Financial Linkages across Korean Banks

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Abstract

This paper assesses the interconnectedness across Korean banks using three alternative methodologies. Two methodologies utilize high frequency financial data while the third uses bank balance sheet data to assess banks’ bilateral exposures, systemically vulnerable banks, and systemically risky banks. The analysis concludes that while Korean banks are interconnected, both the financial risk and contagion risk from such interconnectedness have declined significantly in the aftermath of the global financial crisis.

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I. INTRODUCTION

The global financial crisis has demonstrated once again how the interconnections between banks can become a powerful channel for the transmission of shocks across financial institutions and to the real economy. The systemic consequences of such interconnectedness underscore the importance of understanding the exposures of financial institutions to each other as a key step in maintaining the overall stability of the financial system and addressing emerging vulnerabilities preemptively.

In this paper, we apply three methodologies to assess the financial linkages across Korean banks and to quantify the risks that this interconnectedness may pose on other banks and on the system as a whole. The network approach utilizes bank balance sheet data, and the co-risk and distress dependence analyses rely on analyzing high frequency data to exploit the information embedded in CDS spreads and stock prices. The findings indicate that Korean banks are interconnected. However, there appears to be no bank that by itself poses systemic threats to the rest of the system, and no bank is highly vulnerable to distress in another bank. Moreover, the analysis suggests both the financial risks that banks pose and the contagion risk that they create have declined significantly in the aftermath of the global financial crisis.

- The network approach can trace financial exposures that banks can create on each other by utilizing bank balance sheet data. In particular, by breaking down the assets of a bank, it can extract the financial exposure of a bank to another bank in the system, and measure financial institutions’ resilience to a domino effect triggered by financial distress in one institution. This approach focuses on two types of risks: (i) credit risk, which is created by the credit losses that the default of one bank generates in other banks; and (ii) the funding loss since banks in the system need to find funding sources other than the defaulting bank. Results from the network approach suggest that even the default of a bank, which would create the largest credit-plus-funding loss for another bank in the system, would not create a significant risk for any of the domestic banks that we analyze. On average, the largest loss that a bank would have suffered, should another bank that it is most exposed were to default, peaked at about 16 percent of its capital by the end of 2008, and declined to less than 11 percent by the third quarter of 2009. Further, writing off the largest credit-plus-funding shock from bank’s capital would still allow Korean banks to maintain BIS capital adequacy ratios (Basel II basis) above the 8 percent regulatory minimum. One caveat to this conclusion is that this analysis only tested for a knock-off effect of a particular bank in the system, and results may differ for combined bank failures.

- The co-risk model estimates the rise in the default probability of a bank based on the default probability of another bank in the system. In this methodology, we estimate a 95th quantile regression model by utilizing credit default swap (CDS) spreads of Korean banks and some other common global and domestic risk measures, such as the TED-spread. We then calculate the conditional credit risk of Korean banks based on the coefficients

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estimated from this model. The findings of this methodology indicate that average conditional co-risk of Korean banks moves in line with risk aversion in international financial markets. The outbreaks of major international financial events were the main factors moving the conditional default probability of Korean banks. Domestic events had smaller effects on the conditional default probabilities, indicating that Korean banks are highly integrated with global financial markets. Currently, the average co-risk of Korean banks remains slightly above pre-crisis levels. The results also show that banks with lower financial stability indicators, such as the ones with higher CDS spreads, are the ones most vulnerable to systemic risk.

Distress dependence is the last approach that we utilize in this paper to assess the financial risks among Korean banks, as depicted earlier by Segoviano and Goodhart (2009). In this methodology, we utilize stock market prices and estimated default probabilities of Korean banks in order to quantify the level of distress that a bank, or a group of banks, can pose on another bank in the system or on the whole system. The results from this analysis indicate that financial inter-linkages are quite strong in Korea; however, the systemic risk that a bank can pose to the whole system is limited. Similar to the results found in the co-risk approach, distress dependence of banks follows major global financial events, and the level of this dependence in the system has declined significantly in the aftermath of the global financial crisis.

There are several advantages of using these methodologies over standard financial stability indicators (FSI). First, these methodologies allow us to quantify the risks imposed by financial interconnectedness. The FSIs, on the other hand, can only provide bank-by-bank risk in the system, but not the risks posed by financial linkages across banks. And these inter-linkages are particularly important considering that financial losses can increase nonlinearly depending on the concentration/diversification of bank portfolios. Unexpected losses by a bank can trigger knock-on effects on the whole system, as experienced during the global financial crisis.

Second, two of the three methodologies that we use in this paper, i.e., co-risk and distress dependence, use market-based indicators and are forward-looking. In other words, they reflect investors’ assessment of the financial health of a specific bank, and the domestic and global developments that would affect its prospects. Traditional FSIs, on the other hand, are based on the book value of a bank, and hence are static and backward-looking. Also, given that market-based indicators are available on a much higher frequency, such as the daily frequency that we use for the co-risk and distress dependence analyses, they can provide a risk assessment for a bank, or for the whole banking system, on a virtually real-time basis. On the other hand, FSIs are based on bank balance sheets, and are available on a monthly basis, and in addition there is also a lag of time between the actual observed data and the time

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4 The average conditional co-risk rose from the pre-crisis average of 5 percent to around 10 to 15 percent during 2007–08 and peaked close to 25 percent immediately after the failure of Lehman Brothers, before dropping subsequently.

5 However, one should also mention that investors’ assessment of financial risk may not necessarily imply the true riskiness of a bank. In some instances prices may deviate from the underlying fundamentals, such as due to liquidity constraints, and perceived distress may rise with some local/global events rather than the institution itself becoming more risky.
that it is reported. Given that defaults happen during times of elevated volatility, it is important to be able to monitor changes in banking distress on a high frequency basis.

This paper has the following structure. The following three sections respectively provide the analyses and findings of network, co-risk, and distress dependence approaches; and the last section concludes.

**II. NETWORK APPROACH**

The network approach is a useful methodology to assess the financial institutions’ resilience to an adverse credit shock or a liquidity squeeze. It can also track domino effects throughout the system triggered by financial distress in a particular institution. This approach quantifies direct linkages between financial institutions by measuring losses for an individual financial institution induced by the default of another one, based on bilateral exposures between financial institutions. It also detects channels through which credit and funding risks spread to the whole financial system. While previous banking system stress testing studies have relied mainly on using the BIS’ cross-country bilateral exposures data, this analysis utilizes interbank exposure data, which is a preferable way to assess the actual risk arising from the linkages between financial institutions or vulnerabilities of financial institutions to financial distress. With this analysis, we also aim to identify systemically important financial institutions which are large enough to lead to defaults in other banks.

**A. Data**

We estimate the level of distress in the Korean banking system by using bilateral interbank exposure data for 18 Korean banks which comprise the bulk of the banking system. The analysis is applied for end-2008, end-2009 and September 2010. These dates are chosen to trace the effects of the global financial crisis on the Korean banking sector over time. We construct an inter-bank exposure matrix based on the data on the bilateral exposures of Korean banks. Banks’ assets (including assets denominated in foreign currencies) cover bank bonds receivable, loans, call loans, bonds purchased under repurchase agreements, accounts receivable, and derivative assets. Liabilities (including liabilities denominated in foreign currencies) are deposits, CD, bank bonds payable, borrowings, call money, bonds sold under repurchase agreements, accounts payable, and derivative liabilities.

**B. Methodology**

We simulate the losses for an individual bank in the network due to financial shocks induced by the default of another bank. Financial shocks are classified into credit shocks and credit-plus-funding shocks. Table 1 provides in detail the basic concepts on credit and funding

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6 For instance, if the default of Bank A leads to the default of counterparty Bank B which leads to default by Bank C.

7 Any other exposure of banks to each other that is not recorded in the bilateral exposure data, such those that are off-balance sheet, cannot be captured by this methodology.

8 The dataset covers Busan, Citibank Korea, Daegu, Hana, Industrial Bank of Korea, Jeju, Jeonbuk, KDB, Kookmin, Korea Exchange, Korea Exim, Kwangju, Kyongnam, NongHyup, SC First, Shinhan, Suhyup, and Woori.
shocks. Finally we detect the knock-on effects by comparing the simulated losses of an individual bank and its ability to absorb these losses.

### Table 1. Financial Shock Scenarios

<table>
<thead>
<tr>
<th>Basic Concept</th>
<th>Credit shock</th>
<th>Credit-plus-funding shock</th>
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<tbody>
<tr>
<td><strong>Basic Concept</strong></td>
<td>In the case of default of a particular bank, other banks suffer credit losses due to the bad assets of the defaulting bank.</td>
<td>In the case of default of a particular bank, other banks suffer a liquidity squeeze, as well as a credit loss. A liquidity squeeze indicates that banks need to find a new funding source for the financing that they used to obtain from the defaulting bank. In the process of replacing the funding from the defaulting bank, they suffer additional losses due to fire sales of assets and etc.</td>
</tr>
<tr>
<td><strong>Loss Formula</strong></td>
<td>Direct Credit Loss: [ Exposure\ at\ Default\ (EAD) = \text{related to a defaulting bank} ]</td>
<td>Funding Loss: Funding previously granted by the defaulting bank [ (1 - \text{rollover ratio}) \times \text{haircut} ]</td>
</tr>
</tbody>
</table>
|                       | Loss given Default (LGD): 100%. We assume that a bank needs to write off all of its exposure due from the defaulting bank | \[ Rollover rate: \]
|                       | **Korean Won:** 65% (Same April 2009 GFSR)                                                        | **Foreign Currency:** 30% (Average rollover ratio during 97 Asian crisis)                  |
| Default Criteria       | When a bank’s loss is greater than its total capital, the bank is judged to be in default.        | (same as in the left column)                                                                |
C. Results

Credit shock simulation

This simulation focuses on the transmission of a pure credit shock assuming that all institutions are able to roll over their funding needs. It concludes that the losses of other banks from the default of any one bank in the system are likely to be easily absorbed with their own capital. Figure 1 shows the ratio of the largest loss of a bank relative to its capital due to credit shock induced by the default of another bank in the Korean banking system, which is defined as following:

\[
\frac{\text{Max}(\text{loss of a bank } i \text{ due to default of bank } j)}{\text{Bank’s total capital}}
\]

where bank j is one of the other banks in the Korean banking system.

In Figure 1, there is no bank whose loss from the default of another individual bank is greater than its total capital. For instance, the ratio of the largest credit-shock-loss-to-capital, for a Korean bank, lies between 4 percent and 33 percent, and the mean of these ratios has declined from the end of 2008 through September 2010, although the minimum-maximum range has widened at the end of September 2010.

Figure 2 provides the BIS capital adequacy ratios (CARs), based on Basel II, after reflecting the largest loss of a bank due from the default of another bank in the system. From 2008 to 2010, the average CAR of Korean banks has exceeded 12 percent. Moreover, there is no bank in the system whose CAR falls below the 8 percent regulatory minimum BIS capital ratio after the shock. Therefore, the default of a single bank does not appear to be a significant source of serious distress in the Korean banking system looking at the credit shock channel.

Credit-plus-funding shock simulation

In this section we estimate the joint impact of credit losses and liquidity shocks on the Korean banking system. For funding losses, we assume a 50 percent haircut due to the fire sale of assets, a 65 percent rollover ratio for interbank debt nominated in Korean won and a 30 percent rollover ratio for interbank debt nominated in foreign currencies.

Figure 3 presents the ratios of the largest credit-plus-funding losses for Korean banks, due to the default of another bank in the system, as a ratio of their capital. These results show that even the largest credit-plus-funding loss is unlikely to cause any of the Korean banks in the system to default. Figure 3 also shows that even the largest loss for any Korean bank is no more than 40 percent of the bank’s total capital. Specifically, credit-plus-funding shock losses of the banks in the system range from 5 percent to 36 percent of their total capital. The average ratio of credit-plus-funding shock losses of the banks in the system has declined to less than 11 percent from about 16 percent since the end of 2008.

Figure 4 provides the BIS capital adequacy ratios of Korean banks after deducting the largest
Figure 1. Credit Shock on Bank Capital

*Ratio of the largest loss of a bank relative to its capital due to a credit shock induced by the default of another bank in the system*

Note: Inter-quartile range is the range between the first and the third quartile.

Figure 2. BIS Capital Ratio after a Credit Shock

*BIS capital ratio after deducting the largest loss of a bank from its capital due to a credit shock induced by the default of another bank in the system*
Figure 3. Credit-Plus-Funding Shock on Bank Capital

*Ratio of the largest loss of a bank relative to its capital due to a credit-plus-funding shock induced by the default of another bank in the system*

Source: BOK Staff Calculation

Figure 4. BIS Capital Ratio after a Credit-Plus-Funding Shock

*BIS capital ratio after deducting the largest loss of a bank from its capital due to a credit-plus-funding shock induced by the default of another bank in the system*

Source: BOK staff calculation
credit-plus-funding shock from their capital. The results show that Korean banks are well-capitalized, and the capital adequacy ratios of all the banks in the system remain higher than the 8 percent required minimum.

D. Summary

The results of the network approach show that Korean banks are well-capitalized: they can absorb losses due to the default of another bank in the system. Simulations demonstrate that there is no bank in the system whose loss absorption capacity would be seriously damaged due to the default of another bank in the system, to the extent that all of its capital would be exhausted. In other words, there would be no bank in the system that would be forced to default due to the default of another bank.

The network approach finds no systemically important financial institution, which may create distress on other banks in the system, or no systemically vulnerable bank, which is likely to default due to financial troubles in another bank. All Korean banks exceeded the 8 percent minimum BIS capital ratio, even after simulating the largest amount of losses due to the hypothetical default of any other bank in the system.

These results are underpinned by several factors:

- First, in the Korean banking system, direct interbank exposures are not excessively large relative to banks’ capital. Instead, households and the corporate sector tend to be the major funding source of Korean banks, while nondeposit liabilities are diversified across a range of funding sources. This has been further reinforced since December 2009 when Korea’s Financial Supervisory Service announced the adoption of the loan-to-deposit (LTD) regulation to limit wholesale funding.

- Second, Korean banks are well capitalized and thus have strong loss-absorption capacity. In fact, the median CAR of Korean banks has exceeded 12 percent during the period under study, which covers extreme tail events such as the global financial crisis and the market uncertainty related to the fiscal and financial troubles in the euro area. Further, the average CAR of Korean banks increased from 12.7 percent as of end-2008 to 14.6 percent as of end-2009.

III. Co-Risk Approach

The co-risk approach measures systemic risk by considering spillover effects. It is a variation of the Value-at-Risk (VaR), and has various advantages over the traditional VaR. The main

9 However, this finding should not be generalized to the case where more than one bank is in distress at the same time.

10 LTD regulation is imposed on the ratio of won-denominated loans to deposits, and banks are required to maintain their LTD ratio at or below 100 percent. As of end-September 2010, the average LTD of 15 Korean banks that are subjected to this regulation stood at 96.5 percent, down from 115.9 percent in December 2009.

11 This is a very popular methodology used for measuring financial risk by estimating the probability of portfolio losses based on historical price trends and volatilities. We follow the approach of Adrian and Brunnermeier (2009).
one (see Brunnermeier et al. (2009)), is that it provides one of the best approaches to assess
the implications of endogenous risk since it captures the rise in the overall risk for a financial
institute conditioned on the fact that another one is in distress. Also, the co-risk approach
provides useful information to monitor the systemic risk induced by the default of a bank in
the system.

However, this approach does have some limitations. As pointed out by Lee (2010), the co-
risk measured by quantile regression estimations may be biased due to tail dependence. In the
corisk approach, we estimate a reduced form model, but this may not reflect all the critical
information relevant for an individual bank’s default, given that the default risk is fatter at the
tail of a distribution. Further, the co-risk model measures systemic risk from the distribution
of asset returns for the financial institution of interest; this does not necessarily reflect the
true financial status of that institution, but only the risk priced in by financial markets.

A. Data

We utilize daily financial data from January 1, 2006 through March 31, 2011 to calculate the
corisk of 11 Korean Banks: seven commercial banks (Citibank Korea, Hana, Kookmin,
Korea Exchange, SC First, Shinhan, and Woori) and four specialty banks (Industrial Bank of
Korea, Korea Development, Korea EXIM and Nonghyup). The main data of interest are the
credit default swap (CDS) spreads of these 11 banks. In order to control for common global
and domestic risk factors, we utilize additional variables, including:

- The TED spread, to measure risk aversion in international financial markets.13
- The spread between one-year KORIBOR and Korean Treasury bond yields, to control for
the credit risk in domestic inter-bank markets.
- The spread between three-year Korean and U.S. Treasury bond yields, to capture the
difference between the domestic and international interest rates.

B. Methodology

The co-risk model is estimated by measuring the increase in the default risk of a bank in the
event of the default of another bank in the system. This methodology helps us capture both
the direct and indirect linkages between two financial institutions.

The methodology is a variation of the VaR methodology, where the unconditional VaR of
bank \( i \), measured at \( \tau \)th critical level percentile, can be expressed as the following:

\[
\Pr(R_i \leq \text{VaR}_i^\tau) = \tau
\]  

Then, the co-VaR of bank \( i \) measured by \( \text{CoVaR}_{ij}^\tau \) under the condition of \( \text{VaR}_j^\tau \) can be
shown as

---


13 This is the spread between the dollar interbank rate and the corresponding U.S. Treasury bill rate.
Pr\left(R_i \leq \text{CoVaR}_{ij}^\tau \mid \text{VaR}_j^\tau\right) = \tau \quad (1.2)

There are various methodologies to measure the co-risk between two financial institutions. We follow Adrian and Brunnermeier (2009), measuring the Co-VaR by relying on quantile regression estimations. In order to estimate the co-risk between Korean banks, we first establish a 95th quantile regression model by using the daily CDS spreads of 11 Korean banks and the common global and domestic risk factors.

In the regression model, the CDS spread of a particular bank, for which the co-risk is estimated, is used as the dependent variable and credit risks, i.e. the CDS spreads, of the remaining 10 other banks and the common risk factors as the independent variables. We estimate this model by using the quantile regression below.

\[ \text{CDS}_i = \alpha + \sum_{k=1}^K \beta_{i,k} R_k + \beta_{ij} \text{CDS}_j \quad (1.2) \]

where, \( \text{CDS}_i, \text{CDS}_j \) are the CDS spreads of bank \( i \) and bank \( j \) for \( (i \neq j) \)
\( R_k (k = 1, \cdots, K) \) are the common risk factors
\( \alpha, \beta_{i,k}, \beta_{ij} (k = 1, \cdots, K) \) are \( \tau \)th quantile regression coefficients

The coefficients estimated from the quantile regression analysis represent the sensitivity of a particular bank to the credit risks due from other banks in the system, which are set as independent variables at the 95th quantile level. In other words, this regression analysis allows us to estimate the credit risk of a particular bank, conditional on the default of another one bank at the 95th quantile CDS spread.

Estimated coefficients from the 95th quantile regression model are compared with the 95th quantile CDS spreads of the same bank to produce the conditional co-risk, shown by the percentage increase or decrease of this coefficient relative to the latter.

We then construct a matrix, in which the cells are conditional co-risks between two banks, to assess both the bilateral direct and indirect linkages and also the systemic importance and vulnerability of a particular bank.

\[
\begin{bmatrix}
0 & \text{Co-risk}_{i2} & \cdots & \text{Co-risk}_{iN} \\
\text{Co-risk}_{2i} & 0 & \ddots & \vdots \\
\vdots & \ddots & \ddots & \text{Co-risk}_{Ni} \\
\text{Co-risk}_{N_i} & \cdots & \text{Co-risk}_{N_{iN-1}} & 0
\end{bmatrix} \quad (1.3)
\]

where, \( \text{Co-risk}_{ij} \) means co-risk of bank \( j \) conditional on the distress of bank \( i \).

Based on the risks estimated from this matrix, we will focus on three indicators to measure systemic linkages in the Korean banking system.
• **Systemic vulnerability** of a particular bank is the extent to which a bank is vulnerable due to the distress of other banks in the system. Systemic vulnerability of a particular bank is expressed as 
\[ \frac{\sum_j C\cdot Risk_{dj}}{N-1} \]

• **Systemic importance** of a particular bank shows how much distress in a particular bank exposes vulnerabilities to other banks in the system. Systemic importance of a particular bank is calculated as the average of cells in the column of conditional co-risk matrix, expressed as 
\[ \frac{\sum_j C\cdot Risk_{dj}}{N-1} \]

• **Systemic risk** is defined as the linkage of default risk among the banks in the system, which we calculate from the average of all elements in the co-risk matrix, 
\[ \frac{\sum_j \sum_i C\cdot Risk_{ij}}{M(N-1)} \]

A rise in any of the ratios above indicates higher levels of risk.

### C. Results

The estimated conditional co-risk measure of 11 Korean banks is presented in Table 2 and Table 3. Table 2 presents the conditional co-risk for end-September 2010, to capture the recent risk level in the system and Table 3 shows the co-risk measures during October 13, 2008, when the average co-risk of Korean banks reached their highest level following the collapse of Lehman Brothers a month before.

The conditional co-risk matrices in Table 2 and Table 3 are presented as percentage changes in the distress level of a bank stemming from the bankruptcy of another bank. A rise in these measures indicates an increase in the level of the risk.

In these matrices, each row features the percentage conditional credit risk (or a rise in the CDS spreads) of the bank listed in that row, induced by distress from a bank listed in the corresponding column, but only when the CDS spreads are at their 95th percentile. For instance, Table 2 shows that the credit risk of bank 1 (listed in the first row) conditional on the default risk of bank 2 (listed in the second column) is 3.2 percent higher than that corresponding to the 95th percentile of bank 1’s own CDS distribution, which is estimated from the quantile regression.

The estimated conditional co-risk measures indicate low levels of risk for Korean banks in recent times. Further, as presented in Table 3, the level of distress among Korean banks, even considering the period when systemic risk peaked across Korean banks during the global financial crisis, was much more contained than what was experienced in other parts of the world. The peak in the distress level across Korean banks was lower than the estimated conditional co-risk measures between large complex financial institutions (LCFIs) overseas.\(^{14}\)

---

\(^{14}\) According to the IMF’s calculation, the conditional co-risk measure between LCFIs was between -6 and 617 percent in March 2008.
Table 2. Conditional Co-risk Estimates, as of end of September 2010
(In percent)

<table>
<thead>
<tr>
<th></th>
<th>bank 1</th>
<th>bank 2</th>
<th>bank 3</th>
<th>bank 4</th>
<th>bank 5</th>
<th>bank 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>bank 1</td>
<td>—</td>
<td>3.2</td>
<td>6.0</td>
<td>4.9</td>
<td>-6.2</td>
<td>-2.6</td>
</tr>
<tr>
<td>bank 2</td>
<td>16.4</td>
<td>—</td>
<td>12.0</td>
<td>10.5</td>
<td>1.5</td>
<td>12.6</td>
</tr>
<tr>
<td>bank 3</td>
<td>2.6</td>
<td>1.1</td>
<td>—</td>
<td>2.0</td>
<td>-5.5</td>
<td>0.9</td>
</tr>
<tr>
<td>bank 4</td>
<td>4.2</td>
<td>0.4</td>
<td>1.6</td>
<td>—</td>
<td>-6.4</td>
<td>-0.7</td>
</tr>
<tr>
<td>bank 5</td>
<td>30.2</td>
<td>20.9</td>
<td>27.1</td>
<td>29.0</td>
<td>—</td>
<td>30.0</td>
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<td>bank 6</td>
<td>3.8</td>
<td>-0.2</td>
<td>6.9</td>
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Source: Bank of Korea (BOK) staff calculations.

Note: Each cell in the table reports the co-risk measure corresponding to the bank listed in the rows (e.g., bank 1) and conditional on the bank listed in the columns (e.g., bank 2). The co-risk measure of bank 1 conditional on bank 2 is calculated as the percentage difference between bank 1’s estimated credit default swap (CDS) spread and bank 1’s observed CDS spread at the 95th empirical percentile. The estimated CDS spread of bank 1 is obtained by using bank 2’s 95th empirical percentile CDS spread as an input in the 95th quantile regression of bank 1 on bank 2. For instance, the co-risk measure of 3.2 percent for bank 1 conditional on bank 2 implies that the CDS spread of bank 1, at its 95th percentile value, increases by 3.2 percent if the CDS spread of bank 2 is at its 95th percentile value. The larger the co-risk measure, the more vulnerable bank 1 is to bank 2 or the more important bank 2 is to bank 1.
Table 3. Conditional Co-risk Estimates, as of October 13, 2008
(In percent)

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Source: BOK staff calculations.
The results from the co-risk analysis indicate that the average conditional co-risk for the 11 Korean banks changed over time reflecting developments in international financial markets (Figure 5). The risk rose following major distress events in financial markets such as Bear Stearns’ bankruptcy and Lehman Brothers filing for court protection; and fell with the normalization of the markets. The average conditional co-risk was low, at approximately 5 percent, before the outbreak of the U.S. subprime mortgage crisis. It then fluctuated between 10 to 15 percent during the second half of 2007 and first half 2008, when major international financial events had occurred, such as the suspension of fund redemptions by BNP Paribas (August 2007) and the bankruptcy of Bear Stearns (March 2008).

![Figure 5. Changes in the Average Conditional Co-risk](source)

D. Summary

The findings of the co-risk analysis show that risks due to linkages in the Korean banking system were lower before the outbreak of the global financial crisis. Moreover, the risks from these linkages follow major financial market events. For instance, the estimated level of risk rose during the bankruptcy of Bear Stearns and Lehman Brothers, and has normalized since then. Domestic events, such as the default of some Mutual Savings Banks in early 2011, on the other hand, had smaller effects on the conditional distress probabilities, indicating that Korean banks’ high integration with the global financial markets is a more important source of risk.

The results also show that banks with higher CDS spreads are the ones which are more sensitive to distress coming from other banks in the system. Fluctuations in the domestic and/or international financial market conditions would be more of a concern for these banks, also affecting their capacity to pay foreign debt. However, one should also mention that this may also be due to investors selling the most risky assets first, including CDSs, when there is a shock to the system.
IV. DISTRESS DEPENDENCE APPROACH

The distress dependence approach, based on the methodology introduced by Segoviano and Goodhart (2009), is another approach to quantify the risks created by financial interconnectedness. In particular, it allows measurement of the level of distress that a bank, or a group of banks, creates on another bank or on the whole banking sector.15

There are several advantages of using this methodology over solely looking at other high frequency indicators, such as the CDS spreads or stock prices of banks. First, distress dependence methodology contains the information obtained from daily high frequency indicators, such as the stock prices. Hence, a rise in the distress level of a particular bank gets automatically embedded into the distress dependence statistic calculated for that bank. Second, this methodology allows us to quantify distress levels that a bank can generate on another bank in the system, whereas looking at the daily movements in the CDS-spreads, for instance, provides an assessment of the financial risk only of a particular bank. A rise in the level of distress faced by a particular bank also does not necessarily reveal the true level of distress that a bank poses on another one in the system.

Second, given that conditional default dependencies may follow nonlinear processes, the distress dependence approach allows us to calculate default probabilities much closer to the levels generated by the data, rather than looking at the mere correlation coefficients of daily financial indicators.

Third, this methodology allows us to estimate the distress level generated on the system due to troubles coming from a bank or group of banks. This is also quite important to capture the system-wide risk more accurately.

Nevertheless, this methodology also has its limitations, and the major caveat of this approach is that it reveals the investor’s perception of risk on the distress level of a bank or the banking system. The investor, or market’s perception of risk, may not always reveal the true default probability of a bank.16

A. Data

For the estimations we use daily financial data on the stock prices and expected default frequencies (EDF) of 11 Korean banks from 2006 to March 31, 2011. Both the stock price and EDF statistics are obtained from Moody’s KMV data base. These 11 banks are: Busan, Cheju, Daegu, Hana, Industrial Bank of Korea, Jeonbuk, Kookmin, Korea Development, Korea Exchange, Shinhan and Woori.

15 Further, the joint default probability assumptions used in this methodology can help to estimate tail risk events better than the previous methodology.

16 For instance, as experienced during the global financial crisis, markets did not price in default of major financial institutions such as the Lehman Brothers.
B. Methodology

Following Segoviano and Goodhart (2009), the model can be broken down into two steps. In the first step, we estimate the default probability of each bank in the system, and in the second step, we estimate the banking stability multivariate densities.

Step 1: Estimating default probability of banks

In estimating the default probability of Korean banks, we will rely on the contingent claims approach (CCA), which is based on the option pricing literature.\(^{17}\) The contingent claims analysis in our framework links the market value of bank assets to its equity value. First, we start by utilizing traditional accounting methodology to establish the link between a bank’s equity and assets, and then we will move to the option pricing models to calculate the implied market value and the volatility of a bank’s assets.

According to the traditional T-accounting system, one can relate the asset value of a bank to the sum of its liabilities and equity, which is given by equation (1) below.

\[
A(t) = E(t) + L(t)
\]  

(1)

Bank liabilities possess risk and in the case of default some of the debt holders may not receive full payment. We could model this expected loss, the amount that debt holders would lose in case of a bank default, as an implicit put option, \(P(t)\), which is equal to the difference between risky bank liabilities, \(L(t)\), and the default-free value of bank liabilities, which is equivalent to the default point of a bank, \(DP(t)\). If the market value of a bank’s assets falls below this level, then the bank defaults. The dynamics of a default-free bank liability can be modeled like a bond which yields the risk-free interest rate of \(r\). Then, we can express the risky bank liability \(L(t)\) as follows.

\[
L(t) = DP e^{-r(T-t)} - P(t)
\]  

(2)

In relation to our analysis, the contingent claims analysis links the equity value of a bank, which is modeled as a call option, to the asset value of that bank, which is the underlying security. Then, equity can be expressed as the following stochastic function \(E[t, A(t)]\), where \(t\) denotes time, and \(A\) stands for assets. In this model \(A(t)\) follows a geometric Brownian motion with drift \(\mu_A\) and volatility \(\sigma_A\) with the following stochastic diffusion process:\(^{18}\)

\[
dA = A[\mu_A dt + \sigma_A dW]
\]  

(3)

In equation (3), \(dW\) stands for a Wiener process with mean zero and variance one.

\(^{17}\)The CCA approach relies on the efficient markets assumption that the equity markets reflect all information available at a given point in time. In other words, the market assesses the stock price of company given its expected profit structure, its competitive position, the business and macroeconomic environment etc.

\(^{18}\)The geometric Brownian motion of a process \(X\) can only have positive values, which is the relevant case for modeling the dynamics of a bank’s assets, as assets can only have positive values.
The option pricing literature states that the value of equity depends on the dynamics of asset value over time; and we can model it based on the diffusion process of assets. Applying Itô’s lemma to the equity value of a bank, one can write the partial differential equation of the equity price dynamics as follows.

\[
dE(t, A) = \frac{\partial E}{\partial t} dt + \frac{\partial E}{\partial A} \mu A dt + \sigma A dW + \frac{1}{2 \sigma^2 A^2} \frac{\partial^2 E}{\partial A^2} A^2 dt
\]

(4)

The Black-Scholes-Merton model asserts that for a risk-free portfolio, the partial differential equation of equation (4) can be written as following.

\[
\frac{\partial E}{\partial t} + \frac{1}{2} \frac{\partial^2 E}{\partial A^2} \mu A + \frac{1}{2} \frac{\partial^2 E}{\partial A^2} \sigma^2 A^2 = rE
\]

(5)

The CCA approach models bank equity as a contingent claim, because its value depends on the value of bank assets and the default-free value of bank liability, which is equivalent to the strike price of an option. The argument is as follows. The stockholders of a bank have a contingent claim on the residual value of a bank’s assets, which is equal to the value of total assets minus debt. However, equity is a junior claim, meaning that in the case of default, when the market value of a bank’s assets is less than its debt obligations, then the equity holders cannot receive any positive amount. Then, we can model equity as the following call option, where

\[
E = \max[A - DP, 0]
\]

(6)

Equation (6) reveals that the value of a bank’s equity is equal to the value of its assets, denoted by A, minus the current debt obligations of that bank, or the default point, denoted by DP, only if the bank can generate enough cash to cover its current debt obligations, in other words \( A \geq DP \); if not, it is equal to null.

Imposing the boundary condition given by equation (6) at time \( T \) provides the closed form solution to the partial differential equation given in equation (5). This can be expressed as the following, in equation (7).

\[
E = AN(d_1) - DP e^{-rT} N(d_2)
\]

(7)

---

19 A variable \( X \) follows an Itô process if its stochastic differential equation is given by:

\[
dX = \mu(t, X) dt + \sigma(t, X) dW
\]

Itô’s lemma states that a function \( f(t, X) \) follows an Itô process, when the variable \( X \) follows an Itô process, and \( f(t, X) \) is once differentiable in \( t \), and twice in \( X \). Then the dynamics of \( f(t, X) \) is given by the following process:

\[
df(t, X) = \frac{\partial f}{\partial t} dt + \frac{\partial f}{\partial X} dX + \frac{1}{2} \frac{\partial^2 f}{\partial X^2} \sigma^2(t, X) dt
\]
In equation (7), \( r \) is the risk-free interest rate; \( T \) is the time to maturity of the risk-free debt, \( DP \); \( N(d) \) is a cumulative probability distribution function for a standard normal variable; and \( \sigma_A \) is the standard deviation of a bank’s assets. \( A \) is the current market value of the underlying asset – namely, the bank assets, and \( DP \) (the default-free value of bank liabilities) is the strike price of the put option.

As derived by Itô’s lemma, the relationship between the volatility of a bank’s assets and the volatility of its equity is given by the following equation:

\[
E \sigma_e = A \sigma_A N(d_1) = A \sigma_A \frac{\partial E}{\partial A}
\]

(8)

The difference between the CCA approach and option pricing is in terms of estimating the unknowns. In the CCA approach, one actually knows the equity information, through bank capitalization data, but not the asset value or the implied asset volatility. The option pricing equations (7) and (8) then provide a system of equations which allows calculating the implied asset value and volatility by forward iteration.

The distance to default measure is defined as the distance between the logarithm of the asset values and the logarithm of the risk-free liabilities of a bank, normalized by the standard deviation of its assets; and it shows how many standard deviations the bank is away from defaulting on its debt. Then, for a risk-free debt of maturity \( T \), we can measure it by the ratio of \( d_2 \), given in equation (7), to the volatility of a bank’s assets, \( \sigma_A \). In other words, the distance to default is measured by:

\[
DD = \frac{\ln\left(\frac{A}{DP}\right) + \left(r + \frac{1}{2} \sigma_A^2\right)T}{\sigma_A \sqrt{T}}
\]

(9)

The probability of default on the other hand is measured by calculating the cumulative distribution function of \( N(-DD) \) by relying on risk-neutral pricing.\(^{20}\) As an example, if a bank is two standard deviations away from defaulting on its one year maturing debt, then its probability of default is equal to 2.28 percent since \( N(-2) = 0.0228 \). The probability of default

\(^{20}\) Note that distance to default measure also eliminates the problem that the probability distribution of asset values is not lognormal, because the distance to default measure is normalized by the volatility of the asset values.
increases as the market leverage of a bank or the volatility of the market value of bank assets increase.\textsuperscript{21}

Figure 6 below provides a simple presentation of the linkages between the book value of bank assets, the implicit asset value and the distance to default.

Similar to the CCA approach, Moody’s provide a dataset based on the expected default frequency (EDF) of corporations. However, in order to calculate the probability of default, this database uses an empirically determined function, which is based on historical data, to map distance to default measure to probability of default. For instance, if the historical data shows that, on average, two out of 100 companies defaulted within a year time, amongst the companies which are three standard deviations away from default; then, this function assigns 2 percent to the EDF credit measure for a company with a distance to default of three.

The advantage of using the EDF credit measure rather than the cumulative normal distribution is that the EDF credit measure assigns more realistic default probabilities to a company. The normal cumulative distribution assigns lower default probabilities to a company which is three or more standard deviations away from default, compared to the EDF credit measure. The historical default data indicates that the probabilities of the cumulative normal distribution are rather on the optimistic side.

The disadvantage of using the EDF measure, on the other hand, is that one cannot know explicitly the model and the parameters used for estimations. Nevertheless, since the EDF provides a more realistic default probability, in this paper, we will rely on the EDF statistics, rather than the normal distribution as a mapping function.

\textsuperscript{21} Market leverage, as defined in the Moody’s KMV data base, is the ratio of risk-free debt (or the default point) to the market value of a company’s assets.
Figure 7 presents the stock price and the expected default frequency of large and small Korean banks, where the size is based on the loan book value of these banks. As expected, larger banks have higher book value and lower default frequency, since as the market perceives a smaller default probability for the larger banks.

Figure 7. Stock Price and Default Probabilities of Korean Banks

Source: Moody’s KMV.
Step 2: Estimating banking stability multivariate density (BSMD)

In this section, we estimate the level of distress in the Korean banking system, by combining the individual distress levels that we estimated in the previous section. In other words, and as presented in, we combine the individual default probabilities of Korean banks into a portfolio of banking system stability indicators.

By utilizing the consistent information multivariate density optimizing (CIMDO) methodology, as presented in Segoviano (2006), we estimate the banking system as a portfolio of multivariate density functions (BSMD), where the CIMDO density function is obtained by minimizing a conditional default probability function as given in

\[
L(p, q) = \int \int p(x, y) \ln p(x, y) dxdy - \int \int p(x, y) \ln q(x, y) dxdy \\
+ \vartheta_1 \left[ \int \int p(x, y) \varphi_{\varphi_{x}} dxdy - PoD^x_t \right] \\
+ \vartheta_2 \left[ \int \int p(x, y) \varphi_{\varphi_{y}} dxdy - PoD^y_t \right] + \mu \left[ \int \int p(x, y) dxdy - 1 \right]
\]

Where, \(PoD^x_t\) is the probability of default of bank x at time t, and the expected default frequency statistics are used to approximate for the default probabilities. The data on x and y are derived from the stock prices of bank x and bank y.

Minimizing the above density function yields the following multivariate distribution,

\[
p(x, y) = q(x, y) \exp \left\{ - \left[ 1 + \hat{\mu} + \left( \vartheta_1 \varphi_{\varphi_{x}} \right) + \left( \vartheta_2 \varphi_{\varphi_{y}} \right) \right] \right\}
\]
Default dependence estimated from the copula functions are multivariate distributions, where the marginal probabilities are uniform on the \([0,1]\) interval. These density functions allow us to calculate the distress dependence over time, reflecting the change in the expected default frequency of banks.

C. Results

Based on the results calculated from the banking stability multivariate density function, we construct three main indicators of financial distress for the Korean banking system. These indicators help us assess the level of distress due from a bank or group of banks on other banks in the system.

**Distress dependence among Korean banks**

Distress dependence statistic shows the default probability of bank \(x\) conditional on the default probability of bank \(y\), which can be represented as:

\[
P(X \geq x_i^*, Y \geq y_j^*)
\]

\[
P(Y \geq y_j^*)
\]

In Table 4, we provide the distress dependence matrix across Korean banks for two time periods: October 2008 and March 2011. The first date corresponds to the period when market risks reached an elevated level in Korea, and the second date shows the most recent bilateral risk exposures across Korean banks.

In the distress dependence matrix, each row presents the distress on the row bank conditional on the probability that the column bank defaults. For instance, the top row in Table 4 reveals that during October 2008, the highest level of exposure was due from the conditional default probability of Bank 7.

In general, larger banks are systemically more important, as conditional default probabilities due to stress at these banks, i.e. Bank 10 and Bank 11, are the largest as presented in Table 4.

Last, in line with the decline in global risk aversion from October 2008 to March 2011, the conditional distress statistics for the majority of Korean banks fell.

**Banking stability index**

Banking stability index (BSI) is estimated by calculating the expected number of bank defaults conditional on the default of at least one other bank in the system. For instance, expected number of bank defaults, for a system of two banks, can be expressed as following,

\[
\frac{P(X > x_i^*) + P(Y > y_j^*)}{1 - P(X < x_i^*, Y < y_j^*)}
\]
Table 4. Distress Dependence Matrix (October 2008 and March 2011)

<table>
<thead>
<tr>
<th>Oct-08</th>
<th>bank 1</th>
<th>bank 2</th>
<th>bank 3</th>
<th>bank 4</th>
<th>bank 5</th>
<th>bank 6</th>
<th>bank 7</th>
<th>bank 8</th>
<th>bank 9</th>
<th>bank 10</th>
<th>bank 11</th>
</tr>
</thead>
<tbody>
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<td>bank 1</td>
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<td>0.64</td>
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<td>0.24</td>
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<td>0.70</td>
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<tr>
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<td>0.06</td>
<td>0.06</td>
<td>0.07</td>
<td>0.06</td>
<td>0.09</td>
<td>0.06</td>
<td>0.08</td>
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<td></td>
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<td>0.74</td>
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<td>0.08</td>
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<td>—</td>
<td>0.32</td>
<td>0.10</td>
<td>0.37</td>
<td>0.11</td>
<td>0.31</td>
<td>0.44</td>
<td>0.38</td>
</tr>
<tr>
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<td>0.42</td>
<td>0.50</td>
<td>—</td>
<td>0.15</td>
<td>0.84</td>
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<td>0.22</td>
<td>—</td>
<td>0.25</td>
<td>0.15</td>
<td>0.23</td>
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<td>0.29</td>
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<tr>
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<td>0.23</td>
<td>0.29</td>
<td>0.41</td>
<td>0.09</td>
<td>—</td>
<td>0.09</td>
<td>0.31</td>
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<td>0.39</td>
<td>0.73</td>
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<td>0.69</td>
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<td>0.57</td>
<td>0.14</td>
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<td>0.58</td>
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<tr>
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<td>0.44</td>
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<th>bank 7</th>
<th>bank 8</th>
<th>bank 9</th>
<th>bank 10</th>
<th>bank 11</th>
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<td>0.77</td>
<td>0.19</td>
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<td>0.44</td>
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<tr>
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<td>0.18</td>
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<td>0.15</td>
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<td>0.07</td>
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<td>0.70</td>
<td>0.18</td>
<td>0.32</td>
<td>0.61</td>
<td>—</td>
</tr>
</tbody>
</table>

Source: IMF staff calculations.
Figure 9 provides the expected number of bank defaults in the Korean banking system from January 2007 through March 2011. The BSI index of the Korean banking system before the outbreak of the global financial crisis was around 1.4, which was similar to that of major U.S. and European banks.\(^\text{22}\)

The BSI index worsened significantly during the elevated distress levels in global financial markets, reaching to a peak at around 3, indicating that three banks may default conditional on the distress of another bank. This ratio rose to more than 3.5 for the major U.S. and European banks during the same period.\(^\text{23}\)

![Figure 9. Expected Number of Bank Defaults 2007–11](image)

The BSI index started falling together with the decline in global risk aversion, and it stayed around 1.9, which is still higher than the pre-crisis levels. This finding is also consistent with the findings from the co-risk approach.

**Joint probability of distress**

Joint probability of distress is a tail event study for Korean banks. This statistic provides us an assessment of the probability of all the banks in the system defaulting. For instance, for a system of two banks, one can calculate the joint probability of distress as follows,

\[
\int_{x}^{\infty} \int_{y}^{\infty} p(x, y) dx dy
\]

\(^\text{22}\) See Goodhart and Segoviano (2009).

\(^\text{23}\) See Goodhart and Segoviano (2009).
Figure 10 provides the estimates of the joint probability of default statistic for Korean banks from January 2007 through March 2011. This statistic is rather small for Korean banks. Even though the statistic spiked quite rapidly during the global financial crisis, reflecting the nonlinear and larger increases experienced by the individual banks, it never reached levels that would have stressed the whole financial system.

![Figure 10. Joint Probability of Default, 2007–11](image)

Source: IMF staff calculations.

**D. Summary**

The distress dependence approach helps us quantify the level of distress that a bank, or a group of banks, can pose on another bank in the system or to the whole system. Results indicate that Korean banks are interconnected; however, the systemic risk that a bank can pose on the whole system is quite limited. Similar to the results found with the co-risk approach, distress dependence of the banking system follows major global financial events.

**V. Conclusion**

In this paper, we apply three different methodologies to quantify the risks based on financial interconnectedness of Korean banks. These methodologies respectively are the network, co-risk and distress dependence approaches. The findings of our analyses indicate that Korean banks are interconnected; however, both the financial risk that they possess and the contagion risk that they create on each other have declined significantly in the aftermath of the global financial crisis.

The network approach measures the financial exposures that banks create on each other by analyzing bank balance sheet data, by focusing on two types of risks. The first is credit risk, and incorporates the credit losses that a bank needs to write off due to the default of another
bank in the system. Second is the funding loss in addition to the credit risk, since default of a bank necessitates other banks in the system to find other funding sources to replace the funding obtained from the defaulting bank. Results from this analysis indicate that default of no single Korean bank generates significant distress on other banks in the system. Further, writing off the credit-plus-funding risk from the banks’ capital still leaves Korean banks with BIS capital ratios higher than the 8 percent required minimum.

The co-risk model estimates the rise in the default probability of a bank based on the default probability of other banks in the system. The findings of this methodology indicate that major international financial events, such as the bankruptcy of Lehman Brothers, were the main factors moving the conditional default probability of Korean banks. Domestic events changed the conditional default probabilities relatively less, indicating that Korean banks’ high integration to the global financial markets is a more important source of distress.

The distress dependence approach helps us quantify the level of distress that a bank, or a group of banks, can pose on another bank in the system or to the whole system. Results indicate that Korean banks are connected; however, the systemic risks that a bank can pose onto the whole system are limited. Similar to the results found with the co-risk approach, distress dependence of banks follow major global financial events.
REFERENCES


London School of Economics).

