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Abstract

This paper formulates a novel modeling framework that delivers: (a) forecasts of indicators of systemic real risk and systemic financial risk based on density forecasts of indicators of real activity and financial health; (b) stress-tests as measures of the dynamics of responses of systemic risk indicators to structural shocks identified by standard macroeconomic and banking theory. Using a large number of quarterly time series of the G-7 economies in 1980Q1-2010Q2, we show that the model exhibits significant out-of-sample forecasting power for tail real and financial risk realizations, and that stress testing provides useful early warnings on the build-up of real and financial vulnerabilities.

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I. Introduction

The 2007-2009 financial crisis has spurred renewed efforts in systemic risk modeling. Bisias et al. (2012) provide an extensive survey of the models currently available to measure and track indicators of systemic financial risk. However, three limitations of current modeling emerge from this survey. First, almost all proposed measures focus on (segments of) the financial sector, with developments in the real economy either absent, or just part of the conditioning variables embedded in financial risk measures. Second, there is yet no systematic assessment of the out-of-sample forecasting power of the measures proposed, which makes it difficult to gauge their usefulness as early warning tools. Third, stress testing procedures are in most cases sensitivity analyses, with no structural identification of the assumed shocks.

Building on our previous effort (De Nicolò and Lucchetta, 2011), this paper contributes to overcome these limitations by developing a novel tractable model that can be used as a real-time systemic risks’ monitoring system. Our model combines dynamic factor VARs and quantile regressions techniques to construct forecasts of systemic risk indicators based on density forecasts, and employs stress testing as the measurement of the sensitivity of responses of systemic risk indicators to configurations of structural shocks.

This model can be viewed as a complementary tool to applications of DSGE models for risk monitoring analysis. As detailed in Schorfheide (2010), work on DSGE modeling is advancing significantly, but several challenges to the use of these models for risk monitoring purposes remain. In this regard, the development of DSGE models is still in its infancy in at least two dimensions: the incorporation of financial intermediation and forecasting. In their insightful review of recent progress in developments of DSGE models with financial intermediation, Gertler and Kyotaki (2010) outline important research directions still unexplored, such as the linkages between disruptions of financial intermediation and real activity. Moreover, as noted in Herbst and Schorfheide (2010), there is still lack of conclusive evidence of the superiority of the forecasting performance of DSGE models relative to sophisticated data-driven models. In addition, these models do not typically focus on tail risks. Thus, available modeling technologies providing systemic risk monitoring tools based on explicit linkages between financial and real sectors are still underdeveloped.

Contributing to fill in this void is a key objective of this paper.

Three features characterize our model. First, we make a distinction between systemic real risk and systemic financial risk, based on the notion that real effects with potential adverse welfare consequences are what ultimately concerns policymakers, consistently with the definition of systemic risk introduced in Group of Ten (2001). Distinguishing systemic financial risk from systemic real risk also allow us to assess the extent to which a realization of a financial (real) shock is just amplifying a shock in the real (financial) sector, or originates in the financial (real) sector. Second, the model produces real-time density forecasts of indicators of real activity and financial health, and uses them to construct forecasts of indicators of systemic real and financial risks. To obtain these forecasts, we use a dynamic factor model (DFM) with many predictors combined with quantile regression techniques. The choice of the DFM with many predictors is motivated by its superior forecasting performance over both univariate time series specifications and standard VAR-type models (see Watson, 2006). Third, our design of stress tests can be flexibly linked to
selected implications of DSGE models and other theoretical constructs. Structural identification provides economic content of these tests, and imposes discipline in designing stress test scenarios. In essence, our model is designed to exploit, and make operational, the forecasting power of DFM models and structural identification based on explicit theoretical constructs, such as DSGE models.

Our model delivers *density forecasts* of any set of time series. Thus, it is extremely flexible, as it can incorporate multiple measures of real or financial risk, both at aggregate and disaggregate levels, including many indicators reviewed in Bisias et al. (2012). In this paper we focus on two simple indicators of real and financial activity: real GDP growth, and an indicator of health of the financial system, called FS. Following Campbell, Lo and MacKinlay (1997), the FS indicator is given by the return of a portfolio of a set of systemically important financial firms less the return on the market. This indicator is germane to other indicators of systemic financial risk used in recent studies (see e.g. Acharya et al., 2010 or Brownlee and Engle, 2010).

The joint dynamics of GDP growth and the FS indicator is modeled through a dynamic factor model, following the methodology detailed in Stock and Watson (2005). Density forecasts of GDP growth and the FS indicator are obtained by estimating sets of quantile auto-regressions, using forecasts of factors derived from the companion factor VAR as predictors. The use of quantile auto-regressions is advantageous, since it allows us to avoid making specific assumptions about the shape of the underlying distribution of GDP growth and the FS indicator. The blending of a dynamic factor model with quantile auto-regressions is a novel feature of our modeling framework.

Our measurement of systemic risks follows a risk management approach. We measure systemic *real* risk with GDP-Expected Shortfall (GDPES), given by the expected loss in GDP growth conditional on a given level of GDP-at-Risk (GDPaR), with GDPaR being defined as the worst predicted realization of quarterly growth in real GDP at a given (low) probability. Systemic *financial* risk is measured by FS-Expected Shortfall (FSES), given by the expected loss in FS conditional on a given level of FS-at-Risk (FSaR), with FSaR being defined as the worst predicted realization of the FS indicator at a given (low) probability level.

*Stress-tests* of systemic risk indicators are implemented by gauging how impulse responses of systemic risk indicators vary through time in response to structural shocks. The identification of structural shocks is accomplished with an augmented version of the sign restriction methodology introduced by Canova and De Nicolò (2002), where aggregate shocks are extracted based on standard macroeconomic and banking theory. Our approach to stress testing differs markedly from, and we believe significantly improves on, most implementations of stress testing currently used in central banks and international organizations. In these implementations, shock scenarios are imposed on sets of observable variables, and their effects are traced through —behavioral” equations of certain variables of interest. Yet, the —shocked” observable variables are typically *endogenous*: thus, it is unclear whether we are shocking the symptoms and not the causes. As a result, it is difficult to assess both the qualitative and quantitative implications of the stress test results.

We implement our model using a large set of quarterly time series of the G-7 economies during the 1980Q1-2010Q1 period, and obtain two main results. First, our model provides
significant evidence of out-of-sample forecasting power for tail real and financial risk realizations for all countries. Second, stress tests based on this structural identification provide early warnings of vulnerabilities in the real and financial sectors.

The remainder of the paper is composed of four sections and an Appendix. Section II outlines the model setup, estimation, forecasting, and stress testing. Section III describes the implementation of the modeling framework on data for the G-7 countries, focusing on estimation and forecasting. Section IV presents an example of a stress testing procedure. Section V concludes. The Appendix summarizes the banking model used for identification, a description of the data and supplementary Figures.

II. The Model

Following Stock and Watson (2005), the dynamics of \( N \) series \( X_n \), indexed by \( i \in N \), with \( N \) large, is modeled with a Dynamic Factor Model (DFM) described by the following equations:

\[
X_n = \lambda_t(L)f_t + \gamma_t(L)X_{n-1} + v_n \tag{1}
\]

\[
f_t = \Gamma(L)f_{t-1} + \eta_t \tag{2}
\]

Each series \( X_n \) is a function of a vector of dynamic factors \( f_t \), of its own lags and idiosyncratic errors \( v_n \), which are assumed to be uncorrelated at all leads and lags. Equation (2) describes the dynamics of these factors through a VAR. Under the assumption that dynamic factors have finite lags up to \( p \), and defining the vector of static factors with \( F_t = [f_t', f_{t-1}', \ldots, f_{t-p-1}'] \), one obtains the static form representation of the DFM:

\[
X_n = \Lambda_tF_t + \gamma_t(L)X_{n-1} + v_n \tag{3}
\]

\[
F_t = \Phi(L)F_{t-1} + G\eta_t \tag{4}
\]

Matrix \( \Phi(L) \) includes \( \Gamma(L) \) and 0’s, while \( G \) is a matrix of coefficients of dimension \( r \times q \), where \( r \) is the number of static factors and \( q \) that of dynamic factors. In our application, we assume \( r = q \), which implies \( \Phi(L) = \Gamma(L) \) and \( G = I \). Thus, Equation (4) is equivalent to Equation (2). As shown in Stock and Watson (2005), Equations (3)-(4) can be transformed in a (restricted) Factor-Augmented VAR (FAVAR) representation of the DFM akin to that adopted by Bernanke, Boivin, and Eliasz (2005).

In this paper, we focus on predictions of two of the \( N \) series: real GDP growth, denoted by \( GDPG_t \), and the FS indicator, denoted by \( FS_t \):

\[
GDPG_t = \Lambda^{R}F_t + \gamma^{R}_t(L)GDPG_{t-1} + v^{R}_t \tag{5}
\]

\[
FS_t = \Lambda^{F}F_t + \gamma^{F}_t(L)FS_{t-1} + v^{F}_t \tag{6}
\]
To construct density estimates of $GDP_G$ and $FS_I$, we first estimate quantile auto-regressions (Koenker, 2005) of the form (5) and (6), with estimates of the static factors $\hat{F}_t$ as conditioning variables. Denote with $\tau \in (0,1)$ a particular quantile, and with a "hat" estimated quantile coefficients. Quantile estimates of (5) and (6) for each $\tau \in \{1,2,\ldots,99\}$ are:

$$GDP_Q(\tau) = \hat{\alpha}_1(\tau) + \hat{\lambda}^{R'}(\tau)\hat{F}_t + \hat{\gamma}_R(\tau)(L)GDP_{t-1},$$

$$FS_Q(\tau) = \hat{\alpha}_2(\tau) + \hat{\lambda}^{'R'}(\tau)\hat{F}_t + \hat{\gamma}_F(\tau)(L)FS_{t-1}.$$  

For low values of $\tau \in (0,1)$, the Value-at-Risk (VaR) measures $GDP_{aR}$ and $FS_{aR}$ are the fitted values of $-GDP_Q(\tau)$ and $-FS_Q(\tau)$. There are two well known limitations of VaR-type measures: (a) they not take into account the size of tail losses; and (b) they lack "coherence" in the sense of Artzner et al. (1999), since they do not satisfy the sub-additivity property required for consistent risk ordering. A measure overcoming these problems is given by the Expected Shortfall ($ES$). Given a random variable $X$, $ES$ is defined as the expected downside loss at $\tau$ percent probability associated with a fall in $X$ below quantile $\tau$ (see, e.g. Acerbi and Tasche, 2002). Denoting with $E_t$ the expectation operator conditional on information available at date $t$, for any given $\tau \in (0,1)$, our systemic risk indicators are defined as:

$$GDP_{ES}(\tau) = -E_t(GDP_G \mid GDP_G \leq GDP_{aR}(\tau))$$

$$FS_{ES}(\tau) = -E_t(FS_I \mid FS_I \leq FS_{aR}(\tau))$$

### A. Estimation and Forecasting

Estimation and forecasting are accomplished in four steps.

**Step 1: Choice of number of factors and lags**

We compute static factors, and choose the number and lags of the factor VAR (Equation (4)), as follows. First, we use principal components to extract all factors with eigenvalues greater than 1, in number $R$. Second, we order factors according to their explanatory power of the variance of the data, and construct the set of factors $\tilde{F} = \{(F_{r=1}), \ldots, (F_1, F_2, \ldots, F_{r=R})\}$. Lastly, as in Stock and Watson (2002), we choose the number of lags $L$ and the number of static factors $r$ that maximize the Bayesian Information Criterion (BIC) for Equations (5)-(6), estimated for each set of factors in $\tilde{F}$ and with one, two, three, and four lags. Thus, the optimal number of lags $L^*$ and the number of static factors $r^*$ yield the maximum BIC criterion among $4xR$ specifications of Equations (5)-(6).
Step 2: Quantile Estimation

The \( r^* \) estimated factors with lags \( L^* \) are used as regressors of the quantile auto-regressions (7) and (8) for \( \tau = 1, 2, ..., 99 \). Estimated quantile regressions may generally exhibit —crossings” of the conditional quantile functions. Such —crossing”, if and when it occurs, implies that the key assumption that distribution functions are monotonically increasing is violated. As stressed by Koenker (2005, Ch. 8), this problem is likely to be more severe for quantile auto-regressions. Crossing can be the result of a mis-specification of the model, which in turn can adversely affect its forecasting performance.

We address the problem of potential —crossing” by adopting the rearrangement procedure introduced by Chernozukhov et al. (2010). By rearranging original quantile estimates into monotone quantile estimates, Chernozukhov et al. (2010) show that the resulting quantile curves are closer to the true quantile curves in finite samples. By construction, these rearranged quantile curves do not exhibit crossing. We implement this rearrangement procedure by re-ordering at each date the quantiles originally estimated via (7) and (8) whenever crossing occurs. These sorted quantiles are our final quantile estimates.

Step 3: Density Estimation and Construction of Systemic Risk Indicators

The quantile estimates provide discrete density estimates at each date. To obtain continuous densities and compute expected shortfalls, we proceed as follows. Recall that given a continuous probability distribution \( F \) of a random variable \( X \), the quantile corresponding to probability \( \tau \), denoted by \( Q(\tau) \), is also equal to \( Q(\tau) = F^{-1}(\tau) \), where \( F^{-1}(\tau) = \inf(x \mid F(x) \geq \tau) \) is the generalized inverse of \( F \). Then, the expected shortfall of \( X \) can be expressed as:

\[
ES(\tau) = -\frac{1}{\tau} \int_{0}^{\tau} F^{-1}(y)dy = -\frac{1}{\tau} \int_{0}^{\tau} Q(y)dy 
\]

(11)

To estimate \( Q(\tau) \) as a continuous functions of \( \tau \in (0,1] \), we regress the series of the 99 discrete quantiles at each date on a polynomial function of order \( m \)—with \( m \) selected to maximize the \( R^2 \) of these regressions at each date—obtaining \( \hat{Q}_i(\tau) = \sum_{i=0}^{m} \hat{a}_i \tau^i \), where —hats” denote estimated coefficients. Therefore, expected shortfall estimates are given by:

\[
ES_i(\tau) = -\frac{1}{\tau} \int_{0}^{\tau} \hat{Q}_i(y)dy = -\frac{1}{\tau} \int_{0}^{\tau} \sum_{i=0}^{m} \hat{a}_i y^i dy = -(\hat{a}_0 + \hat{a}_1 \frac{\tau}{2} + \hat{a}_2 \frac{\tau^2}{3} + ... + \hat{a}_m \frac{\tau^{m-1}}{m}) 
\]

(12)

This procedure is similar to several methods aimed at estimating tails of distributions based on extreme value theory (EVT). These methods are typically based on estimates of Hill indicators obtained employing subsets of observations of the data relatively close to the tail of interest. The underlying assumption is that unconditional densities are generated by a wide family of distributions with supports that are unbounded below.² Our estimation procedure

differs from these methods: we do not impose distributional assumptions on our real and financial indicators, which have supports that are bounded below, and use information on the entire distribution of interest through the full range of discrete quantile estimates.

**Step 4: Multi-step Forecasts of Systemic Risk Indicators**

In this last step, we construct \( k \) quarters-ahead forecasts of systemic risk indicators. As quantile estimates are linear in factors and lagged variables, we follow Stock and Watson (2002) by using \( k \) quarters-ahead quantile projections. Using (7) and (8) (with estimated factors denoted with a "—hat") these \( k \) quarters-ahead quantile projections are:

\[
GDPGQ_{t+k}(\tau) = \hat{\alpha}_1^k(\tau) + \hat{\Lambda}_k^R(\tau) \hat{F}_t + \gamma_{R}^k(\tau)(L)GDPG_{t-1} \tag{13}
\]

\[
FSQ_{t+k}(\tau) = \hat{\alpha}_2^k(\tau) + \hat{\Lambda}_k^{FS}(\tau) \hat{F}_t + \gamma_{FS}^k(\tau)(L)FS_{t-1} \tag{14}
\]

Finally, applying the procedure described in Step 3, we obtain \( k \) quarters-ahead forecasts of expected shortfalls of GDP growth and the FS indicator, denoted by \( GDPES_{t+k}(\tau) \) and \( FSES_{t+k}(\tau) \) respectively.

**B. Stress Testing**

We define stress testing as the measurement of the sensitivity of responses of systemic risk indicators to configurations of structural shocks. These responses are obtained as impulse responses and variance decompositions of our systemic risk indicators to identified structural shocks. In this paper we present a version of this stress testing procedure, based on a particular identification procedure, and simple metrics of changes in the sensitivity of impulse responses and variance decompositions of systemic risk indicators to different configurations of structural shocks. However, we should stress that our stress testing procedure is fairly general, since it can be implemented using different identification schemes and metrics of sensitivities of responses of systemic risk indicators to shocks.

Our identification strategy is based on an augmented version of a sign restriction methodology used by Canova and De Nicolò (2002). As detailed in Canova (Ch. 4, 2007), identification through sign restrictions can be carried out through a variety of linearized DGSE models that have a VAR representation, and are also implementable in the context of Bayesian VARs (see Del Negro and Schorfheide, 2010). In our case, a theoretical model will impose sign restrictions on the responses of certain sets of observable variables in equation (3) to (orthogonalized) shocks to factors. Our procedure differs from that of Canova and De Nicolò (2002) in two respects: orthogonal innovations are extracted from the factor VAR rather than from a low dimensional VAR, and sign restrictions are derived from both aggregate dynamic macroeconomic theory and a simple banking model.\(^3\)

Canova and De Nicolò (2002) show that the following sign restrictions can be derived from a wide class of general equilibrium monetary macroeconomic models with different micro-

\(^3\) An application of a version of the sign restriction methodology in the context of FAVAR models which focuses on monetary shocks is in Ahmadi and Uhlig (2009)
foundations. If a positive temporary orthogonal innovation represents a positive transitory aggregate supply shock, then it should generate transitory weakly positive output responses and weakly negative transitory responses in inflation. On the other hand, if it is a real aggregate demand shock, it should generate weakly positive transitory responses in output and inflation.

To identify demand and supply shocks to bank credit, we use the simple partial equilibrium model by Boyd, De Nicoló and Loukoianova (2010)—briefly summarized in the Appendix—where sign restrictions of interest are obtained using measures of bank credit growth and changes in loan rates. The restrictions implied by this banking model are as follows. If there is a positive transitory shock to the demand for bank credit (e.g. because of a positive technology shock to firms generating an increase in demand for investment, or an increase in the quality of investment prospects), then we should observe a transitory increase in bank credit growth and an increase in loan rates. We call a shock generating these responses a positive credit demand shock. Conversely, if there is a positive transitory shock to the supply of bank credit (e.g. banks expand their assets through an increase in bank debt and/or capital), then we should observe a transitory increase in bank credit growth coupled with a decline in loan rates. We call a shock generating these responses a positive credit supply shock. Of course, negative shocks have the signs of these responses all reversed.

Note that real aggregate demand or supply shocks can affect the underlying drivers of the supply and demand for bank credit simultaneously. For example, a negative aggregate demand shock can induce firms and household to decrease their demand for bank credit. In a simple diagram of bank credit demand and supply, this would be represented by a leftward shift of the demand for bank credit, which would result in a decline in loan rates ceteris paribus. At the same time, the adverse wealth effects of a negative aggregate demand shock may induce investors to reduce their supply of funds to banks, or banks could reduce their supply of credit as they may become increasingly capital-constrained or risk averse: this would result in a leftward shift in the supply of credit ceteris paribus. Which effect dominates on net will be reflected in movements in loan rates and bank credit growth. If negative credit demand shocks dominate, then loan rates and bank credit growth should decline, while the converse would hold if negative credit supply shocks dominate.

Table A below summarizes the responses of GDP growth, inflation, bank lending growth, and changes in loan rates in response to positive structural shocks implied by standard aggregate macroeconomic models and a partial equilibrium banking model.

Table A. Signs of theoretical responses of key variables to positive shock combinations

<table>
<thead>
<tr>
<th>Shocks</th>
<th>GDP growth</th>
<th>Inflation</th>
<th>Bank Credit Growth</th>
<th>Change in Lending Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate Supply + Bank Credit Demand</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Aggregate Supply + Bank Credit Supply</td>
<td>+</td>
<td>-</td>
<td>+</td>
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<tr>
<td>Aggregate Demand + Bank Credit Demand</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Aggregate Demand + Bank Credit Supply</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>
Thus, identification of structural shocks is conducted by checking whether orthogonal innovations of the factor VAR produce responses of the four variables considered that match the signs of the responses implied by a class of aggregate macroeconomic models and a banking model.

III. IMPLEMENTATION: ESTIMATION AND FORECASTING

We implemented our modeling procedure using quarterly time series of the G-7 economies for the period 1980Q1-2010Q1. All series are taken from Datastream. For each country, the vector of quarterly series $X_t$ in equation (1) includes between 50 and 80 series detailed in the Appendix. These series are classified into three groups. The first group comprises equity markets data, including prices, price/earnings ratios and dividend yields for the entire market and by sector. The second group includes financial, monetary and banking variables related to credit conditions, namely: interest rates for different maturities, monetary policy rates, bank prime rates and interbank rates, bank lending, and monetary aggregates. The third group includes price and quantity indicators of real activity. This set of variables includes capacity utilization, the unemployment rate, industrial production, a consumer price index, and house prices.

A. Estimation

Following the steps outlined previously, we estimated static factors of each variable by principal components according to the procedures described in Stock and Watson (2005). Our factor and lag selection criteria resulted in between 4 and 7 static factors depending on a country dataset, leading us to choose five factors for each country, and one lag in each country. Estimated factors were used as independent variables of quantile regressions specified with one lag. Quantile crossing occurred in only about one percent of dates in each country. Lastly, $m=4$ was the best value of the polynomial approximation to obtain continuous densities, with the $R^2$ associated with the relevant regressions at each date and country not lower than 0.96.

Figure 1 reports the time series of in-sample estimates of GDPES and FSES series at 20 and 5 percent probability levels, and the relevant eight-step forecasts as of 2010Q1 for the U.S., These systemic risk fan charts compactly summarize the range of expected tail real and financial prospects for a given probability range.

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4 As a cross-check, we also estimated the number of factors using the Bai and Ng (2002) criterions as applied to equation (2), obtaining a similar number of static factors for each country dataset.
Figure 1. United States: Systemic Risk Fan Charts

Note that spikes in \( GDP\text{ES} \) correspond to every recession episode, but their magnitude differs across episodes, with the spike in 2008Q4-2009Q1 being the largest experienced since the 1980s. Interestingly, spikes in \( F\text{S}\text{ES} \) do not necessarily match spikes in \( GDP\text{ES} \), suggesting that the co-movements in the left tails of real activity and financial health are time-varying. Note that the difference between ES indicators at 5 percent and 20 percent probability track changes in the expected shortfalls associated with changes of the size (or fatness) of the left tail. Indeed, the size of the left tail increases at each spike, and the extent to which that occurs indicates that expected shortfalls differ markedly for real activity and financial stress. These observations apply as well to the systemic risk fan charts of the other six countries reported in Appendix Figure Set 1.

Figure 2 depicts one-step density forecasts of GDP growth and the FS indicator for 2010Q3, compared to those estimated in the quarter including Lehman’s collapse (2008Q3). This figure provides a rather striking illustration of the importance of changes in the tails due to systemic events. Although, as expected, the densities in the crisis period (2008Q3) are to the left of the post-crisis period (2010Q3), the most relevant differences of the two densities for both indicators are in the left tails.

Figure 2. United States: Density Forecasts
B. Forecasting Accuracy

We evaluate the accuracy of density forecasts both in-sample and out-of-sample applying standard tests proposed in the literature. By implication, this evaluation is a test of the forecasting performance of our systemic risk indicators.

Assessing the accuracy of density forecasts amounts to test whether the estimated density is "close" to the true unobserved density. As shown in Diebold, Gunther and Rey (1998), a series of estimated quantiles accurately captures the actual distribution of a variable $X_i$ if the series of Probability Integral Transforms (PIT) $z_i$, defined as the series of quantiles of the probability distribution that correspond to each observation in $X_i$, satisfies two properties: a) the series $z_i$ is identically and independently distributed (independence), and b) the series $z_i$ is distributed uniformly over the unit interval (uniformity).

To test these properties, we constructed PITs for both our real activity and FS indicators for each of the seven countries. To check independence, we tested whether autocorrelations of these series up to eight lags were significantly different from 0 for each of the seven countries. We found that for all countries and both indicators these autocorrelations are not significantly different from 0 at standard confidence levels, suggesting that our model generates PITs consistent with the independence property. To check uniformity, we followed Diebold, Gunther and Rey’s (1998) suggestion to compare graphically our density estimates to a uniform density on the unit interval, and compute confidence intervals under the null of i.i.d. uniform distribution, decile by decile. For all countries, uniformity is satisfied for most deciles, with few exceptions either for some right tails or middle deciles. Overall, these preliminary tests suggest that density estimates appear satisfactory in both dimensions, although there is room for improvements especially in the uniformity dimension.

We conducted formal tests assessing the accuracy of one-step density forecasts in-sample and out-of sample. This latter test is clearly the most important, as it ultimately gauges the usefulness of our model as a risk monitoring tool. Given the relatively low number of observations in our application, we resorted to non-parametric methods. Specifically, we used standard "goodness-of-fit" tests for categorical data based on the Pearson’s Q statistics. For small samples, the Pearson’s Q statistics is approximately distributed as a chi-square with $k-1$ degrees of freedom, where $k$ is the number of categories or partitions of the data.

**In-sample one-step forecasting accuracy**

To assess in-sample fit, we partitioned the unit interval in regions delimited by two specific quantile ranges, where we used (in-sample) quantile estimates. The first partition includes 4 regions delimited by the estimated quantiles: $[<Q5,Q5-Q10,Q10-Q20,>Q20]$. This partition

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5 For a survey of tests of accuracy of density forecasts, see Corradi and Swanson, 2006).

6 For details on these tests, see for example De Groot and Schervish (2002). For a review of applications in financial risk management, see Campbell (2005).
is designed to test whether the fraction of actual realizations of GDPG and FS are close to the left-tail of the actual (unobserved) distribution. A perfect matching of the estimated and the actual distribution would result in 5 percent of observations falling in the first region (<Q5), 5 percent in the second region (Q5-Q10), 10 percent in the third region (Q10-Q20), and 20 percent in the fourth region. In this case, a Q statistics not greater than the .95 percentile of the chi-square distribution with 3 degrees of freedom (equal to 7.815) would lead to not reject (or to accept) the null that the fit is good. The second partition includes 6 regions delimited by the estimated quantiles: [<Q10,Q10-Q25,Q25-Q50,Q50-Q75,Q75-Q90,>Q90]. This partition is designed to test whether the fraction of actual realizations of GDPG and FS are close to the entire actual (unobserved) distribution. A perfect matching of the estimated and the actual distribution would result in 10 percent of observations falling in the first region (<Q10) and the last region (>Q90), 15 percent in regions Q10-Q25 and Q75-Q90, and 25 percent in regions Q25-Q50 and Q50-Q75. In this case, a Q statistics not greater than the .95 percentile of the chi-square distribution with 5 degrees of freedom (equal to 11.07) would lead to accept the null that the fit is good. As shown in Table 1, the tests of in-sample goodness of fit show that for all countries, for both real and financial indicators, and for both tests for the tail and the entire distribution, the model delivers (in-sample) density estimates with a good fit.

**Out-of-sample one-step forecasting accuracy**

To assess out-of-sample fit, we focused on out-of-sample goodness of fit on the left-tail, partitioning the unit interval into two regions: [<Q20,>Q20]. We considered four forecasting horizons, from one quarter to four quarters ahead. Recursive forecasts of quantiles were computed in "simulated" real-time, starting in 1999Q1. In each forecasting quarter, up to quarter 2008Q4, we re-estimated the entire model using only observations up to that quarter, but kept the selection of the number of factors and lags fixed. In this case, a Q statistics not greater than the .95 percentile of the chi-square distribution with one degree of freedom (equal to 3.84) would lead to accept the null that the prediction is accurate. As shown in Table 2, out-of-sample predictions are generally satisfactory. For two countries, the U.S. and the U.K., predictions are good for both variables and all forecasting horizons. For GDPG, there is only one rejection at some horizon for Canada, Japan and France, and two for Italy. By contrast, there is only one rejection for FS (Germany). Overall, these results support the potential ability of our model to forecast future developments in systemic real and financial risks. Yet, is this forecasting accuracy useful to issue reliable early warnings? We tackle this issue next.


Here we report perhaps the most demanding evaluation of the model’s ability to serve as a risk monitoring tool: we assess if the model signals increased systemic risks prior to historical declines in real activity and increased financial stress during 2008.

Figure 3 reports forecasts of systemic real risk (GDPES5) and financial risk (FSES5) at five percent probability using data up to 2007Q4, for k=1,2, 3 and 4 (i.e. 2008Q1, 2008Q2, 2008Q3 and 2008Q4). As of 2007Q4, the forecasts for the US and Canada indicated a sharp
predicted increase of both systemic real and financial risks in 2008. Forecasts of systemic real risk indicators for the other countries indicated values close to those recorded at the forecasting date, while systemic financial risk were forecast higher in Japan and France. Overall, these forecasts indicate an ability of the model to deliver warning signals about impending risks, since these risks later materialized.

**Figure 3. Out-of Sample Multi-Step Forecasts**
(Forecasting quarter: 2007Q4)

A further indication of the ability of the model to generate informative early warnings signals is illustrated as follows. After a surge experienced in March 2008, during the entire second quarter of 2008 most financial risk indicators in advanced economies (such as CDS spreads) returned to levels witnessed on the onset of the crisis in the summer of 2007 (see BIS, 2008, pp.1-2). On the real side, global growth was projected to slow down moderately (see IMF World Economic Outlook, 2008). In sum, as of the end of the second quarter of 2008, the substantial ease in risk indicators in the financial sector suggested a decline in systemic financial risk, whereas growth prospects, although revised downward, were not generally judged as implying imminent systemic real risk.

However, a very different picture would have emerged from the forecasts of our model. Using only data available as of end of the second quarter of 2008, Figure 4 illustrates forecasts of GES5 and FSES5 for the U.S four quarters ahead. Systemic real risk is forecast to jump up considerably in 2008Q3 and remains elevated for all other subsequent quarters. Systemic financial risk forecasts essentially follow the same patterns, although the changes do not appear as dramatic as those for systemic real risk.
Thus, the model would have given strong early warnings of increasing systemic real and financial risks also in 2008Q2, when such warnings could not be inferred by looking simply at market developments and real data releases. As it is well known, what happened in 2008Q3-Q4 confirmed the prediction of heightened systemic real and financial risks. This is further evidence of the ability of our model to serve as a useful risk monitoring tool.

IV. IMPLEMENTATION: A STRESS TESTING EXAMPLE

A. Identification

The identification procedure outlined previously was implemented following three steps. First, we selected an orthogonal decomposition of the Factor VAR. Second, for each country, we computed impulse responses of GDP Growth, Inflation, Bank Lending Growth and first differences in Loan Rates using Equations (3). Lastly, we checked whether the joint signs of the responses of these variables conformed to the signs predicted for different shocks by the basic macroeconomic and banking models summarized in Table A.

As a benchmark orthogonalization, we chose a Choleski decomposition with factors ordered according to their explanatory power of the common variations in the data, with factor 1 ordered first, factor 2 second, and so on. The simple assumption underlying this choice is that the casual ordering implied by this decomposition reflects the relative importance of factors in explaining variations in the data, and each idiosyncratic component of the observable variables does not affect any of the factors at impact.  

We examined alternative decompositions with inverted ordering of the variables, obtaining similar signs of the responses of each of the observable variables to shock to orthogonalized innovations. We also examined the covariance matrix of innovations of the VAR of each country, and such matrices appeared approximately diagonal in all cases: this indicates that the ordering of variables in the VAR is not likely to change results under the casual ordering selected, and also suggests that our results are robust to other orthogonal decompositions—

(continued…)
Appendix—Figure Set 2 reports impulse responses of GDP growth, Inflation, Bank Lending Growth and changes in Lending Rates for each of the G-7 countries. Strikingly, the response of all variables to all shocks at impact or for at least up to two quarters after impact is either strictly positive (in most cases) or non negative (in few cases). Hence, according to Table A, all orthogonalized shocks in these economies can be identified as combinations of aggregate demand shocks and bank credit demand shocks.

The finding of aggregate demand shock as the predominant drivers of real cycles in the G-7 economies is consistent with the findings by Canova and De Nicolò (2003), who used a small dimension VAR for the G-7 countries, but implemented a full search for shocks interpretable according to aggregate macroeconomic theory in the entire space of non-recursive orthogonalizations of the VAR of each country. Our results are also consistent with recent work by Arouba and Diebold (2010), who find shocks interpretable as demand shocks as the dominant source of aggregate fluctuations in the U.S. The finding that aggregate bank credit demand shocks are the predominant drivers of cycles in bank credit growth supports the conjecture that slowdowns in aggregate bank credit growth are primarily, although not exclusively, driven by downturns in real activity. Recent U.S. evidence by Berrospide and Edge (2010) is also consistent with our results.

Notably, the five identified aggregate demand and bank credit demand shocks are not all the same, as they have a differential impact on GDP growth, inflation, bank lending growth and changes in loan rates within as well as between countries. This suggests that the sectors of the economy where they originate are different. Indeed, as shown in Table 2, the variance decompositions of the four variables VAR in each country indicate that the variance explained by each shock varies across both variables and countries, with most shocks resulting relevant in each country.

**B. A Simple Stress Test**

Changes across time in the impulse response function and variance decompositions of our systemic risk indicators can give a measure of changes in the resilience of the real and financial sides of the economy, and the interdependences between systemic real and financial risks.

To preview, Figure 5 reports of the impulse responses of GDPES and FSES measures to negative identified shocks. As it may be expected, all these shocks have a negative impact on the measures of systemic real and financial risk, generating positive co-movements between the systemic risk indicators. The behavior of the impulse responses of GDPES and FSES is qualitatively very similar in each country, although magnitude and persistence of these

---

8 The only exception is the shock associated with the third factor for Canada, whose responses do not satisfy any of the sign restrictions in Table A, and thus results unidentified.

9 Note that the third shock implies a negative response of FSES at impact, but its response jumps up to positive territory immediately after from the first quarter on.
shocks differ. As shown in Table 5, the relevant variance decompositions indicate that these are significant in magnitude for each shock, and there are significant cross country variations.

**Figure 5. Impulse Responses of GDPES5 and FSES5 for the United States**

Our stress-testing procedure aims at gauging whether our stress tests signal lower resilience to structural shocks in the G-7 economies *prior* to the 2007Q3, which is the quarter during which the 2007-2008 crisis began.

Figure 6 shows the *difference* of the cumulative impact of the impulse response functions up to eight quarters of GDPES and FSES to one-standard deviation shock estimated for the whole sample period before the crisis (1980Q1-2007Q2), and since the mid 1990s (1993Q2-2007Q2). Estimation of the factors were accomplished separately for the two samples, since the standard deviation of all five shocks in the second estimation period is not larger than that for the whole period, a positive difference indicates a larger cumulative adverse impact in the last period compared to the whole sample.
Figure 6 contains two results that could have been useful for policymakers prior to the financial crisis. First, the sensitivity of GDPES to most shocks has decreased in the last decade relative to the whole sample period in all countries except in the U.S., and to some extent Japan and the U.K. This is particularly remarkable, given the reduction in real growth volatility during the “Great Moderation” period in the U.S. Second, the sensitivity of FSES to most shocks in the last decade decreased in most cases except again the U.S., and in part in the U.K. Despite the decline in financial markets volatility during this period, the stress test signals increased systemic financial risk primarily in the U.S. In sum, the U.S. economy is the main country that exhibits a positive difference in the cumulative impact of impulse responses for both GDPES and FSES indicators: contrary to common perceptions, the U.S. economy had increased its vulnerability to shocks both on the real and financial sides—in absolute terms as well as relative to the other G-7 economies—in the years preceding the 2007-2009 financial crisis.

V. CONCLUSION

This paper has presented a modeling framework that can be used as a tool for positive analysis as well as a systemic risk monitoring system. Our empirical implementation of the model using G-7 country data shows that it delivers useful early warning signals about developments in systemic real and financial risk owing to its significant out-of-sample forecasting ability, and allows us to conduct informative stress tests.
One important advantage of our modeling framework is its flexibility: the model can embed in a unified framework different and/or multiple measures of systemic risk, different identification procedures, and different stress testing designs. Another important advantage of our model is its amenability to be further developed in important directions while keeping its basic structure unchanged. Two such developments are already part of our research agenda. The first is an extension of our framework to the simultaneous modeling of countries and regions of the world. This would allow us to expand the set of positive questions that the model can address, and provide risk monitoring tools of systemic risk interdependencies across countries. The second development would include the use of more disaggregated data, together with a richer set of theoretical constructs as identification tools in order to design more detailed stress tests.
Table 1. In-Sample Goodness-of-Fit

Each column reports the fraction of observations falling in the region delimited by each estimated quantile. Significance of the Q-statistics at a 5 percent confidence level is reported in boldface.

<table>
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<tr>
<th>GDPG</th>
<th>&lt;Q5</th>
<th>Q5-Q10</th>
<th>Q10-Q20</th>
<th>&gt;Q20</th>
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Table 2. Out-of-Sample Goodness of Fit

Each column reports the Q statistics corresponding to the forecast horizon k (in quarters). Significance of the Q-statistics at a 5 percent confidence level is reported in **boldface**.

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Table 3. Variance Decomposition of GDP Growth, Inflation, Bank Lending Growth and Changes in Loan Rates

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<th>Bank Credit Growth</th>
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<td>0.14 0.17 0.33 0.03 0.74 0.26</td>
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N.B.: Boldfaced values denote estimates significantly different from 0 at 5 percent confidence levels.
Table 4. Variance Decomposition of GDPES and FSES to Identified Shocks

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<td>0.01</td>
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N.B.: Boldfaced values denote estimates significantly different from 0 at 5 percent confidence levels.
Appendix

A Banking Model

The banking model by Boyd, De Nicolo’ and Loukoianova (2010) used for the identification of demand and supply shocks to bank credit can be briefly summarized as follows. The economy is composed of a —government‖ and three classes of risk-neutral agents: entrepreneurs, depositors, and banks. The banks in the model raise insured deposits, make risky loans, and hold risk free government bonds. The deposit insurer bails out the banks when they fail.

Entrepreneurs have no initial resources but have access to identical risky projects that require a fixed amount of date $t$ investment, standardized to 1, and yield a random output at date $t+1$. At date $t$ the investment in a project yields $Y$ with probability $P_{t+1} \in (0,1)$, and 0 otherwise. Since the probability of success $P_{t+1}$ is a random variable independent across entrepreneurs, and its realization is observed by them at date $t+1$, entrepreneurs make their date $t$ decisions on the basis of their conditional expectations of $P_{t+1}$, denoted by $E_tP_{t+1}$.

Making assumptions about heterogeneity of entrepreneurs with respect to their opportunity costs, and denoting with $R^L$ the loan interest rate, BDNL derive the inverse loan demand function:

$$R^L(X_t, E_tP_{t+1}) = Y - (E_tP_{t+1})^{-1}X_t \quad (A1)$$

Depositors invest all their funds in a bank at date $t$ to receive interest plus principal at date $t+1$. Deposits are fully insured, so that the total supply of deposits does not depend on risk, and is represented by the upward sloping inverse supply curve

$$R^D(Z_t) = \alpha Z_t \quad (A2)$$

where $Z_t$ denotes total deposits. The slope of this function is a random variable whose realization is observed at date $t$.

Banks collect insured deposits, and pay a flat rate insurance premium standardized to zero. On the asset side, banks choose the total amount of lending and the amount of bonds. One-period bonds are supplied by the government in amounts specified below. For simplicity, we assume that only banks can invest in bonds. A bond purchased at date $t$ yields a gross interest rate $r_t$ at date $t+1$. In both loan and deposit markets banks are symmetric Cournot-Nash competitors. Banks are perfectly diversified in the sense that for any positive measure of entrepreneurs financed, $P_{t+1} \in (0,1)$, is also the fraction of borrowers whose project turns out to be successful at date $t+1$. Banks observe the realization of $P_{t+1}$ at date $t+1$. Hence, as for the entrepreneurs, banks make their date $t$ decisions on the basis of their conditional expectations $E_tP_{t+1}$.

The government supplies a fixed amount of bonds to the market, denoted by $B$. The government also guarantees deposits. Whenever bank deposits payments cannot be honored in part or in full, the government will pay depositors all the claims unsatisfied by banks, with the payments being financed by issuing additional bonds.
Denoting \( p \equiv E, P, \), an equilibrium is a total amount of loans, bonds and deposits \( Z \), bond interest rates, loan rates, deposit rates, and government interventions such that: a) the banking industry is in a symmetric Nash equilibrium; b) the bond market is in equilibrium; and c) the government meets its commitment to deposit insurance. The following table summarizes changes in the equilibrium total lending and the loan rate in response to an adverse credit demand shock or an adverse credit supply shock, which is used in Table A.

<table>
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<th>Adverse Shocks</th>
<th>Bank Credit Growth</th>
<th>Change in Lending Rates</th>
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<tr>
<td><strong>Credit Demand</strong></td>
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<td></td>
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<tr>
<td>( p ) or ( Y ) decrease</td>
<td>( - )</td>
<td>( - )</td>
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<tr>
<td><strong>Credit Supply</strong></td>
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<tr>
<td>( \alpha ) increases</td>
<td>( - )</td>
<td>( + )</td>
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Appendix Figure Set 1. Systemic Risk Fan Charts

Canada: Systemic Risk Fan Charts
forecast date: 2010Q1

Systemic Real Risk
gesX = GDP Expected Shortfall with probability X

[Graph showing time series of systemic real risk for Canada]

Systemic Financial Risk
fesX = FS Expected Shortfall with probability X

[Graph showing time series of systemic financial risk for Canada]

Japan: Systemic Risk Fan Charts
forecast date: 2010Q1

Systemic Real Risk
gesX = GDP Expected Shortfall with probability X

[Graph showing time series of systemic real risk for Japan]

Systemic Financial Risk
fesX = FS Expected Shortfall with probability X

[Graph showing time series of systemic financial risk for Japan]
United Kingdom: Systemic Risk Fan Charts
forecast date: 2010Q1

Systemic Real Risk
\[ \text{ges}_X = \text{GDP Expected Shortfall with probability } X \]

Systemic Financial Risk
\[ \text{fes}_X = \text{FS Expected Shortfall with probability } X \]
Appendix Figure Set 2. Impulse Responses of GDP Growth, Inflation, Bank Lending Growth and Change in Lending Rates to Shocks to Factors and Own Shock

United States

GDP Growth

Inflation

Bank Lending Growth

Loan Rate

Appendix Figure Set 2. Impulse Responses of GDP Growth, Inflation, Bank Lending Growth and Change in Lending Rates to Shocks to Factors and Own Shock

United States

GDP Growth

Inflation

Bank Lending Growth

Δ Loan Rate

95% CI

orthogonalized irf

Graphs by irfname, impulse variable, and response variable
Figure Set 2 (cont…)

Impulse Responses of GDP Growth, Inflation, Bank Lending Growth and Change in Lending Rate to Shocks to Factors and Own Shock

Canada

GDP Growth

Inflation

Bank Lending Growth

Δ Loan Rate

Graphs by irfname, impulse variable, and response variable

Graphs by irfname, impulse variable, and response variable

Graphs by irfname, impulse variable, and response variable

Graphs by irfname, impulse variable, and response variable
Impulse Responses of GDP Growth, Inflation, Bank Lending Growth and Change in Lending Rate to Shocks to Factors and Own Shock

Japan

Figure Set 2 (cont.)

GDP Growth

Inflation

Bank Lending Growth

Δ Loan Rate
Figure Set 2 (cont…)

Impulse Responses of GDP Growth, Inflation, Bank Lending Growth and Change in Lending Rate to Shocks to Factors and Own Shock

United Kingdom

**GDP Growth**

![Graphs showing impulse responses of GDP growth for different models.](image1)

**Inflation**

![Graphs showing impulse responses of inflation for different models.](image2)

**Bank Lending Growth**

![Graphs showing impulse responses of bank lending growth for different models.](image3)

**Δ Loan Rate**

![Graphs showing impulse responses of change in loan rate for different models.](image4)
Figure Set 2 (cont…)

Impulse Responses of GDP Growth, Inflation, Bank Lending Growth and Change in Lending Rate to Shocks to Factors and Own Shock

France

GDP Growth

Inflation

Bank Lending Growth

Δ Loan Rate

Graphs by irfname, impulse variable, and response variable

Graphs by irfname, impulse variable, and response variable

Graphs by irfname, impulse variable, and response variable

Graphs by irfname, impulse variable, and response variable
Figure Set 2 (cont...)

Impulse Responses of GDP Growth, Inflation, Bank Lending Growth
and Change in Lending Rate to Shocks to Factors and Own Shock

Germany

**GDP Growth**

**Inflation**

**Bank Lending Growth**

**Δ Loan Rate**
Figure Set 2 (cont...)

Impulse Responses of GDP Growth, Inflation, Bank Lending Growth and Change in Lending Rate to Shocks to Factors and Own Shock

Italy

GDP Growth

Inflation

Bank Lending Growth

Δ Loan Rate

Graphs by irfname, impulse variable, and response variable

95% CI orthogonalized irf
List of Variables

All variables below are extracted for each country in the G-7 group during the 1980.Q1-2010Q1 period. The frequency of all series is quarterly. Data transformations are implemented to make all series stationary. Series transformations are indicated as follows: $\Delta \ln = \log$ level difference; $\Delta \text{levels} = \text{level difference}$.

**Equity Markets**

*Equity indices, Price Earnings ratios and Dividend yields:*

- Market $\Delta \ln$
- Oil & gas $\Delta \ln$
- Chemicals $\Delta \ln$
- Basic resources $\Delta \ln$
- Construction & Materials $\Delta \ln$
- Industrial goods and services $\Delta \ln$
- Auto and Parts $\Delta \ln$
- Food and Beverages $\Delta \ln$
- Personal and Household goods $\Delta \ln$
- Health Care $\Delta \ln$
- Retail $\Delta \ln$
- Media $\Delta \ln$
- Travel and leisure $\Delta \ln$
- Telecom $\Delta \ln$
- Utilities $\Delta \ln$
- Banks $\Delta \ln$
- Insurance $\Delta \ln$
- Financial services $\Delta \ln$
- Technology $\Delta \ln$

**Credit Conditions**

- Policy rate $\Delta \text{levels}$

*Treasury bonds:*

- 2 YR $\Delta \text{levels}$
- 3 YR $\Delta \text{levels}$
- 5 YR $\Delta \text{levels}$
- 7 YR $\Delta \text{levels}$
- 10 YR $\Delta \text{levels}$
- 30 YR $\Delta \text{levels}$
- Money base $\Delta \ln$
- Money supply M1 $\Delta \ln$
- Interbank rate $\Delta \text{levels}$
- Prime rate charged by banks (month AVG) $\Delta \text{levels}$
- Bank Lending $\Delta \ln$
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<td>Unemployment rate</td>
<td>Δlevels</td>
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<td>Industrial production-total index</td>
<td>Δln</td>
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<tr>
<td>CPI all items</td>
<td>Δln</td>
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<tr>
<td>Capacity utilization</td>
<td>Δlevels</td>
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<tr>
<td>Housing market index</td>
<td>Δlevels</td>
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REFERENCES


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LeBaron, Blake, 2009, —Robust Properties of Stock Returns Tails,” Brandeis University, Working Paper..


