Abstract

This paper investigates the consequences of pricing to market for exchange rate pass-through and real exchange rate dynamics across different patterns of trade under market segmentation. Under two-way, intraindustry trade—where home prices display greater linkage with those of foreign competitors—domestic and export prices exhibit lower pass-through and greater destination-specific adjustment compared to intersectoral trade. With both trade patterns, pricing-to-market behavior intensifies the degree of persistence in the real exchange rate under nominal rigidities, and allows monetary shocks to have permanent effects on relative prices when goods markets remain segmented.

JEL Classification Numbers:

E12, F12, F41

1/ Southeast Asia and Pacific Department, International Monetary Fund. Work on this paper began while the author was a lecturer in the Woodrow Wilson School of the Public and International Affairs and was completed when he was in the Research Department. The author would like to thank Michael Knetter, Laurence Ball, and IMF Research Department Seminar participants for helpful comments and discussions. The views expressed and the errors remaining are the author’s sole responsibility.
## Contents

<table>
<thead>
<tr>
<th>Summary</th>
<th>iii</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Introduction</td>
<td>1</td>
</tr>
<tr>
<td>II. Model of Pricing to Market</td>
<td>5</td>
</tr>
<tr>
<td>III. Pass-Through and Pricing to Market</td>
<td>10</td>
</tr>
<tr>
<td>IV. Relative Prices and Dynamics</td>
<td>13</td>
</tr>
<tr>
<td>V. Conclusions</td>
<td>19</td>
</tr>
</tbody>
</table>

### Text Tables

1. Domestic and Export Demand Functions Under Monopolistic Competition 6
2. Optimal Price-Setting Rules 9
3. Real Exchange Rate Dynamics 17

### List of Figures

1. Exchange-Rate Pass-Through and the Pattern of Trade 10a
2. Pricing to Market and the Pattern of Trade 12a

Appendix. The Basic Model 20

References 24
Summary

Since the abandonment of fixed exchange rates in 1973, foreign exchange markets have experienced a great deal of turbulence. However, the large swings in exchange rates under the era of floating have not brought about comparable variation in inflation or disinflation rates as one would expect with pass-through. Instead, volatile nominal exchange rates have translated into volatile real exchange rates in the post-Bretton Woods world, while prices locally have remained remarkably stable.

In an attempt to understand these developments, this paper develops a model of goods market segmentation wherein firms may systematically price discriminate to stabilize prices and quantities across market destinations. Motivated by a preponderance of empirical evidence disavowing the law of one price, the model details some of the economic implications of pricing-to-market behavior. Specifically, the framework examines some of the macroeconomic consequences of market segmentation and shows the resulting behavior of international prices to be broadly in line with the stylized facts. The results include both cross-sectional implications regarding nominal prices and time-series implications regarding relative prices.

Across different trade patterns, significant variation exists in the degrees of pass-through and pricing to market depending upon the degree of intraindustry trade and the substitutability between domestic and foreign goods. Across time, dynamic adjustment in prices suggests that nominal and real exchange rates move together over the short run and over the longer run as well to the extent that markets remain segmented. Overall, pricing to market provides a potentially important source of local price stickiness and real exchange rate variability and persistence.
I. Introduction

Any plausible theory of real exchange rate determination must be able to resolve two prominent stylized facts regarding international relative prices. First, at an aggregate level, real exchange rate movements tend to be very persistent. Second, at the micro level, individual prices tend to be sticky in terms of local currency. 1/ Of course, open-economy models in the Keynesian tradition have long since emphasized sticky prices in goods and labor markets. Perhaps the most well-known example is the Dornbusch (1976) overshooting model, wherein sticky goods prices in conjunction with flexible, forward-looking exchange rates lead to real exchange-rate overshooting and transitory deviations from purchasing power parity (PPP).

However, by invoking the law of one price, this literature incorporates price stickiness at the point of origin rather than at the market destination. In other words, the standard Keynesian approach generates price-level inertia in terms of national currency (seller's currency) rather than local currency (buyer's currency). Furthermore, by relying on the law of one price, these models typically cannot generate the degree of persistence that is observed in actual real exchange rate time-series. Not only do real and nominal exchange rates move together over the short term, they often move together over much longer horizons as well—sometimes over many years.

The law of one price of course has had a long-standing tradition in international economics. Interpreted as a condition for spatial arbitrage, the law of one price asserts that the price of a good be the same in all locations when measured in a common currency. Extending this condition to all traded goods, PPP theory posits an aggregate version of the law of one price, applied at the level of national price indices. The theory maintains that the price of entire baskets of goods should be equal (or proportional) across countries. Together, PPP and the law of one price have served as the basis for some of the most-widely held theories in international economics, greatly influencing the discourse on exchange rate determination.

Meanwhile, empirical evidence supporting the law of one price has remained elusive. Numerous empirical studies have consistently documented the absence of "one price" in practice, net of transport costs, tariffs, and other trade impediments. 2/ In combination with the very mixed evidence on PPP, 3/ some have begun to call into question those theories that rely

---

1/ Comparing the relative price of different goods within the same country versus the relative price of same good across different countries, Engle (1993) finds the former measure to be less variable in all but a few cases such as energy prices and primary commodities. Moreover, the second relative price tends to be several times more variable on average, confirming that the local prices prevailing in a given market destination remain comparatively quite stable.


3/ See Breuer (1994) for a recent survey of the PPP literature.
upon the concept of spatial arbitrage in goods. 1/ Consequently, given its dubious validity, the law of one price has been subject to modification, and alternative approaches have received closer attention. 2/ In a departure from the orthodox view, some have suggested systematic violations to one price as manifested by the phenomenon of pricing to market.

With pricing to market, forces that would normally assure spatial arbitrage are absent, allowing corresponding prices to diverge across markets. This view—which is adopted in this paper—essentially breaks from standard models by favoring market segmentation rather than integration, and thereby acknowledging the presence of economic barriers and structural rigidities that restrict convergence in inter-market prices. Pricing to market may thus provide an alternative to the traditional view that differential rates of adjustment in goods prices versus asset prices alone underlie persistence in the real exchange rate.

Much of the empirical evidence on market segmentation has drawn from the experience of United States in the 1980s. In particular, several studies have focused on the behavior of U.S. import prices during the period of massive appreciation 3/ and subsequent depreciation 4/ of the dollar.

1/ Krugman (1989, p. 43) summarizes this dissatisfaction:

"[W]e must now admit that international Keynesianism, while more like reality than international monetarism, itself turns out to have a problem: it does not go far enough in rejecting international arbitrage. Not only does the Law of One Price fail to hold at the level of aggregate national price indices...it doesn't even hold at the level of individual goods."

2/ Aizenman (1984) illustrates that when transport or information costs impede arbitrage over the very short term, PPP holds up to constant plus white noise. However, it is the persistence of relative-price movements that dominates the time-series of the real exchange rate which requires further explanation.

3/ During the dollar upswing between 1980 and 1985, Dornbusch (1987, p. 104) observes the following fact regarding U.S. import prices:

"[T]he order of magnitude of the decline [in import prices] remains relatively small compared to the change in relative unit labor costs. With a change in relative unit labor costs of more than 40 percent, the decline in the relative price in most cases was less than 20 percent. That is not at all out of line with the theory once some degree of 'pricing to the American market' is taken into account."

4/ For the ensuing period from 1985-87 when the dollar fell precipitously, Hooper and Mann (1987) find that import-price increases, in percentage terms, were well short of the nominal depreciation. Krugman (1989) reports a similar finding in the specific case of Japanese exports in manufactures, finding that export prices (in dollars) were relatively stable in the destination market despite sharply rising unit labor costs (in dollars) at the point of origin.
For example, Krugman (1987) examines U.S. import prices during the period of the dollar's rise. Constructing dollar price indices for U.S. import bundles and comparing them to the dollar price of the same bundle exported to third-country markets, he concludes that more than 30 percent of the real appreciation of the dollar was reflected in a divergence between these prices.

In light of the empirical support for the proposition that certain markets may be segmented, the theoretical avenues regarding the issue of pricing to market have not yet been fully developed. Typically, models of pricing to market present static, partial equilibrium results from the perspective of an exporting industry or firm. 1/ Hence, these models cannot address the dynamic price interactions and adjustment underlying the evolution of relative prices and the real exchange rate at the macroeconomic level.

Using a general equilibrium model of monopolistic competition and market segmentation, this paper investigates the macroeconomic consequences of pricing to market for real exchange rate behavior across different patterns of trade. Allowing producers to price discriminate between local and foreign markets, the monetary model examines exchange rate pass-through and pricing-to-market behavior under segmented markets, and relative-price dynamics under nominal rigidities. By also incorporating the structure of trade, the paper reexamines the cross-sectional implications of intersectoral versus intraindustry trade for macroeconomic adjustment in contrast to the findings in Faruqee (1992).

The central result can be summarized as follows. The pattern of industry specialization and trade largely determines the degree of strategic complementarity or price linkage between producers from different countries. In particular, under intraindustry trade—wherein home and foreign products are close substitutes—there exists a higher degree of linkage between domestic and foreign prices prevailing in the same market than under intersectoral (cross-industry) trade. Consequently, domestic and export prices exhibit greater responsiveness to exchange-rate fluctuation, a lower degree of pass-through, and a greater degree of pricing to market under two-way trade.

In a setting of imperfect integration, 2/ a greater degree of price linkage across borders translates into stronger mean-reversion in the real exchange rate. The intuition is as follows. With menu costs and

---

1/ A partial exception is Delgado (1991) which develops a dynamic menu cost model of pricing to market, albeit in a partial equilibrium setting.

2/ Under imperfect integration in world markets for goods and services, countries differ in their national consumption patterns and in the units of account in which they set prices—favoring both their own goods and currency. However, the law of one price still equates currency-adjusted prices across markets. See Faruqee (1992). For a general discussion of imperfect integration see Krugman (1989).
price-staggering, prices display inertia in terms of national currency (unit of account) through a lack of coordination among decentralized price-setters. In an open economy, however, producers are concerned with relative prices both at home and abroad. With the law of one price, the added concern with the foreign-currency price of output compels price-setters to partly overcome the coordination failure underlying domestic price inertia. Prices become less sticky in terms of national currency in order to reduce variation in terms of foreign currency. Hence, the stronger international linkage in prices under two-way trade reduces the variability and persistence of real exchange rate fluctuation compared to intersectoral trade. However, once the assumption of spatial arbitrage is removed, this comparative result no longer holds true.

Once export prices detach from domestic prices with segmented markets, prices are in effect set in terms of local currency rather than national currency, and price linkages predominantly exist within market rather than across markets. If home and foreign prices are inertial in the same unit of account, the pattern of trade no longer affects the comparative degree of variability and persistence in relative prices. Instead, the level of persistence becomes solely a function of the frequency and timing of price adjustment. As a result, market segmentation allows international relative prices to exhibit equally persistent deviations across diverse patterns of trade, despite inherent differences in the substitutability between home and foreign goods.

Comparative issues aside, pricing to market ensures greater price stability in terms of local currency than otherwise, regardless of the trade pattern. Consequently, segmented markets allow greater inertia in the price-level domestically and slower adjustment in relative prices internationally than under the law of one price. Thus, pricing to market provides an important propagation mechanism for explaining the large and protracted swings in real exchange rates characteristic of the post-Bretton Woods era. Moreover, in the presence of market segmentation, monetary shocks have lasting effects on relative prices. Without goods arbitrage, absent are the economic forces that would otherwise guarantee the equivalent price of a basket of goods across countries. Instead of long-run PPP, monetary disturbances have permanent effects on the real exchange rate in the absence of one price.

Persistence and permanence in relative prices have important time-series implications for exchange rates. When monetary shocks have persistent effects on relative prices due to price-level inertia, nominal and real exchange rates move together in the short run as has been emphasized in earlier sticky-price models. When monetary shocks have "permanent" effects on relative prices, nominal and real exchange rates move together over the longer run as well—so long as markets remain segmented and the law of one price systematically fails.

The paper is organized as follows. Section II develops a two-country model of monopolistic competition and market segmentation under intraindustry and intersectoral trade. Using this static framework, Section
III addresses the issues of exchange rate pass-through and pricing to market. A dynamic version of the model is developed in Section IV. Incorporating nominal rigidities into the analysis, this section examines the dynamic adjustment of relative prices and the time-series behavior of the real exchange rate. Section V offers some concluding remarks.

II. Model of Pricing to Market

Consider a world economy consisting of a home and foreign country, with each country composed of \( n \) producer-consumers respectively. These individual agents produce and sell a differentiated good in order to purchase and consume the variety of goods made by all agents, taking others' prices as given. Meanwhile, each agent acts as a monopolistic competitor, setting price and choosing a level of production depending on the demand that the individual producer faces. Building on the constant elasticity of substitution (CES) approach of Dixit and Stiglitz (1977), I focus on two basic variants of this central framework: the cases of intersectoral and intraindustry trade. 1/

Under intersectoral trade, countries specializes and trade at the industry level according to international differences in comparative advantage. This pattern of trade—often associated with the exchange between North and South—emerges as a result of underlying differences in relative factor proportions and gains from cross-industry trade. Thus, this framework essentially embodies the traditional Heckscher-Ohlin view of international trade. Although in this context, monopolistic competition characterizes each sector rather than perfectly competitive industries.

Under intraindustry trade, the premise is quite different. Countries are now essentially identical with respect to factor composition, yet still gain from specialization and trade at the variety level due to scale economies and product differentiation. This modern interpretation of trade offers an explanation for the two-way, intraindustry exchange often observed between OECD countries, but left unaccounted for under the standard factor proportions theory. Thus, the second case embeds some of the recent developments in the noncompetitive theory of international trade. 2/

Under both trade patterns, the local and export demands facing an individual home producer \( i \) are given in Table 1. 3/ Note that monetary variables with asterisks are expressed in units of foreign currency, and \( E \) is the nominal exchange rate expressing the domestic price of foreign currency. Also in Table 1, \( \epsilon > 1 \) represents the constant elasticity of substitution between any two varieties from the same industry, while \( \alpha \) and \( \beta \) are taste parameters, measuring the extent to which countries symmetrically favor their own goods in consumption. These latter two measures roughly

---

1/ See Appendix for details and the basic set-up of the model.
2/ See for example Helpman and Krugman (1985).
3/ See Appendix for details.
capture the nominal expenditure shares allocated to locally-produced goods from each country. Assuming that countries predominantly consume the goods that they produce themselves, I specify that \(0.5<\alpha, \beta<1\) under local goods preference.

Table 1. Domestic and Export Demand Functions Under Monopolistic Competition

<table>
<thead>
<tr>
<th>Pattern of Trade</th>
<th>Home Market</th>
<th>Foreign Market</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intersectoral Trade</td>
<td>(Y_i = \left( \frac{P_i}{P} \right)^{-\epsilon} \left( \frac{M}{P} \right) )</td>
<td>(Y_i = \left( \frac{P_i}{P} \right)^{-\epsilon} \left( 1-\alpha \right) \left( \frac{EM^*}{P} \right) )</td>
</tr>
<tr>
<td>Intraindustry Trade</td>
<td>(Y_i = \left( \frac{P_i}{Q} \right)^{-\epsilon} \left( \frac{M}{Q} \right) )</td>
<td>(Y_i = \left( \frac{P_i}{EQ^<em>} \right)^{-\epsilon} \left( 1-\beta \right) \left( \frac{M}{EQ^</em>} \right) )</td>
</tr>
</tbody>
</table>

From Table 1, demand for output \(Y_i\) in each market under interindustry trade is a function of two components: relative price and aggregate demand. Demand for a particular variety is a decreasing function of its price \(P_i\) relative to other prices in the industry, where \(P\) is the relevant index of producer prices set by producer \(i\) and his \((n-1)\) fellow compatriots. Second, product demand for each variety depends on aggregate demand facing the

\[Q = (P)^{\alpha(EP^*)^{1-\alpha}}\]

where \(\alpha\) is the exact expenditure share on home goods. Under intraindustry trade, the home CPI and expenditure share on home goods are:

\[Q = \left[ \beta P^{1-\epsilon} + (1-\beta) EP^*^{1-\epsilon} \right]^{1-\epsilon} \quad \text{and} \quad \bar{\alpha} = \frac{\beta}{\beta + (1-\beta)(EP^*/P)^{1-\epsilon}}.\]

Given relative producer prices in general equilibrium, \(\beta\) is chosen so that \(\bar{\alpha} = \alpha\), keeping expenditure patterns identical across trade patterns.
industry in each country, where real money balances measure real aggregate demand in each market through a simple quantity equation relationship (see Appendix).

In the case of intraindustry trade, variety demand is once again a function of relative price and aggregate demand. However, since foreign goods now compete in the same industry, the relative-price component is a function of the individual price relative to other home and foreign industry prices. This modification in demand is seen by the ratio of $P_j$ to the domestic CPI $Q$ at home and the foreign CPI $EQ*$ abroad, where the consumer price indices and producer price indices naturally coincide under trade in one industry. Moreover, real money balances also reflect the appropriate deflator to capture real aggregate demand in the relevant market. Note that the demands under two-way trade assume from the outset that country-size is not an issue ($n=n*$). Otherwise, a measure of relative country size would also enter the demand under intraindustry trade, further distinguishing it from the case of intersectoral trade. 1/

Introducing market segmentation into the analysis, I also assume that each producer can price discriminate across markets, setting a different price in each location if the agent so chooses. This assumption represents the critical point of departure from Faruqee (1992), where the law of one price applied. Now with segmented markets, the implicit arbitrage mechanisms that assured the law of one price are absent. Consequently, each home and foreign producer can price to each market separately, depending on the market conditions that prevail in each location.

In standard models of market segmentation, firms price to market by making destination-specific adjustments to price-cost margins in response to exchange rate changes. However, in this CES framework, markups are constant, closing the usual channel through which prices may systematically differ. Hence, to ensure that producers have incentive to price discriminate given their ability to do so, I also assume that costs are separable (and convex) in the production of domestic and export good. 2/

2/ There are many justifications for the premise of differential costs. If there exist market-specific costs in transportation, distribution, production, advertising and/or servicing, then costs can differ at the margin for the home and exported good. For example, foreign markets may require different product specifications and/or have different governmental regulations which differentiate costs of production; producing the export good may even take place in the destination country itself, involving a completely separate plant and production run. These and similar explanations may also help explain why markets are actually segmented in the first place.
Otherwise, producers would always choose the same price for each market under either pattern of trade. 1/

Qualitatively, pricing behavior under cost separability is similar to the pricing-to-market responses under differential markups. Usually, firms reduce markups in markets where the currency has weakened in order to stabilize price in terms of local currency. 2/ In this context, exchange-rate movements elicit a very similar response. For example, a depreciation of the foreign currency lowers demand for home exports which are now more expensive abroad. Consequently, faced with reduced demand and cost, home producers respond by lowering export prices in units of home currency in order to offset the price increase in terms of foreign currency. Producers still desire local price stability, but now through a marginal-cost channel rather than the markup channel.

Note that this simple representation of price discrimination places no boundaries on the degree of divergence between prices for the same good. Clearly, this is an extreme assumption. More likely, there would exist some neutral band—likely based upon the level of transport or adjustment costs—within which price differentials would persist. Beyond that range, "gray markets" would emerge as agents found it profitable to circumvent the producer's own distribution channels. However, for small enough demand discrepancies (large enough adjustment costs) this representation is certainly a reasonable assumption.

As for notation, I designate the home and foreign destinations as markets 1 and 2 respectively. So, for example, the prices set by home producer \( i \) and foreign producer \( j \) for the home market are denoted by: \( P^1_i \) and \( P^1_j \), expressed in units of national currency. Solving the producer's problem, by maximizing real revenues from domestic and export sales minus the utility costs of production (see Appendix), yields the optimal price for each agent for each market destination.

Table 2 presents the optimal price-setting rule (ignoring constants) for each markets and under both trade patterns from the perspective of the representative home producer. Note that lowercase letters denote logarithms of variables \( (q = \ln Q) \). From Table 2, note that in every case the optimal (log) price \( \bar{P}^i \) in units of home currency 3/ is a weighted average of local

---

1/ Constant differential markups could be introduced into this CES framework by assuming differential elasticities of substitution across markets \( (\varepsilon^* \neq \varepsilon^*) \). In that case, there would exist a constant degree of pricing to market.

2/ Typically, with differential markups, demand is less convex than the constant elasticity case. See Marston (1990).

3/ Krugman (1984) finds that most countries invoice exports in terms of domestic currency when relative country-size differences are not significant. The exception is LDC exports which are predominantly invoiced in U.S. dollars.
money, consumer prices at home, and local producer prices for the industry, with all weights being strictly positive. 1/

Table 2. Optimal Price-Setting Rules

<table>
<thead>
<tr>
<th>Pattern of Trade</th>
<th>Home market</th>
<th>Foreign market</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intersectoral trade</td>
<td>$p_1^-1 = \pi m + \theta q + (1-\pi-\theta)p^1$</td>
<td>$p_2^-2 = \pi (e+m^*) + \theta q + (1-\pi-\theta)p^2$</td>
</tr>
<tr>
<td>Intraindustry trade</td>
<td>$p_1^-1 = \pi m + (1-\pi)q$</td>
<td>$p_2^-2 = \pi (e+m^<em>) + \theta q + (1-\pi-\theta)(e+q^</em>)$</td>
</tr>
</tbody>
</table>

Under intersectoral trade, the price rule for the home market places a weight $\pi$ on domestic money which represents nominal aggregate demand in that market. An increase in aggregate demand raises demand for each individual variety, effecting an increase in price. The weight placed on domestic consumer prices captures a real-income effect. Since each producer is also a consumer—ultimately concerned with utility—an increase in domestic consumer prices in general translates into a loss of real purchasing power and a desire to raise nominal price in response. The weight placed on industry prices $1-\pi-\theta$ captures a relative-price effect. An increase in industry prices lowers producer $i$'s relative price, raising the agent’s product demand and price.

Under intraindustry trade, the price rule for domestic prices has the same general form. Once again, producers are concerned about domestic consumer prices to the extent $\theta$. However, now, the relative-price coefficient $1-\pi-\theta$ capturing the positive interaction between industry prices is also placed on the home CPI under two-way trade. Together with the

1/ Based on taste parameters, the coefficients in Table 2 are given by:

$$\pi = \frac{\gamma}{\varepsilon(\gamma-1)+1} \quad \text{and} \quad \theta = 1-\varepsilon\pi,$$

where both coefficients and their sum are between $(0,1)$. ©International Monetary Fund. Not for Redistribution
real-income effect, the weight placed on \( q \) sums to \( 1-\pi \) in Table 2. So whereas price-setters place a weight \( 1-\pi \cdot \theta \) on \( p^I \) under cross-industry trade, that weight now falls on \( p^I, p^{I^*} \) and \( e \) (subsumed in \( q \)). Hence, there exists a greater degree of linkage between home and foreign producer prices prevailing in the domestic market. In the terminology of Cooper and John (1988), there exists a greater degree of strategic complementarity between home and foreign pricing decisions under intraindustry trade. 1/

The intuition behind this result is relatively straightforward. Home and foreign varieties are closer substitutes under two-way trade, so that movements in international relative prices have larger consequences for individual demands. Hence, home price-setters are more concerned with foreign prices set for the home market when choosing their own price. The same conclusion holds true for export prices as well. Home export prices under two-way trade display greater influence and interaction with foreign local prices, since the relative-price effect also includes foreign industry prices and the nominal exchange rate.

III. Pass-Through and Pricing to Market

Using the static framework of the previous section, the effects of various disturbances on nominal prices can be measured given a particular calibration of the model. These comparative static exercises are carried out in this section. In particular, I calculate the responsiveness of the optimal price to an exchange-rate disturbance in all four cases, given that all other variables remain unchanged.

The first measure of interest is the degree of exchange-rate pass-through. Pass-through is defined as the percentage change in home prices—measured in terms of foreign currency—resulting from a change in the nominal exchange rate. If the foreign-currency price of a good changes one for one with the exchange-rate, the degree of pass-through is one. In other words, the exchange rate change is fully reflected in the price change while the home-currency price remains unaffected. At the other extreme, if a price responds fully to offset the change in the exchange rate, leaving the foreign-currency price unaffected, the degree of pass through is zero. Mathematically, this sensitivity measure is calculated (in absolute value) by \( |\frac{\partial p_i^m}{\partial e} - 1| \in (0,1) \), for \( m=1,2 \).

Figure 1 displays the degree of pass-through in home prices, holding expenditure shares fixed and allowing the elasticity of substitution to vary, where dashed, and solid lines represent intraindustry and intersectoral trade, respectively. Note that the pass-through responses shown also apply to foreign producers as well through symmetry. 2/

---

1/ See Faruqee (1992) for further discussion.
2/ Pass-through abroad is defined as \( |\frac{\partial p_i^{m^*}}{\partial e} + 1| \) for \( m=1,2 \).
Figure 1. Exchange-rate Pass-through and the Pattern of Trade
that when the elasticity of substitution is exactly one, the two types of trade are identical. This result follows from the characterization of international trade subsumed in preferences.

Explicitly, preferences are Cobb-Douglas across goods from different industries, while CES preferences apply to goods within an industry. Hence, when countries engage in one-way trade across sectors there generally exists a lower degree of substitutability between home and foreign goods. Meanwhile, with two-way intraindustry trade, the CES specification posits that home and foreign goods are closer substitutes--unless ε equals unity and this distinction in utility vanishes. 1/ Conversely, as ε increases, the disparity between the patterns of trade widens in terms of how agents perceive domestic goods compared to foreign goods.

This characterization of preferences embodies the basic idea behind the Armington model. 2/ The separability assumption under the Cobb-Douglas specification in utility makes the distinction that goods from different industries--appropriately defined--belong to separate commodity classes, representing distinct products. At the industry level, each product is then further differentiated into many varieties with a constant elasticity of substitution between them.

Several results from this framework can be inferred from Figure 1. Naturally, the degree of pass-through is lower in export prices regardless of the trade structure since export demand is always more sensitive to exchange rate fluctuation than domestic demand for the home good. In contrast, note that in case of the law of one price, pass-through in domestic and export prices in each case must be identical by definition.

Also in Figure 1, as ε increases, the degree of pass-through generally declines under two-way trade, but increases under intersectoral trade. The intuition behind this latter result follows from the pervious discussion of preferences and trade. As the elasticity of substitution increases, producers display greater strategic complementarity with their industry rivals. This stronger linkage in prices translates into greater sensitivity to any changes in relative prices. As industry varieties become closer substitutes, price-setters under intersectoral trade become increasingly concerned with keeping prices in line with other resident producers. At that margin, exchange rate fluctuations are of no consequence for relative prices (same unit of account). Hence, producers are more content to allow a greater degree of exchange rate pass-through.

As for intraindustry trade, as the elasticity of substitution increases, price-setters prefer to keep price more closely in line with those of industry competitors as before, but now the industry includes

1/ Applying L'Hôpital's rule to preferences described in the Appendix verifies this equivalence result in the limit as ε→1.
2/ In Armington (1969), goods are imperfect substitutes according to country rather than industry.
foreign producers in each destination. Consequently, home producers display a greater responsiveness to exchange rate fluctuations through offsetting changes in their home-currency prices to minimize movements in relative prices. Figure 2 displays the degree of price discrimination under each trade pattern resulting from an exchange rate disturbance. This degree of pricing to market is measured by the percentage deviation between the domestic price and export price for the same good as a percentage of the change in exchange rate. 1/ Mathematically, this measure is given (in absolute value) by $|\frac{\partial P^1_t}{\partial e} - \frac{\partial P^2_t}{\partial e}|$.

Once again, when $\epsilon=1$, the two patterns of trade are really the same and the degree of pricing to market is also identical. As the elasticity of substitution increases, producers under intra-industry trade decide to price discriminate across markets to a greater extent. The reasoning is as follows. Stronger price linkages within the industry, motivate home price-setters to keep price more closely in line with competitors, including foreign producers, in each destination (see again dashed lines in Figure 1). However, since demand abroad for the domestic product is more sensitive to the rate of exchange, movements in the value of currency effect a greater response in prices set for the foreign market than for the home market, increasing the extent of price discrimination. This result is seen in Figure 2 by an increasing degree of pricing to market under two-way trade.

As for intersectoral trade, the total absence of an import-competing industry in each country greatly limits producer concerns regarding international relative prices. Furthermore, as $\epsilon$ increases, closer ties between industry price-setters who happen to be from the same country further reduce the level of price discrimination. This is seen by a decreasing degree of pricing to market in Figure 2. As the relative-price effect receives greater weight compared to the aggregate demand and income effects, exchange rate changes are increasingly ignored in both market destinations. This finding is clearly at one extreme.

From the setup of the model, complete specialization at the industry level characterizes intersectoral trade between two nations that could not be more different. In sharp contrast, complete overlap in industry production represents intra-industry trade between identical trading partners. In an intermediate structure between these two boundary cases, each country would produce predominantly in one sector or the other, but at least have some presence in both. In that instance, the typical degree of pricing to market would be increasing in $\epsilon$, lying somewhere in between the limiting cases shown in Figure 2.

1/ An alternate but related definition of pricing to market used elsewhere when considering multiple markets is the discrepancy between various export prices for a given producer. See for example Krugman (1987) and Knetter (1993).
Figure 2. Pricing to Market and the Pattern of Trade

Pricing to Market

Intraindustry

Intersectoral

ε
This page intentionally left blank
Interpreted as a boundary range, Figure 2 illustrates the significant variation in pricing-to-market behavior associated with various patterns of international trade. In a multi-sector setting, the same general result would obtain. In those sectors where the degree of intraindustry trade is high, price-setters would display a greater degree of pricing to market and a lower degree of pass-through. Recent empirical evidence documenting the degree of cross-industry variation in pricing-to-market behavior is reported by Knetter (1992). Examining various export prices for the G3 and the United Kingdom, Knetter (1992) finds that industry is the critical dimension explaining the variation in the pattern of pricing-to-market practices within and across countries.

IV. Relative Prices and Dynamics

This section extends the model to incorporate dynamics. In the presence of nominal frictions such as menu costs, producers do not adjust price continuously. Instead, price-setters respond to changing market conditions only intermittently. Moreover, when decentralized producers adjust price, they do so at different times through a lack of coordination. This lack of synchronization in pricing decisions and revisions induces inertia in the overall price-level. When price-setters move at different times, agents who respond and change price do not adjust completely in an effort to maintain relative prices with those who wait. Consequently, the repercussion of demand shocks may extend long after the initial impulse.

To capture this overlapping adjustment process, a simple staggered timing structure for price-setting is imposed. Specifically, I assume that half of all firms, home and foreign, revise their domestic and export prices in even periods when necessary, while the remaining half adjust in odd periods. I Taking the frequency and timing of intermittent price changes as given, I proceed to examine the comparative dynamics that emerge from this common staggering structure under contrasting patterns of trade and market segmentation.

Dropping the i subscript and representing price-setters by their time-location instead, the dynamic optimal price rule (without discounting) for a particular market--domestic or export--under both trade patterns is:

\[ x_t = 0.5 \bar{p}_t + 0.5 \bar{p}_{t+1}, \]

(1)

\[ \text{1/ See Ball and Cecchetti (1988), and Ball and Romer (1989) for further discussion on the optimal timing and frequency of price adjustment and explanations for the existence of price staggering.} \]
where $x_t$ is the actual price set for the periods $t$ and $t+1$ by home firms moving at time $t$. 2/ Also in equation (1), $\bar{p}_t$ and $\bar{p}_{t+1}$ represent the actual and expected optimal price at time $t$ and $t+1$, where expectations are formed rationally, based on all the information available at time $t$. Hence, time-dependent pricing leaves prices predetermined for two periods. Furthermore, the even-odd timing structure introduces a synchronized adjustment similar to Taylor (1979). Illustrating the overlapping nature of prices, the index of home producer prices prevailing in the domestic market takes the (approximate) form: $p_t^1 = 0.5x_{t-1}^1 + 0.5x_t^1$.

Substituting the static price rules from Table 2 and their corresponding foreign counterparts into the dynamic price-setting equation (1) defines a system of equations with four state variables, consisting of home and foreign local prices and export prices, where each price equation takes the form of a second-order stochastic difference equation as a result of the simple two-period overlapping structure of price adjustment. With the focus on real exchange rate dynamics, the four-variable system can be reduced to one difference equation in relative prices, given the behavior of the three forcing variables consisting of home and foreign money and the nominal exchange rate.

To proceed, note that the consumption-based real exchange rate $r^*$ defined simply as the currency-adjusted ratio of CPIs in each country--can be written (in logs) as a weighted measure of relative local and export prices: $r^* = \alpha(e+p^2-p^1) + (1-\alpha)(p^2-e^*+p^1)$. 3/ Using this measure of the real exchange rate and defining a measure of domestic-foreign price differentials: $d^* = \alpha(x^2-x^1) + (1-\alpha)(x^2-x^1)$, the difference equation governing the dynamics of price differentials under both patterns of trade can be written as:

---

1/ Agents are not risk neutral here, and equation (1) omits a risk premium that is a function of the conditional distribution of all nominal variables. For example, if money, prices, and the exchange rate are log-normal, the risk premium is a constant (comprised of variance and covariance terms) and can be ignored. Alternatively, the dynamics can be interpreted as deviations from a (stochastic) trend reflecting time-varying risk--which has very little impact on relative prices through symmetry across home and foreign price-setters.

2/ In a general sense, one can view equation (1) as the outcome of minimizing a quadratic loss function defined by squared deviations in actual price from optimal price over the period for which prices are predetermined.

3/ With two-way trade, this definition serves as a linear approximation. Out of steady state, spending patterns are constant under the approximation, neglecting the (typically second-order) effects of relative price movements on budget shares under intraindustry trade. See also footnote 1, page 6.
where

\[ d_t = \frac{\psi_0}{2}(d_{t-1} + d_{t+1}) + \frac{\psi_1}{2}(m_{t+1}^* - m_t^* + t_{m_t^*} - t_{m_{t+1}^*}) + \frac{\psi_2}{2}(-\epsilon_{t+1} - \epsilon_{t+1}) \]  

(2)

where

\[ \psi_0 = \frac{1 - \pi - 2\theta(1 - \alpha)}{1 + \pi + 2\theta(1 - \alpha)}, \psi_1 = \frac{2\pi}{1 + \pi + 2\theta(1 - \alpha)}, \psi_2 = \frac{4(1 - \alpha)(\theta(2\alpha - 1) - \pi)}{1 + \pi + 2\theta(1 - \alpha)}. \]

Note that in the equation in (2), the "homogeneity" condition: \( \psi_0 + \psi_1 + \psi_2 = 1 \) is not satisfied. The implication of this result is that monetary disturbances will have permanent effects on relative prices (long-run non-neutrality). 1/ Hence, PPP fails to hold under market segmentation. 2/

Solving the second-order difference equation in (2), the (non-explosive) fundamental solution is given by:

\[
d_t = \lambda d_{t-1} + \frac{\lambda \psi_1}{\psi_0} \sum_{j=0}^{\infty} \lambda^j(t_{m_{t+j}}^* - t_{m_{t+j+1}}^* + t_{m_{t+j+1}} - t_{m_{t+j}}^* + t_{m_{t+j}} - t_{m_{t+j+1}}^*) + \sum_{j=0}^{\infty} \frac{\lambda \psi_2}{\psi_0} \lambda^j(-\epsilon_{t+j} - \epsilon_{t+j+1})
\]

where

\[
\lambda = \frac{1 - \sqrt{\pi + 2\theta(1 - \alpha)}}{1 + \pi + 2\theta(1 - \alpha)} \in (0, 1).
\]

Using the fact that \( r = e + 0.5d + 0.5d_{-1} \), the dynamic solution for the real exchange rate is:

\[
r_t = \lambda e_{t-1} + \frac{\lambda \psi_1}{2\psi_0} \left( \sum_{j=0}^{\infty} \lambda^j(t_{m_{t+j}}^* - t_{m_{t+j+1}}^* + t_{m_{t+j}} - t_{m_{t+j+1}}^*) + \right.
\]

\[
+ \sum_{j=0}^{\infty} \lambda^j(-\epsilon_{t+j} - \epsilon_{t+j+1}) + \left. \right) + \frac{\lambda \psi_2}{2\psi_0} \left( \sum_{j=0}^{\infty} \lambda^j(-\epsilon_{t+j} - \epsilon_{t+j+1}) + \sum_{j=0}^{\infty} \lambda^j(-\epsilon_{t+j} - \epsilon_{t+j+1}) \right) + e_t - \lambda e_{t-1}.
\]

1/ Real quantities are of course also affected. In general equilibrium, the composition of \((\log)\) output is related to the steady-state real exchange rate by: \( r = \alpha(y_1 - y_2^*) + (1 - \alpha)(y_1^* - y_2) \).

2/ Furthermore, one can show that \( \psi_0 + \psi_1 + \psi_2 < 1 \), unless \( 2(\alpha - 1) = \gamma - 1 \) in which case homogeneity obtains. In the knife-edge condition, the left-hand side is bounded above by zero (local goods preference) while the right-hand side is bounded below by zero (rising marginal costs).
As alluded to, the dynamics governing the behavior of the real exchange rate in equation (4) apply to both patterns of trade. 1/ At first glance, this equivalency result may seem a bit surprising given the comparative differences shown earlier in Section III. However, the findings are mutually consistent. In fact, the differences regarding the behavior of nominal prices described earlier are what guarantee the equivalence result regarding the behavior of relative prices.

For illustration, compare the optimal price rules for the home exporter shown in Table 2. The single difference across trade patterns is found in the relative-price component. Under intraindustry trade, the home producer is concerned with competing foreign prices, the exchange rate, and other home export prices, whereas that same agent under cross-industry trade is concerned only with other home export prices.

Under the law of one price, the added concern with the foreign-currency price of output would compel price-setters in the first instance to reduce the degree domestic price inertia resulting from menu costs and price staggering. Prices would be less sticky in terms of national currency in order to reduce variation in terms of foreign currency, leading to less variable and persistent deviations in the real exchange rate under two-way trade. 2/ However, as indicated by equation (4), once the assumption of spatial arbitrage is removed, this comparative difference regarding the behavior relative prices no longer holds true.

Instead, once export prices detach from domestic prices, goods prices are in effect set in terms of local currency rather than national currency since price linkages predominantly exist within market destinations rather than across them. As a result, under two-way trade, the destination prices set by industry rivals from different countries are inertial in the same unit of account, as if they were from the same country, which happens to be the case of intersectoral trade. Consequently, real exchange rate dynamics are the same across differing patterns of trade, ceteris paribus, and depend only upon the timing structure of price adjustment.

Overall, pricing-to-market behavior increases the degree of persistence in the real exchange rate compared to the setting of one price. Regardless of the trade pattern, if price-setters effectively stabilize price in terms

1/ The behavior of the nominal exchange rate is also identical across both trade patterns. Given money market clearing and balanced trade (see Appendix), the (log) nominal exchange rate is given by:

\[ e_t = m_t - m_t^* \]

Comparing this expression to equation (4) highlights the fact that asset market prices and goods markets prices adjust at differential rates. By further specifying the law of motion for \( m \) and \( m^* \) (forcing variables), one can then obtain closed form solutions based on (4)

of local currency, destination prices become less responsive to exchange rate disturbances. Pricing to market in effect allows price-setters to keep export prices more stable in terms of foreign currency without requiring that domestic prices be more flexible in terms of home currency, unlike the case of one price. Consequently, the real exchange rate displays greater variability and persistence. 1/ For example, with an import share equal to a fourth, the degree of persistence under intersectoral trade would be twice that under the law of one price, and the corresponding increase under two-way trade would be even larger. 2/

Based on equation (4), Table 3 presents a sample of computed persistence values for the real exchange rate across different degrees of openness $\alpha$ and substitutability $\varepsilon$. Note that a decrease in $\alpha$, representing an increase in expenditure on imports as a share of national income, reflects an increase in openness.

<table>
<thead>
<tr>
<th>Openness $\alpha$</th>
<th>Persistence $\lambda$</th>
<th>Permanence</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\varepsilon = 2$</td>
<td>$\varepsilon = 6$</td>
</tr>
<tr>
<td>0.99</td>
<td>0.40</td>
<td>0.51</td>
</tr>
<tr>
<td>0.95</td>
<td>0.35</td>
<td>0.46</td>
</tr>
<tr>
<td>0.90</td>
<td>0.29</td>
<td>0.40</td>
</tr>
<tr>
<td>0.80</td>
<td>0.21</td>
<td>0.33</td>
</tr>
<tr>
<td>0.70</td>
<td>0.14</td>
<td>0.26</td>
</tr>
<tr>
<td>0.60</td>
<td>0.09</td>
<td>0.21</td>
</tr>
</tbody>
</table>

1/ In a closed-economy context, Ball and Romer (1990) show that real rigidities—such as efficiency wages—reinforce the effects of nominal rigidities, inducing a greater degree of persistence in domestic prices. In an open economy, pricing to market provides the source of real rigidity—allowing firms to stabilize relative prices in each market—which increases the degree of stickiness in local prices (in terms of local currency) and magnifies the degree of persistence in the real exchange rate.

2/ The equivalent solution for intersectoral trade under one price for the degree of inertia is $(2\alpha-1)\lambda$ compared to $\lambda$ under market segmentation. And in a closed economy ($\alpha=1$), the dynamics are the identical in the two instances. Meanwhile, when $\alpha=0.75$, inertia under one price is half that under pricing to market, and for intraindustry trade the increase in persistence can be shown to be larger still. See Faruqee (1992).
Also, persistence—reflecting the degree of inertia in the real exchange rate—is measured by the autoregressive parameter seen in equation (4). Finally, the last column in the table measures the lasting change in the real exchange rate resulting from a permanent monetary disturbance. This permanence factor measures the change in the steady-state value of the real exchange as a percentage of the change in the equilibrium nominal exchange rate. \(1/\) So for example, if \(\alpha = 0.9\), a 10 percent increase in the domestic money stock (and proportional nominal depreciation) would induce a steady-state real depreciation of 2 percent. \(2/\)

In Table 3, notice that persistence declines with increasing openness. The basic reasoning is as follows. As the import share rises, a nominal depreciation leads to greater CPI inflation, and real income is thus affected more by exchange rate changes. Since producers are also consumers concerned with their purchasing power, greater concern with the foreign-currency value of revenue reduces price stickiness in terms of local currency, thereby reducing inertia in the real exchange rate

\[(\theta(1-\alpha)^t \Rightarrow \lambda^t).\]

Also in Table 3, persistence increases with the elasticity of substitution between goods. The argument follows similarly as above only via the relative-price channel rather than the income channel. A higher degree of substitutability between varieties implies that demand is more sensitive to a change in relative price (see Table 1). As producers become more concerned with their price relative to the industry, efforts to stabilize price in terms of local currency in each market intensify. The resulting increase in local price stickiness induces greater overall persistence in the real exchange rate.

Lastly, permanence is an increasing function of openness. As import expenditure shares increase, the asymmetric effects of a lasting domestic monetary expansion on prices across segmented markets become larger. More simply, the degree of permanence rises with increasing openness since the effects of pricing to market increase with international trade. In a multi-sector setting, this finding can be interpreted in the context of tradability. In sectors where a product is essentially not traded \((\alpha = 1)\), the effects of pricing to market on the long-run real exchange rate are small. Finally, note that the permanent effects of monetary shocks are result of market segmentation and not nominal rigidity and would exist even if prices were perfectly flexible.

Persistence and permanence in relative prices have important time-series implications for exchange rates. As with earlier sticky-price

---

\(1/\) Provided that market segmentation exists in steady state.

\(2/\) The model thus includes a role for monetary factors in determining equilibrium real exchange rates, so long as markets remain segmented. See Krugman (1990) for a recent discussion on real determinants of equilibrium exchange rates.
models, differential rates of adjustment in goods prices versus asset prices allow nominal and real exchange rates to move closely together in the short run. Following an initial impulse, the real exchange rate then eventually self-corrects towards equilibrium, reflecting the persistent effects of monetary disturbance on relative prices. The current framework also further predicts that the nominal and real exchange rate move together over the longer run as well, I/ since monetary shocks have permanent effects on the nominal rate of exchange and on equilibrium relative prices to the extent that goods markets remain segmented. Alternatively, if the law of one ultimately price holds in the long run, the "permanence" factor can be viewed as long-term persistence in the real exchange rate, extending beyond the period required for prices to adjust.

V. Conclusions

With the collapse of Bretton Woods and the abandonment of fixed exchange rates in 1973, the two decades since have seen a great deal of turbulence in foreign exchange markets. Yet the large swings in exchange rates under the era of floating have not brought about large movements in inflation or disinflation as one would expect with pass-through. Instead, volatile nominal exchange rates have translated into volatile real exchange rates in the post-Bretton Woods world, while prices locally have remained remarkably stable.

In an attempt to understand these developments, this paper develops a model of market segmentation and pricing to market, with important macroeconomic consequences for the behavior of international prices broadly in line with the stylized facts. Motivated by a preponderance of the empirical evidence disavowing the law of one price, the model details some of the economic implications of pricing-to-market behavior. The results include both cross-sectional implications regarding nominal prices and time-series implications regarding relative prices.

Across different patterns of trade, significant variation exists in the degree of pass-through and pricing to market depending upon the degree of intraindustry trade and the substitutability between home and foreign goods. Across time, dynamic adjustment in prices suggest that nominal and real exchange rates move together over the short run and over the longer run as well to the extent that markets remain segmented. Overall, pricing to market provides a potentially important source of local price stickiness and real exchange rate persistence.

I/ See Adams and Chadha (1991) for empirical evidence.
The Basic Model

For the home country, producer-consumer i's utility function is given as follows:

\[ U_i = \left( \frac{C_i^h}{Q} \right)^{\frac{\mu}{\mu-1}} \left( \frac{M_i}{Q} \right)^{1-\frac{1}{\mu}} - \frac{1}{\gamma} \left( \frac{Y_i^h}{Q} \right)^{\gamma} - \frac{1}{\gamma} \left( \frac{Y_i^f}{Q} \right)^{\gamma} - F_i - zD_i; \quad 0 < \mu < 1, \gamma > 1, \]  

(A1)

where \( C_i \) is a consumption basket of home and foreign goods, \( M_i \) represents money holdings of home currency (no currency substitution), \( Q \) is the domestic consumer price index, and \( Y_i^h, Y_i^f \) are agent i's the level of output for the domestic and export markets. \( F_i \) denotes fixed costs in the production of a single variety, while \( z \) captures the "menu cost" for changing price. Also in equation (A1), \( D_i \) is a decision dummy variable, equaling one if agent i changes his or her own price and zero otherwise. Lastly, \( \mu \) and \( 1-\mu \) represent the constant expenditure shares of goods and money, while \( \gamma \) measures the elasticity of marginal disutility with respect to output.

In the case of intersectoral trade, agents have CES subutilities over home and foreign varieties of goods, respectively. Explicitly, \( C_i \) is given as:

\[ C_i = \left( \frac{C_i^h}{\alpha} \right) \left( \frac{C_i^f}{1-\alpha} \right)^{1-\alpha} ; \quad \frac{1}{2} < \alpha < 1, \]  

(A2)

where \( C_i^h \) represents i's consumption basket of all home goods:

\[ C_i^h = (n)^{1/\epsilon} \left[ \sum_{j=1}^{n} \left( \frac{C_i^{hj}}{\epsilon} \right)^{\epsilon} \right]^{1-\epsilon} ; \quad \epsilon > 1, \]  

(A3)

and where \( C_i^f \) represents i's consumption basket of all foreign goods:

\[ C_i^f = (n)^{1/\epsilon} \left[ \sum_{j=1}^{n} \left( \frac{C_i^{fj}}{\epsilon} \right)^{\epsilon} \right]^{1-\epsilon} ; \quad \epsilon > 1. \]  

(A4)

In these last two subutility expressions, \( C_i^{hj} \) and \( C_i^{fj} \) represent agent i's consumption levels of home good \( j \) and foreign good \( j \), while \( \epsilon \) measures the constant elasticity of substitution between any two home or any two foreign varieties, respectively.

Consider now the case of intraindustry trade. Home and foreign goods no longer belong to separate commodity groups as countries exchange goods...
within the same industry. To modify preferences accordingly, I replace equations (A2) through (A4) with:

$$C_i = (n)^{1/1-\epsilon} \left[ \beta^{1/\epsilon} \sum_{j=1}^{n} \left( C_{ij}^h \right)^{\epsilon-1/\epsilon} + (1-\beta)^{1/\epsilon} \sum_{j=1}^{n} \left( C_{ij}^f \right)^{\epsilon-1/\epsilon} \right] \epsilon^{1-1/\epsilon}; \quad \frac{1}{2} < \beta < 1, \quad (A5)$$

where $\beta$ measures home goods preference. Although agents consider home and foreign varieties to be of the same product type, residents in each country still prefer local goods.

The budget constraint, identical in both instances, completes the formulation of the consumer's problem facing the home agent:

$$\sum_{j=1}^{n} P_j C_{ij}^h + \sum_{j=1}^{n} E P_j^* C_{ij}^f + M_i = I_i, \quad (A6)$$

where $P_j$ is the price of home good $j$ (in home currency) and $P_j^*$ is the price of foreign good $j$ (in foreign currency) prevailing in the home market (designated later by a superscript $1$), $E$ is the nominal exchange rate (home currency price of foreign currency), and $I_i$ is agent $i$'s level of nominal wealth.

**Intersectoral Trade**

Solving the consumer's problem by maximizing (A1), given equations (A2) through (A4), with respect to $C_{ij}^h, C_{ij}^f,$ and $M_i$ subject to (A6) yields the following individual demand functions for domestic goods, imports and money:

$$C_{ij}^h = \left( \frac{P_j}{P} \right)^{-\epsilon} \left( \frac{\alpha \mu I_i}{nP} \right) \text{ for } j=i ... n; \text{ where } P = \left[ \frac{1}{n} \sum_{j=1}^{n} P_j^{1-\epsilon} \right]^{1/1-\epsilon}, \quad (A7)$$

and

$$C_{ij}^f = \left( \frac{P_j^*}{P^*} \right)^{-\epsilon} \left( \frac{(1-\alpha) \mu I_i}{nEP^*} \right) \text{ for } j=1 ... n; \text{ where } P^* = \left[ \frac{1}{n} \sum_{j=1}^{n} P_j^{*1-\epsilon} \right]^{1/1-\epsilon}, \quad (A8)$$

and

$$M_i = (1-\mu)I_i \quad (A9)$$
Intraindustry Trade

Again, solving the consumer's problem by maximizing (A1) subject to (A6), but given (A5) instead, one can write the individual demands for each home and foreign variety under two-way trade as:

\[ C_{ij}^h = \left( \frac{P_j}{Q} \right)^{-\varepsilon} \left( \frac{\beta \mu I_i}{nQ} \right) \text{ for } j=1 \ldots n, \]  

(A10)

and

\[ C_{ij}^e = \left( \frac{EP_j^*}{Q} \right)^{-\varepsilon} \left( \frac{(1-\beta)\mu I_i}{nQ} \right) \text{ for } j=1 \ldots n. \]  

(A11)

Money demand remains unchanged from that under Interindustry trade in (A9). Summing up individual demands for each home variety in (A7) or (A10) over all home consumers along with the equivalent export demands over foreign consumers yields the product demands facing the representative domestic producer as a function of relative price and real wealth at home and abroad.

The Producer's Problem

Producer i's revenues plus his or her initial money holdings make up the individual's nominal wealth: \( I_i = P_1^1 Y_{i1} + P_2^2 Y_{i2} + M_i \). Using this definition of wealth, the indirect utility function (ignoring menu costs) is:

\[ U_i = \frac{P_1^1 Y_{i1} + P_2^2 Y_{i2}}{Q} - \frac{1}{\gamma} Y_i^1 \gamma - \frac{1}{\gamma} Y_i^2 \gamma - F_i + \frac{M_i}{Q}. \]  

(A12)

For stability, "marginal cost"--in terms of the marginal disutility of output--must be non-decreasing, requiring \( \gamma - 1 \geq 0 \). Hence, scale economies in production in the model refer to decreasing average rather than marginal costs. The producer's problem can be stated as maximizing the modified profit function (A12) with respect to each price given demand for output in each market shown in Table 1. The explicit solutions to the producer's problem are shown (in logs) in Table 2.

General Equilibrium

The following conditions characterize general equilibrium under both patterns of trade.

Money Market Equilibrium

Using the money demand function in (A9) and the definition of wealth, domestic money market equilibrium is given by:
\[ M = \sum \bar{M}_i = \sum M_i = (1-\mu)\left(\sum I_i\right) = \frac{1-\mu}{\mu} \sum \left(p_1^1 Y_1 i + p_1^2 Y_2 i\right). \]  

(A13)

where the total money stock held by home agents equals domestic aggregate money demand and is proportional to nominal GNP. Based on this quantity equation relationship at home and similarly abroad, one can derive the demand facing each producer in both the local and export market shown in Table 1, where \( \frac{M}{I-\mu} \) is equal to GNP per capita.

**Goods Market Equilibrium**

In symmetric equilibrium at the industry level, identical producers set identical prices. Consequently, the following relative prices are unity in general equilibrium: \( \frac{P_1^1}{P_1} = \frac{P_1^2}{P_2} = 1 \) and correspondingly for foreign producer prices. Adding up product demands in Table 1--given relative producer prices in equilibrium--yields an income-expenditure equality condition for the home country:

\[ p_1^1 Y_1 + p_1^2 Y_2 = QC, \]  

(A14)

where quantity variables without i subscripts indicate measures summed over all home agents (e.g., \( C = \sum C_i \)). Note that goods market equilibrium in (A14) equates GNP at market prices with aggregate consumption, requiring balanced trade (\( NX = 0 \)) in the absence of capital mobility.

**Exchange Rate Equilibrium**

Given goods and money market clearing and balanced trade \( (EP^1 = P_2 Y^2) \), the nominal exchange rate in equilibrium is given by:

\[ E = \frac{M}{M^*} \]  

(A15)

The rate of exchange adjusts to ensure balance of payments equilibrium. In symmetric equilibrium at the country level, national money supplies are assumed to be equal and, hence, local and export prices respectively are also equal at home and abroad. Thus, the initial symmetric steady-state equilibrium has both the nominal and real exchange rate equal to unity: \( E = R = 1 \).
References


