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Optimal Monetary and Fiscal Policy with Limited Asset Market Participation

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Optimal Monetary and Fiscal Policy with Limited Asset Market Participation

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Abstract

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This paper characterises the jointly optimal monetary and fiscal stabilisation policy in a new Keynesian model that allows for consumers who lacking access to asset markets consume their disposable income each period. With full asset market participation, the optimal policy relies entirely on the interest rate to stabilise cost-push shocks and government expenditure is not changed. When asset market participation is limited, there is a case for fiscal stabilisation policy. Active use of public spending raises aggregate welfare because it enables a more balanced distribution of the stabilisation burden across asset-holding and non-asset-holding consumers. The optimal response of government expenditure is sensitive to the financing scheme and whether the policymaker has access to a targeted transfer that can directly redistribute resources between consumers.

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## Contents

I. Introduction .......................................................................................... 3

II. The Baseline Model .............................................................................. 5
   A. Households ......................................................................................... 5
   B. Firms and Price Setting .................................................................... 6
   C. Fiscal Policy ....................................................................................... 7
   D. Aggregation and Market Clearing ...................................................... 7
   E. Steady State and Linearisation ........................................................... 7

III. Equilibrium, Calibration and Determinacy ........................................... 9
   A. Equilibrium ......................................................................................... 9
   B. Calibration ......................................................................................... 10
   C. Determinacy ....................................................................................... 10

IV. Optimal Policy ..................................................................................... 11
   A. Social Welfare ................................................................................... 12
   B. Optimal Monetary Policy with Exogenous Fiscal Policy ................. 12
   C. Jointly Optimal Monetary and Fiscal Policy ..................................... 14

V. Extensions .......................................................................................... 18
   A. CRRA Preferences ........................................................................... 18
   B. Targeted Transfers ........................................................................... 19
   C. Alternative Financing Assumptions ................................................... 21

VI. Conclusion ......................................................................................... 24

Appendix ................................................................................................. 26
   A. Derivation of the Baseline Model ....................................................... 26
   B. Derivation of the Social Welfare Function ....................................... 27
   C. Solving for Optimal Policy ................................................................. 29
   D. Extensions ......................................................................................... 30
   E. The 'non-Keynesian' Case ................................................................. 31

References ............................................................................................. 33

Figures

Figure 1. Determinacy in the baseline model .......................................... 11
Figure 2. Optimal feedback coefficients for different values of $\lambda$ ........ 15
Figure 3. Impulse responses to a persistent cost-push shock in the baseline model. 16
Figure 4. Impulse responses to a persistent cost-push shock with CRRA utility, targeted transfers and equal lump-sum tax financing .......... 20
Figure 5. Impulse responses to a persistent cost-push shock with goverment debt. 23
Figure 6. Determinacy for the 'non-Keynesian case'. .............................. 32
I. Introduction

The current financial crisis has led to renewed interest in using fiscal policy as a stabilisation tool. The standard new Keynesian model used to analyse stabilisation policy, however, does not contain fiscal policy (Clarida et al. 1999, Woodford 2003). Recently, a number of studies have added fiscal policy to this setup to explore whether it should assist optimal monetary policy in the stabilisation of shocks.¹

Strikingly, when fiscal policy is confined to setting government expenditure there is no role for an active fiscal stabilisation policy in the standard model (Eser et al. 2009).²

Intuitively, the optimal policy mix relies on monetary policy to stabilise cost-push shocks because nominal inertia in price setting is the main distortion of the model and the real interest rate is more effective in controlling inflation than government expenditure. As government expenditure is inactive, the optimal policy is identical to Clarida et al. (1999).

In this paper we show that the optimal stabilisation policy involves active use of government expenditure when an additional distortion, limited asset market participation, is introduced. Following Gali et al. (2004) we deviate from the representative-agent assumption of the standard model and develop a model in which a fraction of consumers continues to smooth inter-temporally (the ‘asset holders’) while a new group of consumers has no access to asset markets (the ‘non-asset holders’). The inclusion of non-asset holders – who consume their current disposable income each period – is motivated by a number of studies documenting deviations from the permanent income hypothesis. Campbell and Mankiw (1989), for example, show that aggregate consumption behaves as if 40-50 percent of the US population simply consumed their current income. Using micro data, Johnson et al. (2006) find substantial deviations from the permanent income hypothesis.

In our baseline model the optimal policy response to a positive cost-push shock involves an increase in government spending with limited asset market participation. The intuition is that with asset market restrictions (i) a utilitarian policymaker cares about the distribution of losses among asset and non-asset holders, and (ii) monetary and fiscal policy affect the two groups of consumers differently. While contractionary monetary policy reduces the consumption of non-asset holders more than that of asset holders, an increase in government expenditure raises the consumption of non-asset holders and reduces that of asset holders. Neither (i) or (ii) hold with full asset market participation. Compared to a setup that abstracts from fiscal policy, the active use of public expenditure allows a reallocation of the stabilisation burden across consumers that raises aggregate welfare. While tight monetary policy ensures inflation stabilisation, expansionary fiscal policy plays a redistributional role.


²If fiscal policy has a direct effect on prices – by varying distortionary tax rates – the optimal policy involves an active fiscal policy in the stabilisation of shocks (Benigno and Woodford 2003).
The literature has focused on two related implications of limited asset market participation.

First, Gali et al. (2004) and Bilbiie (2008) show that the determinacy properties of monetary-policy rules can change dramatically when the share of non-asset holders becomes large, weakening or even overturning the Taylor principle. Further, Bilbiie (2008) explores the implications of limited asset market participation for optimal monetary policy. As the presence of non-asset holders strengthens the link between interest rates and aggregate demand – and hence makes monetary policy more effective – the optimal interest-rate response to inflation falls with the share of non-asset holders.³

Second, this literature shows that the introduction of non-asset holders alters the propagation of government spending shocks. In representative-agent models a positive public expenditure shock leads to lower private consumption, because its financing implies a negative wealth effect (Linneman and Schabert 2003). Empirical evidence, at odds with this prediction, mostly finds a positive effect of government expenditure shocks on private consumption (Blanchard and Perotti 2002). Adding non-asset holders to the standard model can account for this finding (Gali et al. 2007). With nominal rigidities the fiscal shock can raise the real wage, leading to higher consumption by non-asset holders. With a high enough share of non-asset holders, this increase can outweigh the negative consumption response of the asset-holders.⁴ While this literature has emphasised the consequences of limited asset market participation for the effects of government spending shocks, the implications for optimal fiscal policy have not yet been explored.

To obtain a closed-form solution of the optimal policy, our baseline model⁵ adopts logarithmic utility and assumes that government expenditure can be financed with a proportional profit tax. The optimal policy continues to increase government expenditure in response to positive cost-push shocks when we allow for CRRA utility and government debt financing – unless the steady-state level of debt is large. The optimal response of government expenditure is, however, sensitive to two other extensions. First, if the policymaker has access to a targeted lump-sum transfer – which can perfectly redistribute resources between consumers – the optimal policy no longer involves a change in government expenditure and the resulting equilibrium is identical to that with full asset market participation. Second, the optimal policy reduces government expenditure in response to positive cost-push shocks if financed by an equal lump-sum tax on both consumers. By cutting government expenditure and rebating the available resources to the consumers, the policymaker attempts to replicate the targeted transfer policy.

The paper is structured as follows. Sections II and III introduce and discuss the baseline model. Section IV characterises the optimal stabilisation policy and section V considers extensions of the baseline model. Section VI concludes.

³If the share of non-asset holders becomes ‘too high’, higher real interest rates raise aggregate demand (Bilbiie 2008). In this ‘non-Keynesian’ region, determinacy requires the Taylor principle to be violated.
⁴In the ‘non-Keynesian’ region, a positive public spending shock reduces aggregate consumption and output (Bilbiie and Straub 2004, Rossi 2007).
⁵As we consider the ‘non-Keynesian’ region to be of little practical relevance, it is only discussed briefly in Appendix E. The optimal policy still increases public spending and the Taylor principle is violated.
II. The Baseline Model

We extend the model of Bilbiie (2008) to include endogenous fiscal policy. This setup is a generalisation of the standard new Keynesian model without capital (Woodford 2003) that introduces agents who consume their labour income each period (Gali et al. 2004). In addition to a continuum of households, the model contains a continuum of monopolistically competitive producers who set prices in a staggered fashion a la Calvo. We assume that the policymaker sets the nominal interest rate and government expenditure to stabilise cost-push shocks.\(^6\)

To focus on the role of government expenditure, we abstract from distortionary taxes and assume that public spending is financed by a proportional profit tax.\(^7\) The baseline model further adopts logarithmic utility in consumption and abstracts from lump-sum transfers to the non-asset holders – ensuring constant hours of the non-asset holders. These assumptions will be relaxed in section V.

A. Households

Drawing on Gali et al. (2004) and Bilbiie (2008), we assume that an exogenous fraction \(1 - \lambda\) of consumers have unrestricted access to asset markets (the ‘asset holders’). These consumers hold nominal one-period assets and receive company profits in the form of dividends. The remaining \(\lambda\) consumers, in contrast, are shut out from asset markets (the ‘non-asset holders’).\(^8\) We assume that differences between consumers arise from their respective capacity to asset markets, rather than from differences in preferences.

1. Asset holders

The proportion \((1 - \lambda)\) of asset holders choose consumption \(C_{O,t}\), hours worked \(N_{O,t}\) and nominal asset holdings \(A_{O,t}\) by solving the following optimisation problem:

\[
\max_E E_t \sum_{s=t}^{\infty} \beta^{s-t} \left( \ln C_{O,s} + \chi \ln G_s - \frac{N_{O,s}^{1+\varphi}}{1 + \varphi} \right)
\]

subject to the budget constraint:

\[
P_t C_{O,t} + R_{t}^{-1} A_{O,t+1} = A_{O,t} + (W_t N_{O,t} + P_t D_{O,t})
\]

\(^6\)We follow the literature in focusing on cost-push shocks because technology and preference shocks can be perfectly offset through variations in the natural rate of interest (Woodford 2003).

\(^7\)A proportional profit tax is non-distortionary in our log-linearised model because it does not contain capital. Throughout the paper we abstract from time-varying distortionary tax rates, as these would play an active role in the optimal policy even with full asset market participation (Benigno and Woodford 2003).

\(^8\)In Gali et al. (2004) some agents do not hold physical capital and consume their current labor income. They refer to these as ‘rule-of-thumb’ or ‘non-Ricardian’ consumers. We follow Bilbiie (2008) in assuming that some consumers do not have access to asset markets and label these ‘asset holders’.
where $R_t$ is the gross nominal return on assets purchased a time $t$, $G_t$ is government expenditure and $W_t$ is the nominal wage. Asset holders receive post-tax firm profits in form of real dividend payments $D_{O,t}$. The first-order conditions are:

$$R_t^{-1} = E_t(Q_{t,t+1})$$  \hspace{1cm} (2)

$$N^{c}_{O,s}C_{O,s} = \frac{W_t}{P_t}$$  \hspace{1cm} (3)

where the stochastic discount factor $Q_{t,t+1}$ is given by:

$$Q_{t,t+1} = \beta \frac{C_{O,t}}{C_{O,t+1}} \frac{P_t}{P_{t+1}}$$  \hspace{1cm} (4)

2. Non-asset holders

The remaining proportion $\lambda$ of non-asset holders choose consumption $C_{R,t}$ and hours $N_{R,t}$ in each period $t$ to maximise:

$$\max \left( \ln C_{R,t} + \chi \ln G_t - \frac{N_{R,t}^{1+\varphi}}{1+\varphi} \right)$$  \hspace{1cm} (5)

subject to the condition that consumption equals income

$$P_tC_{R,t} = W_tN_{R,t}$$  \hspace{1cm} (6)

The first order condition is given by:

$$N^{c}_{R,t}C_{R,t} = \frac{W_t}{P_t}$$  \hspace{1cm} (7)

For simplicity we assume our consumers to have identical preferences. Also, because both types of consumers supply labour of an identical type, there is a uniform wage.

B. Firms and Price Setting

A continuum of monopolistically competitive firms produce differentiated intermediate goods which are used as inputs by a perfectly competitive firm producing a single final good. The final good is produced with a constant returns technology,

$$Y_t = \left[ \int_0^1 Y_t(i) \frac{\varepsilon}{\varepsilon-1} di \right]^{-\varepsilon},$$

where $Y_t(i)$ denotes the quantity of intermediate good $i$ used as an input and $\varepsilon$ denotes the elasticity of substitution between differentiated goods. The perfectly competitive firm maximises profits $P_t Y_t - \int_0^1 P_t(i) Y_t(i) di$, where $P_t$ is the price index for final goods and $P_t(i)$ denotes the prices for intermediate goods, yielding the set of demand schedules:

$$Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t$$  \hspace{1cm} (8)
while the price index is given by \( P_t = \left( \int_0^1 P_t(i)^{1-\varepsilon} \, di \right)^{1/(1-\varepsilon)} \). The production function for a monopolistically competitive firm producing intermediate good \( i \) is:

\[ Y_t(i) = N_t(i) \tag{9} \]

Firms producing intermediate goods are assumed to set nominal prices in a staggered fashion à la Calvo.\(^9\) Each firm resets its price with a fixed probability \( 1 - \gamma \) in each period. A firm resetting its price in period \( t \) will set \( P_t(i) \) to maximise the discounted sum of its future real profits:

\[
E_t \sum_{s=0}^{\infty} \gamma^s Q_{t,t+s} (1 - \Lambda_{t+s}) \left[ \frac{P_t(i)}{P_{t+s}} Y_{t+s}(i) - (1 - \zeta) \frac{W_{t+s}}{P_{t+s}} Y_{t+s}(i) \right] \tag{10}
\]

subject to (8), where \( \Lambda_t \) denotes a proportional profit tax and \( \zeta \) denotes a steady-state employment subsidy (which will be discussed in section IV).

C. Fiscal Policy

The allocation of government spending across goods is determined by minimising total cost \( \int_0^1 P_t(i) G_t(i) \, di \), implying a demand relationship \( G_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} G_t \). In the baseline model, the government is assumed to finance its expenditure with the profit tax, \( G_t = \Lambda_t O_t \), where \( O_t \) denotes aggregate real profits. Real dividends, \( D_t = (1 - \Lambda_t) O_t \), are rebated to the asset holders.

D. Aggregation and Market Clearing

Aggregate variables for consumption and hours worked are respectively given by \( C_t = \lambda C_{R,t} + (1 - \lambda) C_{O,t} \) and \( N_t = \lambda N_{R,t} + (1 - \lambda) N_{O,t} \). Market clearing requires that all dividends be paid to asset holders, \( D_t = (1 - \lambda) D_{O,t} \), all assets be held by asset holders, \( A_t = (1 - \lambda) A_{O,t} \), and the income identity to hold, \( Y_t = C_t + G_t \).

E. Steady State and Linearisation

The dynamics of the system is studied by taking a log-linear approximation of the equilibrium conditions around its steady state. Following Gali et al. (2004) we assume that steady-state consumption is equal across groups, \( C = C_O = C_R \), an outcome that is ensured by a steady-state lump-sum tax on profits which eliminates steady-state dividend

\(^9\)We follow the literature in adopting Calvo pricing because of analytical tractability and despite its empirical problems (see, for example, Rudd and Whelan 2007).
payments to the asset holders (see Appendix A.1). Given homogenous preferences, hours are also identical in steady state, $N = N_O = N_R$.

We then take a (log-)linear approximation of the equilibrium conditions around this steady state, denoting the log-deviation of a variable from its steady state with a small-case letter $(x_t = \log (X_t / X) \simeq (X_t - X) / X)$, while $\pi_t = \log (P_t / P_{t-1})$ and $\omega_t = \log ((W_t / P_t) / (W / P))$. The log-linearisation of the aggregation rule for any variable $x_t$ yields $x_t = \lambda x_{R,t} + (1 - \lambda) x_{O,t}$. Linearisation of the asset holder’s first order conditions (2) and (3) produces:

$$c_{O,t} = E_t c_{O,t+1} - (n_t - E_t \pi_{t+1}) \quad (11)$$
$$\omega_t = \varphi n_{O,t} + c_{O,t} \quad (12)$$

Log-linearisation of the first order conditions of the non-asset holders (6) and (7) yields:

$$c_{R,t} = \omega_t + n_{R,t} \quad (13)$$
$$\omega_t = \varphi n_{R,t} + c_{R,t} \quad (14)$$

The log-linearised income identity is given by:

$$y_t = \theta c_t + (1 - \theta) g_t \quad (15)$$

where $\theta$ is the steady-state share of private consumption in output, which is related to $\chi$ (see Appendix B.1). Linearisation of the production function (9) produces:

$$n_t = y_t \quad (16)$$

The log-linear approximation of the first order condition of the price setting problem (10), together with the definition of the price level, results in a standard New Keynesian Phillips curve (see Appendix A.2):

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \omega_t + \mu_t \quad (17)$$

where $\kappa = \frac{(1 - \beta_2)(1 - \gamma)}{\gamma}$. The Phillips curve is standard as steady-state consumption levels are equal across groups and because the proportional profit tax is non-distortionary in the log-linearised model. Following Clarida et al. (1999) we have included a ‘cost push’ shock, $\mu_t$, which is assumed to follow an exogenous first-order autoregressive process,\(^\text{10}\)

$$\mu_t = \rho \mu_{t-1} + \xi_t$$

where $\rho \in [0, 1)$ and $\xi_t$ is a white noise process.

\(^\text{10}\)The cost-push shock can stem from changes in the elasticity of substitution between goods, resulting in a variable markup (see Appendix A.2).
III. Equilibrium, Calibration and Determinacy

A. Equilibrium

Combining (13) and (14), we see that the labour supply is constant for the non-asset holders:

\[ n_{R,t} = 0 \]

This is because with logarithmic utility the income and substitution effects for these agents cancel exactly to leave hours worked unchanged. Total supply of labour is therefore given by \( n_t = (1 - \lambda) n_{O,t} \). Consumption of the non-asset holders then tracks the real wage to exhaust the budget constraint:

\[ c_{R,t} = \omega_t \quad (18) \]

Notice that \( n_R \) is only constant if no lump-sum transfers to non-asset holders take place, \textit{and} if utility in consumption is logarithmic. Both of these assumptions will be relaxed in section V, implying variable hours for non-asset holders.

Combining the linearised aggregation rules for consumption and hours with (12), (15) and (18) we obtain an expression for the real wage:

\[ \omega_t = (1 + \theta \alpha) c_{O,t} + \alpha (1 - \theta) g_t \quad (19) \]

with

\[ \alpha = \frac{\varphi}{1 - \lambda (1 + \varphi \theta)} \]

Given the central role played by the real wage (equal to both real marginal cost and \( c_R \)), let us discuss its determination in some detail. For shares of non-asset holders that are not ‘too high’ \((\lambda < \lambda^* = 1/(1 + \varphi \theta))\) such that \( \alpha > 0 \) the real wage depends positively on both \( c_O \) and \( g \). A fall in \( c_O \) – for example brought about by a monetary tightening – has two effects on the real wage. Lower \( c_O \) reduces the real wage directly (and one-for-one with logarithmic utility) through its effect on \( n_O \) via the asset holder’s intratemporal optimality condition, (12). A reduction in \( c_O \) furthermore has an indirect effect on the real wage because it reduces the demand for labour. Together the direct and indirect effects imply that the real wage changes more than one-for-one with \( c_O \) (i.e. \( 1 + \theta \alpha > 1 \)). As \( \lambda \) rises (but remains below \( \lambda^* \)), the effect of \( c_O \) on the real wage becomes larger as the indirect effect gains strength. This is because the required aggregate adjustment of labour supply is borne by fewer asset holders and the real wage has to change by more in response to changes in labour demand to clear the market. Monetary policy therefore becomes more powerful in controlling the real wage with higher \( \lambda \).

While government expenditure does not have a direct effect – because utility is separable, \( g \) does not enter (12) – it raises the real wage indirectly through an increase in labour
demand.\textsuperscript{11} As the indirect effect strengthens with fewer asset holders, fiscal policy also becomes more powerful in affecting the real wage.

Notice that the sign of the effects of both monetary and fiscal policy reverses when $\lambda > \lambda^*$. In this ‘non-Keynesian’ region, an increase in the real interest rate and a reduction in government expenditure both raise aggregate demand (Bilbiie 2008, Bilbiie and Straub 2004, Rossi 2007). As we consider this region to be of little practical relevance, we concentrate on the conventional case with $\lambda < \lambda^*$. For completeness the ‘non-Keynesian’ case is briefly discussed in Appendix E.

\section*{B. Calibration}

In our simulations the quarterly discount rate, $\beta$, is set to 0.99, implying an annual steady-state interest of about 4 percent. Following Gali and Monacelli (2005) we assume $\varepsilon = 6$, which is consistent with a 20 percent steady-state markup, and $\varphi = 1$, which implies a unit labour supply elasticity. We set $\gamma = 0.75$, consistent with an average period of one year between price adjustments. Following Gali and Monacelli (2008) we parameterise the steady-state consumption to output ratio as $\theta = 0.75$. In line with empirical estimates (Campbell and Mankiw 1989, Bilbiie et al. 2008) and earlier calibrations (Bilbiie and Straub 2004, Gali et al. 2007) we set $\lambda = 0.4$ in our baseline calibration.$\textsuperscript{12}$ This value ensures conventional policy effects, as the above calibration implies a threshold of $\lambda^* = 0.58$. The cost-push shock is assumed to be persistent ($\rho = 0.75$) with a standard deviation of 0.005 (Ireland 2004, Woodford 2003).

\section*{C. Determinacy}

As determinacy issues can arise with limited asset market participation (Gali et al. 2004), we briefly discuss them for the baseline model. We assume that monetary and fiscal policy feed back onto current-period inflation using the following rules, respectively:\textsuperscript{13}

\begin{align*}
    r_t &= \phi^g_\pi \pi_t \\
    g_t &= \psi^g_\pi \pi_t
\end{align*}

Substituting for (20) and (21), equations (11), (15), (17) and (18) can be expressed as a two-dimensional system:

\[
    E_t X_{t+1} = \Pi^{-1} (\Gamma X_t + \varepsilon_t)
\]

\textsuperscript{11} Changes in $g$ also affect the real wage through their effect on $c_O$ via the wealth effect of their financing. While small for transitory increases, this effect strengthens for more persistent changes.

\textsuperscript{12} Bilbiie and Straub (2004) set $\lambda = 0.4$ while Gali et al. (2007) choose $\lambda = 0.5$ for their baseline calibration.

\textsuperscript{13} We consider current-inflation rules for consistency with our closed-form solution in section IV. Gali et al. (2004) and Bilbiie (2008) also consider the determinacy properties of expected-inflation rules.
Figure 1: Determinacy in the baseline model. Dark areas indicate uniqueness, light areas indicate indeterminacy.

\[ \Phi = \begin{pmatrix} \Phi_1 & \Phi_2 \\ \Phi_3 & \Phi_4 \end{pmatrix} \]

with \( X_t = (\pi_t, c_{O,t})' \), \( \varepsilon_t = (\mu_t, 0)' \) and

\[
\Pi = \begin{pmatrix} \beta & 0 \\ 1 & 1 \end{pmatrix}, \quad \Gamma = \begin{pmatrix} 1 - \kappa \alpha (1 - \theta) \psi_\pi & -\kappa (1 + \theta \alpha) \\ \phi_\pi & 1 \end{pmatrix}
\]

With two non-predetermined variables, determinacy requires that both eigenvalues of \( \Pi^{-1} \Gamma \) lie outside the unit circle.

Figure 1 reports the simulated determinacy boundaries for our baseline calibration. When government expenditure is exogenous \((\psi_\pi = 0)\) and policy effects are conventional \((\lambda < \lambda^*)\), determinacy is ensured if the Taylor principle is satisfied (Bilbiie 2008). With endogenous government expenditure, the Taylor principle remains a sufficient condition for determinacy for \( \lambda < \lambda^* \), unless government expenditure rises strongly in response to inflation. For large positive \( \psi_\pi^* \) the Taylor principle is too weak a criterion (Figure 1, right-hand panel) and determinacy requires the real interest rate to rise more aggressively in response to inflation because government expenditure puts upward pressure on prices.

**IV. Optimal Policy**

We now turn to the stabilisation of cost-push shocks under optimal discretionary policy. After deriving the social loss function, we will solve for optimal policy with exogenous fiscal policy in section B and optimising fiscal policy in section C. To obtain insightful closed-form solutions, we initially abstract from persistent cost-push shocks and set \( \rho = 0 \). Throughout we will only discuss values of \( \lambda \) that ensure conventional policy effects \((\lambda < \lambda^*)\) and determinacy of the optimal policy.

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14 For a discussion of the determinacy properties of the ‘non-Keynesian’ region \( \lambda > \lambda^* \) see Appendix E.

15 The optimal commitment policy offers similar insights into our model (results available upon request).
A. Social Welfare

To derive a social welfare function for policy analysis we make two assumptions. First, following Woodford (2003), we assume that the distortion caused by monopolistic competition in steady state is eliminated by a production subsidy, financed by a steady-state lump-sum tax on firms (see Appendix B.1). As this lump-sum tax eliminates profits in steady state, it simultaneously ensures identical consumption levels across the two groups (i.e. it is the same lump-sum tax we referred to above). Second, following Bilbiie (2008), we assume that the policymaker is utilitarian and maximises a convex combination of the period utility functions of the asset holders, \( U_O \), and non-asset holders, \( U_R \), with the respective weights determined by \( \lambda, U_s = (1 - \lambda) U_{O,s} + \lambda U_{R,s} \). The social welfare function can then be represented (up to second order) by (see Appendix B.2):

\[
\frac{1}{2} E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[ \frac{\varepsilon}{\kappa} \pi_s^2 + \left( \frac{\varphi + \lambda}{1 - \lambda} \right) y_s^2 + (1 - \theta) g_s^2 + \theta (1 - \lambda) c_{O,s}^2 + \theta \lambda c_{R,s}^2 \right]
\]  

(22)

where we omit terms independent of policy and terms of higher order than two. The relative weight attached to aggregate output fluctuations rises with \( \lambda \), as only asset holders adjust their labour supply. The social loss also depends on the volatility of public and private consumption. Importantly, with two types of consumers, the distribution of private consumption volatility matters.

B. Optimal Monetary Policy with Exogenous Fiscal Policy

With exogenous fiscal policy the policy problem consists of minimising the social loss function (22), subject to (15), (17), (19) and \( g_t = 0 \). We construct a Lagrangian:

\[
H_t^x = \frac{1}{2} E_t \sum_{t=s}^{\infty} \beta^{t-s} \left( \frac{\varepsilon}{\kappa} \pi_t^2 + \left( \frac{\varphi + \lambda}{1 - \lambda} \right) y_t^2 + \theta (1 - \lambda) c_{O,t}^2 + \theta \lambda c_{R,t}^2 \right)
\]

\[ + L_t^R \left( c_{R,t} - (1 + \theta \lambda) c_{O,t} \right) + L_t^\pi \left( \pi_t - \beta E_t \pi_{t+1} - \kappa c_{R,t} - \mu_t \right) \]

\[ + L_t^y \left( y_t - \lambda \theta c_{R,t} - (1 - \lambda) \theta c_{O,t} \right) \}

Differentiating with respect to \( \pi, y, c_O, c_R, L_R^R, L_\pi \) and \( L_\gamma \) yields seven first order conditions (see Appendix C.1). Under optimal discretionary policy and with white noise

\[ ^{16}\text{The numerical solution of the optimal policy was obtained using the approach of Soederlind (1999). The welfare outcome is expressed in terms of percent of steady-state consumption gained.} \]
cost-push shocks we have $L^\pi_{t-i} = \pi_{t+i} = c_{j,t+i} = 0$, for $i > 0$.\footnote{Under optimal discretionary policy, expectations at the time of optimisation are taken as given (Currie and Levine 1993) and with a white noise cost-push shock there are no dynamics beyond period $t$.} Given the linear-quadratic setup of the problem, the optimal instrument rule can be expressed as a linear function of current-period inflation (see Appendix C.1 for the full solution to the problem):

$$ r_t = \phi^\pi_n \pi_t $$

(23)

with

$$ \phi^\pi_n = \frac{\varepsilon (1 + \theta \varphi) \varphi}{\alpha \theta \Theta} $$

where $\Theta = 1 - \lambda + \theta (\varphi + \lambda (1 + \theta \varphi^2)) > 0$.

With full asset market participation, the optimal policy collapses to the standard case of Clarida et al. (1999) with $y_t = \theta c_t$. The first order conditions reduce to the familiar ‘lean against the wind’ policy, $\pi_t = -\frac{1}{\varphi} y_t$. For $\lambda = 0$ equation (23) simplifies to $\phi^\pi_n = \frac{\varepsilon}{\theta}$. The optimal policy therefore fulfils the Taylor principle (as $\varepsilon > \theta$ holds for reasonable calibrations).

With non-asset holders, the optimal policy continues to obey the Taylor principle – and hence ensures determinacy – if the share of non-asset holders is not too large, $\lambda < \hat{\lambda}$, where:

$$ \hat{\lambda} = \frac{(\varepsilon - \theta) (1 + \theta \varphi)}{\theta^2 (1 + \theta \varphi^2) + \varepsilon (1 + \theta \varphi)^2 - \theta} < \lambda^s $$

Two features of the optimal policy are important.

First, the responsiveness of $\varphi$ to inflation under optimal policy is always larger than that of $c_O$:

$$ c_{R,t} - c_{O,t} = -\frac{\varepsilon \varphi (1 + \theta \varphi)}{\Theta} \pi_t $$

As clarified in the discussion of (19) above, the real wage – and hence $c_R$ – contracts by more than the initial fall in $c_O$. Along the optimal policy path, the non-asset holders will therefore suffer a greater contraction in consumption than the asset holders. This implies that monetary policy becomes more powerful as the share of non-asset holders rises. However, as the distribution of consumption across agents increasingly matters with larger $\lambda$, monetary policy actions simultaneously become more costly. It follows from (23) that the optimal response to inflation falls with the share of non-asset holders (i.e. $\partial \phi^\pi_n / \partial \lambda < 0$). Furthermore, even though monetary policy becomes more effective, aggregate consumption is contracted by less and inflation is higher under optimal policy with larger $\lambda$ because of the distributional costs of reducing aggregate consumption.\footnote{It can be shown that $\delta \pi_t / \delta \lambda > 0$ and $\delta c_i / \delta \lambda > 0$, see Appendix C.1.}

Secondly, given our assumption of logarithmic utility, any change in aggregate hours is exclusively borne by the asset holders:

$$ n_{O,t} = -\frac{\varepsilon (1 + \theta \varphi)}{\Theta} \pi_t $$
As non-asset holders do not adjust their labour supply, \( n_O \) has to fall more strongly the higher is \( \lambda \) to achieve any given reduction in aggregate hours. (We will show in section V that the asset holders will continue to reduce their hours more sharply than the non-asset holder when we allow for more general preferences).

These two observations imply that the optimal disinflation path involves an unbalanced distribution of losses across agents: while the labour supply of the asset holders is more volatile than \( n_R \), the consumption of the non-asset holders is more volatile than \( c_O \).

Monetary policy therefore has important distributional effects with limited asset market participation.

Simulations of the optimal policy coefficients in Figure 2 (solid line) show that \( \phi^x_\pi \) falls with \( \lambda \) where we again restrict ourselves to values of \( \lambda \) for which the Taylor principle is ensured. Figure 3 depicts the impulse responses under optimal monetary policy for a persistent cost-push shock with \( \lambda = 0.4 \) (solid line) and \( \lambda = 0 \) (dotted line). In both cases, optimal policy raises the real interest rate, which reduces the consumption of the asset holders. Labour supply contracts and the real wage falls to reduce inflation. With limited asset markets, the consumption of non-asset holders clearly contracts more than that of asset holders. Given this distributional cost, optimal policy with non-asset holders raises the real interest rate by less than with full asset market participation.

### C. Jointly Optimal Monetary and Fiscal Policy

Let us now consider the role of fiscal policy. The constrained loss function is written as:

\[
H^g_t = \frac{1}{2} E_t \sum_{t=s}^{\infty} \beta^{t-s} \left\{ \frac{\xi}{K} \pi_t^2 + \left( \frac{\varphi + \lambda}{1 - \lambda} \right) y_t^2 + \theta \left( 1 - \lambda \right) c_{O,t}^2 + \theta \lambda c_{R,t}^2 + (1 - \theta) g_t^2 \right\} + L^R_t \left( c_{R,t} - (1 + \theta \alpha) c_{O,t} - \alpha (1 - \theta) g_t \right) + L^\pi_t \left( \pi_t - \beta E_{t+1} \pi_t - \kappa c_{R,t} - \mu_t \right) + L^y_t \left( y_t - \lambda \theta c_{R,t} - (1 - \lambda) \theta c_{O,t} - (1 - \theta) g_t \right)
\]

Differentiating with respect to the same variables as before and \( g \) yields eight first order conditions. Appendix C.2 reports these and the solution of the system. Assuming a white noise cost-push shock, the optimal instrument rules can be expressed as:

\[
\begin{align*}
  r_t & = \phi^o_\pi \pi_t \\
  g_t & = \psi^o_\pi \pi_t
\end{align*}
\]

where

\[
\begin{align*}
  \phi^o_\pi & = \frac{\varepsilon (1 + \varphi - \lambda \theta (1 + \varphi (2 + \theta \varphi)))}{\theta (1 + \varphi (1 + \theta \lambda \varphi))} \\
  \psi^o_\pi & = \frac{\lambda \varepsilon (1 + \varphi (1 + \theta \varphi))}{1 + \varphi (1 + \theta \lambda \varphi)}
\end{align*}
\]
Figure 2: Optimal feedback coefficients for different values of $\lambda$. $W$ denotes the welfare gain of optimising fiscal policy.
Figure 3: Impulse responses to a persistent cost-push shock in the baseline model.
With $\lambda = 0$ the coefficients simplify to $\phi^o = \xi \bar{\zeta}$ and $\psi^o = 0$. The optimal fiscal policy is therefore inactive and the monetary policy is identical to the setup with exogenous fiscal policy (see also Stehn and Vines 2008, Eser et al. 2009). As this is an important result, let us discuss its intuition in some detail.

The optimal policy relies entirely on monetary policy with full asset market participation because it is a much more effective tool in stabilising shocks than fiscal policy. To see this, consider the stabilisation of a positive cost-push shock which requires a contraction of the real wage. From (19) we see that this can be achieved by raising interest rates to contract $c_O$ (reducing the real wage by $(1 + \theta \alpha) c_O$) and/or by cutting government expenditure (cutting the real wage by $\alpha (1 - \theta) g$). The key is that the contraction in $c_O$ has both a direct effect (on labour supply) and an indirect effect (via labour demand) on the real wage, while $g$ has only an indirect effect (via labour demand). Therefore, reducing $c_O$ by one unit has a larger effect on the real wage than reducing $g$, $(1 + \theta \alpha) > \alpha (1 - \theta)$.

Movements in both $c_O$ and $g$, however, carry a direct cost (through their respective squared terms in welfare) and an indirect cost (through the variability of output). Given this ‘comparative advantage’ of monetary over fiscal policy in controlling the real wage, the optimal policy will rely predominantly on contracting $c_O$. Government spending is not used at all because the use of $c_O$ to reduce inflation increases the cost of using $g$: a fall in $c_O$ reduces output, and so makes any cut in $g$ more costly such that no reduction in $g$ is desirable once the optimal reduction in $c_O$ has taken place (see Stehn and Vines 2009 for details). The optimal policy therefore leaves $g$ unchanged and the dynamics of the system are identical to the case with exogenous fiscal policy. This result is robust to an open economy setting, wage inertia and, under certain restrictions, also inflation persistence and government debt (Eser et al. 2009).

With limited asset market participation this result breaks down and fiscal policy becomes active. The optimal policy raises both the real interest rate and government expenditure with $\lambda > 0$ (see (24) and (25)). This optimal policy mix – of contractionary monetary policy and expansionary fiscal policy – appears counter-intuitive.

Increasing government spending is optimal because it dampens the fall in output and ensures more stable $n_O$, which, in turn, improves on the uneven distribution of welfare losses across agents. The increase in $g$, however, also puts upwards pressure on the real wage and hence prices. To counter the resulting inflationary pressures, the policymaker raises the real interest rate by more than with exogenous fiscal policy:

$$\phi^o - \phi^x = \frac{\lambda \varepsilon (1 - \theta) \Psi(\varphi + \lambda \Psi)}{\Psi \Theta} \geq 0$$

where $\Psi = 1 + \varphi (1 + \theta \varphi) > 0$, and $x^o$ and $x^x$ denote variable $x$ under optimal and exogenous fiscal policy, respectively. As the interest rate is increased by more, the optimal

---

19 Because of comparative advantage, both the optimal response of $c_O$ and of $g$ imply the same $\delta c_O/\delta \pi$. And once monetary policy is set such that $c_O = -\frac{\pi}{\bar{\zeta}}$, then the effect of $c_O$ on the optimal choice of $g$ will exactly cancel the effect of $\pi$ on the optimal choice of $g$ and there will be no change in $g$.

20 We again restrict ourselves to the region where the optimal policy is determinate, for which $\lambda < \hat{\lambda}$ is a sufficient condition.

21 The negative wealth effect associated with the increase in government expenditure also reduces $c_O$. 
policy is determinate for a larger share of non-asset holders than with exogenous fiscal policy (see dashed line in Figure 2). It follows that the contraction of $c_O$ is always larger with optimising fiscal policy than with exogenous fiscal policy:

$$
c^0_O - c^x_O = -\frac{\lambda \varepsilon (1 - \theta) (\phi + \lambda \Psi)}{(1 + \phi (1 + \theta \lambda \varphi))} \Theta \pi_t
$$

As monetary policy has a comparative advantage in affecting the real wage, however, a small additional fall in $c_O$ is sufficient to offset the upward pressure on the real wage resulting from the increase in $g$. The increase in $g$ hence outweighs the fall in $c_O$ such that aggregate demand and hours worked increases (see also dashed line in Figure 3). As a result, the contraction of $n_O$ is always smaller with optimising fiscal policy than with exogenous fiscal policy:

$$
n^0_{O,t} - n^x_{O,t} = -\frac{\lambda \varepsilon (1 - \theta) \Psi}{(1 + \varphi (1 + \theta \lambda \varphi))} \Theta \pi_t
$$

The cost of this policy mix compared to that with exogenous fiscal policy – namely higher volatility of $c_O$ and $g$ – is outweighed by the benefits of more stable $n_O$. As the share of non-asset holders rises, both the optimal fiscal response to inflation and the additional monetary tightening become larger (i.e. $\partial \psi^0 / \partial \lambda$, $\partial (\phi^x - \phi^0) / \partial \lambda > 0$). This active fiscal policy raises welfare above the level achieved with an exogenous government spending policy – especially for persistent shocks (Figure 2).

V. Extensions

We now explore to what extent the main result of the baseline model – that the optimal policy involves active use of government expenditure with limited asset market participation – continues to hold when a number of simplifying assumptions are relaxed. First, instead of logarithmic utility we consider a more general utility function over private and public consumption. The main effect of this generalisation is that $n_R$ becomes variable, and that aggregate hours adjustments are no longer borne by the asset holders alone. Second, we explore the optimal policy when the policymaker has access to transfers that can be directly targeted to the non-asset holders. Finally, we consider two alternatives to the assumption that a profit tax is available to finance government expenditure; a financing scheme that levies equal lump-sum taxes on both agents and endogenous debt accumulation.

A. CRRA Preferences

We start by considering a CRRA utility function, instead of the logarithmic utility function. Period utility is now given by ($j = O, R$):

$$
U_{j,s} = \frac{C_{j,s}^{1-\sigma}}{1-\sigma} + \chi \frac{C_{s}^{1-\sigma}}{1-\sigma} - \frac{N_{j,s}^{1+\varphi}}{1 + \varphi}
$$

(26)
where $\sigma$ is the inverse of the elasticity of intertemporal substitution. Appendix D.1 summarises the new first order conditions and the log-linearisation of the model. The new system is given by:

\begin{align*}
c_{O,t} &= E_t c_{O,t+1} - \frac{1}{\sigma} (r_t - E_t \pi_{t+1}) \\
n_{O,t} &= \frac{1}{\varphi} \omega_t - \frac{\sigma}{\varphi} c_{O,t} \\
c_{R,t} &= (1 + \eta) \omega_t \\
n_{R,t} &= \eta \omega_t \\
\omega_t &= \Upsilon (1 - \lambda) (\sigma + \varphi \theta) c_{O,t} + \Upsilon \varphi (1 - \theta) g_t
\end{align*}

where $\eta = (1 - \sigma) / (\varphi + \sigma)$ and $\Upsilon = (1 - \lambda (\varphi \theta + \sigma) (1 + \eta))^{-1}$. Most importantly, the labour supply of the non-asset holders, $n_R$, is now no longer constant. For empirically plausible calibrations ($\sigma > 1$), $n_R$ depends negatively on the real wage, because a higher real wage raises consumption and leisure. The real wage depends on $c_O$ and $g$ as before. Appendix B.2 shows that the social loss function is now given by:

\begin{equation}
W_s = \frac{\varepsilon}{\kappa} n_s^2 + \sigma (1 - \theta) g_s^2 + \theta \sigma ((1 - \lambda) c_{O,s}^2 + \lambda c_{R,s}^2) + (1 + \varphi) ((1 - \lambda) n_{O,t}^2 + \lambda n_{R,t}^2) - y_s^2
\end{equation}

The main difference to the logarithmic utility baseline is that the terms in $y$, $n_O$ and $n_R$ enter separately.

The policy problem now consists of choosing $c_O$ and $g$ to minimise (32), subject to (27)-(31). As no convenient closed-form solution is available, we will consider simulations of the optimal policy. Figure 4 (crossed line) shows that the introduction of CRRA preferences leaves the baseline result qualitatively unchanged. As a higher $\sigma$ raises the cost of consumption volatility in the social loss function, both $c_O$ and $c_R$ are contracted less than in the baseline, at the cost of higher inflation. Given the lower inter-temporal elasticity of substitution, the real interest rate has to be raised by more to reduce consumption. As before, government expenditure is raised in response to the shock – although less than before due to the variations in $n_R$.

### B. Targeted Transfers

We now assume that, in addition to setting public expenditure, the government can target a nominal lump-sum transfer payment $L_{R,t}$ to the non-asset holders, financed by a nominal lump-sum tax on the asset holders, $T_{O,t}$. The asset holder’s budget constraint is given by:

\begin{equation}
P_t C_{O,t} + R_t^{-1} A_{O,t+1} = A_{O,t} + (W_t N_{O,t} + P_t D_{O,t}) - T_{O,t}
\end{equation}

The non-asset holder’s problem consists of maximising (26) subject to:

\begin{equation}
P_t C_{R,t} = W_t N_{R,t} + L_{R,t}
\end{equation}
Figure 4: Impulse responses to a persistent cost-push shock with CRRA utility, targeted transfers and equal lump-sum tax financing.
Appendix D.1 summarises the first order conditions and the log-linearisation of the model. As in the previous section, \(c_O\) and \(n_O\) are given by (27) and (28), respectively. The targeted transfer is financed by a lump-sum tax on the asset holders, \(t_{O,t} = \lambda/(1 - \lambda)\). The remaining system is described by:

\[
c_{R,t} = (1 + \eta)\omega_t + \frac{\varphi}{\varphi + \sigma}l_{R,t} \tag{34}
\]
\[
n_{R,t} = \eta\omega_t - \frac{\sigma}{\varphi + \sigma}l_{R,t} \tag{35}
\]
\[
\omega_t = \gamma(1 - \lambda)(\sigma + \varphi\theta)c_{O,t} + \gamma\varphi(1 - \theta)g_t + \gamma\lambda\frac{\varphi(\sigma + \varphi\theta)}{(\sigma + \varphi)}l_{R,t} \tag{36}
\]

The targeted transfer raises \(c_R\) and lowers \(n_R\). The real wage now depends on \(c_O\), \(g\) and \(l_R\).

The introduction of targeted transfers has drastic consequences for the optimal policy mix. When the policymaker has access to a targeted transfer, in addition to setting monetary policy and government expenditure, the optimal instrument rules are given by (see Appendix D.2):

\[
r_t = \frac{\varepsilon}{\theta}\pi_t \tag{37}
\]
\[
g_t = 0 \tag{38}
\]
\[
l_{R,t} = \varepsilon(1 + \varphi)\pi_t \tag{39}
\]

The optimal policy raises transfers to the non-asset holders in response to rising inflation, while the interest rate and government expenditure responses are identical to the \(\lambda = 0\) case. The resulting equilibrium is the same as that in the full asset market participation setup, in which there is no role for active government expenditure (see Appendix D.2, dashed line in Figure 4).

The limited asset market participation setup effectively collapses to the standard representative agent model since with the help of the targeted transfer the policymaker can replicate the optimal consumption – and hours – path of an economy in which all agents have asset market access. As distributional costs are eliminated, the optimal policy with targeted transfers (the representative agent case) delivers a tighter disinflation and a higher level of welfare than without targeted transfers (the limited asset market participation case).\(^{22}\) This policy mix, however, requires access to a time-varying lump-sum transfers and the identification of asset and non-asset holders.

\[\text{C. Alternative Financing Assumptions}\]

Up until now we have assumed that government expenditure can be financed by a proportional profit tax, which is non-distortionary in our model. We now explore the

\(^{22}\text{For our baseline calibration } (\lambda = 0.4, \rho = 0.75), \text{ the availability of targeted transfers raises welfare by a substantial 0.35 percent of steady-state consumption.}\]
implications of two alternative financing means: an equal lump-sum tax on the two consumer groups and the accumulation of government debt.

1. Equal Lump-Sum Tax

Let us first consider an equal lump-sum tax on both agents instead of the proportional profit tax. To analyse this case we use the setup of the targeted transfers in the previous section and set \( g_t = t_{O,t} = -l_{R,t} \). While \( t_{O} \) is non-distortionary – and essentially the same as the proportional profit tax because dividends are rebated to asset holders – a lump-sum tax on the non-asset holders, \(-l_{R}\), affects \( c_{R} \) one-for-one. The policy problem is the same as in the previous section, except that the policymaker only chooses \( c_{O} \) and \( g \) while internalising the new financing constraint, \( g_t = -l_{R,t} \).

Figure 4 (dotted line) presents simulations of the optimal policy. Interestingly, while the optimal fiscal policy is still active with the new financing scheme, government expenditure is reduced in response to the positive cost-push shock. This is because the policymaker sets government expenditure in an attempt to replicate the targeted transfer to the non-asset holders discussed above, i.e. cut public expenditure and transfer the resulting resources to consumers (compare the dotted and dashed lines). While the asset holders simply save the windfall, the non-asset holders raise \( c_{R} \) and reduce \( n_{R} \) as was the case with the targeted transfer. These results suggest that lump-sum transfers – even if untargeted – are a powerful tool for redistributing welfare losses compared to government expenditure.

2. Government Debt

Second, we assume that the government can issue public debt to finance its expenditure. In doing so we abstract from the proportional profit tax and the equal lump-sum tax and return to the case of logarithmic utility \((\sigma = 1, T_{O,t} = L_{R,t} = 0)\). The evolution of one-period nominal debt is given by:

\[
\Delta B_{t+1} = (1 + r_t) \left( B_t + P_t g_t - \tau P_t Y_t \right)
\]

where \( \tau \) is a constant labor-income tax rate that is levied on both agents. Linearisation yields:

\[
b_{t+1} = r_t + \frac{1}{\beta} \left( b_t - \pi_t + \frac{(1 - \theta)}{B} g_t - \frac{\tau}{B} y_t \right)
\]

where we defined the real debt stock is as \( B_t = \bar{B}_t / P_{t-1} \) and \( B \) as the steady-state ratio of debt to output, which determines the steady-state tax rate, \( \tau = (1 - \beta) B + (1 - \theta) \). All other log-linearised equations of the model remain unchanged. An important choice is now the calibration of \( B \) (Leith and Wren-Lewis 2007). We will compare a low debt \((B = 0)\) with a ‘high’ debt economy \((B = 0.2)\).\(^{23}\)

\(^{23}\)With one-period debt, the entire stock of debt is refinanced every quarter at the short-term interest rate, \( r \), giving monetary policy large leverage over the debt stock. While \( B = 0.2 \) implies an annualized debt-to-GDP ratio of only 5 percent, it is a high amount of short-term public debt.
Figure 5: Impulse responses to a persistent cost-push shock with government debt.
The introduction of government debt clearly alters the determinacy properties of the model. Determinacy now not only depends on the monetary-policy response to inflation, but also on the fiscal feedback to debt (Leeper 1991, Kirsanova and Wren-Lewis 2007). For $\lambda < \lambda^*$ determinacy is ensured if monetary policy fulfils the Taylor principle and fiscal policy responds to the debt stock in a stabilising manner.

The policy problem now consists of the minimisation of (22) subject to (15), (17), (19) and (40). Notice that, with two state variables in the system, the optimal feedback coefficients are no longer functions of inflation alone. In the low-debt economy the government continues to increase government expenditure initially and then reduces government expenditure to control debt (dashed line in Figure 5). The positive response of government expenditure to the cost-push shock, however, ceases to be optimal with higher steady-state debt (dotted line). The reason is as follows.

Leith and Wren-Lewis (2007) have shown that, following a shock, time consistency requires government debt to be returned to its pre-shock level. As the interest rate rises in response to the shock and output contracts, the debt stock rises. The higher the steady-state level of debt, the more debt accumulates for a given increase in the interest rate, and the harder it will be for the policymaker to increase government expenditure, as debt needs to be returned to its pre-shock level. Therefore, while for low steady-state levels of debt the optimal policy still involves an increase in government expenditure (though less than with lump-sum taxes), government spending is cut for higher ratios of debt. Depending on the calibration, the introduction of debt may hence overturn the positive response of government expenditure to cost-push shocks.

This conclusion, however, is dependent on studying discretionary policies – as we have done throughout this paper. The optimal commitment policy need not return debt to its pre-shock level (Benigno and Woodford 2003) and, therefore, would continue to increase government expenditure in response to positive cost-push shocks for high-debt calibrations (results available upon request).

VI. Conclusion

Using an extension of the standard New Keynesian model, we have shown that the inclusion of limited asset market participation creates a case for active fiscal stabilisation policy. With full asset market participation, the optimal policy relies entirely on raising the interest rate to stabilise cost-push shocks, while government expenditure is inactive. When asset market participation is constrained, optimal fiscal policy becomes active.

In the baseline model the optimal policy raises government spending and increases interest rates by more than with exogenous fiscal policy. The combination of contractionary monetary and expansionary fiscal policy to stabilise a positive mark-up shock appears counter-intuitive. It is optimal, however, because an increase in public expenditure enables a more balanced distribution of welfare losses across the two groups of consumers during
disinflation – and thereby raises social welfare. While tight monetary policy ensures the stabilisation of inflation, expansionary fiscal policy plays a redistributional role.

We then showed that the optimal policy mix is sensitive to three extensions. First, government expenditure would not be used actively in an economy with non-asset holders if the policymaker had access to a targeted lump-sum transfer. If such an instrument were available, the optimal policy would replicate the full asset market participation case. In practice, means-tested benefits are likely to achieve some degree of targeting and hence perform part of the redistributional role of fiscal policy. Second, we showed that the optimal policy reduces government expenditure in response to positive cost-push shocks if financed by an equal lump-sum tax on both agents. By cutting government expenditure and rebating the available resources to the consumers, the policymaker attempts to replicate the targeted transfer policy. Finally, government expenditure (under optimal discretionary policy) would not be expanded in an economy with high steady-state debt. Intuitively, a policymaker in such an economy would find it prohibitively difficult to expand public spending during periods of falling tax revenues and rising interest payments.

In this paper we have not addressed two other interesting extensions. First, we argued that the optimality of an increase in government expenditure relies on the monetary authority’s ability to respond to the additional inflationary pressures from the fiscal expansion. One would therefore expect optimal fiscal policy to abandon its redistribution role and act to stabilise inflation in economies with a fixed exchange rate or a binding zero bound for the nominal interest rate. Second, we have not studied the implications of distortionary taxation, which have a direct effect on prices. We are planning to explore both of these extensions in future work.
A. Derivation of the Baseline Model

1. Steady State

From the Euler equation (2) the steady-state interest rate is given by \( R = 1 + r = \beta^{-1} \).

The first order condition of (10) determines the steady-state real wage, \( \frac{w}{P} = \frac{1 - \lambda}{\varepsilon} \), and profits in steady state, \( O = \frac{1}{\varepsilon}Y \). Using the budget constraints of each group, consumption levels are given by:

\[
C_O = \frac{(\varepsilon - 1)}{\varepsilon}Y = C_R + D_O
\]

Equality of steady-state consumption across the two groups holds when steady-state profits, and hence dividends, are zero. This can be achieved by a steady-state lump-sum tax that eliminates profits.

2. Price Setting

Given the price setting problem, (10), the price that is chosen by firms that are able to reset their prices in period \( t \) is given by:

\[
\tilde{P}_t = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t \sum_{s=0}^{\infty} \gamma^s Q_{t,t+s} (1 - \Lambda_{t+s}) (1 - \varkappa) \frac{W_{t+s}}{P_{t+s}} P^\varepsilon_{t+s} Y_{t+s}}{E_t \sum_{s=0}^{\infty} \gamma^s Q_{t,t+s} (1 - \Lambda_{t+s}) P_{t+s}^{-1} P^\varepsilon_{t+s} Y_{t+s}}
\]

In equilibrium \( Q_{t,t+s} = \beta^s \left( \frac{C_{O,t}}{C_{O,t+s}} \right) \left( \frac{P_t}{P_{t+s}} \right) \) and hence the expression for the optimal price can be re-written as:

\[
\tilde{P}_t = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t \sum_{s=0}^{\infty} (\gamma \beta)^s \frac{C_{O,t} P_t}{C_{O,t+s} P_{t+s}} (1 - \Lambda_{t+s}) (1 - \varkappa) \frac{W_{t+s}}{P_{t+s}} P^\varepsilon_{t+s} Y_{t+s}}{E_t \sum_{s=0}^{\infty} (\gamma \beta)^s \frac{C_{O,t} P_t}{C_{O,t+s} P_{t+s}} (1 - \Lambda_{t+s}) P_{t+s}^{-1} P^\varepsilon_{t+s} Y_{t+s}}
\]

This expression can be log-linearised to give:

\[
\bar{p}_t = (1 - \gamma \beta) E_t \sum_{s=0}^{\infty} (\gamma \beta)^s [\omega_t + \mu - \nu]
\]  \hspace{1cm} (41)

where \( \bar{p}_t \) is the log of the optimal price set by those firms that reset their price at time \( t \), \( \nu = -\log (1 - \varkappa) \) and \( \mu = -\log \left( \frac{\varepsilon}{\varepsilon - 1} \right) \). The price index evolves according to:

\[
P_t = \left[ \gamma P_{t-1}^{\varepsilon} + (1 - \gamma) \tilde{P}_t^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}
\]  \hspace{1cm} (42)

Combining (41) and (42) yields (17) in the main text (see Gali and Monacelli (2005) for details).
B. Derivation of the Social Welfare Function

In this section we present the derivation of the social welfare function for the case of CRAA utility, which nest the baseline model for $\sigma = 1$.

1. The Social Planner’s Problem and the Flexible-Price Equilibrium

To determine the subsidy, $\kappa$, we contrast the social planner’s problem with the outcome under flexible prices. The social planner determines the allocation of consumption and production of goods in the economy to maximise utility, (26), subject to the production function and the income identity. The first order conditions for $Y$ and $G$ can, respectively, be written as $(Y_t - G_t)^{-\sigma} = Y_t^*$ and $(Y_t - G_t)^{-\sigma} = \chi G_t^{-\sigma}$, which in turn implies $C_t^{-\sigma} = \chi G_t^{-\sigma}$. In the steady state, these reduce to

$$(C^*)^{-\sigma} = (Y^*)^\varphi = (N^*)^\varphi = \chi (G^*)^{-\sigma}$$

where superscript ‘\*:’ denotes the efficient steady-state level. Substituting for $\frac{C_t^\sigma}{\chi} = \theta$ one can show that $\theta = \frac{\chi^{\frac{1}{1+\varphi}}}{\chi^{\frac{1}{1-\sigma}}}$. In the flexible-price equilibrium, profit maximisation implies that firms equate marginal costs with marginal revenues:

$$\left(1 - \frac{1}{\varepsilon}\right) = (1 - \kappa) (N_t^\varphi) (C_t^\varphi)$$

where superscript ‘\*:’ denotes the flexible-price level. If $\kappa = \frac{1}{\varepsilon}$, then hours in the flexible-price equilibrium are identical to those in the efficient steady state, $(N_t^\varphi) = (C_t^\varphi)$. If the government then implements spending in line with the social planner’s problem in steady state, then the flexible price steady state is the same as the efficient output level.

2. Second-order Approximation

Individual welfare for type $j$ ($j = O, R$) in period $t$ is:

$$U_{j,s} = \frac{C_{j,s}^{1-\sigma}}{1-\sigma} + \chi \frac{C_s^{1-\sigma}}{1-\sigma} - \frac{N_s^{1+\varphi}}{1+\varphi}$$

The second-order approximation to $U_{j,s}$ can be written as (ignoring terms independent of policy and of order higher than two):

$$U_{j,s} = C_j^{1-\sigma} \left(c_{j,s} + \frac{1}{2} (1 - \sigma) c_{j,s}^2\right) + \chi G_{1-\sigma} \left(g_s + \frac{1}{2} (1 - \sigma) g_s^2\right) - N_j^{1+\varphi} \left(n_{j,t} + \frac{1}{2} (1 + \varphi) n_{j,t}^2\right)$$
Using $U_s = (1 - \lambda) U_{O,s} + \lambda U_{R,s}$ aggregate welfare is given by:

$$U_s = C^{1-\sigma} \left[ \lambda c_{R,s} + (1 - \lambda) c_{O,s} + \frac{1}{2} \lambda (1 - \sigma) c_{R,s}^2 + \frac{1}{2} (1 - \lambda) (1 - \sigma) c_{O,s}^2 \right] + \chi G^{1-\sigma} \left[ g_s + \frac{1}{2} (1 - \sigma) g_s^2 \right] - N^{1+\varphi} \left[ \lambda n_{R,s} + (1 - \lambda) n_{O,s} + \frac{1}{2} \lambda (1 + \varphi) n_{R,t}^2 + \frac{1}{2} (1 - \lambda) (1 + \varphi) n_{O,t}^2 \right]$$

We notice that $n_t = y_t + \Delta_t$, where $\Delta_t = \ln \left( \int_0^1 \left( \frac{p(i)}{p} \right)^{-\varepsilon} \, di \right)$ is price dispersion as in Woodford (2003). We then take a second-order approximation of the income identity (15), and re-arrange:

$$\lambda c_{R,s} + (1 - \lambda) c_{O,s} = \frac{1}{\bar{\theta}} y_s + \frac{1}{2\bar{\theta}} y_s^2 - \frac{(1 - \theta)}{\theta} g_s - \frac{1}{2} (1 - \lambda) c_{O,s}^2 - \frac{1}{2} \lambda c_{R,s}^2 - \frac{1}{2} \frac{(1 - \theta)}{\theta} g_s^2$$

Using (43) we can show that the steady-state employment subsidy ensures $C^{1-\sigma} = \theta N^{1+\varphi}$ and $\chi G^{1-\sigma} = (1 - \theta) N^{1+\varphi}$, which allows us to eliminate linear terms in the social welfare function. Combining previous expressions we obtain:

$$U_s = \frac{1}{2} N^{1+\varphi} \left\{ y_s^2 - \frac{1}{2} \sigma (1 - \theta) g_s^2 - \frac{1}{2} \sigma \left[ \lambda c_{R,s}^2 + (1 - \lambda) c_{O,s}^2 \right] - \frac{1}{2} \lambda (1 + \varphi) n_{R,t}^2 + \frac{1}{2} (1 - \lambda) (1 + \varphi) n_{O,t}^2 \right\} - \Delta_t$$

Using $\sum_{s=0}^\infty \beta^s \Delta_s = \frac{1}{2} \frac{\varepsilon}{K} \sum_{s=0}^\infty \beta^s \pi_s^2$ (see Woodford 2003) we obtain:

$$U_s = \frac{\varepsilon}{K} \pi_s^2 + \sigma (1 - \theta) g_s^2 + \theta \sigma (1 - \lambda) c_{O,s}^2 + \lambda c_{R,s}^2 + (1 + \varphi) \left( (1 - \lambda) n_{O,t}^2 + \lambda n_{R,t}^2 \right) - y_s^2$$

For $\sigma = 1$ we have $n_{R,t} = n_{R,t}^2 = 0$ and using $n_t = (1 - \lambda) n_{O,t}$ the expression simplifies to (22) in the main text.
C. Solving for Optimal Policy

1. Optimal Monetary Policy with Exogenous Fiscal Policy

The first order conditions are given by:

\[
\frac{\delta H^x_t}{\delta \pi_t} = \frac{\varepsilon}{\kappa} \pi_t + L^\pi_t - L^\pi_{t-1} = 0
\]
\[
\frac{\delta H^x_t}{\delta y_t} = \left( \frac{\varphi + \lambda}{1 - \lambda} \right) y_t + L^y_t = 0
\]
\[
\frac{\delta H^x_t}{\delta c_{O,t}} = \theta (1 - \lambda) c_{O,t} - (1 + \theta \alpha) L^R_t - (1 - \lambda) \theta L^y_t = 0
\]
\[
\frac{\delta H^x_t}{\delta c_{R,t}} = \theta \lambda c_{R,t} + L^R_t - \kappa L^\pi_t - \lambda \theta L^y_t = 0
\]
\[
\frac{\delta H^x_t}{\delta L^R_t} = c_{R,t} - (1 + \theta \alpha) c_{O,t} = 0
\]
\[
\frac{\delta H^x_t}{\delta L^y_t} = y_t - \lambda \theta c_{R,t} - (1 - \lambda) \theta c_{O,t} = 0
\]

and (17). Using \( L^\pi_{t-i} = \pi_{t+i} = c_{j,t+i} = 0 \), for \( i > 0 \) we obtain the equilibrium outcomes as a linear function of the cost-push shock (the only state variable):

\[
\pi_t = \frac{\theta \Theta}{\kappa \varepsilon (1 - \lambda)(1 + \theta \varphi)^2 + \theta \Theta} \mu_t, \quad c_{O,t} = -\frac{\varepsilon \varphi (1 + \theta \varphi)}{\alpha (\kappa \varepsilon (1 - \lambda)(1 + \theta \varphi)^2 + \theta \Theta)} \mu_t
\]
\[
c_{R,t} = -\frac{\varepsilon \varphi (1 + \theta \varphi) (1 + \alpha \theta)}{\alpha (\kappa \varepsilon (1 - \lambda)(1 + \theta \varphi)^2 + \theta \Theta)} \mu_t, \quad y_t = (1 - \lambda) n_{O,t} = -\frac{\varepsilon \theta (1 + \theta \varphi) (1 - \lambda)}{\kappa \varepsilon (1 - \lambda)(1 + \theta \varphi)^2 + \theta \Theta} \mu_t
\]

where \( \Theta = 1 - \lambda + \theta (\varphi + \lambda (1 + \theta \varphi^2)) > 0 \).

2. Jointly Optimal Monetary Policy and Optimal Fiscal Policy

The first order conditions are given by:

\[
\frac{\delta H^o_t}{\delta \pi_t} = \frac{\varepsilon}{\kappa} \pi_t + L^\pi_t - L^\pi_{t-1} = 0
\]
\[
\frac{\delta H^o_t}{\delta y_t} = \left( \frac{\varphi + \lambda}{1 - \lambda} \right) y_t + L^y_t = 0
\]
\[
\frac{\delta H^o_t}{\delta c_{O,t}} = \theta (1 - \lambda) c_{O,t} - (1 + \theta \alpha) L^R_t - (1 - \lambda) \theta L^y_t = 0
\]
\[
\frac{\delta H^o_t}{\delta c_{R,t}} = \theta \lambda c_{R,t} + L^R_t - \kappa L^\pi_t - \lambda \theta L^y_t = 0
\]
\[
\frac{\delta H^o_t}{\delta g_t} = (1 - \theta) g_t - \alpha (1 - \theta) L^R_t - (1 - \theta) L^y_t = 0
\]
and (15), (17) and (19). Using $L_{t-i}^\pi = \pi_{t+i} = c_{j,t+i} = 0$, for $i > 0$, we can express the evolution of the economy under optimal policy as:

$$\pi_t = \frac{\theta (1 + \varphi (1 + \theta \lambda \varphi))}{\Omega} \mu_t, \quad c_{O,t} = -\frac{\varepsilon (1 + \varphi - \lambda \theta (1 + \varphi (2 + \theta \varphi)))}{\Omega} \mu_t$$

$$c_{R,t} = -\frac{\varepsilon ((1 + \theta) (1 + \varphi) - \lambda \theta (1 + \varphi (2 + \theta \varphi)))}{\Omega} \mu_t, \quad y_t = (1 - \lambda) n_{O,t} = -\frac{\varepsilon \theta (1 - \lambda) (1 + \varphi)}{\Omega} \mu_t$$

where $\Omega = [\theta (1 + \varphi + \varepsilon \kappa ((1 + \varphi) (\theta^{-1} - \lambda) + \varphi (1 - \lambda + \varphi (1 - \theta))) + \lambda \theta \varphi^2)] > 0$.

### D. Extensions

This section summarises the derivation of the model and the optimal policy problem with CRAA utility and targeted transfers.

#### 1. The Extended Model

While (2) remains unchanged, (4) and (3) are replaced by, respectively:

$$Q_{t,t+1} = \beta \left( \frac{C_{O,t}}{C_{O,t+1}} \right)^\sigma \frac{P_t}{P_{t+1}}$$

$$N_{O,s}^\varphi C_{O,s} = \frac{W_t}{P_t}$$

Log-linearisation yields (27) and (28) in the main text. The first order condition of the non-asset holders is given by:

$$N_{R,s}^\varphi C_{R,s} = \frac{W_t}{P_t}$$

Log-linearisation of this condition and the new budget constraint (33) yields:

$$\omega_t = \varphi n_{R,t} + \sigma c_{R,t}$$

$$c_{R,t} = \omega_t + n_{R,t} + l_{R,t}$$

Combining (44) and (45) produces (34). Substituting this expression into (45) yields (35). Combining these expressions with the linearised aggregation rules for consumption and hours and the linearised income identity produces (36) in the main text. Setting $l_{R,t} = 0$ in equations (34)-(36) yields (29)-(31).

#### 2. Optimal Policy with Targeted Transfers

When targeted transfers are available, the policymaker chooses $c$, $g$ and $l_R$ to minimise the following constrained loss function:
Solving the resulting first-order conditions produces the following equilibrium:

\[
\pi_t = \frac{\theta}{\theta + \varepsilon \kappa (1 + \theta \varphi)} \mu_t, \quad c_{O,t} = c_{R,t} = -\frac{\varepsilon}{\theta + \varepsilon \kappa (1 + \theta \varphi)} \mu_t
\]

\[
y_t = n_{O,t} = n_{R,t} = -\frac{\varepsilon \theta}{\theta + \varepsilon \kappa (1 + \theta \varphi)} \mu_t
\]

### E. The ‘non-Keynesian’ Case

In this appendix we very briefly discuss the ‘non-Keynesian’ region, for which the share of non-asset holders is sufficiently high (\(\lambda > \lambda^*\)) to overturn the conventional effects of monetary and fiscal policy. Bilbiie (2008)’s reasoning is as follows. Changes in the real interest rate, through inter-temporal substitution in \(c_{O,t}\), affect the real wage which, in turn, leads to variations in profits and thus dividend payments to the asset holders. If the share of non-asset holders is sufficiently high, the potential variations in profit income are strong enough to offset the interest rate effects on the demand of asset holders. An increase in the real interest rate then has an inverted effect on aggregate demand. For the same reason the effects of government expenditure can be inverted in this region (Bilbiie and Straub 2004, Rossi 2007).

The key point we make is that the main conclusion of the baseline model – that the optimal fiscal policy raises government expenditure in response to higher inflation – continues to hold in the ‘non-Keynesian’ region.

#### 1. Determinacy Issues

We assume that monetary and fiscal policy are set by (20) and (21), respectively, and set \(\lambda = 0.65 > \lambda^*\). As Bilbiie (2008) has shown, determinacy requires a reduction in the real interest rate – and hence the violation of the Taylor principle – with exogenous fiscal policy (Figure 6).\(^{24}\) This occurs because in a ‘non-Keynesian’ economy the real interest rate

\(^{24}\)As in Bilbiie (2008), determinacy is also ensured if the feedback on inflation becomes very large (\(\phi^*_\pi > 10\)), empirically implausible values which we do not report.
rate has an inverted effect on aggregate demand. When an endogenous fiscal response is introduced, determinacy continues to require a reduction in the real interest rate provided government expenditure reacts weakly to inflation. The Taylor principle is restored only if $\psi^e_\pi$ is strongly negative.

2. Optimal Policy

As in Bilbiie (2008), the optimal policy reduces the real interest rate in response to higher inflation in the ‘non-Keynesian’ region ($\alpha < 0$) with exogenous fiscal policy (see (23) in the main text). While the jointly optimal policy continues to violate the Taylor principle (see (24)) the optimal fiscal policy continues to raise government expenditure in response to higher inflation (see (25)). Given its contractionary effect, an increase in government expenditure helps reduce the real wage which, in turn, allows the interest rate to be cut by less than with exogenous fiscal policy and leads to an improvement in welfare.
References


