Are They All in the Same Boat?  

Thomas Helbling and Tamim Bayoumi
This paper reviews the international business cycle among Group of Seven (G-7) countries since 1973 from two angles. An examination of business cycle synchronization among these countries using simple descriptive statistics shows that synchronized slowdowns have been the norm rather than the exception and that the slowdown in 2000-2001 largely followed patterns seen in the past. The paper also identifies the international business cycle with an asymptotic dynamic factor model. Two global factors explain roughly 80 percent of the variance in G-7 output gaps at business cycle frequencies. The factor model decomposes the "common part" of national output fluctuations into two factors, one capturing the average G-7 cycle and one that corrects for phase and amplitude differences. We also found some evidence supporting the hypothesis that global shocks were the main force behind the slowdown in 2000-2001.

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I. INTRODUCTION

The worldwide growth slowdown in 2000–2001 has refocused attention on the international business cycle linkages. The main reason for this renewed interest is the unexpected breadth of this slowdown, which was initially expected to remain largely confined to the United States. These expectations appear to have reflected the seemingly benign business cycle linkages during the 1990s, when the timing of classical recessions among the Group of Seven (G-7) countries was strikingly dispersed. As a result, aggregate output of industrial countries had recorded continuous growth while it had fallen in earlier recession episodes such as the early 1970s. Against this background, the almost simultaneous downturn in the major economies was widely considered unusual.

Naturally, the unexpectedly strong degree of synchronization in the slowdown during 2000–2001 has raised many questions. Is the observed synchronized slowdown unusual? Are spillovers stronger than in the past? Does the fact that inventories and fixed investment contributed more to the U.S. slowdown than private consumption matter for the transmission? Is the synchronized slowdown the result of global shocks or of increased spillovers of country-specific shocks? Has the increasing international economic interdependence, especially in financial markets, enhanced underlying international business cycle linkages?

This paper tries to answer some of these questions. Its main goal is to put recent events in perspective by documenting some quantitative aspects of international business cycle linkages among the G-7 countries since 1973, when the generalized floating of the major currencies was introduced. The focus is on two issues in particular. First, a few stylized facts on international business cycle linkages among the G-7 countries are established by analyzing four dimensions of business cycle linkages. The results show that from a historical perspective, synchronized slowdowns are the norm rather than the exception, and that events in 2000–2001 should not have come as a surprise. In establishing the stylized facts, the paper partly builds on traditional business cycle concepts such as peaks and troughs—concepts which have experienced a revival following recent work by Harding and Pagan (2001 and forthcoming). Second, the paper identifies and quantifies the international business cycle in a G-7 panel dataset, building on recently developed asymptotic dynamic factor models. These models are a natural choice for the empirical investigation of international business cycle linkages since common factors in output fluctuations and aggregate demand fluctuations across countries are their quintessential implication.

The paper is organized as follows. Section II focuses on four simple questions concerning international business cycle linkages. Section III briefly reviews the recently developed so-called asymptotic dynamic factor models. Section IV then applies such a model to find the common factors in G-7 output fluctuations. Section V focuses on the robustness of the results with regard to the cross-sectional dimension of the panel dataset and on the strength of international business cycle linkages in key demand components. The last section concludes and discusses policy implications.
II. SOME STYLISTED FACTS ON G-7 BUSINESS CYCLE LINKAGES

Common elements in business cycle fluctuations across countries—international business cycle linkages in short—have long been noted in the literature on business cycles. This section lays out some of the main stylized facts, based on commonly used statistics to characterize business cycle linkages and business cycle chronologies.

A. Do Recessions and Expansions in G-7 Countries Coincide?

Naturally, one would expect that with significant international business cycle linkages, the timing of recessions and expansions would be similar among G-7 countries. In what is now often called classical business cycle analysis—attributed to early business cycle research at the National Bureau of Economic Research (NBER) in the United States from the 1930s to the 1950s—the timing of recessions and expansions is determined by turning points—peaks in the case of a recession and troughs in the case of expansions. The full set of turning points over a period of time constitutes a business cycle chronology.

Determining the actual extent of synchronicity in peaks and troughs in G-7 countries is complicated by the absence of consistent and generally recognized dates for peaks and troughs in aggregate economic activity. Replicating the NBER methodology is difficult because of the significant degree of judgment involved. Fortunately, however, Harding and Pagan (2001 and forthcoming) have shown that peaks and troughs in the NBER's reference cycle can be approximated closely by applying a simplified version of the Bry-Boschan algorithm to quarterly U.S. real GDP data. On this basis, and since real GDP is widely

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2 See, for example, Haberler (1937).

3 See Burns and Mitchell (1946) for an early tract, Moore (1983) and Zarnowitz (1992) for studies based on the NBER methodology, and Harding and Pagan (2001) for a recent overview of modern and classical business cycle analysis.

4 The NBER determines peaks and troughs in aggregate economic activity on the basis of peaks and troughs in a number of indicators. As peaks and troughs in the various series differ, informed judgment is needed to reconcile the conflicting dates to generate the so-called reference cycle, a synthetic series whose peaks and troughs describe the business cycle. To date, an NBER business cycle dating committee performs this task and determines peaks and troughs for the United States, which are generally recognized as "official" business cycle dates.

5 Bry and Boschan (1971) demonstrated how their dating algorithm closely approximates peaks and troughs that NBER researchers had identified in the monthly series used to determine the reference cycle. King and Plosser (1994) applied the original Bry-Boschan algorithm to a monthly, interpolated series of real GDP. Artis, Kontolemis, and Osborn (1997) applied a modified Bry-Boschan algorithm to monthly industrial production data for a number of countries.
accepted as a good indicator of aggregate economic activity (Stock and Watson, 1999),
consistent dates for peaks and troughs for the G-7 countries were computed with this
algorithm.\

How synchronized have recessions and expansions been in the past? Figure 1, where
the chronology of peaks and troughs in G-7 business cycles is depicted, illustrates how
recessions in G-7 countries during 1973–2000 tended to cluster in four periods. Comparing
turning points in other G-7 countries against those in the United States suggests the
following:

- For the 5 countries that experienced a recession, the 1974/75 recessions were by far
  the most synchronized, not only in terms of the timing of peaks and troughs—and
  hence the length of recessions—but also in terms of recovery duration—the time
  needed for real GDP to reach or exceed the previous peak level after a recession.
  Despite the depth of the 1973-75 recessions, Canada and Japan did not experience a
classical recession.

- The recessions in the early 1980s were also closely synchronized, except for those in
  the United Kingdom. As in the 1974/75 episode, not all countries suffered from a
classical recession during this period (France was spared in 1981/82, Italy in 1980).

- The recessions in the early 1990s were the least synchronized given the widespread
differences in the timing of peaks and troughs and recovery durations. However,
unlike earlier episodes, when some countries did not experience a recession, all
countries went through a recession during this episode.

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6 The simplified Bry-Boschan algorithm involves three steps. First, the algorithm defines
local peaks and troughs in the log of real GDP:

\[
\text{peak at } t = \{(y_{t-2}, y_{t-1}) < y_t > (y_{t+1}, y_{t+2})\},
\]

\[
\text{trough at } t = \{(y_{t-2}, y_{t-1}) > y_t < (y_{t+1}, y_{t+2})\}.
\]

Second, alternation of peaks and troughs is ensured by picking the maximum peak or
minimum trough in case of repeated peaks or troughs. Third, minimum durations of full
cycles (5 quarters) and phases (2 quarters) are guaranteed by censoring rules. The main
difference with regard to the original algorithm is that the local peaks and troughs are directly
derived from the raw data rather than from a sequence of filtered data with subsequent
refinement.
Figure 1: G-7 Cycle Synchronization: Classical Cycles

(Periods of U.S. Recessions are Shaded)
Overall, the picture that emerges is that of generally synchronized recessions, and to a somewhat lesser extent, synchronized recoveries. Also noteworthy is the fact that the United States experienced classical recessions around the time other countries were in recession, suggesting that global cycles are intimately linked with United States cycles.

A glance at Figure 1 also shows that the 2000–2001 slowdown seems to resemble earlier episodes of synchronized recessions from at least three angles. First, the largest economies (in this case the United States, Japan, and Germany) all experienced a classical recession. Second, not all G·7 economies slid into recession. Third, the dispersion of peaks across the countries in recession was small (2 quarters).

A more formal way to measure, the extent to which recessions and expansions in two countries $i$ and $j$ are broadly concurrent, is the concordance statistics that was recently proposed by Harding and Pagan (2001). The nonparametric statistics determines the number of periods, as a proportion of the number of periods in the sample, during which two economies $i$ and $j$ are in the same state. It is defined as:

$$C_q = \frac{1}{T} \sum_{t=1}^{T} (S_q S_q') + S_q S_q'$$

where $T$ denotes the number of observations, and $S_q$ is an indicator variable for the state of the economy in country $i$ in period $t$. If the country is in an expansion, the variable takes the value 1. In a recession, $S_q$ becomes 0. As an analytical solution to the distribution of the test statistics does not seem to exist, McDermott and Scott (2000) derived the distribution of the test statistics with Monte Carlo simulations.

The concordance statistics are shown in Table 1. Out of 21 measures, 19 are significant at the 5 percent and 10 percent levels, suggesting that expansions and recessions in G·7 countries generally coincide. Not surprisingly, the 2 exceptions concern bilateral concordance between Japan and other G·7 countries, suggesting that among business cycle developments in G·7 countries, those in Japan are the most detached. The fact that Japan did not experience any classical level recession during the 1970s and 1980s reflects the country’s high average growth rate during this period. Under these conditions, even large adverse shock only led to a sharp fall in GDP growth but not to a decline in the level. This dependence of the classical business cycle chronologies on the underlying trend growth rate is a weakness of the classical business cycle concept, as noted inter alia by Stock and Watson (1999), and limits its applicability in cross-country analysis. Overall, the empirical evidence for the post Bretton Woods period suggests that with the exception of Japan, a recession in one G·7 country is, on average, likely to coincide with a recession in other G·7 countries.7

7 This conclusion seems robust with regard to changes in the sample period, as indicated by the concordance statistics for the period 1960–99 that were calculated by McDermott and Scott (2000) for all G·7 countries but France.
Table 1. Concordance Statistics for Expansions and Recessions, 1973Q1–2001Q4

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<td>0.91 **</td>
<td>0.99 **</td>
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Source: Authors’ calculations.

1/ Entries that are significant at the 5 percent level are marked with **; entries that are significant at the 10 percent level are marked with *. Significance levels were calculated with the response surface parameters provided by McDermott and Scott (2000).

B. Do Growth Cycles in G-7 Countries Coincide?

Classical business cycle fluctuations are characterized by cycles of relatively long duration and an asymmetry in the duration of recessions and expansions—with the latter lasting on average roughly 5 times as long as the former. Hence, chances of finding significant concordance statistics in relatively short samples seem high. For analysts of international business cycles, concordance in the timing of expansions and recessions captures only one dimension of business cycle linkages. For policymakers, for example, knowing only the direction of output comovements is not a comforting basis for decision-making. Any countercyclical policy measure requires some information of the magnitude of output comovements. Also, small macroeconomic shocks may not lead to recessions or strong booms but may have international repercussions. Hence, in addition to the general direction of fluctuations, the issue of whether fluctuations more generally are similar across countries is also of considerable interest.

Comparing magnitudes of macroeconomic fluctuations in general leads to the domain of growth cycles since conventional indicators of aggregate economic activity are nonstationary variables while statistical measurement of amplitudes requires stationary

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8 The critical values of the test statistics depend on the coefficient of variation in the case of trended series. For the coefficient of variations found in the sample series, the average critical value is about 0.8 for the 5 percent significance level, suggesting that business cycle phases need to coincide only 80 percent of the time. Given the asymmetry in the length of recessions and expansions, the concordance criterion does not seem stringent for standard significance levels.
variables as inputs. Empirical analysis in this domain, therefore, needs to cross the "minefield" of detrending. As methodological issues related to detrending are not the focus of this essay, a pragmatic stance is taken. All series are detrended by using the approximate bandpass filter proposed by Baxter and King (1999), which is a combination of a low-pass filter to eliminate flow frequency components of 32 quarters or more and a high-pass filter to eliminate high frequency components of 5 quarters or less. Hence, business cycle frequencies are those in the interval of 6 to 32 quarters. Given that detrending remains a subject of controversy, another detrending method—log first differences—was also used, and differences with the bandpass filter noted where appropriate.

Growth cycles also have turning points that can be dated by a simplified Bry-Boschan algorithm as well. As can be seen in Figure 2, growth cycles are shorter in duration and less asymmetric with regard to the duration of expansions and recessions. A comparison of Figures 1 and 2 suggests that the timing of major growth contractions—those that overlap with classical recessions in major G-7 countries—is more synchronized than the beginning of classical recessions. Specifically, these contractions are highly synchronized insofar as they begin within a window of 1 or 2 quarters in all G-7 countries. Hence, unlike classical recessions, all G-7 countries suffer from growth contractions when others experience major growth contractions. Most notably, even Japan experienced growth recessions in the early 1970s or early 1980s. However, in the case of minor growth contractions (e.g., those occurring in the mid-1980s or mid-1990s), the timing does not seem as synchronized. This

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9 See Canova (1998) for a recent survey.

10 For the analysis in section IV and V, only the low frequency components of 32 quarters or more will be eliminated with a low-pass filter to obtain stationary series that still contain the fluctuations with frequencies of 5 quarters or less, which are also of interest. While the bandpass filter is now widely used, it has recently been criticized by Trimbur and Harvey (2000) and Murray (2001), who note that the filter may be inconsistent with some structural models of trend and cycle.

11 While there is little to gain from smoothing quarterly log-level GDP series, as noted by Harding and Pagan (2001), the smoothing of detrended quarterly log GDP series (output gaps) is essential for the algorithm to pick up only turning points that seem significant in terms of their deviations from trend. In line with the original Bry-Boschan algorithm, the initial set of turning points was determined with centered 3-quarter moving averages of output gaps.

12 This fact was noted by Harding and Pagan (forthcoming).
Figure 2: G-7 Cycle Synchronization: Growth Cycles

(Periods of U.S. Growth Recessions are Shaded)
suggests that the transmission of large shocks is quite fast, which is highly plausible in the case of global shocks.

Nevertheless, there are also marked differences among growth cycles. Most strikingly, the end of major growth contractions is less synchronized. Hence, the length of growth cycle recessions differs across countries, which could suggest that shocks, either directly or through transmission, hit the G-7 economies at similar times but that the response of each economy to the shocks varies, possibly reflecting, among other factors, differences in policy responses, and in economic and financial structures. Despite these differences in timing of troughs, concordance statistics suggests that growth cycles across G-7 countries are generally similar in their timing. Except for the bilateral concordance between Japan and the United Kingdom, all statistics are significant at the 5 percent level (Table 2).\(^1\)

Table 2. Concordance Statistics for Growth Cycles, 1973Q1–2001Q4 1/

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Source: Authors' calculations.

1/ Entries that are significant at the 5 percent level are marked with **; entries that are significant at the 10 percent level are marked with *. Significance levels were calculated with the response surface parameters provided by McDermott and Scott (2000).

As is evident from Figure 2, growth cycles peaked in all G-7 countries in 2000, suggesting the beginning of a synchronized growth contraction in these countries. From a historical perspective, this high degree of synchronization across G-7 countries is not unusual, as the cross-country comparison of growth cycle peaks in earlier episodes has

\(^1\) Once again, the concordance criterion does not seem stringent for standard significance levels. For growth cycles, the concordance statistics needs to be above about 0.61 to be statistically significant at the 5 percent level for any two series to be more correlated than two random walk series (without a drift). Hence, even if growth cycles were out of phase about 40 percent of the time, they would still be considered concordant.
shown. The high degree of synchronization is consistent with the hypothesis that the slowdown in 2000 was caused by the coincidence of a number of global shocks (IMF, 2001).

C. Are Direction and Magnitudes of Output Fluctuations Similar Across G-7 Countries?

International business cycle linkages extend beyond turning point synchronization in classical or growth cycles. They are likely to result also in similar magnitudes of output fluctuations, especially in the case of global shocks. Empirical evidence based on correlation coefficients confirms that direction and magnitude of output fluctuations around potential output in G-7 countries are indeed closely related, as one would expect. The average bilateral correlation among G-7 output gaps at business cycle frequencies during 1973–2001 is 0.51 (top panel of Table 3).

The strength of business cycle linkages across G-7 countries varies noticeably, however, despite generally large positive correlations. Specifically, looking at the correlation coefficients that are larger than the mean reveals three clusters of strong cross-country business cycle linkages. The first cluster includes the United States, Canada, the United Kingdom, and to a lesser extent, Germany. In this regard, it is remarkable how much more Germany’s growth fluctuations are connected with those in the “Anglo-Saxon” axis when compared with France or Italy. The second cluster, not surprisingly, includes Germany, France, and Italy whose economies are closely linked in the context of the European Union and monetary cooperation. The third cluster, rather surprisingly, comprises Japan and Germany, possibly with France and the United Kingdom. Overall, the evidence corroborates the notion of generally strong international business cycle linkages. Nevertheless, their strength varies across countries, as evidenced by the existence of correlation clusters, which is consistent with the notion that countries are hit by similar shocks, but that their effects vary considerably across countries.

Correlation coefficients based on the log growth rates of real GDP are similar but are slightly weaker (the average correlation coefficient is 0.42). Interestingly, there is only evidence for two correlation clusters, one comprising the Anglo-Saxon countries (and, possibly, Germany), and another one comprising the continental European G-7 countries. The correlations between output growth in Japan and the continental European G-7 countries are now below average.

Business cycle linkages are clearly dynamic in nature. Depending on the nature of the shocks, the international transmission of their effects from one country through trade and financial channels likely involves lags. Static correlation coefficients—even for bandpass-filtered series—may understate the extent of cross-border business cycle linkages. What is needed are correlation measures that allow for some dynamics. Spectral-based measures are helpful in this respect since they do not only allow for a dynamic relationship between two series \( x_t \) and \( y_t \) but provide also for the identification of the frequencies that are most important in accounting for the overall correlation patterns found in the data. Specifically, the charts in Figure 3 show the coherencies between any two (stationary) output series in G-7 countries.
Table 3. Cross-Correlations of Outputs in G-7 Countries, 1973Q1 – 2001Q4 1/
(Shaded entries are correlation coefficients for the first differences of log outputs while the other entries are for output gaps)

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Source: Authors' calculations.

1/ Output gaps at business cycle frequencies (6 - 32 quarters) were computed with the approximate bandpass filter proposed by Baxter and King (1999). For the first differences, correlation coefficients were calculated on the basis of the spectral density matrix at frequency zero. For output gaps, the correlations coefficients reflect contemporaneous correlations.
Figure 3. Pairwise Output Coherences Among G-7 Countries


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Source: Authors’ calculations. Note: Output gaps (above diagonal) are 4-year-50term log output series. Frequency is 3rd six point.
countries. The coherence between two series \( x \) and \( y \) at frequency \( \pi \) is usually interpreted as proportion of the variance of the series \( y \) at frequency \( \pi \) that is explained by the regression of \( y \) on leads and lags of \( x \). In this sense, it can be interpreted as a frequency-based \( R^2 \) measure. Accordingly, even if \( x \) and \( y \) were in different phases of a cycle, their dynamic correlation would still be detected. Like conventional \( R^2 \) measures, coherences are bound to fall into the interval \([0,1]\).

Do the dynamics matter for the measurement of the strength of business cycle linkages? The charts in Figure 3 imply that the picture provided by static correlation coefficients is accurate insofar as the magnitudes and rankings of coherences are comparable to those of the correlation coefficients reported in Table 3.\(^{14}\) This suggests that the dynamics do not matter in this narrow sense. Nevertheless, in a more conventional sense, dynamics matter, as one would expect. The charts in the figure show that cross-country output linkages are particularly strong at business cycle frequencies. In many cases, the coherences are either negatively sloped (from lower to higher frequencies) or hump-shaped with higher values for business cycle frequencies. Hence, the strength of comovements at low to medium business cycle frequencies, that is, between 3 to 8 years, is typically higher than at short-term business cycle frequencies (1\( \frac{1}{2} \) to 3 years). This pattern is consistent with asymmetries in transmission lags. Indeed, the phase angles in the cross-spectrum of bilateral pairs of output series (not shown) indicate that U.S. output fluctuations at business cycle frequencies are roughly coincident with those of Japan, the United Kingdom, and Canada while they lead those in the continental European countries. By implications, those of Japan, the United Kingdom, and Canada are also leading with respect to these countries.\(^{15}\) In some cases, coherences at high frequencies also seem important, indicating the existence of significant short-term output linkages. These short-term linkages are especially relevant for the United States and Canada and for the linkages among continental European countries. This suggests either a faster transmission of shocks or common shocks that affect mostly countries within a cluster, which is highly plausible given the close proximity of these economies.

D. Are International Business Cycle Linkages Stable?

The striking dispersion of classical business cycle peaks and troughs during the early 1990s could mean that the structure of international business cycle linkages has changed or is unstable over time. Examining variations in bilateral correlation coefficients over time suggests that within the three (two) clusters of especially strong output links, correlations

\(^{14}\) The output gaps in the above-diagonal charts are low-pass filtered log output series, unlike the ones used in the calculation of the correlation coefficients in Table 3.

\(^{15}\) German output fluctuations at business cycle frequencies are roughly coincident with those of France and Italy, except for short-term business cycle frequencies, where a slight German lead is implied by the data.
have remained roughly stable over time (lower panels in Table 3). However, transatlantic and transpacific business cycle linkages or, in other words, linkages among the three major currency areas, seem less stable. Correlation coefficients between the United States and Japan or the United States and the Continental European countries for the period 1973Q1–1989Q4 (covering 4 growth cycles in the United States according to our chronology) are substantially higher than those for 1973–2001 (covering 6 U.S. growth cycles). Moreover, output correlations among the three major G-7 countries are quite similar during the 1970s and 1980s, unlike during the 1990s.

The lower correlations for 1973–2001 compared to 1973–89 are consistent with two hypotheses about the nature of the changes in international business cycle linkages. The first one is that the early 1990s were exceptional insofar as a rare constellation of large country-specific shocks led to a rather unusual dis-synchronization of output fluctuations even though the underlying structure of linkages remained unchanged. The second hypothesis would be that of more persistent if not permanent changes in the structure of business cycle linkages. To assess the relevance of these hypotheses, a set of correlation coefficients was computed for a sample that excludes data for the period of the fifth U.S. growth cycle (according to our chronology), that is, for a data sample from 1973–2001 that excludes observations for the period 1990Q1–1994Q2. Comparing these correlations coefficients with those for 1973–89 shows that except for those involving Japan, the two sets of coefficients are very similar. Asymptotic $\chi^2$ test for the equality of two correlation matrices suggest that the null hypothesis cannot be rejected at standard marginal significance levels for low pass-filtered and log first differenced output series. For bandpass-filtered output series, the test is rejected at the 5 percent but not at the 1 percent level (the marginal significance level of the test statistics is 0.037).

The evidence shown in the charts in Figure 4, where the coherences for the periods 1973–89 and 1973–89 plus 1994Q3–2001 are compared, corroborates the conclusion of rather similar correlation structures, as bilateral coherences for the two periods overlap in all cases except for those involving Japan. Hence, the first hypothesis of less connected business cycles because of large country-specific shocks during the early 1990s is relevant for the case of transatlantic linkages involving the continental European G-7 countries. The second hypothesis may be relevant for Japan.

16 The correlation coefficients for the subsample periods that are reported in Table 3 were corrected by the Forbes-Rigobon (1999) procedure, as the higher output volatility during 1973–89 compared to 1990–2001 may lead to an upward bias the correlation coefficients for the first subperiod.

17 See Jennrich (1970) for the specification of the test and Anderson (1958) on the computation of the variance-covariance matrix of an estimated correlation matrix.
Figure 4. Pairwise Output Coherences Among G-7 Countries II

Solid lines: 1973-2000; ex 3rd US growth cycle. Otherwise the set up is the same as in Figure 3.

Source: Author calculations. Note: Output gaps (above diagonal) are low-pass filtered by output series. Frequency in units in years.
Overall, empirical evidence supports the notion that in many ways, international business cycle linkages among G-7 countries have remained unchanged during most of the period since the introduction of the generalized floating among the major currency areas in 1973. In terms of U.S. growth cycles, the linkages have been similar during 5 out of 6 cycles, except for Japan. From this perspective, the synchronized slowdown in 2000-01 does not seem surprising either.

III. ASYMPTOTIC DYNAMIC FACTOR MODELS: A SYNOPSIS

While interpretation and measurement of international business cycle linkages vary among researchers, many macroeconomists would probably agree that it is best understood as a small set of factors that are common to all countries and that explain a substantial fraction of fluctuations in major macroeconomic aggregates. The common factors themselves reflect a combination of global shocks affecting all countries and country-specific disturbances with significant spillover effects. It is, therefore, quite natural to examine international business cycle linkages with dynamic factor models.

Linear factor models decompose an \( n \)-dimensional vector of time series \( X_t \) into a small number of orthogonal common factors \( F_t \), where \( F_t \) is a \( q \)-dimensional vector with \( q \ll n \), and an \( n \)-dimensional vector \( \Xi_t \) of idiosyncratic errors that are orthogonal to \( F_t \) at all times.\(^{18}\) Typically, the models are also dynamic as \( X_t \) depends not only on \( F_t \) but also on \( k \) lags of \( F_t \). Formally, the canonical linear dynamic factor model can be written as:

\[
X_t = \Psi_t + \Xi_t = A(L)F_t + \Xi_t
\]

where \( X_t \) is a column vector of \( n \) observations such that \( X_t = [x_{t1}, x_{t2}, \ldots, x_{tn}] \), \( t=1,\ldots,T \), where \( \Psi_t \) is a column vector of \( n \) common factors such \( \Psi_t = [\psi_{t1}, \psi_{t2}, \ldots, \psi_{tn}] \), one for each element of \( X_t \), and where \( \Xi_t \) is a vector of \( n \) error terms. The common factors themselves are linear combinations of \( q \) generic factors contained in the column vector \( F_t \) such that \( F_t = [f_{t1}, f_{t2}, \ldots, f_{tq}] \). \( A(L) \) is an \( n \times q \) coefficient matrix—the factor loadings—at lag \( L \) that maps the generic factors into the common factors. Hence, while the generic factors \( f_{ti} \) are identical across \( X_t \), the coefficient matrix \( A(L) \) allows for a series-specific response, so that the common factors in \( \Psi_t \) differ across \( X_t \), reflecting differences in factor dynamics. In the remainder of the paper, we will distinguish between factors, which are elements in \( F_t \), and common factors. With regard to the latter, we may also be interested in subsets of \( \Psi_t \). For example, we will refer to \( A_1(L)f_{t1} \) as the first common factor.

\(^{18}\) See Sargent and Sims (1977) and Geweke (1977) for the seminal papers on dynamic factor models in economic applications.
Initially, the identification of the common factors required severe restrictions on the elements of $\Xi$, which could neither be serially correlated nor cross-correlated. Over the years, researchers have demonstrated how the common factors could be identified and estimated with less restrictive assumptions about the elements of $\Xi$. Recently, Forni and others (2000) and Stock and Watson (1999b) have shown that with large cross-sections (and large samples in the time dimension), the restrictions on the elements of $\Xi$ can be relaxed considerably.\(^{19}\) For large $n$ and $T$, there can be some cross-correlation among the elements of $\Xi$, since on average, the effects of these idiosyncratic variations on the variation of the elements in $X$ will be zero. This relaxation is advantageous for the application to the common component in G-7 business cycle fluctuations since there could be intra-European fluctuations that are distinct from common fluctuations in all the G-7 countries.

The estimators proposed by Forni and others (2000) and Stock and Watson (1999b) are both nonparametric and based on principal components analysis. They are easy to implement since the determination of the common components only involves linear algebra rather than the maximization of complicated multivariate likelihood functions, as in the case of parametric approaches.\(^{20}\) The Stock and Watson estimator is based on principal components in the time domain while that of Forni and others is based on principal components in the frequency domain. Specifically, the Forni estimator uses the first $q$ eigenvectors of the spectral density matrix of the vector process $X_{st}$ selected frequencies in the interval $[-\pi, \pi]$ to determine the common factors. With an inverse Fourier transform of the coefficients, the common factors are recovered in the time domain. The frequency-domain approach of Forni and others is more flexible than that of Stock and Watson, as it allows explicitly for leads and lags in the dynamic relationship between the input series and the common factors.\(^{21}\) Given that U.S. output appears to be leading output fluctuations in continental European G-7 countries, this flexibility in the modeling of the factor propagation mechanism across countries should prove to be an advantage in our application.

For empirical applications, the frequency domain-based estimator of Forni and others (op. cit.) suffers from the fact that it implies two-sided filtering of the observations in $X$. This reduces the number of fitted values. Moreover, and perhaps more importantly, forecasting is difficult, given the two-sided filter. Fortunately, however, Forni and others (2002) have shown how their approach can be adapted to generate consistent forecasts. Specifically, they develop a one-sided estimator that can be used to generate the optimal linear forecast of the common component $\Xi$, given information up to time $t$. The idiosyncratic component can also

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\(^{19}\) See also Reichlin (2002) for an excellent survey.


\(^{21}\) Presumably, the matrix $\bar{X}$ in the Stock-Watson model could be extended to include leads as well.
be forecast using linear time series methods. Hence, the optimal linear, h-ahead forecast of $X_t$ follows as:

$$X_{t+h}^P = \Psi_{t+h}^P + \Xi_{t+h}^P$$

where $\Psi_{t+h}^P$ and $\Xi_{t+h}^P$ are the optimal forecasts of $\Psi_{t-h}$ and $\Xi_{t+h}$, respectively. With regard to the latter, Reichlin (2002) has suggested using univariate, autoregressive models for the forecasts, given the small cross-sectional correlation among the elements of the idiosyncratic component. Reichlin also noted that the one-sided estimator can also be used to re-estimate the within-sample common component by setting $h$ equal to zero, which is similar to the smoothing of Kalman filter-based estimates.

A general drawback of principal components-based approaches is the lack of tests that could guide the specification of the model in terms of the number of common factors and the number of leads and lags in the estimation of the spectral density matrix. Moreover, issues of temporal stability have not yet been addressed either. With regard to the choice of the number of factors, Forni and others recommend determining a cut-off criterion for the fraction of the space spanned by the spectral density matrix that a significant common factor would need to explain. With respect to the number of lags (leads) to be used in the calculation of the spectral density matrix, they have recommended to use fixed rules such as

$$p = \text{round}(\sqrt{T}/4) \quad \text{or} \quad p = \text{round}(2\sqrt{T}/3).$$

IV. COMMON FACTORS IN G-7 OUTPUT FLUCTUATIONS

In a first step, the analysis of common factors in G-7 output fluctuations will be based on quarterly data for real GDP at market prices for the period 1973Q1–2001Q4 (117 observations). While the cross-sectional dimension of this sample is small ($n=7$), which risks contaminating the estimated common factors with idiosyncratic noise, it is nevertheless informative to proceed with this step, mainly for three reasons. First, the direction and magnitude of output comovements across countries are widely used indicators of business cycle linkages. Second, comparing the results from a small panel with those from a larger panel, which will be done subsequently, allows one to assess the performance of asymptotic factor models in applications with small cross-sectional dimensions. Third, the focus on output comovements alone also allows for some intuitive graphic analysis of sources and stability of international business cycle linkages, unlike for the results obtained with large panel datasets.

The common factors and components were estimated with the two-sided, nonparametric estimator proposed by Forni and others (2000), as summarized above. As is common in factor model applications, the detrended real GDP series were standardized to

$$2^2 \text{Stock and Watson (1999b) showed however, that with } n >> T, \text{ some limited time variation in the factor loadings can be accommodated.}$$
avoid the results being affected systematically by cross-country differences in the magnitude of output variability. For the calculation of the spectral density matrices, 3 lags were used based on the rules recommended by Forni and others (op. cit.) To check for the sensitivity with regard to the detrending method, both output gaps derived with a low-pass filter (applied to the natural logarithm of real GDP at market prices) and first differences of log real GDP were used. A reference to the vector process $X_t$ will from now on refer to either the 7 output gap series or the 7 log growth rate series.

A. How Many Common Factors and How Much Do They Explain?

With the international business cycle defined as a small set of factors that explain a substantial fraction of countries' output fluctuations, an immediate question is that of the number of factors that characterize output linkages among G-7 countries. To answer this question, analyzing the cumulative sums of the eigenvalues of the spectral density matrices of the vector of the real GDPs at various frequencies $\lambda$ is helpful (Figure 5). Each eigenvalue reflects a common factor; their sums are measures of their cumulative contribution to explaining the space spanned by the spectral density matrices at these frequencies. The first factor on average explains roughly 60 percent of joint output fluctuations at business cycle frequencies, which is remarkable and certainly supports the notion of common elements in cross-country output fluctuations. The explanatory power of the first factor declines for high frequency components in output fluctuations, which is consistent with the earlier finding that output comovements are especially strong at low to medium business cycle frequencies. The second factor also contributes substantially to explaining joint output fluctuations. Together, the first two common factors explain roughly 80 percent of the joint output fluctuations at business cycle frequencies. The third factor raises the ratio to about 90 percent. The explanatory power of the other factors is small, a finding which again is consistent with our notion of the international business cycle as a small set of common factors.

How should the number of common factors $q$ to be used in the calculation of the common components be determined? Forni and others (2000) have suggested selecting the number of factors on the basis that the $q$-th factor should account for at least 5 percent, on average, of the space spanned by the spectral density matrices. However, in the current context, where significance should follow from substantive economic considerations, the question is whether 5 percent is not too low a threshold for a factor to explain substantial cross-country variation, especially in a sample with only 7 cross-sectional elements.

In view of this ambiguity, it is useful to base the decision on how much each factor contributes to explaining output fluctuations in each country. Figure 6 illustrates this for each of the first 4 common factors, all of which explain more than 5 percent of the overall
Figure 5. Dynamic Eigenvalues
(Eigenvalues of Spectral Density Matrices of X_t At Indicated Frequency)

Source: Authors' calculations.
variation in \( X_t \). The charts show the square of the average cohesion at business cycle frequencies between output gaps or log growth rates and each of the four factors in each country. This measure can be interpreted as the \( R^2 \) measure in a bivariate regression, in which a common factor at a frequency \( \lambda \) is the explanatory variables for the \( \lambda \) frequency component of the output gap or the log growth rate.\(^{23}\) From now on, we will refer to this measure as the average factor \( R^2 \).

For output gaps, the charts in Figure 6 suggest the following:

- Strikingly large differences in the goodness of fit of the common factor model arise across countries. Average factor \( R^2 \) values of more than 0.6 for all four factors are found for the United States, the United Kingdom, and Canada. This finding corroborates the earlier finding of a correlation cluster comprising these three countries. Surprisingly, the four factors also explain a good part of output fluctuations in Japan. For the continental European countries, the common factors have less explanatory power, most notably for France and Italy.

- Comparing the \( R^2 \) measures across countries suggests that only the first two factors are consistently important in explaining output gap fluctuations. The explanatory content of the third and the fourth factor is typically rather small. This can be taken as evidence for the interpretation of the first two factors as the truly global factors.

- The global factors do not seem relevant for Italy, where only the third factor appears to have explanatory power. While this third factor also contributes marginally to fluctuations in the other G-7 countries except for the United States and Canada, it appears to be primarily a Japanese factor.

\(^{23}\) The cohesion measure at frequency \( \lambda \) is defined as:

\[
\rho_{xy}(\lambda) = \frac{C_{xy}(\lambda)}{\sqrt{S_x(\lambda)S_y(\lambda)}}
\]

where \( C_{xy}(\lambda) \) denotes the cospectrum between time series \( x \) and \( y \) at this frequency and where \( S_x(\lambda) \) denotes the spectrum of series \( x \). As usual, it is sufficient to calculate \( \rho_{xy}(\lambda) \) for \( \lambda \in [0,\pi] \). The cohesion is the square root of the "real" part of the coherence. The computation of the squared quadrature spectrum, the "imaginary" part of the coherence, is redundant since in the generalized dynamic factor model, the common components are already projections on leads and lags of the factors, i.e., of the fundamentals \( F_t \).
Figure 6. Average Factor R-Squares at Business Cycle Frequencies
(Model Estimated with Log Growth Rates: Solid Bar; Model Estimated with Output Gaps: Shaded Bar)

Source: Authors' calculations.
• The fourth factor appears to pick up a common factor in the cycles of France and the United Kingdom, given the relatively higher $R^2$ values for this factor in the two countries.

Overall, we conclude that the first two factors are the global factors. Hence, if the focus on the analysis were primarily on analyzing the global factors, selecting the first two factors for the computation of the common component for each country would seem sufficient. If the focus were primarily on the goodness of fit, four factors would be needed for the dynamic factor model to explain a substantial part of fluctuations also in Italy, and to a lesser extent, in France. Since we are primarily interested in the global factors, we decided to proceed with two generic factors in the computation of the common factors.

In general, the common factors explain a somewhat larger share of fluctuations in output gaps than in output growth rates, which is not surprising given that differencing amplifies the weight of short-term frequencies in the filtered series compared to bandpass-filtered series. All results point to the fact that common factors have more explanatory power at business cycle frequencies than at irregular, short-term frequencies. Nevertheless, the ranking of the goodness of fit among countries generally remains unaffected by the detrending method, except for the United Kingdom, where a stark difference in the goodness of fit between output gaps and log first differences can be noted.

The sums of the average factor $R^2$ measures for the first two common factors are remarkably similar in their relative ranking and their magnitudes with the correlations between monthly industrial production growth and a corresponding common component in a sample of industrial countries obtained by Lumsdaine and Prasad (1999). In particular, Lumsdaine and Prasad find that the common component (which in our case is given by the sum of the first two common factors) is much more correlated with fluctuations in Canada and the United States than with those in any other countries, which matches our results. This finding is in notable contrast with the results obtained by Gregory, Head, and Raynauld (1997), who find that their world factor explains a much lower share of the fluctuations in Canadian and U.S. output when compared to other G-7 countries.

### B. Geography and Dynamics of International Business Cycle Linkages

Since the factors are essentially dynamically weighted averages of the raw output series contained in $X_t$, the “country composition of the factors” contains useful information about the dynamics of the international business cycle. To extract this information, average factor $R^2$ measures at business cycle frequencies for each of the two common factors in each country were computed. Unlike above, the bivariate regression would involve the common factor at frequency $\lambda$ of a certain country as the dependent variable in bivariate regressions on the $\lambda$ frequency component of the output gap in G-7 countries. The charts in Figure 7 suggest that the first factor is a global factor, as all 7 output gaps generate sizable $R^2$ statistics. Nevertheless, there is noticeable “Anglo-Saxon” bias, as the highest $R^2$ measures are typically registered for output gaps of the United States, the United Kingdom, and Canada. This is particularly evident for the countries outside the Anglo-Saxon cluster. For the Anglo-
Figure 7. Average Factor R-Squares, by Country, at Business Cycle Frequencies
(First Factor: First Bar; Second Factor: Second Bar)

Source: Authors' calculations.
Saxon countries, an interesting asymmetry emerges insofar as the first factor also picks up output fluctuations in the continental European G-7 countries. The second factor is more difficult to describe in general terms because the relative distribution of the R² statistics across countries varies for each country. In Japan and Germany, for example, the relative weight of the own-country output gap is noticeably larger. Comparing the geography of factor R² statistics for output gaps and log growth rates shows that the detrending method does not appear to matter.

How should one interpret the factors? Could the first factor be interpreted as the "global" factor that mainly reflects global shocks while the second factor, characterized by differences among countries, would capture country-specific shocks? Since dynamic factor models are essentially atheoretical, they do not lend themselves to straightforward interpretation. However, indirect evidence can give some clues. A first clue follows from a comparison of the first and second common factors with a GDP-weighted aggregate G-7 output gap (Figure 8). The comparison shows how the first factor in each country closely matches the "average" G-7 cycle. The second factor "controls" for some of the country-deviation from this average cycle, both in terms of cycle amplitude and phase. This decomposition of the common components into an average and a difference factor is intuitive and is reminiscent of solution techniques in some two-country open economy macro models.²⁴

Exploring the temporal stability of the estimated common factors provides another clue. In the absence of formal tests for the temporal stability of the factor loadings as described above, an attempt was made to explore temporal stability informally by investigating the implications of estimating the model for the period 1973Q1-1990Q1 instead of 1973Q1-2001Q4. One can rationalize this exercise by asking what conclusions on international business cycle linkages an econometrician would have drawn if he had posed this question to himself at the outset of the last U.S. recession in the second or third quarter of 1990. Arguably, this is an ex post rationalization for the break date, although at the time, signs of a slowdown were apparent.²⁵ Analysis along the same lines as above shows how fewer factors explain a larger proportion of joint output fluctuations when compared to the period 1973-2001, suggesting that international business cycle linkages were stronger during this period. Three common factors explain most of the space spanned by the spectral density matrices of the vector process X (Table 4). Analysis along the same lines as above once again suggests that there are two truly global factors while comparing these two factors and that the common component in each country can be decomposed into an average and a difference component, both results that were also found for the entire sample. Hence, the business cycle asymmetries of the early 1990s did not result in a different factor decomposition but rather resulted in a deterioration of the factor model's fit.

²⁴ See, for example, Tumovsky (1986).

²⁵ See, for example, IMF (1990).
The analysis of forecast errors provides further clues about the nature of the common factors. Figure 9 depicts the one-step ahead forecast errors for the first and second common factor in all G-7 countries. These forecast errors could be viewed as estimates of the shocks that hit the G-7 economies during the sample period. The charts clearly illustrate the role of shocks to the first common factor during growth contractions, when the forecast errors are similar in direction and magnitude. Also, shocks to the first factor are large compared to the errors for the second factors, a finding that is consistent with the notion of synchronized recessions and growth contractions.\textsuperscript{26} The early 1990s were different. Shocks to the first factor appear smaller in size and shocks to the second factor also mattered. In fact, the differences in the signs of the forecast errors for the second factor after the onset of the U.S. growth recession in the 1989 (depicted by vertical lines in the chart) illustrate the dis-synchronization during this period well. In addition, the forecasts errors for the idiosyncratic components in Germany and Japan during this period are also large compared to those for the common factors, which corroborates the hypothesis of dis-synchronization.

Table 4. Factor Contribution to Explaining the Variation in G-7 Output Gaps, 1973–89 1/ (Cumulative Sums of Averages of Normalized Eigenvalues at indicated Frequencies)

<table>
<thead>
<tr>
<th>Eigenvalues</th>
<th>Business Cycle Frequencies (6–32 quarters)</th>
<th>Business Cycle and Irregular Frequencies (32 quarters and less)</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>0.662</td>
<td>0.643</td>
</tr>
<tr>
<td>Second</td>
<td>0.844</td>
<td>0.825</td>
</tr>
<tr>
<td>Third</td>
<td>0.927</td>
<td>0.915</td>
</tr>
<tr>
<td>Fourth</td>
<td>0.965</td>
<td>0.960</td>
</tr>
<tr>
<td>Fifth</td>
<td>0.984</td>
<td>0.980</td>
</tr>
<tr>
<td>Sixth</td>
<td>0.995</td>
<td>0.992</td>
</tr>
<tr>
<td>Seventh</td>
<td>1.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Source: Authors' calculations.
1/ As measured by the spectral density matrices of the output gaps. See text for details and information about how to interpret the entries.

\textsuperscript{26} Indeed, the forecast errors for the first common factor are strongly correlated (average of 0.92) while those for the second common factor are, on average, barely correlated (average of 0.14).
Figure 8. Average G–7 Output Gap and Common Factors, by Country
(Two-Sided Estimation of Common Factors)

Source: Authors' calculations.
Figure 9. One-Step-Ahead Forecast Errors in Common Factors
(Forecast error for first factor depicted in lines; bars represent errors to second factor)

Source: Authors' calculations.
because of idiosyncratic shocks (Figure 10). A comparison of shocks to the first common factor and the idiosyncratic components also shows that shocks to the first factor matter during synchronized contractions while shocks to the idiosyncratic component are small. As an aside, we would also add that using log first differences generates qualitatively similar results.

Naturally, despite all the clues, our estimated common factors do not lend themselves to an obvious structural interpretation in terms of the source of the underlying shocks. Nevertheless, the fact that the generalized dynamic factor model decomposes the "common part" of national growth cycles into two factors, one capturing the average G-7 cycle and one that corrects for phase and amplitude differences, is remarkable and, in our opinion, allows for some conjectures about the nature of the shocks, in particular with regard to those behind the most recent slowdown. More specifically, we claim that the relative strength of shocks to the common factors and the idiosyncratic component allows for some conjectures about the nature of shocks. For example, if the shocks to the first common factor across all G-7 output gaps are much larger than the shocks to the other components (second common factor or idiosyncratic components) at time t, one may safely conjecture that these shocks reflect a global shock in this period. On the other hand, if shocks to the second common factor or, especially, idiosyncratic components appear more important, country-specific shocks appear more likely as a source of disturbance. In this regard, it is instructive to compare the slowdown in 2000-01 with earlier episodes. As Figures 9 and 10 show (the vertical lines depict peak quarters in U.S. output gaps in 1979, 1989, and 2000—the 1973 peak coincides with the left y-axis), the largest shocks in 2000-01 were clearly those to the first common factor. This constellation is very similar to the 1973-75 and 1979-80 episodes. On this basis, we conjecture that global shocks were the main force behind the most recent growth contraction. In the 1989-90 episode, relatively large idiosyncratic shocks and shocks to the second common factor are consistent with the hypothesis of cycle dis-synchronization in the early 1990s due to large country-specific shocks.

V. COMMON FACTORS FROM A LARGER PANEL

In a second step, common factors in G-7 output fluctuations were re-estimated with a larger panel data set that includes quarterly data for real GDP at market prices, real private consumption, private fixed residential investment, other private fixed investment, and exports of goods and factor services for the period 1973Q1-2001Q4. The cross-sectional dimension of this sample is larger (n=35), which should improve the statistical qualities of the estimated common components. Accordingly, the section focuses on whether the cross-sectional dimension matters for the main findings. In addition, including the most important private

27 Idiosyncratic shocks are the one-ahead residuals from a vector autoregression of the idiosyncratic component $\Xi_t$ based on the reestimation of the common factor with the one-sided filter proposed by Forni and others (2001).
Figure 10. One-Step-Ahead Forecast Errors, by Country
(Line depicts errors in first common factor; bars represent errors in idiosyncratic component)

Source: Authors' calculations.
sector demand components and exports also allows one to gauge how cross-country linkages operate through specific domestic demand components and to what extent exports are affected by these linkages. The analysis proceeds along the same lines as in the last section and compares the results. Since the detrending methods used did not matter for the principal findings of the last section, the discussion in this section will only be based on low pass-filtered output series (output gaps).

How does the larger cross-sectional dimension affect the contribution of each common factor to explaining the space spanned by the spectral density matrices of the vector process $X_t$ at various frequencies? The first two common factors explain 61 percent of the variations at business cycle frequencies compared with about 80 percent in the small panel (Table 5). Each of the first five factors explains more than 5 percent of the overall variation in $X_t$; together they account for about 84 percent. At business cycle and irregular, short-term frequencies, the first two factors roughly account for about 50 percent of the variance (69 percent in the previous section). The earlier finding that the explanatory content of the first few factors decreases rapidly at the irregular frequencies extends to the larger panel as well. Also, as above, the explanatory power of the first two factors was larger during 1973-89, they accounted for 70 percent of the space spanned by the spectral density matrices at

<table>
<thead>
<tr>
<th>Factor</th>
<th>Cumulative Sums of Averages of Normalized Eigenvalues at indicated Frequencies</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Business Cycle Frequencies (6-32 quarters)</td>
</tr>
<tr>
<td></td>
<td>First: 0.444</td>
</tr>
<tr>
<td></td>
<td>Second: 0.611</td>
</tr>
<tr>
<td></td>
<td>Third: 0.713</td>
</tr>
<tr>
<td></td>
<td>Fourth: 0.782</td>
</tr>
<tr>
<td></td>
<td>Fifth: 0.837</td>
</tr>
<tr>
<td></td>
<td>Sixth: 0.876</td>
</tr>
<tr>
<td></td>
<td>Seventh: 0.903</td>
</tr>
</tbody>
</table>

Source: Authors’ calculations.

As measured by the spectral density matrices of the output gaps. See text for details and how to interpret the entries.

As above, the common factors and components were estimated used the nonparametric approach of Forni and others (2000). All raw variables were first transformed into natural logarithms, detrended, and standardized. For the calculation of the spectral density matrices, 3 lags were used based on the fixed rules recommended by Forni and others (2000).
business cycle frequencies. Overall, however, the main finding remains that a small number of
factors explain the lion share of fluctuations in key macroeconomic variables.

Does the larger panel help to improve the fit of the generalized dynamic factor model
with regard to output gaps? Interestingly, the results vary. For Germany, France, and Canada,
the fit—as measured by the sum of the factor $R^2$ measures for the first five common factors
with regard to output gaps (at business cycle frequencies)—clearly improves compared with
the small panel of the previous section (Figure 11). In the case of Germany and Canada, the
improvement is dramatic (by more than 0.3). In the United States, Italy, and the United
Kingdom, the fit is about unchanged, while in the case of Japan, it deteriorates slightly. The
larger panel, therefore, changes the ranking in the goodness of fit of the model for output gaps.
Common factors now explain a larger share of output gap fluctuations in Germany than in
Japan, which is consistent with the results from the earlier correlation analysis. These results
suggest that taking into account a broad set of dynamic linkages is important in modeling
common factors in output fluctuations.

In the last section, it was shown that the generalized dynamic factor model decomposes
the common factors in national growth cycles into two factors, one capturing the average G-7
cycle and one that corrects for phase and amplitude differences. Does this result still hold with
the larger panel? The answer, in short, is yes. The comparison between the average G-7 output
gap and the first common factors in the large and the small cross-section sample shows that the
average difference between the two first factors is negligible (Figure 12). This is an important
result, as it suggests that much of the analysis with regard to common shocks and their role in
synchronized growth contractions remains relevant.

The factor decomposition into an average and a difference component is less evident
for other demand components. For example, the first common factors for private residential
fixed investment are not closely related to the G-7 average of this aggregate. This is not
surprising in light of the evidence shown in Figure 11, where the first common factor is
usually important for explaining fluctuations in output and private consumption in all countries
whereas for the other variables, its importance differs greatly across countries. In Germany, the
first factor matters for private gross fixed capital formation other than residential investment
and even for residential investment. In Japan, Italy, and Canada, it matters for exports of goods
and factor services. In France, the first factor matters about as much as for output as it does for
residential investment. As a result, the G-7 averages for private gross fixed capital formation,
be it residential or other, and exports have little explanatory power. This is yet another piece of
evidence suggesting that the structure of the G-7 economies and their responses to shocks must
vary greatly.

VI. CONCLUSIONS

During 2000–2001, the seven major advanced economies experienced another broadly
synchronized growth slowdown. The breadth of synchronization was widely considered
surprising, as prior expectations appear to have been based on the experience with seemingly
benign international business cycle linkages during the 1990s. We argue that the synchronous
Figure 11. Average Factor R-Squares at Business Cycle Frequencies, by Series

Source: Authors' calculations.
Figure 12. G-7 Output Gap and First Common Factors, by Size of Cross Section
(Two-Sided Estimation of Common Factors)

Source: Authors’ calculations.
slowdown in the G-7 countries should have come as less of a surprise. On the contrary, from a historical perspective, synchronized slowdowns have been the norm rather than the exception since 1973, when the generalized floating of the major currencies was introduced. Our empirical results suggest that the most recent slowdown was not only typical with regard to its synchronized timing but also with regard to the underlying shocks. Specifically, we found evidence that global or common shocks were the source of the slowdown. Such shocks have been associated with all major growth contractions but one in the G-7 countries since 1973.

International business cycle linkages manifest themselves not only in the synchronous timing of contractions but also more generally in strong output comovements over time, as evidenced by correlation measures in the time and frequency domains. Nevertheless, it should be noted that the strength of business cycle linkages is far from being uniform and varies noticeably across G-7 countries. We find evidence for two clusters of particularly strong cross-country business cycle linkages. The first cluster includes the United States, Canada, the United Kingdom, and, to a lesser extent, Germany, while the second cluster includes Germany, France, and Italy. We interpret this as another piece of evidence for the notion that countries are hit by similar shocks, but that these shocks’ effects vary considerably across countries. We also found evidence for the hypothesis that for transatlantic business cycle linkages between the United States (and Canada and the United Kingdom) and the continental European G-7 countries, the early 1990s were exceptional insofar as a rare constellation of large, country-specific shocks led to a rather unusual dis-synchronization of output fluctuations while the underlying structure of linkages remained unchanged. In the case of Japan, however, the strength of output linkages with other G-7 countries appears to have decreased during the entire decade.

Common factors in output fluctuations across countries are the quintessential reflection of international business cycle linkages. Results based on the generalized dynamic factor model proposed by Forni and others (2000) suggest that two global factors explain roughly 80 percent of the variance in G-7 output gaps at business cycle frequencies. We also show how the factor model decomposes the “common part” of national output gap or growth cycles into two factors, one capturing the average G-7 cycle and one that corrects for phase and amplitude differences. Explorations into the temporal stability of the estimated common factors show that business cycle asymmetries, such as those of the early 1990s, do not affect the fundamental structure of the factor decomposition but rather the overall explanatory power of the second factor (which generally increases as differences from the average cycle become more pronounced) and the overall goodness of fit (which generally decreases).

The synchronous slowdown in activity reconfirms an old insight into international business cycle linkages, namely that their strength varies over time, depending on the nature, magnitude, and origin of disturbances that affect each economy. Experience has shown that global disturbances and disturbances in the United States have generally been associated with strong linkages in the past. The slowdown in 2000–2001 is also a reminder that global developments such as real crude oil prices and the performance of individual countries often
contain useful information on international business cycle linkages in the future. Accordingly, they should be carefully assessed by analysts and policymakers alike.

The results of the dynamic factor model also provide some tentative evidence for the hypothesis that global shocks were the main force behind the slowdown in 2000–2001. However, further research into the nature of shocks driving the common component of G-7 output fluctuations is needed. What is the nature of the global shocks? Do they reflect common policies? Empirical research on these questions is needed for a deeper understanding of the international business cycle linkages.
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