The paper discusses the case against a laissez faire approach to resource allocation and develops a model of supply bottlenecks. It argues that: (1) once budget constraints are hardened and credit markets begin to function appropriately, externalities associated with production bottlenecks and adjustment costs—other considerations aside—provide a case for subsidizing the costs of critical inputs for the state sector but not the new private sector; (2) the optimal subsidy declines as the private sector grows; and (3) the subsidy should be "financed" by taxing wage income in the state sector, which will strengthen incentives for workers to move.

JEL Classification Numbers:
D20, H23, L50, P40

1/ Mr. Aizenman is Professor of Economics at Dartmouth College. He was a Visiting Scholar in the Research Department when this paper was written.
## Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summary</td>
<td>iii</td>
</tr>
<tr>
<td>I.   Introduction</td>
<td>1</td>
</tr>
<tr>
<td>II.  An Analytic Framework</td>
<td>5</td>
</tr>
<tr>
<td>1.   Production</td>
<td>5</td>
</tr>
<tr>
<td>2.   Prices of intermediate products</td>
<td>9</td>
</tr>
<tr>
<td>3.   Resource allocation: the one period case</td>
<td>10</td>
</tr>
<tr>
<td>4.   Comparative statics</td>
<td>13</td>
</tr>
<tr>
<td>5.   Optimal investment with adjustment costs:</td>
<td>14</td>
</tr>
<tr>
<td>the two period case</td>
<td></td>
</tr>
<tr>
<td>III. Policy Conclusions</td>
<td>16</td>
</tr>
<tr>
<td>Appendix I. On the Economics of Bottlenecks</td>
<td>20</td>
</tr>
<tr>
<td>Appendix II. Derivations</td>
<td>26</td>
</tr>
<tr>
<td>References</td>
<td>30</td>
</tr>
<tr>
<td>Text Charts</td>
<td></td>
</tr>
<tr>
<td>Figure 1</td>
<td>14a</td>
</tr>
<tr>
<td>Figure 2</td>
<td>14a</td>
</tr>
<tr>
<td>Figure 3</td>
<td>16a</td>
</tr>
<tr>
<td>Appendix I Charts</td>
<td></td>
</tr>
<tr>
<td>Figure 4</td>
<td>20a</td>
</tr>
<tr>
<td>Figure 5</td>
<td>24a</td>
</tr>
<tr>
<td>Figure 6</td>
<td>24a</td>
</tr>
</tbody>
</table>
Summary

This paper discusses the case against countries using a laissez-faire approach to resource allocation in restructuring the inherited state industrial sector. The analysis focuses on externalities associated with supply bottlenecks and adjustment costs. While much of the concern with bottlenecks has centered on impediments to international trade, bottlenecks can also arise whenever the requirements for certain inputs to production are stochastic (such as needs for energy sources or spare parts) and the opportunity cost of holding inventories is high. These conditions are likely to prevail in the state industrial sector—whose creditworthiness is currently limited by its outdated production technologies—once budget constraints are hardened and credit markets begin to function effectively.

In modeling the externalities associated with production bottlenecks and considering the policy implications in the presence of adjustment costs, the paper recognizes that producers have incentives to enter into pooling arrangements, supported potentially by market mechanisms, for reallocating stocks of critical inputs. Such arrangements, however, do not suffice to eliminate the externalities. Moreover, the externalities rise in a highly nonlinear manner (for example, exponentially) as critical inputs become more scarce.

Although many other factors need to be considered in designing policies to influence resource allocation. The analysis suggests, first, that once budget constraints are hardened and credit markets begin to function appropriately, the externalities associated with production bottlenecks and adjustment costs provide a case for subsidizing the costs of critical inputs for the state industrial sector but not for the new private sector. Second, the appropriate policy has an important time dimension, with the optimal subsidy declining as the private sector grows. Finally, countries should "finance" the subsidy by taxing the wage income generated in the state sector, which will strengthen incentives for workers to move out of that sector. It is also suggested that the provision of such subsidies be governed largely by rules rather than by discretion and that eligibility requirements be made conditional on maintaining wage restraint and meeting prespecified benchmarks in restructuring and in other enterprise reforms.

Although financing requirements constrain the size of the subsidies that can be provided to the state sector without undermining macroeconomic stability, countries should view the amount of financing to raise as a fundamental policy choice in designing their reform strategy. Their willingness and ability to finance a gradual or moderate-speed contraction of the state industrial sector—and thereby to avoid a rapid contraction of that sector—may be crucial in maintaining popular support for the transformation effort and making it credible that the reform program can be sustained. This in turn may be crucial for obtaining the financial support of domestic savers and foreign private investors.
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I. Introduction

At the outset of the transition to a market-oriented economy, much of the population is employed by large state-owned enterprises, the mobility of the workforce is generally low, many enterprises are engaged in production activities that cannot be sustained without financial support from the state, and much of the physical capital stock is too obsolete or immobile to shift into profitable new activities. The policy authorities thus face the challenge of designing a strategy for extensively shrinking or eliminating the state industrial sector over the medium run without generating during the transition a degree of economic hardship that is too severe and widespread to sustain the political support for, and credibility of, the transformation effort.

This paper focuses on one of the many important dimensions of the strategy for transforming the state industrial sector--namely, the strategy for attempting to ensure that the allocation of resources toward state enterprises is appropriate. The issues that arise need to be addressed in a multiperiod framework, focusing not only on the appropriate allocation of resources in the short run, but also on ways to ensure that resource allocation promotes the transformation process and encourages labor and capital to shift into relatively productive activities at an appropriate speed.

1/ For discussions of other dimensions of the problem of transforming the state industrial sector, see for example, Bruno (1992), Dooley and Isard (1992), Fischer (1992), Sachs (1991), and van Wijnbergen (1992). One other dimension of the problem is to find ways of creating effective ownership and control, which requires both a privatization process that leaves enterprises under effective private control and the introduction of procedures and incentives to insure appropriate behavior on the part of management and workers in the period until privatization occurs. Another dimension involves collecting and processing information required for creditors, potential private owners, or other decision makers both to assess the financial performance of state-owned enterprises (SOEs) in their initial activities and to evaluate the range of prospects for restructuring the enterprises. A third dimension is to find effective ways of hardening enterprise budget constraints, which requires mechanisms to curtail automatic transfers to SOEs from the general government as well as mechanisms to discourage the provision of passive financing to SOEs by banks and other enterprises. In addition, progress in downsizing the state industrial sector can be greatly enhanced through measures aimed at strengthening the social safety net, increasing labor mobility, and otherwise reducing the adjustment costs borne by workers, and through measures aimed at improving economic infrastructure, strengthening the legal and institutional framework for commercial activity, opening the economy to foreign investment, and otherwise enhancing the attractiveness of new private capital formation.
The starting point is to recognize that a number of factors argue against a laissez faire approach to resource allocation. Some of these factors tend to distort resource allocation away from new private enterprises. Such factors include perverse incentives for individual banks (whether state-owned or private) to roll over nonperforming loans, as well as incentives for established enterprises to behave as a group in ways that make it difficult for the banking system to impose hard credit constraints. Other factors, such as adjustment costs and supply bottlenecks, may create a tendency for resource allocation toward state enterprises to fall short of the socially-optimal level. In particular, various types of "adjustment costs" limit the speed with which labor and capital can be absorbed in a productive way into the new private sector, which raises the prospect of high rates of unemployment in the short run if resource allocation to the state industrial sector is curtailed sharply. Furthermore, to the extent that the state industrial sector is particularly prone to supply bottlenecks linked to the availability of resources, a sharp curtailment of resource allocation to the state industrial sector raises the prospect of socially-excessive output losses.

The balance of these considerations depends, essentially by definition, on the extent to which enterprises face hard budget constraints and credit markets function appropriately. Experience suggests that, without hard budget constraints and appropriately functioning credit markets, resource allocation to the state industrial sector may well exceed the socially optimal level and restrain severely the expansion of the new private sector. By contrast, as this paper argues, if countries were to succeed in establishing hard budget constraints and appropriately functioning credit markets, resource allocation to the state industrial sector might well fall considerably below the socially optimal level. This prospect may partly explain why it has been difficult in practice to establish the credibility of hard budget constraints in transforming economies. Conversely, it suggests that efforts to make hard budget constraints credible and to establish appropriately functioning credit markets can be facilitated by a proper move away from a laissez faire approach to resource allocation.

The paper develops a model of the externalities associated with supply bottlenecks and analyzes their implications for resource allocation in the presence of adjustment costs. The analysis suggests that: (1) to the extent that enterprises face hard budget constraints and credit markets function appropriately, production bottlenecks are likely to arise in the state industrial sector; (2) the externalities associated with production bottlenecks and adjustment costs provide a case, ceteris paribus, for subsidizing the costs of critical inputs for the state industrial sector but not for the new private sector; (3) the appropriate policy in this context has an important time dimension, with the optimal subsidy declining as the new private sector grows; and (4) the subsidy should be "financed" by taxing

the wage income generated in the state industrial sector, which will strengthen incentives for workers to move out of the state sector.

While the prospect of large output losses arising from bottlenecks in obtaining supplies of critical inputs to production has raised alarm, much of the concern has been focused on bottlenecks to international trade. Such concern is particularly relevant for the countries of the former Council for Mutual Economic Assistance (CMEA) and, within that, the former Soviet Union (FSU), where to a large extent the production sectors inherited from the past are comprised of highly-specialized state enterprises that historically have been heavily reliant on other enterprises within the FSU or the former CMEA for intermediate inputs. Although the extensive division of labor may have generated substantial productivity benefits in the past, the multi-stage character of extensively specialized production processes—particularly where different stages of production are now concentrated in different countries—has created the potential for widespread supply bottlenecks and substantial production losses. Indeed, the sharp output declines experienced in recent years by the countries of the former CMEA have been partly associated with a substantial shrinking of trade among these countries, and there is much anecdotal evidence that the same phenomenon has been experienced by the states of the FSU. As many have emphasized, the international community can play a major role in using its technical expertise and negotiating power to assist in the design of policies and institutional mechanisms to mitigate the bottlenecks in cross-border trade—not only those bottlenecks arising from the nature of trade restrictions and customs procedures, but also those arising from inadequacies in financial payments and settlement systems.

It would be misleading, however, to suggest that obstacles to international trade are the only major source of supply bottlenecks in transforming economies. In general, bottlenecks can arise whenever the requirements for certain inputs to production are stochastic (such as the needs for energy sources or spare parts) and the opportunity cost of holding inventories is high. This source of bottlenecks merits particular attention in the context of the transforming economies, where efforts are being made to harden budget constraints and to establish appropriately functioning credit markets. Other things equal, such efforts are likely to restrict severely the availability of credit to the state industrial sector, where existing production patterns are largely dictated by investment decisions made in the past, when energy and other primary products were available at relatively low prices, and when a high degree of political and economic integration within the FSU and former CMEA made it much more sensible to invest in large-scale highly-specialized production units. To a large extent the curtailment of credit to the state sector is appropriate, since there are substantial welfare gains to be reaped from shifting production away from existing facilities to new best practice technologies. But the

1/ See, for example, Nordhaus (1992) and Wolf (1992).
adjustment requires new investment and will take time. Meanwhile, the curtailment of credit to the state sector, in the absence of appropriate countervailing policies, may well contribute to relatively low inventories of critical inputs in that sector, and thus to a relatively high incidence of production bottlenecks.

In analyzing this phenomenon and considering its policy implications, the model developed in this paper recognizes that producers have incentives to enter into pooling arrangements, supported potentially by market mechanisms, for reallocating stocks of critical inputs among the pool. It is demonstrated, however, that such arrangements do not suffice to eliminate the externalities associated with production bottlenecks. An interesting implication is that the externalities rise in a highly nonlinear manner (e.g., exponentially) with increases in the probability that production needs will exceed the average level of inventories, and hence with increases in the opportunity cost of holding inventories.

It is noteworthy that the issue of policy intervention to address the externalities associated with production bottlenecks has received little attention in the industrialized countries. /* This may well reflect the nonlinear nature of the externalities in combination with the fact that the industrialized countries do not have extensive sunk investments in outdated capital vintages. The latter fact implies that most producers in the industrial countries are relatively creditworthy and therefore able to maintain relatively appropriate levels of inventories. As a result, the nonlinear nature of the externalities suggests that the social costs of production bottlenecks in the industrial countries are relatively low.

In considering appropriate policies for reducing the probability and the expected costs of production bottlenecks in the transforming economies, several points from the literature deserve emphasis. The first is that bottlenecks in industries producing intermediate products can have far more costly effects on aggregate economic activity than bottlenecks in industries

/* See, however, U.S. General Accounting Office (1981) for a study conducted in the aftermath of the relatively severe capacity shortages that arose in the United States at times during the 1970s.
producing finished goods. The second is that the probability of bottlenecks needs to be analyzed in an intertemporal framework.

The rest of the paper is organized as follows. Section II begins by characterizing the relevant externalities in a one-period context, taking the structure of production as given. It then introduces investment opportunities and the process of allocating resources over time in a manner that promotes the growth of a new private sector and leads to the phasing out of the old state industrial sector. Section III addresses the implications for policies to subsidize the state industrial sector and for other policy measures. To streamline the presentation, the analytic framework underpinning the characterization of production bottlenecks is developed in Appendix I, and the derivations of most of the other analytic results are provided in Appendix II.

II. An Analytic Framework

1. Production

Consider an economy with two production sectors: X and Y. X is the state industrial sector inherited from the past; Y is the newly emerging private sector. In each sector output is produced with labor (L) and raw material (M). As the initial shock to the economy, central planning has been abandoned and prices have been "liberalized." Consequently, the prices of final outputs and raw material, which are all assumed to be tradable, are determined exogenously on world markets. Intermediate goods, which are produced in some subsectors of X, are assumed to be highly specialized and useful only for purposes of production in other subsectors of X. Under the relative prices that producers currently face, which involve a higher relative price of raw material than that prevailing prior to the shock, the technologies employed in X are no longer best-practice technologies; but the

2/ In our analysis the intertemporal dimension of policies is linked primarily to the contraction of the state industrial sector over time. By contrast, the discussion of intertemporal considerations in the literature on bottlenecks in industrialized countries has stressed that the capacity to supply critical inputs will evolve over time in response to the incentives created by excessive or deficient profits, relative price changes, and so forth. The industrial-country literature has also emphasized that shocks impinging on productive capacity decisions and the probability of bottlenecks at one stage of production can have indirect effects on capacity decisions and the probability of bottlenecks at subsequent stages. For example, in the United States during the 1950s, 60s, and early 1970s, capacity growth in the paper industry outpaced demand growth, and the relatively cheap price of paper had spillover effects on the printing industry; see U.S. General Accounting Office (1981), p. 15.
costs of adjusting technologies or shifting the physical capital stock into sector Y are assumed to be prohibitive. Thus, the expansion of production using efficient technologies requires new investment and a sacrifice of current consumption.

We concentrate attention on the case for policy measures to influence indirectly the allocation of resources between sectors X and Y. In doing so, we choose to abstract from issues of policy intervention in response to monopoly power. Accordingly, it is assumed that production in each sector is carried out by a large number of identical firms, and that resource allocation is the outcome of perfectly competitive market processes.

Sector X is modeled in a manner that recognizes the high degree of specialization in the state industrial sectors of eastern Europe and the former Soviet Union. There are assumed to be n subsectors—corresponding to different industries or, under careful interpretation, to different countries. Production in each subsector requires inputs of intermediate products from other subsectors, as well as inputs of labor and raw material. Thus, if production or output deliveries are reduced by bottlenecks in one subsector, other subsectors will also be forced to reduce production. In modeling the specialization of production in this way, we depart in some respects from the literature catalyzed by Ethier (1982), in which it is assumed that final output is assembled from various specialized components that are each produced without any requirements for intermediate inputs. 1/

We focus first on the economy during a single period of time. Sector Y is assumed to consist of \( \theta_y \) identical firms that combine labor and raw material using technologies that exhibit diminishing marginal productivity for each input. Total output in sector Y is described by

\[
Y = \theta_y \left( \frac{M_y}{\theta_y} \right)^\beta \left( \frac{L_y}{\theta_y} \right)^\gamma
\]

with \( 0 < \beta, \gamma < 1 \) and \( \beta + \gamma \leq 1 \)

where \( M_y \) and \( L_y \) are the total inputs of raw materials and labor used by sector Y, with each identical firm using a fraction \( 1/\theta_y \) of the sector total.

For sector X, consider the case in which production is sequential, with subsector 1 providing intermediate inputs to subsector 2, which provides

\[
1/\text{The bulk of the literature that followed Ethier's contribution has been framed in a nonstochastic world, and has concentrated on demonstrating the relevance of endogenous division of labor for explaining the gains from trade. By contrast, our interest is to address issues that arise in a stochastic world, and to emphasize that the expected gains from a greater division of labor must be weighed against the expected losses from a greater potential for production bottlenecks.}
\]

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intermediate inputs to subsector 3, and so forth. 1/ Although a sequential production process implies in reality that a production stoppage in subsector \( k \) affects subsectors \( k+1, \ldots, n \) with a lag, it simplifies the notation to assume that the entire sequence takes place during a single period of time. 2/

For subsector \( k \) of \( X \), define

\[
\phi_k = \theta_k a_k \left\{ \frac{M_k}{\theta_k} \right\}^\beta \left\{ \frac{L_k}{\theta_k} \right\}^\gamma
\]

where \( \theta_k \) is the number of identical firms and \( M_k, L_k \) are the total inputs of raw material and labor used by the subsector, with \( 0 < \beta, \gamma < 1 \) and \( \beta + \gamma \leq 1 \). Each firm in subsector \( k \) faces some probability of incurring a production bottleneck, in which case the output of the firm is zero. For each of the firms, let \( Q_k \) denote the probability that no production bottleneck arises. The probabilities for individual firms are assumed to be identical (since the firms are identical) but independent. Letting \( X_k \) denote the quantity of output from subsector \( k \) and \( X_{k,r} \) the output of a representative firm in subsector \( k \), it is assumed that

\[
X_{1,r} = \begin{cases} 
\phi_1/\theta_1 & \text{with probability } Q_1 \\
0 & \text{with probability } 1-Q_1 
\end{cases}
\]

and for \( k=2, \ldots, n \)

\[
X_{k,r} = \begin{cases} 
\left[ \left( X_{k-1}/\theta_k \right)^\alpha + \left( \phi_k/\theta_k \right)^\alpha \right]^{1/\alpha} & \text{with probability } Q_k \\
0 & \text{with probability } 1-Q_k 
\end{cases}
\]

Recall that the constant-elasticity-of-substitution specification in (4) includes the special cases in which \( X_{k-1} \) (intermediate inputs) and \( \phi_k \) (labor

---

1/ While it seems technically feasible to extend the analytic framework to more complicated cases in which the input/output matrix for the subsectors of \( X \) is not diagonal, the qualitative nature of the policy implications would not be affected.

2/ The assumed correspondence between production sequences and time periods has no bearing on the substance of either the "single sequence" analysis or the "multiple sequence" analysis that will come later. The substantive results derive from the fact that a production stoppage or shortfall in subsector \( k \) leads to production stoppages or shortfalls of consistent duration or magnitude in subsectors \( k+1, \ldots, n \).
and raw material) are combined in a Leontief production function \((\alpha=-\infty)\), in a Cobb-Douglas production function \((\alpha=0)\), and in an additive production function with infinite elasticity of substitution \((\alpha=1)\). Note also that, according to (4), all the output from subsector \(k-1\) is used as intermediate inputs for subsector \(k\), and that firm \(r\) in subsector \(k\) is left with nothing to sell in the event of a production bottleneck.

In modeling the probability of production bottlenecks, it is assumed that \(M\) and \(L\) are inputs that must be obtained at the beginning of the production period. The production environment is considered to be uncertain in the sense that the amount of raw material needed to avoid production stoppages is stochastic. Thus, the probability of production bottlenecks depends fundamentally on the amount of raw material inputs on hand at the start of the production period relative to the scale of output. It is recognized that firms within a given subsector have incentives to enter into pooling arrangements for quickly obtaining additional "raw material" (including such things as spare parts, energy sources, and construction materials) from other firms in the subsector if it turns out that the other firms have more than they need. It is assumed, however, that firms cannot obtain additional raw material within the production period from sources outside their subsector. 1/

Appendix I develops an analytic framework based on these assumptions, which derives the reduced-form specification: 2/

\[
Q_{k,r} = Q_k(m_{k,r}, m_k)
\]

where \(m_{k,r} = M_{k,r}/L_{k,r}\), \(m_k = M_k/L_k\). While the representative producer chooses \(m_{k,r}\), the bottleneck probability it faces also depends on \(m_k\), which it takes as given. This is the source of the externality. Let \(\eta_{k,r}\) and \(\eta_k\) denote the elasticities of \(Q_k\) with respect to \(m_{k,r}\) and \(m_k\). Although these elasticities would be zero in the absence of production bottlenecks, within the framework developed in Appendix I it is shown that

\[
\eta_k = \frac{m_k}{Q_k} \frac{\partial Q_k}{\partial m_k} > 0; \quad \eta_{k,r} = \frac{m_{k,r}}{Q_k} \frac{\partial Q_k}{\partial m_{k,r}} > 0
\]

1/ Although it is convenient for purposes of the calculus to limit the size of pooling arrangements to subsectors, the main thrust of the analysis would not be affected by enlarging the pooling arrangements -- e.g., to all enterprises within the country. One rationale for focusing on subsector-wide pooling arrangements is that firms in different subsectors are likely to have different types of input needs.

2/ Note that since the scale of output depends on the scales of the two inputs, it is convenient to assume that \(Q_{k,r}\) depends on ratios of raw material to labor rather than ratios of raw material to the scale of output.
Using conditions (3) and (4), the expected values of the outputs at successive stages of the sequence can be written as

$$EX_1 = Q_1 \phi_1$$

$$EX_2 = Q_2 [(Q_1 \phi_1)^\alpha + (Q_2 \phi_2)^\alpha]^{1/\alpha} = [(Q_2 Q_1 \phi_1)^\alpha + (Q_2 \phi_2)^\alpha]^{1/\alpha}$$

and, in general, 1/

$$EX_k = \left[ \sum_{j=1}^{k} (Q(j) \phi_j)^\alpha \right]^{1/\alpha} \quad \text{for} \quad Q(j) = \frac{k}{i=j} Q_i$$

Note that allocational decisions affecting the probability of a bottleneck at any particular stage of the production process also affect the expected values of the outputs at all subsequent stages.

2. Prices of intermediate products

It is useful at this point to characterize the transfer prices of the nontradable intermediate inputs that will emerge from competitive market-clearing processes. To do so, we focus here on a two-stage production sequence in which the price of final (stage-2) output, as determined exogenously on world markets, is unity. We are interested in determining the price $P_1$ that would clear the market for $X_1$ if the demand for $X_1$ came entirely from the producers of $X_2$. Under the assumption of perfect competition, firms in subsector 2, which individually take $P_1$ as given, would demand inputs of $X_1$ up to the point where the expected marginal revenue was equal to the marginal cost $P_1$. As shown in Appendix II, the market-clearing price is closely approximated by

$$P_1 = Q_2 \left[ 1 + \left( \frac{\phi_2}{Q_1 \phi_1} \right)^\alpha \right]^{(1-\alpha)/\alpha}$$

It is instructive to note that, in the limiting case with zero probability of production bottlenecks (i.e., $Q_1 = Q_2 = 1$), $P_1 > 1$. This essentially reflects the gains from the specialization of production. 2/

More generally, if firms in subsector 2 faced a low probability ($Q_2$) of

1/ This result reflects the independence of the probabilities of production bottlenecks and the law of large numbers.

2/ It is implicitly assumed here that a single-stage process for producing final output would take the same form as $\phi_1$ or $\phi_2$. 

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avoiding production bottlenecks, the market clearing $P_1$ could be less than unity. 1/

3. Resource allocation: the one period case

The next steps are to consider how resources will be allocated in a laissez faire environment in which competitive firms maximize expected profits, and then to compare the competitive allocation with the allocation that is socially optimal given a particular social objective function. The focus here remains on a two-stage production process, with the extension to $n$ stages described in Appendix II. Firms in both stages take as given the prices of raw material, $P_m$, and the wage rate of labor, $W$.

Producers in subsector 1 individually chose $L_{1,r}$ and $M_{1,r}$ (for $r=1,\ldots,n_1$) to maximize their expected profits; as shown in Appendix II, the optimizing decisions of the identical individual firms imply:

$$L_1 = M_1 = \frac{s_1 E(X_2)}{\frac{1}{W} (\gamma \eta_{1,r})}$$

(9) \hspace{1cm} (10)

where

$$E(X_2) = Q_2 [(Q_1 \Phi_1)^\alpha + (\Phi_2)^\alpha]^{1/\alpha}$$

(11)

$$s_1 = \frac{(Q_1 \Phi_1)^\alpha}{(Q_1 \Phi_1)^\alpha + (\Phi_2)^\alpha}$$

(12)

1/ The latter possibility reflects an assumption that there is no way to market $X_1$ other than through sales to firms in subsector 2. For the case in which $X_1$ could also be sold to domestic households or exported, the price paid by subsector 2 would not fall below the price offered by other prospective purchasers. It can be verified that our discussion continues to hold in this case. The main effect of allowing producers at stage 1 to export or sell to domestic consumers is to shift the costs associated with bottlenecks toward stage 2 (or with $n$ stages of production, toward the downstream stages).
Producers in subsector 2 choose $X_{1,2}^r$, $L_{2,r}$, and $M_{2,r}$ (for $r = 1, \ldots, n$) to maximize their expected profits. This implies:

\begin{align}
(13) \quad L_2 &= \frac{E(X_2)}{w} (s_{21} - \eta_{2,r}) \\
(14) \quad M_2 &= \frac{E(X_2)}{p_m} (s_{22} + \eta_{2,r})
\end{align}

where

\begin{equation}
(15) \quad s_{21} = 1 - s_1 = \frac{(\phi_2)^\alpha}{(Q_1 \phi_1)^\alpha + (\phi_2)^\alpha}
\end{equation}

It is shown in Appendix II that

\begin{equation}
(16) \quad s_1 = \frac{p_1 E(X_1)}{E(X_2)}
\end{equation}

Thus, $s_1$ and $s_2$ can be interpreted as shares of the expected value of output from the final stage of production, with $s_1$ corresponding to the ratio of the expected value of subsector 1 output to the expected value of subsector 2 output.

Consider next the socially optimal allocation of resources when the social welfare function has the form

\begin{equation}
(17) \quad V = \delta [C + \omega (L - L)] + (1 - \delta)L
\end{equation}

where $C$ is the expected level of final consumption, $L$ is total employment in the economy ($L_1 + L_2 + L_m$), and $L$ is the full employment level of $L$ (or the endowment of leisure). Note that for $\delta = 1$, $\omega$ can be interpreted as the shadow price of leisure (labor) relative to final consumption, and welfare can be viewed to depend simply on the consumption of goods (and services) and leisure. In the more general case $0 < \delta < 1$, condition (17) also allows for the possibility that employment is seen as providing direct social or political benefits, over and above its implications for consumption and leisure. Although our analysis concentrates for the most part on the case $\delta = 1$, policy authorities have clearly exhibited strong reluctance to shut down enterprises with large numbers of employees, and the case $\delta < 1$ provides a crude way of incorporating that reluctance into the social objective function.
Another noteworthy result is that the size of the externality associated with production bottlenecks increases from one stage of production to the next, other things equal. Appendix II derives this result.

Under specification (17), it is shown in Appendix II that the socially optimal allocation is:

\[
L_1^* = \frac{s_1 E(X_2)}{W} (\gamma - \eta_1^*)
\]

\[
M_1^* = \frac{s_1 E(X_2)}{P_{m,1}} (\beta + \eta_1^*)
\]

\[
L_2^* = \frac{E(X_2)}{W} (s_2 \gamma - \eta_2^*)
\]

\[
M_2^* = \frac{E(X_2)}{P_{m,2}} (s_2 \beta + \eta_2^*)
\]

where

\[
\eta_k = \eta_k + \eta_{k,r} \quad \text{for } k = 1, 2
\]

\[
W' = W - \frac{1-\delta}{\delta}
\]

\[
P_{m,k}' = P_m - \frac{1-\delta}{\delta} \cdot \frac{\delta(L_1 + L_2 + L_y)}{\delta M_k}
\]

Now compare the competitive allocations and the socially optimal allocations. Notice first that, in the limiting case in which allocational decisions have no effect on the probability of bottlenecks \((\eta_1 = \eta_2, \eta_1, \eta_2, \eta_1 = 0)\), in which employment receives no direct weight in the social welfare function \((\delta=1)\), and in which the market price and shadow price of labor are the same \((W = W' \text{ and hence } W = \omega)\), the competitive allocations and the optimal allocations coincide. Next notice that in the presence of bottlenecks, since \(0 < \eta_{k,r} < \eta_k\) and \(P_{m,k} < P_m\), the competitive allocations of \(M_1\) and \(M_2\) are always less than optimal. By contrast, the competitive allocations of \(L_1\) and \(L_2\) may be either greater or less than optimal, depending on whether the externality associated with reducing unemployment (as reflected in \(W' \leq W\)) outweighs the externality associated with reducing the probability of production bottlenecks (as reflected in \(0 < \eta_{k,r} < \eta_k\)).

Another noteworthy result is that the size of the externality associated with production bottlenecks increases from one stage of production to the next, other things equal. Appendix II derives this result.
for the case of n stages of production. For the two-stage case, it can be seen from the above conditions for the $M_k, M_k^*, \text{and } \eta_k$ that (for $\delta=1, W=\omega, M_1^* - M_2$ is proportional to $s_1 \eta_1$ whereas $M_2^* - M_2$ is proportional to $\eta_2$; accordingly, for $\eta_1=\eta_2, (M_2^* - M_2)/(M_1^* - M_1) = 1/s_1 > 1$. The intuition for this result comes from condition (7). For the case in which $\phi_1$ and $\phi_2$ are equally elastic with regard to $M_1$ and $M_2$ respectively, the presence of bottlenecks implies that the elasticity of $E_X$ with respect to $M_2$ exceeds the elasticity of $E_X$ with respect to $M_1$, since raising the probability of avoiding a bottleneck at stage two reduces the probability not only of sacrificing production at stage 2, but also of wasting the production from stage 1 that is used as inputs to stage 2.

4. Comparative statics

Figures 1 and 2 summarize the single-period comparative statics of the labor market and the demand for raw material. In Figure 1, the horizontal distance $O'O$ reflects the supply of labor. The demands for labor in sectors X and Y are measured to the right and the left of points $O$ and $O'$, respectively. The initial equilibrium is characterized by the solid demand curves ($L_X^*L_X^*$ and $L_Y^*L_Y^*$), the locations of which are drawn for a given price of $M$. The impact of the switch to a new regime is a substantial increase in the price of $M$, inducing a drop in the use of $M$ and corresponding declines in the demands for labor. Hence, the demand curves associated with competitive allocation will shift toward their origins to locations depicted by the broken lines. This shift will give rise to unemployment unless the wage rate is sufficiently flexible downward. Part of the loss of output, employment, and income can be avoided, however, through policy intervention to address the externalities associated with production bottlenecks in a manner that achieves the optimal allocation; such intervention will shift the demand curve for sector X to $L_X^*L_X^*$.

Figure 2 focuses on the demand for $M$ by sector X. Point A represents the initial position, where demand for raw material at the relatively low initial price is assumed to be sufficiently large to eliminate the possibility of production bottlenecks. With the increase in the relative price of raw material from $P_0^M$ to $P_1^M$, demand contracts and bottlenecks become probable. The light dotted line describes the hypothetical level of demand that would emerge if firms naively assumed $Q=1$. The broken curve is the relevant demand locus when firms take into account the probability of production bottlenecks, and B describes the amount of raw material that sector X would purchase at the price $P_1^M$ in a laissez faire environment. The dark solid line represents the optimal demand for raw material, which internalizes the bottleneck externality. Point C describes the optimal…

1/ For sake of the present illustration, it is assumed to be meaningful to focus on the demand curves associated with competitive allocation prior to the price shock.
Appendix I describes more extensively the nonlinear nature of the externality and the optimal subsidy. 

5. **Optimal investment with adjustment costs: the two period case**

Consider next the process through which the new private sector expands over time and draws resources (labor and raw material) out of the old sector. Our main interest is to characterize how policies should be adjusted dynamically as the transformation of production proceeds.

For simplification, consider the two period case. Investment occurs through an increase in the number of firms, and it is assumed that investment is warranted only in sector Y. The social objective function for the two period case is taken to be

\[
\Omega = V(1) + \frac{V(2)}{1+\rho}
\]

where \(V(1), V(2)\) are specified as in (17) for the first and second periods, and \(\rho\) is the time rate of discount.

The introduction of investment requires a corresponding sacrifice in consumption of final output during the first period. It is assumed that the marginal cost of creating new firms is an increasing function of the number of new firms, reflecting adjustment costs; in particular, the total cost of investment is assumed to be a quadratic function of the number of new firms. Thus, expected consumption in period 1 is represented as

\[
C(1) = E Y_2(1) + Y(1) - K[(\Delta Y_y) + \frac{\xi}{2} (\Delta Y_y)^2] + P_m [R_1-M(1)]
\]

where \(\Delta Y_y = Y(2) - Y(1)\) is the number of new firms in sector Y, \(R_1\) denotes the amount of raw material that is produced domestically during period 1, and \(P_m [R(1)-M(1)]\) can be viewed as income from net exports of raw material. 

Expected consumption during period 2 is represented as

\[
C(2) = E Y_2(2) + Y(2) + P_m [R(2)-M(2)]
\]

The socially optimal level of investment is found by maximizing \(\Omega\) with respect to \(\Delta Y_y\). Under various conditions, the socially optimal \(\Delta Y_y\) might differ from the amount of investment that would occur in a laissez-faire environment. This would be the case if the wage rate was different than the social shadow price of labor \((W-W')\), or if investors in a laissez-faire

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1/ Appendix I describes more extensively the nonlinear nature of the externality and the optimal subsidy.

2/ We treat \(R\) as exogenous and abstract from any costs of producing it.
environment formed biased expectations of period-2 prices and wages, perhaps because they failed to perceive the effects of policies on these variables.

Consider first the case in which private investment decisions are not distorted—in particular, the case in which $\delta=1$ and $W=\omega$ (such that the wage rate equals the social shadow price of labor), and in which producers in sector Y pay an exogenous unsubsidized world market price for M. As shown in Appendix II, the solution is

$$\Delta \theta = \frac{1}{K} \frac{\beta}{1+\rho} \left( \frac{\Pi_{y,r}(2)}{K} \right)$$

(28)

Here, $\Pi_{y,r}(2)$ is the profit level of the representative firm in sector Y during period 2; under condition (1)

$$\Pi_{y,r} = a_y \left( \frac{M_{y,r}}{L_{y,r}} \right)^\beta \left( \frac{M_{y,r}}{L_{y,r}} \right)^\gamma - P \cdot M_{y,r} - W_{y,r}$$

Investment thus depends positively on the present value of expected profits and negatively on adjustment costs. For the case in which the social objective function attaches a non-zero weight to employment ($\delta<1$) and there is unemployment, the term $\left( (1-\delta)/(\delta(1+\rho)) E L_{y,r}(2) \right)$ must be added to the numerator on the right-hand-side of (28), where $E L_{y,r}(2)$ is the expected employment level of the representative firm in sector Y during period 2; in addition, the expression for expected profits ($E \Pi_{y,r}(2)$) must be modified by substituting $\omega$ for $W$. The net effect of these modifications to (28) may imply that, under a policy of laissez faire, the rate of new investment in sector Y would fall short of the socially optimal rate, after taking into account the direct effect of additional employment in sector Y on social welfare.

Regardless of whether $\Delta \theta_y$ corresponds to the socially optimal rate, as the number of firms in sector Y increases, the demand for labor will expand. Thus, with reference to Figure 1, the curve $L_{y,r}$ will shift to the left over time, tending initially to eliminate unemployment and, eventually, to push up the wage rate. Once investment begins to have an influence on the wage rate, the bottleneck externalities in sector X will begin to decline, even with an unchanged number of producers in sector X. In particular, the higher relative price of labor will lead firms to reduce production through reductions in the use of both M and L. (This will also happen if labor starts to move out of the state sector in response to after-tax wage

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1/ Also assume that private investors in sector Y form unbiased expectations about the period-2 wage rate, taking into account the sensitivity of the wage rate both to the level of investment and to any period-1 policies to address production bottlenecks in sector X. The individual investor nevertheless assumes that the wage rate is independent of its own actions.
differentials induced by policies implemented to "finance" the subsidies.) Moreover, the decline in raw material use by sector X will be proportionately less than the declines in employment and output, thereby reducing the probability of raw material shortages. The latter effect implies that the optimal subsidy per unit raw material purchased by sector X will decline at the same time that the total demand for raw material by sector X declines.

This brings out the important intertemporal dimension of policy to address production bottlenecks. Such policy needs to focus both on measures to insure that investment in new firms and technologies proceeds over time at the socially optimal rate $\lambda$ and on the provision of appropriate subsidies to old firms to insure that the allocation of production inputs during each period of time is socially optimal. As the new private sector expands and draws labor out of the old state industrial sector, the size of the optimal unit subsidy to producers in the old sector shrinks to zero, and the total amount of revenue spent on subsidies (or foregone) by the government shrinks proportionately faster. This is illustrated in Figure 3 by the shift in the optimal allocation curves from $M^1_X$ to $M^2_X$ and the corresponding decline in the optimal subsidy from $u(1)$ to $u(2)$.

### III. Policy Conclusions

As the transformation efforts proceed in the former centrally planned economies, the discussion of various policy issues is becoming more focused. The debate over rapid reform versus gradualism is leading to growing recognition that in many areas early liberalization and rapid institutional/structural reforms may be both feasible and desirable, but that it is important to guard against the dangers of an excessively rapid output decline in the state industrial sector. This in turn is bringing wider recognition of the perils of a laissez faire approach toward the state industrial sector.

To a large extent, these dangers stem from supply bottlenecks and adjustment costs—two problems that have been highlighted in this paper. The state industrial sector inherited from the past is highly specialized, with many enterprises engaged in producing intermediate goods. This implies a risk of sizeable spillover effects from bottlenecks at any particular stage of production, or from obstacles to international trade where final output requires imported raw materials or intermediate inputs. The prospect of large output losses in the state industrial sector would not be such a concern if factors of production were mobile. But "adjustment costs" in the

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1/ Policy measures to reduce "adjustment costs" can also be welfare improving.

2/ It is also generally appreciated that the reform process, including its speed, needs to be conditioned to economic and political circumstances on a country by country basis.
FIGURE 3
transforming economies are high in the sense that much of the existing capital stock cannot easily be adapted to new activities and much of the existing labor force is immobile.

Experience suggests that, until budget constraints are hardened and credit markets begin to function appropriately, resource allocation to the state industrial sector is likely to exceed the socially optimal level and constrain the expansion of the new private sector. By contrast, this paper points to the danger that resource allocation to the state industrial sector might fall considerably short of the socially optimal level if countries were successful in establishing hard budget constraints and appropriately functioning credit markets. The latter prospect implies that it may be very difficult to sustain hard budget constraints and appropriately functioning credit markets under a laissez faire approach to resource allocation.

The analytic framework developed in the paper has emphasized two types of externalities in this context: one associated with production bottlenecks, the other associated with the social costs of unemployment. It has also emphasized that the externalities associated with production bottlenecks are highly nonlinear in nature and likely to be much more important in the former centrally planned economies than in advanced market economies. In particular, once credit markets begin to function appropriately, production bottlenecks are likely to be a prevalent phenomenon in the state industrial sector, where outdated technologies make it difficult for enterprises to obtain credit, thereby inducing them to hold relatively low inventories of critical production inputs.

One implication of the externalities—ceteris paribus—is that, along with establishing hard budget constraints and appropriately functioning credit markets, the optimal policy involves subsidizing the costs of critical inputs for the state industrial sector, where existing production patterns are largely dictated by investment decisions made in the past. Such subsidies should not be provided for the new private sector, since it is important to insure that investment in new facilities is directed toward technologies that are optimal under prevailing relative prices. The rationale for subsidization is both to reduce the probability of production bottlenecks and to avoid the undesirable effects on employment of any discrepancy between the wage rate and the social shadow price of labor. The case for subsidization, however, cannot be left unqualified. The important caveat is that a policy of subsidizing the state industrial

\[1\] Of course, as discussed in Section I, policies for addressing these externalities should not be designed without taking account of other factors that may tend to bias resource allocation toward the state industrial sector. Nor should the policy strategy for resource allocation be designed in isolation from the broader policy strategy for transforming the state industrial sector through privatization, steps to improve the management of state enterprises, and various reforms to the economic and legal environment.
sector will not necessarily be superior to a laissez faire regime if the subsidy mechanism is poorly designed. Perhaps more to the point, a laissez faire attitude toward the state industrial sector is not likely to be politically viable in practice, which implies that the relevant policy task is to choose an appropriate form of subsidy mechanism.

In this connection, the general wisdom that has developed emphasizes at least five important points: 1/ (i) the provision of subsidies should be guided largely by rules rather than discretion; (ii) the rules should define the subsidies ex ante, with enterprises taught to adhere to budget constraints established ex ante; (iii) there should be a well-defined and time-consistent strategy for phasing out the subsidies over time; (iv) the subsidies should not extend to the new private sector; and (v) the provision of subsidies should not be allowed to give rise to large monetary or fiscal imbalances.

The model developed in this paper suggests a form of subsidy that is consistent with points (i)-(iv): in particular, a subsidy on the price paid by the state industrial sector for critical inputs to production ("raw material"). 2/ In addition, it suggests that the size of the subsidies (per unit input) should decline to zero over time. Beyond this, admittedly, the model provides limited help in addressing the practical issues of identifying which critical inputs to subsidize, and of deciding the appropriate size of the subsidies. 3/

Efforts to address the externalities associated with supply bottlenecks should not focus on subsidization policies alone. To a large extent, the incidence of production bottlenecks within the FSU and former CMEA countries may reflect impediments to international trade in raw materials and intermediate inputs. Such impediments can be associated either with trade restrictions and customs procedures or with deficiencies in financial

1/ The first four of these points were stressed, for example, by János Kornai in a seminar presented during December 1992 at the International Monetary Fund on the evolution of financial discipline in post-socialist systems.

2/ Where the wage rate differs from the social shadow price of labor, the model also suggests subsidizing or taxing the wage rate paid by the state industrial sector.

3/ Although the model suggests that, in theory, the optimal subsidy per unit of critical inputs will generally differ in size for different subsectors of the state industrial sector, in practice a system of differential subsidies could create arbitrage incentives that would distort the effectiveness of the system. Thus, it may well be desirable not to subsidize costs at different rates for different state enterprises in the same region. Different subsidy rates for different regions might be considered, however, if there are regional differences in the probability of bottlenecks or in the social costs of unemployment.
payments and settlement mechanisms. Policies and institutional reforms to address these sources of bottlenecks can have substantial payoffs.

With regard to subsidization policies, one matter of serious concern is to prevent such policies from contributing to unwarranted wage increases or unproductive investments in the state industrial sector, or to delays in proceeding with restructuring efforts. This concern may suggest that the eligibility of a state enterprise to continue receiving subsidies should depend on it maintaining wage restraint and meeting various prespecified benchmarks in progressing with restructuring and other enterprise reforms.

Another matter of serious concern is to ensure that any subsidies provided to the state industrial sector are adequately "financed" to prevent them from leading to large monetary or fiscal imbalances. In this regard, wage restraint across the state industrial sector as a whole--or a tax on incomes earned in the state sector--is likely to be very important for maintaining macroeconomic stability. This is obviously the first best policy when the rationale for subsidization stems from circumstances in which the wage rate faced by state enterprises exceeds the social shadow price of labor. However, there is also a strong case for taxing the income of the workers in the state sector to provide "financing" when the rationale for subsidizing the state sector is to reduce the incidence of production bottlenecks. 1/ Not only will the subsidy policy tend to reduce the incidence of bottlenecks in the short run, but also, the differential between after-tax wages in the state sector and the new private sector will tend to increase the speed with which workers seek to move out of the state sector over time.

Although the financing constraint limits the size of the subsidies that can be provided to the state industrial sector without undermining macroeconomic stability, the amount of "financing" to raise for this purpose can be viewed as a fundamental policy choice in designing the reform strategy. The willingness and ability of countries to "finance" a gradual or moderate-speed contraction of the state industrial sector--and thereby to avoid a rapid contraction of the state sector--may be crucial in maintaining popular support for the transformation effort and making it credible that the reform program can be sustained. This in turn may be crucial for obtaining the financial support of domestic savers and foreign private investors. 2/

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1/ This, in a sense, amounts to taxing the additional income that is generated when subsidies to the state industrial sector lead to reductions in production bottlenecks and in unemployment.

2/ As a case in point, the impressive economic performance in China over the past 14 years, which has been facilitated by high rates of domestic savings and foreign capital inflows, has reflected policies aimed, inter alia, at achieving an orderly contraction of the state industrial sector.
On the Economics of Bottlenecks

The purpose of this Appendix is to provide a detailed example of the factors explaining the probability of production bottlenecks. The paper relies on the reduced form equation (5), which assumes that the probability of a bottleneck for the representative producer depends both on its own use of raw materials and on the average sector wide use. This Appendix derives equation (5) from a fully specified model.

Our framework highlights several key aspects of bottlenecks that should hold for other examples. First, the behavior of bottlenecks and the externalities associated with them are highly nonlinear: while bottlenecks are nonexistent when supplies of inputs are sufficiently high, they come to life for low supply levels, and scarcity intensifies them. Second, pooling and exchange arrangements, supported potentially by market mechanisms, are very useful in reducing the severity of bottlenecks. Third, rarely will such arrangements suffice to deal optimally with bottlenecks, so they should be supplemented by proper policies.

To develop the analysis, consider the behavior of a given producer in stage k. For notational simplicity we suppress the k index. The amount of raw material per worker (M/L) required for production is a random variable, which we denote by e and refer to as a minimum requirement. Hence, if the actual allocation of M/L for a representative producer is m:

\begin{align}
Q_r = \begin{cases} 
1 & \text{for } e \leq m \\
0 & \text{for } e > m
\end{cases}
\end{align}

The probability distribution of e is denoted by f(e), for 0 ≤ e ≤ d. To facilitate exposition we consider a continuous version of the model. The representative producer purchases raw material and hires workers ex-ante. If the realization of its minimum requirement is e₀ (see Figure 4), the producer needs only a fraction of its raw material to produce the planned output. Hence, such a producer has an excess supply of raw material (per worker) of m-e₀. If its realized minimum requirement exceeds m, it is not able to produce.

Consider first the symmetric equilibrium, where all producers are similar ex-ante, each holding m. Note that producers will benefit from a pooling arrangement, which will facilitate the transfer of materials from producers with excess supply to those with excess demand. Suppose that all producers in a given sector participate in such a pooling agreement, and that the heterogeneous needs of different sectors (or transaction costs) limit the pool to that sector.
FIGURE 4
The equilibrium can be characterized by an allocation rule that allows producers to buy up to $g$ units from the pool, where $g$ is determined by a clearing condition:

\[
\frac{m}{d} \theta \int (m-e)f(e)de + \theta \int mf(e)de = \theta \int (e-m)f(e)de \\
0 \quad m+g \\
m 
\]

The left hand side of (A2) stands for the supply to the pool, being the sum of the excesses of producers whose realizations of the minimum requirement are at points such as $e_0$ and $e_2$ (see Figure 4). The right hand side is the demand from the pool, stemming from producers at points such as $e_1$. Alternatively, (A2) can be rewritten as

\[
\frac{m+g}{m} = \int \theta e f(e)de \\
0 
\]

Equation (A3) determines the value of $g$ as an implicit function of $m$. It has a simple interpretation: the equilibrium achieved with the pool is equivalent to the one obtained if all producers pool ex-ante their raw material and reallocate the pool ex-post to those with the lowest realizations of the minimum requirement. Note from (A3) that, when the material held by the representative producer equals the expected value of the minimum requirement $e$, the pool solves completely the idiosyncratic uncertainty (i.e., if $m = E(e)$, where $E$ stands for the expectations operator, then $g = d - m$, and $Q = 1$). Thus, in the presence of effective pooling, bottlenecks occur only if $m$ is below $E(e)$. Henceforth we assume that this condition applies.

We turn now to characterizing the behavior of a marginal producer. The representative producer takes $g$ as given, being determined by the sector-wide average allocation of material, denoted by $m$ (i.e., $g = g(m)$, being determined by (A3)). The decision variable for the representative

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1/ Throughout our discussion we assume that the stochastic minimum requirements are distributed independently across the producers. Thus, all shocks are idiosyncratic such that $f(e)$ can be used to characterize the realized distribution of $e$ across producers.

2/ We can envision various mechanisms that will support the pooling system. One possibility is a mechanism where the pool auctions the material to the highest bidder and redistributes all profits to producers in a uniform manner.
producer is his own allocation $m_r$, which determines his probability of successful production ($Q_r$) according to

$$Q_r = \int_0^{m_r+g} f(e) \, de$$

(A4)

Applying (A4) we infer that, in the symmetric equilibrium (where $m_r = m$, $Q_r = Q$) the various elasticities of the probability that no bottleneck will occur are given by:

$$\eta_r = \frac{\partial \log Q_r}{\partial \log m_r} = \frac{m_r}{Q_r} f(m_r+g)$$

(A5)

$$\eta = \frac{\partial \log Q_r}{\partial \log m} = \frac{m}{Q_r} f(m+g) \frac{\partial g}{\partial m}$$

(A6)

$$\bar{\eta} = \eta + \eta_r = \frac{m}{Q} f(m+g)(1 + \frac{\partial g}{\partial m})$$

(A7)

The gap between $\bar{\eta}$ and $\eta_r$ is a measure of the externality generated by the presence of bottlenecks, and is proportional to $\partial g/\partial m$. This gap can be viewed as a version of a congestion externality: the atomistic producer overlooks the external effect of a marginal increase in its allocation, which marginally increases $g$ for all producers. From (A3) we infer that

$$\frac{\partial g}{\partial m} = \frac{1}{(m+g)f(m+g)} - 1$$

(A8)

Hence:

$$\bar{\eta} = \frac{m}{m+g} \frac{1}{Q}$$

(A9)

and

$$\frac{\eta - \eta_r}{\eta} = \frac{1 - \frac{m+g}{(m+g)f(m+g)}}{\frac{1}{(m+g)f(m+g)} - 1} = \frac{\partial g}{\partial m}$$

(A10)

The last equation measures the percentage difference between the total elasticity of the probability $Q$ with respect to the allocation and the elasticity of the probability $Q$ as perceived by the representative producer. Thus, (A10) measures the relative magnitude of the bottleneck externality.

To assess the policy impact of this externality consider the case of two production stages, as described by equations (9)-(24) of the paper. The policy maker may design a policy to internalize the externality by adopting
the proper set of subsidies cum taxes. The optimal policy should equate the first order conditions facing the representative producer with those facing the social planner. For example, let \( u_k \) denote the optimal subsidy to the price of raw material in stage \( k \), and assume that \( \delta = 1 \). Optimal policies should satisfy:

\[
\begin{align*}
\frac{(\beta + \bar{\eta}_1) s_1 E(X_{2\,1})}{P_m} &= \frac{(\beta + \eta_{1\,r}) s_1 E(X_{2\,r})}{P_m (1 - u_1)} \\
\frac{(s_2 \beta + \bar{\eta}_2) E(X_{2\,r})}{P_m} &= \frac{(s_2 \beta + \eta_{2\,r}) s_1 E(X_{2\,r})}{P_m (1 - u_2)}
\end{align*}
\]

From which we infer that

\[
(13) \quad u_1 = \frac{\eta_1}{\beta + \bar{\eta}_1}; \quad u_2 = \frac{\eta_2}{s_2 \beta + \bar{\eta}_2}
\]

A similar procedure allows us to determine the optimal subsidies/taxes on the cost of labor employed in sector \( X \).

To obtain further insight, we turn now to two specific distributions. For the uniform distribution, the corresponding density function is \( f(e) = 1/d \). Applying (A1)-(A13) we infer that

\[
\begin{align*}
(14) \quad g &= \sqrt{2dm - m} \\
(15) \quad (\bar{\eta} - \eta_r)/\eta_r &= \sqrt{\frac{d}{2m} - 1} \\
(16) \quad u_1 &= \frac{1 - \frac{2m_1}{\beta + 1}}{d} \\
(17) \quad u_2 &= \frac{1 - \frac{2m_2}{s_2 \beta + 1}}{d}
\end{align*}
\]

Figure 5a summarizes the dependency of \( g \) and the bottleneck externality on the allocation \( m \). Note the nonlinear nature of bottleneck externality: the externality (as measured by \( \frac{\partial g}{\partial m} - (\bar{\eta} - \eta_r)/\eta_r \)) "explodes" as the scarcity of \( m \) becomes acute. Figure 5b summarizes the dependency of the optimal policies on \( m \).
To verify the robustness of the above result, we turn now to the case of a truncated normal distribution, where the distribution is truncated at both ends and normal over the relevant range defined between 0 and d. The mean of the resource requirement is d/2. The corresponding density function is given by

\[
(A18) \quad f(e) = \exp \left[-\frac{1}{2} \left(\frac{e-d/2}{\sigma}\right)^2\right] \int_0^{d/2} \exp \left[-\frac{1}{2} \left(\frac{e-d/2}{\sigma}\right)^2\right]
\]

for \( \sigma > 0 \), \( 0 \leq e \leq d \)

While this case is more involved, it can be verified that, as long as the standard deviation is relatively large (\( d < \sqrt{2} \sigma \)), the characteristics of the bottleneck externality are similar to the case of a uniform distribution, as summarized in Figure 6a. If, however, the standard deviation is relatively small (\( d > \sqrt{2} \sigma \)), the behavior of the bottleneck externality is non-monotonic, as shown in Figure 6b. This can be seen by noting that

\[
(A19) \quad \text{sign} \left[ \frac{\partial ((\bar{\eta} - \eta_r)/\eta_r)}{\partial m} \right] = \text{sign} \left[ \frac{m+g}{\sigma} \left( \frac{m+g}{\sigma} - \frac{d}{2\sigma} \right) - 1 \right]
\]

Applying to this condition the fact that bottlenecks occur if and only if \( m+g < d \), we infer that for \( d < \sqrt{2} \sigma \) the curve \((\bar{\eta} - \eta_r)/\eta_r\) is downward sloping for all the relevant values of \( m \) (0 \( \leq m \leq d/2 \)), and that for \( d > \sqrt{2} \sigma \) the curve has a unique internal minimum for \( m \) that must exceed \( d/4 \). In both cases, for small allocations of raw material the results are identical to those for the uniform distribution: the externality explodes for acute shortages and is downward sloping for relatively small values of raw material. For \( d > \sqrt{2} \sigma \) the \( g \) curve is upward sloping (but bounded above) for allocations of \( m \) that approach the mean of the distribution (\( d/2 \)).

We close the discussion with several remarks that put the above model into broader perspective. Our analysis should be viewed as applicable in the short run. In the long run, the distribution determining the minimum input requirement is endogenously determined, and may be pushed leftward by the proper investment. The relatively wasteful production patterns (low output/input ratios) in existing plants in Eastern European countries may be viewed as the outcome of a long history where raw materials were abundant at artificially low prices. While the long-run welfare gains associated with switching to more efficient production technologies are potentially

\[\text{I/ This can be seen visually in Figure 4 by applying the properties of a density function. In particular for a general distribution, (A8) can be represented as the ratio of areas } [A-B]/[B+C], \text{ since the sum of areas } A \text{ and } C \text{ is unity and the sum of areas } B \text{ and } C \text{ is } (m+g)f(m+g).\]
staggering, in the short run the state enterprises are operating with technologies that fitted (at best) the past. As our paper illustrates, operating the old technologies under the new relative prices may invoke bottlenecks that call for the proper policies. It should be emphasized that these externalities are not the outcome of price rigidities, but of vintage effects: old technologies may be ill fitted to new relative prices. This explains also why the needed policies are confined to state enterprises. New private enterprises have the advantage of using capital and technologies that should fit the new relative prices. In fact, this argument can be pushed further--any attempt to extend the set of policies to new, privately-owned enterprises will diminish welfare by reducing the incentives of the new enterprises to adopt proper technologies. Hence, the asymmetric treatment of the two sectors in our model is the outcome of sunk cost considerations: the old sector is a declining industry, whereas the new sector is at the expansionary stage. Underlying this conclusion is recognition of the irreversible nature of past investment, which cannot be adapted to the new environment. The bottom line is that economic liberalization and the transition to market-oriented decision making should be supplemented with the proper policies that target the existing, declining industries.
Derivations

The purpose of this Appendix is to review the derivations of the key equations from the paper.

Equations (8) and (16):

In the competitive case, the problem facing the representative stage-2 producer is:

\[ \text{MAX} \left[ E(X_{2,r}) - P_1 X_{1,r}^d - W L_{2,r} - P_{\text{m}2,r} M_{2,r} \right] \]

\( \{L_{2,r}, M_{2,r}, X_{1,r}^d\} \)

where, using (3)-(4),

\[ E(X_{2,r}) = Q_{2,r} \left[ \left( X_{1,r}^d \right)^\alpha + \left[ a_2 M_{2,r} (L_{2,r})^\gamma \right]^{1/\alpha} \right] \]

and \( X_{1,r}^d \) is the representative stage-2 demand for stage-1 input. The first order condition for this optimization is

\[ P_1 = \frac{\partial E(X_{2,r})}{\partial X_{1,r}^d} \]

Applying (B2)-(B3) we get

\[ P_1 = Q_{2,r} \left[ 1 + \left( \frac{a_2 (M_{2,r})^\beta (L_{2,r})^\gamma}{X_{1,r}^d} \right)^{1/\alpha} \right]^{(1-\alpha)/\alpha} \]

Note that in an equilibrium with symmetric firms

\[ X_{1,r}^d = \frac{E(X_{1,r})}{\theta_{2}} = \frac{Q_1 \phi_1}{\theta_{2}} \text{ and } a_2 (M_{2,r})^\beta (L_{2,r})^\gamma = \frac{\phi_2}{\theta_{2}} \text{. Applying these equations to (B4) we infer (8). Equation (16) is inferred by applying (B4) and (7).} \]
Equations (9) and (10):

The problem facing the representative stage-1 producer is:

\[(B5) \max \left[ P_1 E(X_1, r) - WL_1, r - P_m M_1, r \right] \]

where

\[(B6) E(X_1, r) = Q_1, r a_1(M_1, r)^{\beta(L_1, r)^{\gamma}} \]

In solving this maximization problem, the producer recognizes that \(Q_1, r\) is affected by his own use of raw material, but treats the industry wide average level of material utilization as given. This yields:

\[(B7) P_m = \frac{P_1 a_1(M_1, r)^{\beta(L_1, r)^{\gamma}}}{M_1, r} \left[ \frac{\partial Q_1, r}{\partial M_1, r} \cdot m_1, r + \beta Q_1, r \right] \]

Recall that in a symmetric equilibrium: \(s_1 = P_1 E(X_1)/E(X_2); a_1(M_1, r)^{\beta(L_1, r)^{\gamma}} = \Phi_1/\theta_1;\) and \(Q_1, r = Q_1.\) Applying these facts to (B7) we obtain (10). A similar procedure leads to (9).

Equations (13) and (14):

The problem facing the representative stage-2 producer is characterized by (B1). Optimizing with respect to \(M_2, r\) we obtain

\[(B8) P_m = \frac{E(X_2, r)}{Q_2, r} \frac{\partial Q_2, r}{\partial M_2, r} \frac{1}{L_2, r} + \frac{\beta E(X_2, r)}{M_2, r} \left[ \frac{a_2(M_2, r)^{\beta(L_2, r)^{\gamma}}}{\alpha} \right] \]

Note that in a symmetric equilibrium: \(a_2(M_2, r)^{\beta(L_2, r)^{\gamma}} = \Phi_2/\theta_2; X_{1, r} = Q_1 \Phi_1/\theta_2;\) and \(Q_2, r = Q_2.\) Applying these facts to (B8) we infer (14). The same procedure applied the choice of \(L_2, r\) yields (13).

Equations (18)-(21):

In deriving the optimal allocation it is useful to note that in the symmetric equilibrium \(m_{k, r} = m_k.\) Unlike the representative producer that treats \(m_k\) as exogenously given, the optimal allocation recognizes the above
equality. Applying (11) and (17) we obtain that the optimal use of materials in the two stages is characterized by

(B9) \[ \delta \left[ \frac{E(X_2)}{Q_1 \Phi_1^\alpha} \left( \frac{\partial Q_2}{\partial M_1} + \frac{\partial Q_2}{\partial M_2} \right) + \frac{\partial (L_1 + L_2 + L_y)}{\partial M_1} \right] - p_m \]

and

(B10) \[ \delta \left[ \frac{E(X_2)}{Q_2 L_2} \left( \frac{\partial Q_2}{\partial M_2} + \frac{\partial Q_2}{\partial M_1} \right) + \frac{\beta E(X_2)}{M_2} \left( \frac{\Phi_2^\alpha}{Q_1 \Phi_1^\alpha} + \frac{\Phi_2^\alpha}{\Phi_2^\alpha} \right) - p_m \]

Equations (19) and (21) are obtained by applying (11) and (15) to the above equalities and collecting the various terms. The same procedure applied to the choice of labor inputs in the two stages yields (18) and (20).

**Extension to n stages:**

While our analysis was confined to two stages, it can be readily extended to n production stages. Applying (7) and (17) we infer that the optimal demand for labor and material in stage k is given by

(B11) \[ L_k = \frac{E(X_n)}{\bar{w}} \left[ s_k \gamma - \left( \sum_{i=1}^{k} s_i \bar{\eta}_k \right) \right] \]

(B12) \[ M_k = \frac{E(X_n)}{\bar{p}_{m,k}} \left[ s_k \beta + \sum_{i=1}^{k} s_i \bar{\eta}_k \right] \]

where \( s_k = \frac{\left( Q^{(k)} \Phi_k \right)^\alpha}{\sum_{j=1}^{n} \left( Q^{(j)} \Phi_j \right)^\alpha} \)
The demands of the representative producer are obtained from (B11) and (B12), applied to the case in which \((\bar{\eta}_k, W', P_{m,k}')\) are replaced with \((\eta_k, r, W, P_m)\), respectively.

\[
P'_{m,k} = P_m - \frac{1-\delta}{\delta} \sum_{i=1}^{\delta} \frac{\partial (\sum L_i + M_y)}{\delta M_k}
\]
References


Sachs, Jeffrey D., "Crossing the Valley of Tears in East European Reform," Challenge 34 (September-October 1991), pp. 26-34.

