Challenging the Empirical Evidence from Present Value Models of the Current Account

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Abstract

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Under near-singularity conditions typically generated by persistence in current account data the predictions of present value models become extremely sensitive to small sample estimation error. Moreover, traditional Wald tests will distort the likelihood that the model is true. Using OECD data we find that: (i) the Wald test often leads to the wrong inference compared to a valid test; (ii) in all cases posterior distributions of the predicted series and associated correlation coefficients and variance ratios are very wide. In particular, one cannot draw any firm conclusion regarding excess current account volatility.

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Keywords: Current account, present value model, model evaluation

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I. INTRODUCTION

The intertemporal approach to the current account first introduced by Sachs (1981) views net accumulation of foreign assets as a way for domestic residents to smooth consumption intertemporally in the face of idiosyncratic income shocks. Compared to static analyses of the current account that preceded it, this approach assigns an explicit and crucial role to both agents’ expectations and to the time series properties of the shocks to the economy. Under some simplifying assumptions, the intertemporal approach yields an analytical and easily tractable expression for the current account equal to the present discounted value of expected future net income declines.

Starting with Sheffrin and Woo (1990), economists have been eager to put the present value model to the test. In order to construct forecasts of income declines that take into account all relevant information available to the agent—and not just to the econometrician—researchers have relied on insights provided by Campbell (1987) and Campbell and Shiller (1987) in different contexts. If the model is a true representation of the data, then the current account should embed all relevant information necessary to forecast income changes. In practice, this has meant running unrestricted VARs on income changes and the current account. The estimated VAR system is then used to forecast income declines and obtain the associated model-predicted series for the current account. Testing for equality between the estimated and actual series is akin to testing nonlinear restrictions on the VAR parameters.

Many empirical tests of the present value model have reached a similar conclusion: the model is rejected by the data, and this rejection is often due to actual current account series being positively correlated with, but much more volatile than, model-predicted series. Given that the present value model assumes full capital mobility, the finding of excess current account volatility has been used as evidence against Feldstein and Horioka’s famous proposition of limited international capital mobility. In this context, recent papers in the literature have tried to “augment” the model in several directions to generate extra predicted

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2 The intertemporal approach to the current account is surveyed in Obstfeld and Rogoff (1995).

3 Net income refers to income net of investment and government expenditures. In the remainder of the paper we will use the term “income” to refer to net income.


5 Exceptions include Ghosh and Ostry (1995) who find that the model fits well for many developing countries.

6 Ghosh (1995) is a major proponent of this point of view for industrialised countries.
volatility. Bergin and Sheffrin (2000) show that allowing for variable real exchange rates and interest rates improves the fit of the model relative to Australian, Canadian, and British data. Gruber (2004) generates extra volatility in the predicted series by way of habit formation and excess smoothness in consumption. Nason and Rogers (2003) test competing additions to the model and find that exogenous shocks to the world real interest rate best reconcile the extended model with Canadian data.

How robust are the findings? Figure 1 shows actual and predicted current account series for Canada and the UK in two nonoverlapping subperiods. For Canada, the model fits the data very well for the first subperiod, but there is evidence of strong excess volatility in the second. For the UK it is almost the opposite. Recent papers trying to explain excess current account volatility cannot account for this subperiod instability. For example, if nonseparable preferences explain excess volatility, how can we find large excess volatility in some periods but no excess volatility in others? Similarly, if shocks to the world interest rate are the explanation, why are these shocks affecting Canada and the UK at very different times?

This paper argues that the Campbell-Shiller methodology as used in this literature can yield very misleading results. We show that under near-singularity conditions typically generated by persistence in current account data, small sample estimation errors can have disproportionately large effects on the path of the predicted series. Posterior distributions of the predicted series and associated correlation coefficients and variance ratios will be very wide, casting doubt on inference based on a direct comparison of the actual and predicted series. Moreover, we show that the traditional Wald test of the model is not valid in the singularity region. Since the test’s null hypothesis amounts to nonlinear restrictions on the VAR parameters, researchers rely on the Delta method linear approximation to compute the necessary variance-covariance matrix. In short samples the linear approximation becomes less and less precise as one approaches singularity. The Wald test may thus imply a very different likelihood relative to a valid linear F-test, potentially leading to model rejection when the F-test accepts it and vice versa.

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7 Note also that the correlation between the actual and predicted series for the UK goes from negative and large (in absolute value) in the first subperiod to positive and large in the second.

8 These graphs are meant for illustration and cannot be taken as formal evidence against the literature’s findings. However, formal analysis in our paper will confirm that excess volatility is not a robust finding.

9 Note that our discussion will be conducted from a likelihood perspective. We will use the terms “confidence intervals” and “hypothesis tests” as they are used in the literature, but treating them as ways of generating descriptive information about the shape of the likelihood function of the posterior pdf. For example, we interpret “false rejection” by a test as implying that the likelihood is smaller in the neighborhood of the null hypothesis than it actually is.
We illustrate our theoretical points using the same five countries that Obstfeld and Rogoff (1996) chose to discuss the literature: Belgium, Canada, Denmark, Sweden, and the UK. Results from annual and quarterly data strongly support our predictions. First, likelihoods differ sharply between the two tests. Indeed, in four out of ten cases the Wald test leads to the wrong inference at traditional confidence levels, rejecting the model where the F-test accepts it and vice versa. For instance, the Wald test strongly rejects the model for all five countries using quarterly data but the F-test accepts it for Belgium and Canada. Second, we generate posterior distributions of the predicted series by taking draws from the multivariate normal given by the VAR parameter estimates and their variance-covariance matrix and reestimating for each draw the predicted series. The 2.5th and 97.5th posterior percentile lines of the model-predicted series are almost without exception very wide compared to the actual series, often spectacularly so. This reflects the fact that under near-singularity, the cross-equation parameters can have very large variances. We also build posterior distributions of the correlation coefficient and variance ratio between actual and predicted current account using similar procedures. When one looks at the distributions of the variance ratio, they are always very dispersed and indicate substantial probability that the actual series is both several times more and several times less volatile than the model-predicted series. Posterior distributions of the correlation coefficient are also very dispersed and show substantial probability that the correlation is close to both 1 and -1. These findings occur regardless of whether the F-test has accepted or rejected the model. Such dispersion makes any claim of excess current account volatility (or its opposite for that matter) very dubious.

It is important to note that our paper is not the first to warn against traditional inference from present value models of the current account. Besides the obvious criticism that output and the current account may not be linear processes, Kasa (2004) has warned against plausible (linear) income processes for which the model-consistent current account may not reflect all relevant information to forecast income changes. As Kasa acknowledges, however, such processes are plausible but very hard to distinguish empirically from processes that pose no problem to the methodology. Our criticism of the literature is more general since our case rests on conditions of near-singularity, which are easy to verify and almost always met in the data. The implications of near-singularity for the model’s predictions can also be confirmed with the data.

Finally, the issues highlighted in this paper are potentially relevant in other areas of the literature which also use the Campbell-Shiller methodology. One example is the expectations theory of the term structure of interest rates, where researchers often rely on bivariate VAR’s in short rate changes and the long/short yield spread to construct the present value model-predicted series of the spread (see among others Campbell and Shiller (1987), Campbell and Shiller (1991) and Hardouvelis (1994)). Our paper is organized as follows. Section II explains how the present value model is tested and exposes the pitfalls of the empirical methodology under near-singularity. Section III presents the empirical results based on OECD data. Section IV concludes.

Indeed, this was the area of research where the Campbell-Shiller methodology was first introduced.
II. TESTING THE PRESENT VALUE MODEL

A. The Tests

In its simplest form, assuming quadratic utility on the part of the representative agent, a constant real return on a single internationally traded bond, and a subjective discount factor equal to the inverse of the (gross) return, the present value model (PVM) predicts that the current account is given by:

\[ CA_t = -E_t \sum_{i=t+1}^{\infty} \left( \frac{1}{1 + r} \right)^{i-t} \left[ Y_t - Y_{t-1} \right] \]  

(1)

where \( r \) is the (constant) real interest rate and \( Y \) is income net of government spending and investment. That is, permanent shocks to income should have no effect on the current account. Positive transitory shocks should raise the current account on impact. Anticipated future income increases will lower the current account.

Assume that the joint data generating process of income changes (\( \Delta Y \)) and the current account (\( CA \)) is given by an unrestricted \( l \) order VAR, where \( l \) is the number of lags. Define the following \( 2l \) column vector of data \( X_t = [\Delta Y_t, \ldots, \Delta Y_{t-l+1}, CA_t, \ldots, CA_{t-l+1}] \). Then:

\[ X_t = CX_{t-1} + u_t \]

where \( C \) is the companion matrix and \( u_t \) is a \( 2l \) column vector of zero-mean homoskedastic errors \([u_{\Delta Y}, 0, \ldots, 0, u_{CA}, 0, \ldots] \). In this setup, the forecast of the income change \( k \) periods ahead is given by:

\[ E_t \Delta Y_{t+k} = AC^k X_t \]

where \( A \) is the \( 2l \) row vector \([1 \ 0 \ 0 \ldots 0] \). Using this forecast in the present value relation (1) yields the following expression for the time \( t \) value of current account predicted by the model:

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11 It is trivial to relax the last two assumptions and still get a very tractable expression for the current account. See Ghosh (1995).

12 As mentioned earlier, the intuition for obtaining forecasts of expected income declines from a VAR containing the current account comes from Campbell and Shiller (1987). If the model is correct then the current account should contain all the relevant information used by agents to form such forecasts.
\[ CA_{p,t} = KX_t, \]  

(2)

where \[ K = -\frac{AC}{1 + r} \left[ I - \frac{C}{(1 + r)} \right]^{-1} \]

One then obtains the estimate \( \hat{CA}_{p,t} \) by replacing \( C \) with its empirical estimate \( \hat{C} \) in (2).

Correlation coefficients and volatility ratios between \( \hat{CA}_{p,t} \) and the actual current account series can serve as preliminary tests of the model. More formally, testing whether \( \hat{CA}_{p,t} \) and \( CA_t \) are equal is akin to testing whether \( K \) equals the \( 2l \) vector \( T \) whose elements are all zero except for the \( (l+1) \)th element, which equals one.\(^{13}\) An estimate of the variance-covariance of \( K \) is needed to perform such a test. Since \( K \) is a non-linear function of the VAR parameters, researchers approximate its variance-covariance by \( JVJ' \), where \( V \) is the variance-covariance matrix of the VAR parameters and \( J \) is the Jacobian of \( K \). This is the Delta method linear approximation of the variance-covariance. Then, the statistic:

\[ W = (K - T) \ast \left[ JVJ' \right]^{-1} \ast (K - T)' \]

has an asymptotic chi-square distribution with \( 2l \) degrees of freedom. This is the Wald test of the cross-equation restrictions of the model typically used in the literature.

B. Problems Under Near-Singularity

From equation 2, we can see that \( CA_{p,t} \) is a linear function of the inverse of \( M = \left[ I - \frac{C}{(1 + r)} \right] \).

Were \( M \) to have at least one eigenvalue close to zero (i.e. were \( C \) to have at least one eigenvalue close to \( 1 + r \)), a small error in the estimated VAR parameters would translate into potentially very large deviations in the inverse of \( M \) and hence on the path of the model predicted series. This would make any statistical inference based on a comparison of the actual and model predicted series dubious. Posterior distributions of the predicted series and associated correlation coefficients and variance ratios will be very wide.

Besides excess sensitivity, near-singularity can lead to false rejection and false acceptance of the model using the traditional Wald test. If in short samples the Delta method does not yield a good approximation of the variance-covariance matrix of \( K \), the statistic \( W \) in equation 3 will be far from following its asymptotic distribution. It turns out that in the near-singularity

\(^{13}\) More precisely, this is a joint test of the model and of the assumption that the data generating process of income changes and the current account is given by the unrestricted VAR.
region mentioned above, the Delta method does greatly distort this variance-covariance in short samples.

To see the problem, assume for simplicity that the element in $K$ we are trying to test for is proportional to $\frac{1}{c}$, where $c$ is some parameter to be estimated. Figure 2 shows what happens when the true value $c_0$ is close to zero. In this example, the small sample estimate $\hat{c}$ falls a bit further from zero but its probability interval still contains the true value. However, the interval $[\hat{K}_{L,\theta}, \hat{K}_{R,\theta}]$ computed by linear approximation will be “too small” given the steepness of the curve and may therefore not include the true value $K_0$. Clearly, the problem arises because the slope of $\frac{1}{c}$ changes rapidly in the neighborhood of 0. Also of note, distortion is a short sample issue since the interval around $\hat{c}$ shrinks as sample size increases. We can now see from equations 2 and 3 that the reasoning in Figure 2 can be extended to the Chi-square test of the present value model. Under near-singularity, the Delta method would produce a variance-covariance matrix which is “too small” and a $W$ statistic which is “too large”, leading to a false rejection of the model. Figure 3 shows the opposite problem. Here, the estimate $\hat{c}$ falls a bit closer to zero and its interval excludes $c_0$. However, the interval around $\hat{K}$ will be “too large” given the steepness of the curve and will include $K_0$.

Translating this to the Chi-square test, the Delta method would produce a variance-covariance matrix which is “too large” and a $W$ statistic which is “too small”, leading to a false acceptance of the model.

The relevant question now is: how common is the near-singularity issue in practice? Unfortunately, the answer is “very common.” Table 1 shows for each country sample one estimated eigenvalue of the VAR companion matrix. These eigenvalues are above 0.9 for all five countries in quarterly data, and in three out of five cases in annual data. In fact, the eigenvalues are often close to $1 + r$ which is the critical value for singularity. VAR estimations show that the coefficients on lagged income changes are often small while coefficients on the lagged current account are high. It is current account persistence that is generating near-singularity in our data.

It is useful to bear in mind that we follow a Bayesian approach in the paper. From a classical perspective, the VAR parameters have only an asymptotic justification. Moreover, when the unit root case is approached with sample size held fixed, the usual OLS intervals become less and less precise in the sense of coverage probability. It is therefore not theoretically impossible that the Delta method would yield an interval that is at least as accurate. From a Bayesian perspective, the usual OLS confidence intervals are correct descriptions of the posterior under a flat prior, and the Delta method does indeed distort a true posterior probability interval.

C. A Valid Test

Consider now the following alternative test. Let’s define $R_t = CA_t - (1 + r)CA_{t-1} - \Delta Y_t$ and $I_{t-1}$ as the information set containing all the values of $CA_{t-1}, ..., CA_{t-n}, \Delta Y_{t-1}, ..., \Delta Y_{t-n}$, as well as
the \( t - 1 \) or previous values of any other variable. Equation 1 implies that \( E_t(r_t | I_{t-1}) = 0 \).

One can thus regress \( R_t \) on \( I_{t-1} \) with the appropriate number of lags and do a simple F-test on the joint nullity of the coefficients of all the regressors in the information set. This test is fully valid as it does not rely on a (bad) linear approximation.\(^{14}\) One can then “measure” the distortions in the standard Wald test of the \( K \) vector by comparing the posterior probabilities that the model is true given by the Wald and the F tests. The F-test has been used by the literature but much less so than the Wald test.\(^{15}\) The likely reason is that the usual methodology has the advantage of yielding a model-predicted series which can then be compared directly with the data.

### III. Empirical Results

In this section we turn to country data. We take the same five small open economies that Obstfeld and Rogoff (1996) chose to discuss tests of the current account using the Campbell-Shiller methodology. These countries are: Belgium, Canada, Denmark, Sweden, and the United Kingdom. We use data at both annual and quarterly frequencies. The data appendix details the data sources, and discusses data construction and other estimation issues. As was mentioned before, the eigenvalues of the variance-covariance matrix in Table 1 show that near-singularity is a real issue in most country samples considered. In this context, we study each aspect of the Campbell-Shiller methodology as used in this literature.

#### A. Tests of the Model

Table 1 gives the likelihood of the Wald and F tests expressed as one minus the \( cdf \) of the test statistic. The distortion associated with the Wald test can be very large. The Wald statistic suggests levels of significance that are up to eighty-eight percentage points different from their actual level as given by the F-test. The average distortion across the ten country samples equals thirty three percentage points. Most strikingly perhaps, in four out of ten cases the Wald test leads to the wrong inference at the traditional 95 percent level of

\(^{14}\) Here again the analysis is conducted from a likelihood perspective. A classical approach would run into the issue created by the possible existence of a unit root (which is a special form of persistence). If \( CA \) has a unit root, this does not change the fact that \( R_t \) is by construction stationary under the null, indeed serially uncorrelated. But some of the right-hand side variables in the regression might include unit roots. The full set of right-hand side variables coefficients, when tested as a group, would generate an F-statistic whose asymptotic sampling distribution under the null is not the standard F-distribution. However, from a likelihood perspective and under Gaussianity assumptions of the residuals the likelihood’s shape is itself Gaussian regardless of whether unit roots are present. The F-test does not need special interpretation in the presence of unit roots. Moreover, the F-statistic is exact in small samples for the reasons previously discussed.

\(^{15}\) Exceptions include Otto (1992) who only uses the F-test and Nason and Rogers (2003) who use both.
confidence (i.e., the Wald test rejects the model when it should have been statistically accepted, or vice versa). In particular, the Wald test always rejects the model with quarterly data while the F-test accepts it for two out of five countries (Belgium and Canada). Put together, these results illustrate how misleading the Wald test commonly used in the literature can be.

B. The $K$ Vector

The literature uses several other ways to assess the model’s fit. To start, some papers informally compare the estimated $K$ vector with its theoretical value (see Obstfeld and Rogoff (1995, 1996)). However, the coefficients of the $K$ vector can be very imprecisely estimated in the presence of singularity created by persistence, regardless of whether the model is or is not a good representation of the data. As we know, $K$ is a non-linear function of the parameters of the companion matrix of the VAR. In the region of the aforementioned singularity, a small difference in the coefficients of the estimated companion matrix translates into a very large difference in the coefficients of the $K$ vector. This can be seen in Figure 2 as $\hat{c}$ approaches zero.

This prediction is borne by the data. For each country sample, we generate 10,000 draws from the multivariate Normal distribution given by the estimated VAR parameters and their associated variance-covariance matrix.\footnote{In the case of the UK quarterly data we generated 5,000 draws for computational reasons.} For each of these draws we compute the associated $K$ vector, giving us a (Bayesian) posterior distribution of the $K$ vector representative of estimation uncertainty.\footnote{Nason and Rogers (2003) also compute measures of the dispersion in the $K$ vector coefficients, albeit using different statistical methods.} Figures 4 and 5 present the ex-post distribution of the $l+1$th coefficient of $K$, which is supposed to be equal to one under the null. For all country samples, the variance of the coefficient is very large. Even when the model is consistent with the data as determined by the F-test, there is a high probability that the coefficient will be far from its theoretical value. The coefficient can easily be negative.\footnote{To see why a large and negative coefficient can occur, note in Figure 2 that the probability interval around $\hat{c}$ can encompass small and negative values and hence imply large and negative values of $K$.} Since by construction $CA_{p,j} = KX_j$, the large posterior variance of the $K$ vector has strong implications for the path of the predicted current account, its variance, and its correlation with actual data, as we will now discuss.

C. Graphical Analysis

The literature has often drawn inference by comparing the paths of the actual and predicted current account during economically significant periods. For example, Sheffrin and Woo

\begin{itemize}
\item \footnote{In the case of the UK quarterly data we generated 5,000 draws for computational reasons.}
\item \footnote{Nason and Rogers (2003) also compute measures of the dispersion in the $K$ vector coefficients, albeit using different statistical methods.}
\item \footnote{To see why a large and negative coefficient can occur, note in Figure 2 that the probability interval around $\hat{c}$ can encompass small and negative values and hence imply large and negative values of $K$.}
\end{itemize}
(1990) note that the model underestimates UK current account deficits generated by the first oil shock. Ghosh (1995) draws similar conclusions for Japan following the second oil shock. To evaluate the robustness of such inference, Figures 7 and 8 plot the 2.5th and 97.5th percentiles of the posterior distribution of the predicted current account. We obtain this distribution by recalculating the predicted series for each of the 10,000 values of $K$, and the plotted lines correspond to the 2.5th and 97.5th percentiles of the distribution at each point in time.\(^{19}\) Consistent with the finding that $K$ has very large variance in the singularity region, the percentile lines are typically very wide compared with the actual series, often dramatically so. Returning to our previous example, it is true for instance that actual deficits following the first oil shock exceed the (single sample estimate of the) model predicted series (see Figures 6 or 8). Yet particularly in quarterly data the percentile lines easily encompass those deficits, showing that the conclusion that the model underestimated the deficits is unwarranted.

**D. Variance Analysis**

The literature has often emphasized that actual current account series are typically more volatile than the model’s predictions (see among others Ghosh (1995), Obstfeld and Rogoff (1996), Nason and Rogers (2003), and Gruber (2004)).\(^{20}\) In fact, Ghosh (1995) uses excess current account volatility as evidence against Feldstein and Horioka’s claim that international capital markets are not highly integrated.

Figures 10 and 11 plot the posterior distributions of the intertemporal variance ratio.\(^{21}\) Variance ratios are often very dispersed, which is a direct consequence of the dispersion in the $K$ vector in the singularity region. Even when the model is consistent with the data the predicted current account can still be much more or much less volatile than the actual. For example, the model is strongly consistent with Belgian annual data yet there is a 40 percent probability that the predicted current account is over four times as volatile as the actual. There is also a 20 percent probability that the predicted current account displays less than a fourth of the volatility of the actual series. Similar observations hold for the other data sets which failed to reject the model. Also, the data does not support claims that the current account is excessively volatile. In our samples, the probability that the predicted series is

\(^{19}\)Cashin and McDermott (1998a) and Hall et al. (2001) generate confidence bands using bootstrapping techniques. However, Sims and Zha (1995) show that Bayesian bands are often better even by classical criteria.

\(^{20}\)We noted in the introduction some inconsistencies regarding excess current account volatility findings.

\(^{21}\)For each draw $i$ from the multivariate Normal, we construct the predicted current account $CA_{p,t}^i = K'X_t$ for all $t$ in our time range, where $X_t$ is the time $t$ vector of data. For each draw $i$ the variance ratio is calculated as the intertemporal variance of $CA_{p,t}^i$ over the $t$ range divided by the intertemporal variance of the actual series.
more volatile than the actual is often large. It averages 44 percent over our 10 samples, ranging from 11 percent for Belgian quarterly data to over 97 percent for Swedish quarterly data. Note that one cannot conclude that the current account is less volatile than the model predictions either.

### E. Correlation Analysis

Despite the supposed failure of the model to match current account volatility, some authors have claimed that the model has value in that the correlation between actual and predicted series tends to be quite high (see Obstfeld and Rogoff (1996) and Nason and Rogers (2003) among others). Figures 12 and 13 show the posterior distributions of the in-sample correlation between actual and predicted series. These distributions are once again very wide reflecting dispersion in the $K$ vector, casting doubt on the above claim. Also, correlation values are in no way indicative of the model’s statistical validity. Most strikingly in the case of Belgian and Danish annual data the valid F-test accepts the model, yet there is over 45 percent probability that the correlation lies between $-1$ and $-0.9$. Conversely, for Swedish annual data there is a 37 percent probability that the correlation will exceed 0.95 even though the test has rejected the model. Finally, the distributions often cluster around one and minus one which can also be explained by the behavior of $K$ under near-singularity. To see why, consider the case of one lag in estimation. Then:

$$CA_{p,t} = \hat{k}_1 \Delta Y_t + \hat{k}_2 CA_t$$

and

$$corr(\hat{CA}_p, CA) = \frac{\text{cov}(\hat{k}_1 \Delta Y + \hat{k}_2 CA, CA)}{\sqrt{\text{Var}(\hat{k}_1 \Delta Y + \hat{k}_2 CA)} * \text{Var}(CA)}$$

If $\hat{k}_2$ is positive (negative) and very large relative to $\hat{k}_1$, then $\hat{CA}_p$ is mostly driven by $\hat{k}_2 CA$ and the correlation will tend to one (minus one).

### IV. Concluding Remarks

In this paper we have shown how misleading the Campbell-Shiller methodology can be when used on present value models of the current account. Our discussion casts doubt on the results found in this large empirical literature, including extensions to the present value model evaluated using similar techniques. In particular, our statistical analysis does not support any conclusion of excess current account volatility. The paper implies that the F-test and not the traditional Wald test should be used to formally test the model. Also, inference should in no case be limited to the sample estimate of the predicted path of the current account. It is essential that ex-post distributions of the predicted series and associated correlation coefficients and variance ratios be systematically constructed. Given the potentially high variance of the cross-equation parameters in the singularity region, it is likely that researchers will often find it difficult to make useful inferences about the relationship between the data and the model’s predictions.
REFERENCES


DATA APPENDIX

All our data are from the International Financial Statistics of the International Monetary Fund. The periods covered are indicated in the table below, noting that for each country we use the longest available sample in IFS.22

Net output and current account are defined as: \( Y_t = GDP_t - G_t - I_t \) and \( CA_t = GNP_t - C_t - I_t - G_t \) and are expressed in real, per capita terms. Corresponding IFS series are as follows: GNP: gross national income (line 99a); G: government consumption (line 91f); I: sum of private gross fixed capital formation (line 93e) and increase/decrease in stocks (line 93i); C: household consumption (line 96f); and GDP: gross domestic product (line 99b). For conversion into real, per capita terms we use GDP volume in 1995 or 1996 terms (line 99b) and population (line 99z).

<table>
<thead>
<tr>
<th>Country</th>
<th>Annual Data</th>
<th>Quarterly Data</th>
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</table>

As has been standard practice in the literature (see, e.g., Campbell (1987) or Sheffrin and Woo (1990)), we remove the means from the current account and from the first difference in net output (we only test the dynamic restrictions of the theory). We set annual and quarterly real interest rates to 4 percent and 1 percent respectively. For the VAR we use the number of lags selected by the Akaike information criterion. Our results are robust to changes in the number of lags or in the value of the real interest rate.

Finally, some authors assume that the discount factor is not equal to the inverse of the gross real interest rate (see Ghosh (1995)). In such a case, the current account equation includes a consumption-tilting parameter which needs to be estimated. We followed this procedure as a robustness check. The estimated consumption-tilting parameters are usually close to one (their value when the discount factor is equal to the inverse of the gross real interest rate). The resulting series display similar properties as before, and our results remain robust to this specification.

22 For Belgium we cut the sample in 1998 as there is a break in the data following the adoption of the euro in 1999.
<table>
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<td>United Kingdom</td>
<td>Annual</td>
<td>1948–2002</td>
<td>2</td>
<td>0.67</td>
<td>2.8%</td>
<td>0.1%</td>
</tr>
<tr>
<td>Belgium</td>
<td>Quarterly</td>
<td>1980–1998</td>
<td>3</td>
<td>0.98</td>
<td>0.7%</td>
<td>43.7%*</td>
</tr>
<tr>
<td>Canada</td>
<td>Quarterly</td>
<td>1948–2002</td>
<td>8</td>
<td>0.94</td>
<td>2%</td>
<td>38.1%*</td>
</tr>
<tr>
<td>Denmark</td>
<td>Quarterly</td>
<td>1988–2002</td>
<td>4</td>
<td>0.93</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Sweden</td>
<td>Quarterly</td>
<td>1990–2002</td>
<td>4</td>
<td>0.97</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>Quarterly</td>
<td>1955–2002</td>
<td>4</td>
<td>0.94</td>
<td>0%</td>
<td>0%</td>
</tr>
</tbody>
</table>

Notes: The figures for the tests correspond to one minus the cdf of the test statistic. A star denotes model acceptance by the F-test at the 5 percent level of confidence. For each sample the number of lags was selected using the Akaike criterion.
Figure 1. Actual (-) and Predicted (--.) Series: Subsample Estimation
Note: $c_0$ is the true value of the parameter, $\hat{c}$ the empirical estimate, and $[\hat{c}_L, \hat{c}_R]$ the probability interval around $\hat{c}$. The Delta method yields the probability interval $[\hat{K}_{L,\delta}, \hat{K}_{R,\delta}]$ around $\hat{K}$ rather than the correct $[\hat{K}_L, \hat{K}_R]$. 

Figure 2. False Rejection in the Singularity Region
Note: $c_0$ is the true value of the parameter, $\hat{c}$ the empirical estimate, and $[\hat{c}_L, \hat{c}_R]$ the probability interval around $\hat{c}$. The Delta method yields the probability interval $[\hat{K}_L, \hat{K}_R]$ around $\hat{K}$ which includes $K_0$ even though $[\hat{c}_L, \hat{c}_R]$ excludes $c_0$. 
Figure 4. Posterior Distributions of the \((l+1)^{th}\) Coefficient of the \(K\) Vector: Annual Data

Note: A star denotes model acceptance by the F-test at the 5 percent level of confidence.
Figure 5. Posterior Distributions of the \((l+1)\)\textsuperscript{th} Coefficient of the \(K\) Vector: Quaterly Data

Note: A star denotes model acceptance by the F-test at the 5 percent level of confidence.
Figure 6. Actual (-) and Predicted (--) Series: Annual Data

Note: A star denotes model acceptance by the F-test at the 5 percent level of confidence.
Figure 7. Actual (-), Predicted (--), and Percentile Lines (Bold): Annual Data

Note: Bold lines correspond to the 2.5th and 97.5th percentiles of the posterior distribution of the predicted series. A star denotes model acceptance by the F-test at the 5 percent level of confidence.
Figure 8. Actual (-) and Predicted (-->) Series: Quarterly Data

Note: A star denotes model acceptance by the F-test at the 5 percent level of confidence.
Figure 9. Actual (-), Predicted (--), and Percentile Lines (Bold): Quarterly Data

Notes: Bold lines correspond to the 2.5th and 97.5th percentiles of the posterior distribution of the predicted series. A star denotes model acceptance by the F-test at the 5 percent level of confidence.
Figure 10. Posterior Distributions of the Variance Ratio: Annual Data

Notes: The variance ratio is expressed as the variance of the predicted series over that of the actual. A star denotes model acceptance by the F-test at the 5 percent level of confidence.
Figure 11. Posterior Distributions of the Variance Ratio: Quarterly Data

Notes: The variance ratio is expressed as the variance of the predicted series over that of the actual. A star denotes model acceptance by the F-test at the 5 percent level of confidence.
Figure 12. Posterior Distributions of the Correlation Coefficient: Annual Data

Note: A star denotes model acceptance by the F-test at the 5 percent level of confidence.
Figure 13. Posterior Distributions of the Correlation Coefficient: Quarterly Data

Note: A star denotes model acceptance by the F-test at the 5 percent level of confidence.