume that they die/exit with constant probability. In particular, we assume that each entrepreneur survives until the next period with probability γ . Entrepreneurs who “die” in period t are not allowed to purchase capital, but instead simply consume their accumulated resources and depart from the scene. Therefore, entrepreneurial consumption in each period will be:

$$C_t^e = (1 - \gamma) \mathcal{Y}_{t+1}^k K_{t+1}^j \left(1 - \Gamma(\bar{\omega}_{t+1}^j) \right), \quad (20)$$

where $\mathcal{Y}_{t+1}^k K_{t+1}^j \left(1 - \Gamma(\bar{\omega}_{t+1}^j) \right)$ is the share of finished capital going to entrepreneurs in each period. Symmetrically, entrepreneurs who survive accumulate net worth as follows:

$$NW_{t+1}^j = \gamma \mathcal{Y}_{t+1}^k K_{t+1}^j \left(1 - \Gamma(\bar{\omega}_{t+1}^j) \right). \quad (21)$$

Since $\mathcal{Y}_{t+1}^k = Q_t (1 + R_{t+1}^k)$, net worth is positively related to the price and the stock of capital. In contrast, as noted by [Faia and Monacelli \(2007\)](#), the aggregate return on finished capital R_{t+1}^k has an ambiguous impact on net worth. On the one hand, an increase in R_{t+1}^k generates a higher return for each unit of finished capital owned by entrepreneurs. On the other hand, an increase in R_{t+1}^k also generates an increase in the external finance premium, as shown in equation (16), which contributes to the risk premium and therefore reduces net worth.

D. Final Good Sector

The aggregate final good Y_t is produced by perfectly competitive firms. It requires assembling a continuum of intermediate goods (whose problem is derived below), indexed by i , via the aggregate production function:

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}. \quad (22)$$

Profit maximization yields typical demand functions:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t, \quad (23)$$

for all i , where $P_t = \left(\int_0^1 P_t(i)^{-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}$ is the price index consistent with the final good producers earning zero profits.

E. Intermediate Firms

Each household owns an equal share of the intermediate-goods producing firms. Each firm assembles labor (supplied by the workers) and finished entrepreneurial capital to operate a constant return to scale production function $\Phi(\cdot)$ for the variety i of the intermediate good:

$$Y_t = \Phi(\exp(A_t), N_t(i), K_t(i)) \quad (24)$$

where A_t is a productivity shifter common to all firms (i.e., total factor productivity), which will be of crucial importance for the definition of our macro uncertainty shock (section II.H).

Each firm i has monopolistic power in the production of its own variety and therefore has leverage in setting the price. In so doing it faces a quadratic cost equal to:

$$\frac{\omega_p}{2} \left(\frac{P_t(i)}{P_{t-1}(i)} - \pi \right)^2 \quad (25)$$

where π is the steady-state inflation rate and where the parameter ω_p measures the degree of nominal price rigidity. The higher ω_p the more sluggish is the adjustment of nominal prices. In the particular case of $\omega_p = 0$, prices are flexible.

The problem faced by each monopolistic firm is to choose the sequence of factors of production $\{K_t(i), N_t(i)\}_{t=0}^{\infty}$ and prices $\{P_t(i)\}_{t=0}^{\infty}$ that maximize expected discounted real profits:

$$\max_{\{K_t(i), N_t(i), P_t(i)\}_{t=0}^{\infty}} \mathbb{E}_t \sum_{t=0}^{\infty} \frac{\beta^t}{P_t} \left(P_t(i) Y_t(i) - (W_t N_t(i) + Z_t K_t(i)) - \frac{\omega_p}{2} \left(\frac{P_t(i)}{P_{t-1}(i)} - \pi \right)^2 \right), \quad (26)$$

subject to (24) and (23). Denoting by $\{mc_t\}_{t=0}^{\infty}$ the sequence of Lagrange multipliers associated with this problem and by $\tilde{p}_t \equiv P_t(i)/P_t$ the relative price of variety i , the optimality

conditions read:

$$\begin{aligned}
\frac{W_t}{P_t} &= mc_t Y_{n,t}, \\
\frac{Z_t}{P_t} &= mc_t Y_{k,t}, \\
0 &= Y_t \tilde{p}_t^{-\varepsilon} ((1 - \varepsilon) + \vartheta mc_t) - \omega_p \left(\pi_t \frac{\tilde{p}_t}{\tilde{p}_{t-1}} - \pi \right) \frac{\pi_t}{\tilde{p}_{t-1}} + \\
&\quad + \omega_p \left(\pi_{t+1} \frac{\tilde{p}_{t+1}}{\tilde{p}_t} - \pi \right) \pi_{t+1} \frac{\tilde{p}_{t+1}}{\tilde{p}_t^2}
\end{aligned} \tag{27}$$

where $Y_n = \partial Y / \partial N$, $Y_k = \partial Y / \partial K$, $\pi_t = P_t / P_{t-1}$ is the gross inflation rate, and where we have suppressed the superscript i , since all firms employ an identical capital to labor ratio in equilibrium. Note that the Lagrange multiplier mc_t can be interpreted as the real marginal cost of production. In a symmetric equilibrium where $\tilde{p}_t = 1$, equation (27) can be written as a forward-looking Phillips curve:

$$(\pi_t - \pi) \pi_t = \beta \mathbb{E}_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} (\pi_{t+1} - \pi) \pi_{t+1} \right\} + Y_t \frac{\varepsilon}{\omega_p} \left(mc_t - \frac{\varepsilon - 1}{\varepsilon} \right). \tag{28}$$

F. Monetary Policy

Monetary policy is assumed to be conducted by means of an interest rate reaction function, constrained to be linear in the logs of the relevant arguments:

$$\frac{1 + R_t^n}{1 + R^n} = \left(\frac{1 + R_{t-1}^n}{1 + R^n} \right)^{\phi^r} \left(\frac{1 + \pi_t^n}{1 + \pi} \right)^{(1 - \phi^r) \phi^\pi} \left(\frac{1 + Y_t}{1 + Y_{t-1}} \right)^{(1 - \phi^r) \phi^y}. \tag{29}$$

The parameter $\phi^r \in [0, 1)$ generates interest-rate smoothing, $\phi^\pi > 0$ and $\phi^y \geq 0$ control the responses to deviations of inflation from target π and from output growth respectively. Given the inflation target π , the steady-state nominal interest rate, R^n , is determined by the equilibrium of the economy.

G. Market Clearing

Final goods' market equilibrium requires final good's production be allocated to private consumption of households and entrepreneurs, investment, and to resource costs that originate from both the adjustment of prices and from the banks' monitoring of entrepreneurial activ-

ity:

$$Y_t = C_t + C_t^e + I_t + \frac{\omega_p}{2} (\pi_t - \pi)^2 + \mu G(\bar{\omega}) \frac{\mathcal{Y}_t^k}{P_t} K_t. \quad (30)$$

H. Sources of Uncertainty in the Model

Three exogenous processes are assumed to drive the dynamics of our model economy. As is standard in the literature, we assume that the log-level of Total Factor Productivity (TFP) follows an autoregressive process:

$$A_t = \rho^A A_{t-1} + e^{W_t} \sigma^A \varepsilon_t^A, \quad (31)$$

where ε_t^A follows a $\mathcal{N}(0, 1)$ process and the parameter σ^A is the standard deviation of innovations to A_t (i.e., the TFP shock). The parameter σ^A is pre-multiplied by an additional process, e^{W_t} , which acts as a shifter of the standard deviation of A_t . We refer to e^{W_t} as the stochastic volatility of TFP. We also assume that W_t follows an autoregressive process of the type:

$$W_t = \rho^W W_{t-1} + \sigma^W \varepsilon_t^W, \quad (32)$$

where ε_t^W follows a $\mathcal{N}(0, 1)$ process and the parameter σ^W is the standard deviation of innovations to W_t .

By allowing the variance of TFP shocks to rise, the probability of events that are distant from the mean increases. In the face of an increase in uncertainty, economic agents are likely to modify their behavior even though the mean outcome is unchanged (i.e., there are no first moment shocks to TFP). We define *macro uncertainty shocks* as exogenous changes in the variance of TFP (i.e., movements in W_t) that do not affect its level.

The third exogenous process and second source of uncertainty in our model is the dispersion of idiosyncratic entrepreneurial productivity. As introduced by [Dorofeenko, Lee, and Salyer \(2008\)](#) and [Christiano, Motto, and Rostagno \(2014\)](#), we allow the variance of the idiosyncratic shocks to vary over time. Note that, if ω is log-normally distributed with $\omega \sim \log \mathcal{N}(1, \sigma_\omega^2)$, then the log of ω is normally distributed, i.e. $\log(\omega) \sim \mathcal{N}(M, S^2)$, where M and S^2 are the mean and the variance of the underlying normal distribution. For technical purposes, it is easier to model the variance of the underlying Normal distribution, which (after fixing the mean of ω to 1) is defined as $S^2 = \log(1 + \sigma_\omega^2)$. As in [Christiano, Motto, and Rostagno \(2014\)](#), we

model the log-deviation of S_t from its steady-state value as:

$$\log\left(\frac{S_t}{\bar{S}}\right) = \rho^S \log\left(\frac{S_{t-1}}{\bar{S}}\right) + \sigma^S \varepsilon_t^S, \quad (33)$$

where ε^S follows a $\mathcal{N}(0, 1)$ and σ^S is the standard deviation of innovations to S_t .

Therefore, when S_t increases, the dispersion of entrepreneurial outcomes increases too. Despite leaving the mean of the outcomes unaffected, an increase in S_t will affect the entrepreneurial loans market. Intuitively, higher dispersion of returns implies, *ceteris paribus*, a higher probability of entrepreneurial bankruptcy. Given the information asymmetry between banks and entrepreneurs and the costly state verification, this will affect the level of lending rates and, therefore, capital demand. We refer to exogenous movements in S_t as *micro uncertainty shocks*.

III. CALIBRATION AND SOLUTION METHODOLOGY

In this section we describe how the parameters of the model and the time series properties of the exogenous processes are pinned down.¹⁰ We then discuss the methodology we use to solve and simulate the model.

A. Parameters of the model

The time unit is a quarter. The model's parameters that are not associated with the exogenous processes (31)-(33) are presented in Table 1 and motivated below.

We first fix two parameters that help to pin down the solution of the entrepreneurial problem defined in section II. The annual steady-state inflation, π , is set to 2 percent; and the time discount factor, β , is set to 0.994 so as to target an annualized average real risk-free rate of interest of 2.4 percent, similar to Fernandez-Villaverde, Guerron-Quintana, and Rubio-Ramirez (2010).

Turning to the parameters related to the credit friction, we calibrate those to obtain reasonable steady-state values for key financial variables, namely the external finance premium and the

¹⁰In the next Sections we will also consider alternative values for some key parameters to shed light on the propagation mechanisms.

Table 1. PARAMETERS VALUES OF THE MODEL

<i>Parameter</i>	<i>Description</i>	<i>Value</i>	<i>Source/Target</i>
μ	Monitoring cost	0.25	Carlstrom and Fuerst (1997)
γ	Survival probability	0.985	Christiano, Motto, and Rostagno (2014)
α	Capital share	0.3	Labor share of 70%
δ	Depreciation rate	0.025	Investment/output of 18%
β	Discount factor	0.994	Annual real rate of 2.4%
ρ	Risk aversion	2	Standard value
ν	Inv. Frisch elasticity	1	Christiano, Motto, and Rostagno (2014)
τ	GHH scaling factor	2.5	Steady-state hours ($N = 1/3$).
ε	Goods' elast. of subst.	10	Mark-up of 11%
θ	Price stickiness	105	Calvo price stick. of 0.75
ϕ_k	Investment adj. cost	30	Output. Volatility
π	Steady state inflation	2%	Fernandez-Villaverde and others (2015)
$\bar{S}$	St. Dev. of idiosyncratic prod.	0.225	Ext. fin. premium of 188 bps
ρ_r	Int. rate smoothing	0.8	Policy rate persistence
ρ_y	Response to output	0.3	Born and Pfeifer (2014)
ρ_π	Response to inflation	1.8	Born and Pfeifer (2014)

entrepreneurial default rate. We set the steady-state value of the quarterly survival rate of entrepreneurs γ to 0.985, the same value used by [Christiano, Motto, and Rostagno \(2014\)](#) and fairly similar to the value originally used by BGG; the monitoring cost μ to 0.25 as in [Carlstrom and Fuerst \(1997\)](#), and close to the value estimated by [Christiano, Motto, and Rostagno \(2014\)](#) at 0.21; and, finally, the steady-state value of the standard deviation of the idiosyncratic productivity $\bar{S}$ to 0.225, slightly lower but very close to the value estimated by [Christiano, Motto, and Rostagno \(2014\)](#) (namely, 0.26). This parametrization yields reasonable values for our target variables. The quarterly, steady-state probability of default is around 1 percent, very close to the 0.974 percent value used in [Carlstrom and Fuerst \(1997\)](#) and [Fisher \(1999\)](#), and not far from the original 0.75 percent value used by BGG; finally, the implied steady-state external finance premium is of about 188 basis points, almost identical to the value used by [Carlstrom and Fuerst \(1997\)](#). Moreover, the steady-state value of leverage ratio implied by the above calibration is of about 2—the same value used in BGG.

Turning to household preferences, we set the relative disutility of labor effort, τ , so that the value of hours worked is equal to 1/3 in the steady state, as is common in the literature. Also, the coefficient of risk aversion in the utility function, ρ , is fixed to 2 as in [Fernandez-Villaverde and others \(2015\)](#), while the inverse of the Frisch elasticity of labor supply, ν , is fixed to 1 as in [Christiano, Motto, and Rostagno \(2014\)](#).

The production technology is assumed to be Cobb-Douglas with constant returns to scale. Without deviating from the standard values used in the literature, we set the quarterly aggregate capital depreciation rate, δ , to 0.025 and the capital's share, α , to 0.3.¹¹ The elasticity of substitution across varieties in the CES aggregator, ε , is set to 10, consistent with a price markup of roughly 11 percent, as in [Born and Pfeifer \(2014\)](#). Regarding the degree of nominal price rigidity, we follow [Faia and Monacelli \(2007\)](#), who show that it is possible to build a mapping between the frequency of price adjustment in the Calvo–Yun model and the price adjustment cost parameter, ω_p . This is done by log-linearizing equation (28) and obtaining an expression for the elasticity of inflation to the real marginal cost which can be compared with empirical studies on the New-Keynesian Phillips curve, such as [Gali and Gertler \(1999\)](#) and [Carlstrom, Fuerst, and Paustian \(2010\)](#) who consider a Calvo-Yun model. The price adjustment parameter, ω_p , is chosen such that, in an equivalent Calvo price-setting model, prices are fixed for 4 quarters on average.

We set the coefficients of the interest rate reaction function similar to the estimated values in [Born and Pfeifer \(2014\)](#), namely 1.8 for the coefficient on inflation, ϕ^π , and 0.25 for the coefficient on output growth, ϕ^y . We set the interest-rate smoothing parameter, ϕ^r , to 0.8 to match the persistence of the effective federal funds rate.

B. Exogenous Processes

We now describe how we estimate the time series properties (i.e., persistence and standard deviation) of the three exogenous processes in our model, namely aggregate TFP as well as macro and micro uncertainty. Table 2 summarizes the parameter values we use, while the details are reported below.

We start with micro uncertainty. As a proxy for the time-varying dispersion of the idiosyncratic productivity of entrepreneurs we use a time series of the cross-sectional standard deviation of establishment-level TFP innovations. This measure of uncertainty, labelled σ_t^{micro} , comes from [Bloom and others \(2012\)](#), who recover it using annual data from the Census panel of manufacturing establishments over the 1972–2009 sample period.¹² To compute this

¹¹These parameter values imply the following great ratios in steady state: consumption over total output is 76 percent; investment over total output is 18 percent, and entrepreneurial consumption over total output is 6 percent.

¹²The data is available at the following website: <https://people.stanford.edu/nbloom/>. It includes data on over 50,000 establishments from 1972 to 2009. [Bloom and others \(2012\)](#) focus on a sub-set of 15,673 establishments with 25+ years of data.

Table 2. PARAMETER VALUES OF THE EXOGENOUS PROCESSES IN THE MODEL

<i>Parameter</i>	<i>Description</i>	<i>Value</i>	<i>Source/Target</i>
ρ^S	Persistence of Micro Uncert.	0.86	Data
σ^S	St. Dev. of Micro Uncert.	0.023	Data
ρ^A	Persistence TFP	0.98	Data
σ^A	Std. Dev. TFP	0.007	Data
ρ^W	Persistence of Macro Uncert.	0.88	Data
σ^W	St. Dev. of Macro Uncert.	0.140	Data

measure of micro uncertainty, Bloom and others (2012) first calculate establishment-level TFP. Then, they compute serially uncorrelated TFP innovations as the residuals from a regression of $\log(\text{TFP})$ on its lagged value, and a set of year fixed effects and establishment fixed effects. Finally, they compute the cross-sectional standard deviation for each period, i.e. σ_t^{micro} . In what follows we will use the time series properties of σ_t^{micro} to parametrize the micro uncertainty process in (33). To do that, we follow the procedure proposed by Chugh (2016).¹³

Figure 2 (left panel) displays the cross-sectional standard deviation of establishment-level TFP shocks, σ_t^{micro} . The plot of the time series reveals a modest upward trend that is removed with a linear trend to produce a model-consistent measure of micro uncertainty, S_t . The resulting cyclical component of the cross-sectional standard deviation of establishment-level TFP shocks is reported in Figure 2 (right panel). The standard deviation of S_t over the sample period 1972–2009 is 3.83 percent, close to Chugh (2016)’s estimates (3.15 percent). Equation (33) is then estimated with OLS to obtain a point estimate of the autoregressive parameter, ρ^S , which yields 0.56 (t-statistic of 4.03). The standard deviation of the (annual) micro uncertainty innovations, σ^S , is computed to be 3.17 percent to match the standard deviation of 3.83 percent for S_t .

These estimates cannot be used directly in the model as the frequency of establishment-level TFP data is annual whereas the frequency of the model is quarterly. To address this mismatch, we follow Chugh (2016) by assuming sufficient smoothness in the micro uncertainty process and compute the quarterly persistence parameter as $\rho^S = 0.56^{1/4} = 0.86$. The parameter σ^S is then computed as follows: data from the quarterly model is simulated and time-aggregated,

¹³Differently from Chugh (2016) we use a different data set that covers a longer and more recent sample period and a larger number of cross-sectional units. Chugh (2016) uses annual data of plant-level profitability constructed by Cooper and Haltiwanger (2006) from the Longitudinal Research Database (LRD). The data set covers approximately 7,000 large U.S. manufacturing plants over the period 1974–1988.

Figure 2. MICRO UNCERTAINTY: CROSS-SECTIONAL DISPERSION OF ESTABLISHMENT-LEVEL TFP

NOTE. The left panel plots the cross-sectional standard deviation of establishment-level TFP (σ_t^{micro}) from Bloom and others (2012). The right panel plots the log-deviation of σ_t^{micro} from a linear trend, i.e. our proxy for S_t .

and σ^S is changed so that the standard deviation of the micro uncertainty process matches its annual empirical counterpart. This procedure yields $\sigma^S = 0.023$.¹⁴

Our estimate is in line with the values found by similar studies in the literature. Bloom and others (2012) assume a two-point Markov chain process for micro uncertainty, where idiosyncratic volatility is set to a value of 0.039 (which approximately triples in the heightened uncertainty state). Christiano, Motto, and Rostagno (2014) derive the (unanticipated component of) the standard deviation of micro uncertainty innovations directly from their DSGE model through Bayesian estimation techniques using U.S. aggregate macro-financial data and obtain a value for σ^S of 0.07. Chugh (2016) finds a value of 0.037. Finally, Balke, Martinez-Garcia, and Zeng (2017), who estimate σ^S via simulated method moments using aggregate macro-financial data, find a value of 0.025. Despite the different data and sample period used in different studies, our estimates are close to those found elsewhere in the literature.

We estimate the time series properties of TFP and macro uncertainty using aggregate TFP data for the U.S. business sector over the sample period 1972:Q1-2012:Q4.¹⁵ First, we fit an AR(1) process to the log-deviations of the level of TFP from a linear trend, so as to estimate the persistence of the TFP process, ρ^A , and the standard deviation of its innovations, σ^A . Ac-

¹⁴Note that, due to bankruptcy, some of the variations in idiosyncratic TFP would be censored in the data (and more so at times of high uncertainty). As a consequence, our estimates of the standard deviation of micro uncertainty shocks might be biased downward.

¹⁵The data can be downloaded at: <http://www.frbsf.org/economic-research/total-factor-productivity-tfp/>. See Fernald (2012), Fernald and Matoba (2009), and Kimball, Fernald, and Basu (2006). The sample period was chosen to yield estimates that can be compared against the micro uncertainty estimates. The results are very similar if we use more recent data.

cording to this exercise, we set ρ^A to 0.98 and σ^A to 0.007. Our values of both the persistence and the standard deviations of TFP are standard for the time series properties of the Solow residual. Indeed, our parametrization is in line with many similar papers in the literature, such as [Fernandez-Villaverde and others \(2011\)](#), [Bloom and others \(2012\)](#), and [Christiano, Motto, and Rostagno \(2014\)](#).

We then compute the standard deviation of the TFP innovations with a rolling window of eight quarters to compute a proxy for the time-varying volatility of the TFP innovations. The left panel of Figure 3 plots the resulting time series (σ_t^{macro}), which reveals a modest downward trend in volatility during the Great Moderation period.¹⁶ As before, we use log-deviations of σ_t^{macro} from a linear trend as the model-consistent measure of macro uncertainty, i.e. as a proxy for W_t . The cyclical component of our proxy for the stochastic volatility of TFP, W_t , is reported in the right panel of Figure 3. An AR(1) process is estimated for equation (32) resulting in a persistence estimate for ρ^W of 0.88 with standard deviation, σ^W , of 0.140. Our parametrization of the macro uncertainty process yields parameter values that are similar to the ones used by similar studies in the literature (see for example [Caldara and others \(2012\)](#) and [Born and Pfeifer \(2014\)](#)).

Figure 3. MACRO UNCERTAINTY: TIME-VARYING VOLATILITY OF AGGREGATE TFP

NOTE. The left panel plots the standard deviation of quarterly TFP innovations (σ_t^{macro}) obtained using a rolling window of eight quarters. The right panel plots the log-deviation of σ_t^{macro} from a linear trend, i.e. our proxy for W_t .

One important consideration is in order here. The estimation of the time series properties of both micro and macro uncertainty implicitly assumes that all of the observed movement in the uncertainty proxies stems from ‘exogenous’ uncertainty shocks. However, fluctuations in

¹⁶Results are little changed when considering a different size of the rolling window.

uncertainty may arise endogenously as a result of other type of (first moment) shocks.¹⁷ The estimates obtained above should therefore be seen as an upper bound.

C. Solution Method

DSGE models are normally solved by taking a linear or log-linear (i.e., first-order) approximation around their non-stochastic steady-state equilibrium. However, when using first-order approximations, shocks to the variance of the exogenous processes do not play any role, since certainty equivalence holds and the decision rule of the representative agent is independent of shocks' second (or higher) moments.¹⁸

For second (or higher) moments to enter the decision rules of economic agents, a higher approximation to the policy functions is needed. In particular, a 3rd-order Taylor series expansion of the solution of the model, allows for second moments to play an independent role in the approximated policy function. We compute a 3rd-order Taylor series approximation around the non-stochastic steady state of the model using [Dynare 4.3](#).¹⁹

Impulse responses functions (IRFs) are defined as the response of the variables in a dynamic system to an exogenous impulse of a given size. These are often computed using the equilibrium of the dynamic system (i.e., steady state) as an initial condition. This is because, in linear models, IRFs do not depend on the state of the economy when the shock occurs, nor on the sign and size of the shock. However, when using a higher order approximation to the solution of the model, this is not the case anymore and impulse responses computed from the steady state are just one of the many IRFs of the non-linear model. We follow [Fernandez-Villaverde and others \(2011\)](#) and compute IRFs in deviation from the ergodic mean of the data generated by the mode.²⁰

¹⁷See for example [Bachmann and Moscarini \(2011\)](#), [D'Erasmus and Moscoso-Boedo \(2011\)](#), [Cesa-Bianchi, Rebucci, and Pesaran \(2015\)](#), [Berger, Dew-Becker, and Giglio \(2016\)](#).

¹⁸Micro uncertainty shocks (as the ones considered in this paper) and the Knightian uncertainty shocks (as in [Ilut and Schneider, 2014](#)) represent an exception and their impact can be studied with linear methods.

¹⁹As [Fernandez-Villaverde and others \(2011\)](#) show, 3rd-order approximation to the policy functions is sufficient to capture the dynamics of the model, with little gain to using an approximation higher than the 3rd-order.

²⁰We refer the reader to the Online Appendix for details on how the IRFs are constructed.

IV. IMPULSE RESPONSE ANALYSIS

In this section we examine the transmission of both macro and micro uncertainty shocks by analyzing the IRFs to a 1 standard deviation increase in macro (ε_t^W) and micro uncertainty (ε_t^S) in our model. We then focus on the role that key model ingredients play in the transmission and amplification of both shocks, paying attention to sticky prices, credit frictions, monetary policy rules, consumers' preferences (risk aversion as well as different representations of consumption and leisure).

A. Baseline Results

We start by analyzing the IRFs to a 1 standard deviation increase in the macro uncertainty innovation. Under our baseline calibration, this is equivalent to an increase in the standard deviation of TFP shocks of about 15 percent, i.e. from their steady state level of 0.007 to about 0.008. This is shown in panel A of Figure 4.

The shock propagates via a precautionary savings motive which acts to reduce consumption. With sticky prices in the short run, the shock puts downward pressure on firms' marginal costs, resulting in an increase in markups and a reduction in labor demand. As the labor supply schedule is fixed under GHH preferences, the fall in labor demand leads to a fall in hours and in the real wage.²¹ The key insight (as noted by [Basu and Bundick \(2017\)](#), [Fernandez-Villaverde and others \(2011\)](#), [Born and Pfeifer \(2014\)](#)) is that under price stickiness output is demand determined (i.e., firms must satisfy whatever output is demanded at a given price), so that the reduction in consumption acts to reduce aggregate demand. Hence, demand for both labor and capital falls, and investment falls too. The impact on output is small: in our baseline calibration, output falls by less than 0.02 percent with a muted effect on financial variables (e.g., the risk premium increases by less than 1.5 annualized basis point).²²

We turn next to the analysis of micro uncertainty shocks (see panel B of Figure 4). We consider a 1 standard deviation increase in the micro uncertainty innovation, which is equal to

²¹In our baseline calibration —and differently from [Basu and Bundick \(2017\)](#)— the macro uncertainty shock does not generate an impact increase in “precautionary labor supply”, since consumption does not enter the labor supply schedule. We explore the role of different preferences below.

²²This is different from similar studies in the literature (e.g. [Basu and Bundick, 2017](#); [Fernandez-Villaverde and others, 2011](#)), who find larger impacts of similar types of macro uncertainty shocks. The main difference lies in the type of uncertainty shock (i.e., whether to government spending or demand) and the size of the uncertainty shock. Our results are comparable with those reported [Born and Pfeifer \(2014\)](#).

Figure 4. THE IMPACT OF MACRO AND MICRO UNCERTAINTY SHOCKS

NOTE. Impulse response functions (IRFs) to a 1 standard deviation increase in macro uncertainty (panel A) and in micro uncertainty (panel B). The IRFs are computed with respect to the ergodic mean of the variables of interest. All responses are in percent, except for the risk premium which is in basis points. The unit of the x-axis is quarters.

an increase in micro uncertainty of 2.5 percent. Whilst the macro uncertainty shock operates through the precautionary saving motive, the micro uncertainty shock operates through the cost of external debt and entrepreneurial capital demand. Specifically, higher dispersion of the idiosyncratic shock implies larger returns for some entrepreneurs but larger losses for others. All else equal, this implies a higher bankruptcy rate. With no credit frictions this would have no impact on the model economy, since banks' expected return has not changed and both entrepreneurs and banks are risk neutral. Under asymmetric information, however, the costly state verification problem introduces a wedge (the monitoring cost) in the bank's zero

profit condition: a higher default rate (due to those entrepreneurs experiencing larger negative shocks) increases the expected costs for banks who will in turn charge higher lending rates. This generates a fall in capital demand and hence in investment.²³ As in the macro uncertainty shock, sticky prices ensure the comovement between consumption and investment. Indeed, the reduction in investment (caused by the fall in capital demand) acts to reduce aggregate demand.

In addition to the different transmission mechanism, an important difference between micro and macro uncertainty shocks lies in the magnitude of the response of total output to the shock. As displayed in Figure 4, a 1 standard deviation shock to micro uncertainty leads to a fall of about 0.8 percent of total output, an impact which is almost 40 times larger than the macro uncertainty shock. This larger impact does not apply only to output: the risk premium now increases by more than 60 basis points and net worth falls by 1 percent.

B. Inspecting the Transmission Mechanism of Uncertainty Shocks

The discussion above suggests that there are few key factors that can affect the transmission mechanism of uncertainty shocks: household preferences and the severity of the credit friction are closely related to the transmission channels for macro and micro uncertainty, respectively; as well as the degree of price stickiness and the response of monetary policy. We now turn to the analysis of these different amplification mechanisms.

1. Price stickiness

As a number of earlier papers have documented, price stickiness is crucial for the propagation of uncertainty shocks: it generates comovement between consumption and investment as well as amplifying the impact of the shocks (Basu and Bundick (2017), Fernandez-Villaverde and others (2011), Born and Pfeifer (2014)). Figure 5 shows this observation as it compares the IRFs from the baseline calibration (dark circles) with the IRFs from a flexible price version of the model (light squares) obtained by setting the price stickiness parameter as $\omega_p \simeq 0$.

The flex price economy is particularly useful because it highlights how micro and macro uncertainty shocks propagate. Panel A of Figure 5 shows the IRFs for the macro uncertainty

²³A more detailed description of these mechanisms is explained in the Online Appendix with a simple comparative steady state exercise.

Figure 5. THE IMPACT OF UNCERTAINTY SHOCKS: THE ROLE OF PRICE STICKINESS

NOTE. Impulse response functions (IRFs) to a 1 standard deviation increase in macro uncertainty (panel A) and in micro uncertainty (panel B). The IRFs are computed with respect to the ergodic mean of the variables of interest. All responses are in percent, except for the risk premium which is in basis points. The unit of the x-axis is quarters.

shock. With capital predetermined, output can only change in response to labor movements. With flexible prices (and constant mark-ups) the labor demand schedule is unchanged in response to the uncertainty shock; and with GHH preferences the labor supply schedule is also fixed, which results in unchanged hours and wages. Consequently output is unchanged in response to the shock, which in turn implies that lower consumption is channelled towards

higher investment. This implies no comovement between consumption and investment, contrary to what is observed over the business cycle.²⁴

Differently from the macro shock, which affects consumption, the micro uncertainty shock depresses investment. Increased micro uncertainty generates an increase in the cost of external finance as the expected cost associated with bankruptcies is now larger. Higher lending rates imply lower demand for capital, thereby generating a sharp fall in investment and in the price of capital. As [Christiano, Motto, and Rostagno \(2014\)](#) note, the shock resembles an increase in the tax rate on the return on investment which acts to discourage saving (and hence investment) and boosts consumption or leisure. Symmetrically to what we saw in the case of the macro uncertainty shock, under flexible prices lower investment leads to higher consumption as mark-ups, labor demand and labor supply (under GHH preferences) remain unchanged (see panel B of Figure 5).

Sticky prices are therefore crucial for generating comovement between consumption and investment as well as acting as a powerful amplifying mechanism. Specifically, the higher the degree of price stickiness, the larger is the effect of both macro and micro uncertainty shocks on output.²⁵

2. Credit Frictions

We turn next to the role of the credit friction in the transmission of uncertainty shocks. The financial accelerator mechanism amplifies the impact of any aggregate shock that affects the net worth of entrepreneurs in general equilibrium. For example, in periods where financial market distortions are more severe, uncertainty shocks could have a larger impact on economic activity, risk premia, and asset prices.

To investigate this mechanism further, Figure 6 compares our baseline results (dark circles) against a case where credit frictions are more pronounced (light squares). These alternative cases are computed by modifying the value of the monitoring cost parameter (μ), where a larger monitoring cost implies a more severe credit friction in the economy. In our ‘high

²⁴With non-GHH (separable) preferences and flexible prices the decline in consumption brought about the macro uncertainty shock and the precautionary saving motive, acts to shift labor supply (it expands for a given level of the real wage). This worsens the co-movement problem as hours (and hence output) expand in response to the uncertainty shock. See below for more on separable preferences.

²⁵In a previous version of this paper ([Cesa-Bianchi and Fernandez-Corugedo, 2014](#)), we show that the effect of uncertainty shocks on output almost doubles when using a version of the model calibrated so as to obtain an average probability of changing prices of 5 quarters, instead of 4 quarters as in the baseline.

credit friction' exercise we set $\mu = 0.5$. This implies that the steady-state level of the spread between the lending rate and the risk free rate is now 25 basis points higher than in our base-line; and that a larger fraction of total output is “lost” in monitoring activities.

Figure 6. THE IMPACT OF UNCERTAINTY SHOCKS: THE ROLE OF CREDIT FRICTIONS

NOTE. Impulse response functions (IRFs) to a 1 standard deviation increase in macro uncertainty (panel A) and in micro uncertainty (panel B). The IRFs are computed with respect to the ergodic mean of the variables of interest. All responses are in percent, except for the risk premium which is in basis points. The unit of the x-axis is quarters.

Panel A of Figure 6 shows that, when credit frictions are more pronounced (light squares), macro uncertainty shocks tend to have a larger impact on investment and financial variables (i.e., net worth, the price of capital and the risk premium). As a result, the effect on total out-

put is also larger. Panel B of Figure 6 shows that credit frictions are also important in magnifying the impact of micro uncertainty shocks: when increasing the severity of the credit friction, the impact of a micro uncertainty shock on the risk premium and on investment increases substantially. Investment falls by about 0.15 percent more than in the baseline, and the response of the risk premium is 20 basis points higher than in the baseline. Total output—which falls by about 0.8 percent in our baseline—falls by more than 0.9 percent when the severity of the credit friction is increased. These results show that credit frictions are crucial for the transmission of uncertainty shocks. The mechanism is simple. A large monitoring cost introduces a larger wedge in the banks’ zero profit condition, inducing banks to raise the spread they charge on lending interest rates. It follows that when credit frictions are more severe (i.e., when the monitoring cost is larger) the effect of both macro and micro uncertainty shocks on total output is larger.²⁶

3. Other Ingredients

So far we have highlighted the role played by sticky prices and credit frictions in the transmission of uncertainty shocks. We now consider the impact that three additional mechanisms have on the propagation properties of the model, namely: the role of households’ preferences, risk aversion and monetary policy. Figure 7 compares the output response to a macro and a micro uncertainty shock under our baseline results (dark circles) against versions of the model (light squares) with: (i) log-separable preferences between consumption and leisure as in King, Plosser, and Rebelo (1988) (KPR henceforth); (ii) a higher coefficient of relative risk aversion, namely $\rho = 5$;²⁷ and (iii) a more aggressive monetary policy rule, where we set the coefficient on inflation at $\rho^\pi = 3$.²⁸

(a) Households’ Preferences. We focus first on the role of households’ preferences. Specifically, we consider the role of log-separable preferences as in KPR. With separable preferences in consumption and leisure, the impact of the macro uncertainty shock on output is smaller (panel A of Figure 7). The transmission mechanism of the shock is as noted before: the shock acts to lower consumption via precautionary savings which in turn acts to lower la-

²⁶Note here that, in the limiting case where the monitoring cost is zero (e.g., assuming that both the entrepreneur and the bank could costlessly observe idiosyncratic shocks), the impact of a micro uncertainty shock would also tend to zero.

²⁷The value $\rho = 5$ is at the high end of the plausible values for the coefficient of relative risk aversion. See Carroll (2001) and Carroll (2009).

²⁸The full set of impulse responses are not reported here for brevity, but they are available in the Online Appendix.

bor demand under sticky prices. But with KPR preferences (relative to the baseline case with GHH preferences), the decline in consumption results in a “precautionary” outward shift in labour supply which acts to mitigate the effect of the macro uncertainty shock on hours and hence output. The effect on output depends on the overall impact that the opposing shifts in labor demand and supply have on total hours. Either way, the impact on output is mitigated relative to the case with GHH preferences by the shift in labor supply, implying a weaker response of hours, output, consumption and investment.

Figure 7. THE IMPACT OF UNCERTAINTY SHOCKS ON OUTPUT: THE ROLE OF HOUSEHOLDS’ PREFERENCES, RISK AVERSION, AND MONETARY POLICY

NOTE. Impulse response functions (IRFs) to a 1 standard deviation increase in macro uncertainty (panel A) and in micro uncertainty (panel B). The IRFs are computed with respect to the ergodic mean of the variables of interest. All responses are in percent. The unit of the x-axis is quarters.

For the case of a micro uncertainty shock, the effects on output are also smaller when KPR preferences are used relative to the baseline with GHH preferences. This is again, as in the case of the macro uncertainty shock, largely explained by the labor market response (and in particular the response of labor supply). But there is also a more subtle mechanism at play in this case pertaining to the complementarity of consumption and hours: with GHH preferences consumption and hours are complements, whereas this is not the case with separable prefer-

ences.²⁹ This implies that the decline in hours brought about by the micro uncertainty shock acts to depress consumption further, thereby amplifying the impact of the shock relative to the case when preferences are separable (panel B of Figure 7).

(b) Risk aversion. Next we examine the importance of risk aversion. A larger coefficient of risk aversion makes consumers more prudent (in the [Kimball \(1990\)](#) sense) and increases the strength of the precautionary motive. As a result, macro uncertainty shocks have a larger impact on hours, output and investment (panel A of Figure 7). Higher risk aversion also amplifies the impact of micro uncertainty shocks, although the amplification is relatively small. The main channel behind the amplification is not the precautionary saving motive but rather that higher levels of risk aversion increase the degree of complementarity between consumption and hours, resulting in a larger drop of consumption in response to the drop in hours brought about by the micro uncertainty shock.

(c) Monetary Policy. Finally we consider the importance of the monetary policy rule in propagating the uncertainty shocks. The equilibrium response of the real rate, an important determinant of consumption and investment, depends on how strongly the central bank reacts to the contractionary effects of the uncertainty shock. We compare our baseline, where the coefficient on inflation in the Taylor rule is set to $\rho^\pi = 1.8$, with a more aggressive Taylor rule where $\rho^\pi = 3$. When monetary policy responds more aggressively to inflation, the nominal interest rate falls by more than in the baseline and this translates into a smaller fall in inflation. Lower nominal interest rates and higher inflation imply a lower real interest rate than in the baseline, which therefore dampens the impact on consumption, investment, output and the credit spread of both macro and micro uncertainty shocks

V. UNCONDITIONAL BUSINESS CYCLE PROPERTIES

Conditional IRFs are not sufficient to gauge the importance of uncertainty shocks over the business cycle. In this section we conduct simple numerical experiments where we compute unconditional business cycle properties of some variants of the baseline model. Specifically, we follow [Bachmann and Bayer \(2013\)](#) and compare the baseline model—that features both micro and macro uncertainty shocks, alongside standard aggregate productivity shocks—with variants of the model where we introduce the shocks one at the time.

²⁹See [Chetty \(2006\)](#) and [Monacelli and Perotti \(2008\)](#) for more details on this issue.

Table 3. BUSINESS CYCLE STATISTICS – DATA AND MODEL SIMULATIONS

	(1)	(2)	(3)	(4)
	<i>Data</i>	<i>Baseline</i>	<i>Only TFP</i>	<i>Only Micro Uncertainty</i>
<i>Volatility</i>				
Output	1.55	1.56	1.53	0.34
<i>Volatility of aggregate variables relative to output volatility</i>				
Consumption	0.82	0.81	0.80	1.08
Investment	2.92	1.54	1.58	1.37
Hours	0.91	0.56	0.49	1.44
EFP	0.12	0.17	0.14	0.13
<i>First-order autocorrelation</i>				
Output	0.88	0.76	0.81	0.20
Consumption	0.88	0.76	0.83	0.17
Investment	0.91	0.73	0.77	0.49
Hours	0.95	0.60	0.85	0.21
EFP	0.87	0.83	0.89	0.76
<i>Contemporaneous correlation with aggregate output</i>				
Consumption	0.88	0.99	1.00	0.99
Investment	0.95	0.98	0.99	0.84
Hours	0.82	0.89	0.98	1.00
EFP	-0.46	-0.58	-0.73	-0.60

NOTE. Column (1) reports business cycle statistics for US data over the 1972Q1-2012Q4 sample period. Column (2) refers to simulations from our baseline model, with both micro and macro uncertainty shocks, alongside standard aggregate productivity shocks. Columns (3) and (4) refer to simulations where there is only one shock driving the model economy, i.e. the aggregate TFP shock and the micro uncertainty shock, respectively. All series, from data and model simulations, have been logged and HP-filtered with a smoothing parameter 1600, with the exception of the External Finance Premium (EFP).

Table 3 reports the results associated with the baseline calibration of the model. Column (1) displays the standard deviation, persistence and correlation of some selected US variables. We focus on the cyclical behavior of output, consumption, investment, and hours.³⁰ Moreover, given the focus of the paper on credit frictions, we also consider the spread between BAA and

³⁰We use US data over the sample period 1972:Q1–2012:Q4. Real GDP, private final consumption expenditure, and gross fixed capital formation are from OECD Main Economic Indicators; Hours are from the BLS; the BAA to AAA spread is from Moody.

AAA-rated corporate bond yields, our proxy for the external finance premium.³¹ Column (2) reports the same statistics computed from simulations of the baseline model, i.e. including ‘standard’ aggregate TFP shocks alongside micro and macro uncertainty shocks.³² Despite the relative simplicity of the model and the small number of shocks considered, the model does a good job at matching some key features of the data.

The comparison between column (1) and (2) of Table 3 provides a natural check for the validity of the parametrization strategy for the micro uncertainty shock. Despite the fact that no information about credit spreads has been used to derive the time series properties of micro uncertainty, the dynamics of the external finance premium computed from the model in column (2) are roughly in line with those observed in the data in column (1). Specifically, the persistence of credit spreads in the model is reasonably close to the persistence that we measure in the data. The same is true for the volatility of the credit spread relative to output, while we slightly overshoot the correlation between output and the credit spread.

We then consider an additional version of the model in which the sole exogenous process in the model is TFP (i.e., where the uncertainty shocks are turned off); and one in which micro uncertainty shocks are the only driving force. The business cycle properties of these two variants of the model are reported in column (3) ‘*Only TFP*’ and column (4) ‘*Only Micro Uncertainty*’ of Table 3, respectively. A comparison between the baseline model in column (2) and the ‘*Only TFP*’ model in column (3) reveals that their business cycle properties are almost identical, echoing the findings of [Bachmann and Bayer \(2013\)](#) who argue that uncertainty shocks are not key drivers of the business cycle. Finally, column (4) of Table 3, reports the business cycle statistics for the ‘*Only Micro Uncertainty*’ model. The results show that micro uncertainty shocks alone drive a small, but non trivial share of output volatility (around 20 percent of the output volatility that we observe in the data) but cannot by themselves be the key driver of business cycle fluctuations.

Our estimate of the importance of micro uncertainty shocks falls on the high end of the estimates from the previous studies that use a similar financial accelerator model. [Chugh \(2016\)](#) finds that micro uncertainty shocks drive about 5 percent of GDP volatility, while [Christianiano, Motto, and Rostagno \(2014\)](#) estimate a much more important role of micro uncertainty

³¹Results are similar when using the Excess Bond Premium of [Gilchrist and Zakrajsek \(2012\)](#) as an alternative proxy for the external finance premium.

³²To obtain the moments implied by the model, we simulate the model economy for 2000 periods. We then use the last 164 periods (i.e., the same number of observations that we have in the data, from 1972:Q1 to 2012:Q4) to compute the statistics of the simulated variables (in log-deviation from an HP trend with smoothing parameter 1600).

shocks using macro-financial data, at about 20 percent of GDP volatility.³³ What drives these different results? To answer this question, we explore the role of the different factors we considered in the previous section, namely variants of the model with flexible prices, high credit frictions, KPR preferences, an aggressive monetary policy rule, and a high risk aversion coefficient. In addition to that, we also consider a model in which we set the standard deviation of the micro uncertainty shock to the value estimated by [Christiano, Motto, and Rostagno \(2014\)](#), namely $\sigma^S = 0.07$. For each of these variants, Table 4 reports the standard deviation of output computed with the same set of shocks as in the baseline model, labelled ‘*All Shocks*’ in column (1); with aggregate TFP shocks only, labelled ‘*Only TFP*’ in column (2); and with only the micro uncertainty shock, labelled ‘*Only Micro Uncertainty*’ in column (3). To facilitate the analysis of the results, the first row of Table 4 also reports the business cycle statistics obtained with our baseline calibration (as in Table 3).

Table 4. BUSINESS CYCLE STATISTICS – VARIANTS OF THE BASELINE MODEL

	(1)	(2)	(3)
	<i>All shocks</i>	<i>Only TFP</i>	<i>Only Micro Uncertainty</i>
Baseline	1.56	1.53	0.34
Flex. Price	1.62	1.58	0.01
High Credit Frictions	1.61	1.56	0.47
KPR Preferences	1.12	1.11	0.12
Active Mon. Policy	1.55	1.53	0.19
High Risk Aversion	1.68	1.59	0.44
CMR Micro Shock	1.94	1.53	1.17

NOTE. Column (1) refers to simulations obtained with both micro and macro uncertainty shocks, alongside standard aggregate productivity shocks. Columns (2) and (3) refer to simulations where there is only one shock driving the model economy, i.e. the aggregate TFP shock and the micro uncertainty shock, respectively. The first row reports the results from the baseline model, while the remaining rows correspond to different variants of the model as described in the text. All series from the model simulations have been logged and HP-filtered with a smoothing parameter 1600.

The results are as follows. The degree of price stickiness is crucial for the transmission of uncertainty shocks. Indeed, while in the ‘*Flex prices*’ model with ‘*All Shocks*’ (column (1)) the volatility of output increases slightly relative to the baseline, it falls dramatically when micro uncertainty shocks are the sole source of variation (column(3)). Specifically, in this

³³Given that our model does not feature anticipated shocks, we compare our results with the percentage of the variance of GDP accounted for by the *unanticipated* component of the risk shock in [Christiano, Motto, and Rostagno \(2014\)](#) (see their Table 5).

specification, micro uncertainty shocks account for less than 1 percent of total output volatility. The severity of the credit friction plays a similarly important role for the transmission of uncertainty shocks, as the share of output volatility accounted for by the micro uncertainty shock increases to about 30 percent in the ‘*High Credit Frictions*’ model, compared to 20 percent in the baseline. Differently, KPR preferences dampen both uncertainty shocks and TFP shocks: in the ‘*KPR Preferences*’ model, output volatility falls substantially in column (1), and so does the share of output volatility explained by micro uncertainty (which falls to about 12 percent), as shown in column (3). Monetary policy plays an important role for both uncertainty shocks, with a more aggressive monetary stance lowering their importance. The degree of risk aversion also matters quantitatively, with a higher share of output volatility associated to a higher degree of risk aversion (see the ‘*High Risk Aversion*’ model). Finally, when increasing the standard deviation of the micro uncertainty shocks (‘*CMR Micro shock*’ model), their importance increases by a similar magnitude. This is not surprising since the micro uncertainty shock enters agents’ decision rules linearly.³⁴

VI. CONCLUSIONS

This paper studies the effect of a mean preserving shock to the variance of aggregate total factor productivity (macro uncertainty) and to the dispersion of entrepreneurs’ idiosyncratic productivity (micro uncertainty) in a financial accelerator DSGE model with sticky prices.

The model is disciplined using aggregate data on total factor productivity for the U.S. business sector and disaggregated data on establishment-level total factor productivity. The estimated time series properties of micro uncertainty are in line with previous estimates based on disaggregated micro data, but smaller than (some of) the estimates based on aggregated macroeconomic data: understanding what drives the gap between the two approaches could be the focus of further research in this area.

Our analysis shows that uncertainty shocks, when considered alongside aggregate TFP shocks, do not seem to be a major source of business cycle fluctuations. Micro uncertainty shocks have a larger impact on total output relative to macro uncertainty shocks, and they account for a small (but non-trivial) share of all output volatility in our baseline calibration. Key deter-

³⁴We also checked the role played by investment adjustment costs in driving our results by running a simulation where we set the adjustment cost parameter to zero. Apart from increasing the volatility of investment in the unconditional simulations, the unconditional business cycle statistics are very similar to the baseline.

minants of our estimate are the presence of sticky prices, the severity of credit frictions, the specification of households preferences, and the response of monetary policy.

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APPENDIX A. APPENDIX

A.1. Equilibrium

Define $q_t \equiv Q_t/P_t$, $nw_t \equiv NW_t/P_t$, $z_t \equiv Z_t/P_t$. For a given path for the exogenous processes, a recursive (imperfectly) competitive equilibrium of the model is a sequence of allocations for the endogenous variables that solves the following system of equations.

Euler equation of households:

$$U_{c,t} = \beta(1 + R_t^n) \mathbb{E}_t \left[\frac{U_{c,t+1}}{\pi_{t+1}} \right]. \quad \text{A.1}$$

Labor supply:

$$mc_t Y_{n,t} = -\frac{U_{n,t}}{U_{c,t}}. \quad \text{A.2}$$

Marginal product of capital:

$$mc_t Y_{k,t} = z_t. \quad \text{A.3}$$

Price of capital:

$$q_t = \left[1 - \phi_k \left(\frac{I_t}{K_t} - \delta \right) \right]^{-1}. \quad \text{A.4}$$

Zero profit condition:

$$y_{t+1}^k K_{t+1} \left(\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1}) \right) = (1 + R_t^n)(q_t K_{t+1} - nw_{t+1}). \quad \text{A.5}$$

NK Phillips curve:

$$(\pi_t - \pi) \pi_t = \beta \mathbb{E}_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} (\pi_{t+1} - \pi) \pi_{t+1} \right\} + Y_t \frac{\varepsilon}{\omega_p} \left(mc_t - \frac{\varepsilon - 1}{\varepsilon} \right) \quad \text{A.6}$$

Net worth law of motion:

$$nw_{t+1} = \gamma y_{t+1}^k K_{t+1} (1 - \Gamma(\bar{\omega}_{t+1})). \quad \text{A.7}$$

Entrepreneurs real consumption:

$$C_t^e = (1 - \gamma) (1 - \Gamma(\bar{\omega}_{t+1})) y_t^k K_t. \quad \text{A.8}$$

Aggregate resource constraint:

$$A_t F(K_t, N_t) = C_t + C_t^e + I_t + \frac{\omega_p}{2} (\pi_t - \pi)^2 + \mu G(\bar{\omega}) y_t^k K_t. \quad \text{A.9}$$

Accumulation of aggregate capital:

$$K_{t+1} = (1 - \delta)K_t + I_t - \frac{\phi_k}{2} \left(\frac{I_t}{K_t} - \delta \right)^2 K_t. \quad \text{A.10}$$

Monetary policy:

$$\frac{1 + R_t^n}{1 + R^n} = \left(\frac{1 + R_{t-1}^n}{1 + R^n} \right)^{\phi^r} \left(\frac{1 + \pi_t^n}{1 + \pi} \right)^{(1-\phi^r)\phi^\pi} \left(\frac{1 + Y_t}{1 + Y_{t-1}} \right)^{(1-\phi^r)\phi^y}. \quad \text{A.11}$$

Definition of real income from holding one unit of finished capital:

$$y_t^k = z_t + q_t \left[1 - \delta - \frac{\phi_k}{2} \left(\frac{I_t}{K_t} - \delta \right)^2 + \phi_k \left(\frac{I_t}{K_t} - \delta \right) \frac{I_t}{K_t} \right]. \quad \text{A.12}$$

$$y_{t+1}^k = \frac{(1 + R_{t+1}^k)q_t}{\pi_{t+1}}. \quad \text{A.13}$$

Optimal contract:

$$\frac{1 + R_{t+1}^k}{1 + R_t^n} = \psi_t. \quad \text{A.14}$$

where:

$$\psi_t = \left(\frac{(1 - \Gamma(\bar{\omega}_{t+1}^j)) (\Gamma'(\bar{\omega}_{t+1}^j) - \mu G'(\bar{\omega}_{t+1}^j))}{\Gamma'(\bar{\omega}_{t+1}^j)} + (\Gamma(\bar{\omega}_{t+1}^j) - \mu G(\bar{\omega}_{t+1}^j)) \right)^{-1}. \quad \text{A.15}$$

A.2. Households' Preferences

In the paper we compare two different functional forms for households' preferences, namely log-separable preferences of the [King, Plosser, and Rebelo \(1988\)](#) type, and GHH preferences of the [Greenwood, Hercowitz, and Huffman \(1988\)](#) type. Below, we describe the functional form of those preferences and we show how they affect the households' key equations, namely the Euler equation for consumption and labour supply.

KPP preferences

These preferences are as in [King, Plosser, and Rebelo \(1988\)](#) (KPR henceforth). Agents' utility is log-separable in consumption and labour:

$$\frac{(C_t(1 - N_t)^{\tau^{KPR}})^{(1-\rho)}}{1 - \rho}. \quad \text{A.16}$$

Then:

$$\begin{aligned} U_{c,t} &= C_t^{-\rho} (1 - N_t)^{\tau^{KPR}(1-\rho)}, \\ U_{n,t} &= -\tau^{KPR} C_t^{1-\rho} (1 - N_t)^{\tau^{KPR}(1-\rho)-1}. \end{aligned} \quad \text{A.17}$$

This implies that the Euler equation and labour supply conditions are:

$$\begin{aligned} C_t^{-\rho} (1 - N_t)^{\tau^{KPR}(1-\rho)} &= \beta(1 + R_t^n) \mathbb{E}_t \left[C_{t+1}^{-\rho} (1 - N_{t+1})^{\tau^{KPR}(1-\rho)} \frac{P_t}{P_{t+1}} \right], \\ \frac{W_t}{P_t} &= \tau^{KPR} \frac{C_t}{(1 - N_t)}. \end{aligned} \quad \text{A.18}$$

In this case the Euler equation states that expected consumption growth is a function of the real interest rate and of the growth rate of expected labour. Consumption appears in the labour supply equation, implying that labour supply shifts in response to movements in consumption.

GHH Preferences

These preferences are as in [Greenwood, Hercowitz, and Huffman \(1988\)](#). With this utility function, the amount of hours worked by households will actually affect the amount of utility received from consumption, i.e. the cross-derivative of utility with respect to consumption and labour is unequal to zero.

$$\frac{1}{1-\rho} \left(C_t - \tau^{GHH} N_t^{1+v} \right)^{1-\rho}. \quad \text{A.19}$$

Then:

$$\begin{aligned} U_{c,t} &= \left(C_t - \tau^{GHH} N_t^{1+v} \right)^{-\rho}, \\ U_{n,t} &= -\tau^{GHH} (1+v) N_t^v \left(C_t - \tau^{GHH} N_t^{1+v} \right)^{-\rho}. \end{aligned} \quad \text{A.20}$$

This implies that the Euler equation and labour supply conditions are:

$$\begin{aligned} \left(C_t - \tau^{GHH} N_t^{1+v} \right)^{-\rho} &= \beta(1 + R_t^n) \mathbb{E}_t \left[\left(C_{t+1} - \tau^{GHH} N_{t+1}^{1+v} \right)^{-\rho} \frac{P_t}{P_{t+1}} \right], \\ \frac{W_t}{P_t} &= \tau^{GHH} (1+v) N_t^v. \end{aligned} \quad \text{A.21}$$

In this case, as with KPR preferences, the Euler equation states that expected consumption growth is a function of the real interest rate and of the growth rate of expected labour. But unlike KPR preferences, labour supply is a positive function of only the real wage. Therefore, as the marginal rate of substitution is independent of consumption and only depends on the real wage, there is no wealth effect on the labour supply.