Central Bank Independence and Macro-prudential Regulation

Kenichi Ueda and Fabián Valencia
Central Bank Independence and Macro-prudential Regulation

Prepared by Kenichi Ueda and Fabián Valencia

Abstract

We consider the optimality of various institutional arrangements for agencies that conduct macro-prudential regulation and monetary policy. When a central bank is in charge of price and financial stability, a new time inconsistency problem may arise. Ex-ante, the central bank chooses the socially optimal level of inflation. Ex-post, however, the central bank chooses inflation above the social optimum to reduce the real value of private debt. This inefficient outcome arises when macro-prudential policies cannot be adjusted as frequently as monetary. Importantly, this result arises even when the central bank is politically independent. We then consider the role of political pressures in the spirit of Barro and Gordon (1983). We show that if either the macro-prudential regulator or the central bank (or both) are not politically independent, separation of price and financial stability objectives does not deliver the social optimum.

JEL Classification Numbers:C61, E21, G13

Keywords: Monetary Policy, Macro-prudential Regulation, Central Bank Independence, Time-inconsistency

Authors’ E-Mail Addresses: kueda@imf.org, fvalencia@imf.org

1 The authors thank Larry Ball, Olivier Blanchard, Stijn Claessens, Giovanni Dell’Ariccia, and Luc Laeven for comments and discussions.
## Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I  Introduction</td>
<td>3</td>
</tr>
<tr>
<td>II Model Setup</td>
<td>4</td>
</tr>
<tr>
<td>III Social Planner Benchmark</td>
<td>7</td>
</tr>
<tr>
<td>IV Time inconsistency in a dual-mandate central bank</td>
<td>8</td>
</tr>
<tr>
<td>V  Separation of Objectives Achieves Social Optimum</td>
<td>11</td>
</tr>
<tr>
<td>VI The role of political independence</td>
<td>13</td>
</tr>
<tr>
<td>A Non-Independent Central Bank and Independent Macro-prudential Regulator</td>
<td>14</td>
</tr>
<tr>
<td>B Non-Independent Macro-prudential Regulator and Independent Central Bank</td>
<td>14</td>
</tr>
<tr>
<td>VII Welfare Comparisons</td>
<td>16</td>
</tr>
<tr>
<td>VIII Conclusions</td>
<td>18</td>
</tr>
<tr>
<td>References</td>
<td>20</td>
</tr>
<tr>
<td>Appendices</td>
<td>1</td>
</tr>
<tr>
<td>I  Non-Independent Single Authority</td>
<td>1</td>
</tr>
<tr>
<td>II Distortionary Macro-prudential Regulation</td>
<td>3</td>
</tr>
</tbody>
</table>

List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I Welfare Loss Across Institutional Arrangements</td>
<td>17</td>
</tr>
</tbody>
</table>
I. INTRODUCTION

A growing literature based on models where pecuniary externalities reinforce shocks in the aggregate advocates the use of macro-prudential regulation (e.g. Bianchi (2010), Bianchi and Mendoza (2010), Jeanne and Korinek (2010), and Jeanne and Korinek (2011)). Most research in this area has focused on understanding the distortions that lead to financial amplification and to assess their quantitative importance. The natural next question is how to implement macro-prudential regulation.

Implementing macro-prudential policy requires, among other things, figuring out the optimal institutional design. In this context, there is an intense policy debate about the desirability of assigning the central bank formally with the responsibility of financial stability. This debate has spurred interest in studying the interactions between monetary and macro-prudential policies with the objective of understanding the conflicts and synergies that may arise from different institutional arrangements.

This paper contributes to this debate by exploring the circumstances under which it may be suboptimal to have the central bank in charge of macro-prudential regulation. We differ from a rapidly expanding literature on macro-prudential and monetary interactions, including De Paoli and Paustian (2011) and Quint and Rabanal (2011), mainly in that our focus is on the potential time-inconsistency problems that can arise, which are not addressed in existing work.

Our departure point is the work pioneered by Kydland and Prescott (1977) and Barro and Gordon (1983) who studied how time-inconsistency problems and political pressures distort the monetary authority’s incentives under various institutional arrangements. In our model, there are two stages, in the first stage, the policymaker (possibly a single or several institutions) makes simultaneous monetary policy and macro-prudential regulation decisions. In the second stage, monetary policy decisions can be revised or “fine-tuned” after the realization of a credit shock. This setup captures the fact that macro-prudential regulation is intended to be used preemptively, once a credit shock (boom or bust) have taken place, it can do little to change the stock of debt. Monetary policy, on the other hand, can be used ex-ante and ex-post.

The key finding of the paper is that a dual-mandate central bank is not socially optimal. In this setting, a time inconsistency problem arises. While it is ex-ante optimal for the dual-mandate central bank to deliver the socially optimal level of inflation, it is not so ex-post. This central bank has the ex-post incentive to reduce the real burden of private debt through inflation, similar to the incentives to monetize public sector debt studied in Calvo (1978) and Lucas and Stokey (1983). This outcome arises because ex-post the dual-mandate central bank has only one tool, monetary policy, to achieve financial and price stability.

We then examine the role of political factors with a simple variation of our model in the spirit of Barro and Gordon (1983). We find that the above result prevails if policy is conducted by politically independent institutions. However, when institutions are not politically independent (the central bank, the macro-prudential regulator, or both) neither separate institutions nor combination of objectives in a single institution delivers the social optimum. As in Barro and Gordon (1983), the non-independent institution will use its policy tool at hand to try to generate
economic expansions. The non-independent central bank will use monetary policy for this purpose and the non-independent macro-prudential regulator will use regulation. Which arrangement generates lower welfare losses in the case of non-independence depends on parameter values. A calibration of the model using parameter values from the literature suggest, however, that a regime with a non-independent dual-mandate central bank almost always delivers a worse outcome than a regime with a non-independent but separate macro-prudential regulator. Finally, if the only distortion of concern is political interference (i.e. ignoring the time-inconsistency problem highlighted earlier) all that is needed to achieve the social optimum is political independence, with separation or combination of objectives yielding the same outcome.

From a policy perspective, our analysis suggests that a conflict between price and financial stability objectives may arise if pursued by a single institution. Our results also extend the earlier findings by Barro and Gordon (1983) and many others on political independence of the central bank to show that these results are also applicable to a macro-prudential regulator. We should note that we have abstracted from considering the potential synergies that may arise in having dual mandate institutions. For instance, benefits from information sharing and use of central bank expertise may mitigate the welfare losses we have shown may arise (see Nier, Osinski, Jácome and Madrid (2011)), although information sharing would also benefit fiscal and monetary interactions. However, we have also abstracted other aspects that could exacerbate the welfare loss such as loss in reputation.

The paper is organized as follows. The next section presents the model. Section III presents the Social Planner’s benchmark. Section IV presents the case where a time inconsistency problem arises under a dual-mandate central bank. Section V shows how separation of objectives delivers the social optimum. Section VI shows the solution when we consider political influence. Section VII performs a welfare comparison across regimes. Section VIII concludes.

II. MODEL SETUP

We start by assuming a loss function with three elements: the variance of output, \( y \), inflation, \( \pi \), and leverage, \( \phi \):

\[
L = \frac{a}{2}(y - y^*)^2 + \frac{b}{2}(\pi - \pi^*)^2 + \frac{c}{2}(\phi - \phi^*)^2. \tag{1}
\]

where \( a > 0 \), \( b > 0 \), and \( c > 0 \) denote the weights corresponding to each objective and the starred variables denote the corresponding socially optimal levels. The loss function (1) is taken from Carlstrom, Fuerst and Paustian (2010) who obtain it from a second-order approximation to the social welfare function in a model with nominal rigidities and agency costs in credit markets. A similar loss function is derived in Cúrdia and Woodford (2009) in which an inefficiency in bank screening of loans generates a wedge between borrowing and lending rates.

In general, in a frictionless economy, welfare depends only on output. When a price-rigidity distortion is added, an inefficient allocation of resources appears and, for the same level of output, welfare varies depending on the intensity of this distortion. Therefore, a second term, price
stability, appears in the loss function when these distortions are considered. Financial frictions create a similar distortion in the price of capital (Carlstrom et al. (2010)) or generates an inefficient allocation of resources (Cúrdia and Woodford (2009)). In short, distortive financial frictions are another source of the overall welfare loss and it may vary for the same level of output and inflation, hence the third term appears in our loss function (1). This third term tells us that deviations of actual leverage from some socially optimal level are costly.\(^2\)

In specifying this loss function, we follow closely Carlstrom et al. (2010). In their paper and in the literature in general, the social welfare function is typically approximated by a second-order Taylor expansion (i.e., log-linear-quadratic approximation) around the non-stochastic steady state. Evaluation at the optimal values implies that the terms with first derivatives are equal to zero, and only the terms with second derivatives remain as in (1). The lack of cross-derivatives in the loss function is the result of simplifying assumptions.\(^3\) Note that, because the welfare function arises from an approximation around the steady state, it is suitable to study only small deviations from steady state. Therefore, our focus is on economic fluctuations during normal times but not on crisis times, which entail large shocks.\(^4\)

We now proceed to lay out the remaining equations in the model. Output is given by a standard Lucas supply curve, augmented with the effects of changes in the supply of credit

\[
y = \bar{y} + \alpha(\pi - \pi^e) + \beta\delta,
\]

where \(\bar{y}\) denotes the level of output that would prevail in absence of distortions, \(\pi^e\) denotes expected inflation, and \(\delta\) denotes the change in the amount of debt. We assume that greater availability of credit increases output by allowing more investment and consumption, given \(\bar{y}\). We also assume that \(\delta\) has two components:

\[
\delta = \delta_0 + \epsilon,
\]

where \(\delta_0\) corresponds to the expansion in credit that is controlled for by regulatory actions and \(\epsilon\) is a credit shock with \(E[\epsilon] = 0\) and variance \(\sigma^2_\epsilon\). We think of \(\delta_0\) as the effect of countercyclical regulatory tools such as capital requirements, loan-to-value ratios, and dynamic provisioning. They can be chosen as to allow more or less leverage in the private sector. De Nicoló, Favara and Ratnovski (2012) present an overview of macro prudential tools proposed in the literature. These credit shock reflects uncertainty about the ultimate effect of macro-prudential regulation, for

\(^2\)Bianchi (2010), Bianchi and Mendoza (2010), Jeanne and Korinek (2010), and Jeanne and Korinek (2011) show how excessive leverage arises in the economy from individuals not internalizing the aggregate impact of their financial decisions.

\(^3\)For instance, a linearly homogenous production function, separable utility in credit goods and non-credit goods (Carlstrom et al. (2010)), or the credit constraint independent of inflation (Cúrdia and Woodford (2009)). These assumptions imply that the cross-derivatives become zero.

\(^4\)Not only large shocks reduce the accuracy of the approximations but they may also exacerbate distortions generated by regulation, which during normal times may be quantitatively small. For instance, by constraining credit during the recovery after a large negative shock. The outcome in that case could be similar to what arises in monetary and fiscal interactions when one considers the distortionary effects of taxation (Dixit and Lambertini (2003)). We reflect on this case in the Appendix.
instance by unexpected changes in banks’ or borrowers’ behavior or by capital inflows. They are not meant to be interpreted as credit allocation in a central planning economy.

There is a component of inflation, \( \pi_0 \), that can be controlled by the monetary authority but total inflation is also affected by credit growth.

\[
\pi = \pi_0 + \gamma \delta. \tag{4}
\]

Notice that for simplicity we are not allowing feedback from output to inflation. Therefore, the assumed effect of credit growth on inflation can be understood as stemming from the aggregate demand expansion induced by larger credit availability. There are two policy tools in the model, \( \pi_0 \) and \( \delta_0 \). Expected inflation can be expressed as

\[
\pi^e = \pi_0^e + \gamma \delta^e = \pi_0^e + \gamma \delta_0^e. \tag{5}
\]

We assume that decision-making has two stages, in the first stage, monetary and macro-prudential regulation are decided simultaneously, but before the realization of the credit shock. After the realization of the credit shock, monetary policy decisions can be revised, but macro-prudential regulation cannot. We think of this extra step as “fine-tuning” monetary policy, conceptualizing the fact that monetary policy decisions can be made more frequently than macro-prudential regulation. The purpose of this assumption is to study time-inconsistency problems that can arise similar to those examined in the literature in the case of fiscal and monetary interactions (e.g. Calvo (1978), Lucas and Stokey (1983), and others). In these cases the policymaker has the incentives of reducing the real value of public debt by generating higher inflation.

We define private sector leverage as \( \Phi = D/PY \). \( D \) denotes the stock of nominal debt, \( P \) the price level, and \( Y \) real GDP. Furthermore, we assume that the predetermined level of debt in the economy is given by \( \Phi = D/(P^eY^e) \). This is the total nominal amount of debt before any additional credit expansion due to macro-prudential regulation or credit shocks, evaluated at the expected price and output level. This predetermined level of indebtedness can be understood as an initial condition due to structural factors or to recent macroeconomic developments.

\( \delta \) is the rate of change in the stock of nominal debt. In other words \( D = \overline{D}(1 + \delta) \). We can write the following definition for \( \Phi \)

\[
\Phi = \frac{D}{PY}, \quad \Phi = \frac{\overline{D}(1 + \delta)}{PY}, \quad \Phi = \frac{\Phi P^eY^e(1 + \delta)}{PY},
\]

\[
\ln(\Phi) = \phi \approx \overline{\phi} - (\pi - \pi^e) - (y - y^e) + \delta. \tag{6}
\]

Equation (6) is quite intuitive. Ex-post leverage in the economy is the outcome of surprises in inflation \( -(\pi - \pi^e) \); surprises in output \( -(y - y^e) \); credit growth \( \delta \), which in turn is affected by regulatory actions \( \delta_0 \) and credit shocks \( \epsilon \); and the pre-determined level of leverage \( \overline{\phi} \). This
relationship implies that positive surprises in output and inflation (i.e. lax monetary policy) reduce leverage. Under the assumption of financial risk in the economy being positively correlated with leverage, this implies lower financial risk.\footnote{Equation (6) implies that laxer monetary policy, and thus higher inflation, reduces risk. One could argue that the effect could also go in the opposite direction (i.e. laxer monetary policy increases risk). There is empirical and theoretical work supporting that lax monetary policy increases the riskiness of new bank loans (Jimenez, Ongena, Peydro-Alcalde and Saurina (2007), Ioannidou, Ongena and Peydro (2009), De Nicoló, Dell’Ariccia, Laeven and Valencia (2010), and Valencia (2011)). However, the empirical evidence in these papers also shows that lax monetary policy reduces the risk of outstanding loans, trumping the effect on the riskiness of new loans, if monetary policy remains lax not for too long.}

## III. Social Planner Benchmark

We consider first the case of a social planner who solves the above problem under commitment, that is, $\delta_0^e = \delta_0$ and $\pi_0^e = \pi_0$. We use $y^*$, $\pi^*$, and $\phi^*$ to denote the (possibly constrained) social optimal output, inflation, and leverage in the economy, which reflects also the non-stochastic steady state of the model. The first stage problem can be summarized by

$$\min_{\pi_0, \delta_0} L = E\left[\frac{1}{2}(y - y^*)^2 + \frac{b}{2}(\pi - \pi^*)^2 + \frac{c}{2}(\phi - \phi^*)^2\right],$$

subject to

- $y = y + \beta \delta$,
- $y^e = y + \beta \delta_0^e$,
- $\pi = \pi_0 + \gamma \delta$,
- $\pi^e = \pi_0^e + \gamma \delta_0^e$,
- $\phi = \phi + \delta$,
- $\pi_0^e = \pi_0$,
- $\delta_0^e = \delta_0$.

The first order conditions are:

$$\pi_0 : 0 = b(\pi_0 + \gamma \delta_0 - \pi^*),$$

$$\delta_0 : 0 = a\beta(y^* - y) + b\gamma(\pi_0 + \gamma \delta_0 - \pi^*) + c(\delta_0 + \phi - \phi^*).$$

which yield solutions for $\delta_0$ and $\pi_0$, given by

$$\delta_0^* = \frac{a\beta(y^* - y) + c(\phi^* - \phi)}{c + a\beta^2},$$

$$\pi_0^* = \pi^* - \gamma \delta_0^*.$$
We now examine what happens in the second stage, once the credit shock has been realized and monetary policy can be adjusted. Note that macro-prudential regulation can not be adjusted at this stage so that $\delta_0^*$ is fixed at (8). The objective function is the same as the one shown above, except for the removal of the expectations operator because the shock is now realized. The first order condition is now taken only with respect to $\pi_0$ which is the only policy variable that is revised at this stage:

$$\pi_0 : 0 = b(\pi - \pi^*).$$

The social planner delivers the same level of inflation ex-post and ex-ante. In the first stage, ex-ante monetary policy is to choose the mean level of inflation, $E[\pi_0^{SP}] = \pi^* - \gamma E[\delta_0]$. Because the shock is on average zero, the social planner only fine tunes $\pi_0$ to correct for the realization of the credit shock. That is, the difference between its ex-ante and ex-post choice for $\pi_0$ is $-\gamma \epsilon$, which has an expected value of zero. By definition, the solutions to this problem are socially optimal, with the equilibrium given by

$$y^{SP} = \bar{y} + \beta (\delta_0^* + \epsilon),$$
$$\pi^{SP} = \pi^*,$$
$$\delta_0^* = \phi^* - \bar{\phi},$$
$$\phi^{SP} = \bar{\phi} + \delta_0^* + (1 - \beta) \epsilon.$$

Notice also that $\pi^*$ and $\phi^*$ are not determined in this model and we assume they are given exogenously. Without loss of generality, we set from here on $\pi^* = 0$, $\phi^* > 0$. The non-stochastic steady state level of the model, around which the welfare function is approximated, is then given by

$$y^* = E[y^{SP}] = \bar{y} + \beta \delta_0^*,$$
$$\pi^* = E[\pi^{SP}] = 0,$$
$$\phi^* = E[\phi^{SP}] = \phi^* > 0.$$

The expected welfare loss is given by

$$L^{SP} = \frac{\sigma^2}{2} (a\beta^2 + c(1 - \beta)^2).$$

**IV. TIME INCONSISTENCY IN A DUAL-MANDATE CENTRAL BANK**

Consider now a central bank with a dual mandate on price and financial stability, while the rest of the government (although passively) has a mandate on output stability. This central bank, denoted as DUCB, chooses $\pi_0$ and $\delta_0$ as was the case for the social planner. Two important differences are now introduced: the central bank makes decisions under discretion, taking private expectations as given, and its objective function is composed only of the terms corresponding to inflation and leverage. The sequence of events is the following: i) private agents form expectations, ii)
monetary and macro-prudential policies are decided simultaneously, iii) the credit shock is realized, and finally iv) monetary policy is “fine-tuned” to correct for the realization of the shock.

The objective for the central bank in the first stage is given by

$$L^{DUCB} = \min_{\pi_0, \delta_0} E\left[ \frac{b}{2} \pi^2 + \frac{c}{2} (\phi - \phi^*)^2 \right], \quad \text{(10)}$$

subject to

$$y = \bar{y} + \beta \delta, \quad \text{(11)}$$
$$y^e = \bar{y} + \beta \delta^e, \quad \text{(12)}$$
$$\pi = \pi_0 + \gamma \delta, \quad \text{(13)}$$
$$\pi^e = \pi_0^e + \gamma \delta^e, \quad \text{(14)}$$
$$\phi = \bar{\phi} + \delta. \quad \text{(15)}$$

The first order conditions are given by

$$\pi_0 : 0 = E[b \pi - (1 + \alpha) c (\phi - \phi^*)],$$
$$\delta_0 : 0 = E[b \gamma \pi - c ((1 + \alpha) \gamma - (1 - \beta)) (\phi - \phi^*)],$$

$$= \gamma E\left[ (b \pi - (1 + \alpha) c (\phi - \phi^*)) \right] + E[c (1 - \beta) (\phi - \phi^*)].$$

Imposing rational expectations on the policy variables, \( \delta_0 = \delta^e_0 \) and \( \pi_0 = \pi^e_0 \), in the above equations gives us solutions for \( \pi_0 \) and \( \delta_0 \) that are identical to those of the social planner.

$$\pi_0 = -\gamma \delta_0, \quad \text{(16)}$$
$$\delta_0 = \phi^* - \bar{\phi}. \quad \text{(17)}$$

Notice that the weights on the inflation and financial stability objectives did not play a role in achieving this outcome. Even if the preferences of the central bank differ from those of society, regarding the relative weights on the objectives, the outcome of the first stage is the same as the social planner’s. Dixit and Lambertini (2001) arrive at a similar conclusion in the context of monetary-fiscal interactions in a monetary union: as long as the objectives are aligned, the weighting of these objectives does not matter in the context of a quadratic loss.

We now turn to examine what the DUCB central bank chooses to do after the realization of the credit shock. The objective problem does not change, except for the fact that the expectations operator is no longer needed and the only decision variable is \( \pi_0 \), which implies
\begin{equation}
L^{DUCB} = \min_{\pi_0} b \pi^2 + \frac{c}{2} (\phi - \phi^*)^2,
\end{equation}

subject to the same constraints (11) to (15) as before, which yields the same first order condition as shown above

\[ b \pi = c(1 + \alpha)(\phi - \phi^*). \]

Using the solution for \( \delta_0 \) obtained in the first stage and the definition of \( \phi \) we obtain

\[ \begin{align*}
  b \pi &= c(1 + \alpha)(\phi - \phi^*), \\
  b \pi &= c(1 + \alpha)(\bar{\phi} - (y - y^e) - (\pi - \pi^e) + \phi^* - \bar{\phi} + \epsilon - \phi^*), \\
  b \pi &= c(1 + \alpha)(-(y - y^e) - (\pi - \pi^e) + \epsilon), \\
  b \pi &= c(1 + \alpha)(-(1 + \alpha)(\pi - \pi^e) + \epsilon(1 - \beta)).
\end{align*} \tag{18} \]

from taking expectations of both sides in the above equation we get \( \pi^e = 0 \). Inflation expectations are unaffected by the addition of this new stage. Equilibrium inflation, however, becomes

\[ \pi = \frac{c(1 + \alpha)(1 - \beta)}{b + c(1 + \alpha)^2} \epsilon, \]

which is positive under the assumption of \( \beta < 1.6 \). Unlike the social planner, who would choose in the second stage \( \pi_0 = -\gamma \delta_0 - \gamma \epsilon \) to exactly offset the credit shock to deliver zero inflation in equilibrium, the central bank less than fully offsets the credit shock.

\[ \pi_0^{DUCB} = -\gamma \delta_0 - \gamma \epsilon + \omega \epsilon. \]

It is optimal for the dual central bank to respond less strongly to the inflationary pressures generated by the credit shock than the social planner. Because the DUCB central bank takes private sector expectations as given and it does not care about the level of output it lets inflation rise expecting that the direct impact of inflation on the real value of debt and the impact of higher output because of the resulting surprise in inflation reduce leverage. Consequently, the DUCB central bank repairs private balance sheets by allowing inflation to rise. The central bank does so because it care about financial stability but ex-post it has only one tool to achieve both price and financial stability.

By dividing the numerator and denominator of \( \omega \) by \( c \) and deriving with respect to \( \frac{b}{c} \), which is weight on price stability relative to the weight on financial stability in the objective function, we

\[ \text{This assumption is consistent with the empirical literature, which has generally found an elasticity of GDP growth to credit growth of 0.45 or below (e.g. Ashcraft (2006), Calomiris and Mason (2006), and others).} \]
\[ \frac{\partial \omega}{\partial c} = \frac{-(1+\alpha)(1-\beta)}{b^2 + (1+\alpha)^2} < 0 \] under our assumption of \( \beta < 1 \). The more the central bank cares about price stability, relative to financial stability, the lower the inflation chosen in the second stage.

The economy’s resulting equilibrium is

\[
\begin{align*}
y^{DUCB} &= y^* + \beta \epsilon + \alpha \omega \epsilon, \\
\pi^{DUCB} &= \omega \epsilon, \\
\phi^{DUCB} &= \phi^* + (1 - \beta) \epsilon - \epsilon \omega (1 + \alpha).
\end{align*}
\]

The dual-mandate central bank delivers ex-ante zero inflation, but it delivers positive inflation ex-post in case of a positive credit shock. Because \( \omega \) does not depend on \( \gamma \), this result holds for any value of \( \gamma \). Notice that by assuming that the credit shock is mean-zero, no inflation bias arises ex-ante as in Barro and Gordon (1983). A direct implication of relaxing this assumption is that when we take expectations in Equation (18), expected inflation would not be zero and an inflation bias would appear.

The expected social welfare loss is given by

\[ L^{DUCB} = \frac{a \sigma^2 \omega}{2} (2a \alpha \beta + \omega (a \alpha^2 + b) + c(1 + \alpha)(2(1 - \beta) + (1 + \alpha))) + L^{SP}. \]

It is easy to see in the above equation that the welfare loss is positively related to the variance of credit shocks \( \sigma^2 \), implying that in environments with volatile credit expansion, such as periods of financial liberalization, financial innovation, or large capital inflows, our results are particularly relevant.

**V. Separation of Objectives Achieves Social Optimum**

In the previous section we showed how a dual-mandate central bank delivers a suboptimal outcome. We now show that by separating the price and financial stability objectives in different institutions the social optimum arises. This is a straightforward case, with the central bank in charge only of price stability when it chooses \( \pi_0 \) and the macro-prudential regulator in charge only of financial stability when it chooses \( \delta_0 \).

The objective for the central bank is given by

\[ L^{CB} = \min_{\pi_0} E\left[ \frac{b}{2} \pi_0^2 \right], \]
and the objective for the macro prudential regulator

$$L^F = \min_{\delta_0} E[\frac{C}{2} (\phi - \phi^*)^2],$$  \hspace{1cm} (19)$$
subject to (11) to (15) as before

The first order condition for monetary policy directly gives the social optimum:

$$E[\pi] = 0,$$
$$\pi_0 = -\gamma \delta_0.$$  

The first order condition for $\delta_0$ is given by

$$E[c(\phi - \phi^*)(1 - \beta - \gamma (1 + \alpha))] = 0.$$  

Assuming that $1 - \beta \neq \gamma (1 + \alpha),$\textsuperscript{7} this first order condition reduces to $(\phi - \phi^*) = 0$, that is,

$$E[\phi] = \phi^*.$$  

Together with the optimal solution for $\pi_0$, and after imposing rational expectations, this implies that $\delta_0 = \phi^* - \phi$, which is the social planner’s solution. In the second stage, the central bank has the opportunity to revise its choice for $\pi_0$

$$\min_{\pi_0} \frac{b}{2} \pi^2,$$

subject to the same constraints as before and the first stage solution for $\delta_0$. The problem yields the same first order condition as in the social planner problem

$$\pi = 0,$$

which means $\pi_0 = -\gamma (\delta_0 + \epsilon)$.

The above conditions imply that $E[\pi] = 0$, meaning that private inflation expectations remain unaltered as before. As it was the case for the social planner, the central bank has no incentives to deliver inflation other than the social optimum, and thus its choice for $\pi_0$ in the second stage only corrects for the realization of the credit shock. Therefore, the central bank finds it optimal to deliver zero inflation ex-ante and ex-post and no time inconsistency problem arises.

\textsuperscript{7}This condition is equal to $\frac{d\phi}{d\delta_0} = 1 - \beta - \gamma (1 + \alpha) \neq 0.$
The equilibrium in the economy is given by

\[ y = y^{SP}, \]  
\[ \pi = 0, \]  
\[ \phi = \phi^{SP}. \]  

(20)  
(21)  
(22)

Therefore, the social welfare for the independent agencies and social planner are the same. The result is also robust to a Stackelberg (follower-leader) specification in which monetary policy is chosen after macro-prudential regulation is chosen. This is the case because the problem for the second stage under macro-prudential leadership is identical to the one above, and hence the solution would be the same.

Notice that by construction, a monetary authority separate from a macro-prudential regulator does not internalize the impact of its actions on the level of debt. In turn, a separate macro-prudential regulator does not internalize the impact of its choice for regulation on inflation. In equilibrium, the optimal response from monetary policy is to exactly undue the effects of credit growth on inflation and thus the equilibrium inflation is socially optimal.

VI. THE ROLE OF POLITICAL INDEPENDENCE

In the previous section we showed that separation of objectives is superior to a dual mandate central bank because of a time-inconsistency problem. In this section, we extent our baseline model to incorporate political factors. Following Barro and Gordon (1983), we assume that the government as a whole wishes to stabilize output at the level \( y^* + k \), where \( k > 0 \) can be interpreted as the intensity of political pressures to generate economic expansions. For the social planner, \( k = 0 \).

We abstract from the time-inconsistency problem raised in the previous section to focus solely on political independence issues. We show that separation of objectives achieves the social optimum as long as policy is conducted by politically independent institutions. That is, by institutions that are not affected by political pressures to stimulate output above \( y^* \). In fact, in absence of the time inconsistency issue raised in the previous section, a politically independent dual-mandate (price and financial stability) central bank also delivers the social optimum. When the institutions in charge of policy are not politically independent, the outcome is suboptimal, regardless of whether price and financial stability are pursued by the same institution or by separate entities.

First we show below that a non-independent central bank and an independent macro-prudential regulator is not a socially optimal setting, but neither is an independent central bank and a non-independent macro-prudential regulator. In Appendix I we show that a non-independent central bank in charge of both price and financial stability does not deliver the social optimum either. In short, if the only distortion of concern is political interference, say because the volatility of credit shocks is extremely small, ensuring political independence of institutions should be the central point of the debate.
A. Non-Independent Central Bank and Independent Macro-prudential Regulator

We now show that if the central bank is not politically independent (NICB) the social optimum is not achieved. This follows almost exactly Barro and Gordon (1983). Political motives are introduced by assuming that the policymaker wants to stimulate output for political reasons above what is socially optimal, \( y^* + k \). It is important to clarify upfront that with non-independence we emphasize the intention to overstimulate output. A central bank may still care about output and be politically independent if \( k = 0 \). Because our interest is to show that a welfare loss may arise, it will suffice to show it in the first stage problem. For purposes of this section we only examine the first stage problem and set \( \epsilon = 0 \) so \( \delta = \delta_0 \).

The objective for the central bank is now given by

\[
L^{NICB} = \min_{\pi_0} \alpha \frac{(y - y^* - k)^2}{2} + \frac{b}{2} \pi^2, \tag{23}
\]

and the objective for the macro prudential regulator

\[
L^{MR} = \min_{\delta_0} \frac{c}{2} (\phi - \phi^*)^2, \tag{24}
\]

subject to (11) to (15).

As before, private agents form expectations before decisions by policymakers are made. The first order conditions are given by

\[
\pi_0 : 0 = a\alpha (y - y^* - k) + b\pi, \\
\delta_0 : 0 = c(\phi - \phi^*)(1 - \beta - \gamma(1 + \alpha)).
\]

The equilibrium levels of inflation, output, and debt are given by

\[
y^{NICB} = y^*, \\
\pi^{NICB} = \frac{a\alpha k}{b}, \\
\phi^{NICB} = \phi^*.
\]

Equilibrium output and debt are at the socially optimal levels. However, an inflation bias now appears because political pressures create the incentives for the central bank to generate surprises in inflation in an attempt to stimulate output.

The overall loss is given by

\[
L^{NICB} = \frac{a^2k^2\alpha^2}{2b}. \tag{25}
\]

B. Non-Independent Macro-prudential Regulator and Independent Central Bank

A similar problem than the one shown above arises if the macro-prudential regulator is not independent from a politically motivated rest of the government. Consider the case in which an
independent central bank is exclusively focused on price stability, but now the macro-prudential regulator is not politically independent and has an output stability mandate (NIMR).

The objective for the central bank is given by

$$L^{CB} = \min_{\pi_0} \frac{b}{2} \pi^2,$$

and the objective for the macro prudential regulator

$$L^{NIMR} = \min_{\delta_0} \frac{a}{2} (y - y^* - k)^2 + \frac{c}{2} (\phi - \phi^*)^2,$$

subject to the same constraints (11) to (15).

The first order conditions are given by

$$\pi_0 : 0 = \pi,$$
$$\delta_0 : 0 = a(\beta + \alpha \gamma)(y - y^* - k) + c(\phi - \phi^*)(1 - \beta - \gamma(1 + \alpha)).$$

The solutions are

$$y^{NIMR} = y^* + \frac{\beta k}{\beta + \frac{c}{a}(\frac{1 - \gamma}{\beta + \alpha \gamma} - 1)},$$
$$\pi^{NIMR} = 0,$$
$$\phi^{NIMR} = \phi^* + \frac{k}{\beta + \frac{c}{a}(\frac{1 - \gamma}{\beta + \alpha \gamma} - 1)}.$$

Clearly, when the distortion disappears (i.e. $k = 0$) the social planner’s equilibrium arises again.

The equilibrium debt and output are higher than the social optimum if the denominator of the second term (in each equilibrium value) is positive. A sufficient condition is

$$\frac{d\phi}{d\delta_0} = 1 - \beta - \gamma(1 + \alpha) > 0.$$

This expression corresponds to the increase in leverage from a change in macro-prudential regulation. We assume it is positive because otherwise more credit flows implies lower leverage, implying a Ponzi scheme. In this setting, the central bank has no incentives to generate surprises in inflation and thus the equilibrium level of inflation is the social optimum. However, in achieving this level of inflation the central bank needs to compensate for the actions of the NIMR macro-prudential regulator whose choice of $\delta_0$ is not at the socially optimal level.

\[8\text{We opted for choosing political influences by means of a higher output target than the social optimal, but the same conclusions follow if we assume that political influences appear in the form of generating higher credit availability so that leverage is pushed above the social optimum.}\]
Solving from the first order conditions for $\delta_0$, we obtain

$$\delta_0 = \delta_0^* + \frac{k}{\beta + \frac{\varepsilon}{a} \left( \frac{1}{\beta + \alpha \gamma} - 1 \right)},$$

which implies

$$\pi_0 = \pi_0^* - \frac{k}{\beta + \frac{\varepsilon}{a} \left( \frac{1}{\beta + \alpha \gamma} - 1 \right)}.$$

It pays off to the un-committed macro-prudential regulator to loosen up macro-prudential regulation because she expects higher inflation and output to partially undo the increase in debt. In turn, the monetary authority needs to respond more strongly to contain inflation, given the laxity of regulation. It turns out, however, that the increase in inflation the regulator was expecting never materializes because the monetary authority responds accordingly to achieve the social optimal level of inflation. Consequently, debt and output are excessive. The central bank chooses the socially optimal inflation rate but it does so by maintaining a tighter stance than what the social planner would do.

The overall loss is given by

$$L^{NIMR} = \frac{(a \beta^2 + c)k^2}{2 \left( \beta + \frac{\varepsilon}{a} \left( \frac{1}{\beta + \alpha \gamma} - 1 \right) \right)^2}. \quad (28)$$

Our findings for this section of the paper suggest that the social optimum can be restored by separating the agencies from the rest of the government, which is assumed to be the source of political pressures. In other words, if political pressures are the only distortion, the social optimum arises with politically independent institutions, regardless of whether these institutions have an exclusive or a dual mandate on price and financial stability.

VII. WELFARE COMPARISONS

Table 1 summarizes the welfare loss, relative to the social planner’s loss, qualitatively for the regimes considered, under the two environments explored: when there are political pressures to generate economic expansions and when there are ex-post debt monetization incentives. At a glance, the table shows that if both conditions are present, only a regime with politically independent and separate agencies delivers the social optimum.

We further compare welfare implications numerically for the regimes where a welfare loss arises. We limit this exercise to the cases analyzed in Section VI under political pressures: i) a non-independent single authority (NICOM) for which the solutions are shown in Appendix I, ii) a non-independent central bank and an independent macro-prudential regulator (NICB), and iii) an independent central bank and a non-independent macro-prudential regulator (NIMR).
Table 1. Welfare Loss Across Institutional Arrangements

<table>
<thead>
<tr>
<th>Regime</th>
<th>Welfare Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Debt Monetization Incentives</strong></td>
<td></td>
</tr>
<tr>
<td>A. Independent CB, Independent MR</td>
<td>0</td>
</tr>
<tr>
<td>B. Dual Mandate Central Bank (Price and Financial Stability) (DUCB)</td>
<td>Loss</td>
</tr>
<tr>
<td><strong>Expansionary Political Pressures</strong></td>
<td></td>
</tr>
<tr>
<td>A. Independent CB, Independent MR</td>
<td>0</td>
</tr>
<tr>
<td>B. Non-Independent CB, Independent MR (NICB)</td>
<td>Loss</td>
</tr>
<tr>
<td>C. Non-Independent MR, Independent CB (NIMR)</td>
<td>Loss</td>
</tr>
<tr>
<td>D. Independent and Dual Mandate CB (Financial and Price Stability)</td>
<td>0</td>
</tr>
</tbody>
</table>

The exercise requires calibrating some of the parameters of the model. We start by setting the weights in the welfare function equal to those in De Paoli and Paustian (2011), which correspond to $a = 0.63$, $b = 173$, and $c = 0.17$. These weights show that in existing models with nominal rigidities and financial frictions, the range of parameter values used in the literature imply a much larger weight on price stability than on any of the other two objectives. We also examine the case of equal weights. We assume $k = 0.01$ and $\sigma = 0.01$. The parameter $\alpha = 35$ is taken from Dixit and Lambertini (2001) in which the output equation is the same we use in this paper. Figure 1 plots the resulting welfare loss for the case with expansionary political pressures as a function of the parameters $\beta$ and $\gamma$, for which there are less clear cut values in the literature.

This exercise is oriented towards getting a sense of the ranking among these suboptimal regimes. The top three graphs show the welfare loss when the weights are calibrated as discussed above, whereas the bottom three assume $a = b = c = 1/3$. The solid line corresponds to the NICOM case, the dashed line to the NICB case, and the dotted line to the NIMR case.

Focusing first on the outcomes under calibrated weights. For values of $\gamma$ closer to zero and $\beta < 0.5$, NIMR dominates both NICB and NICOM. Notice that NICOM can also be seen as a non-independent central bank in charge of both price and financial stability. For high values of $\beta$ NIMR still dominates NICOM. Only when macro-prudential regulation has a strong positive or

---

9Faia and Monacelli (2007) do not derive an analytical expression for the welfare function, but they find it is optimal for the authority to try to neutralize a credit distortion, suggesting also that financial conditions enter the welfare function.

10It is also important to note that we conducted this analysis with $\alpha = 5$ and what changes is the level of losses, but not the ranking, which is what we emphasize in our analysis.
negative impact on inflation (i.e. \( \gamma \) equals 1 or -1), NIMR is dominated by NICOM and NICB. Again, for high values of \( \beta \), NIMR dominates NICOM.

Focusing now on the case of equal weights, NIMR dominates NICOM and NICB for the values of \( \gamma \) considered and all but the highest values of \( \beta \) in the range under analysis. In sum, which among these suboptimal regimes dominates depends on parameter values. However, in almost all cases, a non-independent macro prudential regulator dominates a non-independent single authority. In other words, if institutions are not politically independent, it may still be preferable to have separate institutions than a single institution in charge of both, price and financial stability.

\section*{VIII. Conclusions}

We consider macro-prudential regulation and monetary policy interactions to investigate the welfare implications of different institutional arrangements. In our framework, monetary policy can re-optimize following a realization of credit shocks, but macro-prudential regulation cannot be adjusted immediately after the credit shock. This feature of the model captures the ability of adjusting monetary policy more frequently than macro-prudential regulation because macro-prudential regulation is an ex-ante tool, whereas monetary policy can be used ex-ante and ex-post. In this setting, a central bank with a price and financial stability mandate does not deliver the social optimum because of a time-inconsistency problem. This central bank finds it optimal ex-ante to deliver the social optimal level of inflation, but it does not do so ex-post. This is because the central bank finds it optimal ex-post to let inflation rise to repair private balance.
sheets because ex-post it has only monetary policy to do so. Achieving the social optimum in this case requires separating the price and financial stability objectives.

We also consider the role of political independence of institutions, as in Barro and Gordon (1983). Under this extension, separation of price and financial stability objectives delivers the social optimum only if both institutions are politically independent. If the central bank or the macro-prudential regulator (or both) are not politically independent, they would not achieve the social optimum. Numerical analysis in our model suggest however, that in most cases a non-independent macro-prudential regulator (with independent monetary authority) delivers a better outcome than a non-independent central bank in charge of both price and financial stability.
REFERENCES


©International Monetary Fund. Not for Redistribution

________, and ______, 2011, “Macroprudential Regulation Versus Mopping up After the Crash,” *University of Maryland mimeo.*


APPENDIX I. NON-INDEPENDENT SINGLE AUTHORITY

Consider a single authority, not the social planner, with the goal of stabilizing output, inflation, and leverage (NICOM). The authority’s decision problem is similar to the social planner’s, with the difference that this authority cannot commit and its output objective is higher by \( k \). As it was the case for the analysis in Section VI, we focus only on the first stage solutions. The authority chooses policy variables taking agents’ expectations as given, although agents are assumed to form expectations rationally.

\[
\min_{\pi_0, \delta_0} L = \frac{a}{2}(y - y^* - k)^2 + \frac{b}{2}\pi^2 + \frac{c}{2}(\phi - \phi^*)^2, \tag{A1}
\]

subject to the constraints (11) to (15).

The first order conditions are given by

\[
\pi_0 : 0 = a\alpha(y - y^* - k) + b\pi - (1 + \alpha)c(\phi - \phi^*), \\
\delta_0 : 0 = a(\alpha \gamma + \beta)(y - y^* - k) + b\gamma\pi - c((1 + \alpha)\gamma - (1 - \beta))(\phi - \phi^*), \\
= \gamma(a\alpha(y - y^* - k) + b\pi - (1 + \alpha)c(\phi - \phi^*)) + a\beta(y - y^* - k) + c(1 - \beta)(\phi - \phi^*).
\]

Imposing rational expectations, \( \pi^*_0 = \pi_0 \) and \( \delta^*_0 = \delta_0 \) in the above equations allows us to write them as follows:

\[
\pi_0 : a\alpha(y^* - \bar{y} + k) - c(1 + \alpha)(\phi^* - \bar{\phi}) = ((a\alpha - c(1 + \alpha))\delta_0 + b\pi, \\
\delta_0 : a\beta(y^* - \bar{y} + k) + c(1 - \beta)(\phi^* - \bar{\phi}) = ((a\beta^2 + c(1 - \beta))\delta_0.
\]

By eliminating \( \delta_0 \), equilibrium inflation is given by

\[
\pi_{NICOM} = \frac{ac(\alpha + \beta)\left((y^* - \bar{y} + k) - \beta(\phi^* - \bar{\phi})\right)}{b(c(1 - \beta) + a\beta^2)}
\]

or using the definition for \( y^* \) derived in the main part of the paper

\[
\pi_{NICOM} = \frac{ack(\alpha + \beta)}{b(c(1 - \beta) + a\beta^2)}.
\]

Equilibrium inflation is not at the optimum (i.e., zero) unless the numerator is zero, which happens when \( k = 0 \) or when the authority does not care about output \( a = 0 \). This the standard inflation bias that arises in Barro and Gordon (1983) or in Dixit and Lambertini (2003) under discretionary policies. The “distortion” \( k \) creates an incentive to deviate from the socially optimal level of \( \pi_0 \) and the authority attempts to generate surprises in inflation to increase output.
However, these incentives are fully anticipated by private agents and thus inflation is higher than socially desired in equilibrium.

Equilibrium credit growth is too high compared to the social optimum

$$\delta^{\text{COM}} = \delta^* + \frac{a\beta k}{b(c(1 - \beta) + a\beta^2)},$$

and the level of debt is

$$\phi^{\text{COM}} = \phi^* + \frac{\beta ak}{c(1 - \beta) + a\beta^2}.$$

As it was the case for inflation, in absence of the distortion $k$, $\phi = \phi^*$ and the equilibrium would be socially optimal.

Equilibrium output is given by

$$y^{\text{COM}} = y + \beta(\phi^* - \bar{\phi}) + \beta^2 \frac{ak}{c(1 - \beta) + a\beta^2}.$$

This implies that, with the effort of the authority to generate expansions in output, the economy indeed grows more than the social optimum with too high inflation and credit. As before, in absence of the distortion $k$ or when the authority assigns no weight to output, the social optimum level $y^*$ is achieved. The authority’s intentions of expanding output is no longer completely ineffective as it is in the standard framework without financial regulation, which here can be obtained if we set $\beta = 0$. This is because the authority has now macro-prudential regulation as a tool to stimulate output and it is no longer limited to monetary policy.

The welfare loss for this institutional arrangement is given by

$$L^{\text{COM}} = \frac{1}{2b} \left( \frac{ak}{c(1 - \beta) + a\beta^2} \right)^2 \left( (\alpha + \beta)^2 + b\beta^2(a + c) \right).$$
APPENDIX II. DISTORTIONARY MACRO-PRUDENTIAL REGULATION

In this Appendix, we consider the case where a large shock may create an asymmetric loss function. In particular, credit growth itself may be an important factor in the recovery after a crisis (or in the development or catching-up phase). This means that macro-prudential regulation can (potentially) generate distortions. For simplicity, we assume these distortions generate a welfare loss equivalent to $\chi \delta_0$, with $\chi < 0$ to reflect that macro prudential tightening exacerbates the distortion.

We consider the social planner problem first:

$$
\min_{\pi_0, \delta_0} L = \frac{a}{2}(y - y^*)^2 + \frac{b}{2}(\pi - \pi^*)^2 + \frac{c}{2}(\phi - \phi^*)^2 + \chi \delta_0,
$$

subject to

$$
\begin{align*}
y &= y^e = \overline{y} + \beta \delta_0, \\
\pi &= \pi^e = \pi_0 + \gamma \delta_0, \\
\phi &= \overline{\phi} + \delta_0,
\end{align*}
$$

where the starred variables denote the social optimum without distortions defined in the main text. In this case, similar to the key assumption made in Kydland and Prescott (1977), even the social planner cannot achieve the first best (i.e., a constrained social planner).

The first order conditions are:

$$
\begin{align*}
\pi_0 : 0 &= b(\pi_0 + \gamma \delta_0 - \pi^*), \\
\delta_0 : 0 &= a\beta(\overline{y} + \beta \delta_0 - y^*) + b\gamma(\pi_0 + \gamma \delta_0 - \pi^*) + c(\delta_0 + \overline{\phi} - \phi^*) + \chi.
\end{align*}
$$

These equations yield optimal solutions for $\delta_0$ and $\pi_0$, given by

$$
\begin{align*}
\delta^{SP}_0 &= \frac{a\beta(y^* - \overline{y}) + c(\phi^* - \overline{\phi}) - \chi}{c + a\beta^2}, \\
\pi^{SP}_0 &= \pi^* - \gamma \delta^{SP}_0.
\end{align*}
$$

By definition, the solutions to this problem are socially optimal, with solutions for output, inflation, and debt given by

$$
\begin{align*}
y^{SP} &= \overline{y} + \beta \delta^{SP}_0, \\
\pi^{SP} &= \pi^*, \\
\phi^{SP} &= \overline{\phi} + \delta^{SP}_0 = \phi^* - \frac{\chi}{a\beta^2 + c}.
\end{align*}
$$
As before, $\pi^* = 0$ and $\phi^* > 0$.

Now consider the problem for three independent agencies. The objective for the central bank is given by

$$L^{CB} = \min_{\pi_0} \frac{b}{2} \pi^2, \quad \text{(A4)}$$

and the objective for the macro-prudential regulator

$$L^F = \min_{\delta_0} \frac{c}{2} (\phi - \phi^*)^2 + \chi \delta_0, \quad \text{(A5)}$$

subject to the same constraints (11) to (15).

The first order conditions are given by

$$\pi_0 : \pi = 0, \quad \delta_0 : c(\phi - \phi^*)(1 - \beta - \gamma(1 + \alpha)) + \chi = 0.$$ 

This implies the following equilibrium levels of output, inflation, and debt, after imposing rational expectations.

$$y^{IND3-Dist} = y^* + \beta \frac{\chi}{c} \left( \frac{1}{1 - \beta - \gamma(1 + \alpha)} \right),$$

$$\pi^{IND3-Dist} = 0,$$

$$\phi^{IND3-Dist} = \phi^* - \frac{\chi}{c} \left( \frac{1}{1 - \beta - \gamma(1 + \alpha)} \right).$$

With a passive rest of the government and with independence of the monetary and macro-prudential regulator from political pressures, the socially optimal level of inflation is achieved. However, output and debt turn out to be not at their social optimal levels. The macro-prudential regulator chooses $\delta_0$ to compensate for the costs in terms of a higher loss arising from the distortions generated by regulatory actions. However, this incentive is exacerbated by the fact that the regulator does not internalize the fact that the private sector anticipates these incentives. The result is an over compensation with too much debt and output in equilibrium, under the assumption of $1 - \beta - \gamma(1 + \alpha) > 0$ made earlier.

Finally, consider the case of a central bank with a dual mandate, on price and financial stability. The objective for the central bank is given by

$$L^{CB} = \min_{\pi_0, \delta_0} \frac{b}{2} \pi^2 + \frac{c}{2} (\phi - \phi^*)^2 + \chi \delta_0, \quad \text{(A6)}$$

subject to the constraints (11) to (15).
The first order conditions are given by

\[
\begin{align*}
\pi_0 &= 0 = b\pi - (1 + \alpha)c(\phi - \phi^*), \\
\delta_0 &= 0 = b\gamma\pi - c((1 + \alpha)\gamma - (1 - \beta))(\phi - \phi^*) + \chi, \\
&= \gamma(b\pi - (1 + \alpha)c(\phi - \phi^*)) + c(1 - \beta)(\phi - \phi^*) + \chi. \\
\end{align*}
\]

Equilibrium output, inflation, and indebtedness are given by

\[
\begin{align*}
y_{IND2b-Dist} &= y^* - \beta\frac{\chi}{c}\left(\frac{1}{1 - \beta}\right), \\
\pi_{IND2b-Dist} &= -\frac{(1 + \alpha)\chi}{b(1 - \beta)}, \\
\phi_{IND2b-Dist} &= \phi^* - \frac{\chi}{c}\frac{1}{1 - \beta}.
\end{align*}
\]

Clearly, a dual mandate does not deliver the social optimum either. However, this exercise stresses that under distortionary macro-prudential regulation, the superiority of separation of agencies or a dual mandate central bank depends on how important these distortions are and the parameters of the model.